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ABSTRACT

This paper develops a model where firms make state-dependent decisions on both pricing and acquisition of information. It is shown that when information is not perfect, menu costs combined with the aggregate price level serving as an endogenous public signal generate rigidity in price setting even when there is no real rigidity. Specifically, firms reveal their information to other firms by changing their prices. Because the cost of changing price is borne by a firm but the benefit from better information goes to other firms, firms have an incentive to postpone price changes until more information is revealed by other firms via the price level. The information externality and menu costs reinforce each other in delaying price adjustment. As a result, the response of inflation to nominal shocks is both sluggish and hump-shaped. The model can also qualitatively capture a number of stylized facts about price setting at the micro level and inflation at the macro level.

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1 Introduction

One of the central questions in macroeconomics is how small frictions such as menu costs can generate *large* and *long-lasting* real effects in response to nominal shocks. After the seminal contributions of Mankiw (1985) and Akerlof and Yellen (1985), it became clear that small barriers to nominal price flexibility should be complemented with real rigidities (Ball and Romer 1990) to convert small costs of price adjustment at the micro level into significant, persistent fluctuations at the macro level. Real rigidity is usually modeled as an imperfection in factor or good markets. However, available models have difficulty matching some stylized facts about price setting at the macro and/or micro levels. This paper provides an alternative, information-based mechanism for the amplification and propagation of nominal shocks that captures qualitatively many stylized facts.

The key idea of the paper is that in economies with imperfect, dispersed information, macroeconomic variables not only clear markets but also aggregate information by aggregating private actions. Macroeconomic aggregates such GDP, price level, etc. inform economic agents about unobserved fundamentals such as technology, monetary policy and so on. Interestingly, adding a small cost to private actions in such economies can lead to large social multipliers and, more specifically, to significant rigidities in adjustment. In particular, I demonstrate that the combination of (a) the aggregate price level serving as a free *endogenous* public signal and (b) menu costs generates rigidity in price setting in otherwise very flexible economies, i.e., economies without real rigidities (e.g., no monopolistic competition or any sort of interdependence in the demand functions).

To formalize the ideas, I present a model where firms make state-dependent decisions on pricing and acquisition of information. Firms face menu costs when they change prices. In addition, firms have to infer the current state of nominal demand from a sequence of private and public signals. Firms can buy additional signals if they wish. The price level serves as a free public signal through which private information is aggregated and communicated to firms. This public signal is endogenous to the extent that it depends on the individual actions of firms.

Firms reveal their information to other firms by changing their prices. Because information is not perfect, every additional piece of information is valuable and, therefore, every firm has an incentive to observe other firms' actions that convey useful information about unobserved fundamentals. With a richer information set, a firm can obviously set a better price and earn larger profits.

When firms' current prices are sufficiently close to the optimal prices, firms do not reset their prices and, therefore, do not communicate their information to other firms. Importantly, the cost of changing price (the menu cost) and therefore revealing information is borne privately by a firm, while the benefit from better information goes to other firms (recall that the public signal is free) because other firms can utilize the information set implied by the price change of the adjusting firm. Thus, the value of information is different for a firm and for economy as a whole; there is an important *information externality*. Because no firm has an incentive to reveal its information that could be valuable to other firms, private information may get entrapped in private hands and, therefore, the price level may fail to aggregate private information completely.

Most importantly, because of this information externality, firms have an incentive to postpone price changes until more information about unobserved shocks to nominal demand is revealed by other firms via the price level. In other words, every firm wants to free ride on other firms. But if firms change their prices by a little or few firms change prices, the aggregate price level changes by a little which leads to further delaying of price changes at the firm level because a small change in the price level communicates that the probability or the size of the unobserved aggregate shock is small and hence price adjustment is not warranted. Thus the information externality and menu costs reinforce each other in delaying price adjustment and propagation of information in the economy. More precisely, because of the "*wait-and-see*" game played by firms, the response of inflation to nominal shocks is sluggish and hump-shaped. This matches the stylized facts about the behavior of inflation in response to nominal shocks (Christiano, Eichenbaum and Evans 2005) and persistence of aggregate inflation (Fuhrer and Moore 1995, Stock and Watson 1999).

The model also captures qualitatively a number of stylized facts about price setting at the micro level:[1] 1) there is a lack of synchronization in price setting; 2) price changes at the micro level have little inertia; 3) a relatively large fraction of firms change prices in any given period; 4) the average absolute price change is large; 5) the hazard of price adjustment decreases with the duration of the price spell. The model is also consistent with stylized facts about information:[2] 1) firms have imperfect information; 2) acquisition of information is costly; 3) firms often make pricing decisions without utilizing all available information.

The persistence of aggregate inflation in my model sharply contrasts with previous literature on state-dependent pricing. Caplin and Spulber (1987), Caplin and Leahy (1991, 1997), Danziger (1999), Golosov and Lucas (2007) and others find that in economies with state-dependent pricing monetary policy shocks have little or no real effects because of (almost) instantaneous price adjustment. Dotsey, King and Wolman (1999), Willis (2003) and Gertler and Leahy (2006) achieve significant persistence in models with state-dependent pricing only when they introduce real rigidity in price determination at the firm level. Yet, in their models inflation still jumps immediately at the time of the nominal shock

[1] See Lach and Tsiddon (1992), Dhyne et al (2005), Bils and Klenow (2004), Klenow and Kryvtsov (2005), Alvarez et al (2005a), Alvarez et al (2005b), and Fabiani et al (2005), Nakamura and Steinsson (2007).
[2] See Zbaracki et al (2004) and Fabiani et al (2005).

(which reflects the forward-looking nature of pricing decisions) and inflation does not have a hump-shaped response. Instead, inflation declines monotonically, which is very similar to the response in models with time-dependent pricing. In contrast, the present paper shows that aggregate price adjustment is gradual in response to nominal shocks even when there is no explicit strategic complementarity. Specifically, although the intensive margin (average price change) responds strongly at the time of the shock (this is typical for all Ss-type models), the extensive margin (fraction of firms changing prices) responds so weakly on impact that the jump in the intensive margin has almost no effect on the aggregate price level and inflation. In the end, the intensive margin leads the extensive margin but the selection effect typical for state-dependent pricing does not undo the sluggish response of the extensive margin.[3]

Previous studies on sticky information (e.g., Mankiw and Reis 2002, Bonomo and Carvalho 2004, Ball, Mankiw and Reis 2005, Reis 2006) assume flexible prices and time-dependent updating of information. This paper contributes to this literature in two important directions. First, I endogenize the decision to acquire information. Second, I introduce explicit menu costs and consider state-dependent pricing. That is, the decisions to change prices and purchase information are contingent on macroeconomic and firm-specific conditions. I show that these two extensions have important effects on the price-setting policy of firms and the dynamics of inflation. For example, the endogenous acquisition of information tends to accelerate price adjustment after nominal shocks. However, when the economy reaches a steady state, acquisition of information is essentially nil so that firms choose to remain uninformed and are satisfied with a minimum amount of information about the state of the economy. In addition, my model captures the low serial correlation of price changes at the micro level while previous models with sticky information (or rational inattention) do not.[4] In contrast to the literature with time-dependent pricing, my model does not require rule-of-thumb pricing or indexation to generate inflation persistence.

