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THE USE OF EXPECTED FUTURE VARIABLES IN MACROECONOMETRIC MODELS

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ABSTRACT

A more sophisticated expectational hypothesis than is traditionally used in the specification of macroeconometric models is tested in this paper. Economic agents are assumed to use a vector of variables Z_t in forming their expectations for periods $t+1$ and beyond. These expectations may or may not be rational in the Muth sense. The results provide some evidence in favor of the more sophisticated hypothesis, but they are not strong enough to allow much weight to be put on the hypothesis as yet. The evidence in favor of the hypothesis is strongest for households' response to future wages and prices in their consumption and labor supply decisions and for the Fed's response to future inflation rates.

The sensitivity of the policy properties of my macroeconometric model to the more sophisticated hypothesis is also examined in the paper. The properties are not sensitive for a policy action in which government expenditures are changed. They are somewhat sensitive for an action in which personal tax rates are changed. In the latter case the properties are also sensitive to whether or not the policy action is anticipated.

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by

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I. Introduction

It is common practice in traditional macroeconometric model building to use current and lagged values as proxies for expected future values. It is well known that this procedure may not approximate well the way in which expectations are actually formed. The purpose of this paper is to consider an alternative procedure. Economic agents are assumed to use a vector of variables Z_t in forming their expectations for periods $t+1$ and beyond. These expectations may or may not be rational in the Muth sense.

If expectations are rational in the Muth sense, the most efficient estimation technique is full information maximum likelihood (FIML). This technique accounts for the cross equation restrictions that are implied by the rational expectations hypothesis, which is where most of the testable implications of the hypothesis lie. Unfortunately, the estimates are expensive to compute. For nonlinear models the only known computational method is the Fair-Taylor (1983) method, and this method is not currently computationally feasible for large models.

There are limited information alternatives to FIML, and these are the techniques used in this paper. These techniques are relatively inexpensive. They are also more robust than FIML in that unlike FIML they retain their consistency when expectations are not rational. All they require for consistency is that Z_t be among the variables used by the agents in forming their expectations. Agents can use variables other than those in Z_t . Also, the complete set of variables used by the agents need not include all the predetermined variables in the model.

The two main limited information techniques are those of Hansen (1982) and Hayashi and Sims (1983). The application of these methods to the present problem is not completely straightforward, and so the exact way in which these methods were used is explained in Section II. In particular, some of the equations that are estimated in this paper have a first order autoregressive error term (in addition to the moving average error term that arises from the expectational assumptions), and the way in which Hansen's method was set up to handle this problem needs to be explained.

The expectational hypothesis is tested in Section III using my U.S. macroeconometric model (Fair (1984)). The basic version of this model merely uses current and lagged values to proxy for expected future values. The aim of the work in Section III is to see if a more sophisticated expectational hypothesis leads to better results. Section IV examines the sensitivity of the properties of the model to the alternative expectational hypothesis. The properties of the basic version of the model are compared to the properties of the version of the model that uses the more sophisticated hypothesis. Section V contains a brief conclusion.

II. The Limited Information Techniques

The Basic Model

Consider the model

(1) $$y_t = X_{1t}\alpha_1 + {}_{t-1}X^e_{2t+i}\alpha_2 + u_t ,$$

(2) $$u_t = \rho u_{t-1} + \eta_t , \quad t = 1, \ldots, T .$$

X_{1t} is a $1 \times p-1$ vector of non expectational explanatory variables, α_1 is a $p-1 \times 1$ vector of unknown coefficients multiplying these variables, ${}_{t-1}X^e_{2t+i}$ is the expectation of X_{2t+i} made at the end of period $t-1$, α_2 is an unknown coefficient multiplying this variable, and u_t is an error term that is assumed to follow a first order autoregressive process. It is easy to generalize the following discussion to have the model include more than one expectational variable and include a higher order autoregressive process for the error term. Many structural equations in macroeconometric models have autoregressive errors, and this is the reason for including (2) as part of the basic model. The length ahead parameter i is assumed to be fixed. Again, it is easy to generalize the model to include more than one value of i for the same variable. The y and X variables can be nonlinear transformations of endogenous and predetermined variables, and so equation (1) can be nonlinear in variables.

The Case of No Autoregressive Structural Error ($\rho = 0$)

Let the expectation error for ${}_{t-1}X^e_{2t+i}$ be

(3) $${}_{t-1}\varepsilon_{t+i} = X_{2t+i} - {}_{t-1}X^e_{2t+i} ,$$

where X_{2t+i} is the actual value of the variable. Substituting (3) into

(1) yields

$$y_t = X_{1t}\alpha_1 + X_{2t+i}\alpha_2 + u_t - {}_{t-1}\varepsilon_{t+i}\alpha_2 \tag{4}$$

$$= X_t\alpha + v_t ,$$

where $X_t = (X_{1t} \;\; X_{2t+i})$, $\alpha = \begin{pmatrix}\alpha_1\\ \alpha_2\end{pmatrix}$, and $v_t = u_t - {}_{t-1}\varepsilon_{t+i}\alpha_2$.

Consider first the two-stage least squares (2SLS) estimation of equation (4). Let Z_t be a vector of dimension k of first stage regressors. A necessary condition for consistency is that Z_t and v_t be uncorrelated. This will be true if both u_t and ${}_{t-1}\varepsilon_{t+i}$ are uncorrelated with Z_t . The requirement that Z_t and u_t be uncorrelated is the usual 2SLS requirement. The requirement that Z_t and ${}_{t-1}\varepsilon_{t+i}$ be uncorrelated involves an additional assumption, which is that the variables in Z_t have been used (perhaps along with others) in forming the expectation of X_{2t+i} . As noted in the Introduction, this assumption does not require that the expectations be rational. It merely requires that all the information contained in the Z_t variables be used. Given this assumption (and the other standard assumptions that are necessary for consistency), the 2SLS estimator of α in equation (4) is consistent. This estimator (denotes α_{2SLS}) is:

$$\alpha_{2SLS} = \left(X'Z(Z'Z)^{-1}Z'X\right)^{-1} X'Z(Z'Z)^{-1}Z'y , \tag{5}$$

where α_{2SLS} is $p \times 1$, X is $T \times p$, Z is $T \times k$, and y is $T \times 1$. The application of the 2SLS estimator to models of this type is due to McCallum (1976).

The standard formula for the covariance matrix of α_{2SLS} is not correct for i greater than 0 because in this case v_t is serially correlated. If, for example, i is 2, an unanticipated shock in period t will affect ${}_{t-1}\varepsilon_{t+2}$, ${}_{t-2}\varepsilon_{t+1}$, and ${}_{t-3}\varepsilon_t$, and so v_t will be a second order moving average. In general, v_t will be a moving average of order i.[1] Both the Hansen (1982) and Hayashi-Sims (1983) methods account for this moving average process.

Consider first the Hayashi-Sims estimator, which will be denoted α_{HS}. Let V be the covariance matrix of $v = (v_1, \ldots, v_T)'$. v is $T \times 1$ and V is $T \times T$. The idea of Hayashi and Sims is to find an upper triangular matrix W such that $WVW' = I$ and then to transform the data using W. Let $y^* = Wy$ and $X^* = WX$. The HS estimator of α is simply 2SLS applied to y^* and X^*:

$$\alpha_{HS} = \left(X^{*\prime}Z(Z'Z)^{-1}Z'X^*\right)^{-1} X^{*\prime}Z(Z'Z)^{-1}Z'y^* \ . \tag{6}$$

The estimated covariance matrix for α_{HS} is

$$\left(X^*Z'(Z'Z)^{-1}Z'X^*\right)^{-1} \ . \tag{7}$$

Taking W to be upper rather than lower triangular means that the transformations are with respect to current and future values rather than current and past values. This allows the transformed error term Wv to remain uncorrelated with all the current and past values of the variables in Z.

