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Bennett T. McCallum

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Inflation Determination with Taylor Rules: Is New Keynesian Analysis Critically Flawed?

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ABSTRACT

Cochrane (2007) has strongly questioned the basic economic logic of current mainstream monetary policy analysis, arguing that the standard notion --that "determinacy" of a rational expectations (RE) equilibrium suffices to imply that stable inflation behavior will be generated -- is incorrect. This is because New Keynesian (NK) models are typically consistent with the existence of RE paths with explosive inflation rates (in addition to one or more stable paths) that normally do not imply explosions in real variables relevant for transversality conditions. Consequently, the usual logic does not imply the absence of explosive inflation. That result does not, however, justify negative conclusions about NK analysis. For there is a different criterion that is logically satisfactory for the purpose at hand. This is the requirement that, to be plausible, a RE solution must satisfy the property of least-squares learnability. Adoption of this criterion, which should be attractive to analysts concerned with actual monetary policy, serves to justify in principle the bulk of current mainstream analysis.

Bennett T. McCallum

Tepper School of Business, Posner 256

Carnegie Mellon University

Pittsburgh, PA 15213

and NBER

bm05@andrew.cmu.edu

1. Introduction

Quite recently, John Cochrane (2007) has strongly questioned the basic economic logic of current mainstream monetary policy analysis as widely practiced and described by Clarida, Gali, and Gertler (2000), King (2000), Svensson and Woodford (2005), Taylor (1999), Woodford (2003a), and many other prominent writers.¹ A central ingredient of this analysis is the assumption that monetary policy is conducted by means of a central-bank policy rule for period-by-period adjustment of a short-term nominal interest rate, with the rule calling for adjustments of this policy rate by more than one-for-one in response to incipient movements in inflation—thereby satisfying a condition that is widely referred to as the Taylor Principle.² Cochrane’s contention is that the standard notion—that “determinacy” of a rational expectations (RE) equilibrium serves to guarantee that stable inflation behavior around target will be generated—is incorrect. His basic reasoning is expressed in the following quote:

“I argue that the Taylor principle, in the context of new-Keynesian models, does not, in fact, determine inflation or the price level. Nothing in economics rules out explosive or “non-local” nominal paths. Transversality conditions can rule out real explosions, but not nominal ones” (Cochrane, 2007, p. 2).

Consideration of Cochrane’s argument indicates that there is much merit in this point. The concept of determinacy relied upon in the standard analysis is uniqueness of a dynamically stable (i.e., non-explosive) RE solution in the model under study (which is

¹ Other notable items in the literature include Taylor (1993), Clarida, Gali, and Gertler (1999), Goodfriend and King (1997), King and Wolman (1996), and Rotemberg and Woodford (1997). Goodfriend (2007) presents a recent overview that links formal analysis and actual central bank practice.

² The principle is often extended to cases in which policy responds also to incipient movements in the output gap. See, e.g., Woodford (2003a, pp. 90-94, 252-261), Taylor (1999).

of course intended to depict reality). But the equations of New Keynesian (NK)³ models are typically consistent with the existence of RE paths with explosive inflation rates, in addition to one or more stable paths, under the usual specifications. These are “nominal explosions” that bring about paths along which real money balances tend to decrease in magnitude, rather than growing without limit. More generally, explosive paths for inflation rates—or nominal interest rates—do not normally imply explosions in real variables relevant for transversality conditions, which are crucial for individual agent optimization. Consequently, the usual logic, in the usual models, does not imply the absence of explosive inflation.

It is my conclusion that this position of Cochrane’s is in principle correct. That position as just stated does not, however, justify Cochrane’s negative conclusions about NK analysis.⁴ For there is a different criterion—in many (but not all) cases implied by determinacy and not generally implying determinacy—that is logically satisfactory for the purpose at hand. This is the requirement that, to be plausible, a RE solution should satisfy the property of learnability, of the type made prominent in the work of Evans and Honkapohja (1999, 2001).⁵ Adoption of this criterion, which should be attractive to any analyst concerned with actual monetary policy issues, serves to justify the bulk of current mainstream analysis in principle. Indeed, as argued in McCallum (2003, 2007), it eliminates some other problematic aspects of the NK analysis.

³ New-Keynesian (NK) models are basically the same as those referred to by Goodfriend and King (1997) as “New Neoclassical Synthesis” (NNS) models. Some would consider the latter label to be more apt, from a historical perspective. Nevertheless, I will follow Cochrane in using the more standard label NK to refer to the models of current mainstream analysis.