I also discuss several extensions of the model. First, I argue that the speed of price adjustment depends on the size of nominal shocks. Specifically, larger shocks have smaller (less than proportional) real effects because agents easily discern large nominal shocks and buy endogenous signals more

[3] Midrigan (2006) considers multi-product firms that have economies of scale in the technology of adjusting price. These economies generate real rigidity (strategic complementarity) internal to a firm (in contrast to previous literature where real rigidity is external to a firm, e.g., market demand externality) and thus nominal shocks can generate real effects larger than in standard models with menu costs. Dotsey and King (2005) achieve a small, cotemporaneous increase and hump shape in the inflation response to a nominal shock by introducing very strong real rigidity (smoothed kinked demand as in Kimball (1995)).

[4] In those models, there are no menu costs, and prices are changed every period. As long as economic fundamentals (e.g., money supply) are serially correlated, price changes at the firm level are also serially correlated. This issue is also relevant for models where firms have a convex cost of adjusting prices (e.g., Rotemberg 1982), use indexation, or set a price path (e.g., Burstein 2006).

frequently thus making inflation less persistent. Likewise, the endogenous acquisition of information can explain costly disinflation in low inflation environments (Gordon 1982) and essentially costless disinflation in hyperinflation economies (Sargent 1986). Second, in my model, firms have heterogeneous information sets and therefore heterogeneous forecasts. This feature of the model is consistent with the observed dispersion of inflation forecasts in the data (e.g., Mankiw, Reis and Wolfers 2004). Third, I argue that my model, if augmented with strategic complementarity, can generate an even greater persistence of inflation. In summary, Table 1 shows how this paper fits within the literature.

The structure of the paper is as follows. In the next section, I lay out the model: specify the process for nominal demand; present the firm's problem for setting prices and updating information; describe the dynamic program of the firm; and explain how to solve it when there is imperfect information. I then define the equilibrium and show how to solve the model. I also discuss briefly the interrelationship between the information externality, menu costs, and endogenous public signals. In section 3, I solve the model numerically and discuss a firm's optimal policy, a firm's optimal reset price, and the equilibrium distribution of the price level. Next, I present comparative statics and analyze the implications of using alternative information and pricing assumptions. I then demonstrate the importance of having both menu costs and the endogenous public signal in generating inflation persistence. Finally, I discuss briefly several extensions in section 4 and present my conclusions in section 5.

2 Model

In this section, I present a theoretical model to formalize the intuition behind the interplay between menu costs, free endogenous public signals and sluggish price adjustment in an economy with imperfect information. In this economy, firms maximize profits as well as decide how much information they need to purchase given private and public signals about unobserved exogenously evolving state of nominal demand. This is a complex problem because firms have to filter endogenous public signals about nominal demand and to optimize prices and acquisition of information simultaneously. I impose sufficient structure to convert this setup into an equivalent dynamic programming problem with observed state variables which can be solved recursively. Specifically, firms will use a simple recursive Bayes rule to filter signals about nominal demand and then use the posterior probabilities as state variables in the optimization problem. Conveniently, filtering and optimization problems are separated so that I can use standard dynamic programming tools to solve for and analyze firm's optimal policies.

2.1 Monetary policy

Monetary policy has two regimes: high (H) and low (L). Each regime is characterized by the level of nominal demand y_t (equivalently, money stock): y_H and y_L. The probability of staying in the same regime next period is exogenous and equal to λ. Hence, the transition matrix is

$$\Lambda = \begin{bmatrix} \lambda & 1-\lambda \\ 1-\lambda & \lambda \end{bmatrix}.$$

This transition matrix (and, hence, the distribution of states) is common knowledge. The symmetry of the transition matrix and a small number of states keep computation manageable.[5] The setup can be easily generalized to multi-state settings. Nominal demand is the only aggregate shock in this economy.

2.2 Firms

I assume that there is a unit measure of firms indexed by $i \in [0,1]$. The reduced-form profit function in regime $j = \{L, H\}$ is given by $\pi_j = \pi^* - \frac{1}{2}(p - p_j^*)^2$ where p_j^* is the optimal price in regime *j*. The high-level assumption of a quadratic profit function reflects the fact that deviations of profit from its maximum value are of second order if a given price is close to optimal and, hence, profits described by flexible demand and cost structures can be reasonably well approximated by a quadratic function in price (Mankiw 1985, Akerlof and Yellen 1985). I assume that it is costly for firms to change price. Specifically, there is a fixed (menu) cost τ for changing prices. Importantly, there is no explicit strategic complementarity in price setting.

Firms observe neither current nor past y_t. Instead, firms observe signals about nominal demand. Specifically, at each time period *t* each firm observes two free signals: private x_{it} and public $\overline{P}_{t-1}$. The private signal x_{it} is equal to $y_t + \varepsilon_{it}$ where ε_{it} is a shock distributed identically and independently across firms and time.[6] Note that private information evolves over time because of the shock ε_{it} and changes in y_t. The other signal $\overline{P}_{t-1} \equiv \int p_{i,t-1} di$ is the aggregate price level in the previous period. The lag reflects the delay in collecting and revealing information.[7] Importantly, $\overline{P}_{t-1}$ is an *endogenous*

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[5] If the duration of the low regime is short, firms may be more reluctant to cut prices than in the case with the equal durations of the low and high regimes.

[6] Alternatively, one can define shocks to the profit function so that firms observe fluctuations in profits instead of direct signals x_{it} about y_t. The latter, however, is more convenient in analyzing the problem. In the present setup, firms do not observe profits directly and maximize profits in expectation.

[7] Making a decision to change price depending on the past price level signal helps to ensure that an equilibrium exists which may be harder to achieve if a continuum of firms condition their decisions on the current price level signal. Intuitively, this fairly standard assumption is necessary to avoid circularity in today's price decision aggregating in today's price level, which is the signal on which those price decisions are themselves based. See Jackson (1991) for a discussion.

signal because the signal is determined by the actions of firms. I assume that $\overline{P}_{t-1}$ is observed by all agents *without noise*. Firms do not observe the actions, prices and signals of any other firm (specifically, there is no hierarchical structure of information) or the cross-sectional distributions of prices, actions and signals; firms communicate only via the price level $\overline{P}_{t-1}$. To reiterate, firms' payoffs do not depend on the price level directly. In this model, the price level plays only an informational role, i.e., the price level contains useful information about the state of nominal demand y_t.

In the present setup firms are assumed to observe only prices. Obviously, one can easily extend the present setup by allowing firms to observe quantities as well as by adding more sources of uncertainty (which includes noise to public signals) to preserve imperfect information. This richer specification would improve the plausibility of the model but it would also make the intuition obscure and the solution more complex without changing the main qualitative results of the paper. Hence, I use the simple specification with the minimal number of observed variables and shocks.

Signals x_{it} and $\overline{P}_{t-1}$ are observed without cost. Firms also have the option of buying an additional real-time private signal z_{it} at a cost of ξ. I assume that this additional signal is equal to $z_{it} = y_t + \zeta_{it}$ where ζ_{it} is a shock distributed identically and independently across firms and time.[8] The decision to acquire additional information is based only on *ex ante* information, i.e., before signal z_{it} is observed. After obtaining this extra signal, the firm can decide if it wants to change its price or not. Hence, purchasing additional information has an important option value because the firm can save the menu cost if its price is close to optimal in light of new information. As I show below, the reason why firms may want to have additional, costly signals is that extra information improves the estimate of the unobserved nominal demand y_t and consequently raises profits in expectation. Obviously, firms do cost-benefit calculations when they decide whether to purchase z_{it}. I introduce the shock z_{it} to study the behavior of the model when firms are allowed to acquire information endogenously.

