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RELATIVE FACTOR PRICE CHANGES  
AND EQUITY PRICES

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Relative Factor Price Changes and Equity Prices

ABSTRACT

This paper suggests that the decline in equity prices, and thus in Tobin's average  $q$ , during the 1970s may be attributable to changes in expected relative factor prices. More specifically,  $q$  is shown to be a negative function of the extent to which current relative factor price expectations differ from those when capital was put in place. Because relative factor prices became more volatile after 1967, the observed decline in average  $q$ , and thus in stock prices, can be explained by the "relative price" hypothesis.

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## INTRODUCTION

A widely documented empirical anomaly of the 1970s is the poor performance of equity prices and consequent decline of average Tobin's  $q$ .<sup>1</sup> While the real value of corporate equity increased 113 percent during the 1957-68 period, it declined 20 percent between 1968 and 1979 in spite of substantial retained earnings. Put another way, every estimate of average  $q$  remains relatively stable until 1968, and then declines approximately 50 percent during the next decade [von Furstenberg (1977), Holland and Myers (1980), President of the U.S. (1983)].

While no shortage of explanations exists for the decline in real equity values, the most viable of these is that the real return on existing capital has declined [Hendershott (1981)]. But why has it declined and is the decline sufficient to produce the observed fall in equity values? Bruno (1984) maintains that the sharp run-up in real materials prices during the 1970s, following a decade of decline, explains the slowdown in manufacturing productivity. A generalization of his argument suggests that unanticipated changes in real factor prices generally (but materials prices most importantly) reduced the profitability of existing capital. Moreover, the reduction appears to have been sufficient to explain the entire decline in average Tobin's  $q$ .

Section I demonstrates that the value of outstanding (vintage) capital declines when relative factor prices change unexpectedly. More specifically, capital values are shown to be a negative function of the extent to which current relative factor price expectations differ from those of earlier periods when capital was put in place.

The data required to simulate the impact of relative factor prices on  $q$  are discussed in Section II. In particular, an aggregate production

function is specified, and input prices are estimated in order to compute constrained and unconstrained cost functions.

Section III contains empirical evidence regarding the importance of unexpected changes in relative factor prices to average  $q$ . First, simulated average  $q$  values are compared graphically with the above cited estimates of average  $q$ . Second, von Furstenberg's  $q$  is shown to be related to our simulated series in the expected way.

## I. THE THEORY

### A. Relative Factor Prices and Economic Obsolescence

The basic argument can be made in terms of a two-factor ex ante production function

$$Y = Y(K, L), \tag{1}$$

that is continuous and strictly quasi-concave.<sup>2</sup> At  $t=0$ ,  $Y$  is the flow of gross output,  $K_0$  is the flow of services from capital equipment and structures, and  $L_0$  is the flow of services from labor. Firm value is maximized by choosing inputs  $L$  and  $K$  so as to

$$\begin{aligned} \min. & \quad k_0 K + w_0 L & (2) \\ \text{s.t.} & \quad (1), \end{aligned}$$

where the prices of capital and labor,  $k_0$  and  $w_0$ , are expected to persist throughout the economic life of  $K_0$ .

Without loss of generality, we set  $Y=1$ . Solving the F.O.C. of (2), the firm's input demands per unit of capacity are

$$\begin{aligned} K_0 &= K(k_0, w_0) & (3) \\ L_0 &= L(k_0, w_0). \end{aligned}$$

Given (3), the firm expects to produce with  $K_0$  assets and incur costs of  $C_0 = k_0 K_0 + w_0 L_0$ . Firm value  $V_0$  is the discounted value of the profit expected from the initial investment

$$V_0 = \sum_{t=1}^{\infty} (1-\tau)(P_0 - C_0)(1-d)^{t-1} / (1+r)^t \quad (4)$$

where  $\tau$  is the statutory marginal corporate tax rate,  $P_0$  is the output price,  $d$  is the economic depreciation rate and  $r$  is the weighted average net real cost of capital. Aspects of the current tax code other than the statutory corporate rate are captured by the capital price  $k_0$  and are discussed below. We assume that the output price  $P$  is set by applying a fixed markup  $N$  to the minimum cost achievable, given expected factor prices.<sup>3</sup> Thus

$$V_0 = (1-\tau)NC_0 / (r+d). \quad (5)$$

At expected prices  $k_0$  and  $w_0$ , the replacement value (cost) of the most efficient technology capable of producing  $Y$ ,  $RC_0$ , equals  $V_0$ . Thus,

$$q_0 = V_0 / RC_0 = 1. \quad (6)$$

Initially, marginal  $q$  and average  $q$  both equal unity because all outstanding capital is new.

Assume that the relative price of labor undergoes an unanticipated change at  $t+1$  that is expected to persist forever. The change in  $w$  produces a new solution to (2). In particular,  $L_1 \gtrless L_0$  as  $w_1 \lesseqgtr w_0$ . Capital demand increases (declines) as  $w$  rises if  $K$  and  $L$  are substitutes (complements). New firms will produce with the new optimal factor mix  $(K_1, L_1)$  at a cost  $C_1 = k_0 K_1 + w_1 L_1$  and will have marginal and average  $q$ 's of unity.

The same will be true of existing firms if they can costlessly change factor mixes. Because the shift entails costs, existing firms will likely not shift to the  $(K_1, L_1)$  mix and their value will decline.

