NBER WORKING PAPER SERIES

MIGRATION-REGIME LIBERALIZATION AND SOCIAL SECURITY: POLITICAL-ECONOMY EFFECT

Assaf Razin
Edith Sand

Working Paper 15013
http://www.nber.org/papers/w15013

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
May 2009

Migration-Regime Liberalization and Social Security: Political-Economy Effect
Assaf Razin and Edith Sand
NBER Working Paper No. 15013
May 2009
JEL No. F2,H0

ABSTRACT

The pay-as-you-go social security system, increasingly burdened by dwindling labor force, can benefit from immigrants whose birth rates exceed those of the native born birth. The paper examines adynamic political-economy mechanism through which the social security system influences the young decisive voter's attitudes in favor of a more liberal immigration regime. A Markov equilibrium with social security consists of a more liberal migration policy, than a corresponding equilibrium with no social security. Thus, the social security system effectively provides an incentive to liberalize migration policy through a political-economy mechanism.

Assaf Razin
Department of Economics
Cornell University
Uris 422
Ithaca, NY 14853
and NBER
ar256@cornell.edu

Edith Sand
Eitan Berglas School of Economics
Tel Aviv University
Tel Aviv 69978,
ISRAEL
edit@post.tau.ac.il

Migration-Regime Liberalization and Social Security: Political-Economy Effect

Assaf Razin*and Edith Sand†

May 27, 2009

Abstract

The pay-as-you-go social security system, increasingly burdened by dwindling labor force, can benefit from immigrants whose birth rates exceed those of the native born birth. The paper examines adynamic political-economy mechanism through which the social security system influences the young decisive voter's attitudes in favor of a more liberal immigration regime. A Markov equilibrium with social security consists of a more liberal migration policy, than a corresponding equilibrium with no social security. Thus, the social security system effectively provides an incentive to liberalize migration policy through a political-economy mechanism.

JEL Classification: F22, H55, J11, P16

1 Introduction

All over the world, the combination of declining birth rates, and rising life expectancy, presents major fiscal challenge to the social security system. From an economic perspective, a rise in the dependency ratio (i.e., the proportion of retirees per worker) increases the number of people drawing from the system, while it decreases the number of contributors to the system. Because Immigrants typically have higher birth rates than the native-born population of the host countries, immigration may help pay PAYG social security system. That is, the inflow of immigrants can help alleviate the current demographic imbalance by influencing the age structure of the host economy, in a way which strengthens

*Tel aviv University, Cornell University, CEPR, NBER and CESifo, Ar256@cornell.edu.
†European University Institute, Edith.Sand@EUI.eu.

the social security system. This paper analyzes a political-economy mechanism through which social security systems influence the degree of liberalization of the immigration policy.

Our analysis of the dynamic interactions between the political and economic decisions is conducted in an analytical framework, developed by Krussel and Rios-Rull (1996) and Krusell, Quadrini, and Rios-Rull (1997). Our paper also follows Forni (2006), who provides a neat analysis of Markov sub-game perfect equilibrium of pay-as-you-go social security system in an overlapping generations model with capital accumulation.

Earlier literature on the political economics of immigration includes Benhabib and Jovanovitch (1996), Scholten and Thum (1996), and Ortega (2005). The present paper draws heavily on Sand and Razin (2007), who analyze a political-economy equilibrium model, in which both migration and taxes interact, focusing on the intergenerational aspects of the social security and migration regimes.

The paper is organized as follows. Section 2 develops the analytical framework. Section 3 presents the political economy equilibrium. In Section 4 we characterize the equilibrium with a social security system. In section 5 we characterize the equilibrium with no social security system. Section 6 concludes.

