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ABSTRACT

This paper proposes a dynamic risk-based model capable of jointly explaining the term structure of interest rates, returns on the aggregate market and the risk and return characteristics of value and growth stocks. Both the term structure of interest rates and returns on value and growth stocks convey information about how the representative investor values cash flows of different maturities. We model how the representative investor perceives risks of these cash flows by specifying a parsimonious stochastic discount factor for the economy. Shocks to dividend growth, the real interest rate, and expected inflation are priced, but shocks to the price of risk are not. Given reasonable assumptions for dividends and inflation, we show that the model can simultaneously account for the behavior of aggregate stock returns, an upward-sloping yield curve, the failure of the expectations hypothesis and the poor performance of the capital asset pricing model.

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1 Introduction

Empirical studies of asset pricing have uncovered a rich set of properties of the time series of aggregate stock market returns, of the term structure of interest rates and of the cross-section of stock returns. Average returns on the aggregate stock market are high relative to short-term interest rates. Relative to dividends, aggregate stock returns are highly volatile. They are also predictable; the return on the aggregate market in excess of the short-term interest rate is predictably high when the price-dividend ratio is low and predictably low when the price-dividend ratio is high. The term structure of interest rates on U.S. government bonds is upward sloping, and excess bond returns are predictable by yield spreads and by linear combinations of forward rates. In the cross-section, stocks with low ratios of price to fundamentals (value stocks) have higher returns than stocks with high ratios of price to fundamentals (growth stocks), despite the fact that they have lower covariance with aggregate stock returns. These facts together are inconsistent with popular benchmark models and therefore represent an important challenge for theoretical modeling of asset prices.[1]

One approach to explaining these properties of asset prices is to propose a fully specified model of investor preferences, endowments and cash flows on the assets of interest. Under this approach, the returns investors demand for bearing risks (the prices of risk) are endogenously determined by the form of preferences and the process for aggregate consumption. These prices of risk in turn determine risk premia, volatility, and covariances on the assets in equilibrium. Models that follow this approach typically have a small number of free parameters and generate tight implications for asset prices. We refer to this as the *equilibrium approach*.

A second approach is to directly specify the stochastic discount factor (SDF) for the economy. Foundational work by Harrison and Kreps (1979) demonstrates that, in the absence of arbitrage, there exists a process (known as a stochastic discount factor) that determines current prices on the basis of future cash flows. Given that such a process exists, this second approach specifies the SDF process directly, without reference to preferences or endowments. The exogenously specified SDF implies processes for the prices of risk which determine asset pricing properties. Models based on the SDF typically have a large number of degrees of freedom and therefore allow for substantial

[1]See Campbell (2003) and Cochrane (1999) for recent surveys of the empirical literature and discussion of these benchmark models.

flexibility in matching asset prices. Indeed, the parameters of the SDF and of cash flows are often backed out from asset prices. We refer to this as the *SDF approach*.

In this paper, we seek to explain the aggregate market, cross-sectional, and term structure facts within a single model. To do so, we combine elements of both approaches described above. We assume that only risk arising from aggregate cash flows is priced directly, thus maintaining the strict discipline about the number and nature of priced factors imposed by the equilibrium approach. We determine the parameters of the cash flow processes based on data from the cash flows themselves. This modeling approach maintains the parsimony that is typical of equilibrium models. However, rather than specifying underlying preferences, we directly specify the stochastic discount factor as in the SDF approach. Our goal is to introduce a small but crucial amount of flexibility in order to explain the facts listed in the first paragraph.

Our model's ability to match the data stems in part from properties of the time-varying price of risk, which results in time-varying risk premia on stocks and bonds. As in Brennan, Wang, and Xia (2004) and Lettau and Wachter (2007), we assume first-order autoregressive (AR(1)) processes for both the price of risk and the real interest rate. To model the nominal term structure of interest rates, we introduce an exogenous process for the price level (Cox, Ingersoll, and Ross (1985), Boudoukh (1993)) such that expected inflation follows an AR(1). Realized inflation can therefore be characterized as an ARMA(1,1). Following Bansal and Yaron (2004) and Campbell (2003), we assume an AR(1) process for the expected growth rate of aggregate cash flows.

We calibrate the dividend, inflation, and riskfree rate processes to their counterparts in U.S data. The price of risk is then calibrated to match aggregate asset pricing properties. Several properties of these processes key to the model's ability to fit the data. First, a volatile price of risk is necessary to capture the empirically demonstrated property that risk premia on stocks and bonds are time-varying. This time-varying price of risk also allows the model to match the volatility of stock and bond returns given low volatility of dividends, real interest rates, and inflation. Second, the real riskfree rate is negatively correlated with fundamentals. This implies a slightly upward-sloping real yield curve. Expected inflation is also negatively correlated with fundamentals, implying a yield curve for nominal bonds that is more upward-sloping than the real yield curve.

Our model illustrates a tension between the upward slope of the yield curve and the value premium. The value premium implies that value stocks, which are short-horizon equity (because their cash flows are weighted more toward the present), have greater expected returns than growth stocks, which are long-horizon equity (because their cash flows are weighted more toward the future). Therefore the "term structure of equities" slopes downward, not upward. However, the very mechanism that implies an upward-sloping term structure of interest rates, namely a negative correlation between shocks to fundamentals and shocks to the real interest rate, also implies a growth premium. We show that correlation properties of shocks to the price of risk are key to resolving this tension. Namely, when the price of risk is independent of fundamentals, the model can simultaneously account for the downward-sloping term structure of equities and the upward-sloping term structure of interest rates.

To summarize, our model generates quantitatively accurate means and volatilities for the aggregate market and for Treasury bonds, while allowing for low volatilities in fundamentals. The model can replicate the predictability in excess returns on the aggregate market, the negative coefficients in Campbell and Shiller (1991) bond yield regressions and the tent-shaped coefficients on forward rates found by Cochrane and Piazzesi (2005). Finally, besides capturing the relative means of value and growth portfolios, our model also captures the striking fact that value stocks have relatively low risk according to conventional measures like standard deviation and covariance with the market. Therefore our model replicates the well-known outperformance of value, and underperformance of growth relative to the capital asset pricing model.

Our paper builds on studies that examining the implications of the term structure of interest rates for the stochastic discount factor. Dai and Singleton (2002, 2003) and Duffee (2002)) demonstrate the importance of a time-varying price of risk for explaining the predictability of excess bond returns. Like these papers, we also construct a latent factor model in which bond yields are linear. Ang and Piazzesi (2003), Bikbov and Chernov (2008) and Duffee (2006) introduce macroeconomic time series into the SDF as factors; in our work macroeconomic time series also are used to determine the SDF. Unlike our work, these papers focus exclusively on the term structure.

We also build on a literature that seeks to simultaneously explain prices in bond and the aggregate stock market (see Bakshi and Chen (1996), Bansal and Shaliastovich (2007), Bekaert, Engstrom, and Grenadier (2006), Buraschi and Jiltsov (2007), Gabaix (2008), Lustig, Van Nieuwerburgh and Verdelhan (2008) and Wachter (2006)). We extend these studies by exploring the consequences of our pricing kernel for a cross-section of equities defined by cash flows. In particular, we show that the model can reproduce the high premium on value stocks relative to growth stocks and the fact that value stocks have a low variance and low covariance with the aggregate market.

Our paper also builds on work that seeks to simultaneously explain the aggregate market and returns on value and growth stocks. Several studies link observed returns on value and growth stocks to new sources of risk (Campbell and Vuolteenaho (2004), Campbell, Polk, and Vuolteenaho (2009), Lustig and Van Nieuwerburgh (2005), Piazzesi, Schneider, and Tuzel (2007), Santos and Veronesi (2006b) and Yogo (2006)). Others more closely relate to the present study in that they model value and growth stocks based on their underlying cash flows (Berk, Green, and Naik (1999), Carlson, Fisher, and Giammarino (2004), Gomes, Kogan, and Zhang (2003), Hansen, Heaton, and Li (2008), Kiku (2006), Lettau and Wachter (2007), Santos and Veronesi (2006a) and Zhang (2005)). Unlike these studies, our study also seeks to explain the upward slope of the nominal yield curve and time-variation in bond risk premia. As we show, jointly considering the term structure of interest rates and behavior of value and growth portfolios has strong implications for the stochastic discount factor.

2 The model

In this section we introduce a model in which prices are driven by four state variables: expected dividend growth, expected inflation, the short-term real interest rate and the price of risk. Appendix A solves a more general model in which prices are driven by an arbitrary number of (potentially latent) factors.

2.1 Dividend growth, inflation, and the stochastic discount factor

The model specified in this section has six shocks, namely, a shock to dividend growth, to inflation, to expected dividend growth, to expected inflation, to the real riskfree rate and to the price of risk. Let $\boldsymbol{\epsilon}_{t+1}$ denote a 6×1 vector of independent standard normal shocks that are independent of variables observed at or before time t. We use bold font to denote matrices and vectors.

Let D_t denote the level of the aggregate real dividend at time t and $d_t = \log D_t$. We assume that the log growth rate of the aggregate dividend is conditionally normally distributed with a time-varying mean z_t that follows a first-order autoregressive process:

$$\Delta d_{t+1} = z_t + \boldsymbol{\sigma}_d \boldsymbol{\epsilon}_{t+1} \tag{1}$$

$$z_{t+1} = (1-\phi_z)g + \phi_z z_t + \boldsymbol{\sigma}_z \boldsymbol{\epsilon}_{t+1}, \tag{2}$$

where $\boldsymbol{\sigma}_d$ and $\boldsymbol{\sigma}_z$ are 1×6 vectors of loadings on the shocks $\boldsymbol{\epsilon}$, and ϕ_z is the autocorrelation. The conditional standard deviation of dividend growth is $\sigma_d = \sqrt{\boldsymbol{\sigma}_d \boldsymbol{\sigma}_d'}$. In what follows we will use the notation $\sigma_i = \sqrt{\boldsymbol{\sigma}_i \boldsymbol{\sigma}_i'}$ to refer to the conditional standard deviation of i and $\sigma_{ij} = \boldsymbol{\sigma}_i \boldsymbol{\sigma}_j'$ to refer to the covariance between shocks to i and to j. For the purpose of discussion, we assume that the autocorrelation of z and of the remaining three state variables are between 0 and 1; thus each variable is stationary and positively autocorrelated.[2] The parameter g can therefore be interpreted as the unconditional mean of dividend growth.

Because we are interested in pricing nominal bonds, we also specify a process for inflation. Let Π_t denote the price level and $\pi_t = \log \Pi_t$. Inflation follows the process

$$\Delta \pi_{t+1} = q_t + \boldsymbol{\sigma}_\pi \boldsymbol{\epsilon}_{t+1}, \tag{3}$$

$$q_{t+1} = (1-\phi_q)\bar{q} + \phi_q q_t + \boldsymbol{\sigma}_q \boldsymbol{\epsilon}_{t+1}, \tag{4}$$

where $\boldsymbol{\sigma}_\pi$ and $\boldsymbol{\sigma}_q$ are 6×1 vectors of loadings on the shocks, $\bar{q}$ is the unconditional mean of inflation and ϕ_q is the autocorrelation. In what follows, all quantities will be expressed in real terms unless it is stated otherwise; multiplying by Π_t converts a quantity from real to nominal terms.

Discount rates are determined by the real riskfree rate and by the price of risk. Let r^f_{t+1} denote the continuously-compounded riskfree return between times t and $t+1$. Note that r^f_{t+1} is known at time t. We assume that

$$r^f_{t+1} = (1-\phi_r)\bar{r}^f + \phi_r r^f_t + \boldsymbol{\sigma}_r \boldsymbol{\epsilon}_t, \tag{5}$$

where $\boldsymbol{\sigma}_r$ is a 6×1 vector of loadings on the shocks, $\bar{r}^f$ is the unconditional mean of r^f_t and ϕ_r is the autocorrelation. The variable that determines the price of risk, and therefore risk premia in this homoscedastic model, is denoted x_t. We assume

$$x_{t+1} = (1-\phi_x)\bar{x} + \phi_x x_t + \boldsymbol{\sigma}_x \boldsymbol{\epsilon}_{t+1}, \tag{6}$$

[2]However, realized dividend growth may be (and in fact will be) negatively autocorrelated.

where $\boldsymbol{\sigma}_x$ is a 6×1 vector of loadings on the shocks, $\bar{x}$ is the unconditional mean of x_t and ϕ_x is the autocorrelation.

To maintain a parsimonious model, we assume that only fundamental dividend risk is priced directly. This assumption implies that the price of risk is proportional to the vector $\boldsymbol{\sigma}_d$ (the formulas in Appendix A allow for a more general price of risk). Other risks are priced insofar as they covary with aggregate cash flows. Besides reducing the degrees of freedom in the model, this specification allows for easier comparison to models based on preferences, such as those of Campbell and Cochrane (1999) and Menzly, Santos, and Veronesi (2004). The stochastic discount factor (SDF) is thus given by

$$M_{t+1} = \exp\left\{-r^f_{t+1} - \frac{1}{2}\sigma_d^2 x_t^2 - x_t \boldsymbol{\sigma}_d \boldsymbol{\epsilon}_{t+1}\right\}.$$

Because the SDF is a quadratic function of x_t, the model is in the essentially affine class (Dai and Singleton (2002), Duffee (2002)). Asset prices are determined by the Euler equation

$$E_t\left[M_{t+1} R_{t+1}\right] = 1, \tag{7}$$

where R_{t+1} denotes the real return on a traded asset. Given the lognormal specification the maximal Sharpe ratio is given by

$$\mathrm{SR}_t = \max \frac{E_t R_{t+1} - R^f_{t+1}}{(\mathrm{Var}_t[R_{t+1} - R^f_{t+1}])^{1/2}} = \frac{(\mathrm{Var}_t[M_{t+1}])^{1/2}}{E_t[M_{t+1}]} = \sqrt{e^{x_t^2 \sigma_d^2} - 1} \approx |x_t| \sigma_d.$$

(see Campbell and Cochrane (1999), Lettau and Uhlig (2002), Lettau and Wachter (2007)).

2.2 Prices and returns on bonds and equities

Real bonds

Let P^r_{nt} denote the price of an n-period real bond at time t. That is, P^r_{nt} denotes the time-t price of an asset with a fixed payoff of 1 at time $t+n$. Because this asset has no intermediate payoffs, its return between t and $t+1$ equals

$$R^r_{n,t+1} = \frac{P^r_{n-1,t+1}}{P^r_{nt}}. \tag{8}$$

The prices of real bonds can be determined recursively from the Euler equation (7). Substituting in (8) for the return implies that

$$E_t\left[M_{t+1} P^r_{n-1,t+1}\right] = P^r_{nt}, \tag{9}$$

while the fact that the bond pays a single unit at maturity implies that $P^r_{0t} = 1$. Appendix C verifies that (9) is satisfied by

$$P^r_{nt} = \exp\{A^r_n + B^r_{rn}(r^f_{t+1} - \bar{r}^f) + B^r_{xn}(x_t - \bar{x})\}. \tag{10}$$

The coefficient on the riskfree rate is given by

$$B^r_{rn} = -\frac{1-\phi^n_r}{1-\phi_r}. \tag{11}$$

The coefficient on the price of risk is given by the recursion

$$B^r_{xn} = B^r_{x,n-1}\phi_x - B^r_{r,n-1}\sigma_{dr} - B^r_{x,n-1}\sigma_{dx}, \tag{12}$$

with boundary condition $B^r_{x0} = 0$. The constant term A^r_n is defined by (A.19). The yield to maturity on a real bond is defined as

$$y^r_{nt} = -\frac{1}{n}\log P^r_{nt} = -\frac{1}{n}\left(A^r_n + B^r_{rn}(r^f_{t+1} - \bar{r}^f) + B^r_{xn}(x_t - \bar{x})\right) \tag{13}$$

and is linear in the state variables.

Equation (10) shows that prices of real bonds are driven by the riskfree rate and by the price of risk. Expected dividend growth and expected inflation do not directly influence the prices of real bonds (though they might influence these prices indirectly through correlations with r^f_{t+1} and with x_t). As (11) shows, an increase in the riskfree rate lowers the bond price. Moreover, the magnitude of the price response to a change in r^f_{t+1} is increasing in maturity. This is the duration effect, and it is driven by the persistence ϕ_r. Because the riskfree rate is persistent, a higher value today suggests that future values will also be high. Because of compounding, the further out the cash flow, the larger the effect a change in the riskfree rate has on the price. As (12) shows, the sign of the effect of the price of risk variable depends on the correlations σ_{dr} and σ_{dx}. The sign and magnitude of the effect of an increase in the price of risk is best understood by examining the formula for risk premia, as we now explain.

Let $r^r_{nt} = \log R^r_{nt}$ be the continuously compounded return on the real zero-coupon bond of maturity n. Because real bond prices are lognormally distributed, r^r_{nt} is conditionally normally distributed. We derive risk premia by taking the logs of both sides of the Euler equation (7) and use the properties of the lognormal distribution to evaluate the expectation. It follows that the risk premia on real zero-coupon bonds satisfy

$$E_t[r^r_{n,t+1} - r^f_{t+1}] + \frac{1}{2}\mathrm{Var}_t(r^r_{n,t+1}) = \mathrm{Cov}_t(r^r_{n,t+1}, \Delta d_{t+1})x_t. \tag{14}$$

Note that the second term on the left hand side of (14) is an adjustment for Jensen's inequality. Equations (10) and (11) imply that

$$\mathrm{Cov}_t(r^r_{n,t+1}, \Delta d_{t+1}) \quad = \quad B^r_{r,n-1}\sigma_{dr} + B^r_{x,n-1}\sigma_{dx}. \tag{15}$$

Risk premia on real bonds are time-varying and proportional to x_t. Given a value for x_t, the level of risk premia is determined by the correlations σ_{dr} and σ_{dx}. Comparing (12) and (15), it is clear that the same variables that drive risk premia influence the coefficients B^r_{xn} with a negative sign. This is not surprising, as B^r_{xn} represents the effects of the price of risk variable on the price of the real bond. When bonds carry positive risk premia, $B^r_{xn} < 0$, which implies that an increase in x_t lowers the price of real bonds. Moreover, if risk premia are increasing in maturity, the greater the maturity, the greater the effect of an increase in x_t on the price.

