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ABSTRACT

Campbell and Cochrane (1999) formulate a model that successfully explains a wide variety of asset pricing puzzles, by augmenting the standard power utility function with a time-varying subsistence level, or "external habit", that adapts nonlinearly to current and past average consumption in the economy. This paper demonstrates, that this comes at the "price" of several unusual implications. For example, we calculate that a society of agents with the preferences and endowment process of Campbell and Cochrane (1999) would experience a welfare gain equivalent to a permanent increase of nearly 16% in consumption, if the government enforced one month of fasting per year, reducing consumption by 10 percent then. We examine and explain these features of the preferences in detail. We numerically characterize the solution to the social planning problem. We conclude that Campbell-Cochrance preferences will provide for interesting macroeconomic modeling challenges, when endogenizing aggregate consumption choices and government policy.

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Campbell and Cochrane (1999, hereafter denoted C-C) formulate a model that successfully explains a wide variety of asset pricing puzzles, including the high equity premium, the procyclical variation of stock prices, and the countercyclical variation of stock market volatility. These remarkable results are achieved by augmenting the standard power utility function with a time-varying subsistence level, or "external habit", that adapts nonlinearly to current and past average consumption in the economy. These preferences have been shown to be useful in a number of additional applications. For example Moore and Roche (2002), and Verdelhan (2008) extend the C-C framework to explain anomalies in foreign exchange markets, and Wachter (2006) addresses the term structure. Wachter (2005) provides efficient ways to calculate asset prices, using these preferences and exogenously given consumption and dividends. Bansal et al. (2007) provide an estimation of the C-C model. A continuous-time version and embedding it into a larger context of long run risk evaluation is in Hansen (2008). Guvenen (2008) has re-interpreted these preferences as arising from agent heterogeneity. Given the achieved breakthrough in matching key asset pricing facts by Campbell and Cochrane and other studies using their preference specification, it is all the more important to fully understand the implications of these modeling choices. This paper demonstrates several unusual implications.

We show that the assertions by Campbell and Cochrane that "more consumption is always socially desirable," and that "habit moves nonnegatively with consumption everywhere" are incorrect. As a consequence, government interventions that occasionally destroy part of the endowment can be welfare improving. Figures 1 and 2 illustrate such an outcome for a one-time endowment destruction that lowers the habit level and compares it to a conventional linear habit formulation, calibrated so that the two models share the same steady state. (Detailed explanations follow in section 2.1.) Figure 1 shows that the decline in habit is much larger under the C-C formulation due to strong nonlinearities away from the steady state. As a result, the welfare loss in the first period is more than compensated for by the welfare gains in future periods when consumption is so much higher than the reduced habit level, in contrast to the conventional linear habit formulation, see Figure 2. Figure 3 depicts the effect on overall welfare depending on the size of the initial endowment destruction. Under the C-C formulation, welfare decreases for miniscule and large endowment destructions, but it increases for moderately sized destructions.[1] Under the conventional linear habit formulation, welfare always falls in response to an endowment destruction at the steady state.

The purpose of this paper is to study these surprising welfare implications of the C-C preferences and assess their quantitative importance.

We use the original discrete-time specification of Campbell and Cochrane rather than moving to a continuous-time specification as in e.g. Hansen (2008). Exploiting the potential for large movements of the surplus consumption ratio are key to the analysis here, but may conceivably disappear in a continuous-time formulation or in other alterations of the original Campbell-Cochrane framework. One way of reading the results in this paper therefore is that the differences between a continuous-time framework and discrete-time framework may be substantial, in this case.

1 The model

The utility function of the representative agent is

$$E_0 \sum_{t=0}^{\infty} \delta^t \frac{(C_t - X_t)^{1-\gamma} - 1}{1-\gamma}, \tag{1}$$

where δ is the subjective time discount factor and X_t is the level of external habit. A conventional linear external habit formulation

[1] To detect welfare losses for miniscule endowment destructions under the C-C habit model, see Figure 4 that magnifies the lower range of destructions in Figure 3.

specifies that

$$X_{t+1} = \mu X_t + \alpha C_t^a, \tag{2}$$

where C^a denotes average consumption by all agents in the economy, and μ and α are parameters.

Campbell and Cochrane (1999) proceed differently. They postulate a process for the economy's *surplus consumption ratio*, $S_t^a \equiv (C_t^a - X_t)/C_t^a$. Using lowercase letters to indicate logs, they assume that the log surplus consumption ratio evolves as a heteroscedastic AR(1) process,

$$s_{t+1}^a = (1-\phi)\bar{s} + \phi s_t^a + \lambda(s_t^a)\left(c_{t+1}^a - c_t^a - g\right), \tag{3}$$

where $\phi \in [0,1)$, g and $\bar{s}$ are parameters, and the function $\lambda(s^a)$ is given by

$$\lambda(s^a) = \begin{cases} \bar{S}^{-1}\sqrt{1-2(s^a-\bar{s})} - 1, & s^a \leq s_{\max}; \\ 0, & s^a \geq s_{\max}; \end{cases} \tag{4}$$

with $s_{\max} = \bar{s} + \left(1 - \bar{S}^2\right)/2$. The parameter $\bar{s}$ is the logarithm of the steady-state surplus consumption ratio $\bar{S}$, and Campbell and Cochrane set g equal to the logarithm of the mean consumption gross growth rate G. It can be shown that the C-C formulation and the conventional linear habit formulation in equation (2) share the same steady state if $\mu = G\phi$ and $\alpha = G(1-\phi)(1-\bar{S})$.

Campbell and Cochrane consider a pure endowment economy. Let Y_t be the per capita endowment in period t. Endowment growth is modeled as an i.i.d. lognormal process,

$$\Delta y_{t+1} = g + \nu_{t+1}, \qquad \nu_{t+1} \sim \text{i.i.d. } \mathcal{N}(0, \sigma^2). \tag{5}$$

The equilibrium outcome in a market economy is that consumption equals endowment, $c_t^a = y_t$. We shall now investigate what a social planner would like to do under free disposal, i.e., $c_t^a \leq y_t$.

2 The social planning problem

A social planner or benevolent government, facing a population with C-C preferences given above, maximizes the expected discounted utility, subject to choosing consumption between zero and current endowment. That is, the only option available to the social planner is to let agents consume a fraction of the endowment in any given period. Let the social planner's choice variable at time t be denoted $\psi_t \equiv c_t - y_t \leq 0$, i.e., the logarithm of the fraction of the endowment that is consumed. The objective function can then be rewritten as

$$E_0 \sum_{t=0}^{\infty} \delta^t \frac{(S_t^a C_t^a)^{1-\gamma} - 1}{1-\gamma} = E_0 \sum_{t=0}^{\infty} \delta^t \frac{\exp\left((1-\gamma)(s_t^a + \psi_t + y_t)\right) - 1}{1-\gamma}, \quad (6)$$

and the law of motion for the log surplus consumption ratio can be expressed as

$$s_{t+1}^a = (1-\phi)\bar{s} + \phi s_t^a + \lambda(s_t^a)\left(\psi_{t+1} - \psi_t + \nu_{t+1}\right), \quad (7)$$

where we have used equation (5) to substitute out for g. From now on, we will leave out the superscript a since outcomes for the representative agent and economy-wide averages are the same to the social planner.