2 Analytical Framework

The economy is populated by overlapping generations of representative individuals, who live for two periods. The tax-transfer system is a "pay as you go", where in every period the government levies a flat tax on the wage income of the young generation and pays social security benefits paid to the old generation. The representative individual makes labour-leisure and saving-consumption decisions, and pays social security taxes, in the first period of her life. The individual retires in the second period. The retired individual receives interest income from private savings (made in the first period, when she was young), and social security benefits. Migrants enter the economy when young, and gain the right to vote only in the next period, when old. They have the same preferences as those of the native born, except from having a higher birth rate. We assume that $n > 0$ is the native-born birth rate, and $m(> n)$ is the birth rate of migrants. On arrival, migrants are fully integrated into the social security system. That is, they pay the social security tax when young, and receive the social security benefits when old. Offspring of immigrants are like native born in all respects (in particular, they have the same birth rate as the offspring of the native born). As is standard in such Diamond type overlapping generations model, the aggregate savings of the current young population generates next period aggregate capital. The latter is used as a factor of production, along with the labour input in the next period. The production function exhibits constant returns to scale. Both the wage rate and the rate of interest, are endogenously determined along the equilibrium path.

The utility of the representative young individual is logarithmic:

$$\begin{aligned} U^y(\tau_t, s_t, b_{t+1}) &= Log(wl_t(1-\tau_t) - s_t - \frac{l_t^{\Psi+1}}{\Psi+1}) + \\ &\quad \beta Log(b_{t+1} + (1+r)s_t) \end{aligned} \tag{1}$$

and the utility function of the representative old individual is given by:

$$U^o(s_{t-1}, b_t) = b_t + (1+r)s_{t-1} \tag{2}$$

where is τ_t is period t tax rate, s_t is period t individual saving, b_{t+1} is period t+1 social security benefits, l_t is period t individual labor supply, w_t is period t wage rate, and r_t is period t interest rate.

The production function is of a Cobb-Douglas form, which is assumed to use both labour and capital as its factors of production:

$$Y_t = N_t^{1-a} K_t^{\alpha}, \tag{3}$$

where K_t is the aggregate amount of capital,γ_t is the ratio of migrants to the young native born population, and $N_t = (1+\gamma_t)l_t$ is period t aggregate labor supply (native born and migrants).

The wage rate and interest rate are determined competitively by the marginal productivity conditions (for simplicity, capital is assumed to depreciate completely at the end of the period):

$$w_t = (1-a)(1+\gamma_t)^{-a} l_t^{-a} k_t^{\alpha} \tag{4}$$

$$r_t = \alpha(1+\gamma_t)^{1-a} l_t^{1-a} k_t^{\alpha-1} - 1, \tag{5}$$

where k_t is capital per (native-born) worker. The balanced government budget constraint is derived as in the previous section:

$$b_{t+1} = \frac{\tau_{t+1} w_{t+1} l_{t+1} [(1+n) + \gamma_t(1+m)](1+\gamma_{t+1})}{(1+\gamma_t)} \tag{6}$$

The saving-consumption decisions of young individuals are made by maximizing their utility while taking the prices and policy choices as given, and the labour-leisure decision is given as in the previous section:

$$s_t = \frac{1}{1+\beta}\left(\beta\frac{\Psi}{\Psi+1}w_t l_t(1-\tau_t) - \frac{b_{t+1}}{1+r_{t+1}}\right) \tag{7}$$

$$l_t^{\Psi} = w_t(1-\tau_t) \tag{8}$$

The market clearing condition requires that net domestic savings generate net domestic investment:

$$s_t = k_{t+1}\left(\frac{1+n+\gamma_t(1+m)}{(1+\gamma_t)}\right) \tag{9}$$

Solving equations (20) and (21) for b_{t+1} and substituting b_{t+1} in equations (15) , we can write the indirect utility function of the young as follows.