Computing α_{HS} is straightforward once an estimate of V is available. Given V, W can be computed numerically, and the rest is simply

[1]Note that it is assumed here that expectations are based on information through period $t-1$, not t. If information through period t were used, the order of the moving average would be $i-1$.

matrix calculations. V can be estimated using the 2SLS estimates of v_t, which are

(8) $$\hat{v}_t = y_t - X_t\alpha_{2SLS}\,, \quad t = 1, \ldots, T\,.$$

Estimates of the diagonal elements of V are $T^{-1}\sum_{t=1}^{T}\hat{v}_t^2$; estimates of the elements once removed from the diagonal are $(T-1)^{-1}\sum_{t=2}^{T}\hat{v}_t\hat{v}_{t-1}$; and so on through estimates of the elements removed i places from the diagonal, which are $(T-i)^{-1}\sum_{t=i+1}^{T}\hat{v}_t\hat{v}_{t-i}$.

Two versions of Hansen's estimator are considered in this paper, denoted α_{H1} and α_{H2}. The estimators differ in the estimation of the M matrix below. The estimators are

(9) $$\alpha_{H1} = \alpha_{H2} = \left(X'ZM^{-1}Z'X\right)^{-1}X'ZM^{-1}Z'y\,,$$

where M is some estimate of $\lim T^{-1}E[Z'vv'Z]$. The estimated covariance matrix of α_{H1} and α_{H2} is

(10) $$T\cdot\left(X'ZM^{-1}Z'X\right)^{-1}\,.$$

The general way of estimating M is as follows. Let $f_t = \hat{v}_t \otimes Z_t$, where $\hat{v}_t$ is computed from (8). Let $R_j = (T-j)^{-1}\sum_{t=j}^{T}f_t f'_{t-j}$, where $j = 0, 1, \ldots, i$, where i is the order of the moving average. The estimate of M is $(R_0 + R_1 + R'_1 + \ldots + R_i + R'_i)$. The estimator based on this way of estimating M is denoted α_{H1}. In many cases estimating M in this way does not result in a positive definite matrix, and so α_{H1} cannot be computed. Fortunately, there is an alternative estimate of M available under the assumption that

$$(11) \qquad E[v_t v_s | Z_t, Z_{t-1}, \ldots] = E[v_t v_s] \quad \text{for} \quad t \geqq s ,$$

which says that the contemporaneous and serial correlations in v do not depend on Z. The HS estimator is based on this assumption. This assumption is implied by the assumption that $E[v_t Z_s] = 0$ for $t \geqq s$ if normality is also assumed. Under this assumption M can be estimated as follows. Let $a_j = (T-j)^{-1}\sum_{t=j}^{T}\hat{v}_t\hat{v}_{t-j}$ and $B_j = (T-j)^{-1}\sum_{t=j}^{T}Z_t Z'_{t-j}$, where $j = 0, 1, \ldots, i$. The estimate of M is $(a_0B_0 + a_1B_1 + a_1B'_1 + \ldots + a_iB_i + a_iB'_i)$. The estimator based on this way of estimating M is denoted α_{H2}.

Hayashi and Sims (1983) show that without more information on the determination of Z, it is not in general possible to determine the relative efficiency of their estimator and Hansen's estimator. Both estimators are consistent under fairly general regularity conditions. Hayashi and Sims show that consistency of their estimator is retained when the population V is replaced with a consistent estimate and that consistency of Hansen's estimator is retained when the population M is replaced with a consistent estimate.

The Case of an Autoregressive Structural Error ($\rho \neq 0$)

Lagging (1) by one period, multiplying through by ρ, and subtracting the resulting expression from (1) yields

$$(12) \quad y_t = \rho y_{t-1} + X_{1t}\alpha_1 - X_{1t-1}\alpha_1\rho + {}_{t-1}X^e_{2t+i}\alpha_2 - {}_{t-2}X^e_{2t+i-1}\alpha_2\rho + \eta_t .$$

Note that this transformation yields a new viewpoint date, $t-2$. Let the expectation error for ${}_{t-2}X^e_{2t+i-1}$ be

(13) $$_{t-2}\varepsilon_{t+i-1} = X_{2t+i-1} - {}_{t-2}X^{e}_{2t+i-1} .$$

Substituting (3) and (13) into (12) yields

(14) $$\begin{aligned} y_t &= \rho y_{t-1} + X_{1t}\alpha_1 - X_{1t-1}\alpha_1\rho + X_{2t+i}\alpha_2 - X_{2t+i-1}\alpha_2\rho \\ &\quad + \eta_t - {}_{t-1}\varepsilon_{t+i}\alpha_2 + {}_{t-2}\varepsilon_{t+i-1}\alpha_2\rho \\ &= \rho y_{t-1} + X_t\alpha - X_{t-1}\alpha\rho + v_t , \end{aligned}$$

where $X_t = (X_{1t} \;\; X_{2t+i})$, $\alpha = \begin{pmatrix}\alpha_1 \\ \alpha_2\end{pmatrix}$, and $v_t = \eta_t - {}_{t-1}\varepsilon_{t+i}\alpha_2 + {}_{t-2}\varepsilon_{t+i-1}\alpha_2\rho$. Equation (14) is nonlinear in coefficients because of the introduction of ρ .

Given a set of first stage regressors, equation (14) can be estimated by 2SLS. The estimators are obtained by minimizing

(15) $$v'Z(Z'Z)^{-1}Z'v = v'Dv ,$$

where v is $T \times 1$ and Z is $T \times k$. Z is the matrix of observations of the first stage regressors. A necessary condition for consistency is that Z_t and v_t be uncorrelated, which means that Z_t must be uncorrelated with η_t , $_{t-1}\varepsilon_{t+i}$, and $_{t-2}\varepsilon_{t+i-1}$. In order to insure that Z_t and $_{t-2}\varepsilon_{t+i-1}$ are uncorrelated, Z_t must not include any variables that were not known as of period $t-2$. This is an important additional restriction in the autoregressive case.[2]

[2]Cumby, Huizinga, and Obstfeld (1983), p. 341, incorrectly assert that the instrument set must be moved backward in time as the order of the moving average process for v_t increases and that higher order autoregressive properties of u_t can be handled merely by appropriate quasi differencing. In fact, the instrument set must be moved backward in time as the order of the autoregressive process increases. It need not be moved backward as the order of the moving average process increases.