Campbell and Cochrane (1999, p. 246) report that the social marginal utility is always positive in their model which would seem to imply that the social planner should set $\psi_t = 0$ in all periods. However, Campbell and Cochrane only prove that an infinitesimal destruction of the endowment leads to a welfare loss. To illustrate our surprising finding that noninfinitesimal destructions can increase welfare under the C-C habit formulation, it is instructive to consider a one-time perturbation from a steady state.

2.1 One-time perturbation from a steady state

Consider an economy in a non-stochastic steady state with endowment and consumption growing at a constant growth rate $G \geq 1$. The parameter restriction that ensures a bounded objective function is

$$\delta G^{1-\gamma} < 1\,. \tag{8}$$

Suppose now that the social planner destroys part of the endowment in one single period, denoted period 0. Thus, we have $\log(C_0/Y_0) = \psi < 0$, and the sequences of the logarithms of consumption and the surplus consumption ratio evolve as follows

$$\begin{aligned}
c_0(\psi) &= y_0 + \psi < y_0\,, \\
c_t(\psi) &= y_t\,, && \text{for all } t \geq 1; \\
s_0(\psi) &= \bar{s} + \bar{\lambda}\psi \;<\; \bar{s}\,, \\
s_t(\psi) &= \bar{s} - \phi^{t-1}\psi\left[\lambda(\bar{s} + \bar{\lambda}\psi) - \phi\bar{\lambda}\right] \;>\; \bar{s}\,, && \text{for all } t \geq 1;
\end{aligned}$$

where $\bar{\lambda} \equiv \lambda(\bar{s})$. Evidently, the representative agent's utility falls in period 0 because both his consumption level and the surplus consumption ratio decline relative to the steady state. But the utilities in all future periods increase due to a higher surplus consumption ratio that asymptotically returns to its steady-state value. The question is whether the discounted sum of these changes in utilities produce a welfare gain or a welfare loss.

After eliminating the constant terms involving $-1/(1-\gamma)$ in the preference specification and dividing through by $\exp\left((1-\gamma)y_0\right)$, the discounted life-time utility of the described perturbation can be expressed as

$$W(\psi) \;\equiv\; \frac{\exp\left((1-\gamma)(s_0(\psi)+\psi)\right)}{1-\gamma} \;+\; \sum_{t=1}^{\infty}\delta^t\,\frac{\exp\left((1-\gamma)(s_t(\psi)+tg)\right)}{1-\gamma} \tag{9}$$

with the derivative

$$\begin{aligned}
W'(\psi) = (1+\bar{\lambda})\exp\left((1-\gamma)(s_0(\psi)+\psi)\right) \\
+ \sum_{t=1}^{\infty}\delta^t\left[\phi^{t-1}s_1'(\psi)\right]\exp\left((1-\gamma)(s_t(\psi)+tg)\right),
\end{aligned} \tag{10}$$

where

$$s'_1(\psi) = -\left[\lambda(\bar{s} + \psi\bar{\lambda}) - \phi\bar{\lambda}\right] - \psi\bar{\lambda}\,\lambda'(\bar{s} + \psi\bar{\lambda}) < 0. \tag{11}$$

The derivative s'_1 is negative since $\psi < 0$ and λ is a decreasing function. Thus, whether welfare marginally increases or decreases at negative values of ψ depends on whether the first or the second term in equation (10) dominates numerically. Appendix A shows that welfare is globally increasing with a conventional linear habit formulation: there, endowment destruction in a steady state always leads to a welfare loss. Appendix B shows that this is also true locally along the steady-state path for the C-C preferences. This should come as no surprise since Campbell and Cochrane (1999, p. 246) prove that the social marginal utility is positive in their model. More specifically, they show that the social marginal utility is positive for infinitesimal perturbations when the endowment follows a random walk. When setting growth equal to zero in our calculations, we have a constant endowment level or a degenerate random walk.

However, the local result for the C-C preferences fails to hold globally. Given Campbell and Cochrane's (1999, Table 1) parameter values as reported in our Table 1, we compute the representative agent's welfare associated with one-time endowment destructions between 0 and 25 percent.[2] Figure 3 shows clearly that there exist endowment destructions that do raise welfare.[3] To understand the mechanisms at work, consider as an example the five-percent endowment destruction in Figures 1 and 2. Under the conventional linear habit formulation in equation (2), habit responds with a one-period lag to the endowment destruction in period 0 and, as can be seen in Figure 1, the resulting

[2]Following Campbell and Cochrane (1999), all numerical analyses in our paper are performed at a monthly frequency.

[3]As noted in footnote 1, one cannot discern in Figure 3 that endowment destructions lead to welfare losses locally around the steady state under the C-C habit model, as proven in Appendix B. But after magnifying the scale in Figure 4, we see that welfare does indeed decrease for endowment destructions less than 0.07 percent.

decline in habit is much less than under the C-C habit model. In addition, under the C-C habit formulation in equation (3), habit moves contemporaneously with consumption changes and according to the solid curve in Figure 1, the habit level now falls in response *both* to the endowment destruction (period 0) and to the subsequent increase in consumption (period 1). Hence, the loss of utility in the period with endowment destruction is mitigated under the C-C habit formulation because of the contemporaneous drop in habit, and future utility gains are magnified by the additional habit decline that is triggered by the consumption hike after the endowment destruction.

The effects upon agents' welfare depend on how the distance between consumption and habit change in different periods as illustrated in Figure 5 where

$$
\begin{aligned}
C_0(\psi) - X_0(\psi) &= \exp\left(y_0 + \psi + s_0(\psi)\right) \ ; \\
C_t(\psi) - X_t(\psi) &= \exp\left(y_t + s_t(\psi)\right) \ , \qquad \text{for all } t \geq 1.
\end{aligned}
$$

For a given $\psi \leq 0$, it is informative to study the marginal change in $(C_t - X_t)$ when perturbing ψ,

$$
\begin{aligned}
\frac{\mathrm{d}\ [C_0(\psi) - X_0(\psi)]}{\mathrm{d}\,\psi} &= (1+\bar{\lambda}) \exp\left(y_0 + \psi + s_0(\psi)\right) \ ; \\
\frac{\mathrm{d}\ [C_t(\psi) - X_t(\psi)]}{\mathrm{d}\,\psi} &= \phi^{t-1} s_1'(\psi) \exp\left(y_t + s_t(\psi)\right) \ , \quad \text{for all } t \geq 1.
\end{aligned}
$$

We can see that the derivative in period 0 gets muted at low values of ψ, i.e., at higher levels of endowment destruction, while the opposite is true for the corresponding derivatives in future periods. In fact, the multiplicative term s_1' as given in equation (11) becomes arbitrarily large and negative when ψ is driven to ever lower values and therefore, the associated loss in $(C_t - X_t)$ for $t \geq 1$, becomes arbitrarily large when reducing the amount of endowment destruction in period 0. This in turn implies that $(C_t - X_t)$ for $t \geq 1$, must take on arbitrarily large values when computed at ever lower values of ψ. Figure 5 depicts the exploding outcome for $(C_1 - X_1)$ when increasing the amount of endowment destruction in period 0. Behind the exploding outcome for $(C_1 - X_1)$ in Figure 5 lies a critical property of the C-C preference specifiation: habit can move negatively with consumption. In our example, the subsequent consumption hike in period 1 does not increase but rather *decreases* the habit level. We will examine this property further below in section 4.