$$\begin{aligned} &V^y(w_t,\tau_t,r_{t+1},\tau_{t+1}) = Log\left(\tfrac{1}{1+\beta}\tfrac{\Psi}{\Psi+1}w_t l_t(1-\tau_t)(1+\beta f(\tau_{t+1}))\right) \\ &+\beta Log\left(\tfrac{\beta}{1+\beta}\tfrac{\Psi}{\Psi+1}w_t l_t(1-\tau_t)(1+\beta f(\tau_{t+1}))(1+r_{t+1})\right) \end{aligned} \tag{10a}$$

where $f(\tau_{t+1}) = \frac{\frac{1-\alpha}{\alpha}\frac{1}{1+\beta}\tau_{t+1}}{1+\frac{1-\alpha}{\alpha}\frac{1}{1+\beta}\tau_{t+1}}$,
such that,

$$k_{t+1} = \frac{\beta}{1+\beta}\frac{\Psi}{\Psi+1}\frac{(1+\gamma_t)w_t l_t(1-\tau_t)(1-f(\tau_{t+1}))}{1+n+\gamma_t(1+m)} \tag{11}$$

$$l_t^{\Psi} = w_t(1-\tau_t) \tag{12}$$

$$l_{t+1}^{\Psi} = w_{t+1}(1-\tau_{t+1}), \tag{13}$$

Now, substituting b_t from equation (20) and k_t from equation (23), and using equation (16), the indirect utility function of the old is:

$$\begin{aligned} &V^o(\gamma_{t-1},k_t,w_t,r_t,\tau_t) = \tfrac{\tau_t w_t l_t[(1+n)+\gamma_{t-1}(1+m)](1+\gamma_t)}{(1+\gamma_{t-1})} + \\ &(1+r_t)k_t\left(\tfrac{1+n+\gamma_{t-1}(1+m)}{(1+\gamma_{t-1})}\right) \end{aligned} \tag{14}$$

such that,

$$l_t^{\Psi} = w_t(1-\tau_t), \tag{15}$$

As expected, the old individual favours a maximizing-revenue level of the social security tax rate (the "Laffer Point"), $\tau^* = \frac{\Psi}{\Psi+1}$, and the largest immigration quota,$\gamma = 1$.

2.1 Political-Economic Equilibrium

The Markov Perfect equilibrium is defined as follows.

Definition 1 *A Markov perfect political equilibrium is a vector of policy decision rules,* $\Pi = (T, G)$*, and private decision rule,* S*, where* $T : [0,1] \longrightarrow [0,1]$*, is the tax policy rule,* $\tau_t = T(k_t)$*, and* $G : [0,1] \longrightarrow [0,1]$*, is the immigration policy rule,* $\gamma_t = G(k_t)$*, and* $S(k_t)$ *is the saving decision rule so that* $k_{t+1} = S(\pi_t, k_t)$*, such that the following functional equations hold:*

(1) $\widehat{\Pi}(k_t) = \arg\max_{\pi_t} V^i(\gamma_{t-1}, \pi_t, \pi_{t+1})$

subject to $\pi_{t+1} = \Pi(\gamma_t, S(\pi_t, k_t))$*.*

(2) $S(k_t) = \frac{\beta}{1+\beta}\frac{\Psi}{\Psi+1}\frac{(1+\gamma_t)w_t l_t(1-\tau_t)(1-f(\tau_{t+1}))}{1+n+\gamma_t(1+m)}$,

with $\tau_{t+1} = T(S(k_t))$*.*

(3) A fixed-point condition requiring that given the next period policy outcome (the vector of policy decision rules- $\Pi(k_{t+1})$*), the maximization of the indirect utility of the current decisive voter, subject to the law of motion of the capital stock, will reproduce the same law of motion,* $\widehat{\Pi}(k_t) = \Pi(k_t)$*, (as in condition (1)).*

This means that in equilibrium, policy variables have to maximize the decisive voter's indirect utility function, while taking into account the law of motion of capital, and the expectations that the next period decision rules depend on next period capital per (native-born) worker, which is equal to period t savings.

3 Equilibrium With Social Security

The Markov sub-game Perfect equilibrium is as follows.