Equity

Let P^d_{nt} denote the time-t price of the asset that pays the aggregate dividend at time $t+n$. We will refer to this asset as zero-coupon equity. In solving for the price, it is convenient to scale P^d_{nt} by the aggregate dividend at time t to eliminate the need to consider D_t as a state variable. The return on this zero-coupon equity claim is equal to

$$R^d_{n,t+1} = \frac{P^d_{n-1,t+1}}{P^d_{nt}} = \frac{P^d_{n-1,t+1}/D_{t+1}}{P^d_{nt}/D_t}\frac{D_{t+1}}{D_t}. \tag{16}$$

Let $r^d_{n,t} = \log R^d_{n,t}$ denote the continuously compounded return. Substituting (16) into the Euler equation (7) implies that P^d_{nt} satisfies the recursion

$$E_t\left[M_{t+1}\frac{D_{t+1}}{D_t}\frac{P^d_{n-1,t+1}}{D_{t+1}}\right] = \frac{P^d_{nt}}{D_t}, \tag{17}$$

with boundary condition $P^d_{0t}/D_t = 1$. Appendix C verifies that (17) is solved by a function of the form

$$\frac{P^d_{nt}}{D_t} = \exp\{A^d_n + B^d_{zn}(z_t - g) + B^d_{rn}(r^f_{t+1} - \bar{r}^f) + B^d_{xn}(x_t - \bar{x})\}. \tag{18}$$

The coefficients on expected dividend growth and the riskfree rate are given by

$$B^d_{zn} = \frac{1-\phi_z^n}{1-\phi_z} \quad B^d_{rn} = -\frac{1-\phi_r^n}{1-\phi_r}. \tag{19}$$

The coefficient on the price of risk satisfies the recursion

$$B^d_{xn} = B^d_{x,n-1}\phi_x - \sigma_d^2 - B^d_{z,n-1}\sigma_{dz} - B^d_{r,n-1}\sigma_{dr} - B^d_{x,n-1}\sigma_{dx}, \tag{20}$$

with boundary condition $B^d_{x0} = 0$. The constant term A^d_n is defined by (A.25). Following logic similar to that used to compute risk premia on zero-coupon bonds, we find that risk premia on zero-coupon equity claims are given by

$$E_t[r^d_{n,t+1} - r^f_{t+1}] + \frac{1}{2}\mathrm{Var}_t(r^d_{n,t+1}) = \mathrm{Cov}_t(r^d_{n,t+1}, \Delta d_{t+1})x_t, \tag{21}$$

where (18) and (19) imply that

$$\mathrm{Cov}_t(r^d_{n,t+1}, \Delta d_{t+1}) = \sigma_d^2 + B^d_{z,n-1}\sigma_{dz} + B^d_{r,n-1}\sigma_{dr} + B^d_{x,n-1}\sigma_{dx}. \tag{22}$$

Equation (18) shows that price-dividend ratios are driven by expected dividend growth, by the real interest rate and by the price of risk. Expected inflation does not directly influence equity valuations. As (19) shows, an increase in expected dividend growth increases prices. Because expected dividend growth is persistent, and because D_{t+n} cumulates shocks between t and $t+n$, the greater is the maturity n, the greater is the effect of changes in z_t on the price. An increase in the real interest rate lowers the equity price, and this effect is greater, the greater is the maturity. The intuition is the same as that for real bonds.

As in the case of real bonds, the effect of a change in the price of risk on equities is more subtle and depends on risk premia. Comparing (20) and (22) indicates that the variables that influence risk premia on equities also govern the evolution of B^d_{xn}. Risk premia on zero-coupon equity are determined by the variance of cash flows, and the covariance of cash flows with shocks to expected dividend growth, to the riskfree rate and to the price of risk. For the model to account for the value premium, risk premia on equities will need to be decreasing in maturity rather than increasing. For this reason, B^d_{xn} will be a non-monotonic function of n. We will discuss risk premia on zero-coupon equities more fully later in the paper.

In our model, the aggregate market portfolio is the claim to all future dividends. Therefore its price-dividend ratio is given by

$$\frac{P_t^m}{D_t} = \sum_{n=1}^{\infty} \frac{P_{nt}^d}{D_t} = \sum_{n=1}^{\infty} \exp\left\{A_n^d + B_{zn}^d(z_t - g) + B_{rn}^d(r_{t+1}^f - \bar{r}^f) + B_{xn}^d(x_t - \bar{x})\right\}. \tag{23}$$

Appendix B describes sufficient conditions on the parameters such that (23) converges for all values of the state variables. The return on the aggregate market equals

$$R_{t+1}^m = \frac{P_{t+1}^m + D_t}{P_t^m} = \frac{(P_{t+1}^m/D_{t+1}) + 1}{P_t^m/D_t}\frac{D_{t+1}}{D_t}. \tag{24}$$

Note that the price-dividend ratio is not an affine function of the state variables.

Nominal bonds

Let P_{nt}^π denote the real price of a zero-coupon nominal bond maturing in n periods. The real return on this bond equals

$$R_{n,t+1}^\pi = \frac{P_{n-1,t+1}^\pi}{P_{nt}^\pi} = \frac{P_{n-1,t+1}^\pi \Pi_{t+1}}{P_{nt}^\pi \Pi_t}\frac{\Pi_t}{\Pi_{t+1}}. \tag{25}$$

Let $r_{n,t+1}^\pi = \log R_{n,t}^\pi$ denote the continuously compounded return on this bond. This asset is directly analogous to the dividend claim above: the "dividend" is the reciprocal of the price level, and the "price-dividend ratio" on this asset is its nominal price $P_{nt}^\pi \Pi_t$.

The Euler equation holds for the real return on this bond; therefore the price satisfies

$$E_t\left[M_{t+1}\frac{\Pi_t}{\Pi_{t+1}}P_{n-1,t+1}^\pi \Pi_{t+1}\right] = P_{nt}^\pi \Pi_t, \tag{26}$$

with boundary condition $P_{0t}^\pi \Pi_t = 1$. Appendix C shows that the recursion (26) can be solved by a function of the form

$$P_{nt}^\pi \Pi_t = \exp\{A_n^\pi + B_{qn}^\pi(q_t - \bar{q}) + B_{rn}^\pi(r_{t+1}^f - \bar{r}^f) + B_{xn}^\pi(x_t - \bar{x})\}. \tag{27}$$

The coefficients on expected inflation and the riskfree rate are given by

$$B_{qn}^\pi = -\frac{1-\phi_q^n}{1-\phi_q} \quad B_{rn}^\pi = -\frac{1-\phi_r^n}{1-\phi_r}. \tag{28}$$

The coefficient on the price of risk satisfies the recursion

$$B_{xn}^\pi = B_{x,n-1}^\pi \phi_x + \sigma_{d\pi} - B_{q,n-1}^\pi \sigma_{dq} - B_{r,n-1}^\pi \sigma_{dr} - B_{x,n-1}^\pi \sigma_{dx}, \tag{29}$$

with boundary condition $B^{\pi}_{x0} = 0$. The constant term A^{π}_{n} is defined by (A.31). Following logic similar to that used to compute risk premia on real bonds, risk premia on nominal bonds are equal to

$$E_t[r^{\pi}_{n,t+1} - r^f_{t+1}] + \frac{1}{2}\mathrm{Var}_t(r^{\pi}_{n,t+1}) = \mathrm{Cov}_t(r^{\pi}_{n,t+1}, \Delta d_{t+1})x_t, \tag{30}$$

where

$$\mathrm{Cov}_t(r^{\pi}_{n,t+1}, \Delta d_{t+1}) = -\sigma_{d\pi} + B^{\pi}_{q,n-1}\sigma_{dq} + B^{\pi}_{r,n-1}\sigma_{dr} + B^{\pi}_{x,n-1}\sigma_{dx}.$$

Real risk premia on nominal bonds are determined by the loadings on expected inflation, the real riskfree rate and the price of risk, along with the covariance of each of these sources of risk with shocks to fundamentals. In addition, risk premia are determined by the covariance of unexpected inflation with fundamentals.

Equation (27) shows that nominal bond prices are driven by expected inflation, the real interest rate and the price of risk. Expected dividend growth does not directly influence nominal bond prices. As (28) shows, an increase in expected inflation lowers bond nominal bond prices at all maturities. This effect is greater, the greater the maturity, because Π_{t+n} cumulates shocks between t and $t+n$. An increase in the real interest rates lowers nominal bond prices at all maturities; the greater the maturity the greater is this effect because of duration. The same variables that determine risk premia govern the evolution of B^{π}_{xn}. Because nominal bonds will have risk premia that are positive and increasing in maturity, B^{π}_{xn} will be negative and decreasing in maturity. That is, an increase in the price of risk will lower prices of nominal bonds, and will have a greater effect on long-term bonds than short-term bonds.

It is also of interest to consider the nominal return on the nominal bond, and the nominal yield. Following Campbell and Viceira (2001), we use the superscript \$ to denote nominal quantities for the nominal bond. The nominal (continuously-compounded) yield to maturity on this bond is equal to

$$y^{\$}_{nt} = -\frac{1}{n}\log\left(P^{\pi}_{nt}\Pi_t\right) = -\frac{1}{n}\left(A^{\pi}_{n} + B^{\pi}_{qn}(q_t - \bar{q}) + B^{\pi}_{rn}(r^f_{t+1} - \bar{r}^f) + B^{\pi}_{xn}(x_t - \bar{x})\right) \tag{31}$$

and, like the yield on the real bond, is linear in the state variables. Finally, we use the notation $R^{\$}_{n,t+1}$ to denote the nominal return on the nominal n-period bond:

$$R^{\$}_{n,t+1} = \frac{P^{\pi}_{n-1,t+1}\Pi_{t+1}}{P_{nt}\Pi_t}.$$

It is also of interest to compute risk premia on nominal bonds relative to the one-period nominal bond (as opposed to the real bond, as in (30)). It follows from the equation for nominal prices (27) that

$$E_t\left[r^{\$}_{n,t+1} - y^{\$}_{1t}\right] + \frac{1}{2}\mathrm{Var}_t(r^{\$}_{n,t+1}) = \mathrm{Cov}_t(r^{\$}_{n,t+1}, \Delta d_{t+1})x_t, \tag{32}$$

where

$$\mathrm{Cov}_t(r^{\$}_{n,t+1}, \Delta d_{t+1}) = B^{\pi}_{q,n-1}\sigma_{dq} + B^{\pi}_{r,n-1}\sigma_{dr} + B^{\pi}_{x,n-1}\sigma_{dx}.$$

This section has shown that risk premia on all zero-coupon assets are proportional to x_t. While there is some conditional heteroscedasticity in the aggregate market that arises from time-varying weights on zero-coupon equity, this effect is small. A natural way to drive a wedge between time-variation in bond and stock premia is to allow for time-varying correlations as in Campbell, Sunderam, and Viceira (2009). For simplicity and to maintain our focus on the slope of the term structures of equity and interest rates, we do not pursue this route here.

2.3 Unconditional term structures of equity and interest rates

Prior to describing the full calibration of the model and results from simulated data, we use the results developed above to describe the model's qualitative implications for risk premia on bonds and stocks. We illustrate the issues by comparing bonds and equity maturing in two periods with those maturing in one period. It follows from (14) and (15) that the unconditional risk premium of the real bond maturing in two periods equals[3]

$$E[r^r_{2,t+1} - r^f_{t+1}] + \frac{1}{2}\mathrm{Var}(r^r_{2,t+1}) = -\sigma_{dr}\bar{x}.$$

The risk premium on the one-period real bond is, by definition, equal to zero. The term σ_{dr} represents the covariance of shocks to the real interest rate with shocks to dividend growth: a negative covariance leads to a positive risk premium on the two-period bond because it implies that bonds pay off in good times (bond prices move in the opposite direction from the riskfree rate). The same term appears in the average spread between the yields of the one and the two-period bond:

$$E[y^r_2 - y^r_1] = -\frac{1}{2}\sigma_{dr} - \frac{1}{4}\sigma^2_r, \tag{33}$$

[3]Because $r^r_{n,t+1}$ is normal and shocks are homoskedastic, $\mathrm{Var}_t(r^r_{n,t+1})$ is constant and equal to $\mathrm{Var}(r^r_{n,t+1})$. This reasoning also holds for zero-coupon nominal bonds and zero-coupon equity.

(see (13) and (A.19)). The second term represents an adjustment for Jensen's inequality and is relatively small.

The unconditional risk premia on one and two-period equity claims are equal to

$$E[r^d_{1,t+1} - r^f_{t+1}] + \frac{1}{2}\mathrm{Var}(r^d_{1,t+1}) = \sigma_d^2 \bar{x} \tag{34}$$

$$E[r^d_{2,t+1} - r^f_{t+1}] + \frac{1}{2}\mathrm{Var}(r^d_{2,t+1}) = \left(\sigma_d^2 - \sigma_{dr} + \sigma_{dz} - \sigma_d^2\sigma_{dx}\right)\bar{x}. \tag{35}$$

While the one-period equity claim is only exposed to unexpected changes in dividends, the two-period equity claim is also exposed to unexpected changes in the interest rate, expected dividend and the price of risk. These risk factors are represented by the covariance terms σ_{dz}, σ_{dr} and σ_{dx}. If these processes are correlated with the priced fundamental dividend factor, the risk premium of the two-period equity claim will be different from the one-period premium. Note that the extent to which two-period equity is driven by x_t depends on the one-period premium. This explains why σ_d^2 multiplies σ_{dx} in (35).

The positive premium of value (short horizon) stocks over growth (long horizon) stocks in the data suggests that the equity term structure is downward sloping. Thus the premium on two-period equity should be less than that on one-period equity. Comparing (33) to (34) and (35) suggests that an upward sloping term structure of interest rates requires interest rate shocks to be negatively correlated with dividend shocks ($\sigma_{dr} < 0$). *Ceteris paribus*, this effect leads also to an upward sloping term structure of equity, which implies a growth premium rather than a value premium.

As shown in Lettau and Wachter (2007), a key parameter for the slope of the equity term structure is the correlation of fundamental dividend risk with shocks to the price of risk process x_t. To understand the role of this correlation, consider the special case of $\sigma_{dx} = 1$ and $\sigma_{dr} = \sigma_{dz} = 0$. In this case the two-period equity claim is riskless, as (35) shows. Recall that returns of zero-coupon equity depend on dividend growth and the change in the price-dividend ratio (see (16)). If $\sigma_{dx} = 1$, positive dividend shocks are associated with positive price of risk shocks. In this special case, the effect on the price-dividend ratio cancels out the effect on the dividend growth rate, creating a perfectly hedged one-period return. This example illustrates a general property of the model. If dividend shocks are associated with positive price of risk shocks ($\sigma_{dx} > 0$), long-term equity tends to be less risky than short-term equity. On the other hand, if $\sigma_{dx} < 0$, the equity term structure tends to be upward sloping, which is inconsistent with the large value premium in the data.

While the correlation σ_{dx} does not enter the formulas for the risk premium and the yield of the two-period bond, it does for bonds of maturities greater than two periods. A negative correlation between interest rates and fundamentals implies that long-term bonds have positive risk premia. Because bond prices are determined by risk premia, it follows that changes in risk premia are another source of risk for these bonds. Holding all else equal, $\sigma_{dx} < 0$ leads to a term structure that is more upward sloping than otherwise. However, as explained above, $\sigma_{dx} < 0$ also leads to higher expected returns on long-term equities relative to short-term equities, the opposite of what cross-sectional asset pricing data suggest. The root of the problem is that duration operates for both bonds and equities; when shocks to discount rates are priced, risk premia on all long-term instruments are driven up relative to short-term instruments.

In the calibration that follows, we show that it is indeed possible to match both the upward slope of the term structure of interest rates and the downward slope of the term structure of equities in a model where the riskfree rate and the risk premium vary. Part of the answer lies in the role of expected dividend growth which appears in the equations for equities above and part of the answer lies in the role of expected inflation which influences risk premia on nominal bonds.

3 Implications for returns on stocks and bonds

To study our model's implications for returns on the aggregate market, on real and nominal bonds, and for portfolios sorted on scaled-price ratios, we simulate 100,000 quarters from the model. Given simulated data on shocks $\boldsymbol{\epsilon}_t$, and on expected dividend growth z_t, expected inflation q_t, the real riskfree rate r_t^f, and the price-of-risk variable x_t, we compute real prices of real bonds given (10), ratios of prices to the aggregate dividend for zero-coupon equity (18), and nominal prices of nominal bonds (27). We also compute a series for realized dividend growth (1) and realized inflation (3).

3.1 Calibration

The model specifies processes for dividends, inflation, the real riskfree rate and the price of risk. We calibrate the inflation parameters to data on inflation, dividend parameters to data on dividends and riskfree rate parameters to data on interest rates. The process for the price of risk and correlations between the price of risk process and the other variables is then determined jointly by the term structure of interest rates and equity prices. Tables 1, 2 and 3 give the calibrated values for the means and autocorrelations, the cross-correlations and the standard deviations respectively.

To calibrate the process for inflation, we use the maximum likelihood estimates of Wachter (2006). As Wachter shows, the likelihood function implied by (3) and (4) is the same as that for an ARMA(1,1) process. This is estimated on quarterly data from the second quarter of 1952 to the second quarter of 2004. The mean of expected inflation is 3.68% per annum, and expected inflation is found to have an annual autocorrelation of 0.78 (equivalent to a quarterly autocorrelation of 0.94). The volatility of expected inflation is 0.35% per annum, while the volatility of unexpected inflation is 1.18% per annum. The correlation between shocks to expected and unexpected inflation cannot be identified from inflation data alone. As in Wachter (2006), we set this correlation equal to 1. This has the benefit of reducing the parameter space (because it reduces the number of shocks by one, and therefore eliminates 5 correlations), and it does not appear to reduce the model's ability to fit the data.

Following Lettau and Wachter (2007), dividend growth is calibrated based on an annual data set of Campbell (2003) that begins in 1890; we update it to 2004 using data from CRSP. The instability of many moments of the aggregate market in recent years makes a long data set especially desirable. Average log dividend growth is set to 1.29%, the average for real growth in log dividends over that period. We assume the volatility of realized dividend growth is equal to 10% a value that falls between estimates in the long data ($\sim$ 14%), and in the post-war sample ($\sim$ 6%). Finally, the standard deviation of z_t is set to be 0.32% per annum. The lack of predictability in dividend growth suggests a standard deviation for z_t that is small relative to the standard deviation of realized dividend growth. As discussed further below, this value implies a reasonable amount of dividend growth predictability (see Table 6). The autocorrelation for z_t and the correlation between shocks to z_t and shocks to dividends is calibrated in the same way as in Lettau and Wachter (2007); namely the consumption-dividend ratio is used as an empirical proxy for z_t. Lettau and Ludvigson (2005) show that if consumption follows a random walk and if the consumption-dividend ratio is stationary, the consumption-dividend ratio captures the predictable component of dividend growth. The consumption-dividend ratio can therefore be identified with z_t up to an additive and multiplicative constant. We therefore take the autocorrelation of z_t to be 0.90, the autocorrelation of the consumption-dividend ratio over the 1890–2004 period. We take the correlation between shocks to z_t and shocks to Δd_t to be -0.83, equal to the correlation between these shocks over the 1890–2004 period.