To calculate the effect on welfare from the higher $(C_1 - X_1)$ and the correlated but slowly decaying future values of $(C_t - X_t)$ for $t \geq 2$, one needs to take into account the curvature of the utility function and compare it to the utility loss in period 0 from the fall in $(C_0 - X_0)$. Thus, while $(C_1 - X_1)$ is ever increasing, welfare takes the hump-shaped form depicted in Figure 3.

The feature in Figure 3, that the log of the optimal endowment destruction is finite, can be shown more generally. Let $\omega = (\psi_0, \psi_1, \ldots, s_0, s_1, \ldots)$ be some (stochastic) path for the decisions and consequently surpluses. Rewrite the objective function (6) as

$$\frac{\exp\left((1-\gamma)(s_0^a + \psi_0 + y_0)\right) - 1}{1-\gamma} + V(\omega, (y_t)) \tag{12}$$

where

$$V(\omega, (y_t)) = E_0 \sum_{t=1}^{\infty} \delta^t \frac{\exp\left((1-\gamma)(s_t^a + \psi_t + y_t)\right) - 1}{1-\gamma} \tag{13}$$

Assume that $V(\omega, (y_t)) > -\infty$. For example, this is the case for a never-destruct-endowment policy of $\psi_t \equiv 0, t \geq 1$. So an agent aiming at maximizing welfare will have no problem satisfying this assumption. Consider now an alternative path, $\tilde{\omega} = (\tilde{\psi}_0, \tilde{\psi}_1, \ldots, \tilde{s}_0, \tilde{s}_1, \ldots)$ and suppose that it improves welfare compared to the original path. There are two cases to consider, depending on γ.

First, assume that $\gamma > 1$. Consider the two parts of welfare in (12). The possible gain in the second part $V(\tilde{\omega}, (y_t)) - V(\omega, (y_t))$ is bounded above by $\delta/((1-\delta)(\gamma-1)) - V(\omega, (y_t))$. However, the loss stemming from current utility will be unbounded as $\tilde{\psi}_0 \to -\infty$. Thus, for any $\tilde{\omega}$ improving welfare compared to the current path, the initial endowment destruction must not drop below some treshold $\underline{\psi}$, which generally depends on the current strategy ω and the current state.

Next, assume that $\gamma < 1$. In that case, the loss from driving $\tilde{\psi}_0 \to -\infty$ is finite and given by $(\exp((1-\gamma)(s_0^a + \psi_0 + y_0)) - 1)/(1-\gamma)$. However, the first term in the continuation value of $V(\tilde{\omega}, (y_t))$ will now rise without bounds in ψ_0. It is easy to see that welfare can be improved upon without bounds, since all subsequent terms for $t \geq 2$ are in expected sum bounded below by $-\delta^2/((1-\delta)(1-\gamma))$. Therefore, for the problem of welfare maximization to be well-posed, we must have $\gamma > 1$.

2.2 Solution to the social planning problem

So far, we have considered only a one-time deviation from a steady state benchmark, and shown that the latter is not optimal for the numerical cases shown. What then is the optimal solution? Let us thus turn to calculating the solution of the social planner's problem and a quantitative assessment of how much welfare can be improved by destroying endowments.

To solve for the optimal allocation, we formulate a dynamic programming problem for the social planner. Let s and ψ be the logarithms of last period's surplus consumption ratio and the fraction of last period's endowment that was consumed. The current value of the endowment shock is denoted ν. The optimum value of the social planner's problem is then[4]

$$v(s,\psi,\nu) = \max_{\tilde{\psi} \leq 0,\, \tilde{s}} \left\{ \frac{\exp((1-\gamma)(\tilde{s}+\tilde{\psi}))}{1-\gamma} + \delta E\left[\exp((1-\gamma)(g+\tilde{\nu}))v(\tilde{s},\tilde{\psi},\tilde{\nu})\right] \right\}$$

[4]As in equation (9), we have rescaled the value function by leaving out the constant terms involving $-1/(1-\gamma)$ in the preference specification, and normalized by dividing through by $\exp((1-\gamma)y)$ where y is the current endowment.

where the maximization is subject to

$$\tilde{s} = (1-\phi)\bar{s} + \phi s + \lambda(s)\left(\tilde{\psi} - \psi + \nu\right), \tag{14}$$

$$\tilde{\nu} \sim N(0, \sigma^2). \tag{15}$$

Since $\gamma > 1$, and if a solution to the planning problem exists, then the value function is nonpositive, $v(s, \psi, \nu) \leq 0$. Note that $\lambda(s) \geq 0$.

Proposition 1 *Let $\gamma > 1$. Assume that*

$$\delta \exp((1-\gamma)g + (1-\gamma)\sigma^2/2) < 1 \tag{16}$$

There is a solution to the social planner's problem. The solution for the decision on the endowment destruction is finite, $\tilde{\psi} > -\infty$.

Proof: *To show that there is a solution, first consider the auxiliary problem, restricting the domain to $s \geq \underline{s}$ for some lower bound $\underline{s} > -\infty$, and truncate the normal distribution $\mid \tilde{\nu} \mid < \bar{\nu}$. For this problem, we claim that the optimal solution $\tilde{\psi}$ is bounded from below, $\psi \geq \psi^*$, where ψ^* may depend on the current state (s, ψ, ν). To see this, note that the first term of the objective function diverges to $-\infty$ as $\tilde{\psi} \to -\infty$. On the other hand, the second term, i.e. the expected future value is bounded by zero from above. Hence, the objective function is bounded. Furthermore, Bellman's sufficient conditions for a contraction mapping apply, due to condition (16). Thus, there is a unique solution to the social planner's problem with a restricted domain. As the domain is expanded, standard arguments show that the solution converges locally uniformly to a solution of the social planner's problem stated above.* •

We will now provide a numerical solution to this problem using Campbell and Cochrane's monthly time scale of their parameterization in Table 1 but without any uncertainty, $\nu = \tilde{\nu} = 0$. The endowment growth shocks will be reintroduced in the next section when we assess the quantitative importance of policy interventions.

In delivering a numerical solution, we had to solve a challenging problem. Consider equation (14) without random shocks, and suppose that $\tilde{\psi} = 0$, while driving $\psi \to -\infty$. Note that then $\tilde{s} \to \infty$. While we have shown, that the optimum (in the previous period) occurs for some finite value of ψ, it still may be the case, that very low ψ's are desirable, and, in fact, they are (in a sense to be made precise below). Low ψ's result in high $\tilde{s}$. It then takes a long time per the autoregression in (14) to return to a range below $s_{\max}$. We found that a typical optimal solution path will occasionally increase $\tilde{s}$ to values just below 16, or, put differently to values $\tilde{S} = \exp \tilde{s} = 9000000$. It should be clear that value function iterations on a grid of S-values, using e.g. a constant step size, which also is reasonably exact for the "standard range" of $S \in [0, S_{\max}]$, where $S_{\max} \approx 0.09$, is numerically infeasible (and this is the sense in which ψ is "very low"). A different numerical strategy is required.