⁴ It should be said that Cochrane does not use the term “critically flawed” that appears in my title. I would think, however, that most exponents of NK analysis would regard a model/policy-rule combination, that does not rule out explosive inflation, to be critically flawed.

⁵ More precisely, the requirement is least-squares (LS) learnability, as a necessary condition for a RE equilibrium to be plausible and therefore of potential economic relevance.

2. Cochrane's Critique

Let me begin by outlining the central aspects of Cochrane's argument in the context of the basic NK model that he uses for his main presentation. It is common, in the NK literature, to utilize a three-equation structure that includes an expectational IS function,⁶ a Calvo-type price adjustment relation, and a Taylor-style policy rule in a system that could be written as

$$(1) \quad y_t = b_0 + b_1(R_t - E_t\pi_{t+1}) + E_t y_{t+1} + v_t \quad b_1 < 0$$

$$(2) \quad \pi_t = \beta E_t \pi_{t+1} + \kappa(y_t - \bar{y}_t) \quad \kappa > 0$$

$$(3) \quad R_t = \mu_0 + (1 + \mu_1)\pi_t + \mu_2(y_t - \bar{y}_t) + e_t$$

where y_t represents output/consumption, \bar{y}_t is its "natural rate" flexible-price value, π_t is inflation, and R_t is the one-period (nominal) rate of interest.⁷ Here v_t and e_t are preference and policy shocks while \bar{y}_t is exogenously generated by an autocorrelated technology process. Cochrane simplifies by assuming full price flexibility so that $y_t = \bar{y}_t$ in each period, which eliminates the Calvo equation (2) and the output gap term in (3). If we also let \bar{y}_t be a constant, then the IS relation (1) becomes

$$(4) \quad 0 = b_0 + b_1(R_t - E_t\pi_{t+1}) + v_t.$$

Then if the shock term is neglected and $r = -b_0/b_1$ is recognized as a constant real rate of interest, we have the relation that Cochrane calls a "Fisher equation." From the perspective of monetary policy, I think it better to describe it as is done here, but there is

⁶ This function represents a consumption Euler equation together with the economy's overall resource constraint.

⁷ The rule is written assuming a zero target rate of inflation. If it were non-zero, then a non-zero constant term would appear in the solution equation (7) below.

no substantive difference relative to Cochrane.⁸ Thus the system at hand reduces to (3) and (4), identical to Cochrane's (1) and (2) if we delete the shock v_t from ours while also setting $\mu_0 = r$ in the policy rule, as a sensible central bank would do.

To solve this simple system we combine (3) and (4) to obtain

$$(5) \quad 0 = b_0 + b_1[\mu_0 + (1+\mu_1)\pi_t + e_t - E_t\pi_{t+1}] + v_t$$

and conjecture that with $v_t = 0$ and $e_t = \rho e_{t-1} + \text{white noise } (|\rho| < 1)$, as in Cochrane (2007), there will be a solution of the form

$$(6) \quad \pi_t = \phi_0 + \phi_1 e_t$$

with expectations therefore obeying $E_t\pi_{t+1} = \phi_0 + \phi_1 \rho e_t$. Substitution in (5) then implies that this solution is

$$(7) \quad \pi_t = 0 - \frac{1}{1+\mu_1-\rho} e_t.$$

The latter is, of course, the "fundamentals" or "MSV" solution. With $|\rho| < 1$, it implies that π_t is negatively related to e_t and that larger values of μ_1 serve to reduce the variability of π_t around its target. Now suppose, however, that instead of (6) one looks for a solution of the form

$$(8) \quad \pi_t = \phi_0 + \phi_1 e_t + \phi_2 \pi_{t-1}.$$

Then $E_t\pi_{t+1} = \phi_0 + \phi_1 \rho e_t + \phi_2(\phi_1 e_t + \phi_2 \pi_{t-1})$ and a second solution, in addition to (7), is

$$(9) \quad \pi_t = (1/\rho)e_t + (1+\mu_1)\pi_{t-1}.$$

Clearly, with $\mu_1 > 0$, as specified by the Taylor Principle, this expression (9) implies an

⁸ My preference is to think of the "Fisher equation" as an identity, one which defines the one-period real rate of interest as a nominal rate minus expected inflation. The item under discussion here is a model of individual saving behavior together with a market-clearing condition that consumption equals output in the aggregate plus the assumption that output is constant.

explosive process for the inflation rate.⁹ That is the type of solution referred to by Cochrane, and it is clear that in the model at hand there is no transversality condition that would rule out this explosive solution for the inflation rate. Further investigation, pertaining to models in which the medium-of-exchange properties of money are explicitly recognized, will appear below in Section 4. Pending that discussion, it appears appropriate to accept, on a preliminary basis, the validity of Cochrane's critique of mainstream monetary policy analysis, with NK models and Taylor-style policy rules.