Proposition 2 *The equilibrium is given by the policy rules, and the saving rate are:*

$$T(k_t) = \begin{cases} \tau(k_t) & if \quad k_t \in [k(\overline{\tau}), k(0)] \\ 0 & otherwise \end{cases} \tag{16}$$

$$G(k_t) = \begin{cases} 1 & if \quad k_t \in [k(\overline{\tau}), k(0)] \\ \gamma^* & otherwise \end{cases} \tag{17}$$

$$S(\tau_t, \gamma_t, k_t, \tau_{t+1}) = \begin{matrix} S(\tau(k_t), 1, k_t, \tau(k_{t+1})) & if \quad k_t \in [k(\overline{\tau}), k(0)] \\ S(0, \gamma^*, k_t, 0) & otherwise \end{matrix} \tag{18}$$

where

$$\gamma^* = \frac{\beta(1-\alpha)\Psi(m-n)-\alpha(1+\Psi)(1+n)(1+\frac{\beta\alpha(1+\Psi)}{\alpha+\Psi})}{\alpha(1+\Psi)(1+m)(1+\frac{\beta\alpha(1+\Psi)}{\alpha+\Psi})},$$

$$S(\tau_t,\gamma_t,k_t,\tau_{t+1}) = \frac{\beta}{1+\beta}\frac{\Psi}{1+\Psi}\frac{1+\gamma_t}{1+n+(1+m)\gamma_t}\frac{(1-\alpha)(1+\gamma_t)^{-\alpha}(k_t)^{\alpha}(1-\tau_t)^{\frac{1+\Psi}{\Psi+\alpha}}}{1+\frac{(1-\alpha)(1-\tau_{t+1})}{\alpha+\Psi}},$$

and

$k(\tau) = ((1+\frac{(1-\alpha)}{\alpha}\tau)^{1+\beta}(1-\tau)^{\frac{\beta(1-\alpha)}{\alpha+\Psi}}\frac{1}{c})^{-(1+\frac{(1+\Psi)\alpha\beta}{\Psi+\alpha})}$ *, for* $\tau = \overline{\tau}$ *and* $\tau = 0$*,*

where c is a constant of integration (see Appendix).

First, we explain why the equilibrium tax function $\tau(k)$ *is decreasing in k, in the range* $k \in [k(\overline{\tau}), k(0)]$*. When the next period tax rate rises there are two conflicting forces at work on tax revenues, as usual . The increase in rates , for a given tax, increases revenues. base On one hand, an increase in period t+1 tax rate, for a given tax base, raises period t+1 tax revenues, and thereby social security benefits. But the tax increase reduces the labor supply and diminishes the tax base. If the tax rate is below the Laffer point, which is always desireable by the voter, the tax revenue must increase with the rise in the tax rate. If the tax revenue, and thereby also the social security benefits, rise in period t+1, the incentive to save in period t diminishes, as required by the Euler fist-order condition. Because aggregate savings in period t are equal to the aggregate capital stock in period t+1, the rise in period t+1 tax rate diminishes the aggregate capital stock in period t+1. Hence tax function* $\tau(k)$ *is decreasing in k, in the range* $k \in [k(\overline{\tau}), k(0)]$*. At the threshold point* $\overline{\tau}$*, the value of* $\tau(k)$ *is driven to zero.*

Turning to the equilibrium migration quota, consider the expression for γ^**.The positive term in the numerator on the right hand side captures the beneficial effect of having larger labor force with the immigrants' offspring. The larger labor force in strengthens the social security system. The negative term in the numerator the expression for* γ^* *captures the wage depressing effect of immigrants, which is harmful to the young decisive voter. Consider as a benchmark the case m=n. In this case the beneficial effect of migration, from the perspective of the decisive voter, which arises from the increase in the period t+1 share of the young working force in the total population, vanishes completely. The wage depression effect dominates, and the migration quota is set equal to zero. If, however, m>n, a beneficial economic effect to bring in migrants does exist. For a sufficiently large gap between m and n, the young decisive voter in period t, anticipating an increase in social security benefits in period t+1, will admit immigrants. In this case* $\gamma^* > 0$*.*

Observe also that there is a positive effect of the aging of the native born on the migration quota, captured by a reduction in n.