The estimator that is based on the minimizing of (15) is Amemiya's (1974) nonlinear 2SLS estimator. In the general case (15) can be minimized using a general purpose algorithm. In the particular case considered here a simple iterative procedure can be used, where one iterates between estimates of α and ρ . Minimizing $v'Dv$ with respect to α and ρ results in the following first-order conditions:

$$(16) \qquad \hat{\alpha} = [(X - X_{-1}\hat{\rho})'D(X - X_{-1}\hat{\rho})]^{-1}(X - X_{-1}\hat{\rho})'D(y - y_{-1}\hat{\rho}) \ ,$$

$$(17) \qquad \hat{\rho} = \frac{(y_{-1} - X_{-1}\hat{\alpha})'D(y - X\hat{\alpha})}{(y_{-1} - X_{-1}\hat{\alpha})'D(y_{-1} - X_{-1}\hat{\alpha})} \ ,$$

where the -1 subscript denotes the vector or matrix of observations lagged one period.[3] Equations (16) and (17) can be easily solved iteratively. Given the estimates $\hat{\alpha}$ and $\hat{\rho}$ that solve (16) and (17), one can compute the 2SLS estimates of v_t , which are

$$(18) \qquad \hat{v}_t = y_t - \hat{\rho}y_{t-1} - X_t\hat{\alpha} + X_{t-1}\hat{\alpha}\hat{\rho} \ , \quad t = 1, \ldots, T \ .$$

Now, given $\hat{v}_t$, one can compute M for Hansen's estimator in either of the two ways discussed above. These calculations simply involve $\hat{v}_t$ and Z_t , $t = 1, \ldots, T$. Given M , Hansen's estimates of α and ρ are obtained by minimizing[4]

$$(19) \qquad v'ZM^{-1}Z'v = v'Cv \ .$$

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[3]Data for period 0 are assumed to exist so that y_{-1} can be taken to be $T \times 1$ and X_{-1} can be taken to be $T \times p$.

[4]The estimator that is based on the minimization of (19) is the 2S2SLS estimator in Cumby, Huizinga, and Obstfeld (1983). Since this estimator is a special case of Hansen's generalized method of moments estimator, I have simply referred to it as Hansen's estimator in this paper.

Minimizing (19) with respect to α and ρ results in the first order conditions (16) and (17) with C replacing D . The estimated covariance matrix is

$$(20) \qquad T\cdot(G'CG)^{-1} ,$$

where $\qquad G = (X - X_{-1}\hat{\rho} \quad y_{-1} - X_{-1}\hat{\alpha})$.

To summarize, Hansen's method in the case of a first order autoregressive structural error consists of: 1) choosing Z_t so that it does not include any variables not known in period t-2 , 2) solving (16) and (17), 3) computing $\hat{v}_t$ from (18), 4) estimating M in one of the two ways, and 5) solving (16) and (17) with C in (19) replacing D . The two estimators, based on the two estimates of M , will be denoted α_{H1R} and α_{H2R} .

The possible application of the Hayashi-Sims estimator to the case of an autoregressive structural error is not pursued here; only α_{H1R} and α_{H2R} have been used for the results in Section III.

As a final note on all the methods, one can iterate further than the discussion in this section has so far indicated. Given $\hat{\alpha}_{HS}$, $\hat{\alpha}_{H1}$, or $\hat{\alpha}_{H2}$, $\hat{v}_t$ in (8) can be recomputed, and then the estimation procedure can be repeated for the new values of $\hat{v}_t$. The new coefficient estimates can then be used to compute new values of $\hat{v}_t$, and so on. The whole process can be repeated until convergence (assuming the process converges). Similarly, given $\hat{\alpha}_{H1R}$ or $\hat{\alpha}_{H2R}$, $\hat{v}_t$ in (18) can be recomputed, M can be recomputed, and then (19) can be minimized for the new estimate of M . This whole process can also be repeated. This further way of iterating has not been used for the results in this paper.

III. Tests of the Expectational Hypothesis

The hypothesis that agents use Z_t in forming their expectations is tested in this section within the context of my macroeconometric model (Fair (1984)). The theory behind the model is that agents make decisions by solving multiperiod optimization problems. Before solving these problems agents must form expectations of future values of a number of variables. The key decision variables for households are consumption and leisure, and the key decision variables for firms are prices, wages, production, investment, and employment demand. A typical estimated equation in the econometric model has a decision variable on the left hand side and variables that are assumed to affect the decision on the right hand side. Current and lagged values of variables are used as proxies for expected future values. The tests in this paper consist of adding future values to the right hand side of these equations. Before discussing the tests, however, the use of lagged dependent variables and the treatment of possible constraints on the decision variables in the model should be explained.

Use of Lagged Dependent Variables

Lagged dependent variables are important explanatory variables in the model. The use of these variables can be justified by appeal to the simple partial adjustment model. Assume that equation (1) in Section II holds with no error term and with y_t^* replacing y_t on the left hand side, where y_t^* is the "desired" value of y_t. Assume that y_t only partially adjusts to y_t^* each period, with adjustment coefficient γ :

$$y_t - y_{t-1} = \gamma(y_t^* - y_{t-1}) + u_t \ . \tag{21}$$

The equation for y_t^* and equation (21) can be combined to yield

$$(22) \qquad y_t = X_{1t}\alpha_1\gamma + {}_{t-1}X^e_{2t+i}\alpha_2\gamma + (1-\gamma)y_{t-1} + u_t \; .$$

This procedure has simply added y_{t-1} to the right hand side of equation (1). If expectations are such that ${}_{t-1}X^e_{2t+i}$ is simply proportional to X_{2t} (say, ${}_{t-1}X^e_{2t+i} = \beta X_{2t}$), then (22) becomes

$$(23) \qquad y_t = X_{1t}\alpha_1\gamma + X_{2t}\beta\alpha_2\gamma + (1-\gamma)y_{t-1} + u_t \; .$$

Equation (23) is in a form that can be estimated. It is not possible to identify β and α_2 separately, but for many problems this is not important.

It is well known that there is another justification for the use of lagged dependent variables, which is that expectations are a geometrically declining function of current and past values. Assume that

$$(24) \qquad {}_{t-1}X^e_{2t+i} = \lambda X_{2t} + \lambda^2 X_{2t-1} + \lambda^3 X_{2t-2} + \dots \; .$$

Equations (1) and (24) yield

$$(25) \qquad y_t = (X_{1t} - X_{1t-1}\lambda)\alpha_1 + X_{2t}\alpha_2\lambda + \lambda y_{t-1} + u_t - \lambda u_{t-1} \; .$$

Equation (25) differs from (23) in that it has a different error term and it has $X_{1t} - X_{1t-1}\lambda$ in place of X_{1t} . In practice these two differences are sometimes ignored, and one appeals rather casually to either the partial adjustment model or the model in which expectations are a geometrically declining function of current and past values as a justification for using the lagged dependent variable as an explanatory variable. It should be clear in the present case, however, that appeal must be made to the partial adjustment model. The basic expectational hypothesis of this paper will be tested by adding future values to equation (22). This procedure is incompatible with the assumption that expectations are a geometrically declining function of current and past values.

Treatment of Constraints

Expectations of individual agents in the model are not assumed to be rational, and expectation errors can lead to situations of disequilibrium. The main form of disequilibrium is that households may be constrained from working as much as they would like. This constraint was handled in the following way in the empirical work. Consider as an example the consumption of services, denoted CS_t . Let $CSUN_t$ denote the consumption of services that the household sector would choose if it were not constrained in how many hours it can work. $CSUN_t$ is a function of the variables that influence the households' optimization problems:

$$(26) \qquad CSUN_t = f(\ldots) ,$$

where the variables in f are assumed to be observed. If the household sector is not constrained, then CS_t equals $CSUN_t$, and there is no estimation problem. If the household sector is constrained, then CS_t is less than $CSUN_t$ if, as in the theoretical model, binding labor constraints cause the household sector to consume less than it would have unconstrained. Now, if one can find a variable, say Q_t , such that

$$(27) \qquad CS_t = CSUN_t + \gamma Q_t , \quad \gamma > 0 ,$$

then one has immediately from (26) and (27) an equation in observed variables. Given this setup, the problem for the empirical work is finding a variable Q_t for which the specification in (27) seems reasonable. The variable Q_t should take on a value of zero when labor markets are tight and households are not constrained and a value less than zero otherwise. When the variable is less than zero, it should be a linear function of the difference between the constrained and unconstrained decision values. For the empirical work Q_t was taken to be a particular nonlinear function of a measure of labor market tightness.