In summary, firms make three decisions: 1) whether to buy information or not, 2) whether to change the price or not, 3) if a price change is needed, what the new price should be. Note that the action space *A* does not depend on what price or information a firm has.

2.3 Filtering

In this section, I outline how firms process information. I put sufficient structure on the problem to ensure a simple recursive solution to firms' optimization problem when information is not perfect. Specifically, I impose a condition such that when firms filter a sequence of private and public signals,

[8] I interpret private signals x_{it} and z_{it} as a firm's own market research not revealed to other firms. In contrast, Veldkamp (2006a) considers economies in which there exist industries supplying the same information to heterogeneous firms. This alternative setup can induce more coordination in pricing decisions across firms.

they can use a simple Bayes rule to infer the unobserved state y_t. The estimate of the current y_t will enter later as a state variable in the dynamic program solved by firms.

To formalize the filtering problem, I collect firm's actions into vector $a \in A$ and define the information vector

$$I_{it} = \begin{cases} (I_{i,t-1}, a_{it}, x_{it}, \bar{P}_{t-1}, z_{it}), & \text{if firm } i \text{ buys signal } z_{it} \text{ at time } t \\ (I_{i,t-1}, a_{it}, x_{it}, \bar{P}_{t-1}), & \text{otherwise} \end{cases}$$

which summarizes the history of actions and signals. Note that the true state of nominal demand is unknown to firms and, hence, y_t is not a part of the information vector. For the same reason, actions, prices, signals, etc. of other firms or moments of cross-sectional distributions of actions, prices, signals, etc. (other than $\bar{P}$) cannot be a part of I_{it}. Firms want to know $\mu_{it} = \Pr(y_t = y_H \mid I_{it})$, which is the posterior probability of being in the high regime given the history of signals and actions contained in I_{it}.

In general, because firms filter endogenous signals, μ_{it} does not evolve recursively and, hence, firms have to recompute the posterior using complete history every period. Furthermore, there are two well-known interrelated technical difficulties when agents filter endogenous public signals. On the one hand, one can show that if firms condition on a sufficiently long history of public signals, public signals are observed without noise, and there is only one source of uncertainty (y_t), then firms can perfectly infer the state of y_t.[9] This sharp result is, however, unrealistic because, as Jonung (1981) and others document, economic agents disagree not only about future macroeconomic conditions but also about past and current macroeconomic conditions which is not possible if public signals are fully revealing. To prevent the public signal from being perfectly revealing, one could add noise to the public signal (e.g., Singleton 1987, Mackowiak and Wiederholt 2006) and/or assume that agents fit misspecified (typically linear ARMA) models to forecast the dynamics of the public signal (e.g., Sargent 1991, Lorenzoni 2005).[10] With these modifications, the properties of the equilibrium change smoothly as one adds conditioning variables but the intuition is clouded because the model becomes a "black box".[11]

On the other hand, if agents have heterogeneous information sets (because public signals are not fully revealing), it is difficult to construct a tractable state space for an agent's problem because the

[9] This is a generic result, see, e.g., Hellwig (1982), Caplin and Leahy (1994). If public signals are perfectly revealing, the model is reduced to Lucas's (1972) island model in the sense that after one period nominal shocks have no real effects.

[10] Noise in the public signals can arise for various reasons such as sampling error, finite number o f agents, limited ability to process information.

[11] I experimented with these options (i.e., allow firms to fit AR specifications to model price level signal and/or contaminate the public signal with noise) and I obtained qualitatively similar results. As long as endogenous public signals are not fully revealing but informative, firms have incentives to "wait and see" what other firms are doing.

state space should include current and future higher order expectations, that is, expectations of other agents (see e.g. Woodford 2003). In general, this requires an infinite state space (so called "infinite regress"). Previous work circumvents this problem by introducing different simplifying assumptions that typically impose short lived information (e.g., Lucas, 1972, Dotsey and King 1986, Singleton 1987) or fit an appropriately-defined Kalman filter with a truncated state space (e.g., Townsend 1983, Sargent 1991). The first option leads to information being fully revealed immediately or shortly after shocks occur. The second option requires, in general, a large state space which makes a firm's dynamic program computationally unmanageable.

Since I want to have the simplest possible setup to clearly illustrate the main points of the paper, I cut the Gordian knot by assuming that agents treat public signals as conditionally independent given y_t.[12] That is, firms think that the inference which is correct for private signals is also correct for public signals. Although this assumption might be a departure from full rationality, I argue in Section 2.7 that the intuition behind the main qualitative results of the paper extends to economies with perfectly rational firms. The benefits of this assumption are twofold. First, the assumption prevents the public signal from being fully revealing. Second, the assumption simplifies computation enormously by minimizing the state space in the firm's dynamic program (see below) and delivering a simple recursive evolution of μ_{it}.

To show that the posterior evolves recursively, let $\phi_j(\overline{P}) \equiv \Pr(\overline{P}_{t-1} = \overline{P} \mid y_{t-1} = y_j)$ in regime $j = \{L, H\}$. Note that ϕ_j depends on the strategies chosen by firms and exogenous uncertainty. Hence, ϕ_j is an *equilibrium* concept: agents believe that ϕ_j describes the distribution, and the resulting distribution of the price level conditional on the state of y_t is consistent with the belief. Next, I define the conditional distribution of the real-time signal x_{it} given the state of nominal demand y_t being j as $\psi_j(x) \equiv \Pr(x_{it} = x \mid y_t = y_j)$, $j = \{L, H\}$. Note that ψ_j are exogenous; that is, ψ_j do not depend on the actions of firms. I assume that $\psi_j, \phi_j, j = \{L, H\}$ are common knowledge for firms.

Suppose for now that a firm observes only signals x_{it} and $\overline{P}_{t-1}$. Then the conditional probability of being in the high regime evolves over time as shown in Proposition 1.

Proposition 1

For firm i, the posterior probability of being in the high regime is given by

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[12] This assumption formally amounts to $\Pr(\overline{P}_{t-1} = \overline{P} \mid y_{t-1} = y_j, \overline{P}_{t-2}, \overline{P}_{t-3}, \ldots) = \Pr(\overline{P}_{t-1} = \overline{P} \mid y_{t-1} = y_j)$.

$$\mu_{it} \equiv \Pr(y_t = y_H \mid I_{it}) = \frac{A_H(\mu_{i,t-1}, x_{it}, \overline{P}_{t-1})}{A_H(\mu_{i,t-1}, x_{it}, \overline{P}_{t-1}) + A_L(\mu_{i,t-1}, x_{it}, \overline{P}_{t-1})} \equiv f(x_{it}, \overline{P}_{t-1}, \mu_{i,t-1}), \tag{1}$$

where

$$A_H(\mu_{i,t-1}, x_{it}, \overline{P}_{t-1}) = \psi_H(x_{it})[\phi_H(\overline{P}_{t-1})\lambda\mu_{i,t-1} + \phi_L(\overline{P}_{t-1})(1-\lambda)(1-\mu_{i,t-1})],$$

$$A_L(\mu_{i,t-1}, x_{it}, y_{t-1}) = \psi_L(x_{it})[\phi_H(\overline{P}_{t-1})(1-\lambda)\mu_{i,t-1} + \phi_L(\overline{P}_{t-1})\lambda(1-\mu_{i,t-1})].$$

Proof: see Appendix 1.