To illustrate this point, say that it is technically impossible to transform  $K_0$  into  $K_1$ ; input substitution is possible for labor, but firms can substitute  $K_1$  only as  $K_0$  decays. Input choice for existing firms is now determined by

$$\begin{aligned} \min. \quad & k_0 K + w_1 L & (7) \\ \text{s.t.} \quad & (1), K = K_0. \end{aligned}$$

Factor demands for the constrained minimization are

$$\begin{aligned} K_1^* &= K(k_0, w_0) & (8) \\ L_1^* &= L(k_0, w_0, w_1). \end{aligned}$$

Firms that have committed themselves to producing with  $K_0$  have costs of  $C_1^* = k_0 K_1^* + w_1 L_1^*$  and a market value of

$$V_1^* = (P_1 - C_1^*)(1 - \tau)/(r + d), \quad (9)$$

where  $P_1 = (1 + N)C_1$ .

Replacement cost at  $t=1$  may be defined in two ways. First, firms may replace the old input set  $(K_0, L_1)$  with  $(K_1, L_1)$ . In a competitive market, inputs are replaced at cost

$$RC_1 = (P_1 - C_1)(1 - \tau)/(r + d), \quad (10)$$

where  $C_1 = k_0 K_1 + w_1 L_1$ . Because new firms cost  $RC_1$ , equation (10) represents an opportunity cost. A second definition of replacement cost assumes that the base period replacement cost  $RC_0$  changes with output prices.

That is, the current replacement cost is  $RC_0 P_1 / P_0$ . This definition is used in all time series estimates of replacement cost.<sup>4</sup> While replacement cost may be defined in either of two ways, our model suggests that the two definitions are equivalent, i.e.,  $RC_0 P_1 / P_0 = RC_1$  because  $NC_1 = P_1 - C_1$ . This is a convenient result because it implies that replacement cost can be estimated with an arbitrarily chosen base period.

Average  $q$  ( $q_1^*$ ) for existing firms is market value [equation (9)] divided by replacement cost [equation (10)] or

$$q_1^* = 1 + N^{-1}(1 - C_1^*/C_1). \quad (11)$$

That is, average  $q$  is negatively related to the cost function ratio  $C_1^*/C_1$ .

Three points are of interest regarding equation (10). First, average  $q$  no longer equals marginal  $q$ . Average  $q$ ,  $q_1^*$ , is less than marginal  $q$ ,  $q_1 = V_1 / RC_1$ , because the constraints found in (6) imply that  $C_1^* > C_1$ . This result is a direct application of Samuelson's (1947) LeChatelier principle and holds for any common production function.<sup>5</sup> The result disputes Hayashi's (1982) conclusion that "marginal  $q$  and average  $q$  are essentially the same" when "the firm is a price-taker." Second, the level of  $q_1^*$  is determined by the extent to which current relative factor prices ( $w_1$ ) diverge from those of previous periods ( $w_0$ ). The tax, depreciation and discount parameters, i.e.,  $\tau$ ,  $d$  and  $r$ , do not effect  $q_1^*$ . As the absolute value of the difference between  $w_0$  and  $w_1$  rises, the cost function ratio rises causing  $q_1^*$  to fall. That is,  $q_1^*$  is inversely related to  $(w_1 - w_0)^2$ .<sup>6</sup> Third,  $q_1^*$  is homogeneous of degree zero in  $(k, w)$ . Therefore, neither inflation nor Hicks-neutral technical

change (k and w rise by the same proportion) effects average q. Only relative factor price changes effect  $q_1^*$ .

Additional constraints on input substitution underscore the above conclusions. For example, a second possible constraint is that capital can only be combined with other inputs in fixed proportions, i.e.,  $K_1=K_0$  and  $L_1=L_0$ . The firm's input demands are now  $K_1^{**}=K_0$  and  $L_1^{**}=L_0$ , so that costs are a linear function of  $w_1$ ,  $C_1^{**}=k_0 k_1^{**}+w_1 L_1^{**}$ , and average q is

$$q_1^{**} = 1 + N^{-1}(1 - C_1^{**}/C_1). \quad (11')$$

Because  $C_1^{**}>C_1^*>C_1$ ,  $q_1^{**}<q_1^*<q_1=1$ . Thus the share price response to a change in relative factor prices rises with the number of constraints on a firm's ability to produce efficiently. Capital with low malleability is "risky" in the sense that its value is especially sensitive to relative factor price fluctuations.

Figure 1a plots cost curves representing the previous optimization problems. At  $w_1=w_0$ , all firms are constrained to using the optimal input set. Hence, the constraints are not effective and all cost curves are tangent. As  $w_1$  either increases or decreases, the effectiveness of the constraints increases. Firms that choose inputs according to (7) move along  $C_1^*$  while those unable to perform any input substitution move along  $C_1^{**}$ . In contrast, costs incurred by new firms lie on  $C_1$ .