3. LS Learnability

Several analysts have argued, however, that for any RE solution to be considered plausible, and thereby relevant for policy analysis, it should be learnable.¹⁰ The basic idea is simple and powerful. It is that in any RE model intended to represent behavior in an actual market economy, the individual agents should be able to learn quantitative details concerning the behavior of variables—which they must forecast for decision-making purposes—from data generated by the economy itself.¹¹ Cochrane (2007, p. 44) briefly disputes that contention, as follows: "... a wide variety of almost philosophical principles have been advocated to prune equilibria. For example, Evans and Honkapohja (2001) advocate criteria based on least-squares (LS) learnability, and McCallum (2003) advocates a 'minimum state variable criterion,' which he relates to learnability. These refinements go beyond the standard definitions of economic equilibria. One may argue that when a model gives multiple equilibria, we need additional selection criteria. I argue

⁹ It appears from (9) that this solution will not be defined in the measure-zero case with $\rho = 0$. But in that case one can add e_{t-1} as an additional state variable in (8) and obtain an infinity of explosive solutions that could be indexed by the start-up value of π_{t-1} .

¹⁰ These writers include Bullard (2006), Evans and Honkapohja (2001, p. 12-13), McCallum (2003, 2007), and (arguably) Woodford (2003a, pp. 261-276; 2003b, p. 1183).

¹¹ For RE to obtain, the implied forecasting relationships must be quantitatively accurate.

instead that we need a different model.” This argument of Cochrane’s has two parts: (i) a dismissal of learnability (justified only by his use of the word “philosophical”) and (ii) advocacy of a “different model,” by which he evidently means the fiscal theory of the price level. I will discuss the latter below, in Section 5. Here I wish to disagree with the suggestion that LS learnability is a spurious principle.

Let me begin by quoting myself: “The position that learnability (and thus E-stability) should be regarded as a necessary condition for the relevance of a RE equilibrium [sic?] begins with the presumption that individual agents must somehow learn the magnitudes of parameters describing the economy’s law of motion from observations generated by the economy; they cannot be endowed with such knowledge by magic. Of course any particular learning scheme might be incorrect in its depiction of actual learning behavior. But in this regard it is important to note that the LS learning process in question assumes that (i) agents are collecting an ever-increasing number of observations on all relevant variables while (ii) the structure is remaining unchanged. Furthermore, (iii) the agents are estimating the relevant unknown parameters (iv) with an appropriate estimator (v) in a properly specified model. Thus if a proposed RE solution is not learnable by the process in question—the one to which the E&H results pertain—then it would seem highly implausible that it could prevail in practice” (McCallum, 2007, p. 1378). Cochrane suggests, to the contrary, that the notion that individual agents should have some way of obtaining the information, necessary to form expectations in the manner hypothesized in the analysis, is in some way “additional” to “standard definitions of economic equilibria.” To me, this suggestion seems misplaced. In standard (i.e., RE) analysis, for the determination of current endogenous variables we specify information

sets that include quantitative features of the system plus, at a maximum, current and past values of relevant variables.¹² That is basically what LS learnability does for current knowledge of the economy's structure. Here, I am assuming that our analysis is based on a model in which there are many individual agents interacting on markets, each agent acting individually and without knowledge of other agents' preferences and technologies (therefore, their demand and supply functions), which can be different from his own—an assumption that is not relaxed even if the analyst assumes (for simplicity) that all agents are alike. I seriously doubt that John Cochrane, a dedicated (as well as skillful) economist, would object to this position, even though it is perhaps "philosophical."¹³