Equation (1) is the filtering equation derived from the Bayes rule. A firm i carries the prior $\mu_{i,t-1}$ from the previous period $t-1$, updates the prior in light of signals x_{it} and $\overline{P}_{t-1}$, and obtains the posterior μ_{it}. Note that the posterior in (1) is recursive and signals x_{it} and $\overline{P}_{t-1}$ work like forcing variables. To see the intuition behind this result, rewrite (1) as follows:

$$\mu_{it} = \frac{\tilde{\psi}(x_{it})\tilde{\phi}(\overline{P}_{t-1})\lambda\mu_{i,t-1} + \tilde{\psi}(x_{it})(1-\lambda)(1-\mu_{i,t-1})}{\tilde{\psi}(x_{it})\tilde{\phi}(\overline{P}_{t-1})\lambda\mu_{i,t-1} + \tilde{\psi}(x_{it})(1-\lambda)(1-\mu_{i,t-1}) + \tilde{\phi}(\overline{P}_{t-1})(1-\lambda)\mu_{i,t-1} + \lambda(1-\mu_{i,t-1})}, \tag{2}$$

where $\tilde{\psi}(x_{it}) = \psi_H(x_{it})/\psi_L(x_{it})$ and $\tilde{\phi} = \phi_H(\overline{P}_{t-1})/\phi_L(\overline{P}_{t-1})$ are nothing but likelihood ratios. It can be easily verified that the posterior moves in the direction of the regime that is more likely to be consistent with the observed signals. If the signals are not informative, i.e., $\tilde{\psi} = \tilde{\phi} = 1$, then $\mu_{it} = \lambda\mu_{i,t-1} + (1-\lambda)(1-\mu_{i,t-1})$.[13] Signals can be contradictory by indicating different odds of being in a given regime (i.e., $\tilde{\psi} < 1 < \tilde{\phi}$ or $\tilde{\phi} < 1 < \tilde{\psi}$) and the direction of the change is then determined by the relative strength of the signals.[14] To be clear, agents do not forget past signals but incorporate them in their estimate of the current state using a recursive Bayes rule.

If the price level is not observed, the filtering equation is

$$\mu_{it} = \frac{\tilde{\psi}(x_{it})[\lambda\mu_{i,t-1} + (1-\lambda)(1-\mu_{i,t-1})]}{\tilde{\psi}(x_{it})[\lambda\mu_{i,t-1} + (1-\lambda)(1-\mu_{i,t-1})] + (1-\lambda)\mu_{i,t-1} + \lambda(1-\mu_{i,t-1})},$$

which corresponds to (2) with $\tilde{\phi} = 1$. A similar simplification is available for the case when signal x_{it} is not observed. Hence, $\tilde{\psi} = 1$ or $\tilde{\phi} = 1$ can be also interpreted as cases when some signals are not observed.

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[13] It is straightforward to show that $E\mu_{i,t+1} = \lambda\mu_{it} + (1-\lambda)(1-\mu_{it})$. See Appendix 1.

[14] If the public signal is stronger than the private signal, there could be an absorbing point (or region) from which the price level cannot escape. One can verify that a necessary condition for μ_{it} to escape from any point is that for any $\overline{P}$ there is $x_H = E(x|y_H)$ such that $\tilde{\psi}(x_H)\tilde{\phi}(\overline{P}) \geq 1$ for $\mu \in [0,1/2]$ and there is $x_L = E(x|y_L)$ such that $\tilde{\psi}(x_H)\tilde{\phi}(\overline{P}) \leq 1$ for $\mu \in [1/2,1]$. In other words, the exogenous signal x has to be sufficiently strong to break the coordination induced by the endogenous signal $\overline{P}$ when x and $\overline{P}$ imply different directions.

Firms can improve their posteriors by buying additional information. Define the conditional distribution of signal z_{it} as $\eta_j(z) \equiv \Pr(z_{it} = z \mid y_t = y_j)$, $j = \{L, H\}$. The distributions η_j are exogenous and known to firms. Similar to (1), one can find that the posterior probability after observing signal z_{it} is given by

$$\mu_{it}^{+} = \frac{\eta_H(z_{it})\mu_{it}}{\eta_H(z_{it})\mu_{it} + \eta_L(z_{it})(1-\mu_{it})} \equiv h(\mu_{it}, z_{it}). \tag{3}$$

The difference between (1) and (3) is that (1) is an *inter*temporal filter, while (3) is an *intra*temporal filter.

2.4 Dynamic program

The information vector I_{it} is the most general state space available to a firm, but the dimension of I_{it} grows over time and, thus, it is not convenient for solving the dynamic program. Under certain conditions, information in I_{it} can be compressed into a statistic of a much smaller dimension. Given conditionally independent signals and the rest of the setup, one can show that the posterior probability μ_{it} is a statistic sufficient for control (e.g., Striebel 1965, 1975, Bertsekas and Shreve 1996); that is, μ_{it} encapsulates all information necessary for decision making. Since the state space for y_t consists of two points in my application, the posterior probability is a scalar and μ_{it} represents a genuine compression of information contained in a large vector I_{it}. Note that a firm's own price is a statistic sufficient for itself because it is observed perfectly.

Using this result, one can reduce the imperfect state information program to an equivalent perfect state information program; that is, convert the original problem with imperfect information about the unobserved state of y_t to an equivalent problem where state variables are observable to a firm. Note that the expected flow payoff is equal to

$$\begin{aligned} E\left[\pi(y_t, p_{it}, a_{it}) \mid I_{it}\right] &= \mu_{it}(a_{it})\pi_H(p_{it}(a_{it})) + (1-\mu_{it}(a_{it}))\pi_L(p_{it}(a_{it})) - \boldsymbol{1}_\tau(a_{it})\tau - \boldsymbol{1}_\xi(a_{it})\xi \\ &\equiv \tilde{\pi}(\mu_{it}(a_{it}), p_{it}(a_{it}), a_{it}), \end{aligned}$$

where a_{it} is the vector of a firm's actions, $p(a_{it})$ indicates that price is both a state and choice variable, $\mu(a_{it})$ indicates that the posterior depends on the intra-temporal decision to buy signal z_{it}, and the indicator functions $\boldsymbol{1}_\tau(a_{it}), \boldsymbol{1}_\xi(a_{it})$ show that the payoff additively and non-stochastically depends on whether a firm pays menu and/or information costs. Hence, the expected payoff depends on the information vector via μ_{it}.

If prices could be reset costlessly every period, the optimal price would be $p_{it}^{\#} = \mu_{it} p_H^* + (1-\mu_{it}) p_L^*$ and the optimized expected profit would be equal to $-\mu_{it}(1-\mu_{it})(p_H^* - p_L^*)^2$.

The profit decreases as μ_{it} approaches to ½ and, if the odds of being in high or low regimes are equal ($\mu_{it} = ½$), the value of information is at its highest. Thus, the firm would be willing to pay up to $\mu_{it}(1-\mu_{it})(p_H^* - p_L^*)^2$ for information about the state of y_t. Alternatively, if the posterior is close to zero or one, the value of additional information is small.

Since the profit function $\pi_j, j = \{H, L\}$, is bounded on $[p_L^*, p_H^*]$ and the loss of deviating from $p_{it}^{\#}$ is bounded by the menu cost, it follows that the expected profit $\tilde{\pi}(\mu_{it}(a_{it}), p_{it}(a_{it}), a_{it})$ is bounded too. Given this result, one can show (e.g., Bertsekas and Shreve 1996) that there exists a stationary value function that has two state variables price p and posterior probability μ (which are both observable) and takes the following form

$$V(\mu_{it}, p_{i,t-1}) = \max_a \left\{ \tilde{\pi}(\mu_{it}(a_{it}), p_{it}(a_{it}), a_{it}) + \beta EV(\mu_{i,t+1}(a_{it}), p_{it}(a_{it})) \right\}. \quad (4)$$

An important feature of reducing the initial program to an equivalent perfect state information program is that the problem is separated into filtering and decision-making. In other words, one computes the probability of being in a given state by filtering as if there is no control and then, given the estimate, one finds an optimal solution to the control problem taking the estimate as if it gives perfect state information.