Figure 1b shows the value function corresponding to each cost function in 1a. The value of a new firm, relative to replacement cost, is always unity. If  $w_1=w_0$ , then the value of an old firm is also unity because constraints on input substitution are not effective. Old firms are cost efficient because  $w_1$  was correctly anticipated. This implies that marginal q equals average q, i.e.,  $q_1=q_1^*=q_1^{**}$ . If  $w_1 \neq w_0$ , then



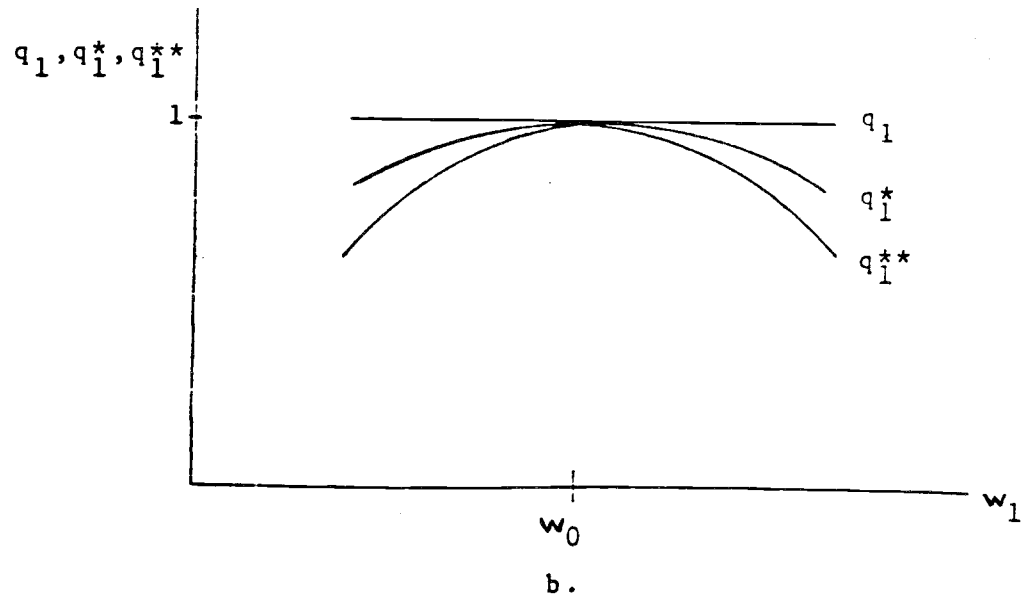
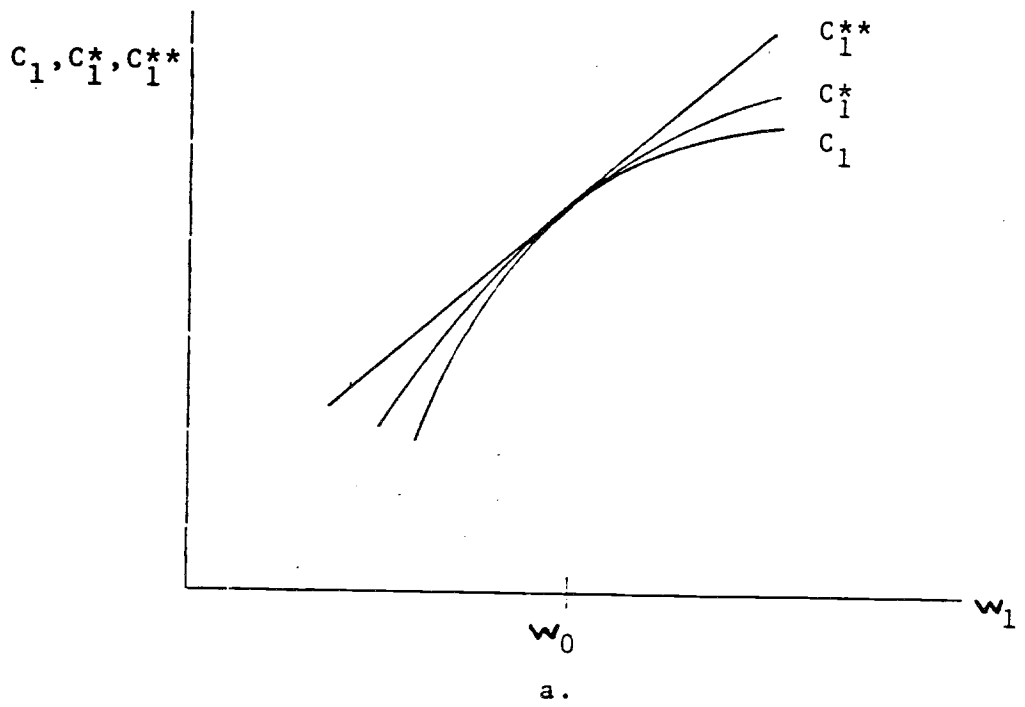


Figure 1  
The Effect of Unexpected Relative Factor Price Changes on Costs and Capital Value

constraints on input substitution are effective. In this case, the value of old firms is less than the value of new firms and average  $q$  is less than marginal  $q$ . Also, the difference between marginal  $q$  and average  $q$  rises (falls) as the number and/or effectiveness of constraints on input substitution rises (falls).

Intuitively, an unanticipated change in relative factor prices renders outstanding production methods economically obsolete, i.e., input combinations become available that can produce a given output more cheaply than combinations currently in use. In a competitive market, the cost advantage held by new input combinations translates into lower output prices, causing the market value of firms using old input combinations to fall.