Data on nominal interest rates are taken from CRSP. The yield on the 90-day Treasury bill represents the short-term nominal yield. Yields of maturities from 1 to 5 years are taken from Fama–Bliss data, which begin in 1952. We choose the mean of the real riskfree rate in order to match the sample mean of the short-term nominal yield over the 1952–2004 period. Our procedure is as follows. From (31) and (A.31) it follows that the mean of the one-period nominal yield is given by

$$Ey_{1t}^{\$} = \bar{r}^f + \bar{q} - \frac{1}{2}\sigma_\pi^2 - \sigma_{d\pi}\bar{x}.$$

Namely, the expected short-term nominal yield is the sum of the real riskfree rate, expected inflation, the negative of one-half times the volatility of realized inflation (a Jensen's inequality adjustment), and an inflation risk premium. The sample mean on the 3-month bill is 1.31% (5.23% per annum). The terms $\bar{q}$ and $\frac{1}{2}\sigma_\pi^2$ are known from the inflation calibration; subtracting the former and adding the latter to 1.31% implies a (quarterly) value of 0.39%. Based on this value for $\bar{r}^f$, we then calibrate $\sigma_{d\pi}$ and $\bar{x}$ as described below, and adjust $\bar{r}^f$ for the inflation risk premium (which turns out to equal 0.15% per quarter). Because the moments of the aggregate market are relatively insensitive to the precise value of $\bar{r}^f$, it is not necessary to repeat this process more than once to obtain the correct value of the nominal yield.[4]

Choosing the autocorrelation and the volatility of the riskfree rate is less straightforward than choosing the level because these parameters are less tightly linked to their counterparts in nominal interest rate data (for example, the volatility of nominal interest rates in the model depends, in a nonlinear way, on the volatilities and autocorrelations of the real riskfree rate, the price of risk and expected and realized inflation). We first choose a set of values to give reasonable implications for the autocorrelation and volatility of nominal interest rates, given a process for x_t. We then

[4]The difference between the simulated value of 5.15% and the mean of 5.23% is due to simulation noise.

re-calibrate the process for x_t based on the new values for r_t^f, and repeat as necessary. Given that the autocorrelation of inflation is 0.78 (in annual terms), the autocorrelation of the real riskfree rate must be higher in order to match the autocorrelations of nominal yields, which are above this value. The autocorrelations and volatilities in the model and in the data are shown in Table 7. An autocorrelation of 0.92 for the riskfree rate results in an autocorrelation for the three-month bond that is somewhat higher than in the data , but that matches the autocorrelations for longer term bonds exactly. Choosing the volatility of the riskfree rate to be 0.19% per annum results in a good fit to volatilities across the maturity spectrum. The volatility of the three-month yield is 2.89% in the model versus 2.93% in the data, while the volatility of the 5-year yield is 2.67% in the model versus 2.72% in the data. The model is therefore able to capture the fact that interest rate volatilities decrease in the maturity of the bond.

The parameters of the process for x_t are chosen to fit moments of stock returns. Like the volatility and persistence of the real riskfree rate, these values are chosen numerically; there is no analytical formula that links these parameters to population moments implies by the model. The average price of risk, $\bar{x}\sigma_d$ is chosen to be 0.85; this generates an average maximal quarter Sharpe ratio of $\sqrt{\exp(0.85^2) - 1} = 1.03$. As shown in Table 10, a high value of $\bar{x}$ allows us to come close to the high Sharpe ratio of the extreme value portfolio (0.58 in the model versus 0.63 in the data). As shown in Table 4, the Sharpe ratio on the market is slightly higher than in the long annual data set (0.40 in the model versus 0.36 in the data). Note that while the extreme value portfolio has the highest Sharpe ratio of the ten portfolios in our cross-sectional calibration, it does not achieve the maximal Sharpe ratio. In order to achieve the maximal Sharpe ratio, its return would need to be perfectly correlated with the dividend shock. However, because some of its payoffs occur in future periods, its return depends, to some degree, on expected dividend growth, real interest rates and the price of risk.[5]

[5]In Lettau and Wachter (2007), we choose a lower value for $\bar{x}\sigma_d$, 0.625. Resetting $\bar{x}$ to this value in the present model implies lower Sharpe ratios and risk premia. Specifically, the Sharpe ratio for the market portfolio is 0.25 and the Sharpe ratio for the extreme value portfolio is 0.35. The term structure has a flatter slope (the difference between the average 5-year and 3-month yield is 1.36%). Because discount rates are lower, the average price-dividend ratio is higher and equal to 35.3. The qualitative implications of the model are unchanged. These results differ from those of Lettau and Wachter (2007) in large part because of the presence of a time-varying real interest rate. This introduces a source of risk which is priced to a lesser extent than dividend risk.

To match the high volatility and predictability of stock returns given the low volatility of fundamentals as described above, the volatility of the price of risk variable must be high. We choose the volatility of shocks to $\sigma_d x_t$ to be 40, implying a volatility of the price-dividend ratio of 0.36 (see Table 4), close to its value of 0.40 in the data. We choose the persistence of x_t to equal 0.85; this implies a persistence of 0.86 for the price-dividend ratio, close to its value of 0.87 in the data. As Table 5 shows, long-horizon stock returns are slightly more predictable than in the data: the regression of the stock returns on the price-dividend ratio has an R^2 of 45% at the one-year horizon, compared with 25% in the 1890-2004 sample and 38% in the 1890-1994 sample. Raising the volatility or the persistence of x_t to match the data counterparts exactly would increase the amount of predictability.

Risk premia in the model are determined by correlations with realized dividends d_t. The correlation between expected inflation q_t and realized dividend growth determines the premium for nominal over real bonds. A value of -0.30 implies that nominal bonds will carry a premium over real bonds, and moreover, that this premium increases in maturity. (because realized inflation and expected inflation are perfectly correlated, realized inflation also must have a correlation of -0.30 with d_t). The correlation between the real riskfree rate and expected dividend growth is also -0.30. This implies an upward sloping real term structure. As explained in detail in Section 3.4, these values represent a compromise between fitting the upward slope of the yield curve and the deviations from the expectations hypothesis. The more negative these correlations are, the greater the slope of the yield curve, and the greater the deviation from the expectations hypothesis. The correlation with expected inflation mainly effects the behavior of short-term yields, while the correlation with the real interest rate mainly effects the behavior of long-term yields. Finally, as in Lettau and Wachter (2007), the correlation between x_t and dividend growth is set to be zero. The implication of this parameter choice is discussed further in Section 3.5.

For simplicity, we assume that most remaining correlations are equal to zero.[6] Exceptions are the correlation between realized and expected inflation (as discussed above) and between expected dividend growth and the price of risk. We allow this latter correlation to be positive based on direct evidence in Lettau and Ludvigson (2005) that expected dividend growth is positively correlated

[6]Correlations between variables not including d_t do not directly impact risk premia and thus have modest implications for the return moments that are the focus of this paper.

with the risk premium on stocks. Indeed, Table 6 shows that at long horizons, the price-dividend ratio predicts dividend growth with a negative sign (though the effect is insignificant). This counter-intuitive result supports the notion that expected dividend growth and discount rates move in the same direction, and that the discount rate effect is stronger than the cash-flow effect. We set the correlation between x_t and z to be 0.35; this reduces the predictability of dividend growth to nearly zero at long horizons despite the persistence of expected dividend growth. Raising the correlation further results in a variance-covariance matrix for which the Cholesky decomposition fails to exist.[7]

In this calibration we have set a number of interaction terms equal to zero. Richer models that are used to estimate the term structure allow arbitrary cross-correlations of shocks and interactions through conditional means. Results from term structure studies (e.g. Dai and Singleton (2003) and Duffee (2002)) suggest that such interactions may be important for fully capturing the dynamics of the term structure of interest rates. Appendix A calculates prices under a more general model that allows for such interactions. Empirically, however, it is not clear how to cleanly identify these parameters with our macro-based approach. Moreover, our simpler model has the advantage that it is easier to interpret. While our model may miss some of the term structure properties captured by the more complex models, it nonetheless seems appropriate for our current purpose.

3.2 Prices and returns as functions of the state variables

Figure 1 shows the factor loadings on each state variable for prices of real bonds, nominal bonds and equity as functions of maturity. As discussed in Section 2, and shown in this figure, the factor loadings on the riskfree rate are negative. An increase in the real riskfree rate decreases prices of all assets. The factor loading on expected inflation is negative for nominal bonds and zero otherwise: An increase in expected inflation decreases nominal bond prices, while leaving other prices unchanged. The factor loading on expected dividend growth is positive for equities and zero otherwise: An increase in expected dividend growth increases stock prices, while leaving other prices unchanged. The magnitude of all of these effects increase in maturity, and the assumption

[7] Intuitively, z_t and d_t are highly negatively correlated. This implies a relation between correlations of variables with z_t and correlations with variables and d_t. As explained below, d_t and x_t have a zero correlation, so the correlation between z_t and x_t cannot be too far from zero.

of AR(1) processes implies that the rate of increase declines exponentially.

Figure 1 also shows that the dynamic effects of changes in the price of risk are subtle and differ qualitatively from the effects of the other processes. For real bonds, B^r_{xn} is negative and decreasing in magnitude, like the coefficient on the riskfree rate. However, in contrast to that of B^r_{rn}, the rate of decrease of B^r_{xn} does not die out exponentially. The reason for this is the interaction between duration and increasing risk premia. At short maturities, the price of risk has little impact (as compared to the riskfree rate) because these assets have very small risk premia. At long maturities the price of risk has large impact (as compared to the riskfree rate) because these assets have large risk premia. Therefore, shocks to x_t have a greater effect at longer maturities than would be suggested by the size of the persistence ϕ_x. Similar comparisons hold for B^π_{xn}, the effect of the price of risk on nominal bonds.

For equities, the factor loading on x_t is not even monotonic. Over a range of zero to ten years, B^d_{xn} decreases in maturity. This is the duration effect: the longer the maturity the more sensitive the price is to changes in the risk premium. After ten years B^d_{xn} increases, and then asymptotes to a level that is lower than B^d_{x0}.[8] This increase is somewhat surprising because it appears to contradict the notion of duration: long-maturity equity should be more sensitive to changes in the risk premium than short-maturity equity. However, because shocks to expected dividend growth are negatively correlated with shocks to realized dividend growth, long-maturity equity acts as a hedge. This effect generates risk premia on long-maturity equity that are relatively low. Because long-maturity equity has lower risk premia, it is less sensitive to changes in x_t.

Figures 2 and 3 show zero-coupon yields as functions of maturities for real and nominal bonds respectively. The figures show yields at their long-run averages, and when the state variables are 2 standard deviations above or below their long-run averages. An increase in either the riskfree rate and the price of risk increases yields at all maturities. The riskfree rate and (in the case of nominal bonds) expected inflation have the greatest effect for short-term yields. In contrast, x_t has very little effect on short-term yields, and much greater effect on medium and long-term yields.

Figure 4 displays ratios of zero-coupon equity prices to aggregate dividends as a function of

[8] On the figure, B^d_{xn} has the appearance of asymptoting to the same level as B^r_{xn}; however B^d_{xn} remains lower than B^r_{xn} even in the limit.

maturity. The figure shows the ratios when state variables are at their long-run averages and when they are two standard deviations above or below these averages. When state variables are equal to their average values, the ratio of price to dividends is decreasing in maturity; longer-maturity equity is worth less, on average, than shorter-maturity equity because the effect of the discount rate dominates the effect of dividend growth.[9] Prices are increasing functions of expected dividend growth z_t, decreasing functions of the real interest rate r_t^f and decreasing functions of x_t. This figure shows that, under our calibration, most of the variation of prices at all maturities comes from variations in risk premia as represented by x_t.

3.3 The aggregate market

Table 4 shows statistics for the aggregate market in simulated and in historical data. In simulated data, we calculate quarterly returns and compound to an annual frequency. We create an annual price-dividend ratio in simulated data by dividing the price by the sum of dividends paid over the previous year.

As Table 4 shows, the volatility and the autocorrelation of the price-dividend ratio are close to those found in the data. This is not a surprise as model parameters were chosen in part to produce reasonable values for these moments. The model implies an average price-dividend ratio of 18.2, while the mean in the data is 25.9. Matching this statistic is a common difficulty for models of this type: Campbell and Cochrane (1999), for example, find an average price-dividend ratio of 18.2. As they explain, this statistic is poorly measured due to the persistence of the price-dividend ratio. The present model fits the volatility of equity returns (20.0% in the model versus 19.4% in the data), though it produces an equity premium that is slightly higher than that in the data (8.1% in the model versus 6.5% in the data). Like the mean of the price-dividend ratio, this number is estimated with substantial noise. The annual autocorrelation of returns is near zero for both model and data.

Table 5 reports results of long-horizon regressions of continuously-compounded excess returns on the log price-dividend ratio. Horizons range from 1 to 10 years. Panel A reports results from simulated data, Panel B reports results from the full data set (1890–2004) and Panel C reports

[9]Any calibration for which the market portfolio is well-defined must have this effect operating for sufficiently high maturities.

results from the 1890–1994 subsample. As found in many previous studies (e.g. Campbell and Shiller (1988), Cochrane (1992), Fama and French (1989) and Keim and Stambaugh (1986)), the price-dividend ratio predicts returns with a negative sign, and this predictability is significant across all horizons at the 5% level. Because so much of the variance in returns is driven by the price of risk x_t, the model generates substantial return predictability. In fact, the R^2 are higher, and the coefficients more negative for the model as compared with the full sample. The model results more closely resemble the 1890–1994 subsample.

Table 6 reports the results of long-horizon regressions of dividend growth on the price-dividend ratio. Evidence indicates that asset prices have little ability to predict dividend growth (Ang and Bekaert (2007), Cochrane (2008), Lettau and Ludvigson (2005) and Lettau and Van Nieuwerburgh (2008)). We replicate this result in our sample as shown in Panel B. In the model, the price-dividend ratio also fails to predict dividend growth; R^2 values from long-horizon regressions do not exceed 2%. This is both because the variation in expected dividend growth is relatively low, and because expected dividend growth is positively correlated with the price of risk. Thus risk premia and expected dividend growth are negatively correlated, leading to less dividend growth predictability than what one would expect given the present-value nature of this model.

3.4 The term structure of interest rates

Means and volatilities of yields

Table 7 shows the implications of the model for means, standard deviations, and annual autocorrelations of nominal and real bond yields. Data moments for bond yields using the CRSP Fama-Bliss data set are provided for comparison. These data are available starting in June of 1952, and are monthly. For the three-month yield, we use the bid yield on the 90-day Treasury bond, also available from CRSP.

Panel A shows that the real yield curve is upward sloping. This occurs because of the negative correlation between the real riskfree rate and fundamentals. Because bond prices fall when the real riskfree rate rises, bond prices fall when growth in fundamentals are low. Therefore long-term real bonds carry a risk premium over short-term real bonds, a risk premium that is reflected in the yield spread.

The negative correlation between the real riskfree rate and fundamentals also drives the nominal term spread. In the case of nominal bonds, there is an additional effect arising from the negative correlation between fundamentals and expected inflation. This negative correlation implies that nominal bond prices fall when fundamentals are low, leading to a positive inflation risk premium (this effect also operates in the models of Piazzesi and Schneider (2006), Wachter (2006)). The model's implications are consistent with empirical evidence that yields on indexed Treasury bonds are increasing in maturity, but that this slope is less than for nominal bonds (Roll (2004)).

The model implies volatilities for nominal bonds that are close to those in the data across all maturities. Volatilities are decreasing in maturity, as in the data. This decrease follows from the stationary autoregressive nature of the underlying processes. The table also shows annual autocorrelations (in the data, these are calibrated based on overlapping monthly observations). The autocorrelations are also similar, though the pattern is flatter in the model (0.85 at the short end, 0.87 at the long end) than in the data (0.80 at the short end and 0.87 at the long end).

Campbell and Shiller (1991) regressions

Table 8 shows the outcome of regressions

$$y^r_{n-h,t+h} - y^r_{nt} = \alpha_n + \beta_n \frac{1}{n-h} \left(y^r_{nt} - y^r_{1t}\right) + e^r_{t+h},$$

for real bonds and

$$y^\$_{n-h,t+h} - y^\$_{nt} = \alpha_n + \beta_n \frac{1}{n-h} \left(y^\$_{nt} - y^\$_{1t}\right) + e^\pi_{t+h}$$

for nominal bonds in simulated and historical data. To compute the values in the table, we take $h = 4$, corresponding to an annual frequency. These "long-rate" regressions are performed by Campbell and Shiller (1991) to test the hypothesis of constant risk premia on bonds.

The relation between risk premia and these regressions can be uncovered using the definition of yields and returns. To simplify the algebra, assume the horizon $h = 1$. For real bonds:

$$r^r_{n,t+1} = y^r_{nt} - (n-1)\left(y^r_{n-1,t+1} - y^r_{nt}\right).$$

Rearranging and taking expectations implies:

$$E_t\left[y^r_{n-1,t+1} - y^r_{nt}\right] = \frac{1}{n-1}\left(y^r_{nt} - y^r_{1t}\right) - \frac{1}{n-1} E_t\left[r^r_{n,t+1} - y^r_{1t}\right]. \tag{36}$$

For nominal bonds, the analogous equation is

$$E_t\left[y^{\$}_{n-1,t+1} - y^{\$}_{nt}\right] = \frac{1}{n-1}\left(y^{\$}_{nt} - y^{\$}_{1t}\right) - \frac{1}{n-1}E_t\left[r^{\$}_{n,t+1} - y^{\$}_{1t}\right]. \tag{37}$$

Thus the coefficient of a regression of changes in yields on the scaled yield spread produces a coefficient of one only if risk premia on bonds are constant. As found by Campbell and Shiller, the data coefficients are not only less than one, they are negative, indicating risk premia on bonds that strongly vary over time.

As Table 8 shows, the model also implies a significant departure from the expectations hypothesis. Coefficients β_n are negative for all maturities. However, the failure of the expectations hypothesis is not as extreme in the model as in the data. This reflects a general limitation of models driven by a single homoscedastic factor. Indeed, Dai and Singleton (2002) find, within the affine class, only a model with three factors driving the price of risk is capable of fully matching the failure of the expectations hypothesis.[10]

Using the model, it is possible to write the coefficients β_n in terms of more fundamental quantities. This sheds light on the aspects of the model that lead to the failure of the expectations hypothesis, as well as tensions inherent in the model. For real bonds, by definition

$$\beta_n = \frac{\mathrm{Cov}(y^r_{n-1,t+1} - y^r_n, y^r_{nt} - y^r_{1t})}{\mathrm{Var}(y^r_{n-1,t+1} - y^r_{1t})}(n-1).$$

Substituting in for changes in yields from (36), and noting that time-$(t+1)$ shocks have zero correlation with time-t yields, we have

$$\begin{aligned}\beta_n &= \frac{\mathrm{Cov}(y^r_{nt} - y^r_{1t} - E_t\left[r^r_{n,t+1} - y^r_{1t}\right], y^r_{nt} - y^r_{1t})}{\mathrm{Var}(y^r_{n-1,t+1} - y^r_{1t})} \\ &= 1 - \mathrm{Cov}(r^r_{n,t+1}, \Delta d_{t+1})\frac{\mathrm{Cov}(x_t, y^r_{nt} - y^r_{1t})}{\mathrm{Var}(y^r_{n-1,t+1} - y^r_{1t})},\end{aligned} \tag{38}$$

where the second line follows from (14). If x_t were constant, then the covariance term in this expression would be zero and $\beta_n = 1$, its value implied by the expectations hypothesis. The deviation from the expectations hypothesis depends on two quantities. The first is $\mathrm{Cov}(r^r_{n,t+1}, \Delta d_{t+1})$, the covariance between bond returns and fundamentals. This determines the average risk premium

[10] In contrast, a single-factor model that allows for significant heteroscedasticity in the state variable can successfully match these data (Wachter (2006)). It is also possible that part of the deviation in the data is reflective of a peso problem (Bekaert, Hodrick, and Marshall (2001)) that is not captured by the model.

on the bond as indicated by (14). The greater are risk premia on bonds, the greater the deviation from the expectations hypothesis. The second term is the coefficient from a regression of x_t on the yield spread. The more risk premia covary with yield spreads, then, the greater the deviation from the expectations hypothesis. Similar reasoning leads to the formula for nominal bonds:

$$\beta_n \;=\; 1 - \mathrm{Cov}_t(r^{\$}_{n,t+1}, \Delta d_{t+1}) \frac{\mathrm{Cov}(x_t, y^{\$}_{nt} - y^{\$}_{1t})}{\mathrm{Var}(y^{\$}_{n-1,t+1} - y^{\$}_{1t})}. \tag{39}$$

Figure 5 displays $\mathrm{Cov}_t(r_{n,t+1}, \Delta d_{t+1})$, $\mathrm{Cov}(x_t, y_{nt} - y_{1t})/\mathrm{Var}(y_{n-1,t+1} - y_{1t})$, and β_n for real and nominal bonds.[11] As Panel A shows, $\mathrm{Cov}_t(r_{n,t+1}, \Delta d_{t+1})$ increases in maturity, reflecting the fact that risk premia increase in maturity and that the term spread is upward-sloping. Risk premia are greater for nominal bonds then for real bonds, and increase faster in the maturity. Despite this, as shown in Panel C, the model implies a greater deviation from the expectations hypothesis for real bonds than for nominal bonds. Moreover, the model predicts coefficients that are roughly constant in maturity over the range of zero to 5 years, while risk premia are upward sloping. The reason is that the upward slope for risk premia is canceled out by a downward slope in $\mathrm{Cov}(x_t, y_{nt} - y_{1t})/\mathrm{Var}(y_{n-1,t+1} - y_{1t})$, which results from the mean-reverting nature of x_t. Moreover, nominal bonds, whose yields are driven by expected inflation as well as by discount rates, have lower values of $\mathrm{Cov}(x_t, y_{nt} - y_{1t})/\mathrm{Var}(y_{n-1,t+1} - y_{1t})$. This explains why the model produces a less dramatic failure of the expectations hypothesis for nominal bonds, despite their higher risk premia.