The value function in (4) is in a generic form and it is now convenient to write the specific dynamic program that a typical firm solves. To facilitate presentation, I show the flow of information, actions, and states in Figure 1. The payoff from not changing the price and not buying the signal z_{it} is equal to

$$V_K(\mu_{it}, p_{i,t-1}) = \tilde{\pi}(\mu_{it}, p_{i,t-1}) + \beta E_{x_{i,t+1}, \overline{P}_t | I_{it}} V(\mu_{i,t+1}, p_{i,t-1}), \quad (5)$$

where $\mu_{i,t+1} = f(x_{i,t+1}, \overline{P}_t, \mu_{it})$ with f given by (1). The expected continuation value is given by

$$E_{x_{i,t+1}, \overline{P}_t | I_{it}} V(\mu_{i,t+1}, p_{i,t-1}) = \iint V(\overbrace{f(x_{i,t+1}, \overline{P}_t, \mu_{it})}^{\mu_{i,t+1}}, p_{i,t-1}) \Theta(x_{i,t+1}, \overline{P}_t, \mu_{it}) dx_{i,t+1} d\overline{P}_t \quad (6)$$

with the density Θ given by (see Appendix 1 for derivations)

$$\begin{aligned}\Theta(x_{i,t+1}, \overline{P}_t, \mu_{it}) = \; & \psi_H(x_{i,t+1})\phi_H(\overline{P}_t)\lambda\mu_{it} + \psi_L(x_{i,t+1})\phi_H(\overline{P}_t)(1-\lambda)\mu_{it} + \\ & \psi_H(x_{i,t+1})\phi_L(\overline{P}_t)(1-\lambda)(1-\mu_{it}) + \psi_L(x_{i,t+1})\phi_L(\overline{P}_t)\lambda(1-\mu_{it}). \quad (7)\end{aligned}$$

The value of changing the price without purchasing the signal z_{it} is given by

$$V_C(\mu_{it}) = \max_p \{ \tilde{\pi}(\mu_{it}, p) - \tau + \beta E_{x_{i,t+1}, \overline{P}_t | I_{it}} V(\mu_{i,t+1}, p) \},$$

where $p_{it}^* = \arg\max_p \{\tilde{\pi}(\mu_{it}, p) - \tau + \beta E_{x_{i,t+1},\overline{P}_t|I_{it}} V(\mu_{i,t+1}, p)\}$ is the optimal reset price, and the expected continuation value is computed analogously to (6). Finally, firms have the option of buying the signal z_{it} and then deciding whether to change their prices or not. The value of this option is equal to

$$V_I(\mu_{it}, p_{i,t-1}) = -\xi + E_{z_{it}|I_{it}} \max\{V_K(\mu_{it}^+, p_{i,t-1}), V_C(\mu_{it}^+)\}, \tag{8}$$

where $\mu_{it}^+ = h(z_{it}, \mu_{it})$ with h given by (3), $p_{it}^\dagger = \arg\max_p \{\tilde{\pi}(\mu_{it}^+, p) - \tau + \beta E_{x_{i,t+1},\overline{P}_t|I_{it}^+} V(\mu_{i,t+1}'^+, p)\}$ is the optimal reset price given μ_{it}^+, $\mu_{i,t+1}'^+ = f(x_{i,t+1}, \overline{P}_t, \mu_{it}^+)$ with f given by (1), $I_{it}^+ = \{I_{it}, z_{it}\}$ is the information set I_{it} augmented with the purchased signal z_{it}, and the expected continuation value is computed analogously to (6). The density for computing $E_{z_{it}|I_{it}}$ is given by $\Xi(z_{it}, \mu_{it}) = \eta_H(z_{it})\mu_{it} + \eta_L(z_{it})(1-\mu_{it})$.

After combining these three options, a firm's value function is

$$V(\mu_{it}, p_{i,t-1}) = \max\{V_K(\mu_{it}, p_{i,t-1}), V_C(\mu_{it}), V_I(\mu_{it}, p_{i,t-1})\}. \tag{9}$$

The solution to the dynamic program (9) gives the optimal response given a firm's state variables μ_{it} and $p_{i,t\text{-}1}$. Denote the optimal policy with Π. Note that, because there is no strategic interaction, no firm has incentives to deviate from Π. One can conjecture that, because of fixed costs, the optimal policy is of Ss type. However, regions of actions are functions of both state variables (i.e., price and posterior) and, thus, cannot be reduced to simple one-dimensional triggers. One can also readily verify that, by assumptions of the problem, the model is symmetric around $\{(\mu, p): p = (p_H^* - p_L^*)\mu + p_L^*\}$, i.e., $V(\mu, p - p_L^*) = V(1-\mu, p_H^* - p)$ for any μ and p.

Given the optimal policy Π, define the law of motion for the price and belief at the firm level as

$$p_{it} = p(p_{i,t-1}, \mu_{i,t-1}, x_{it}, z_{it}, \overline{P}_{t-1}; \theta, \phi_H, \phi_L), \tag{10}$$

$$\mu_{it} = \mu(p_{i,t-1}, \mu_{i,t-1}, x_{it}, z_{it}, \overline{P}_{t-1}; \theta, \phi_H, \phi_L), \tag{11}$$

where the vector $\theta = \{\tau, \xi, \psi_H, \psi_L, \eta_H, \eta_L, \pi_H, \pi_L, \lambda, \beta\}$ collects the structural parameters. The law of motion (10) depends on ϕ_H, ϕ_L because the density Θ and the filtering equation f depend on ϕ_H, ϕ_L. Although ϕ_H, ϕ_L are exogenous to a firm, ϕ_H, ϕ_L are not a part of θ because they are determined in equilibrium. Nominal shocks y_t (aggregate uncertainty) enter (10) through the private signals x and z and the public signal $\overline{P}$.

2.5 Equilibrium

To close the model (see Figure 1), observe that the law of motion for the price level can be computed by aggregating the firm-level law of motion (10):[15]

$$\bar{P}_t = \int p(p_{i,t-1}, \mu_{i,t-1}, x_{it}, z_{it}, \bar{P}_{t-1}; \theta, \phi_H, \phi_L) di \, . \tag{12}$$

Likewise by appropriate aggregation of (10) and (11), define $G : \Gamma_t \to \Gamma_{t+1}$ as the law of motion for $\Gamma_t(\mu, p)$, the joint density of the cross-sectional distribution of posteriors and prices at time *t*. Like (12), *G* depends on the policy Π chosen by firms.