We solve this problem by noting that optimal consumption is equal to endowment for all $s \geq s^*$, where s^* is "one step" above $s_{\max}$ according to equation (14), i.e., where s^* satisfies

$$s^* = \frac{1}{\phi} \left(s_{\max} - (1 - \phi)\bar{s} \right).$$

Indeed, for $s \geq s^*$, one can easily calculate the number of steps it takes to return to the region $s < s^*$, where destruction of the endowment might be optimal, and calculate the utility generated during the transition path to this region.

We impose a grid on destruction choices $(1 - \exp(\psi))$ at step-size $\Delta \exp(\psi) = 0.02$ and use interpolation for s, but "storing" all functions, using a grid for S with step size 0.0002 for $S < S^* = \exp(s^*)$. Above S^*, we use a grid on $\log(S)$, using 10 grid points between each "step" of the AR-process (14).

The resulting decision rule is shown in figure 6. Note the unusual shape of the decision rule that prescribes destruction of the endowment in a triangular-shaped region, with a very steep "canyon" of destruction for S near $S_{\max}$.

Simulations, starting at $S_{\max}$ and no consumption destruction quickly settle down on a regular cycle of consumption destructions, where each cycle typically takes somewhere around 300 periods or months, i.e. approximately 25 years. By inspecting the simulations, we find that a cycle is typically launched as follows. Coming from a high value of s, the surplus consumption ratio eventually drops below s^*, per iteration on (14). Then, there is soon a first period of endowment destruction, and after at most two additional periods, the social planner reverts to full consumption and "launches" s to a high range again, starting the cycle of gradual return to the below-s^*-region.

Besides these typical cycles with brief phases of endowment destruction, there are instances of cycles with more periods of destruction and time spent in the below-s^*-region, before launching s to a higher range. Though, we suspect that this "slow maneuvering" into launch position is just an artifact of our numerical approximation of calculating a value function on a grid, rather than being a feature of the solution to the exact dynamic programming problem. Our conjecture is supported by the following exercise. Given an initial $s \in [s_{\max}, s^*)$, we let the social planner make T consecutive decisions on endowment destruction, while setting consumption exogenously equal to the endowment forever afterwards. Thus, conditional on a launch of s to a higher range, the payoff is approximately the same as in the dynamic program except that we have replaced the future continuation value when drifting below s^* by the value of no more endowment destruction. Since optimal cycles are on average 25 years long, this difference in distant future behavior is heavily discounted and hence, should not matter much for the optimal choice of endowment destruction during the initial launch. The computational advantage of this T-dimensional maximization problem is that it can be solved with standard numerical maximization routines rather than calculations on a grid. When solving this problem for large values of T, we found that the optimal length of the launch phase, i.e., the time spent in the below-s^*-region, never exceeded three periods.

3 Quantitative importance

The findings of the previous section indicate that the recurrent destruction of the endowment is optimal. We now allow for random shocks to consumption growth, using the exact monthly parameterization of Campbell and Cochrane in Table 1. Rather than solve for the optimal policy response, we examine the welfare gains of policies parameterized by (ψ, p), which destroy a fixed fraction $(1 - \exp(\psi)) \in [0, 1]$ of the endowment every p:th period, $p \in \{2, 3, \ldots\}$. The policy experiment starts off with an immediate destruction in the first period and continues in perpetuity. No upper bound on the logarithm of the surplus consumption ratio s_t was imposed in the calculations.

Concerning the initial condition S_{-1}, we pursue two alternatives. First, we use the steady state $S_{-1} = \bar{S}$. Second, we draw S_{-1} from the unconditional distribution depicted in Campbell and Cochrane's Figure 2, derived there for the continuous-time version of the model. We simulate 100 time series of monthly endowments, where the length of each series is 200 years. We compare the average welfare obtained under the policy experiments of periodic destruction to the benchmark "laissez-faire" average welfare measure obtained from the same simulations, when no endowment destruction takes place. The welfare comparison is measured in terms of the percentage increase in consumption needed to make the agents in the laissez-faire economy as well as off as under the policy (ψ, p).

Table 2 contains the results where we find large welfare gains (rather than welfare losses) associated with the policies of periodic destructions. Put differently, a society of agents with Campbell-Cochrane preferences would experience a welfare gain equivalent to a permanent increase of nearly 16% in consumption, if the government enforced one month of fasting per year, reducing consumption by 10 percent then. Note that the government should neither be too timid nor too audacious in its demands of consumption reduction while fasting: the welfare gains at 15 as well as 5 percent reductions are smaller than at 10 percent. Either way, these welfare gains are dramatic.

4 Habit moves negatively with consumption

The economic intuition for the surprising finding that the agents are better off with a cyclical destruction of endowments is to be found in the implied law of motion for the external habit. Campbell and Cochrane (1999, p. 212) claim that habit moves nonnegatively with consumption everywhere. Unfortunately, this is not so. Instead, habit can fall contemporaneously with a rise in consumption even locally around the steady state. After differentiating the law of motion for the surplus consumption ratio in equation (3), we obtain

$$\frac{dx_{t+1}}{dc_{t+1}} = 1 - \frac{\lambda(s_t)}{\exp(-s_{t+1}) - 1}.$$

In the steady state, $s_t = s_{t+1} = \bar{s}$, so the parameterization of the function $\lambda(s)$ in equation (4) guarantees that $dx/dc = 0$ at the steady state. Next, we calculate the second derivative,

$$\frac{d^2 x_{t+1}}{dc_{t+1}^2} = -\frac{\lambda(s_t)^2}{[\exp(-s_{t+1}) - 1]^2} \exp(-s_{t+1}) \leq 0,$$

and the expression is strictly negative at the steady state. This establishes that there is a region around the steady state in which habit moves negatively with consumption.[5]

Based on Campbell and Cochrane's monthly frequency of their parameterization in Table 1, Figure 7 maps out the relationship between

[5]We are thankful to John Cochrane for suggesting this exposition of our argument.

consumption changes and movements in the habit level. In particular, for a given value of last period's surplus consumption ratio, the figure depicts how contemporaneous habit responds to percentage changes in consumption relative to last period's levels. As a numerical reference, the steady-state surplus consumption ratio is 0.057. It can be seen that the habit level moves negatively with consumption for a wide range of consumption increases. Hence, our finding of large welfare improvements associated with the cyclical destruction of endowments can be understood as "investments" in a lower habit level. That is, a period of endowment destruction is most likely to be followed by a rebounce in consumption next period and this consumption growth will often be associated with the strange effect of *lowering* the habit level.[6]

5 Conclusions

We have shown that the habit formulation by Campbell and Cochrane imply, that a social planner could reap large welfare gains by destroying 10 percent of the endowment every 12:th month. To be indifferent to such a policy, the representative agent would have to be compensated by almost 16 percentage points higher consumption for the indefinite future under laissez-faire. These welfare results are most likely connected with a surprising property of the C-C preference specification which is that habit can move negatively with consumption.