4 Equilibrium With No Social Security

In order to emphasize the role of the social security system in the model, we now consider a similar model, but without any transfer payments from the young to the old.

The equilibrium migration policy rule, and the saving rate are:

$$G(k_t) = \gamma^* \tag{19}$$

$$S(\gamma^*) = \frac{\beta}{1+\beta}\frac{\Psi}{1+\Psi}\frac{1+\gamma^*}{1+n+(1+m)\gamma^*}\ \frac{(1-\alpha)(1+\gamma^*)^{-\alpha}(k_t)^{\alpha}}{1+\frac{(1-\alpha)}{\alpha+\Psi}} \tag{20}$$

We can now compare migration policies with and without a social security system. Inspecting the equilibrium migration policies with, and without social security, we can verify that in the former migration policies are either the same, or more liberal, than in the latter regime, depending on the range in which the equilibrium levels of the capital per worker are. The conclusion is that the social security system creates an incentive, through a political-economy mechanism, for a country to bring in migrants.

5 Conclusion

The pay-as-you-go social security system, which in recent time suffers from dwindling labor force, can benefit from immigrants with birth rates that exceed the native-born birth rates. Thus, a social security system provides effectively an incentive, through the political economy mechanism, to liberalize migration policy. The paper examines a political- economic, inter-generational, mechanism through which the social security system affects voter attitudes in favor of more liberal immigration regime. We demonstrate that the Markov equilibrium with social security consists of more liberal migration policies than the corresponding Markov equilibrium with no social security.

The main prediction of the model is that countries with a more comprehensive (Beveridgian-type) social security system, will be more liberal in their migration policies.

Related empirical work (e.g., Cohen and Razin(2009) demonstrates that there exists a statistically significant positive effect of the generosity of the welfare state on the skill composition of migration. Skill migrants, who provides fiscal benefits, are more desirable than unskilled migrants, who constitute a fiscal burden for the welfare state.

6 Appendix

Proof. The proof of the proposition is as follows. Because n>0, the majority resides with the young voters. Thus, the policy decisions concerning the tax rate and migration quotas maximizes the young indirect utility function. (We follow the proof of Forni (2004) to derive the tax policy decision rule.) The policy decision rules are derived by using, as a constraint, the first derivative with respect to the policy variables of the logarithm of the capital accumulation equation. The policy decision rules are:

$$\left(1+\frac{1-\alpha}{\alpha}\tau_t(k_t)\right)^{1+\beta}(1-\tau_t(k_t))^{\frac{\beta(1-\alpha)}{\Psi+\alpha}} = k_t^{-x}c \tag{21}$$

$$\gamma_t = 1 \tag{22}$$

where $x = 1+\frac{(1+\Psi)\alpha\beta}{\Psi+\alpha}$,and c is a positive constant of integration. The policy decision rule of the immigration quotas is at its maximal value, and the policy decision rule of the tax rate is implicitly given in equation (21). Define the following function: $k(\tau) = \left((1+\frac{1-\alpha}{\alpha}\tau)^{1+\beta}(1-\tau)^{\frac{\beta(1-\alpha)}{\alpha+\Psi}}\frac{1}{c}\right)^{-\frac{1}{x}}$. Thus we can rewrite the policy decision rule of the tax rate as: $k(\tau_t) = k_t$. The function $k(\tau)$ is decreasing in τ, for $\tau \in [0,\overline{\tau}]$, where $\overline{\tau} = \frac{\Psi(1+\beta)+\alpha}{\Psi(1+\beta)+\alpha+\beta}$, and increasing in τ, for $\tau \in [\overline{\tau},1]$. Thus, according to equation (21), for every value of capital per (native-born) worker, k_t, there are two solutions for $\tau(k_t)$ in the range $[0,1)$. The solution which satisfies the equilibrium conditions, which is denoted by $\tau(k_t)$, is decreasing in k_t for $k_t \in [k(\overline{\tau}), k(0)]$.