Given these laws of motion and the Markov chain for y_t, one can generate the time series of the price level $\bar{P} \equiv \{\bar{P}_1, \bar{P}_2, ...\}$ and collect the values of the price level in the high and low regimes in $\bar{P}_H \equiv \{\bar{P}_{t_1}, \bar{P}_{t_2}, ...\}$ with $t_j \in \{t : y_t = y_H\}$ and $\bar{P}_L \equiv \{\bar{P}_{s_1}, \bar{P}_{s_2}, ...\}$ with $s_j \in \{s : y_s = y_L\}$. Let ϕ'_H, ϕ'_L be the density functions for $\bar{P}_H$ and $\bar{P}_L$. Clearly, ϕ'_H, ϕ'_L depend on ϕ_H, ϕ_L because the aggregate laws of motion depend on the policy Π chosen by firms which in turn depends on ϕ_H, ϕ_L. For brevity, let Υ be the mapping (updating operator) from ϕ_H, ϕ_L to ϕ'_H, ϕ'_L. In equilibrium, the generated ϕ'_H, ϕ'_L is equal to ϕ_H, ϕ_L; that is, firms' beliefs about ϕ_H, ϕ_L must be consistent with reality as firms perceive it. Hence, equilibrium conditional distributions of the price level (ϕ^*_L, ϕ^*_H) are such that if firms optimize their price setting and filtering given (ϕ^*_L, ϕ^*_H), the conditional distributions of the price level resulting from aggregation of individual firm prices are described by (ϕ^*_L, ϕ^*_H). Formally, an equilibrium is a fixed point of mapping Υ, i.e., $(\phi^*_L, \phi^*_H) = \Upsilon(\phi^*_L, \phi^*_H)$.[16]

2.6 Solution

The difficulty of solving the model lies in the fact that how firms interpret public signals depends on what firms do and, at the same time, the actions of firms depend on how the public signal behaves. Hence, the *distribution* of the aggregate price level conditional on y_t (ϕ_H and ϕ_L) must be determined simultaneously with the optimal pricing and information policies at the firm level. To address this problem, I use simulations to generate and approximate the density functions ϕ_H and ϕ_L. To compute the equilibrium, I perform the following steps:

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[15] Clearly, the price level at time *t* depends not only on the lag of the price level but also on the cross-sectional distributions of firm-level prices ($p_{i,t-1}$) and beliefs ($\mu_{i,t-1}$) as well as exogenous private signals x_{it} and z_{it}. However, firms do not observe these distributions.

[16] Note that, although there is a contraction mapping in the firm's dynamic program, Υ maps function space on function space and there is no contraction on the space of functions ϕ. Hence, there could exist multiple equilibria for ϕ. In simulations, I check if perturbations to equilibrium ϕ converge back to ϕ, that is, ϕ is locally stable.

1) Guess density functions $\phi_H^{(0)}, \phi_L^{(0)}$.

2) Given $\phi_H^{(0)}, \phi_L^{(0)}$, solve the dynamic program for the firm.

3) Given solution in step 2, simulate the model for a large number of periods and firms.

4) Compute the generated distribution of the price level (given money regime) ϕ_L' and ϕ_H'.

5) Update the distribution $\phi_H^{(i+1)} = \omega\phi_H^{(i)} + (1-\omega)\phi_H'^{(i)}$ and $\phi_L^{(i+1)} = \omega\phi_L^{(i)} + (1-\omega)\phi_L'^{(i)}$, where ω is a constant between 0 and 1.

6) Repeat steps 2-5 until convergence (i.e., $\left\|\phi_H^{(i+1)} - \phi_H^{(i)}\right\| < \varepsilon$) is reached.

The procedure can be further simplified by observing that $\phi_L(\overline{P}_t) = \phi_H(1-\overline{P}_t)$. This follows from the symmetry of the problem and, hence, it is enough to consider ϕ_H. To approximate the conditional probability density function ϕ_H, I discretize the interval $[p_L^*, p_H^*]$ into N_b bins.[17] Then, I update the distribution as $\phi_{H,k}^{(i)} = \omega\phi_{H,k}^{(i-1)} + (1-\omega)\phi_{H,k}'^{(i-1)}$ where k denotes the bin.

To solve the firm level dynamic program I use approximation methods. Specifically, I use Gaussian quadratures to compute expectations, approximate the value function with polynomials in price p and posterior μ, and solve the dynamic program using collocation methods (see Miranda and Fackler (2002) for a description of these methods). I set $\omega = 0.98$ to ensure smooth evolution of the conditional density ϕ_H. I simulate the conditional density ϕ_H with 5,000 firms, 5,000 periods and N_b = 100 bins. To reduce the simulation noise in ϕ_H, I use optimal kernel smoothing to evaluate the density at a given point (bin). The starting distribution for ϕ_H is the uniform.

2.7 Discussion: Information externality and price rigidity

When firms set prices, there is no explicit interaction between firms. The only channel of communication between firms is the price level, which is the average price across firms. By altering its price and hence changing the price level, a firm reveals to other firms that there is a change in its belief and information. The price level aggregates firms' prices and, thus, aggregates private information. By observing the price level, firms can infer what other firms think about the economic fundamental y_t. Hence, the price level is an *endogenous* public signal.

Since the price level is observed by all firms, it coordinates the actions of firms. This greater coordination can, in principle, result in multiple equilibria. Indeed, models with elements of herding behavior, bandwagon effects, etc. often emphasize that observed phenomena are manifestations of

[17] Since firm-level prices are bounded between p_L^* and p_H^*, the aggregate price level has the same bounds.

multiple equilibria (bank runs, debt and currency crises, riots, etc.). However, even when the equilibrium is unique, endogenous public signals can qualitatively change the behavior of macroeconomic aggregates by altering the behavior of micro-level agents. In other words, there is an important feedback between the behavior of macroeconomic aggregates and microeconomic behavior.

With menu costs, the price level does not aggregate private information perfectly. If a firm does not change its price, it does not reveal its information.[18] Because firms do not interact strategically, no firm has an incentive to reveal its information that could be valuable to other firms, which is similar in spirit to Grossman and Stiglitz (1976, 1980). Indeed, the cost of revealing information (manifested by price change) is borne by a firm, while the benefit from better information goes to other firms (recall that the public signal is free). Consequently, there is an important information externality and private information may get entrapped, which is similar to Caplin and Leahy (1994), Chamley and Gale (1994), and Chari and Kehoe (2003) where private information also gets entrapped and the endogenous public signal significantly affects the decision to enter into an industry or invest in a country.[19] Furthermore and most importantly, the information externality creates an incentive for firms to postpone their price adjustment until more information is revealed by other firms via the price level. In equilibrium, firms may have weak incentives to change prices because other firms do not change their prices.[20]

Now one can see how the information externality is related to real rigidity. As described by Ball and Romer (1990, p. 184), "Rigidity of prices after a nominal shock is a Nash equilibrium if the gain to a firm from changing its nominal price, given that other nominal prices are unchanged, is less than the cost of changing price. … [R]eal rigidity reduces the gain from adjustment [and] increases the range of nominal shocks for which nonadjustment is an equilibrium." Blanchard and Kiyotaki (1987) show that a market demand externality is the source of real rigidity in the standard models with monopolistic competition in goods markets. A firm does not want to reset its price when other firms keep their prices fixed because the firm can lose its market. In contrast, price rigidity in my model arises endogenously as an equilibrium outcome due to the information externality. A firm observes other firms' fixed prices and makes a conclusion that there is a consensus view that the aggregate

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[18] More precisely, if a firm does not change its price, it is interpreted by other firms that the firm's belief about y_t has not changed enough to justify a price change.

[19] See Chamley (2004) for more examples. One can also link this paper to the literature on global games. See Morris and Shin (2003) for discussion and examples.