#### B. Value Over Time

The two period model presented above assumes that all investment occurs at  $t=0$ . In this case, an unexpected relative factor price shock causes the same proportionate value decline for all capital units. If firms purchase capital over time, then the capital stock at a given point in time consists of the investment that remains from previous periods. That is,

$$K_t = \sum_j I_{t-j} (1-d)^j, \quad (12)$$

where  $I_{t-j} = I(k_{t-j}, w_{t-j})$  is the investment made during  $t-j$ . If factor prices  $k_{t-j}$  and  $w_{t-j}$  change over time, then each  $t-j$  investment is unique in that it minimizes costs for a particular set of expected factor prices. Therefore, the character of the capital stock changes as expected relative factor prices change. As old capital decays and new

capital is purchased, the most recent relative factor price expectations will be disproportionately heavily represented in  $K_t$ . However, there will always be a large number of different types of investment, one for each previous relative factor price expectation.

The effect of a relative factor price shock on observed average  $q$  is complicated by a multi-period capital stock  $K_t$ . For the value model described by equation (11'), firm value relative to replacement cost is now written as

$$Q_t^{**} = \sum_j \lambda_{t-j} q_{t-j}^{**}, \quad (13)$$

where  $\lambda_{t-j} = I_{t-j}(1-d)^j / K_t$  and  $q_{t-j}^{**}$  is the relative value of input combinations chosen in period  $t-j$ . Changes in  $Q_t^{**}$  can occur because the  $\lambda_{t-j}$  weights change or because relative factor price movements cause the  $q_{t-j}^{**}$  to change. If current relative factor price expectations differ from those of previous periods, then  $Q_t^{**} < 1$  because  $q_{t-j}^{**} < 1$  for all  $j$ . Also,  $Q_t^{**}$  will tend to rise over time if relative factor price expectations remain stable. If relative factor price expectations are constant throughout the lifetime of all outstanding capital, then  $Q_t^{**} = 1$ .

In practice,  $Q_t^{**}$  will not always decline when expected relative factor prices change. If the new set of expected relative factor prices is closer to the prices that occurred in some previous period, productive capacity purchased in that period will rise in value. Value only declines for that portion of the capital stock that was installed for expected factor prices that became even less likely owing to the unexpected current change.

## II. PRODUCTION SPECIFICATION AND DATA

Estimates of the effect of relative factor price movement on average  $q$  are found by parameterizing the model and testing it. In particular, a production function is specified, and factor prices estimated, in order to compute constrained and unconstrained cost functions. The simulated cost functions are compared, in conjunction with equations (11') and (13), with observed measures of average  $q$ .

### A. Production Specification

For empirical purposes we assume that firms combine capital, labor, energy (E) and materials (M) in a production function with constant elasticity of substitution  $\sigma$ .<sup>7</sup> That is,

$$Y = [\alpha_K K_t^\rho + \sum_i \alpha_i i_t^\rho]^{1/\rho}, \quad i=L,E,M, \quad (14)$$

where  $\rho=(\sigma-1)/\sigma$  and the  $\alpha_i$  are distribution parameters. The CES function implies that per unit capital demand, as a function of input prices  $k_t$  and  $p_{it}$ , is

$$K_t = [\alpha_K + \sum_i \frac{\alpha_i k_t^{\sigma-1}}{\alpha_K p_{it}^{\sigma-1}}]^{-1/\rho}. \quad (15)$$

The demands for labor, energy and materials have a similar form. The cost function for new capital is  $C_t = k_t K_t + \sum_i p_{it} i_t$ .

The cost function associated with old input combinations is determined by the number and effectiveness of constraints on input substitution. Obviously, we do not know the extent to which firms are constrained in their ability to substitute inputs. We have assumed that all capital is

The cost function associated with old input combinations is determined by the number and effectiveness of constraints on input substitution. Obviously, we do not know the extent to which firms are constrained in their ability to substitute inputs. We have assumed that all capital is putty-clay in order to obtain a lower bound on observed capital values.<sup>8</sup> This assumption implies that equation (13) describes average  $q$ .

#### B. The Data

Three sets of data are used to link relative factor price movement to capital values. First, the production parameters found in equation (14) are specified. Second, a time series of the K, L, E, and M input prices are estimated. Finally, the  $\lambda_{t-j}$  from equation (13) are computed.

The production parameters required for equation (14) are the elasticity of substitution  $\sigma$  and  $\alpha_i$  distribution parameters. Berndt (1976) finds that reliable time series estimates of  $\sigma$  fall in the range 1.15 to 1.25. Therefore,  $\sigma$  is set at 1.2. Cost shares are used as a proxy for the  $\alpha_i$  distribution parameters. Cost share estimates are obtained from industry specific input service cost data constructed by Fraumeni (1979).<sup>9</sup> Fraumeni estimates the prices and quantities of capital, labor, energy and material services, for thirty-six sectors, during the 1958-74 period. Yearly cost shares are computed by aggregating the Fraumeni input service cost data, then dividing each aggregate input cost by the total cost of all inputs.<sup>10</sup>

The price of capital is computed as a weighted sum of the prices of structures and equipment. Data from Berndt and Christenson (1973) suggest that the structures and equipment weights are 0.42 and 0.58. The structures and equipment prices are updated and adjusted versions of user cost of capital estimates made by Hendershott and Hu (1981).<sup>11</sup>

adjusting total man-hours worked for changes in educational attainment and intensity of effort.