The point, of course, is that application of the LS learnability criterion does, in the model at hand, eliminate the explosive solution of equation (9) above. That conclusion is well known from the analyses of Bullard and Mitra (2002) and Honkapohja and Mitra (2001), and is discussed on p. 1161 of McCallum (2003). The relevant analysis can be summarized as follows. Consider linear models that can be written as

$$(10) \quad x_t = AE_t x_{t+1} + Cx_{t-1} + Dz_t$$

where x_t is a $n \times 1$ vector of endogenous variables, the system's exogenous variables z_t being generated by a first-order autoregressive process

$$(11) \quad z_t = Rz_{t-1} + \varepsilon_t$$

with ε_t white noise and R being a stable matrix. Here A and C are $n \times n$, D is $n \times n_1$, and R is $n_1 \times n_1$. Considering fundamental solutions of the form

¹² For some purposes perfect-foresight analysis is useful, of course, but one would not use that assumption in an analysis that is concerned with (e.g.) the variability of asset prices or macroeconomic variables.

¹³ My position could alternatively be expressed as admitting that learnability is not part of "standard definitions" and then arguing that standard definitions are flawed in the context of models appropriate for monetary policy analysis.

$$(12) \quad \mathbf{x}_t = \Omega \mathbf{x}_{t-1} + \Gamma \mathbf{z}_t,$$

standard undetermined-coefficient reasoning establishes that Ω must satisfy the quadratic

$$(13) \quad A\Omega^2 - \Omega + C = 0.$$

There are many matrices that satisfy this quadratic; they result from different orderings of the generalized eigenvalues of a matrix pencil involving parameter matrices A and C ; see McCallum (2007, p. 1380-2). If more than one of the Ω s that satisfies (13) has all its eigenvalues less than 1 in modulus there are multiple stable solutions, i.e., indeterminacy.

For such a system, Evans and Honkapohja (2001, p. 238) report that E-stability (and thus LS learnability) obtains if and only if the following matrices have all eigenvalues with real parts less than 1:

$$(13a) \quad F = (I - A\Omega)^{-1} A$$

$$(13b) \quad \Omega' \otimes F$$

$$(13c) \quad R' \otimes F.$$

It will be noted that, if there are no lagged endogenous variables in the system, then $C = 0$ implying that $\Omega = 0$ and $F = A$. Then the first two conditions amount to the requirement that the eigenvalues of A all have real parts less than 1. In the basic system summarized in (5) above, the fundamentals solution has $\Omega = 0$ and $A = 1/(1 + \mu_1)$. Thus it is clear that the ES learnability requirements (13a-c) are satisfied. By contrast, the non-fundamentals solution (9) yields $\pi_t = (1/\rho)e_t + (1+\mu_1)\pi_{t-1}$, implying that $\Omega = 1 + \mu_1$ and thus that

$$F = (I - A\Omega)^{-1} A = [1 - (1 + \mu_1)/(1 + \mu_1)]^{-1} (1/(1 + \mu_1)) = [1 - 1]^{-1} (1/(1 + \mu_1)) = (1/(1 + \mu_1))/0.$$

Thus in this case $F > 1 + \mu_1$ (an understatement) and at least two of the three conditions (13) are violated. Accordingly, the explosive solution is not learnable. Although it

satisfies the orthogonality conditions for a RE solution, it is implausible that an economic system matching the specification would generate outcomes of the type described by (9).

It should be added explicitly, perhaps, that the analysis of Evans and Honkapohja (2001, pp. 138-9, 235, 238) includes results implying that, with the addition of a few appropriate supplementary conditions, the LS learning process will converge to an explosive path satisfying (9) with probability zero. In that sense, such paths cannot represent plausible outcomes.

4. Medium of Exchange Money

To this point there has been no explicit recognition of money, i.e., an asset that serves as a medium of exchange and thus facilitates transactions in the economy under discussion. It might be thought that such recognition could itself overturn Cochrane's basic argument since recognition of a monetary asset would give rise to a transversality condition pertaining to real money holdings—and transversality conditions tend to rule out certain explosive paths as equilibria because they violate conditions necessary for individual optimality. Cochrane is fully aware of the relevant literature, however, which rules out paths in which the discounted increment to utility provided by future real money balances fails to approach zero as the horizon considered increases indefinitely. His presumption, apparently, is that an explosive inflation rate would tend to reduce real money holdings and, with the $T-1$ power of the discount factor approaching zero, thus to induce the relevant product to approach zero. A basic analysis cited by Cochrane is that of Obstfeld and Rogoff (1983), in which a model with medium-of-exchange money tends to rule out paths along which the price level approaches zero but not paths along which the price level explodes.