[6] To gauge how common this unorthodox habit dynamics is in the laissez-faire economy, we use our stochastic simulations of Campbell and Cochrane's environment in the preceding section to construct an indicator as follows. For any period with consumption growth in excess of the steady-state growth rate, we compute the habit level and compare it to the counterfactual habit level that would have arisen if consumption had only grown from last period at the steady-state rate. Half of all simulated periods experience consumption growth higher than the steady-state rate; of that 50%, roughly one quarter exhibits unorthodox habit dynamics in the sense that the habit level is *reduced* relative to if consumption had only grown at the steady-state rate.

Our findings imply that Campbell and Cochrane's (1999, pp. 245–247) attempt to map their results into a version of the model with internal habit formation must be reconsidered. Households faced with such an internal habit would themselves choose to periodically destroy endowments. Thus, the optimally chosen consumption process would not be the same as the exogenous endowment process.

If the C-C preferences were embedded in an economy with storage or production, it would rationalize outcomes of consumption bunching either chosen by households themselves under internal habit formation or through destabilizing policies by a benevolent government under external habit formation.[7] In calculations not reported here, we follow Campbell and Cochrane's (1999, p. 210) suggestion that their endowment economy can alternatively be closed with a linear production technology. Using the randomly generated endowment sequences in the present paper, we then find large welfare gains from storing roughly 10 percent of the endowment and consuming the savings in a consumption binge every other month. To make the households in a laissez-faire economy that consumes the endowment indifferent to such a policy, consumption would have to be raised by more than 30 percentage points for the indefinite future.

In sum, given the progress made in understanding asset pricing puzzles with the help of Campbell-Cochrane preferences, the next natural step will be to understand the macroeconomic implications, when endogenizing consumption at the aggregate level and when investigating the scope for government intervention. Our analysis has convinced us, that this exercise will provide interesting challenges.

[7]By contrast, Ljungqvist and Uhlig (2000) report on how welfare can be improved through policies of consumption *stabilization* under catching-up-with-the-Joneses preferences, i.e., the conventional linear external habit formulation. In a productivity-shock driven economy, it is shown that such a consumption externality calls for an optimal tax policy that affects the economy countercyclically via procyclical taxes, i.e., "cooling" down the economy with higher taxes in booms and lowering taxes in recessions to stimulate the economy.

Appendix A

We show that welfare cannot increase by destroying part of the endowment along a steady-state growth path, given that the external habit level is governed by a conventional linear law of motion;

$$X_t = \mu X_{t-1} + \alpha C^a_{t-1} = \alpha C^a_0 \sum_{j=0}^{t-1} \mu^j G^{t-1-j} + \mu^t X_0 \,,$$

where the second equality would hold along the constant growth path. In a steady state, habit is ensured to be less than consumption if the parameters satisfy

$$G > \mu + \alpha, \tag{17}$$

and habit would then grow at the rate G and result in a steady-state ratio $X_t/C_t = \alpha/(G-\mu)$.

Let $\{C_t, X_t\}$ denote the sequence of consumption and habit levels in the steady state, and consider a one-time perturbation where a fraction $\triangle \in [0, 1-\alpha/(G-\mu)) \equiv \Gamma$ of the endowment is destroyed in period 0: $\tilde{C}_0 = (1-\triangle)C_0$, $\tilde{C}_t = C_t$ for all $t \geq 1$; $\tilde{X}_0 = X_0$, $\tilde{X}_t = X_t - \mu^{t-1}\alpha\triangle C_0$ for all $t \geq 1$. Let $\Omega(\triangle)$ denote the welfare associated with a perturbation $\triangle$, i.e., the preferences in (1) are evaluated at the allocation $\{\tilde{C}_t, \tilde{X}_t\}$. Since $\Omega''(\triangle) < 0$ for all $\triangle \in \Gamma$, it suffices to show that $\Omega'(0) < 0$ in order to establish that $\Omega'(\triangle) < 0$ for all $\triangle \in \Gamma$. We can compute

$$\Omega'(0) = -(C_0 - X_0)^{-\gamma} C_0 + \sum_{t=1}^{\infty} \delta^t (C_t - X_t)^{-\gamma} \mu^{t-1} \alpha C_0 \,.$$

After substituting in for the steady-state allocation, a condition for $\Omega'(0) < 0$ is

$$-1 + \sum_{t=1}^{\infty} \delta^t G^{-t\gamma} \mu^{t-1} \alpha < 0 \qquad \Longrightarrow \qquad G^{\gamma}\delta^{-1} > \mu + \alpha \,,$$

which is guaranteed to hold under our parameter restrictions (8) and (17).

Appendix B

We verify that an infinitesimal endowment destruction must also decrease welfare under the C-C preference specification;

$$\begin{aligned} W'(0) &= (1+\bar{\lambda})\bar{S}^{1-\gamma} - \sum_{t=1}^{\infty} \delta^t \phi^{t-1}(1-\phi)\bar{\lambda}\left[\bar{S}G^t\right]^{1-\gamma} \\ &= \frac{1-\phi\delta G^{1-\gamma} + (1-\delta G^{1-\gamma})\bar{\lambda}}{1-\phi\delta G^{1-\gamma}}\bar{S}^{1-\gamma} > 0\,, \end{aligned}$$

where the convergence of the infinite sum and the strict inequality follow from $\phi \in [0,1)$ and parameter restriction (8). Thus, in the neighborhood around the steady-state growth path, welfare is strictly increasing in the fraction of the endowment that is consumed.

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Parameter	Variable	Annual	Monthly
Mean endowment growth (%)	g	1.89	0.1575
Standard deviation of endowment growth (%)	σ	1.50	0.4330
Persistence coefficient	ϕ	0.87	0.9885
Utility curvature	γ	2.00	2.0000
Subjective discount factor	δ	0.89	0.9909
Steady-state surplus consumptio ratio	$\bar{S}$	0.057	0.0571

Table 1: Parameters from Campbell and Cochrane (1999, Table 1) who report annualized values but, as in our study, perform all numerical analyses at a monthly frequency.

	Endowment destruction, $1 - \exp(\psi)$				
p	0.01	0.05	0.10	0.15	0.20
2	4.89 (4.19)	8.36 (7.00)	13.66 (11.76)	8.68 (13.77)	-8.18 (5.56)
6	2.48 (2.08)	9.08 (7.60)	15.45 (13.09)	9.65 (15.03)	-8.13 (5.96)
12	1.37 (1.16)	9.03 (7.60)	15.91 (13.40)	9.86 (15.26)	-8.12 (6.04)
24	0.72 (0.62)	8.52 (7.28)	15.89 (13.40)	9.89 (15.28)	-8.12 (6.06)
120	0.24 (0.21)	4.95 (4.27)	13.40 (11.12)	8.51 (14.06)	-8.15 (5.97)

Table 2: Expected welfare gain when switching from the laissez-faire outcome to a government policy indexed by (ψ, p), measured by the percentage increase in consumption needed to attain the same expected utility under no government intervention. A government policy (ψ, p) stipulates that a fraction $1 - \exp(\psi)$ of the endowment is destroyed every p:th month. The numbers without parentheses refer to unconditional welfare gains where the initial surplus consumption ratio is drawn from its unconditional distribution, and the numbers within parentheses are welfare gains conditional upon the initial surplus consumption ratio equal to its steady-state value.