Because Divisia price indexes for energy and materials are only available on a limited basis, these prices are computed with data from the Crude Materials Price Index. The price of materials is found by deleting crude petroleum used for energy related products from the Crude Non-Food Materials Less Fuel Index.<sup>12</sup> The price of energy is estimated by adding crude petroleum used for energy related products to the Crude Fuel Index.<sup>13</sup> One measure of the quality of our factor price proxies is their correlation with the factor prices estimated by Fraumeni (1979). The correlation coefficient of the energy price series is 0.96; that for the two materials prices is 0.91.

Figure 2 plots our five real factor prices (recall that the price of capital is an average of the equipment and structures prices). The calculated real factor prices are more volatile after 1967 than before. As expected, real energy prices rose dramatically during the 1970s. However, the prices of structures, equipment and materials also fluctuated significantly.

Expected real prices are computed by extrapolating current real price growth rates into the future. A number of real price series were constructed. The maximum decline of  $Q_t^{**}$  is sensitive to the length of time current price changes are assumed to continue into the future. If the current rate of real price change is maintained for a relatively short future period, then  $Q_t^{**}$  falls by a relatively small amount (5-10%). Conversely, if the current rate of change is extrapolated for a lengthy period,  $Q_t^{**}$  declines sharply (50-70%). A series which declines roughly in line with the observed decline in 2 (see Figure 3 below) is obtained

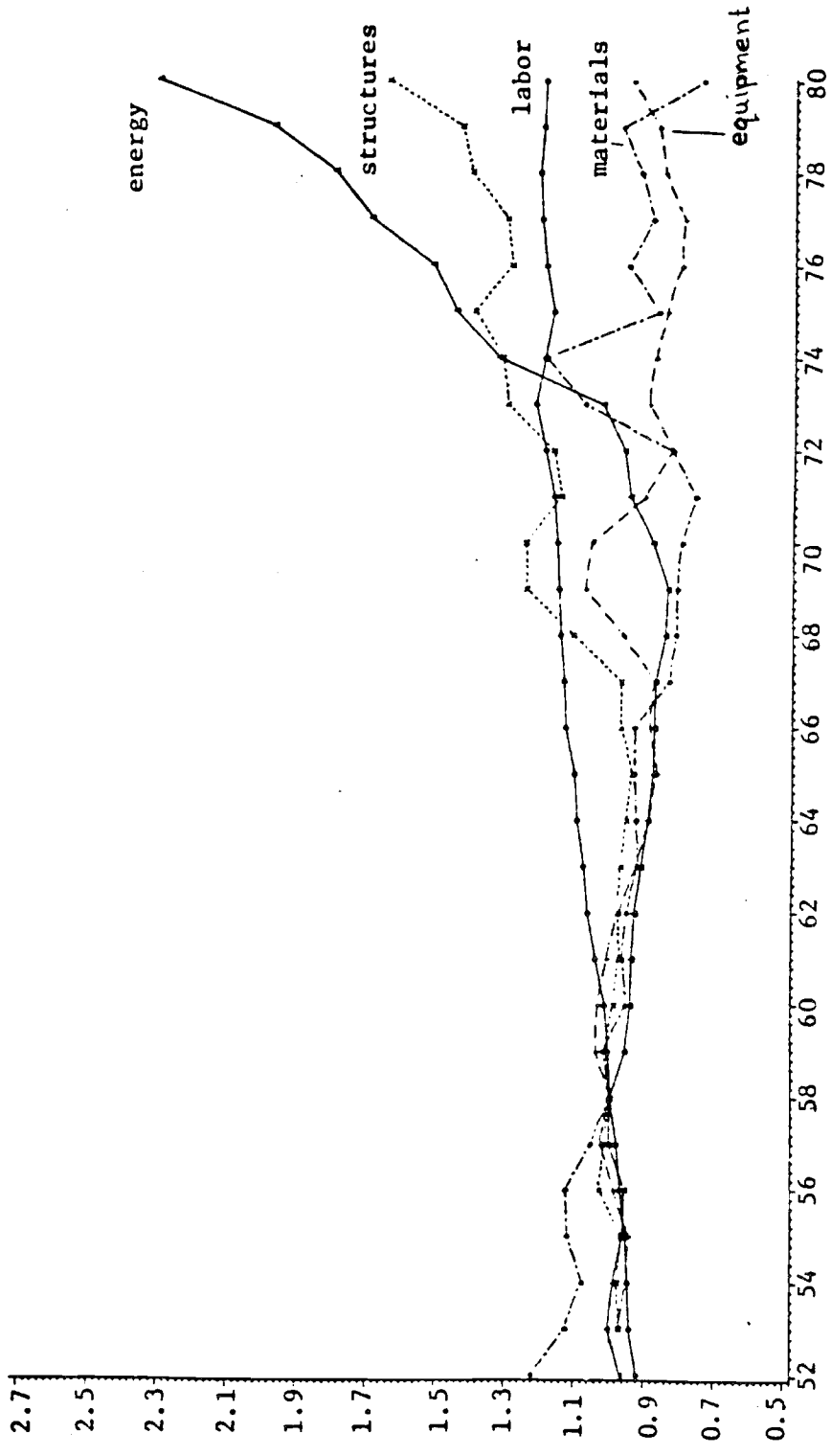


Figure 2

Real Factor Prices, 1952-1980

when real prices are assumed to grow at the current real price growth rate for four years, at one-half of the current growth rate for the following three years, and at one-fourth the current rate in the succeeding three years.<sup>14</sup> (Due to a two to three month reporting lag in aggregate data, each current price is computed from data recorded for the previous quarter.)