Household Equations

Explanatory variables in the decision equations for the household sector that are meant to be proxies for expected future values are: 1) the after-tax nominal wage rate, 2) the price level, 3) the after-tax short-term or long-term interest rate, and 4) after-tax nonlabor income. These variables enter the equations with either no lag or a lag of one quarter, depending on which gave the better results. Other variables in the equations include the initial value of wealth, the labor constraint variable, and the lagged dependent variable. The equations are estimated by 2SLS, where the first stage regressors include the main predetermined variables in the model.[5] The tests in this paper consist of trying future values of the relevant explanatory variables in place of the current or lagged values. The variables used for Z_t are simply the first stage regressors used for the original estimates.[6]

The after-tax nominal wage rate and the price level are entered separately in the household equations. Since these variables are highly

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[5] See Fair (1984), Table 6-1, for the list of first stage regressors for each equation. The number of first stage regressors per equation varies from 34 to 43.

[6] The only exception to this is for equations in which the structural error term follows a first order autoregressive process. In this case all one-period lagged endogenous variables were moved back one period.

collinear, it is not possible to include, say, the current value of each variable and a future value of each variable in the equations together and expect to get sensible results. What was done instead was simply to include each set of values (lagged, current, or future) separately and observe how the properties of the estimated equations change as different values are included.

It is not clear in the present context how the estimated equations should be compared. If the FIML technique could be used, one could merely compare the likelihood values. For the limited information techniques, however, there is no obvious measure of goodness of fit. The procedure used in this paper is to compare the equations on the baiss of the t-statistics of the coefficient estimates of the variables in question. This procedure is somewhat crude, but in most cases it should give a fairly good idea of the explanatory importance of each variable. The results for the household sector are summarized in Table 1.

All the equations in Table 1 except the IH_h equation have been estimated by the H2 technique. The IH_h equation has a first-order autoregressive structural error, and it has been estimated by the H2R technique. The H1 and H1R techniques did not work in the sense that the estimates of M were not positive definite. In no case with a moving average error of order one or greater was M positive definite. On the other hand, no computational problems were encountered in the use of H2 and H2R. In particular, the method discussed in Section II of computing H2R worked well. For example, in the case of H2R applied to the IH_h equation for the wage and price variables led one period, the initial solution of equations (16) and (17) required 13 iterations. The second solution (using C in place of D in (16) and (17)) required 14 iterations. The tolerance

TABLE 1. Estimates for the household sector. Numbers are t-statistics. Estimation period: 1954 I - 1983 I.

Dependent Variable							
	Wage and Price Explanatory Variables Each pair entered separately						
	t-1	t	t+1	t+2	t+3	t+4	t+1 & t+2
CS	0.73,-0.52	0.57,-0.40	1.30,-1.08*	1.12,-0.87	0.56,-0.35	0.30,-0.07	1.25,-1.01
							t+1 & t+2 & t+3
CN	0.54,-0.43	1.43,-1.33	2.12,-2.04*	1.88,-1.75	1.62,-1.47	0.69,-0.52	1.91,-1.80
							t+1 & t+2 & t+3 & t+4
CD	2.03,-1.17	2.45,-1.63	2.34,-1.52	2.61,-1.77	3.28,-2.37	3.68,-2.67*	3.20,-2.34
IH_h	0.91,-0.59	0.56,-0.40	-0.34,1.13	-0.64,1.61	-0.97,2.22	-1.67,3.20	
							t+1 & t+2 & t+3 & t+4
L2	3.77,-3.45	3.22,-2.93	3.14,-2.93	3.52,-3.43*	3.62,-3.17	3.52,-3.03	3.50,-3.29
							t+1 & t+2 & t+3 & t+4
L3	3.86,-4.00	4.02,-4.71	4.34,-4.47	4.55,-4.67	4.72,-4.83	4.91,-5.02	4.95,-5.07*
							t+1 & t+2 & t+3 & t+4
LM	0.49,-1.09	0.67,-1.26	0.77,-1.34	0.88,-1.49	0.98,-1.56	1.15,-1.69*	0.93,-1.49
	Interest Rate Explanatory Variables Each value entered separately						
	t-1	t	t+1	t+2	t+3	t+4	
CS	-2.40	-2.50	-1.45	-1.05	0.24	2.21	
CN	-2.01	0.02	2.03	2.71	2.12	2.66	
CD	-6.32	-6.73	-5.39	-4.49	-3.18	-2.48	
IH_h	-3.02	-2.46	-2.77	-1.22	-0.50	1.30	
	Nonlabor Income Explanatory Variable Each value entered separately						
	t-1	t	t+1	t+2	t+3	t+4	
CN	0.33	2.11	2.43*	1.58	1.95	1.48	

Notes: Estimation technique is H2 for all equations except IH_h, where it is H2R.

"t+1 & t+2" means that the coefficients of each variable for periods t+1 and t+2 were constrained to be equal, and similarly for the others.

*Specification used for Version 2 of the model in Section IV.

Notation: CS = consumer expenditures for services
CN = consumer expenditures for nondurable goods
CD = consumer expenditures for durable goods
IH_h = housing investment, household sector
L2 = labor force of females 25-54
L3 = labor force of all others except males and females 25-54
LM = number of moonlighters

level between successive estimates of ρ was .001.[7]

The data base used in Fair (1984a) was updated through 1984 I for the results in this paper. The estimation period for all the equations in Table 1 was 1954 I - 1983 I. The estimation period had to end four quarters before the end of the data set because future values of up to four quarters ahead were needed for the estimates in Table 1. It should also be noted that the order of the moving average process of the error term was increased as the length ahead of the future values was increased. For $t+1$ the order was one, for $t+2$ the order was two, and so on. For $t-1$ and t the order was zero.

The estimates using the HS technique were very close to the estimates using the H2 technique, and there seemed to be no need to present both sets of results here. For example, the $t+1$ wage and price t-statistics for the CS equation were 1.45 and -1.23 for the HS estimator, which compare to 1.30 and -1.08 in Table 1. For the CN equation the values were 2.06 and -1.98, which compare to 2.12 and -2.04 in Table 1. The $t+4$ wage and price t-statistics for the CD equation were 3.46 and -2.44, which compare to 3.68 and -2.67 in Table 1. It seems clear that similar conclusions would be drawn using either the H2 or HS technique, and so for simplicity only the H2 results are presented here.

Consider first the wage and price results in Table 1. All equations except the IH_h equation have better t-statistics for values led one or more periods than for current or lagged values. An asterisk in the table indicates my choice for the best lead for the equation. These

[7] All the estimates in this paper were computed using the Fair-Parke (1984) program.

are the specifications that have been used for Version 2 of the model in Section IV. The final pair of t-statistics for each equation in the table is for a specification in which coefficients of certain lead values are constrained to be equal. Except for the L3 equation, this specification was not an improvement over the best individual lead value. The coefficient estimates for the IH_h equation for t+1 and beyond are not sensible because they are of the wrong sign.