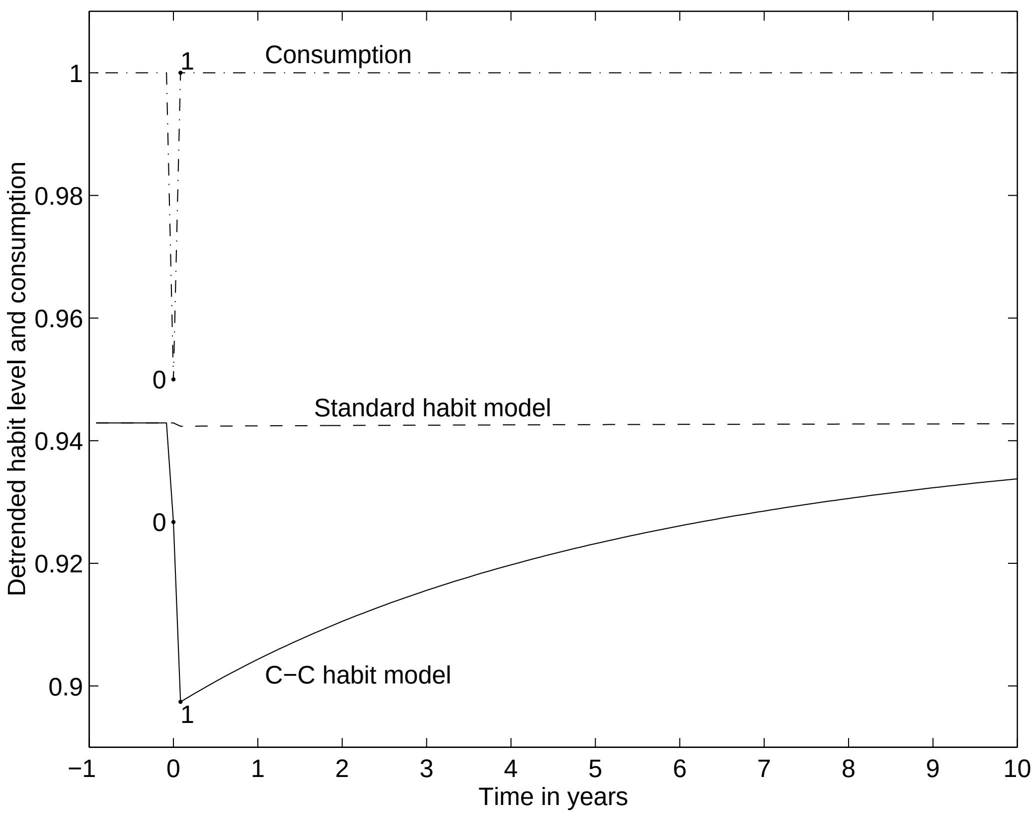

Figure 1: Detrended consumption and habit level associated with a five-percent endowment destruction in period 0. The dash-dotted curve depicts the consumption time series that bounces back in period 1. The solid and dashed curve show the habit time series for the C-C habit model and the standard habit model, respectively. (Parameter values from Table 1 at a monthly frequency but without any uncertainty, $\sigma = 0$.)

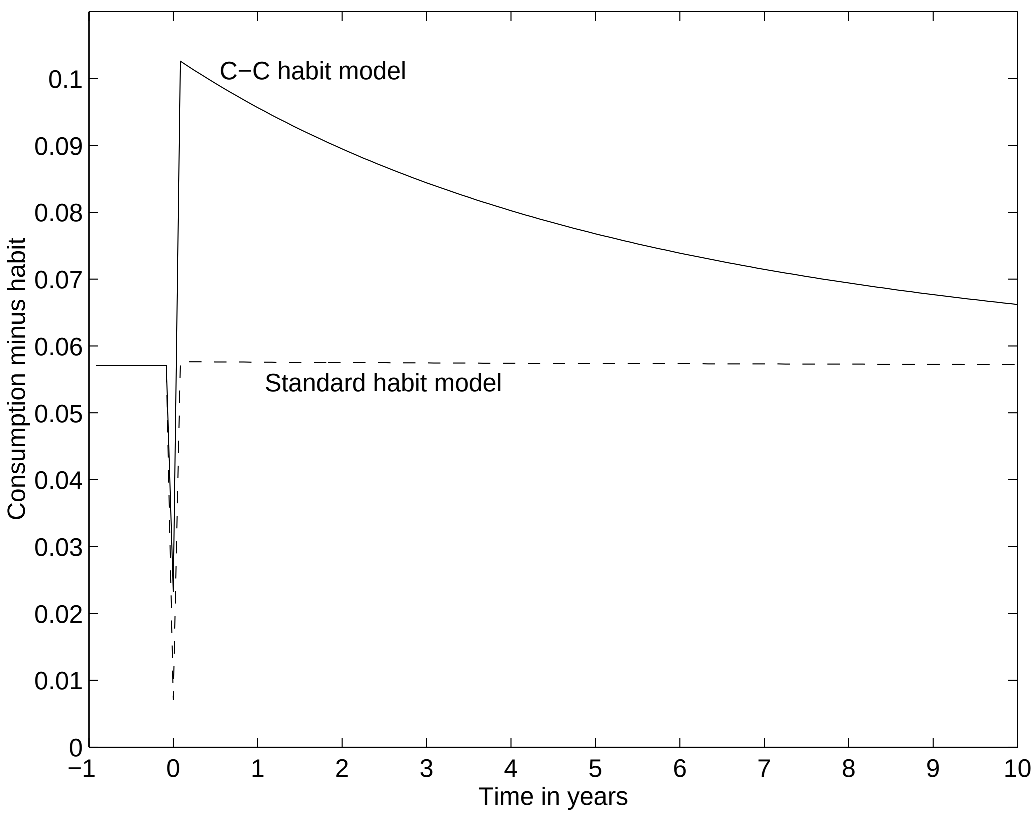

Figure 2: Difference between consumption and habit level associated with a five-percent endowment destruction in period 0. The solid and dashed curve refer to the C-C habit model and the standard habit model, respectively. Detrended time series of consumption and habit levels are taken from Figure 1. (Parameter values from Table 1 at a monthly frequency but without any uncertainty, $\sigma = 0$.)