The  $\lambda_{t-j}$  weights are calculated from capital stock and investment series used in the Federal Reserve-MIT-Penn (FMP) econometric model. The FMP investment series (expenditures for equipment and structures) is taken directly from the NIA while the capital stock is interpolated from annual stocks computed by the BEA. The economic service lives of equipment and structures are assumed to equal sixteen years and fifty years.

Two problems must be resolved before equation (13) can be estimated. First, the economic lifetimes of equipment and structures imply that a significant portion of the factor price sample is required to initially specify  $Q_t^{**}$ . Indeed, the fifty year effective lifetime of structures is longer than our time series of expected factor prices. Thus, it is never possible for the entire capital stock to be represented in (13). A second problem is that the single-period capital price  $q_{t-j}^{**}$  tends to decline with  $j$ . Because  $Q_t^{**}$  is a weighted sum of the  $q_{t-j}^{**}$ ,  $Q_t^{**}$  is negatively related to the maximum lag length. Therefore, the maximum lag length must be held constant for the entire  $Q_t^{**}$  time series in order to avoid a bias that changes over time.

To finesse the above problems, we assume that the value of the most recent ten years of capital stock is a constant proportion of the value of all capital. As a result, a ten-year sample of expected factor prices may be used to compute an estimate of  $Q_t^{**}$  that is proportional



to, but typically greater than, observed capital prices because the component  $q_{t-j}^{**}$  tend to decline with  $j$ . For the assumed depreciation rates, approximately 58% of the outstanding capital stock is represented in  $Q_t^{**}$ . The first quarter that all expected factor prices are available is 1954:1; thus the first estimate of  $Q_t^{**}$  is for 1964:1.

### III. EMPIRICAL ANALYSIS

The aggregate average  $q$ ,  $Q_t^{**}$ , is calculated by combining equations (11') and (13) with the above described data and an estimate of  $N$ . An estimate of  $N$  is inferred from equation (5). We assume  $\tau=0.48$ ,  $d=0.08$ ,  $r=0.07$  and that an investor purchases \$1.00 of capital when all factor prices are unity. The input demands described in equation (15) are used to calculate  $C_0=3.2$ . Thus,  $N=0.09$ .

Figure 3 plots the simulated  $Q_t^{**}$  and three estimates of average Tobin's  $q$ . The average  $q$  estimates include two series computed with yearly data [President of the U.S. (1983), Holland and Myers (1980)] and one with quarterly data [von Furstenberg (1977)].<sup>15</sup> The yearly data are plotted in the second quarter of each year.

Significant differences exist between the absolute magnitudes of alternative estimates of average  $q$ . For example, von Furstenberg's  $q$  is similar to  $Q_t^{**}$ , but both are far less than the yearly  $q$  estimates. However, our interest is with the movement of  $q$  over time, especially during the 1970s, and the overall trend of  $Q_t^{**}$  is very similar to the trend of the three average  $q$  estimates. All estimates are relatively high during the 1960s, decline and rise during the early 1970s, then decline throughout most of the middle and late 1970s. The vintage capital price  $Q_t^{**}$  was relatively stable from 1963 to 1973, declined