The interest rate results in Table 1 do not show any evidence that future values are better than current or lagged values.[8] The coefficient estimates are of the wrong sign for t+3 and t+4 for the CS equation, for all the leads for the CN equation, and for t+4 for the IH_h equation. Otherwise, the t-statistics for the future values are smaller in absolute value than those for the current or lagged values. For the nonlabor income variable, only the results for the CN equation had a future value better than a current or lagged value.

The wage and price results thus provide some evidence in favor of the more sophisticated expectational hypothesis for households. One should be careful, however, not to make too much of the results. In general the results are fairly close across lags, current values, and leads, and in some cases even the best results are not significant by conventional statistical standards.

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[8]The wage and price explanatory variables that were used for the interest rate regressions for the CS, CN, and CD equations are the ones indicated by an asterisk in Table 1. For the IH_h equation the t-1 values of the wage and price variables were used.

Firm Equations

There are six main estimated equations of the firm sector, explaining: 1) the price level, 2) the wage rate, 3) production, 4) investment, 5) number of workers employed, and 6) number of hours worked per worker. The price and wage equations have been extensively examined elsewhere (see Fair (1984c)), and these equations will not be considered in this paper. For the remaining four equations, the current value of sales appears in the production equation as in part a proxy for expected future sales, and current and lagged values of production appear in the investment, employment, and hours equations as in part proxies for expected future production. For the tests in this paper future values of sales are added to the production equation and future values of production are added to the other three equations. The results are summarized in Table 2.

The results for the firm sector provide no evidence in favor of the more sophisticated expectational hypothesis. In the production equation future values of sales are not significant in the three specifications tried. In the investment equation the three future values of production are significant when the coefficients are constrained to be equal, but the coefficient estimates are of the wrong sign. (The three lagged values of production are marginally significant in the investment equation when their coefficients are constrained to be equal.) In both the employment and hours equations the future values are not significant except for the two-quarter-ahead value in the employment equation, which has a coefficient estimate of the wrong sign. The results are thus uniformly negative for the firm sector.

TABLE 2. Estimates for the Firm Sector. Coefficient estimates and t-statistics (in parentheses) are presented. Estimation period: 1954 I - 1983 I

Explanatory Variables

Production Equation

Sales	(1)	(2)	(3)	(4)
t	1.053	1.064	1.045	1.076
	(13.06)	(8.63)	(8.31)	(10.29)
t+1		-.026	.099	-.029[a]
		(-0.29)	(0.75)	(-1.13)
t+2			-.216	-.029[a]
			(-1.88)	(-1.13)
t+3			.068	-.029[a]
			(0.66)	(-1.13)

Investment Equation

Production	(1)	(2)	(3)	(4)	(5)
t-3	.016	.017[a]			
	(1.10)	(1.96)			
t-2	.015	.017[a]			
	(1.06)	(1.96)			
t-1	.020	.017[a]			
	(1.35)	(1.96)			
t	.102	.103	.107	.115	.127
	(6.11)	(6.37)	(6.60)	(5.49)	(7.48)
t+1				-.011	-.016[a]
				(-0.42)	(-2.22)
t+2				.045	-.016[a]
				(1.65)	(-2.22)
t+3				-.088	-.016[a]
				(-3.72)	(-2.22)

Employment Equation

Production	(1)	(2)	(3)	(4)
t-2	-.037			
	(-1.03)			
t-1	.035	.078	.067	.109
	(0.59)	(1.67)	(1.52)	(2.64)
t	.339	.333	.345	.373
	(7.21)	(6.86)	(7.86)	(8.09)
t+1			-.003	.041
			(-0.06)	(0.74)
t+2				-.174
				(-2.76)
t+3				-.071
				(-1.12)

Hours Equation

Production	(1)	(2)	(3)	(4)	(5)
t-1	.020				
	(0.65)				
t	.123	.130	.123	.118	.140
	(4.00)	(4.64)	(3.26)	(2.96)	(4.63)
t+1			.019	.071	-.004[a]
			(0.44)	(1.24)	(-0.31)
t+2				-.072	-.004[a]
				(-1.13)	(-0.31)
t+3				.014	-.004[a]
				(0.28)	(-0.31)

Notes: Estimation technique is H2 for the Investment and Hours equations and H2R for the Production and Employment equations.

[a]Coefficients constrained to be equal within the equation.

Term Structure and Stock Market Equations

There are two term structure equations in the model, one explaining the bond rate and one explaining the mortgage rate. In both equations current and lagged values of the short-term interest rate are taken as proxies for expected future values. (The three-month Treasury bill rate is used as the short-term interest rate in the model.) For the tests in this paper future values of the bill rate were added to these equations. The results are summarized in Table 3.

In the bond rate and mortgage rate equations it is difficult to distinguish between the use of lagged values and future values of the bill rate. In the bond rate equation the current and one-quarter-lagged values are significant when they are included together in the equation. When the lagged values are dropped and three future values are included, the current value looses its significant and the first two future values are significant. The specification that was chosen to be used for Version 2 of the model is the one with the first three future values included. In the mortgage rate equation the current value is always significant, and the specification that was chosen for Version 2 is the one with the current value and the first three future values included.[9]

In the stock price equation the current and one-period lagged values

[9] In earlier work (Fair (1979) and Chapter 11 in Fair (1984a)) I have experimented with a version of my model in which there are rational expectations in the bond and stock markets. This work is different from the present case in that in the earlier work the term structure and stock price equations were not estimated. In the case of the term structure equations, for example, the expectations theory of the term structure of interest rates was merely imposed on the model along with the assumption that expectations are rational in the Muth sense. In the present study the equations that have been used for Version 2 are all estimated equations.

TABLE 3. Estimates of the Term Structure and Stock Market Equations. Coefficient estimates and t-statistics (in parentheses) are presented. Estimation period: 1954 I - 1983 I

Explanatory Variables					
	Bond Rate Equation				
Bill Rate	(1)	(2)	(3)	(4)	(5)*
t-2	.049	.040	.006		
	(1.92)	(1.29)	(0.16)		
t-1	-.215	-.191	-.142		
	(-5.35)	(-3.59)	(-2.22)		
t	.315	.272	.200	.023	
	(10.40)	(4.09)	(2.39)	(0.57)	
t+1		.029	.158	.295	.328
		(0.73)	(1.67)	(3.83)	(5.94)
t+2			-.084	-.206	-.229
			(-0.96)	(-2.31)	(-2.69)
t+3			-.003	.056	.064
			(-0.06)	(1.15)	(1.32)
	Mortgage Rate Equation				
Bill Rate	(1)	(2)	(3)	(4)*	
t-2	-.061	-.037	-.016		
	(-1.54)	(-0.83)	(-0.33)		
t-1	.056	-.001	-.008		
	(0.99)	(-0.01)	(-0.10)		
t	.214	.312	.317	.309	
	(5.05)	(3.46)	(3.03)	(7.07)	
t+1		-.063	-.074	-.077	
		(-1.24)	(-0.65)	(-0.95)	
t+2			-.043	-.045	
			(-0.38)	(-0.44)	
t+3			.048	.052	
			(0.80)	(0.90)	
	Stock Market Equation				
After-Tax Cash Flow	(1)	(2)	(3)		
t-1	3.57	2.94	2.44		
	(1.92)	(1.64)	(1.41)		
t	4.37	5.84	6.41		
	(1.92)	(2.26)	(2.73)		
t+1		-4.01	-1.51		
		(-1.39)	(-0.54)		
t+2			-4.72		
			(-1.93)		
t+3			-5.38		
			(-2.17)		

Notes: Estimation technique is H2.