Cochrane and Piazzesi (2005) regressions

Finally, we ask whether the model can explain the findings of Cochrane and Piazzesi (2005). Cochrane and Piazzesi regress annual excess bond returns on a linear combination of forward rates, where the forward rate for loans between periods $t+n$ and $t+n+h$ is defined as the difference between the log price of the nominal bond maturing in $n-h$ periods and the the log price of the nominal bond maturing in n periods:

$$f^{\$}_{nt} = \log(P^{\pi}_{n-h,t}\Pi_t) - \log(P^{\pi}_{nt}\Pi_t) = \log P^{\pi}_{n-h,t} - \log P^{\pi}_{nt}.$$

[11]While (36) and (37) can be interpreted at any frequency and are run at an annual frequency in the data and the model for Table 8, (38) and (39) require that the frequency be the same as the frequency at which the model is simulated, namely, quarterly. The implied differences for the coefficients β_n are very slight.

In what follows, we take $h = 4$ so that the forward rate is annual. We refer to n as the forward rate maturity. Cochrane and Piazzesi show that the regression coefficients on the forward rates form a tent-shape pattern as a function of maturity (see also Stambaugh (1988)). Moreover, they show that a single linear combination of forward rates has substantial predictive power for bond returns across maturities.

These results offer support for our model's assumptions in that they imply that a single predictive factor drives much of the predictability in bond returns. In our model, that factor is represented by the latent variable x_t. Forward rates, like bond prices, are linear combinations of factors; therefore some linear combination of forward rates will uncover x_t. The model therefore predicts that some linear combination of forward rates will be the best predictor of bond returns, and that the regression coefficients for bonds of various maturities should be the same up to a constant of proportionality (because the true premia are all proportional to x_t).

We replicate the Cochrane and Piazzesi (2005) analysis in our simulated data. We report results for forward rates with $n = 1, 3$ and 5 years but the results are robust to alternative choices. Figure 6 shows the regression coefficients as a function of the forward rate maturity. As this figure shows, the model reproduces the tent shape in regression coefficients.[12] Table 9 reports R^2-statistics in the model and in the data. From monthly Fama–Bliss data (beginning in 1952 and ending in 2004), we construct overlapping annual observations. The R^2-statistics in the model are smaller than those in the data (16% versus 24% for the 5-year bond), but still economically significant.[13]

Given the 3-factor affine structure of the model, it is straightforward to solve for the linear combination of any three forward rates that is proportional to the price of risk x_t. Appendix D gives an analytical formula for these regressions coefficients, and shows that they must either form a tent or a "V"-shape. For example, if we use the 1, 3 and 5-quarter forward rates, and assume that the horizon for forward rates is one quarter, the linear combination

$$-\phi_q^2\phi_r^2 f_{1t} + (\phi_q^2 + \phi_r^2) f_{3t} - f_{5t} \tag{40}$$

[12]The regression coefficients are larger in magnitude than those shown in Cochrane and Piazzesi (2005); this occurs because the correlation between bond returns in our model is greater than that in the data.

[13]The differences between these R^2-statistics and those reported in Cochrane and Piazzesi (2005) are due to a difference in sample period; their sample begins in 1964 whereas ours begins in 1952. In Section 3.6, we report results for both periods.

equals x_t up to a constant of proportionality. The shape arises in part from the fact that forward rates are highly correlated. The coefficient on the first and the third forward rates must be the opposite sign of that on the middle forward rate in order to undo the effects of expected inflation q_t and the riskfree rate r_t^f. Because q_t and r_t^f enter into the equation for forward rates with the same sign at all maturities, undoing their effects requires that the coefficients reverse in sign.

Whether the shape is a tent or a "V" depends on the pattern of forward rate sensitivities to x_t. Simulation results suggest that a tent shape occurs as long as x_t is not extremely persistent (i.e. not more persistent than both r_t^f and q_t). The derivation in Appendix D give some insight into why this might be true. Intuitively, if x_t is extremely persistent, then the forward rate with the greatest maturity among the three regressors will also be relatively sensitive to x_t. In this case, a "V" shape results because the linear combination that exactly replicates x_t loads positively on the forward rate of greatest maturity. However, if the persistence of x_t lies between that of r_t^f and q_t, the middle forward rate will be relatively sensitive to x_t. In this case, a tent shape results because the linear combination that exactly replicates x_t loads positively on the middle forward rate. The preceding results in this section suggest that this is the most empirically relevant case because it is the case that also allows the model to capture facts about equity returns. Perhaps surprisingly, the model implies a tent-shape even if the persistence of x_t is below that of r_t^f and q_t. As discussed in Section 3.2, the response of the yield curve to a change in x_t depends on the pattern of bond risk premia, which in turn depend on the persistences of r_t^f and q_t. Therefore a change in x_t can have a large effect on intermediate-maturity bonds even if x_t is not very persistent itself. For this reason, the tent shape is typical of the model (in the sense that it holds for a variety of realistic calibrations), while the "V"-shape is the exception.

3.5 The cross-section of equities

This section shows the implications of the model for portfolios formed by sorting on price ratios. Following Lynch (2003) and Menzly, Santos, and Veronesi (2004), we exogenously specify a share process for cash flows on long-lived assets. For each year of simulated data, we sort these assets into deciles based on their price-dividend ratios and form portfolios of the assets within each decile. We then calculate returns over the following year. This follows the procedure used in empirical studies of the cross section (e.g. Fama and French (1992)). We then perform statistical analysis on the portfolio returns.

We specify our share process so that assets pay a nonzero dividend at each time (implying that the price-dividend ratio is well-defined), so that the total dividends sum up to the aggregate dividend of the market (so that the model is internally consistent), and so that the cross-sectional distribution of dividends, returns, and price ratios is stationary. The continuous-time framework of Menzly, Santos, and Veronesi (2004) allows the authors to specify the share process as stochastic, and yet keep shares between zero and one. This is more difficult in discrete time, and for this reason we adopt the simplifying assumption that the share process is deterministic. We assume the same process as in Lettau and Wachter (2007): shares grow at a constant rate of 5% per quarter for 100 quarters, and then shrink at the same rate for the next 100 quarters. Lettau and Wachter show that these parameters imply a cross-sectional distribution of dividend and earnings growth similar to that in the data.

More precisely, consider N sequences of dividend shares s_{it}, for $i = 1, \ldots, N$. For convenience, we refer to each of these N sequences as a firm, though they are best thought of as portfolios of firms in the same stage of the life cycle. As our ultimate goal is to aggregate these firms into portfolios based on price-dividend ratios, this simplification does not affect our results. Firm i pays s_{it} of the aggregate dividend at time t, $s_{i,t+1}$ of the aggregate dividend at time $t+1$, etc. Shares are such that $s_{it} \geq 0$ and $\sum_{i=1}^{N} s_{it} = 1$ for all t (so that the firms add up to the market). Because firm i pays a dividend sequence $s_{i,t+1}D_{t+1}, s_{i,t+2}D_{t+2}, \ldots$, no-arbitrage implies that the ex-dividend price of firm i equals

$$P_{it}^{F} = \sum_{n=1}^{\infty} s_{i,t+n} P_{nt}^{d}.$$

Let $\underline{s}$ be the lowest share of a firm in the economy, and assume without loss of generality that firm 1 starts at $\underline{s}$, namely $s_{11} = \underline{s}$. We assume that the share grows at a constant rate g_s until reaching $s_{1,N/2+1} = (1+g_s)^{N/2}\underline{s}$ and then shrinks at the rate g_s until reaching $s_{1,N+1} = \underline{s}$ again. At this point the cycle repeats. All firms are ex-ante identical, but are "out of phase" with one another: As firms move through the life-cycle, they slowly shift (on average) from the growth category to the value category, and then revert back to the growth category. Firm 1 starts out at $\underline{s}$, Firm 2 at $s_{21} = (1+g_s)\underline{s}$, Firm $N/2$ at $s_{N/2,1} = (1+g_s)^{N/2-1}\underline{s}$, and Firm N at $s_{N1} = (1+g_s)\underline{s}$. The variable $\underline{s}$ is such that the shares sum to one for all t.[14] We set the number of firms to 200, implying a

[14]That is, $\sum_{i=1}^{N} s_{it} = \underline{s} + (1+g_s)^{N/2}\underline{s} + 2\sum_{i=1}^{N/2-1}(1+g_s)^{i}\underline{s} = 1$.

200-quarter, or equivalently, 50-year life cycle for a firm. These share processes fully define the firms in the economy.

Panel A of Table 10 shows moments implied by the model. We compute the expected excess return, the volatility of the excess return and the Sharpe ratio. We also compute the abnormal return relative to the CAPM (α_i), and the coefficient on the market portfolio (β_i) from a time series regression of expected excess portfolio returns on expected excess market returns. Panel B shows counterparts from the data when portfolios are formed on the book-to-market ratio. Monthly data from 1952–2004 are from Ken French's website. Lettau and Wachter (2007) show that very similar results occur when portfolios are formed on earnings-to-price or cash-flow-to-price ratios.

Comparing the first line of Panel A with that of Panel B shows that the model matches most of the spread between expected returns on value and growth stocks. In both the model and the data, the expected excess return is about 6% per annum for the extreme growth portfolio. In the model, the extreme value portfolio has an expected excess return of 10%, compared with 11% in the data. Comparing the second line of Panel A with that of Panel B shows that, in the model, the risk of value stocks is lower than that of growth stocks, just as in the data. Sharpe ratios increase from about 0.3 for the extreme growth portfolio to about 0.6 for the extreme value portfolio.

More importantly, the model is able to match the value *puzzle*. Even though the model predicts that value stocks have high expected returns, value stocks in the model have lower CAPM βs than growth stocks. The CAPM α in the model is -2.5% per annum for the extreme growth portfolio and rises to 3.3% per annum for the extreme value portfolio. The corresponding numbers in the data are -1.7% per annum and 4.7% per annum.

These results for value and growth stocks may at first seem counter-intuitive, especially given the implications of the model for the term structure of interest rates. The term structure results in the previous section show that long-run assets require higher expected returns than short-run assets. The results in this section show that the opposite is true for equities. For equities, it is the short-run assets that require high expected returns.

The model resolves this tension between the downward sloping term structure of equities and the upward-sloping term structure of interest rates by the dividend process, the inflation process, and the price-of-risk process x_t. As implied by the data, expected dividend growth is negatively correlated with realized dividend growth. This makes growth stocks a hedge and reduces their risk premium relative to what would be the case if, say, expected inflation were constant. Moreover, expected inflation is negatively correlated with realized dividend growth. This makes long-term nominal bonds riskier than short-term nominal bonds and riskier than real bonds.

The prices of inflation and dividend risks are important for accounting for the combined behavior of equities and bonds. However, they are not sufficient. As the discussion in Section 2.3 indicates, characteristics of the price-of-risk process x_t are also crucial. Because equities carry a higher risk premium than bonds, they are more sensitive to changes in x_t in the sense that a greater proportion of their variance comes from x_t than from r_t^f as compared to both real and nominal bonds. In our specification, variation in the price of risk is itself unpriced. This implies variability in returns on growth stocks (on account of duration), but, at the same time, low expected returns because this variability comes in the form of risk that the representative investor does not mind bearing.

3.6 Interactions

We now examine the model's implications for interactions between the aggregate market, the term structure of interest rates and the cross-section of equities. We consider four state variables in the data and in the model: the price-dividend ratio, the yield spread, the linear combination of forward rates that best predicts bond returns and the value spread. We also consider three excess returns: the return on the market portfolio over the short-term bond, the return on the 5-year nominal bond over the short-term bond and the return on the value portfolio over the growth portfolio. We calculate cross-correlations of the four state variables, cross-correlations of the three excess returns, and predictive regressions of each excess return on each state variable.

Prior to discussing the details of our results, we briefly summarize our findings. While the model largely succeeds at capturing the ability of equity state variables to predict equity returns and bond state variables to predict bond returns, the model implies correlations between these two markets that are higher than what the data imply. This occurs no doubt because of the large role played by the price of risk x_t: x_t is an important driver of both bonds and equities in the model because of the substantial predictability present in both markets. Because the same price of risk determines risk premia for both bonds and equities (see Section 2), these assets are highly correlated in the model. The lack of correlation in the data presents a puzzle, at least from the point of view of a model in which risk premia are driven by a single state variable.

We construct the prices and return series using monthly data from 1952–2004 (because Fama–Bliss data on bond yields begin in June of 1952, this is the earliest starting point we consider for all of the series in this section). We also consider results for the 1964–2004 subperiod.[15] The price-dividend ratio in the data is constructed by dividing the price of the value-weighted CRSP index by the dividends paid over the previous year. The yield spread is the 5-year yield (from Fama-Bliss data) minus the 3-month yield (equal to the bid yield on the 90-day Treasury bond). Both yields are nominal and continuously-compounded. We create a forward-rate factor following the approach of Cochrane and Piazzesi (2005), namely, we compute the average excess holding period return on bonds of maturities ranging from 2 to 5 years and regress it on annual forward rates with maturities ranging from 1 to 5 years. The value spread is defined as in Cohen, Polk, and Vuolteenaho (2003). That is, we start with the six portfolios formed by first sorting firms into two portfolios by size and then into three portfolios by the book-to-market ratio (see Fama and French (1993)). The value portfolio then consists of the portfolio that equally weights the portfolio of large stocks with high book-to-market ratios and the portfolio of small stocks with high book-to-market ratios. The growth portfolio is likewise formed from the portfolio of large stocks with low book-to-market ratios and the portfolio of small stocks with low book-to-market ratios. The value spread is the difference between the log book-to-market ratio of the value portfolio and the log book-to-market ratio of the growth portfolio. Data on these portfolios are from Ken French's website.

The return on the value-weighted CRSP index represents the market return. We construct the return on the 5-year nominal bond using yields on the 4 and 5-year bonds from Fama-Bliss data. To form excess returns, we subtract the return on the 90-day Treasury bill. We construct the value minus growth return using returns on the value portfolio and the growth portfolio as defined in the previous paragraph. All returns are continuously compounded, and we form overlapping annual (and five-year) observations from the monthly data.

We construct the price-dividend ratio and yield spread in the model as described in previous sections. We construct the linear combination of forward rates as in the data, except that (to

[15]Cochrane and Piazzesi (2005) emphasize the 1964-2004 sample because of concerns about the quality of data on bond yields prior to 1964.

avoid co-linearity) we use the 1, 3 and 5-year forward rates rather than all five forwards. The value spread is the dividend-price ratio on the extreme value portfolio minus the dividend-price ratio on the extreme growth portfolio. The market return and bond return were defined previously; we subtract from these returns the real return on the one-quarter nominal bond. The value minus growth return is formed using the return on the extreme value portfolio and the return on the extreme growth portfolio. All returns are real and continuously compounded. The model is simulated at a quarterly frequency. From these quarterly observations, we create an annual time series of state variables and annual returns.

Table 11 shows the cross-correlations between the state variables in the model and in data from 1952–2004 and from 1964–2004. The table shows that the price-dividend ratio and the yield spread are negatively correlated in the model (because increases in x_t positively impact the price-dividend ratio but negatively impact the yield spread), but slightly positively correlated in the data. The price-dividend ratio is also negatively correlated with the linear combination of forward rates (not surprisingly, because the linear combination of forward rates is perfectly correlated with x_t). This correlation is close to zero in the 1952–2004 sample and negative in the 1964–2004 sample (though not as negative as predicted by the model). The model correctly accounts for the strong positive correlation between the price-dividend ratio and the value spread. This positive correlation results from the fact that the market and the value spread both respond negatively to increases in x_t and r_t^f and positively to increases in z_t (note that growth firms are more sensitive than value firms to changes in these variables). The correlation between the value spread and the yield spread is small and negative in the model and small and positive in the data. The correlation between the value spread and the linear combination of forwards is negative in both the model and the data, though the model correlation is larger in magnitude (-0.32 versus -0.17 in the 1964–2004 sample). This correlation is driven by the fact that the value spread is negatively correlated with x_t.

Table 12 shows the cross-correlations between the three returns in the model and in data from 1952–2004 and 1964–2004. The excess return on the market and on bonds are positively correlated in the model and in the data, though the model correlation is higher than in the data (0.83 versus 0.23 in the 1964–2004 sample). This positive correlation occurs because both bond and stock returns are driven to a large extent by x_t. Likewise, the model predicts a negative correlation between bond returns and the value-minus-growth portfolio. However, the model correctly captures the moderately negative correlation between the value minus growth return and the market return (-0.44 in the model and -0.47 in the data). It may at first seem surprising that the model can match this negative correlation: after all, both the equity premium and the value premium depend on x_t with a positive sign. However, this correlation is determined to a large degree by unexpected, rather than expected returns. Positive shocks to x_t and r_t^f lead to negative market return shocks, while positive shocks to z_t lead to positive market return shocks. These factors also influence the value-minus-growth return, but with the opposite sign because they affect growth firms more than value firms. The correlation is not perfectly negative because of the role of shocks to realized dividends, which influence the market portfolio and the value-minus-growth portfolio in the same direction.

Tables 13–16 show the outcomes from predictive regressions of each state variable on the three returns. We consider return horizons of 1 and 5 years. Table 13 reports regressions of the three returns on the lagged price-dividend ratio. As discussed above, the price-dividend ratio predicts excess returns on the market in both the data and the model. However, the model implies that the price-dividend ratio should predict excess returns on bonds, a fact that does not hold up in the data. Finally, the price-dividend ratio predicts returns on the value-minus-growth strategy with a negative sign in the model, but fails to predict this return in the data.

Table 14 repeats the exercise for the yield spread. The model's predictions are in line with the data in that the yield spread is capable of predicting both market and bond excess returns in the model and in the data with the correct sign (however, the effect for bonds is insignificant at longer horizons). The model produces the correct sign for the value minus growth portfolio at the 1-year horizon, though the R^2-statistic is greater in the model than in the data. In the data (but not in the model), the sign of the relation reverses at the 5-year horizon. However, the effect is insignificant when the full sample is used and marginally significant for the 1964–2004 subsample. Table 15 reports results for the linear combination of forward rates. The results in this table are similar to those for the yield spread.[16]

Table 16 reports regressions of the returns on the value spread. The model correctly captures the sign and degree to which the value spread predicts the aggregate market return in the data.