The solution for the policy variables given in equations (21) and (22), will be proved to satisfy the first order conditions of the problem. The young voter's indirect utility function under the assumption that next period decisive voter is young, which sets next period policy decision rules for the tax rate and immigration quotas to be $\tau_{t+1} = \tau(k_{t+1})$,and $\gamma_{t+1} = 1$ respectively, can be written in its Lagrangian form as follows:

$$\begin{aligned}&L(k_t) = A+(1+\beta)Log\left((1-\alpha)k_t^{\alpha}(1+\gamma_t)^{-\alpha}(1-\tau_t)\right)^{\frac{1+\Psi}{\Psi+\alpha}}+\\&(1+\beta)Log[(1+\beta f(\tau(k_{t+1}))+\beta Log\alpha\left((1-\alpha)k_{t+1}^{-\Psi}2^{\Psi}\left(1-\tau\left(k_{t+1}\right)\right)\right)^{\frac{1-\alpha}{\Psi+\alpha}}\\&-\lambda_1(k_{t+1}-\tfrac{\beta}{1+\beta}\tfrac{\Psi}{\Psi+1}\tfrac{(1+\gamma_t)\left((1-\alpha)k_t^{\alpha}(1+\gamma_t)^{-\alpha}(1-\tau_t)\right)^{\frac{1+\Psi}{\Psi+\alpha}}(1-f(\tau(k_{t+1}))}{1+n+\gamma_t(1+m)})\\&-\lambda_2(\tau_t-1)-\lambda_3(-\tau_t)-\lambda_4(\gamma_t-1)-\lambda_5(\gamma_t)\end{aligned} \tag{23}$$

The Kuhn-Tucker conditions are:

$$\frac{\partial L}{\partial \tau_t} = 0 = -\frac{1+\Psi}{\Psi+\alpha}\frac{1+\beta}{1-\tau_t} - \lambda_1 \frac{1+\Psi}{\Psi+\alpha}\frac{k_{t+1}}{1-\tau_t} - \lambda_2 + \lambda_3 \tag{24}$$

$$\frac{\partial L}{\partial \gamma_t} = 0 = -\alpha\frac{1+\Psi}{\Psi+\alpha}\frac{1+\beta}{1+\gamma_t} + \lambda_1 \frac{k_{t+1}}{1+\gamma_t}\left(\frac{n-m}{1+n+\gamma_t(1+m)} - \alpha\frac{1+\Psi}{\Psi+\alpha}\right) - \lambda_4 + \lambda_5 \tag{25}$$

$$\begin{aligned} \frac{\partial L}{\partial k_{t+1}} \quad = \quad 0 = & \left(\frac{\beta(1+\beta)}{1+\beta f(\tau(k_{t+1}))} - \frac{\lambda_1 k_{t+1}}{1-f(\tau(k_{t+1}))}\right)\frac{\partial f(\tau_{t+1})}{\partial \tau_{t+1}}\frac{\partial \tau(k_{t+1})}{\partial k_{t+1}} \\ & - \frac{\beta(1-\alpha)}{\Psi+\alpha}\frac{1}{1-\tau\left(k_{t+1}\right)}\frac{\partial \tau(k_{t+1})}{\partial k_{t+1}} + \frac{1}{k_{t+1}}\left(-\beta\frac{\Psi(1-\alpha)}{\Psi+\alpha}\right) - \lambda_1 \end{aligned} \tag{26}$$

$$k_{t+1} = \frac{\beta}{1+\beta}\frac{\Psi}{\Psi+1}\frac{(1+\gamma_t)w_t l_t(1-\tau_t)(1-f(\tau(k_{t+1}))}{1+n+\gamma_t(1+m)} \tag{27}$$

$$\tau_t - 1 \leq 0,\ \lambda_2 \geq 0 \text{ and } \lambda_2(\tau_t - 1) = 0 \tag{28}$$

$$-\tau_t \leq 0,\ \lambda_3 \geq 0 \text{ and } \lambda_3(-\tau_t) = 0 \tag{29}$$

$$\gamma_t - 1 \leq 0,\ \lambda_4 \geq 0 \text{ and } \lambda_4(\gamma_t - 1) = 0 \tag{30}$$

$$-\gamma_t \leq 0,\ \lambda_5 \geq 0 \text{ and } \lambda_5(\gamma_t) = 0 \tag{31}$$