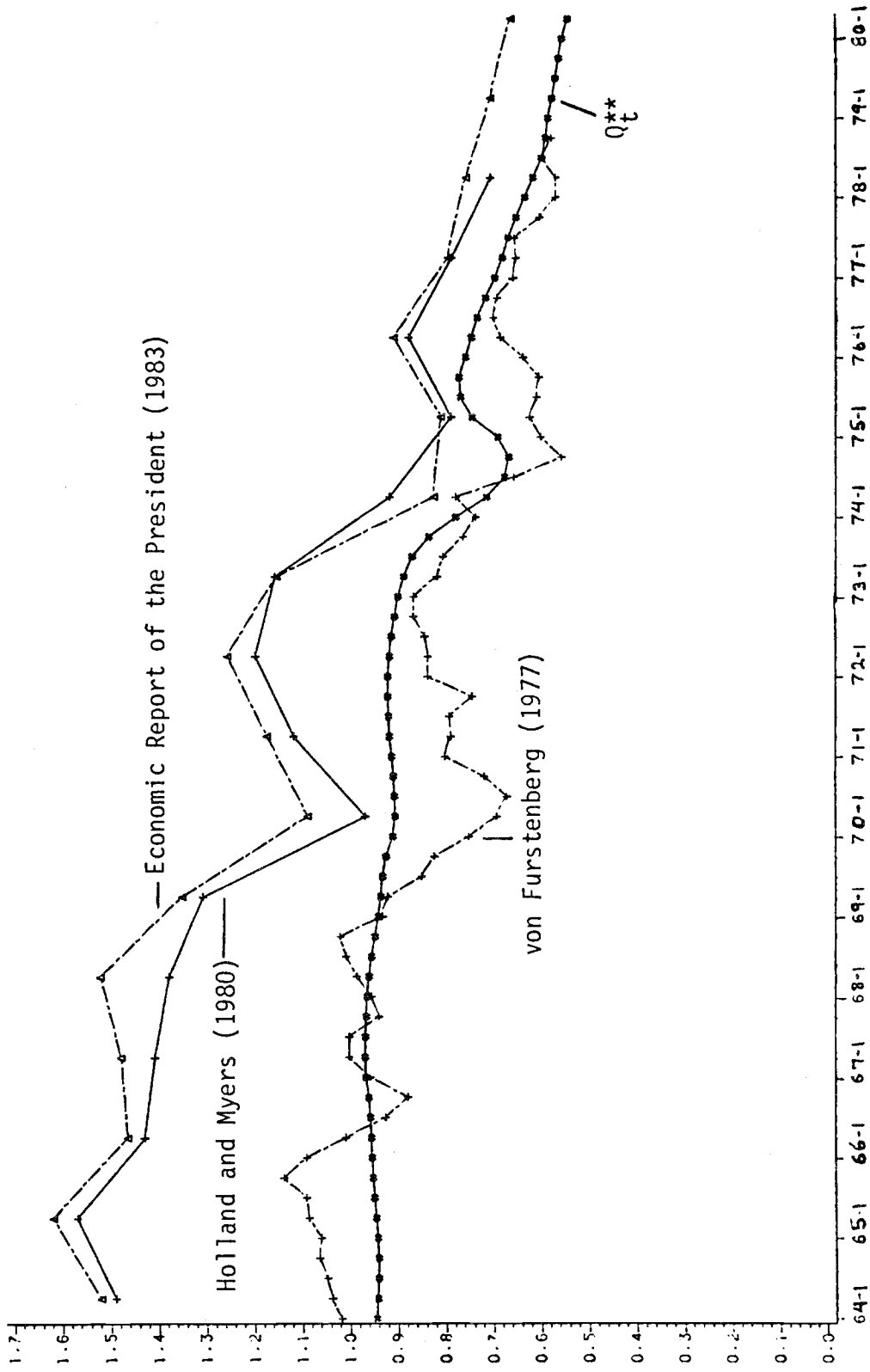


Figure 3

Alternative Estimates of Average Tobin's q, 1964:1-1980:2

sharply in 1974, rose in 1975, then declined from 1976 on. The trend of  $Q_t^{**}$  after the early 1970s is especially similar to the trend of the three average  $q$  estimates.

Alternative simulations of average  $q$  (not shown here) had a trend similar to the Figure 3 estimate. All series peaked during the 1966-1968 period, declined and rebounded between 1973 and 1975, then declined thereafter. As noted above, the final decline was larger the further recent price changes were extrapolated into the future. The variable most responsible for the downward trend was materials prices. Energy prices had very little effect on trend due to the small share of energy in total cost (see footnote 10). This result is consistent with Bruno's (1984) finding that unexpected increases in real materials prices caused the productivity slowdown.

The statistical relation between  $Q_t^{**}$  and an estimate of average  $q$  is found by regressing von Furstenberg's  $q$  ( $VQ_t$ ) on  $Q_t^{**}$ . The OLS estimate is:

$$VQ_t = -0.167 + 1.159Q_t^{**} \quad \bar{R}^2=0.690 \quad D.W.=0.24 \quad (16)$$

(0.087) (0.101)

where the numbers in parentheses are standard errors. The  $Q_t^{**}$  coefficient is significantly greater than zero and indistinguishable from unity. However, the Durbin-Watson statistic is very low. A GLS estimate of the relation between  $VQ_t$  and  $Q_t^{**}$  is:

$$VQ_t = 0.122 + 0.793Q_t^{**} \quad \rho=0.89 \quad \bar{R}^2=0.931 \quad D.W.=1.58 \quad (17)$$

(0.207) (0.250)

The  $Q_t^{**}$  coefficient is smaller in (17) than (16) but is within a standard error of unity and is still statistically greater than zero.<sup>16</sup> We

conclude that relative factor price movements may have been the primary determinant of the observed decline of average  $q$ .

### CONCLUSION

This paper has developed and tested an hypothesis regarding the determination of average Tobin's  $q$ , namely that unanticipated relative factor price changes caused previously optimal outstanding capital to become suboptimal. As a result, the value of the claims that represent ownership of that capital has declined.

Because estimates of five real factor prices were relatively constant prior to 1973 and then increased sharply in volatility, our simulated average  $q$  is relatively constant prior to 1973 and then declines dramatically. With extrapolative price expectations, the decline parallels the movement in previous estimates of Tobin's  $q$ . Materials, rather than energy, price changes are the prime cause of the decline. A simple regression confirms the comparability of the declines in von Furstenberg's  $q$  and our simulated value.

## FOOTNOTES

<sup>1</sup>Tobins's  $q$  is defined as the market value of a firm's liabilities divided by the replacement cost of its assets. The variable has traditionally been used as a tool of investment theory [Brainard and Tobin (1968), Yoshikawa (1980), Hayashi (1982), Fischer and Merton (1984)]. More recent work has utilized  $q$  as a link between the theory of finance and such diverse topics as production theory [Berndt and Fuss (1982)], industrial organization [Lindenbergh and Ross (1981), Lustgarten and Thomadakis (1983)] and taxes [Feldstein (1981)].

<sup>2</sup>Continuity is a convenient assumption because it allows us to consider very small changes in factor prices. One alternative would be to assume that output is produced by a series of activities of the form

$$Y = \min (K/a_K, L/a_L)$$

$$Y = \min (K/b_K, L/b_L)$$

·  
·  
·

where  $a_i, b_i, \dots$  are constants. In this case, firms produce  $Y$  with the cheapest activity, i.e., the input demands are corner solutions. If relative factor prices change, then firms switch to new activities in a discrete fashion.