[16]The size of the predictive coefficients are larger in the model than in the data because the linear combination of forward rates is smoother.

In both model and data, the value spread has little ability to predict bond returns. In the model, the value spread predicts the return on the value minus growth portfolio with a negative sign, though the effect is economically small (the R^2 is 7% at a five-year horizon). In the data however, the value spread predicts the value-minus-growth return with a positive sign.

How should we think about the wedge between model and data when it comes to the value spread's ability to predict the value-minus-growth return? One reason for the discrepancy may arise from the construction of the value spread in the data, a construction which favors small stocks. Other methods of constructing the value spread that weight large stocks more heavily do not have a statistically significant ability to predict the value minus growth return. Given that our results may be best interpreted as a theory for large stocks (given that even the value stocks in the model are large and well-diversified), it may be that this deviation in predictive ability is not a significant failing.

A closer look at the model also indicates that the sign of the relation may not be an intrinsic property of the model, but may rather depend on the precise definition of value and growth. The value spread is negatively correlated with x_t in our calibration because the growth portfolio is more sensitive to changes in x_t than is the value portfolio. This is why the value spread predicts the value-minus-growth return with a negative sign. However, one could construct a value spread in the model that is positively related to x_t. As shown in Figure 1, the effect of x_t on the price-dividend ratio reverses for long-maturity equity: medium-maturity equity loads more on x_t than does short-maturity, but long-maturity equity loads less on x_t than does medium-maturity equity. Our current construction of firms implies that the extreme growth firm consists primarily of medium-maturity equity, since long-maturity equity has a low value and receives little weight. A construction that put more weight on long-maturity equity could produce a different result.

To summarize, the model generates strong predictions for the cross-correlations of price-based variables and excess returns. In general, the model predicts greater positive and lower negative correlations than are found in the data because of the assumption that a single process drives risk premia. Some of the model's predictions are born out. In particular, the model correctly predicts the sign of the correlation between the price-dividend ratio and the value spread, and the excess return on the market and the value-minus-growth return. The model also correctly predicts that term structure variables (such as the yield spread and a linear combination of forward rates) can predict excess returns on both bonds and stocks. Other predictions of the model, such as the fact that the value spread predicts the value-minus-growth return with a negative sign are not in line with the data and point towards directions for future research.

4 Conclusion

This paper has shown that properties of the cross-section of returns, the aggregate market and the term structure of interest rates can all be understood within a single framework. We introduced a parsimonious model for the pricing kernel capable of accounting for the behavior of value and growth stocks, nominal bonds, and the aggregate market. At the root of the model are dividend, inflation, and interest rate processes calibrated to match their counterparts in the data. Time-varying preferences for risk, modeled using a first-order autoregressive process for the price of risk, capture the observed volatility in equity returns and bond yields, as well as time-varying risk premia in the equity and the bond market.

Our model highlights a challenge for any model that attempts to explain both bonds and the cross-section of equities. The upward-sloping yield curve for bonds indicates that investors require compensation in the form of a positive risk premium for holding high-duration assets. However, data on value and growth stocks imply the opposite: investors require compensation for holding value stocks, which are short-horizon equity. Our model addresses this tension by specifying a real riskfree rate that is negatively correlated with fundamentals and a price of risk shock that has zero correlation with fundamentals. We hope that future work will suggest microeconomic foundations for these specifications.

Appendix

A General model

Let $\boldsymbol{H}_t$ be an $m \times 1$ vector of state variables at time t and let $\boldsymbol{\epsilon}_{t+1}$ be an $(m+2) \times 1$ vector of independent standard normal shocks. We assume that the state variables evolve according to the vector autoregression

$$\boldsymbol{H}_{t+1} = \boldsymbol{\Theta}_0 + \boldsymbol{\Theta} \boldsymbol{H}_t + \boldsymbol{\sigma}_H \boldsymbol{\epsilon}_{t+1}, \tag{A.1}$$

where $\boldsymbol{\Theta}_0$ is $m \times 1$, $\boldsymbol{\Theta}$ is $m \times m$, and $\boldsymbol{\sigma}_H$ is $m \times (m+2)$. Assume that the aggregate dividend D_{t+1} follows the process (1) and that the price level Π_{t+1} follows the process (3). However, expected dividend growth, expected inflation, the riskfree rate and the price of risk will be general affine functions of the underlying state vector:

$$\begin{aligned} z_t &= \delta_0 + \boldsymbol{\delta}' \boldsymbol{H}_t \\ q_t &= \eta_0 + \boldsymbol{\eta}' \boldsymbol{H}_t \\ r^f_{t+1} &= \alpha_0 + \boldsymbol{\alpha}' \boldsymbol{H}_t \\ x_t &= \xi_0 + \boldsymbol{\xi}' \boldsymbol{H}_t, \end{aligned}$$

where δ_0, η_0, α_0 and ξ_0 are scalars and $\boldsymbol{\delta}$, $\boldsymbol{\eta}$, $\boldsymbol{\alpha}$ and $\boldsymbol{\xi}$ are $m \times 1$ vectors. Assume that the intertemporal marginal rate of substitution takes the form

$$M_{t+1} = \exp\left\{ -r^f_{t+1} - \frac{1}{2} \|\boldsymbol{\lambda}\|^2 x_t^2 - x_t \boldsymbol{\lambda}' \boldsymbol{\epsilon}_{t+1} \right\}. \tag{A.2}$$

The price of risk is therefore $x_t \boldsymbol{\lambda}$. In the main text, we impose the restriction $\boldsymbol{\lambda} = \boldsymbol{\sigma}_d'$.

We describe the solution method for the case of zero-coupon equity. Consider the recursion (17), and conjecture that the solution takes the form

$$\frac{P^d_{nt}}{D_t} = \exp\{A^d_n + \boldsymbol{B}^d_n \boldsymbol{H}_t\}, \tag{A.3}$$

where A^d_n is a scalar and $\boldsymbol{B}^d_n$ is $1 \times m$. Substituting (A.3) into (17) and expanding out the expectation implies

$$E_t \left[\exp\left\{ -\alpha_0 - \boldsymbol{\alpha}' \boldsymbol{H}_t - \frac{1}{2}(\xi_0 + \boldsymbol{\xi}' \boldsymbol{H}_t)^2 \|\boldsymbol{\lambda}\|^2 - (\xi_0 + \boldsymbol{\xi}' \boldsymbol{H}_t) \boldsymbol{\lambda}' \boldsymbol{\epsilon}_{t+1} + \delta_0 + \boldsymbol{\delta}' \boldsymbol{H}_t + \boldsymbol{\sigma}_d \boldsymbol{\epsilon}_{t+1} + A^d_{n-1} + \boldsymbol{B}^d_{n-1} \left(\boldsymbol{\Theta}_0 + \boldsymbol{\Theta} \boldsymbol{H}_t + \boldsymbol{\sigma}_H \boldsymbol{\epsilon}_{t+1} \right) \right\} \right] = \exp\left\{ A^d_n + \boldsymbol{B}^d_n \boldsymbol{H}_t \right\}.$$

It follows from properties of the lognormal distribution that

$$\exp\left\{ -\alpha_0 - \boldsymbol{\alpha}' \boldsymbol{H}_t - \frac{1}{2}(\xi_0 + \boldsymbol{\xi}' \boldsymbol{H}_t)^2 \|\boldsymbol{\lambda}\|^2 + \delta_0 + \boldsymbol{\delta}' \boldsymbol{H}_t + A^d_{n-1} + \boldsymbol{B}^d_{n-1} \left(\boldsymbol{\Theta}_0 + \boldsymbol{\Theta} \boldsymbol{H}_t \right) + \frac{1}{2} \left(\boldsymbol{\sigma}_d - (\xi_0 + \boldsymbol{\xi}' \boldsymbol{H}_t) \boldsymbol{\lambda}' + \boldsymbol{B}^d_{n-1} \boldsymbol{\sigma}_H \right) \left(\boldsymbol{\sigma}_d - (\xi_0 + \boldsymbol{\xi}' \boldsymbol{H}_t) \boldsymbol{\lambda}' + \boldsymbol{B}^d_{n-1} \boldsymbol{\sigma}_H \right)' \right\} = \exp\left\{ A^d_n + \boldsymbol{B}^d_n \boldsymbol{H}_t \right\}.$$

Matching coefficients implies:[17]

$$\begin{aligned}
\boldsymbol{B}_n^d &= -\boldsymbol{\alpha}' + \boldsymbol{\delta}' + \boldsymbol{B}_{n-1}^d \boldsymbol{\Theta} - (\boldsymbol{\sigma}_d + \boldsymbol{B}_{n-1}^d \boldsymbol{\sigma}_H)\boldsymbol{\lambda}\boldsymbol{\xi}' \qquad (A.4)\\
A_n^d &= -\alpha_0 + \delta_0 + A_{n-1}^d + \boldsymbol{B}_{n-1}^d \boldsymbol{\Theta}_0 - (\boldsymbol{\sigma}_d + \boldsymbol{B}_{n-1}^d \boldsymbol{\sigma}_H)\boldsymbol{\lambda}\xi_0 + \\
&\qquad \frac{1}{2}\boldsymbol{\sigma}_d \boldsymbol{\sigma}_d' + \boldsymbol{B}_{n-1}^d \boldsymbol{\sigma}_H \boldsymbol{\sigma}_d' + \frac{1}{2}\boldsymbol{B}_{n-1}^d \boldsymbol{\sigma}_H \boldsymbol{\sigma}_H' \boldsymbol{B}_{n-1}^{d\,\prime}, \qquad (A.5)
\end{aligned}$$

with $\boldsymbol{B}_0^d = \boldsymbol{0}_{1\times m}$ and $A_0^d = 0$. Note that the terms that are quadratic in $\boldsymbol{H}_t$ cancel.

Note that the recursion for real bonds (9) takes the same form as the recursion for equities (17), except that there is no dividend growth term. We can therefore apply (A.4) and (A.5), provided that we replace δ_0 with 0, $\boldsymbol{\delta}_1$ with $\boldsymbol{0}_{m\times 1}$ and $\boldsymbol{\sigma}_d$ with $\boldsymbol{0}_{1\times(m+2)}$. Therefore, real bond prices satisfy

$$P_{nt}^r = \exp\{A_n^r + \boldsymbol{B}_n^r \boldsymbol{H}_t\}, \qquad (A.6)$$

where A_n^r is a scalar and $\boldsymbol{B}_n^r$ is a $1 \times m$ vector satisfying

$$\begin{aligned}
\boldsymbol{B}_n^r &= -\boldsymbol{\alpha}' + \boldsymbol{B}_{n-1}^r \boldsymbol{\Theta} - \boldsymbol{B}_{n-1}^r \boldsymbol{\sigma}_H \boldsymbol{\lambda}\boldsymbol{\xi}' \qquad (A.7)\\
A_n^r &= -\alpha_0 + A_{n-1}^r + \boldsymbol{B}_{n-1}^r \boldsymbol{\Theta}_0 - \boldsymbol{B}_{n-1}^r \boldsymbol{\sigma}_H \boldsymbol{\lambda}\xi_0 + \frac{1}{2}\boldsymbol{B}_{n-1}^r \boldsymbol{\sigma}_H \boldsymbol{\sigma}_H' \boldsymbol{B}_{n-1}^{r\,\prime}, \qquad (A.8)
\end{aligned}$$

with $\boldsymbol{B}_0^r = \boldsymbol{0}_{1\times m}$ and $A_0^r = 0$.

To price nominal bonds, note that the recursion (26) takes the same form as the equity recursion (17), except that growth in dividends is replaced by the inverse of inflation. Therefore, we can again apply (A.4) and (A.5), provided that we replace δ_0 with $-\eta_0$, $\boldsymbol{\delta}$ with $-\boldsymbol{\eta}$ and $\boldsymbol{\sigma}_d$ with $-\boldsymbol{\sigma}_\pi$. Therefore, the nominal price of the nominal bond satisfies

$$P_{nt}^\pi \Pi_t = \exp\{A_n^\pi + \boldsymbol{B}_n^\pi \boldsymbol{H}_t\}, \qquad (A.9)$$

where A_n^π is a scalar and $\boldsymbol{B}_n^\pi$ is a $1 \times m$ vector satisfying

$$\begin{aligned}
\boldsymbol{B}_n^\pi &= -\boldsymbol{\alpha}' - \boldsymbol{\eta}' + \boldsymbol{B}_{n-1}^\pi \boldsymbol{\Theta} - (-\boldsymbol{\sigma}_\pi + \boldsymbol{B}_{n-1}^\pi \boldsymbol{\sigma}_H)\boldsymbol{\lambda}\boldsymbol{\xi}' \qquad (A.10)\\
A_n^\pi &= -\alpha_0 - \eta_0 + A_{n-1}^\pi + \boldsymbol{B}_{n-1}^\pi \boldsymbol{\Theta}_0 - (-\boldsymbol{\sigma}_\pi + \boldsymbol{B}_{n-1}^\pi \boldsymbol{\sigma}_H)\boldsymbol{\lambda}\xi_0 + \\
&\qquad \frac{1}{2}\boldsymbol{\sigma}_\pi \boldsymbol{\sigma}_\pi' - \boldsymbol{B}_{n-1}^\pi \boldsymbol{\sigma}_H \boldsymbol{\sigma}_\pi' + \frac{1}{2}\boldsymbol{B}_{n-1}^\pi \boldsymbol{\sigma}_H \boldsymbol{\sigma}_H' \boldsymbol{B}_{n-1}^{\pi\,\prime}, \qquad (A.11)
\end{aligned}$$

and $\boldsymbol{B}_0^\pi = \boldsymbol{0}_{1\times m}$ and $A_0^\pi = 0$.

B Convergence of the market price-dividend ratio in the general model

This Appendix derives conditions that guarantee the convergence of the price-dividend ratio, assuming the general model in Appendix A. The results can be specialized to the model in Section 2 using the

[17] Because $\boldsymbol{\xi}'\boldsymbol{H}_t$ and $\boldsymbol{\lambda}'(\boldsymbol{\sigma}_d + \boldsymbol{B}_{n-1}^d \boldsymbol{\sigma}_H)'$ are each scalars,

$$\begin{aligned}
\boldsymbol{\xi}'\boldsymbol{H}_t \boldsymbol{\lambda}'(\boldsymbol{\sigma}_d + \boldsymbol{B}_{n-1}^d \boldsymbol{\sigma}_H)' &= \boldsymbol{\lambda}'(\boldsymbol{\sigma}_d + \boldsymbol{B}_{n-1}^d \boldsymbol{\sigma}_H)'\boldsymbol{\xi}'\boldsymbol{H}_t\\
&= (\boldsymbol{\sigma}_d + \boldsymbol{B}_{n-1}^d \boldsymbol{\sigma}_H)\boldsymbol{\lambda}\boldsymbol{\xi}'\boldsymbol{H}_t.
\end{aligned}$$

definitions in Appendix C. Let $\boldsymbol{K}_1 = \boldsymbol{\Theta} - \boldsymbol{\sigma}_H \boldsymbol{\lambda}\boldsymbol{\xi}'$ and $\boldsymbol{K}_2 = -\boldsymbol{\alpha}' + \boldsymbol{\delta}' - \boldsymbol{\sigma}_d \boldsymbol{\lambda}\boldsymbol{\xi}'$. Then (A.4) can be rewritten as

$$\boldsymbol{B}_n^d = \boldsymbol{B}_{n-1}^d \boldsymbol{K}_1 + \boldsymbol{K}_2.$$

The limit of $\boldsymbol{B}_n^d$ as n goes to infinity is the fixed point of this equation. As long as the eigenvalues of $\boldsymbol{K}_1$ have absolute value less than 1, a fixed point exists (see Hamilton (1994, Chapter 10)). In this case $\boldsymbol{I}_m - \boldsymbol{K}_1$ is invertible, and the fixed point is

$$\overline{\boldsymbol{B}} = \boldsymbol{K}_2(\boldsymbol{I}_m - \boldsymbol{K}_1)^{-1}.$$

Now assume that the eigenvalues of K_1 have absolute value less than one. In the general case, the price-dividend ratio is given by (23), where P_{nt}^d/D_t takes the general form (A.3). Define

$$\bar{A} = -\alpha_0 + \delta_0 + \overline{\boldsymbol{B}}\boldsymbol{\Theta}_0 - (\boldsymbol{\sigma}_d + \overline{\boldsymbol{B}}\boldsymbol{\sigma}_H)\boldsymbol{\lambda}\xi_0 + \frac{1}{2}\boldsymbol{\sigma}_d\boldsymbol{\sigma}_d' + \overline{\boldsymbol{B}}\boldsymbol{\sigma}_H\boldsymbol{\sigma}_d' + \frac{1}{2}\overline{\boldsymbol{B}}\boldsymbol{\sigma}_H\boldsymbol{\sigma}_H'\overline{\boldsymbol{B}}',$$

It follows from (A.5) that for sufficiently large N,

$$A_n^d \approx \bar{A}n + \text{constant} \quad \text{for } n \geq N,$$

where the constant does not depend on n. Therefore

$$\sum_{n=N}^{L} \exp\left\{A_n^d + \boldsymbol{B}_n^d \boldsymbol{H}_t\right\} \approx \exp\left\{\text{constant} + \overline{\boldsymbol{B}}\boldsymbol{H}_t\right\} \sum_{n=N}^{L} \exp\left\{\bar{A}n\right\}.$$

As long as $\bar{A} < 0$, the right hand side approaches a finite limit for $L \to \infty$.

C Solution to the model in Section 2

The model in Section 2 can either be solved directly, or by applying the formulas in Appendix A under appropriate restrictions. The general model in Appendix A reduces to the model in Section 2 if

$$\boldsymbol{\delta} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \; \boldsymbol{\eta} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \; \boldsymbol{\alpha} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \; \boldsymbol{\xi} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \tag{A.12}$$

and if

$$\boldsymbol{\Theta} = \begin{bmatrix} \phi_z & & & \\ & \phi_q & & \\ & & \phi_r & \\ & & & \phi_x \end{bmatrix}, \quad \boldsymbol{\sigma}_H = \begin{bmatrix} \boldsymbol{\sigma}_z \\ \boldsymbol{\sigma}_q \\ \boldsymbol{\sigma}_r \\ \boldsymbol{\sigma}_x \end{bmatrix}, \tag{A.13}$$

where $\boldsymbol{\Theta}$ is a diagonal matrix. Further, set $\boldsymbol{\Theta}_0 = \boldsymbol{0}_{4\times 1}$ so that $g = \delta_0$, $\bar{q} = \eta_0$, $\bar{r}^f = \alpha_0$ and $\bar{x} = \xi_0$. Label the elements of the vectors $\boldsymbol{B}_n^r$, $\boldsymbol{B}_n^d$ and $\boldsymbol{B}_n^\pi$ as follows:

$$\begin{aligned} \boldsymbol{B}_n^r &= [B_{zn}^r, B_{qn}^r, B_{rn}^r, B_{xn}^r] \\ \boldsymbol{B}_n^d &= [B_{zn}^d, B_{qn}^d, B_{rn}^d, B_{xn}^d] \\ \boldsymbol{B}_n^\pi &= [B_{zn}^\pi, B_{qn}^\pi, B_{rn}^\pi, B_{xn}^\pi]. \end{aligned} \tag{A.14}$$

We continue to assume that the price of risk is given by the general form (A.2); the formulas in Section 2 can be obtained by setting $\boldsymbol{\lambda} = \boldsymbol{\sigma}_d'$.