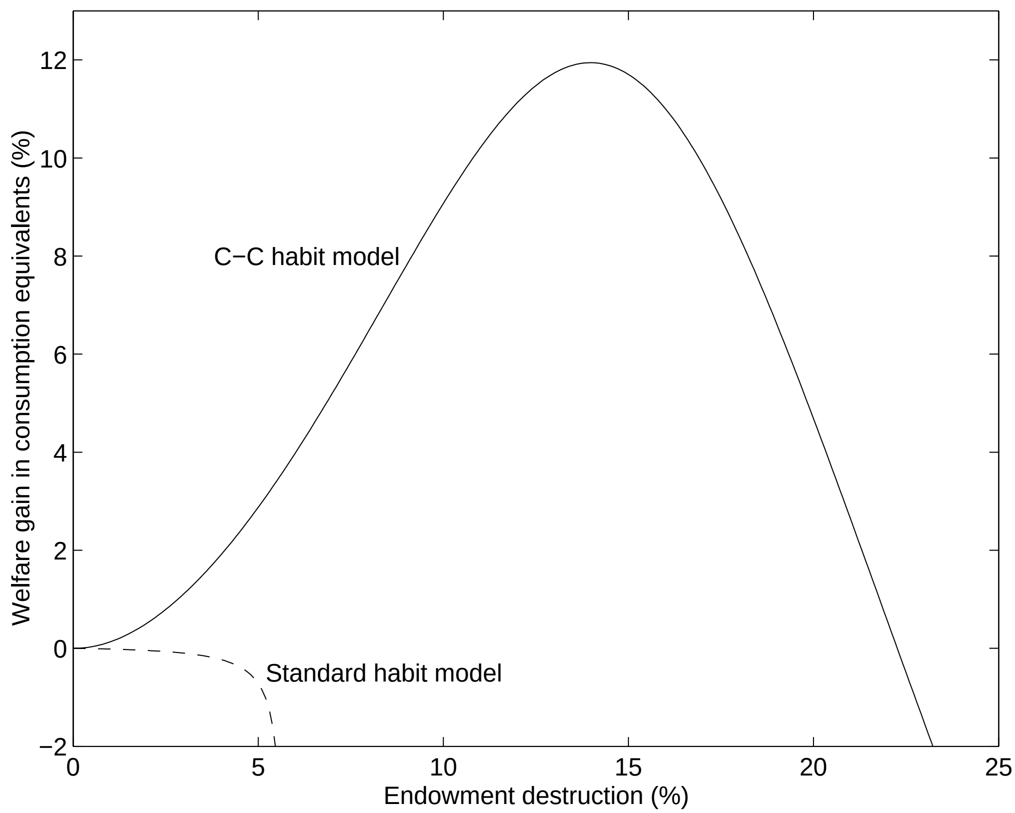

Figure 3: Welfare gain associated with a one-time endowment destruction, measured by the permanent percentage increase in consumption needed to attain the same utility without any destruction. Along a non-stochastic steady-state growth path, a fraction between 0 and 25 percent of the endowment is destroyed in one single period. The solid and dashed curve depict the welfare gain associated with such a destructive policy including the utility loss of the initial endowment destruction under the C-C habit model and the standard habit model, respectively. Note that utility is not defined in the standard habit model for endowment destructions that exceed the surplus consumption ratio in the steady state, $\bar{S} = 0.057$. (Parameter values from Table 1 at a monthly frequency but without any uncertainty, $\sigma = 0$.)

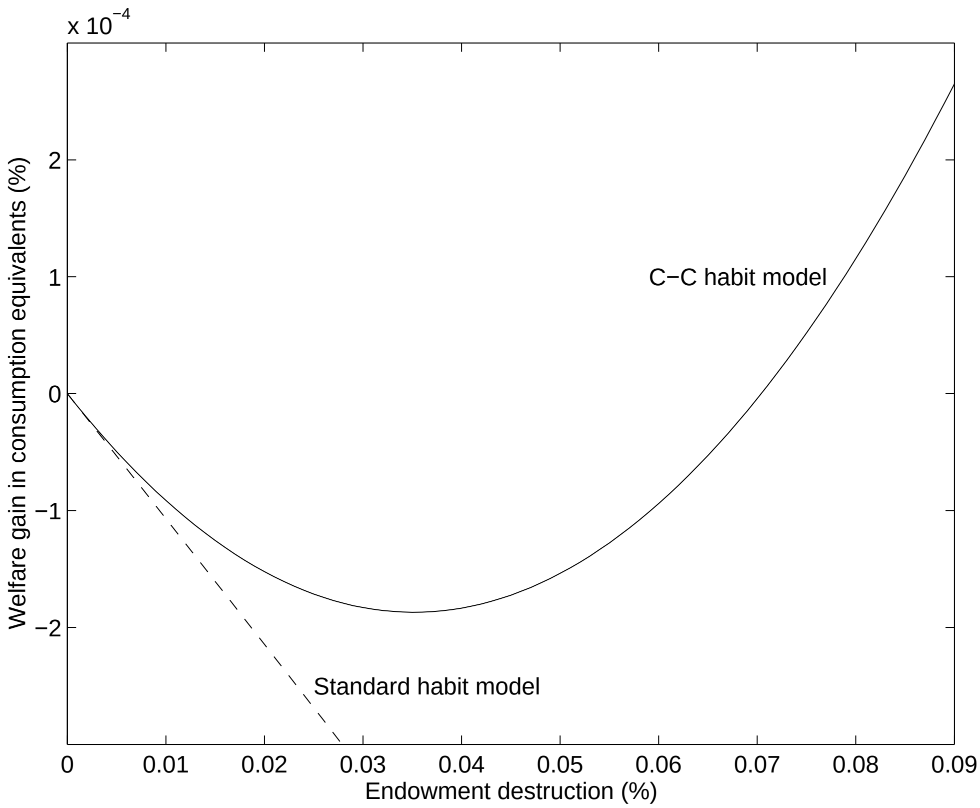

Figure 4: Welfare gain associated with a one-time endowment destruction, measured by the permanent percentage increase in consumption needed to attain the same utility without any destruction. The figure is a magnification of the lower range of endowment destructions in Figure 3. Along a non-stochastic steady-state growth path, a fraction between 0 and 0.09 percent of the endowment is destroyed in one single period. The solid and dashed curve depict the welfare gain associated with such a destructive policy including the utility loss of the initial endowment destruction under the C-C habit model and the standard habit model, respectively. (Parameter values from Table 1 at a monthly frequency but without any uncertainty, $\sigma = 0$.)