[20] In his presidential address, Charles Schultze (1985) makes a similar point with respect to wages: "... nominal wages in each firm will be adjusted in line with observed changes in prices and wages generally. But this process of wage determination does not generate prompt and flexible adjustment of aggregate nominal wages to nominal shocks. A change in the average level of wages is the product of changes in wages by individual firms. To the extent that individual wage adjustment must wait on changes in the average, aggregate nominal wage flexibility will not be a characteristic of the system." Ball and Cecchetti (1988) use similar logic to explain endogenous staggering of pricing decisions. In contrast to this paper, Ball and Cecchetti treat the frequency of price adjustment as exogenous.

shock is not likely or small and therefore there is no need to change the price. But if price is not changed, then this consensus view (belief) becomes self-enforcing since each firm does the same exercise.

The practical implication of the information externality is as follows. When there is a nominal shock, few firms may choose to change their prices because most firms want to see what other firms think about fundamentals. By waiting another period, a firm can get a better idea about the current state of aggregate demand by observing the price level that reveals information available to other adjusting firms. In addition, by waiting another period, a firm can save menu and/or information costs, that is, in light of new information the current price may be just fine. But if individual prices change only by little, the price level then changes by a small increment. This smaller change in the price level communicates to firms that nominal demand has probably not changed and consequently there is no need to change price. If so, there is little price adjustment at the firm level in subsequent periods. This circularity makes price adjustment sluggish and this is the place where the information externality slows down the response. In summary, although there is no built-in real rigidity, there is an *endogenous* rigidity induced by the information externality.[21]

An important question is whether results derived later in the paper stem from the assumed conditional independence of public signals. Indeed, it is not optimal for firms to make this assumption and, hence, one should interpret firms' behavior as boundedly rational. However, there are several reasons to believe that, when agents are fully rational, this assumption may be a sensible description for optimal behavior and that results presented in the next section survive.

For example, this assumption is a reasonable approximation to optimal behavior if firms are informationally constrained as in Woodford (2008) or there is a cost of recomputing the posterior using the entire history every period (and it is less costly to recompute the posterior recursively). The cost of using suboptimal rules varies with fundamental parameters (e.g., the average duration of the regime).[22] Most importantly, if it is costly to use sophisticated forecasting/interpretation rules, the decision to use these sophisticated rules is similar to the decision to buy additional information. Then, as discussed below, no firm is going to use these costly rules at the time of the nominal shocks because every firm is going to wait to see what other firms are doing, which is the wait-and-see game. Hence, the delayed

[21] The focus of this paper is on a menu cost economy, but a similar intuition applies to models with time-dependent pricing and firms filtering an endogenous public signal because non-adjusting firms do not change their prices and, hence, do not reveal their private information. In those models, however, private information gets entrapped by the virtue of the time-dependent pricing rather than endogenously. Exogenously determined price change slows down learning about nominal shocks but time-dependent pricing cannot generate an equilibrium in which price adjustment is slow due to a firm's decision to wait until more information is revealed by price adjustment of other firms. Therefore, signal extraction can make adjustment of prices slower but inflation necessarily jumps at the time of the nominal shock.

[22] Given parameter values considered in Section 3, the lost profit varies between 5 and 30%.

response of inflation survives if firms can choose to use superior but costly forecasting rules. Also as shown in the next section, the persistence and hump shape of the inflation response to nominal shocks is not an artifact of bounded rationality. When I set the menu cost to zero to eliminate the incentives to postpone price adjustment, the response of inflation to nominal shocks does not exhibit strong persistence or the hump shape.

Finally, I present and solve a simple three period model with fully rational agents in Appendix 2 and show that the wait-and-see game survives and generates sluggish hump-shaped inflation response to nominal shocks. An important lesson from this exercise is that although I consider an economy where public signals are not fully revealing because firms fit misspecified models to interpret public signals, the intuition behind the wait-and-see incentive extends to economies with perfectly rational firms and fully revealing public signals. Even if firms are perfectly rational, they have an incentive to free ride on other firms and postpone price adjustment. To emphasize, this incentive works even when there is no built-in real rigidity of any kind. It is the possibility to observe actions of other firms that creates a stimulus to wait which changes the qualitative shape of the aggregate inflation response.

3 Results

In this section, I calibrate and solve the model, present the equilibrium distribution of the price level, and describe the optimal policy for pricing and acquisition of information. I show that the model can qualitatively match the stylized facts about price setting at the macro and micro levels. Comparative statistics demonstrate the effect of alternative parameter values on equilibrium outcomes. In the final subsection, I vary informational assumptions to show the importance of the interplay between menu costs and the aggregate price level serving as an endogenous public signal.

3.1 Parameterization

The following parameters must be assigned values: 1) the menu cost τ; 2) the cost of additional information ξ; 3) persistence of nominal demand λ; 4) discount factor β; 5) p_L^*, p_H^* optimal prices in each regime; 6) distribution of ε and ζ (noise in signals x and z); 7) nominal demands y_H and y_L. Because the model is very abstract, it is difficult to map data directly to the model. For this reason, the choice of parameter values should be taken as a crude approximation aimed at showing the properties of the model.

I assume that a quarter is the duration of the period and set the discount factor to 0.99, which corresponds to a 4% annual interest rate.[23] Without loss of generality (the scale of prices and profits is not relevant), I set $p_L^* = 0$, $p_H^* = 1$ and $\pi^* = 1$. Zbaracki et al (2004) report that the physical (menu) cost of changing price is 0.71% of the annual net profit margin. The cost of collecting and processing information is 4.61% of the annual net profit margin. Because in the case of Zbaracki et al changes in information and prices happen about once a year, the implied instantaneous cost of changing price and information is respectively 2.85% and 18.44% of the quarterly net profit margin. Some studies (e.g., Slade 1998, Willis 1999) report much larger menu costs. In addition, Zbaracki et al (2004) consider a relatively large firm where gathering and processing information is perhaps more complicated than for a typically much smaller firm. Hence, I calibrate $\tau = 0.035$ and $\xi = 0.145$. It follows that the cost of changing information and price is $\tau + \xi = 0.180$.

I assume that $\varepsilon_{it} \sim iid\ N(0, \sigma_\varepsilon^2)$ and $\zeta_{it} \sim iid\ N(0, \sigma_\zeta^2)$. Parameters y_H and y_L only determine the conditional means of the signals x and z and, hence, affect the action of firms only via the likelihood ratios in the filtering equation. Because the log of the likelihood ratio is proportional to $(y_H - y_L)$ divided by the variance of the noise in the signal, I set $y_L = 0$ and $y_H = 0.5$ without loss of generality. I set $\sigma_\varepsilon = 0.7$ and $\sigma_\zeta = 0.25$ which appears to be consistent with how much precision professional forecasters can get relative to naïve forecasts. Finally, I set λ=0.95 which corresponds to policy changing (on average) every 1/(1-0.95)=20 periods (i.e., 5 years). This duration is roughly consistent with the estimate reported in Schorfheide (2005).

3.2 Baseline

Figure 2 presents the regions of actions as a function of the two state variables: price and posterior. A firm's price inherited from the previous period is on the horizontal axis. A firm's posterior for being in the high regime is on the vertical axis. The band around the diagonal is the region where it is optimal to keep the price unchanged and to avoid buying additional information. This band of inaction reflects the fact that, if the price is close to optimal given the posterior, it is not worth changing it. It is optimal to change the price without buying new information if the price is far from the optimal, and there is little uncertainty about the current state (i.e., the posterior is close to one or zero). Finally, it is

[23] If the time period shrink to zero (i.e., time becomes continuous), there is no "wait and see" game because there is no "another period" in continuous time (see Chamley (2004) for details). If one reduces the duration of the time period (e.g., seconds instead of quarters) and does not reduce the precision of the signal, then adjustment is going to be completed faster in terms of absolute time (e.g., it takes seconds not quarters). If one reduces the informativeness of signals per unit of time, then adjustment does not change very much. That is, if the amount of information contained in per-second signals accumulated over one quarter is equal to the amount of information contained in one per-quarter signal, then aggregate dynamics is unlikely to change.

generally optimal to buy additional information if the posterior is close to ½, a situation with the greatest uncertainty about the current state. Note that it is not optimal to change price or purchase additional information when price is close to ½ and posterior is close to ½. At this point, the price is close to optimal given the posterior and by waiting one or more periods (until there is more certainty), a firm can save the cost of changing price or buying information.