For real bonds, (A.7) and (A.8) imply that

$$\begin{aligned}
B^r_{zn} &= B^r_{z,n-1}\phi_z && \text{(A.15)}\\
B^r_{qn} &= B^r_{q,n-1}\phi_q && \text{(A.16)}\\
B^r_{rn} &= -1 + B^r_{r,n-1}\phi_r && \text{(A.17)}\\
B^r_{xn} &= B^r_{x,n-1}\phi_x - \boldsymbol{\sigma}^r_{(n)}\boldsymbol{\lambda} && \text{(A.18)}\\
A^r_n &= -\bar{r}^f + A^r_{n-1} - \boldsymbol{\sigma}^r_{(n)}\boldsymbol{\lambda}\bar{x} + \frac{1}{2}\|\boldsymbol{\sigma}^r_{(n)}\|^2, && \text{(A.19)}
\end{aligned}$$

where

$$\boldsymbol{\sigma}^r_{(n)} = B^r_{r,n-1}\boldsymbol{\sigma}_r + B^r_{q,n-1}\boldsymbol{\sigma}_q + B^r_{z,n-1}\boldsymbol{\sigma}_z + B^r_{x,n-1}\boldsymbol{\sigma}_x$$

is the vector of loadings on the shocks for the return on the n-period real bond. The boundary conditions are $B^r_{z0} = B^r_{q0} = B^r_{r0} = B^r_{x0} = A^r_0 = 0$. Equations (A.15) and (A.16) together with the boundary conditions imply that $B^r_{zn} = B^r_{qn} = 0$ for all n. The solution to (A.17) is given in the main text. The solution to (A.18) is

$$B^r_{xn} = \frac{\boldsymbol{\sigma}_r\boldsymbol{\lambda}}{1-\phi_r}\frac{1-\phi^n_\lambda}{1-\phi_\lambda} - \frac{\boldsymbol{\sigma}_r\boldsymbol{\lambda}}{1-\phi_r}\frac{\phi^n_r - \phi^n_\lambda}{\phi_r - \phi_\lambda}, \tag{A.20}$$

where $\phi_\lambda = \phi_x - \boldsymbol{\sigma}_x\boldsymbol{\lambda}$.

In the case of equities, (A.4) and (A.5) imply that

$$\begin{aligned}
B^d_{zn} &= 1 + B^d_{z,n-1}\phi_z && \text{(A.21)}\\
B^d_{qn} &= B^d_{q,n-1}\phi_q && \text{(A.22)}\\
B^d_{rn} &= -1 + B^d_{r,n-1}\phi_r && \text{(A.23)}\\
B^d_{xn} &= B^d_{x,n-1}\phi_x - \boldsymbol{\sigma}^d_{(n)}\boldsymbol{\lambda} && \text{(A.24)}\\
A^d_n &= -\bar{r}^f + g + A^d_{n-1} - \boldsymbol{\sigma}^d_{(n)}\boldsymbol{\lambda}\bar{x} + \frac{1}{2}\|\boldsymbol{\sigma}^d_{(n)}\|^2, && \text{(A.25)}
\end{aligned}$$

where

$$\boldsymbol{\sigma}^d_{(n)} = \boldsymbol{\sigma}_d + B^d_{r,n-1}\boldsymbol{\sigma}_r + B^d_{q,n-1}\boldsymbol{\sigma}_q + B^d_{z,n-1}\boldsymbol{\sigma}_z + B^d_{x,n-1}\boldsymbol{\sigma}_x$$

is the vector of loadings on the shocks for the return on n-period zero-coupon equity. The boundary conditions are $B^d_{z0} = B^d_{q0} = B^d_{r0} = B^d_{x0} = A^d_0 = 0$. Equation (A.22) together with the boundary condition implies that $B^d_{qn} = 0$ for all n. The solutions to (A.21) and (A.23) are given in the main text. The solution to (A.24) is

$$B^d_{xn} = \left(-\boldsymbol{\sigma}_d\boldsymbol{\lambda} + \frac{\boldsymbol{\sigma}_r\boldsymbol{\lambda}}{1-\phi_r} - \frac{\boldsymbol{\sigma}_z\boldsymbol{\lambda}}{1-\phi_z}\right)\frac{1-\phi^n_\lambda}{1-\phi_\lambda} - \frac{\boldsymbol{\sigma}_r\boldsymbol{\lambda}}{1-\phi_r}\frac{\phi^n_r - \phi^n_\lambda}{\phi_r - \phi_\lambda} + \frac{\boldsymbol{\sigma}_z\boldsymbol{\lambda}}{1-\phi_z}\frac{\phi^n_z - \phi^n_\lambda}{\phi_z - \phi_\lambda}. \tag{A.26}$$

In the case of nominal bonds, (A.10) and (A.11) imply that

$$\begin{aligned} B^{\pi}_{zn} &= B^{\pi}_{z,n-1}\phi_z && \text{(A.27)}\\ B^{\pi}_{qn} &= -1 + B^{\pi}_{q,n-1}\phi_q && \text{(A.28)}\\ B^{\pi}_{rn} &= -1 + B^{\pi}_{r,n-1}\phi_r && \text{(A.29)}\\ B^{\pi}_{xn} &= B^{\pi}_{x,n-1}\phi_x - \boldsymbol{\sigma}^{\pi}_{(n)}\boldsymbol{\lambda} && \text{(A.30)}\\ A^{\pi}_{n} &= -\bar{r}^f - \bar{q} + A^{\pi}_{n-1} - \boldsymbol{\sigma}^{\pi}_{(n)}\boldsymbol{\lambda}\bar{x} + \frac{1}{2}\|\boldsymbol{\sigma}^{\pi}_{(n)}\|^2, && \text{(A.31)} \end{aligned}$$

where

$$\boldsymbol{\sigma}^{\pi}_{(n)} = -\boldsymbol{\sigma}_{\pi} + B^{\pi}_{r,n-1}\boldsymbol{\sigma}_r + B^{\pi}_{q,n-1}\boldsymbol{\sigma}_q + B^{\pi}_{x,n-1}\boldsymbol{\sigma}_x$$

is the vector of loadings on the shocks for the return on the n-period nominal bond. The boundary conditions are $B^{\pi}_{z0} = B^{\pi}_{q0} = B^{\pi}_{r0} = B^{\pi}_{x0} = A^{\pi}_{0} = 0$. Equation (A.27) together with the boundary condition implies that $B^{\pi}_{zn} = 0$ for all n. The solutions to (A.28) and (A.29) are given in the main text. The solution to (A.30) is

$$B^{\pi}_{xn} = \left(\boldsymbol{\sigma}_{\pi}\boldsymbol{\lambda} + \frac{\boldsymbol{\sigma}_r\boldsymbol{\lambda}}{1-\phi_r} + \frac{\boldsymbol{\sigma}_q\boldsymbol{\lambda}}{1-\phi_q}\right)\frac{1-\phi_{\lambda}^n}{1-\phi_{\lambda}} - \frac{\boldsymbol{\sigma}_r\boldsymbol{\lambda}}{1-\phi_r}\frac{\phi_r^n - \phi_{\lambda}^n}{\phi_r - \phi_{\lambda}} - \frac{\boldsymbol{\sigma}_q\boldsymbol{\lambda}}{1-\phi_q}\frac{\phi_q^n - \phi_{\lambda}^n}{\phi_q - \phi_{\lambda}}. \tag{A.32}$$

D Cochrane-Piazzesi regressions

The forward rate for loans between periods $t+n$ and $t+n+h$ is given by the difference in log nominal prices of nominal bonds

$$f^{\$}_{nt} = \log P^{\pi}_{n-h,t}\Pi_t - \log P^{\pi}_{nt}\Pi_t.$$

Let

$$C_{qn} = B^{\pi}_{q,n-h} - B^{\pi}_{qn}$$

and likewise for C_{rn} and C_{xn}. It follows from the formula for nominal bond prices (27), that

$$f^{\$}_{nt} = C_{qn}q_t + C_{rn}r^f_{t+1} + C_{xn}x_t. \tag{A.33}$$

It follows from (28) that

$$C_{qn} = \phi_q^{n-h}\frac{1-\phi_q^h}{1-\phi_q}, \qquad C_{rn} = \phi_r^{n-h}\frac{1-\phi_r^h}{1-\phi_r}. \tag{A.34}$$

The formula for C_{xn} is more complicated, but can be calculated from (A.32). Equation A.33 can be written in matrix form as

$$\boldsymbol{C}\begin{bmatrix} q_t \\ r^f_{t+1} \\ x_t \end{bmatrix} = \boldsymbol{f}_t \tag{A.35}$$

where

$$\boldsymbol{C} = \begin{bmatrix} C_{qn_1} & C_{rn_1} & C_{xn_1} \\ C_{qn_2} & C_{rn_2} & C_{xn_2} \\ C_{qn_3} & C_{rn_3} & C_{xn_3} \end{bmatrix}, \quad \boldsymbol{f}_t = \begin{bmatrix} f^{\$}_{n_1t} \\ f^{\$}_{n_2t} \\ f^{\$}_{n_3t} \end{bmatrix},$$

for three forward rate maturities $n_3 > n_2 > n_1$.

We now solve for the linear combination of forward rates that is proportional to x_t. Accordingly, let $\boldsymbol{\theta}$ be a 3×1 vector such that $\boldsymbol{\theta}' \boldsymbol{f}_t = x_t$. It follows from (A.35) that

$$\begin{aligned}
\boldsymbol{\theta} &= [0\ 0\ 1]\, \boldsymbol{C}^{-1} \\
&= \frac{1}{|\boldsymbol{C}|}[C_{qn_2}C_{rn_3} - C_{qn_3}C_{rn_2},\ C_{rn_1}C_{qn_3} - C_{rn_3}C_{qn_1},\ C_{qn_1}C_{rn_2} - C_{qn_2}C_{rn_1}], \\
&= \frac{1}{|\boldsymbol{C}|}\frac{\phi_q^{-h}\phi_r^{-h}(1-\phi_q^h)(1-\phi_r^h)}{(1-\phi_q)(1-\phi_r)}[\phi_q^{n_2}\phi_r^{n_3} - \phi_q^{n_3}\phi_r^{n_2},\ \phi_q^{n_3}\phi_r^{n_1} - \phi_q^{n_1}\phi_r^{n_3},\ \phi_q^{n_1}\phi_r^{n_2} - \phi_q^{n_2}\phi_r^{n_1}] \quad \text{(A.36)}
\end{aligned}$$

where $|\boldsymbol{C}|$ denotes the determinant of $\boldsymbol{C}$. Assume $\phi_q \neq \phi_r$. Because $n_3 > n_2 > n_1$, it follows that the first and third element of $\boldsymbol{\theta}$ must take the opposite sign from the second element of θ. Therefore, $\boldsymbol{\theta}$ must either have a tent or "V"-shape.

Whether $\boldsymbol{\theta}$ takes the form of a tent or a "V" depends on the sign of the determinant $|\boldsymbol{C}|$. The formula for the determinant of a 3×3 matrix implies that $|\boldsymbol{C}|$ is equal to a positive constant times

$$C_{xn_1}(\phi_q^{n_2}\phi_r^{n_3} - \phi_q^{n_3}\phi_r^{n_2}) + C_{xn_2}(\phi_q^{n_3}\phi_r^{n_1} - \phi_q^{n_1}\phi_r^{n_3}) + C_{xn_3}(\phi_q^{n_1}\phi_r^{n_2} - \phi_q^{n_2}\phi_r^{n_1})$$

Consider the case of $\phi_r > \phi_q$ (which holds in our calibration). It follows from (A.36) that $\boldsymbol{\theta}$ has a tent shape if and only if $|\boldsymbol{C}|$ is negative. This will occur when C_{xn_2} is large relative to C_{xn_1} and C_{xn_3}, namely when the effect of x_t is largest at intermediate maturities. Simulation results show that this tends to occur when ϕ_x is less than ϕ_r. Long-maturity forward rates are then driven more by ϕ_r. Even if ϕ_x is less than ϕ_q, it turns out that short-maturity forward rates are driven more by ϕ_q, because the effect of a change in x_t tends to be determined by a combination of ϕ_x and the autocorrelation of the most persistent source of risk that is correlated with fundamentals.

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Table 1: State variable means and autocorrelations

State variable	Unconditional mean	Autocorrelation
Expected dividend growth z_t	1.29%	0.90
Expected inflation q_t	3.68%	0.78
Real riskfree rate r_t^f	0.96%	0.92
Price of risk $\sigma_d x_t$	0.85	0.85

Notes: Means of expected dividend growth, expected inflation and the riskfree rate are in annual terms (i.e. multiplied by 4). Autocorrelations for all state variables are in annual terms (i.e. raised to the 4th power). The model is simulated at a quarterly frequency.

Table 2: Conditional cross-correlations of shocks

Variable	$\Delta\pi_t$	z_t	q_t	r_{t+1}^f	x_t
Δd_t	-0.30	-0.83	-0.30	-0.30	0
$\Delta\pi_t$		0	1.00	0	0
z_t			0	0	0.35
q_t				0	0
r_{t+1}^f					0

Notes: The table reports conditional cross-correlations of shocks to dividend growth Δd_t, inflation $\Delta\pi_t$, expected dividend growth z_t, expected inflation q_t, the riskfree rate r_{t+1}^f and the price-of-risk variable x_t. The model is simulated at a quarterly frequency.

Table 3: Conditional standard deviations of shocks

Variable	Δd_t	$\Delta \pi_t$	z_t	q_t	r_t^f	$\sigma_d x_t$
Conditional standard deviation	10.00	1.18	0.32	0.35	0.19	40.00

Notes: The table reports conditional standard deviations of shocks in annual percentage terms (i.e. multiplied by 200) for dividend growth Δd_t, inflation $\Delta \pi_t$, expected dividend growth z_t, expected inflation q_t, the riskfree rate r_t^f and the scaled price-of-risk variable $\sigma_d x_t$. The model is simulated at a quarterly frequency.

Table 4: Aggregate price and return moments

	Model	Data
$E(P^m/D)$	18.16	25.95
$\sigma(p^m - d)$	0.36	0.40
AC of $p^m - d$	0.86	0.87
$E[R^m - R_1^\pi]$	8.09%	6.51%
$\sigma(R^m - R_1^\pi)$	20.02%	19.42%
AC of $R^m - R_1^\pi$	-0.05	0.01
Sharpe ratio of market	0.40	0.33

Notes: P^m/D refers to the price-dividend ratio of the aggregate market, where the aggregate dividend D equals the sum of dividends paid over the previous year. $p^m - d$ refers to $\log(P^m/D)$. $R^m - R_1^\pi$ refers to the annual return on the market portfolio in excess of the annual return on the short-term nominal bond, where both returns are measured in real terms. $\sigma(\cdot)$ refers to the standard deviation. AC refers to the annual autocorrelation. Data are annual from 1890–2004.

Table 5: Long-horizon regressions: Excess stock returns

	Horizon in years					
	1	2	4	6	8	10
	Panel A: Model					
β_1	-0.19	-0.34	-0.60	-0.80	-0.92	-1.01
R^2	[0.11]	[0.19]	[0.32]	[0.41]	[0.43]	[0.45]
	Panel B: Data from 1890–2004					
β_1	-0.10	-0.21	-0.35	-0.51	-0.72	-0.91
t-stat	(-2.17)	(-2.70)	(-2.35)	(-2.24)	(-2.66)	(-2.97)
R^2	[0.04]	[0.08]	[0.11]	[0.14]	[0.21]	[0.25]
	Panel C: Data from 1890–1994					
β_1	-0.19	-0.36	-0.55	-0.79	-1.02	-1.19
t-stat	(-3.44)	(-3.93)	(-3.10)	(-3.92)	(-5.37)	(-5.44)
R^2	[0.07]	[0.13]	[0.17]	[0.26]	[0.35]	[0.38]

Notes: Continuously-compounded excess returns on the market portfolio are regressed on the lagged log price-dividend ratio. Returns are measured over horizons ranging from 1 to 10 years and are in excess of the return on the short-term nominal bond, where both returns are measured in real terms. For each data regression, the table reports OLS estimates of the regressors, Newey-West (1987) corrected t-statistics (in parentheses), and R^2-statistics (in brackets). For each model regression, the table reports OLS estimates of the regressors and R^2-statistics. Data are annual.

Table 6: Long-horizon regressions: Aggregate dividend growth

	Horizon in years					
	1	2	4	6	8	10
Panel A: Model						
β_1	0.02	0.03	0.05	0.06	0.08	0.10
R^2	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.02]
Panel B: Data from 1890–2004						
β_1	0.02	0.04	-0.02	-0.02	-0.12	-0.24
t-stat	(0.43)	(0.57)	(-0.22)	(-0.11)	(-0.71)	(-1.23)
R^2	[0.00]	[0.01]	[0.00]	[0.00]	[0.01]	[0.04]
Panel C: Data from 1890–1994						
β_1	0.07	0.15	0.09	0.04	-0.13	-0.28
t-stat	(1.18)	(1.51)	(0.70)	(0.19)	(-0.64)	(-1.31)
R^2	[0.02]	[0.04]	[0.01]	[0.00]	[0.01]	[0.04]

Notes: Log growth rates of the aggregate dividend are regressed on the lagged log price-dividend ratio. Dividend growth is measured over horizons ranging from 1 to 10 years. For each data regression, the table reports OLS estimates of the regressors, Newey-West (1987) corrected t-statistics (in parentheses), and R^2-statistics (in brackets). For each model regression, the table reports OLS estimates of the regressors and R^2-statistics. Data are annual.

Table 7: Moments of zero-coupon bond yields

Maturity (years)	0.25	1	2	3	4	5
	Panel A: Real bonds					
Mean	0.91	1.05	1.23	1.40	1.56	1.71
Standard deviation	1.95	1.89	1.83	1.79	1.75	1.71
AC(1)	0.92	0.92	0.92	0.92	0.92	0.91
	Panel B: Nominal bonds					
Mean	5.15	5.53	5.98	6.38	6.73	7.04
Standard deviation	2.89	2.80	2.73	2.70	2.68	2.67
AC(1)	0.85	0.85	0.86	0.86	0.86	0.87
	Panel C: Data					
Mean	5.23	5.59	5.80	5.98	6.11	6.19
Standard deviation	2.93	2.93	2.87	2.80	2.76	2.72
AC(1)	0.80	0.82	0.84	0.85	0.86	0.87

Notes: Each panel displays means, standard deviations, and 1-year autocorrelations of bond yields. Yields are in annual percentage terms. Panel A displays moments of real yields in the model, Panel B displays moments of nominal yields in the model and the Panel C displays moments of nominal yields in monthly data from 1952–2004.