*Specification used for Version 2 of the model in Section IV.

of after-tax cash flow are used as proxies for expected future values. The results of adding future values to this equation are also summarized in Table 3. The results do not support the use of future values. The future values have coefficient estimates of the wrong sign.

Interest Rate Reaction Function

The behavior of the Federal Reserve is explained by an interest rate reaction function in the model. The left hand side variable is the bill rate. The right hand side variables include: 1) the rate of inflation, 2) a measure of labor market tightness, 3) the rate of growth of real output, and 4) the lagged value of the rate of growth of the money supply. The equation is a "leaning against the wind" equation in the sense that as these four variables increase the Fed is estimated to allow short term interest rates to increase. For the tests in this paper future values of the rate of inflation, the measure of labor market tightness, and the rate of growth of real output were tried in the equation. The results are summarized in Table 4.

The current values of the labor market tightness variable and the real output growth variable gave the best results. For the labor market tightness variable the t-statistics are lower for the future values, and for the real growth variable the coefficient estimates are of the wrong sign for values $t+2$ and beyond. For the inflation variable the future values are better than the current value. The best results were obtained using the first three future values with their coefficients constrained to be equal (equation (8) in Table 4). This is the specification used for Version 2 of the model. There is thus some evidence in favor of the more sophisticated expectational hypothesis for the Fed's expectations of future

TABLE 4. Estimates of the Interest Rate Reaction Function. Coefficients estimates and t-statistics (in parentheses) are presented. Estimation period: 1954 I - 1983 I

Explanatory Variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)*
Rate of Inflation								
t	.081	.030					.004	
	(2.43)	(0.34)					(0.04)	
t+1		.058	.074				.066	.029[a]
		(0.65)	(1.97)				(0.42)	(3.46)
t+2				.078			-.002	.029[a]
				(3.16)			(-0.01)	(3.46)
t+3					.076		.020	.029[a]
					(3.39)		(0.21)	(3.46)
t+4						.058		
						(2.35)		
Labor Market Tightness								
t	.033	.078					.027	.027
	(3.25)	(1.28)					(3.49)	(3.44)
t+1		-.050	.030					
		(-0.85)	(2.45)					
t+2				.018				
				(2.31)				
t+3					.013			
					(1.78)			
t+4						.002		
						(0.28)		
Real Output Growth								
t	.060	.086					.054	.052
	(2.69)	(2.61)					(2.30)	(2.66)
t+1		-.023	.002					
		(-0.52)	(0.08)					
t+2				-.043				
				(-1.98)				
t+3					-.059			
					(-2.91)			
t+4						-.092		
						(-4.50)		

Notes: Estimation technique is H2.

*Specification used for Version 2 of the model in Section IV.

[a]Coefficients constrained to be equal.

inflation rates.[10]

IV. Fiscal-Policy Effects in Two Versions of the Model

The results in Section III provide some support for the hypothesis that agents use a vector Z_t of variables in forming their expectations of future variable values. In particular, the results support this for households' expectations of future wages and prices (and in one case of future nonlabor income), for financial market participants' expectations of future short term interest rates, and for the Fed's expectations of future inflation rates. The purpose of this section is to examine the sensitivity of fiscal-policy effects to the use of this expectational hypothesis. Two versions of the model are considered. Version 1 is the basic version of the model, where no future values are used as explanatory variables.[11]

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[10] In an earlier study (Fair (1984b)) using an earlier data set, I examined the question whether Fed behavior is influenced by expected future federal government deficits. A number of future values of the ratio of the federal government deficit to GNP were tried in the interest rate reaction function, and the best results were obtained using a lead of four quarters. The equations were estimated by the H2 and HS methods. The coefficient estimate of the deficit variable was around 22.0, with a t-statistic of around 2.5 (see Table 1 in Fair (1984b)). With the updated data set used in this study the best results were still obtained for a lead of four quarters, but the coefficient estimate is now only 11.6, with a t-statistic of 1.27. The results in the earlier study were highly tentative, and the current estimates reinforce this. It is still too early to know how much, if any, the Fed is influenced by expected future deficits. With respect to the policy experiments in Fair (1984b), two values of the coefficient of the deficit variable were used: the estimated coefficient of 21.9 and a value half this size. Given the current estimate of 11.6, the second experiment seems more indicative of what might be the case. The deficit variable has not been included in the interest rate reaction function for the results in this study.

[11] As noted in Section III, the data base in Fair (1984a) has been updated through 1984 I for the results in this paper. The estimation period for Version 1 was 1954 I - 1984 I.

Version 2 uses the equations in Section III that have an asterisk beside them. These equations consist of three consumer expenditure equations, three labor supply equations, two term structure equations, and the interest rate reaction function of the Fed. The remaining equations of Version 2 are the same as those of Version 1.

Four fiscal-policy experiments have been performed. The first two are a sustained increase in federal government purchases of goods in real terms GNP beginning in 1970 I. For experiment 1 the change is unanticipated and for experiment 2 the change is anticipated as of 1968 I. The second two experiments are a sustained decrease in the personal income tax rate beginning in 1970 I. For experiment 3 the change is unanticipated, and for experiment 4 the change is anticipated as of 1968 I. The results are presented in Tables 5 and 6. The row 1 results are for Version 1. Since this version is not forward looking, there is no difference between the unanticipated and anticipated results. The numbers in row 2 are the unanticipated results for Version 2, and the numbers in row 3 are the anticipated results.

The calculation of the results in Tables 5 and 6 will first be described, and then the results will be explained. The calculation of the row 1 results is easy to describe. The actual residuals were first added to all the estimated equations in the model and were taken to be exogenous. (The actual residuals are the residuals that were computed at the time of estimation.) This means that when the model is solved using the actual values of the exogenous variables, a perfect tracking solution is obtained. In other words, the "base" values of the endogenous variables for the experiment are merely the actual values. The policy variable was then changed and the model was solved. The difference between the predicted value of

TABLE 5. Estimated effects of a sustained increase in real government spending beginning in 1970 I