It is possible to object to the model used by Obstfeld and Rogoff on the grounds that its money-in-the-utility-function specification does not represent money's transaction-facilitating properties as adequately as would a shopping-time or transaction-cost model in which time or physical resources used up by transactions depend on both real money holdings and real quantities of transactions. Additional analysis in that direction has been conducted by Gray (1984). This paper does not consider every specification that one might desire, but it goes some substantial way toward justifying the conclusion of Obstfeld and Rogoff (1983, p. 686) that under a regime of pure fiat money “... explosive price-level paths—speculative hyperinflations—can be ruled out only when severe restrictions are placed on individual preferences.”¹⁴

Gray's analysis includes model specifications in which the medium-of-exchange role of money is represented by an explicit transaction-cost function. The main weakness that I can see in her argument is that this function specifies that costs depend only upon real money holdings; I would instead specify that the costs depend also on the quantity of transactions conducted (per unit of time, of course). To represent that type of extension, consider the following setup, which is based in part on McCallum (2001).¹⁵

The economy consists of a large number of similar but independently-acting households, each of which has an intertemporal utility function

$$(14) \quad u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + \dots$$

where c_t is consumption in period t . Each household supplies one unit of labor inelastically each period and uses n_t units of labor (the excess, if any, coming from the

¹⁴ Obstfeld and Rogoff say “preferences” because their way of incorporating transaction costs is to include real money balances in agents’ utility function. In Gray’s models both preference and transaction-cost specifications are relevant.

¹⁵ In McCallum (2001) there are a few “typos” including the following: In equations (26) and (27), $\psi_1(c,m)$ should be $\psi_2(c,m)$.

labor market) to produce $f(n_t, k)$ units of output. Its budget constraint for period t is

$$(15) \quad f(n_t, k) - tx_t - w_t(n_t - 1) = c_t + m_t - (1 + \pi_t)^{-1}m_{t-1} + (1 + r_t)^{-1}b_{t+1} - b_t + \psi(c_t, m_t).$$

Here tx_t is lump-sum taxes, w_t is the real wage, m_t is end-of-period real money balances, $1 + \pi_t = P_t/P_{t-1}$ where P_t is the price level in t , b_{t+1} is real private bonds purchased in t , and r_t is the real rate of interest on these bonds. Finally, transaction costs are given by

$\psi(c_t, m_t)$, where $\psi_1 > 0$, $\psi_{11} < 0$, $\psi_2 < 0$, and $\psi_{22} > 0$. Both f and ψ have the Inada properties, as does u .

In this setting, a household's first-order optimality conditions include

$$(16a) \quad u'(c_t) - \lambda_t[1 + \psi_1(c_t, m_t)] = 0$$

$$(16b) \quad -\lambda_t w_t + \lambda_t f_1(n_t, k) = 0$$

$$(16c) \quad -\lambda_t[1 + \psi_2(c_t, m_t)] + \beta E_t \lambda_{t+1}(1 + \pi_{t+1})^{-1} = 0$$

$$(16d) \quad -\lambda_t(1 + r_t)^{-1} + \beta E_t \lambda_{t+1} = 0$$

where λ_t is the Lagrangian multiplier attached to (15). There is a transversality condition (TC) for bonds,

$$(17) \quad \lim_{T \rightarrow \infty} b_{T+1} \beta^{T-1} \lambda_T = 0$$

and another for money:

$$(18) \quad \lim_{T \rightarrow \infty} m_T \beta^{T-1} \lambda_T = 0.$$

The latter two will be taken as necessary for individual-household optimality.¹⁶

For general equilibrium, we must also have

$$(19) \quad n_t = 1$$

¹⁶ The TC (18) can be thought of as arising, from a T -period version of the optimization problem, as a limiting version as $T \rightarrow \infty$ of the second part of the Kuhn-Tucker condition $\partial L/\partial m_T \leq 0$ plus $m_T \partial L/\partial m_T = 0$, since $\partial L/\partial m_T = -\beta^{T-1} \lambda_T$. (Here L refers to the problem's Lagrangian expression.)

$$(20) \quad b_{t+1} = 0$$

$$(21) \quad m_t = M_t/P_t$$

and the government budget constraint¹⁷

$$(22) \quad tx_t + m_t - (1 + \pi_t)^{-1}m_{t-1} = 0.$$

In the latter we take government consumption to equal zero, for simplicity. Thus money is injected via lump-sum transfers (negative taxes). With the government exogenously setting values of M_t , equilibrium paths for the nine endogenous variables c_t , m_t , λ_t , n_t , w_t , P_t , π_t , r_t , and b_{t+1} are given by equations (15), (16a), (16b), (16c), (16d), (19), (20), (21), and the definition $1 + \pi_t = P_{t+1}/P_t$.