Table 8: Long-rate regressions on bond yields

Maturity (years)	2	3	4	5
	Panel A: Real bonds			
β_n	-0.64	-0.67	-0.68	-0.70
R^2	[0.02]	[0.02]	[0.02]	[0.02]
	Panel B: Nominal bonds			
β_n	-0.60	-0.59	-0.59	-0.59
R^2	[0.02]	[0.02]	[0.02]	[0.01]
	Panel C: Data			
β_n	-0.76	-1.11	-1.50	-1.48
t-stat	(-1.66)	(-2.02)	(-2.42)	(-2.13)
R^2	[0.03]	[0.04]	[0.06]	[0.05]

Notes: The table reports annual regressions of changes in yields on the scaled yield spread for real bonds in the model:

$$y^r_{n-4,t+4} - y^r_{nt} = \alpha_n + \beta_n \frac{1}{n-4}\left(y^r_{nt} - y^r_{1t}\right) + \text{error},$$

and for nominal bonds in the model and in the data:

$$y^\$_{n-4,t+4} - y^\$_{nt} = \alpha_n + \beta_n \frac{1}{n-4}\left(y^\$_{nt} - y^\$_{1t}\right) + \text{error}.$$

y^r_{nt} denotes the annual real yield on the n-quarter real bond and $y^\$_{nt}$ denotes in the annual nominal yield on the n-quarter nominal bond. For each data regression, the table reports OLS estimates of the regressors, Newey-West (1987) corrected t-statistics (in parentheses), and R^2-statistics (in brackets). For each model regression, the table reports OLS estimates of the regressors and R^2-statistics. The maturities of the bonds range from 2 to 5 years. Data are monthly from 1952–2004.

Table 9: R^2-statistics from forward rate regressions

	Maturity in years			
	2	3	4	5
Real bonds	0.14	0.14	0.13	0.12
Nominal Bonds	0.22	0.20	0.18	0.16
Data	0.22	0.23	0.27	0.24

Notes: Annual continuously-compounded excess returns on zero-coupon bonds of maturities ranging from 2 to 5 years are regressed on 3 forward rates in the model and 5 forward rates in the data. Bond returns are in excess of the return on the 1-year bond. In the model, the forward rate maturities are 1, 3 and 5 years. In the data, forward rate maturities are 1, 2, 3, 4 and 5 years. The table reports the resulting R^2-statistics for real bonds in the model, nominal bonds in the model and nominal bonds in the data. Data are monthly and from 1952–2004.

Table 10: Moments of equity portfolio returns

	G				Growth to Value					V	V-G
Portfolio	1	2	3	4	5	6	7	8	9	10	10-1
					Panel A: Model						
$ER^i - R_1^\pi$	5.72	5.90	6.18	6.57	7.09	7.69	8.33	8.92	9.45	10.35	4.63
$\sigma(R^i - R_1^\pi)$	20.72	20.90	21.03	21.05	20.84	20.30	19.45	18.50	17.85	17.86	8.17
Sharpe Ratio	0.28	0.28	0.29	0.31	0.34	0.38	0.43	0.48	0.53	0.58	0.57
α_i	-2.52	-2.44	-2.24	-1.87	-1.30	-0.51	0.47	1.49	2.35	3.33	5.85
β_i	1.02	1.03	1.04	1.04	1.04	1.01	0.97	0.92	0.88	0.87	-0.15
					Panel B: Data						
$ER^i - R_1^\pi$	5.91	6.74	7.38	7.29	8.35	8.62	8.56	10.30	10.32	11.64	5.73
$\sigma(R^i - R_1^\pi)$	17.60	15.87	15.79	15.45	14.64	14.74	14.71	15.09	15.81	18.37	14.93
Sharpe Ratio	0.34	0.43	0.47	0.47	0.57	0.58	0.58	0.68	0.65	0.63	0.38
α_i	-1.72	-0.30	0.40	0.65	2.19	2.35	2.58	4.20	4.02	4.70	6.41
β_i	1.10	1.02	1.01	0.96	0.89	0.91	0.87	0.88	0.91	1.01	-0.10

Notes: In Panel A, firms in simulated data are sorted into deciles based on their dividend-price ratios in each simulation year. Returns are calculated over the subsequent year (portfolio 1 consists of firms with the lowest dividend-price ratios, portfolio 10 with the highest). In Panel B, firms in historical data are sorted into deciles based on their book-to-market ratio. Returns are calculated on a monthly basis and annualized (multiplied by 12 in the case of means and intercepts and $\sqrt{12}$ in the case of standard deviations). Data are monthly from 1952 to 2004. In both panels, $R^i - R_1^\pi$ refers to the return on the ith portfolio in excess of the return on the short-term nominal bond, where both returns are measured in real terms. Intercepts (α_i) and slope coefficients (β_i) are from OLS time-series regressions of excess portfolio returns on the excess market return. Means, intercepts, and standard deviations are reported in percentage terms.

Table 11: Cross-correlation of state variables

	Yield spread	CP factor	Value spread
		Panel A: Model	
Price-dividend ratio	-0.47	-0.73	0.86
Yield spread		0.80	-0.10
CP factor			-0.32
		Panel B: Data from 1952–2004	
Price-dividend ratio	0.17	0.03	0.70
Yield spread		0.69	0.03
CP factor			-0.14
		Panel C: Data from 1964–2004	
Price-dividend ratio	0.18	-0.14	0.78
Yield spread		0.65	0.03
CP factor			-0.17

Notes: The table reports correlations between the log price-dividend ratio on the market portfolio, the spread between the 5-year yield and the 3-month yield on nominal bonds (the yield spread), the linear combination of forward rates constructed to best predict average holding period returns on bonds (the CP factor), and the value spread. In the model, the value spread is defined as the log dividend-price ratio of the value portfolio minus the log dividend-price ratio of the growth portfolio. In the data, the value spread is defined as the log book-to-market ratio of the value portfolio minus the log book-to-market ratio of the growth portfolio. Data are monthly.

Table 12: Cross-correlation of excess returns

	Bond return	V–G return
	Panel A: Model	
Market return	0.83	-0.44
Bond return		-0.28
	Panel B: Data from 1952–2004	
Market return	0.15	-0.33
Bond return		0.15
	Panel C: Data from 1964–2004	
Market return	0.23	-0.47
Bond return		0.20

Notes: The table reports correlations between three continuously compounded annual excess returns: the return on the market portfolio in excess of the return on the short-term nominal bond, the return on the nominal five-year zero-coupon bond in excess of the short-term nominal bond, and the return on the value portfolio in excess of the return on the growth portfolio. Data are monthly.

Table 13: Long-horizon regressions of returns on the price-dividend ratio

	Market return		Bond return		V–G return	
Horizon	1	5	1	5	1	5
			Panel A: Model			
β_1	-0.14	-0.50	-0.06	-0.20	-0.08	-0.31
R^2	[0.07]	[0.23]	[0.09]	[0.22]	[0.15]	[0.19]
			Panel B: Data from 1952-2004			
β_1	-0.11	-0.40	0.02	0.06	0.02	0.01
t-stat	(-1.99)	(-3.37)	(0.89)	(1.02)	(0.40)	(0.18)
R^2	[0.07]	[0.17]	[0.01]	[0.02]	[0.01]	[0.00]
			Panel C: Data from 1964-2004			
β_1	-0.08	-0.28	0.02	0.05	0.02	-0.05
t-stat	(-1.30)	(-2.42)	(0.73)	(0.69)	(0.32)	(-0.52)
R^2	[0.04]	[0.09]	[0.01]	[0.01]	[0.00]	[0.01]

Notes: The table reports regressions

$$\sum_{i=1}^{H} r^e_{t+i} = \beta_0 + \beta_1(p^m_t - d_t) + \text{error},$$

where r^e_{t+1} is either the excess return on the market portfolio, the excess return on the 5-year nominal zero-coupon bond, or the return on the strategy that is long the value portfolio and short the growth portfolio. Returns are measured over horizons of one year and five years. The right hand side variable is the lagged price-dividend ratio on the market. For each model regression, the table reports OLS estimates of the regressors and R^2-statistics (in brackets). For each data regression, the table reports OLS estimates of the regressors, Newey-West (1987) corrected t-statistics (in parentheses), and R^2-statistics (in brackets). Data are monthly.

Table 14: Long-horizon regressions of returns on the yield spread

	Market return		Bond return		V–G return	
Horizon	1	5	1	5	1	5
			Panel A: Model			
β_1	3.15	12.00	1.42	5.35	1.87	7.01
R^2	[0.07]	[0.25]	[0.11]	[0.30]	[0.14]	[0.18]
			Panel B: Data from 1952-2004			
β_1	4.15	12.68	2.48	1.66	2.30	-4.82
t-stat	(1.78)	(3.04)	(3.56)	(0.72)	(1.33)	(-1.76)
R^2	[0.04]	[0.10]	[0.13]	[0.01]	[0.02]	[0.04]
			Panel C: Data from 1964-2004			
β_1	3.79	13.21	2.71	1.90	1.85	-5.52
t-stat	(1.60)	(2.93)	(3.91)	(0.82)	(1.02)	(-2.04)
R^2	[0.04]	[0.14]	[0.16]	[0.01]	[0.02]	[0.06]

Notes: The table reports regressions

$$\sum_{i=1}^{H} r^e_{t+i} = \beta_0 + \beta_1(y^\$_{5t} - y^\$_{1t}) + \text{error}$$

where r^e_{t+1} is either the excess return on the market portfolio, the excess return on the 5-year nominal zero-coupon bond, or the return on the strategy that is long the value portfolio and short the growth portfolio. Returns are measured over horizons of one year and five years. The right hand side variable is the lagged spread between the yield on the five-year nominal zero-coupon bond and the yield on the three-month zero-coupon bond. For each model regression, the table reports OLS estimates of the regressors and R^2-statistics (in brackets). For each data regression, the table reports OLS estimates of the regressors, Newey-West (1987) corrected t-statistics (in parentheses), and R^2-statistics (in brackets). Data are monthly.

Table 15: Long-horizon regressions of returns on the linear combination of forward rates

	Market return		Bond return		V–G return	
Horizon	1	5	1	5	1	5
			Panel A: Model			
β_1	3.80	14.07	1.67	6.18	2.26	8.67
R^2	[0.11]	[0.37]	[0.17]	[0.43]	[0.23]	[0.30]
			Panel B: Data from 1952-2004			
β_1	1.11	2.80	1.46	2.58	0.94	-0.24
t-stat	(1.15)	(1.44)	(4.79)	(3.70)	(1.66)	(-0.21)
R^2	[0.02]	[0.03]	[0.24]	[0.14]	[0.02]	[0.00]
			Panel C: Data from 1964-2004			
β_1	1.53	4.91	1.47	2.31	0.67	-0.85
t-stat	(1.91)	(4.55)	(7.70)	(4.68)	(1.25)	(-0.94)
R^2	[0.05]	[0.15]	[0.34]	[0.16]	[0.02]	[0.01]

Notes: The table reports regressions

$$\sum_{i=1}^{H} r^e_{t+i} = \beta_0 + \beta_1 \boldsymbol{\theta}' \boldsymbol{f}_t + \text{error}$$

where r^e_{t+1} is either the excess return on the market portfolio, the excess return on the 5-year nominal zero-coupon bond, or the return on the strategy that is long the value portfolio and short the growth portfolio. Returns are measured over horizons of one year and five years. The right hand side variable is a lagged linear combination of forward rates on nominal bonds, constructed as in Cochrane and Piazzesi (2005). For each model regression, the table reports OLS estimates of the regressors and R^2-statistics (in brackets). For each data regression, the table reports OLS estimates of the regressors, Newey-West (1987) corrected t-statistics (in parentheses), and R^2-statistics (in brackets). Data are monthly.

Table 16: Long-horizon regressions of returns on the value spread

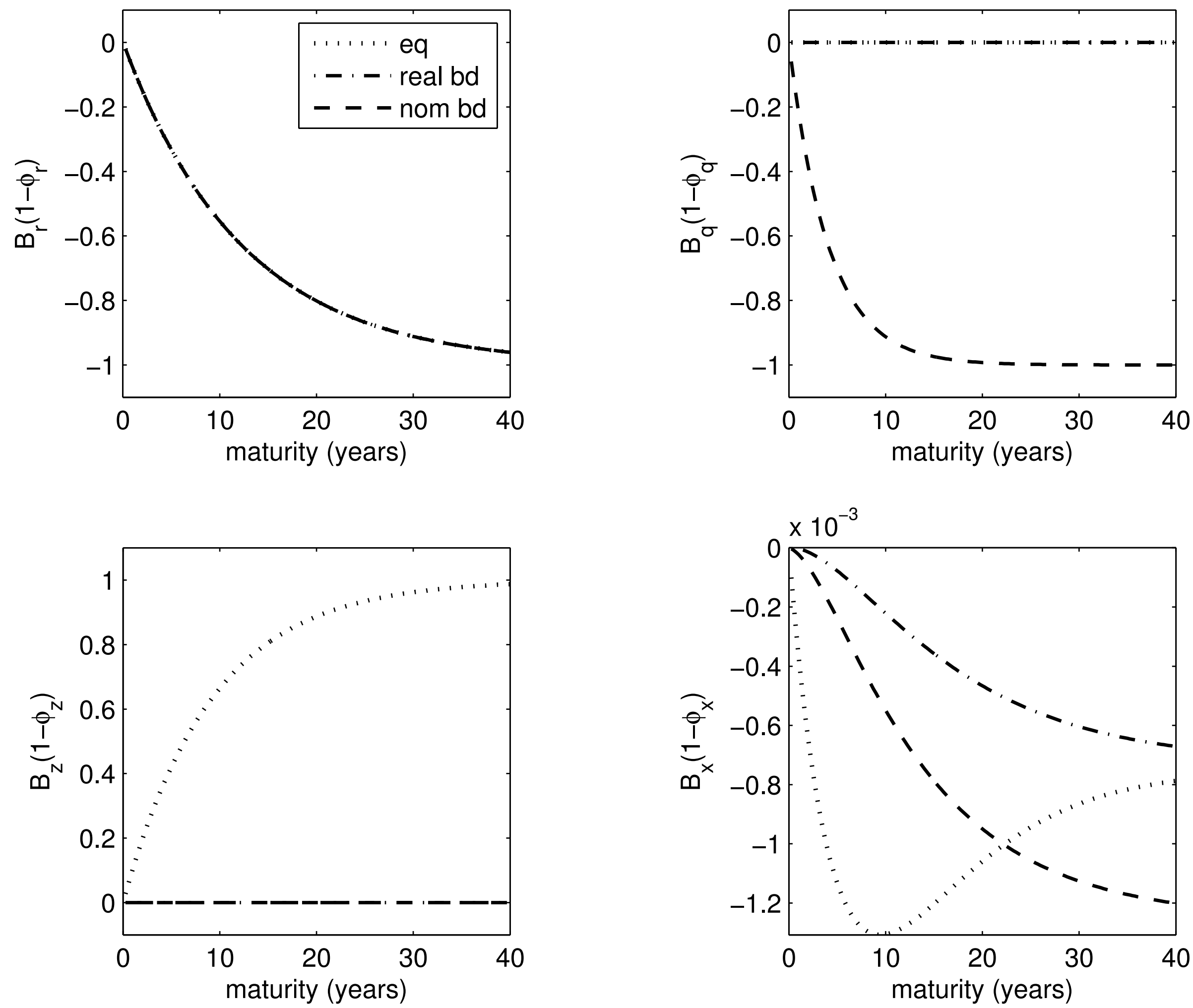
	Market return		Bond return		V–G return	
Horizon	1	5	1	5	1	5
			Panel A: Model			
β_1	-0.20	-0.75	-0.07	-0.24	-0.12	-0.46
R^2	[0.02]	[0.06]	[0.02]	[0.04]	[0.04]	[0.05]
			Panel B: Data from 1952-2004			
β_1	-0.25	-0.63	0.05	0.14	0.23	0.62
t-stat	(-2.07)	(-2.15)	(1.04)	(0.74)	(2.05)	(3.61)
R^2	[0.05]	[0.07]	[0.02]	[0.02]	[0.09]	[0.17]
			Panel C: Data from 1964-2004			
β_1	-0.27	-0.86	0.06	0.22	0.22	0.59
t-stat	(-2.45)	(-2.95)	(1.18)	(1.19)	(1.77)	(3.16)
R^2	[0.08]	[0.14]	[0.02]	[0.05]	[0.09]	[0.15]

Notes: The table reports regressions

$$\sum_{i=1}^{H} r^e_{t+i} = \beta_0 + \beta_1 \left(\text{value spread}\right)_t + \text{error}$$

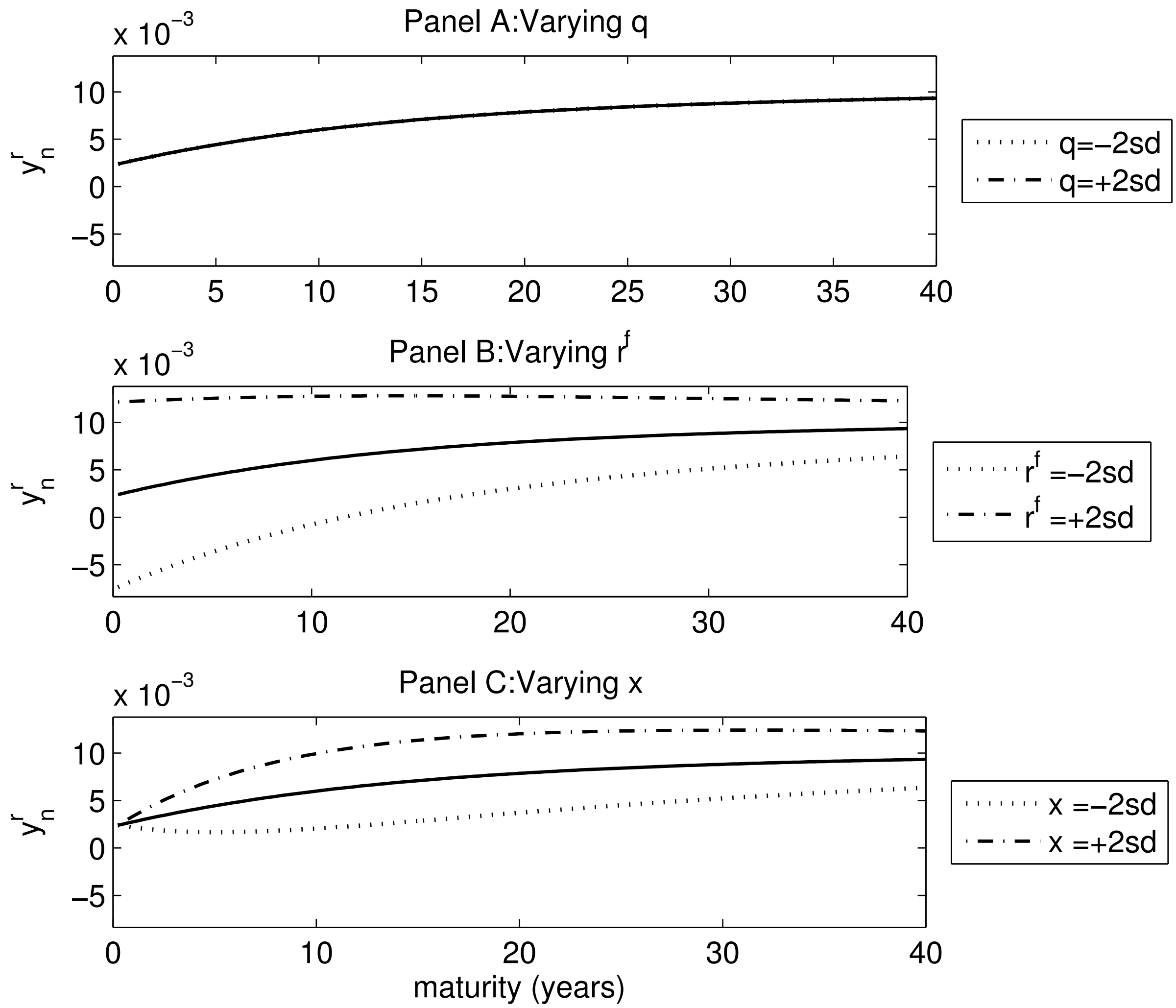
where r^e_{t+1} is either the excess return on the market portfolio, the excess return on the 5-year nominal zero-coupon bond, or the return on the strategy that is long the value portfolio and short the growth portfolio. Returns are measured over horizons of one year and five years. The right hand side variable is the value spread, constructed as the log dividend-price ratio of the value portfolio minus the log dividend-price ratio of the growth portfolio in the model and as in Cohen, Polk, and Vuolteenaho (2003) in the data. For each model regression, the table reports OLS estimates of the regressors and R^2-statistics (in brackets). For each data regression, the table reports OLS estimates of the regressors, Newey-West (1987) corrected t-statistics (in parentheses), and R^2-statistics (in brackets). Data are monthly.