	1968				1969				1970				1971				1972			
	I	II	III	IV	I	II	III	IV	I	II	III	IV	I	II	III	IV	I	II	III	IV
Real GNP																				
1									1.10	1.39	1.43	1.38	1.25	1.13	1.01	.91	.84	.80	.77	.75
2									1.11	1.43	1.51	1.48	1.37	1.25	1.13	1.03	.94	.88	.85	.82
3	-.000	-.001	-.001	-.002	-.002	-.007	-.015	-.016	1.09	1.41	1.49	1.46	1.35	1.24	1.12	1.01	.93	.88	.84	.82
GNP Deflator																				
1									.02	.09	.18	.23	.33	.40	.43	.47	.49	.52	.53	.56
2									.02	.10	.18	.24	.34	.42	.46	.50	.53	.56	.58	.61
3	.000	-.000	-.000	-.000	-.001	-.001	-.002	-.004	.01	.09	.18	.23	.33	.41	.45	.49	.52	.55	.56	.60
Unemployment Rate																				
1									-.06	-.12	-.13	-.12	-.11	-.09	-.08	-.06	-.05	-.05	-.04	-.04
2									-.06	-.11	-.13	-.11	-.11	-.09	-.07	-.06	-.05	-.05	-.05	-.04
3	.000	.000	.000	.000	.001	.002	.003	.004	-.06	-.11	-.12	-.12	-.10	-.09	-.07	-.06	-.05	-.05	-.04	-.04
Bill Rate																				
1									.08	.12	.14	.15	.16	.16	.16	.16	.16	.15	.15	.16
2									.08	.11	.13	.14	.14	.14	.14	.14	.14	.14	.13	.13
3	-.000	-.000	-.000	-.000	-.000	-.000	.001	.006	.08	.11	.13	.14	.14	.14	.14	.14	.14	.14	.13	.13
Bond Rate																				
1									.03	.04	.06	.08	.09	.10	.11	.12	.12	.13	.14	.14
2									.02	.03	.05	.06	.08	.09	.10	.11	.11	.11	.13	.12
3	-.000	-.000	-.000	.000	-.000	.004	-.006	.004	.02	.04	.05	.07	.08	.09	.10	.11	.11	.11	.13	.12
Mortgage Rate																				
1									.02	.05	.07	.09	.10	.12	.13	.13	.14	.15	.15	.15
2									.02	.04	.06	.08	.09	.11	.11	.12	.13	.13	.13	.13
3	-.000	-.000	-.000	.000	.000	.004	.005	.001	.02	.04	.06	.08	.09	.11	.12	.12	.13	.13	.13	.13
Consumption of Services																				
1									-.01	-.04	-.06	-.09	-.12	-.15	-.17	-.19	-.21	-.23	-.25	-.26
2									-.02	-.04	-.07	-.10	-.14	-.18	-.22	-.26	-.29	-.33	-.36	-.38
3	.000	.000	.000	.000	.000	.001	-.001	-.002	-.02	-.05	-.07	-.11	-.15	-.19	-.23	-.27	-.30	-.34	-.37	-.39
Consumption of Nondurables																				
1									.07	.11	.15	.18	.20	.22	.24	.26	.28	.29	.32	.34
2									.09	.18	.27	.34	.39	.42	.45	.46	.47	.47	.48	.48
3	-.002	-.004	-.005	-.005	-.004	-.006	-.008	-.005	.08	.17	.26	.33	.38	.41	.44	.45	.46	.46	.47	.47
Consumption of Durables																				
1									.10	.09	-.04	-.33	-.64	-.98	-1.29	-1.52	-1.73	-1.89	-2.01	-2.05
2									.15	.28	.26	.10	-.18	-.49	-.78	-1.01	-1.21	-1.32	-1.39	-1.44
3	-.000	-.001	-.000	-.002	-.006	-.050	-.100	-.088	.06	.19	.17	.00	-.27	-.57	-.85	-1.08	-1.26	-1.37	-1.43	-1.47
After-Tax Nominal Wage																				
1									.00	.05	.13	.19	.24	.29	.32	.34	.36	.37	.39	.40
2									.00	.05	.12	.19	.24	.29	.33	.35	.37	.39	.41	.42
3	.000	.000	.000	-.000	-.000	-.001	-.002	-.004	-.01	.05	.12	.18	.23	.28	.32	.34	.36	.38	.39	.41

Notes: 1 = Version 1, unanticipated and anticipated,
2 = Version 2, unanticipated,
3 = Version 2, anticipated as of 1968 I.

The change in real government spending was 1.0 percent of real GNP. Let y_{it}^{a} be the value of endogenous variable i for quarter t before the change and let y_{it}^{b} be the predicted value after the change. For real GNP, the GNP deflator, the three consumption variables, and the after-tax nominal wage, the values in the table are $(y_{it}^{b}/y_{it}^{a} - 1)\cdot 100$. For the other variables the values are $y_{it}^{b} - y_{it}^{a}$, where the units are in percentage points.

TABLE 6. Estimated effects of a sustained decrease in the personal income tax rate beginning in 1970 I

	1968				1969				1970				1971				1972			
	I	II	III	IV	I	II	III	IV	I	II	III	IV	I	II	III	IV	I	II	III	IV
Real GNP																				
1									.04	.09	.16	.22	.28	.33	.37	.41	.43	.45	.46	.47
2									.09	.20	.30	.39	.45	.51	.55	.58	.52	.47	.44	.34
3	-.000	-.001	-.003	-.005	.049	.098	.131	.215	.26	.30	.34	.38	.41	.45	.49	.57	.46	.42	.39	.31
GNP Deflator																				
1									.00	.00	.01	.01	.02	.04	.05	.06	.08	.10	.12	.14
2									.00	.01	.02	.02	.05	.07	.10	.11	.13	.15	.17	.19
3	.000	-.000	-.000	-.001	.003	.012	.023	.036	.04	.06	.07	.07	.09	.11	.12	.13	.15	.16	.17	.18
Unemployment Rate																				
1									.01	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02	.03
2									.01	.02	.02	.02	.02	.02	.03	.03	.03	.03	.02	.01
3	.000	.000	.000	.000	.001	.002	.011	.019	.02	.03	.03	.04	.04	.04	.04	.04	.04	.04	.03	.02
Bill Rate																				
1									.00	.01	.01	.02	.03	.03	.04	.05	.05	.06	.06	.07
2									.01	.01	.02	.03	.04	.05	.05	.06	.06	.06	.06	.05
3	-.000	-.000	-.000	.000	.005	.009	.012	.019	.02	.03	.03	.04	.04	.05	.05	.06	.06	.06	.06	.05
Bond Rate																				
1									.00	.00	.00	.01	.01	.02	.02	.03	.03	.04	.04	.05
2									.00	.00	.01	.01	.02	.02	.03	.03	.04	.04	.05	.05
3	-.000	.000	-.000	.000	.001	.002	.005	.007	.01	.01	.02	.02	.03	.03	.03	.04	.04	.04	.05	.05
Mortgage Rate																				
1									.00	.00	.00	.01	.01	.02	.02	.03	.03	.04	.05	.05
2									.00	.00	.01	.01	.02	.03	.03	.04	.04	.05	.05	.05
3	.000	.000	.000	.000	.001	.003	.005	.009	.01	.02	.02	.02	.03	.03	.04	.04	.05	.05	.05	.05
Consumption of Services																				
1									-.00	.03	.06	.10	.13	.17	.20	.23	.27	.30	.33	.36
2									.05	.10	.15	.20	.25	.29	.33	.37	.41	.45	.48	.45
3	.000	.000	.000	.000	-.001	-.004	-.009	.046	.09	.13	.17	.21	.25	.29	.32	.36	.40	.43	.47	.43
Consumption of Nondurables																				
1									.08	.15	.21	.27	.33	.40	.46	.52	.57	.61	.66	.70
2									.17	.29	.40	.50	.59	.68	.75	.81	.86	.89	.92	.76
3	-.002	-.004	-.005	-.005	-.001	-.006	.014	.180	.31	.40	.48	.55	.62	.69	.75	.80	.85	.87	.90	.73
Consumption of Durables																				
1									.14	.27	.39	.53	.59	.67	.69	.68	.67	.65	.61	.54
2									.35	.60	.78	.98	.99	1.06	1.05	1.01	.31	-.15	-.45	-.65
3	-.001	-.005	-.013	-.022	.558	.934	1.179	1.355	1.16	1.03	.96	.98	.89	.90	.87	.84	.15	-.29	-.57	-.74
After-Tax Nominal Wage																				
1									1.65	1.63	1.63	1.66	1.65	1.67	1.66	1.67	1.68	1.72	1.68	1.70
2									1.65	1.63	1.62	1.67	1.65	1.68	1.66	1.68	1.69	1.73	1.70	1.72
3	.000	.000	.000	-.000	-.002	-.001	-.000	-.002	1.65	1.64	1.63	1.67	1.64	1.67	1.65	1.66	1.67	1.70	1.67	1.69

Notes: 1 = Version 1, unanticipated and anticipated,
2 = Version 2, unanticipated,
3 = Version 2, anticipated as of 1968 I.