In this economy, the demand for money relation is, from equations (16c) and (16d),

$$(23) \quad 1 + \psi_2(c_t, m_t) = \frac{1}{1 + R_t}$$

where the nominal interest rate R_t satisfies the Fisher identity

$$(24) \quad 1 + R_t = (1 + r_t)(1 + \pi_{t+1}).$$

(I ask the reader to please include expectation operators where necessary.) With flexible prices, r_t will be a constant and so as π_{t+1} explodes, $1 + \psi_2(c_t, m_t)$ will approach zero.

That is, $-\psi_2(c_t, m_t)$, the marginal benefit from holding money, will approach 1 from below as m_t/c_t declines with the increasing cost of holding money.

To get a feel for the situation, let's consider the specific transaction-cost function used in McCallum (2001), viz.,

¹⁷ By excluding bonds from (22) I am implicitly assuming that bonds are private loans from one household to another.

$$(25) \quad \psi(c, m) = a_1 c (c/m)^{a_2}, \quad a_1 > 0, a_2 > 0$$

in which the average transaction cost has a constant-elasticity relationship to c/m . Note that this function is one in which the average transaction cost grows without limit as real money balances approach zero. But as inflation explodes, $1 + \psi_2(c_t, m_t) = 1 - a_2 a_1 (c/m)^{1+a_2}$ approaches zero and the limiting condition has $c/m = (1/a_2 a_1)^{1/(1+a_2)}$ so m does not approach zero; it approaches a limiting value (say) $m^* > 0$. The calibration adopted in McCallum (2001) has $a_1 = 0.00102$ and $a_2 = 4$, in which case c/m^* equals 3.96. This calibration is designed to pertain to the quarterly frequency, suggesting that real money holdings in this situation are about 1/4 of quarterly spending. In the recent U.S. data, by contrast, the M1 value of c/m is of the order of magnitude of 1, about four times this limiting value.

The relevance of all this for the issue at hand is that the transversality condition (TC) for money holdings does not fail in the model at hand as inflation explodes. The value of λ_t in our model is, from (16a),

$$(26) \quad \lambda_t = \frac{u'(c_t)}{1 + \psi_1(c_t, m_t)} = \frac{u'(c_t)}{1 + (1 + a_2)a_1(c_t/m_t)^{a_2}},$$

But from our calculations in above, we see that the latter does not grow without bound as inflation explodes; the right-hand side of (26) approaches a positive limit. Then with λ_T remaining finite, violation of the TC (18) will not occur, since both m_T and λ_T remain finite while β^{T-1} approaches zero as the horizon T extends indefinitely.

A heuristic but more general argument can be made without reference to any specific transaction technology, as follows. Transversality conditions for money holdings are in general limiting values (as $T \rightarrow \infty$) of the product of three terms: β^{T-1} , m_T , and λ_T ,

where the latter is a Lagrange multiplier on the budget constraint for period T. If this limit is zero, the TC is satisfied; if it is not for some path, that path is not optimal for the agent. Now clearly β^{T-1} approaches zero as T grows, so for the TC to fail, either m_T or λ_T must explode. Exploding inflation tends, however, to drive the former toward small values. So the only way that the TC can fail is for $\lambda_T \rightarrow \infty$ as $T \rightarrow \infty$.

But since λ_T is a multiplier on the budget constraint, with each term in the latter expressed in real terms, λ_T represents the increase in utility made possible by a one-unit decrease in period-T real money holdings, at the margin. This magnitude will tend to increase as money holdings decrease, but it cannot plausibly increase without bound. Marginal utility of consumption may $\rightarrow \infty$ as $c_t \rightarrow 0$, but a model specification that drives consumption to zero (as real money holdings decrease) implies that a barter economy would necessarily feature zero consumption. That seems to be an inadmissible assumption, however, even to an enthusiast for the MOE role of money. This, I believe, is what lies behind Cochrane's assumption that exploding inflation does not imply the failure of any TC.