Figure 1: Model Solution



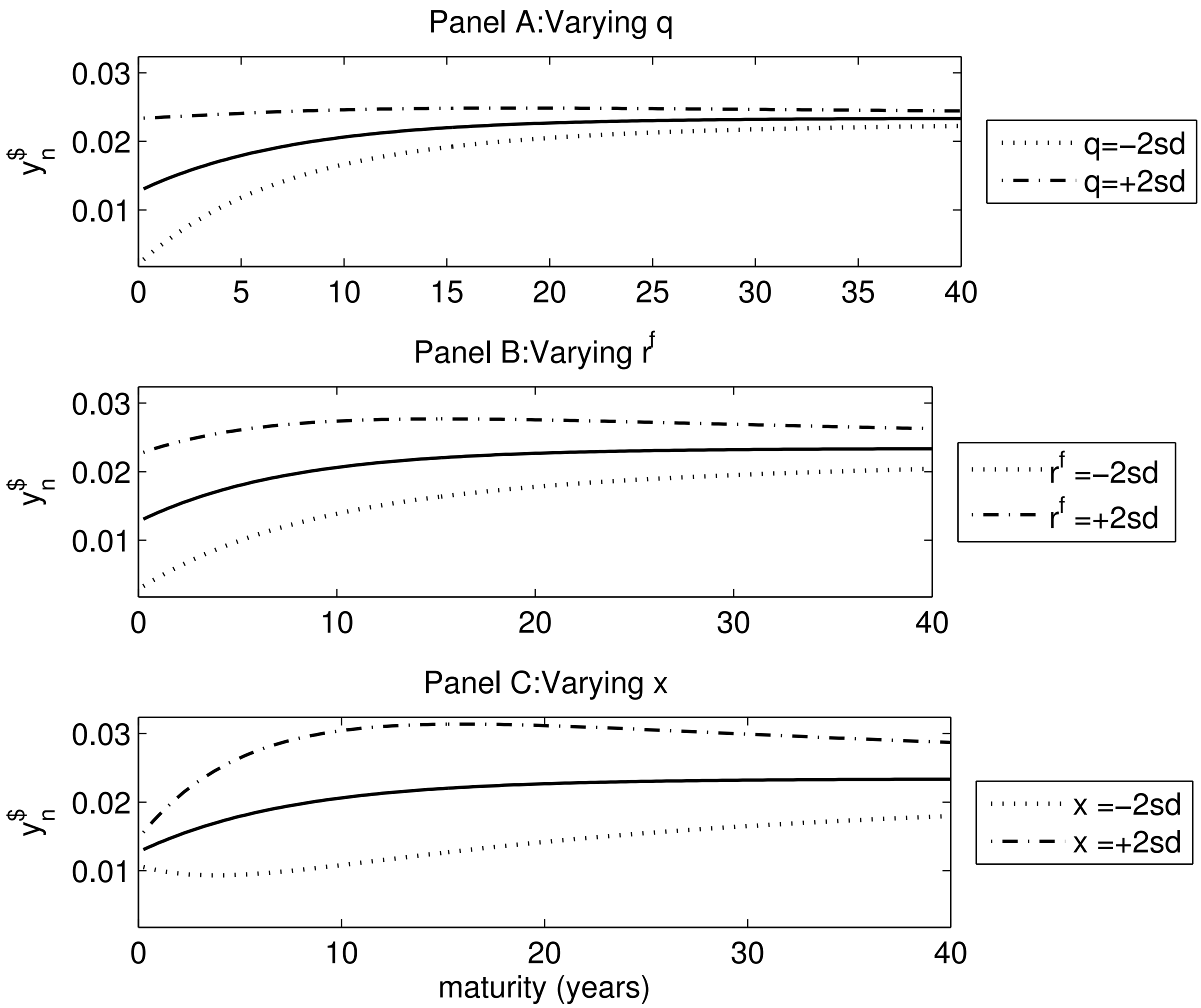
Notes: Solutions to B_{rn}, the sensitivity of prices to the real riskfree rate (top left); to B_{qn}, the sensitivity of prices to expected inflation (top right); to B_{zn}, the sensitivity of prices to expected dividend growth (bottom left); and to B_{xn}, the sensitivity of prices to the price of risk variable. Dotted lines denote the solutions for zero-coupon equity prices expressed in real terms, dashed-dotted lines denote the solutions for real bond prices expressed in real terms, dashed lines denote the solutions for nominal bond prices expressed in nominal terms. The solutions are scaled by the persistence ϕ of the variables. The solution for B_r is identical for all three asset classes. The solution for B_q is identical for equities and real bonds and equal to zero. The solution for B_z is identical for real and nominal bonds and equal to zero.

Figure 2: Yields on Zero-Coupon Real Bonds



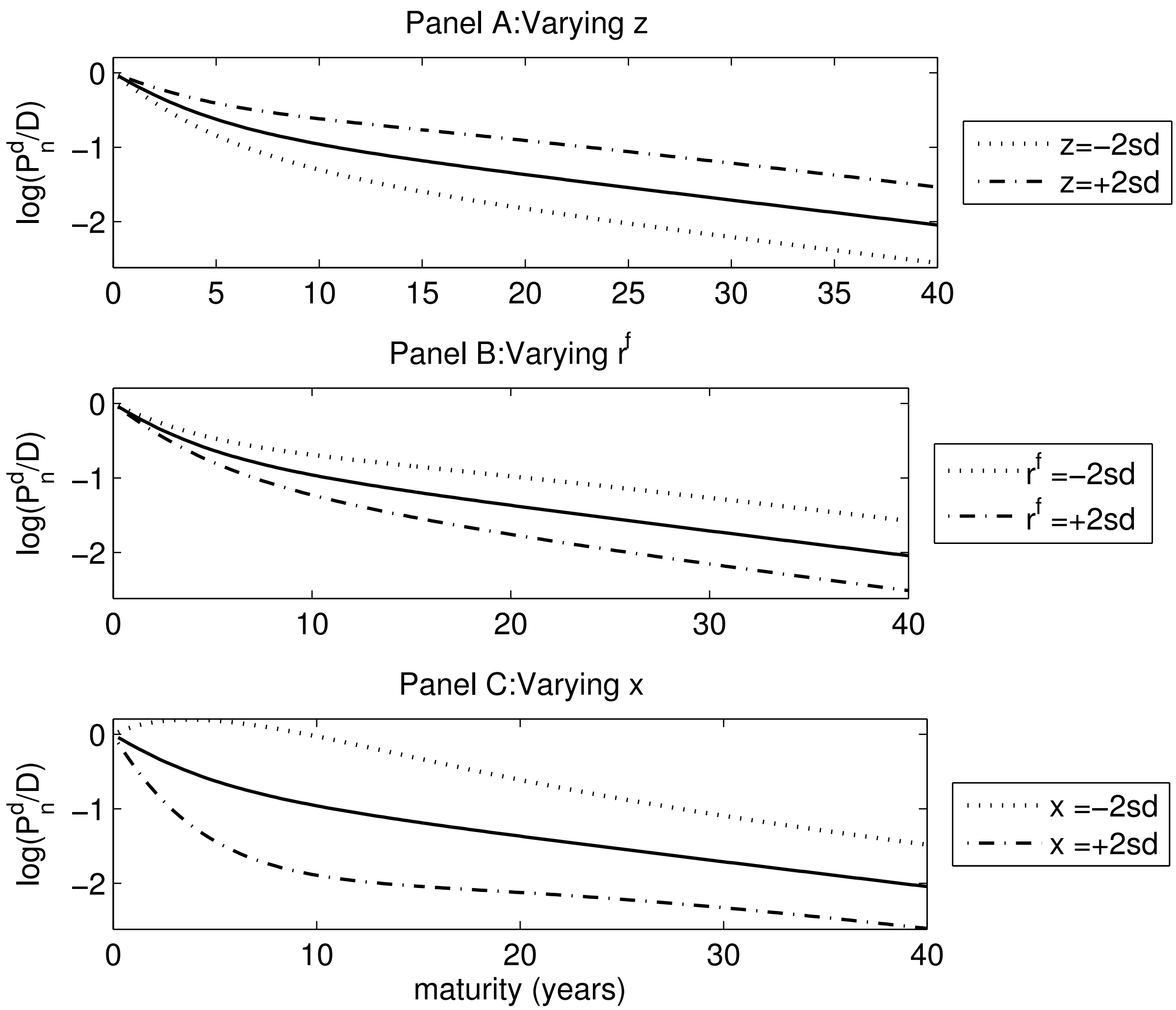
Notes: Panel A shows quarterly yields on real bonds as a function of maturity when the state variables are equal to their long-run mean (solid line), and when expected inflation q_t is equal to the long-run mean plus (dashed-dotted line) or minus (dotted line) two unconditional quarterly standard deviations. All other state variables are kept at their long-run mean. Panel B shows analogous results when the real riskfree rate r_t^f is varied by plus or minus two unconditional quarterly standard deviations. Panel C shows analogous results when the price-of-risk variable x_t is varied by plus or minus two unconditional quarterly standard deviations.

Figure 3: Yields on Zero-Coupon Nominal Bonds



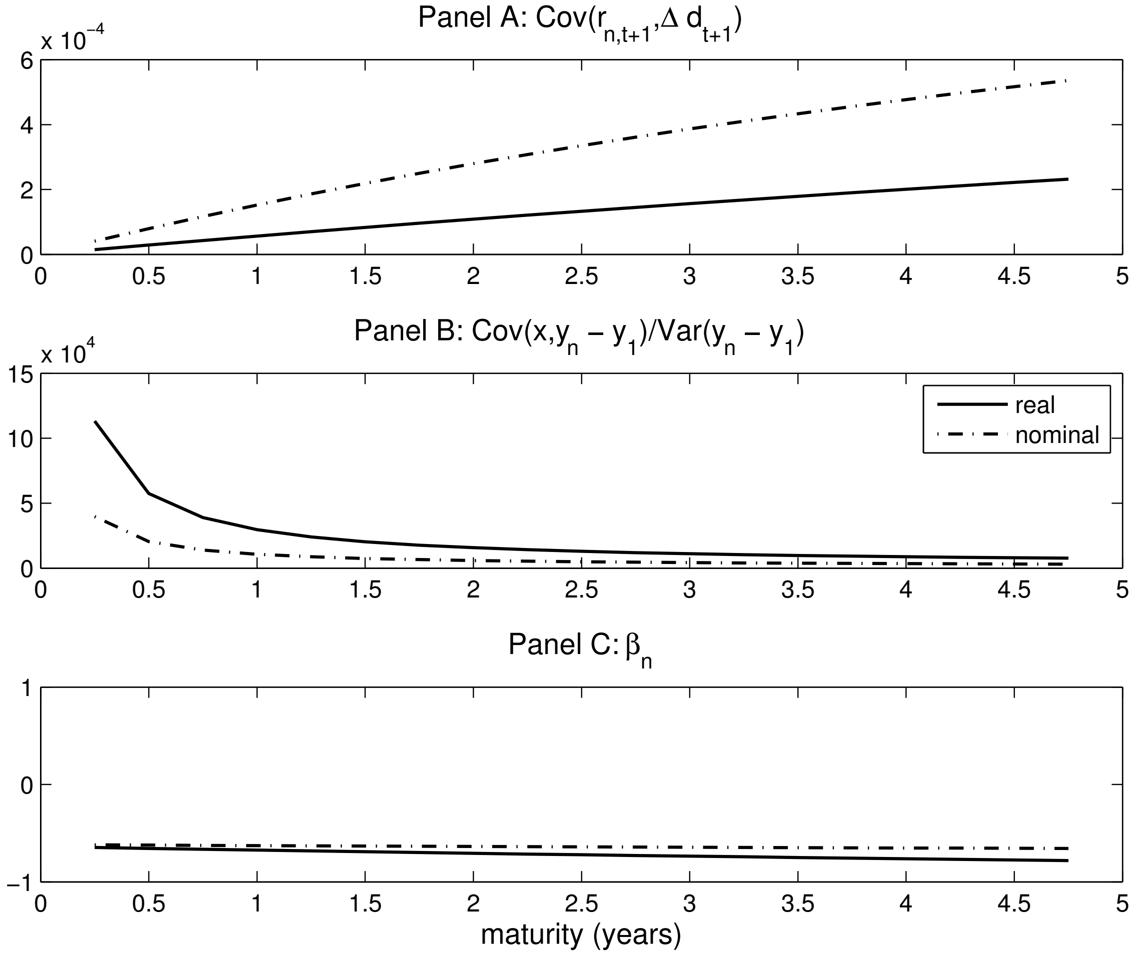
Notes: Panel A shows quarterly nominal yields on nominal bonds as a function of maturity when the state variables are equal to their long-run mean (solid line), and when expected inflation q_t is equal to the long-run mean plus (dashed-dotted line) or minus (dotted line) two unconditional quarterly standard deviations. All other state variables are kept at their long-run mean. Panel B shows analogous results when the real riskfree rate r_t^f is varied by plus or minus two unconditional quarterly standard deviations. Panel C shows analogous results when the price-of-risk variable x_t is varied by plus or minus two unconditional quarterly standard deviations.

Figure 4: Ratios of Prices to Aggregate Dividends for Zero-Coupon Equity



Notes: Panel A shows the log of ratios of zero-coupon equity prices to the aggregate dividend as a function of maturity when the state variables are equal to their long-run mean (solid line), and when expected dividend growth z_t is equal to the long-run mean plus (dashed-dotted line) or minus (dotted line) two unconditional quarterly standard deviations. All other state variables are kept at their long-run mean. Panel B shows analogous results when the real riskfree rate r_t^f is varied by plus or minus two unconditional quarterly standard deviations. Panel C shows analogous results when the price-of-risk variable x_t is varied by plus or minus two unconditional quarterly standard deviations.

Figure 5: Decomposition of Coefficients from Long-Rate Regressions



Notes: Panel A shows the covariance between the return on an n-period bond and fundamentals as a function of maturity. Panel B shows the coefficient from a regression of the price-of-risk variable x_t on the yield spread as a function of the yield maturity. Panel C shows the coefficient β_n from the regression

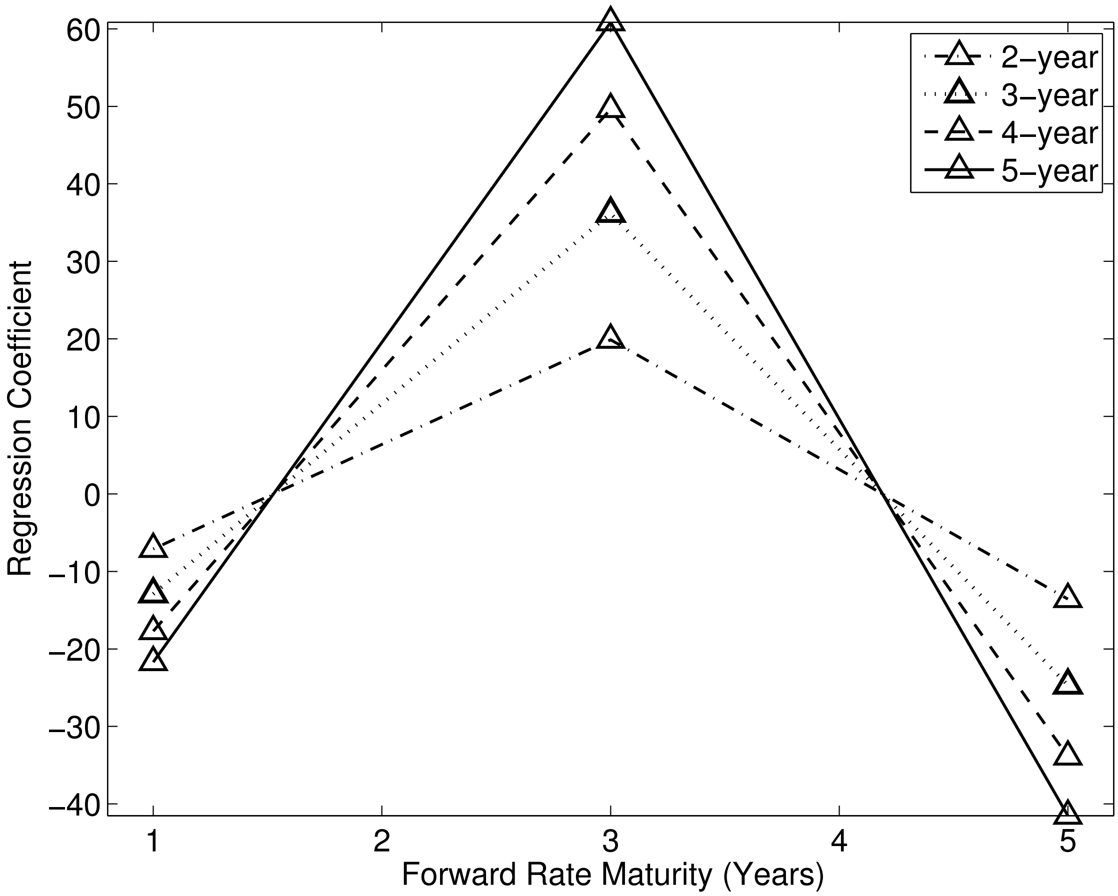
$$y_{n-1,t+1} - y_{nt} = \alpha_n + \beta_n \frac{1}{n-1}\left(y_{nt} - y_{1t}\right) + \text{error},$$

as a function of maturity. Results are shown for real bonds (solid lines) and nominal bonds (dotted lines). The covariance between returns and fundamentals, the coefficient from a regression of x_t on the yield spread and β_n are related by the equation

$$\beta_n = 1 - \mathrm{Cov}(r_{n,t+1}, \Delta d_{t+1}) \frac{\mathrm{Cov}(x_t, y_{nt} - y_{1t})}{\mathrm{Var}(y_{n-1,t+1} - y_{1t})}.$$

The results are from data simulated from the model at a quarterly frequency.

Figure 6: Regressions of Excess Bond Returns on Forward Rates



Notes: Annual returns on 2, 3, 4 and 5-year nominal bonds, in excess of the return on the 1-year bond, are regressed on the 1, 3 and 5-year forward rates in data simulated from the model. The figure shows the resulting regression coefficients as a function of the forward rate maturity.