The tax rate change was made to be comparable to the change in government spending in Table 5 with respect to the initial injection of funds into the system. See Fair (1984a), pp. 312-313, for the exact way in which this was done. The units in this table are the same as those in Table 5.

a variable from this solution and its actual value is the estimate of the response of the variable to the policy change.

Version 2 has expected future variables on the right hand side of some of the estimated equations. Two assumptions have been made for the solution of this version. The first is that agents use my model in forming their expectations, and the second is that the expectations of the exogenous variables in the model are equal to the actual values. These two assumptions imply that agents' expectations of the future values are equal to the model's predictions of them.

The numerical solution of Version 2 is more difficult than that of Version 1 because of the existence of future variables among the explanatory variables. What this means is that future predicted values of the endogenous variables affect current predicted values for Version 2, and so the standard way of solving models period by period cannot be used. One must instead iterate over solution paths of the endogenous variables. The exact method for doing this is presented in Fair and Taylor (1983), and this is the method that has been used for the row 2 and row 3 results. Unlike the estimation method in Fair and Taylor (1983), which, as mentioned in Section I, is expensive, the solution method is fairly routine.[12] A perfect tracking solution for Version 2 was also obtained before the experiments were performed.

A final point about Version 2 should be noted before discussing the results. Although the assumption has been made that the agents' expectations of various future values are equal to the model's predictions of these values, the model is not a rational expectations model as this term

[12]The Fair-Parke program was also used for all the solution work in this paper.

is sometimes used. Within the context of the model the expectations are rational in the Muth sense, but there is nothing in the model, for example, that precludes there from being disequilibrium in the labor market. The labor constraint discussed in Section III can be binding on the household sector. The model is not an equilibrium model even though agents' expectations of some of the variables in the model are rational in the specific sense of Muth.

Consider first the results in Table 5. The main feature of these results is that the values in the three rows are quite close to each other. The main reasons for this are the following. (1) Although the Version 2 interest rate reaction function has the Fed responding to expected future inflation rates one, two, and three quarters out rather than in the current quarter only, it also has smaller estimated coefficients for the labor market tightness variable and the real output growth variable.[13] The net effect of this is that the bill rate increases in row 1 are slightly higher than those in rows 2 and 3. In other words, the Fed leans slightly more against the wind for Version 1 than it does for Version 2.

(2) Although the bond rate and mortgage rate predictions for Version 2 are a function of the bill rate predictions one, two, and three quarters out, the sum of the coefficient estimates across the three quarters is not that large (see Table 3). The net effect is for the bond rate and mortgage rate increases for Version 2 to be about the same as those for Version 1.

(3) Interest rates have an important effect on consumer expenditures in both versions of the model, but given that the interest rate increases

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[13]Equation (8) in Table 4 has a labor market tightness coefficient of .027 and a real growth coefficient of .052. For the Version 1 equation these two coefficients are .035 and .065.

are about the same for the two versions, there are no large differences in effects here. This means that the main differences for consumption, if any, must come from the different responses to prices and wages. It can be seen from the results for the GNP deflator and the after-tax nominal wage, however, that the increases in these two variables are about the same, especially for the first few quarters after the policy change. Since households respond primarily to the difference between the two variables, the effects from price and wage changes for this experiment are small. It is true for nondurable and durable consumption that the increases are larger (or the decreases smaller) for Version 2 than for Version 1, but this is primarily due to the fact that interest rates are slightly lower for Version 2 than for Version 1.

The slightly larger nondurable and durable consumption predictions for Version 2 lead to slightly larger increases in real GNP for Version 2. This in turn leads to slightly larger increases in the GNP deflator for Version 2. The labor supply responses are about the same in the two versions because of the small changes in the real wage, and so the unemployment decreases are about the same.

It makes little difference for Version 2 whether or not the policy change is anticipated. In the anticipated case the bill rate is slightly higher two quarters before the change because of the expected future inflation. The bond rate is slightly higher one quarter before the change and the mortgage rate is slightly higher one, two, and three quarters before the change because of the higher future bill rates. These higher interest rates lead to GNP being slightly lower before the change. This in turn leads to the GNP deflator being slightly lower and the unemployment rate being slightly higher. The changes before the policy changes are, however, quite small.

Consider now the results in Table 6. When the personal income tax rate is decreased, this has a direct positive effect on the after-tax nominal wage. This is thus one of the best experiments for seeing the differences between the two versions of the model because one of the main differences between them is the households' response to after-tax real wage changes. Comparing the increases in the GNP deflator in Table 6 with those in the after-tax nominal wage, it can be seen that the present experiment corresponds to a large increase in the after-tax real wage. The initial increases for consumer expenditures are larger for Version 2 because of the positive households' response to the future real wage increases. This leads to higher initial increases in GNP for Version 2. The interest rate differences are again small for this experiment, and the main differences in effects come from the real wage effects. In this experiment there is also more of a difference between the unanticipated and anticipated results for Version 2. There is more initial expansion for the anticipated results because of the positive response of the households to the future wage increases.

Labor supply in both versions of the model responds positively to real wage increases. This, other things being equal, has a positive effect on the unemployment rate. For the experiment in Table 6 the positive effect dominates in that the unemployment rate changes are positive rather than negative in all three cases. The unemployment rate results are quite close between Versions 1 and 2. The positive real wage response is larger for Version 2, but Version 2 also has more output response, which has a negative effect on the unemployment rate. The net effect is that the changes in the unemployment rate are about the same for the two versions.

V. Conclusion

This study has demonstrated that it is possible with the limited information estimation techniques discussed in Section II and the solution method applied in Section IV to use a more sophisticated expectational hypothesis than has traditionally been done in the specification of large-scale macroeconometric models. Although this is now possible, the results in this paper to some extent leave open the question whether it is necessary. The evidence in favor of the more sophisticated expectational hypothesis is strongest for households' response to wages and prices in their consumption and labor supply decisions. No evidence in favor of the hypothesis for firms could be found. The term structure results are about the same whether lags or future values are used. There is some evidence that the Fed responds to future inflation rates. It is clear that a stronger set of results would need to be found before one could put much weight on the hypothesis.

The results in Section IV are interesting in that they show that the policy responses in a model like mine are not necessarily sensitive to the use of the more sophisticated expectational hypothesis. The results in Table 5 are very close. The differences are larger in Table 6, and it is clear for this policy change that it makes a difference which version of the model one believes and whether or not one assumes that the policy change is anticipated.

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