5. Fiscal Theory of the Price Level

Cochrane, to his credit, does not end his paper with a conclusion that “anything can happen.” Instead, he invokes the “fiscal theory of the price level,” henceforth FTPL. Specifically, he says “... I conclude that the fiscal theory is the only currently-available economic model that can do so,” that is, “can determine the price level or inflation rate in a fiat-money economy with the sort of interest rate targets that we observe central banks to follow” (2007, p. 44). To do this he must, of course, introduce government bonds. That necessity is somewhat unfortunate for several reasons, the most important being that

one should be able to construct a theory of what happens with a Taylor rule in an economy in which there is no government debt other than money. But let us nevertheless take his suggestion and include government bonds. To be distinguishable from money there must be some transaction-facilitating services provided by the latter but not the former (or more such services provided by the latter). Probably Cochrane would not object to that idea.¹⁸

Continuing, then, let us consider the equilibrium that obtains in an economy in which the central bank follows a Taylor rule that satisfies the Taylor Principle—the case with which we are here concerned—while the fiscal authority is setting government spending exogenously and setting (lump sum) taxes in response to the quantity of outstanding bonds according to a rule of the type discussed by Leeper (1991), Sims (1994), Woodford (1995), and Cochrane (2005). The fiscal authority’s rule may be “active” or “passive,” in Leeper’s terminology. In the latter case the rule is a sensible one, i.e., is one that collects enough (lump sum) taxes to decrease the (per capita) debt [assuming no real growth in output] by some positive fraction from one period to the next. An “active” policy, by contrast, either taxes so aggressively that debt is reduced to zero in the “first” period or else so weakly that the debt grows continually from period to period.¹⁹ Neither of these extremes is, I would contend, relevant to actual economies of decently governed nations, not even the profligate USA of today.²⁰

What happens in such a situation, with an active monetary policy and passive

¹⁸ He has, however, discussed the FTPL in the context of an economy with no money. I have argued elsewhere that this approach negates the whole *raison d’être* of the FTPL, which is to provide a logical mechanism of price level determination that contrasts with the monetary approach, a contrast that is ruled out by assumption if the model includes no money. On this topic, see McCallum and Nelson (2005).

¹⁹ It should be noted that Leeper’s terminology terms monetary policy as “active” when it is sensible, and fiscal policy as “active” when it is extremely expansionary or extremely contractionary. Thus it is not the case that there is a symmetry between AM-PF and PM-AF, AM meaning active money, etc.

²⁰ But they will, nevertheless, be considered momentarily.

fiscal policy? Evans and Honkapohja (2007) have examined that case, with results summarized in McCallum and Nelson (2005). In the relevant region, with sensible monetary and fiscal policy, there is a single stable solution and it is learnable. It features exactly the equilibrium assumed in standard analysis of the NK type.

If, however, the fiscal authority adopts (for some perverse reason) an active—i.e., extreme and non-sensible—rule, then there are two cases. First, with overly aggressive tax collections there is no learnable equilibrium. The analysis predicts that this combination of policies would lead to disorderly behavior that rational analysis is poorly equipped to explain. Secondly, with sensible monetary policy and insufficiently strong tax policy, there is no stable solution. (There is a small region in which bonds explode but money and prices are stationary in a learnable equilibrium, and also a still smaller region in which money, bonds, and prices all explode in a learnable equilibrium²¹). In both of these cases, as McCallum and Nelson (2005) explain, the outcome is exactly what traditional monetarist analysis would predict. Thus the FTPL is not, I contend, helpful in explaining price level behavior in a fiat money economy.

5. Conclusions

It is clearly important that the logical foundations of the dynamic models used in current mainstream monetary policy analysis be clearly understood. Accordingly, Cochrane's recent paper makes a substantial contribution by pointing out that determinacy, in the sense of a unique solution that is dynamically stable, does not provide an adequate foundation for analyses of the standard type. It is the case, however, that a more satisfactory criterion, that of least-squares learnability, is available. Basic informational requirements suggest that learnability should be considered necessary for

²¹ See Evans and Honkapohja (2007, pp. 681-682).

the plausibility of any rational expectations equilibrium,²² and acceptance of this view rules out explosive solutions of the type discussed by Cochrane.²³ A useful by-product of such acceptance is that “indeterminacy” ceases to be a problem in other circumstances, such as strenuous inflation-forecast targeting, not discussed by Cochrane (2007).²⁴

²² Indeed, probably, for the definition of an equilibrium.