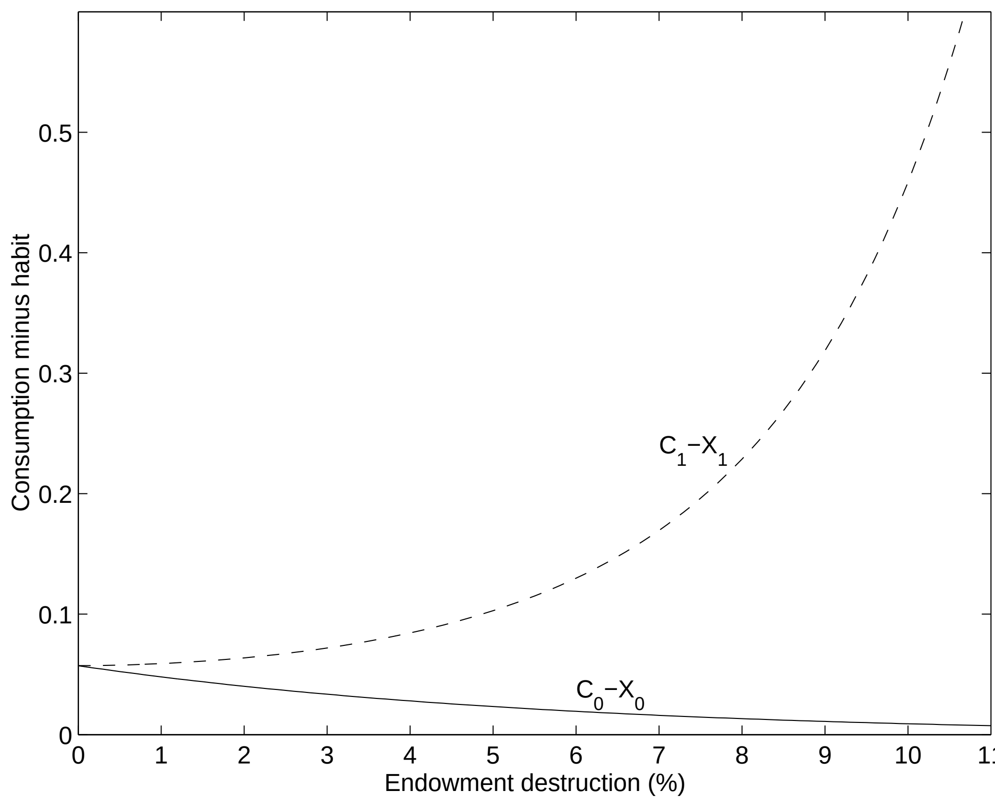

Figure 5: Difference between consumption and habit level under the C-C habit formulation in response to the described one-time endowment destructions of Figure 3. The solid curve depicts the difference between consumption and habit level in the period of the endowment destruction, $C_0 - X_0$, and the dashed curve depicts the detrended difference in the next period, $C_1 - X_1$. (Parameter values from Table 1 at a monthly frequency but without any uncertainty, $\sigma = 0$.)

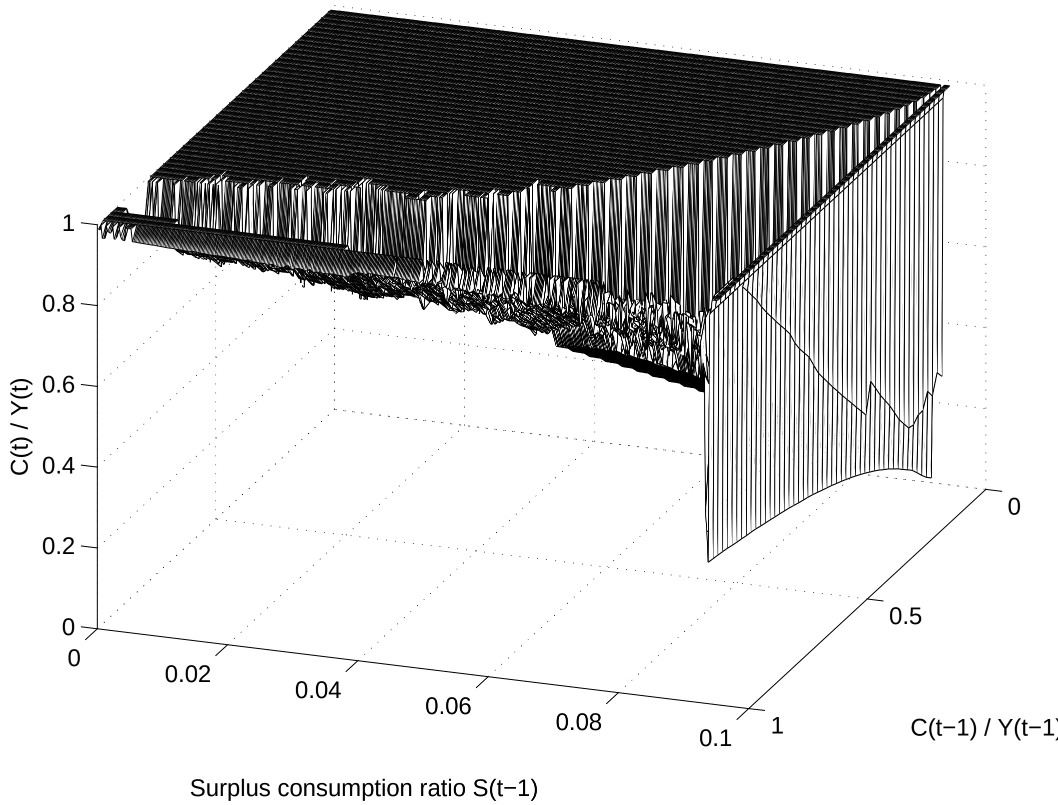

Figure 6: Optimal consumption as a fraction of the endowment, given last period's values of the consumption fraction C_{t-1}/Y_{t-1} and the surplus consumption ratio. We only show the decision rule for $0 \leq S \leq \exp s^*$. (Parameter values from Table 1 at a monthly frequency but without any uncertainty, $\sigma = 0$.)

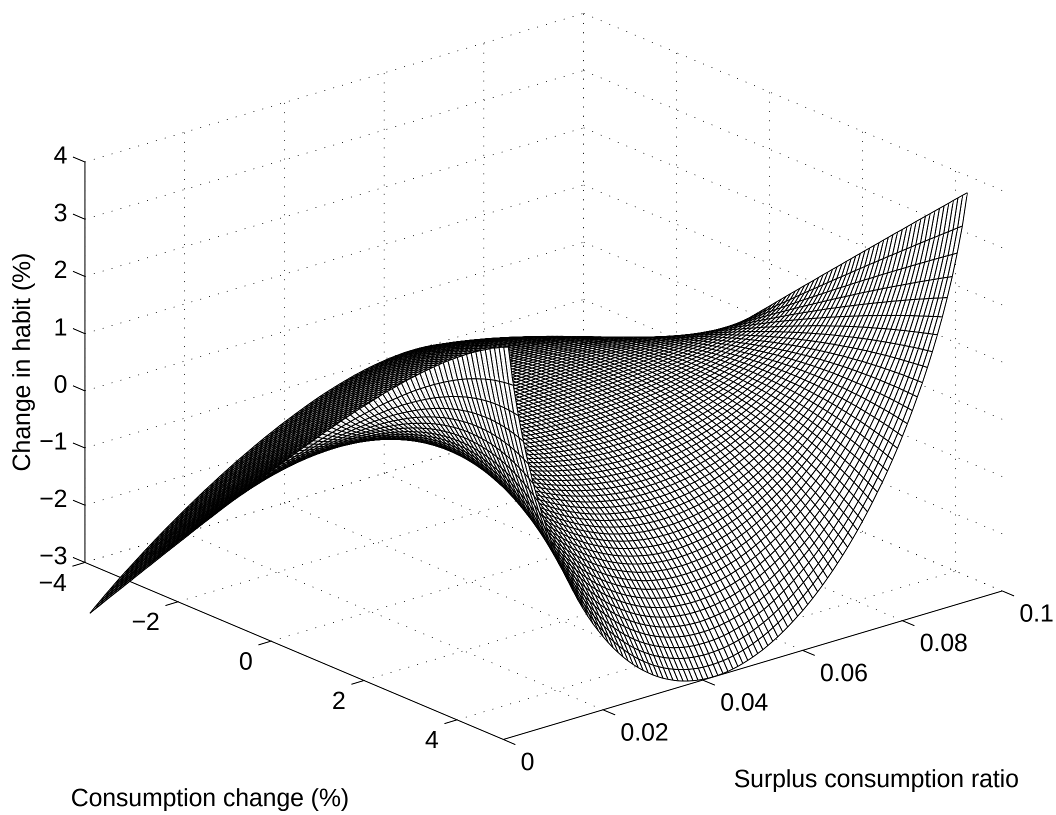

Figure 7: How contemporaneous habit is affected by a consumption change relative to last period's levels, for different values of last period's surplus consumption ratio. (Parameter values from Table 1 at a monthly frequency.)