<sup>3</sup>See Bischoff (1971) or Ando et al. (1975) for examples of the use of this assumption.

<sup>4</sup>An example will illustrate the methodology used to estimate replacement cost. In March 1974 the base sticker price of a Cadillac, Sedan De Ville was \$7,885, whereas six years later it was \$13,282. All estimates of replacement cost assume that the 1980 replacement cost of an undepreciated 1974 Cadillac is \$13,282. No replacement cost estimate adjusts for the fact that the 1980 Cadillac combines inputs in a very different fashion than the 1974 Cadillac.

In actual practice, an aggregate price index would be used to inflate the 1974 Cadillac price. Different aggregate price indexes necessarily lead to different 1980 replacement cost estimates. Salinger and Summers (1983) would use the CPI, Lindenbergh and Ross (1981) would employ the GNP deflator while the BEA would apply a deflator that relates to a class of investments that includes Cadillacs.

<sup>5</sup>The relation between  $C_1^*$  and  $C_1$  can be seen by comparing the respective cost-minimization problems.<sup>1</sup> Because (7) is a constrained form of (2), solving (2) is a necessary, but not sufficient, condition for the solution of (7). Therefore, the unconstrained cost  $C_1$  is never greater than the constrained cost  $C_1^*$ . The minimum difference,  $C_1^* - C_1 = 0$ , occurs when the constraint  $K=K_0$  is "just" binding, i.e., when  $K_1^* = K_1$ .

The LeChatelier principle predicts the above results by suggesting that constraints on a system that is minimizing (maximizing) some objective function can only raise (lower) the value of the function. The fact that  $V_1^* < V_1$  is a consequence of the addition of constraints on input substitution.

<sup>6</sup>An inverse relation between  $q_1^*$  and  $(w_1 - w_0)^2$  implies that a standard variance measure may be used as a proxy for  $q_1^*$ . In particular, variance indexes used to measure relative output price variation [Fischer (1981), Parks (1978)] may also be used to link average  $q$  to relative factor price variation [Elmer (1983)].

<sup>7</sup>The CES function is convenient due to the availability of relevant parameter estimates [Berndt (1976)]. Bruno (1984) uses a similar form.

<sup>8</sup>The assumption that old capital is pure clay represents the most restrictive possible case. Therefore, the assumption induces a downward bias in our simulated capital values (see Figure 3).

However, the pure clay capital model is justified by at least three arguments. First, the choice of any particular subset of input substitution constraints is arbitrary. That is, we have no way of determining which of an infinity of possible constraints is most appropriate. Second, by simulating the most restrictive possible case, we estimate a lower bound to observed capital values. This implies that more realistic capital price estimates can be obtained by computing a weighted average of the simulated capital prices and unity. Finally, the downward bias acts to offset an upward bias that arises from measurement error (discussed below).

<sup>9</sup>We wish to thank Barbara Fraumeni for kindly supplying these data.

<sup>10</sup>The following data compare Elmer's (1983) average cost share estimates with estimates made by Berndt and Wood (1975):

|             | Capital | Labor | Energy | Materials |
|-------------|---------|-------|--------|-----------|
| Elmer       | 0.205   | 0.343 | 0.036  | 0.416     |
| Berndt-Wood | 0.053   | 0.274 | 0.045  | 0.628     |
| (Average)   | 0.129   | 0.308 | 0.040  | 0.522     |

It is clear that significant differences exist between the two estimates. For example, the Berndt-Wood capital share is one-fourth the size of Elmer's capital share and has an absolute magnitude of only 0.053. The Elmer-Fraumeni estimates are computed from a more detailed up-to-date data base than the Berndt-Wood estimates.

<sup>11</sup>A complete description of the factor prices is found in Elmer (1983), Chapter 4.

<sup>12</sup>During the 1970-74 period, approximately 13% of total crude petroleum was used for non-energy related products [OECD Statistics of Energy (1975)]. Therefore, crude petroleum used for energy related product is estimated as 87% of the crude petroleum component of the Crude Non-Food Materials Less Fuel Index.

<sup>13</sup>Removing crude petroleum from the Crude Non-Food Materials Less Fuel Index, then adding it to the Crude Fuel Index, required information on the relative importance of crude petroleum in each index. The Bureau of Labor Statistics has published relative importance data for the years 1957, 1960, 1966 and 1969-present. Our series assumes that the relative importance of crude petroleum figures remained constant prior to 1957 and interpolates for the periods 1958-59, 1961-66 and 1967-68.

<sup>14</sup>If the current average relative factor price is 1.0 and the current quarterly growth rate is 0.01, the expected relative price for next quarter would be  $1.01^1$ , the price expected 17 quarters hence is  $1.01^{16.5}$  and the price expected 29 quarters out is  $1.01^{22.25}$ .

<sup>15</sup>The von Furstenberg  $q$  estimates are a revised and updated version of the original series. We wish to thank George von Furstenberg for supplying these data.

<sup>16</sup>The presence of inventories among corporate assets makes the estimated coefficient on  $Q_t^{**}$  even more acceptable. If the inventory/capital ratio is stable, then the  $Q_t^{**}$  coefficient will be less than unity by the ratio of inventories to inventories plus capital. This ratio was stable from 1960 to 1980 and approximately equal to 0.2. Therefore, the  $Q_t^{**}$  coefficient is expected to equal 0.8.

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