²³ In this paper I have focused on Cochrane’s argument and accordingly have not considered more esoteric classes of sunspot solutions.

²⁴ This position is argued by McCallum (2003). Woodford’s (2003b) comment stresses objections to McCallum’s “minimal state variable” solution concept, but features very little disagreement with his paper’s substantive positions (which do not rely upon that concept).

References

- Bullard, J.B. (2006) "The Learnability Criterion and Monetary Policy." *Federal Reserve Bank of St. Louis Review* 88 (3): 203-217.
- Bullard, J.B., and K. Mitra (2002) "Learning about Monetary Policy Rules," *Journal of Monetary Economics* 49, 1105-1129.
- Clarida, R., J. Gali, and M. Gertler (1999) "The Science of Monetary Policy: A New-Keynesian Perspective," *Journal of Economic Literature* 37, 1661-1707.
- Cochrane, J. H. (2005) "Money as Stock," *Journal of Monetary Economics* 52, 501-528.
- _____ (2007) "Inflation Determination with Taylor Rules: A Critical Review." NBER Working Paper 13409.
- Evans, G.W., and S. Honkapohja (2001) *Learning and Expectations in Macroeconomics*. Princeton University Press.
- _____ (2007) "Policy Interaction, Learning, and the Fiscal Theory of Prices," *Macroeconomic Dynamics* 11, 665-690.
- Goodfriend, M. (2006) "How the World Achieved Consensus on Monetary Policy." *Journal of Economic Perspectives* 21(4), 47-68.
- Goodfriend, M., and R.G. King (1997) "The New Neoclassical Synthesis and the Role of Monetary Policy," *NBER Macroeconomics Annual* 12, 231-283.
- Gray, J.A. (1984) "Dynamic Instability in Rational Expectations Models: An Attempt to Clarify," *International Economic Review* 25, 93-122.
- King, R.G. (2000) "The New IS-LM Model: Language, Logic, and Limits," *Federal Reserve Bank of Richmond Economic Quarterly* 86(3), 45-103.

- King, R.G., and A.L. Wolman (1996) "Inflation Targeting in a St. Louis Model of the 21st Century," *Federal Reserve Bank of St. Louis Review* 78(3), 83-107.
- Leeper, E.M. (1991) "Equilibria under 'Active' and 'Passive' Monetary and Fiscal Policies," *Journal of Monetary Economics* 27, 129-147.
- McCallum, B.T. (2001) "Monetary Policy Analysis in Models without Money," *Federal Reserve Bank of St. Louis Review* 83(4), 145-160.
- _____(2003) "Multiple-Solution Indeterminacies in Monetary Policy Analysis," *Journal of Monetary Economics* 50, 1153-1175.
- _____(2007) "E-Stability vis-a-vis Determinacy Results for a Broad Class of Linear Rational Expectations Models," *Journal of Economic Dynamics and Control* 31, 1376-1391.
- McCallum, B.T., and E. Nelson (2005) "Monetary and Fiscal Theories of the Price Level: The Irreconcilable Differences," *Oxford Review of Economic Policy* 21, 565-583.
- Obstfeld, M., and K.. Rogoff (1983) "Speculative Hyperinflations in Maximizing Models: Can We Rule Them Out?" *Journal of Political Economy* 91, 675-87.
- Rotemberg, J.J., and M. Woodford (1997) "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy," *NBER Macroeconomics Annual* 12, 297-346.
- Sims, C.A. (1994) "A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy," *Economic Theory* 4, 381-399.
- Svensson, L.E.O., and M. Woodford (2005) "Implementing Optimal Policy Through Inflation-Forecast Targeting," in B.S. Bernanke and M. Woodford, eds., Inflation Targeting. University of Chicago Press.

- Taylor, John B. (1993) "Discretion versus Policy Rules in Practice," Carnegie-Rochester Conference Series on Public Policy 39, 195-214.
- _____ (1999) "The Robustness and Efficiency of Monetary Policy Rules as Guidelines for Interest Rate Setting by the European Central Bank," Journal of Monetary Economics 43, 655-679.
- Woodford, M. (1995) "Price Level Determinacy without Control of a Monetary Aggregate," Carnegie-Rochester Conference Series on Public Policy 43-1-46.
- _____ (2003a) *Interest and Prices: Foundations for a Theory of Monetary Policy*. Princeton University Press.
- _____ (2003b) "Comment," Journal of Monetary Economics 50, 1177-1188.