Substituting for λ_1 from equation (26) into equations (24) and (25), we derive the following equations:

$$\frac{\partial L}{\partial \tau_t} = -\lambda_2 + \lambda_3 = 0 \tag{32}$$

$$\frac{\partial L}{\partial \gamma_t} = \frac{(1+\beta)}{1+\gamma_t}\left(\frac{-n+m}{1+n+\gamma_t(1+m)}\right) - \lambda_4 + \lambda_5 = 0 \tag{33}$$

Because $m > n$, from equation (33) we can derive that γ_t has a corner solution. The solution for the tax rate, on the other hand, τ_t, may be bounding or not, meaning that $\tau_t = \tau(k_t) \in [0,1]$[1]. Substituting the solutions for the tax and openness rate into the indirect utility of the young, we obtain that the optimal solution for the openness rate is $\gamma_t = 1$.

The optimal solutions should also satisfy the second order sufficient condition, meaning that the bordered Hessian of the Lagrangian should be negatively defined. Since the solution of the immigration quotas is a corner solution where the largest immigration quota maximizes the young voter's indirect utility function, the bordered Hessian of the Lagrangian is equal to:

$$-g_\tau\left(g_\tau \frac{\partial^2 L}{\partial^2 k_{t+1}} - g_k \frac{\partial^2 L}{\partial k_{t+1}\partial \tau_t}\right) + g_k\left(g_\tau \frac{\partial^2 L}{\partial \tau_t \partial k_{t+1}} - g_k \frac{\partial^2 L}{\partial^2 \tau_t}\right) \tag{34}$$

[1]Note that the utility with $\tau_t = 1$ is equal to minus infinity. Thus, the range for the tax rate is $[0,1)$.

where g_τ and g_k are the derivatives of the constraint of the capital per (native-born) worker from equation (27) with respect to τ_t and k_{t+1} respectively. The bordered Hessian can be rewritten in the following way:

$$(\frac{1+\Psi}{\Psi+\alpha})^2 \frac{1}{(1-\tau_t)^2} \frac{2x(1+\frac{1-\alpha}{\alpha}\tau_t)(1-\tau_t)\left(\frac{1-\alpha}{\alpha}\right)}{\left((1+\beta)\frac{1-\alpha}{\alpha}(1-\tau_t) - \frac{\beta(1-\alpha)}{\Psi+\alpha}(1+\frac{1-\alpha}{\alpha}\tau_t)\right)^2 (1+\frac{1-\alpha}{\alpha}\frac{1}{1+\beta}\tau_t)^2} \tag{35}$$

$$\left(\begin{array}{c} x(1+\frac{1-\alpha}{\alpha}\tau_t)(1-\tau_t)\left(\frac{1-\alpha}{\alpha}\right) + \\ \left((1+\beta)\frac{1-\alpha}{\alpha}(1-\tau_t) - \frac{\beta(1-\alpha)}{\Psi+\alpha}(1+\frac{1-\alpha}{\alpha}\tau_t)\right)(1+\frac{1-\alpha}{\alpha}\frac{1}{1+\beta}\tau_t)(1+\beta) \end{array} \right)$$

Denote by $[\tau_1, \tau_2]$ the range of the tax rate for which the bordered Hessian of the Lagrangian is negatively defined. Since τ_1 is always negative, and tax rate is defined over the range $\tau \in [0, 1]$, the range of the capital optimal solution for the tax rate, $\tau(k_t)$, is defined in the range $k_t \in [k(\overline{\tau}), k(0)]$, where the function $k(\tau)$ is decreasing in τ. ■