Figure 3 plots the optimal reset price as a function of the posterior estimate of the current state of the money regime. The broken line is the frictionless optimal reset price $p^{\#} = p_L^* + (p_H^* - p_L^*)\mu$. By the choice of parameter values, $p^{\#}$ is equal to μ and, hence, the slope of the line is one (45° line). The optimal price is equal to ½ when the posterior is equal to ½. If the posterior is greater than ½ but not close to one, the optimal reset price exceeds the optimal frictionless reset price. On the one hand, the posterior next period is expected to move towards ½ and, hence, there is a stimulus to set prices below μ. On the other hand, there is a greater chance that the regime is high and thus it may stay high for a while. By setting a higher price, a firm can then economize on menu and information costs. Basically, because the profit function is concave (firms are risk-averse), higher moments of the next-period posterior induce firms to set higher prices.

If the posterior is close to one, the optimal reset price is below the 45° line. Recall that the profit function is flat around the optimal price (it is quadratic). In addition, there is a chance that the nominal demand has changed (or another regime is in place now). Hence, the optimal reset price should be smaller than the optimal frictionless reset price. However, if a firm is certain that it is in the high regime, there is no incentive to undercut price (this case appears in comparative statics). The argument for μ less than ½ is symmetric.

Figure 4 presents the equilibrium distribution of the price level conditional on the regime being high. Clearly, the mass of the distribution is shifted to the right because the price level $\overline{P}$ is more likely to be high when y_t is high. However, there is a non-trivial density on values between 0.1 and 0.2. The low density for the intermediate values of the price level reflects the fact that the price level spends little time between the two "steady" states, which are defined as states with an absolute change in the price level less than 0.001 (i.e., essentially zero inflation rate). Because the distribution of the price level is not degenerate, observing the price level does not eliminate uncertainty.

Figure 5 and Figure 6 present typical sample paths of the price level and inflation respectively. The paths clearly indicate the state-dependent nature of the price level and inflation responses to changes in nominal demand. Consider the change in y_t at $T = 138$ when the price level has converged to its stable low-demand value. Immediately after the change, the response is weak and in the first periods after the change the price level increases sluggishly. After some point, however, the price level starts to rise at an increasing rate. Then the change in the price level slows down. Hence, inflation has a hump-shaped response. Figure 7 shows the evolution of the cross-sectional price and posterior distributions. Note that the price distribution reallocates the mass of the firms to the right at a rate slower than the posterior distribution does because price changes are slowed down by the menu cost, while posterior changes are not.

What drives this result? Before the shock, firms are quite confident that they are in the low regime. At the time of the shock, only real-time signals x and z can inform firms about the change in demand. Buying signal z is not likely to be optimal for a firm because the posterior is close to zero. Furthermore, the endogenous price level refers to the previous period. Hence, in the first periods, few firms find it optimal to change price. By waiting until more information is revealed by other firms via $\bar{P}$, a firm can save menu and/or information costs. Since few firms change price, the price level changes relatively little and, therefore, signal $\bar{P}$ continues to indicate that nominal demand is likely to be low. This is the place where the information externality slows down the response. In subsequent periods, as more and more firms change their prices, the price level rises and induces an increasing number of firms to change their prices (see Figure 8 and Figure 9). When the posterior approaches ½, firms choose to purchase additional information and, thus, inflation accelerates further (see Figure 8). As the price level approaches one, the incentive to change prices at the firm level falls because the state is now more certain. Thus, inflation slows down. In summary, inflation has a hump-shaped response with a weak reaction at the time of the change.

Other episodes show that the response of inflation can be much faster and sharper. Consider, for example, regime changes between T = 80 and T = 100. At the time of the regime change, many firms continue to believe in the high demand. Consequently, when real-time signals z and x indicate a shift in policy, many firms embrace this signal and quickly move prices back to the high-demand steady state. This explains why the response of inflation is so quick and large.

Importantly, inflation in this model has two margins: intensive (average price change) and extensive (fraction of firms that change price). Figure 9 plots the paths of these two margins for the episode presented in Figure 5 and Figure 6. After the shock, the intensive margin reacts strongly. In contrast, the extensive margin has a significant sluggishness. Both margins are strongly correlated with inflation (Table 2). However, Table 3 shows that the extensive margin leads both inflation and the intensive margin while the intensive margin is coincident with inflation. This important difference in the behavior of price adjustment margins reflects the state-dependent nature of the problem, i.e., the fact that adjusting firms are further away from the optimal reset price than non-adjusting firms.

Even when the level of inflation converges to zero, approximately 0.5% of firms continue to adjust prices. This churning is determined by the fact that firms receive noisy signals about y_t, and occasionally the signal moves the posterior enough to induce some firms to change price. In the aggregate, however, these individual price movements cancel out. One can have a greater fraction of firms changing price if the menu cost τ is smaller. On average, 5.3% of firms change price in a given period with 2.5% changing price without buying endogenous signal z_{it} and 2.8% after buying z_{it}. The fraction is small relative to the monthly 10-20% fraction reported in Bils and Klenow (2004) and other micro level studies. This is, not surprising, however, because there are very few shocks in this economy. In reality firms face many more shocks: productivity, market structure, transportation, etc. Note that the fraction of firms that adjust prices is significantly less than one thus indicating little synchronization in price setting across firms.

Figure 10 plots the simulated hazard rate of price adjustment given the duration of the price spell. The hazard rate h for spell s is computed as $h(s) = 1 - \sum_{i,l} 1(d_{il} \geq s+1) \Big/ \sum_{i,l} 1(d_{il} \geq s)$ where d_{il} is the duration of l^{th} price spell for firm i. Note that the hazard function generally decreases with the duration of the price spell and flattens for high durations. This pattern is consistent with the empirically observed hazard functions (e.g., Alvarez et al 2005a and Nakamura and Steinsson 2007). In contrast, the hazard function is upward sloping in typical models with fixed cost of adjustment because with the passage of time the price is more likely to hit the boundary of an Ss band. Models with time-dependent price adjustment also have trouble matching the decreasing hazard function as, for example, in the Calvo model where the hazard function is flat. The intuition for the downward sloping hazard function in my model is fairly straightforward. When firms have reset prices recently, the learning about nominal demand y_t is likely to continue. But if firms are likely to get more news about y_t, further price adjustment is likely. On the other hand, if the price has been in place for a long time, the learning is approximately completed and hence the firm is less likely to change the price.

The mean absolute value of price changes is equal to 0.485. In the steady states, the average size of absolute price changes is 0.3. In the data, the average absolute price change is 8-10% (e.g., Klenow and Kryvtsov 2005, Dhyne et al. 2005), which is smaller than $\overline{|\Delta p_{it}|} / p_H^*$ in the model. However, reducing the size of the menu cost can decrease the average size of price changes. Thus, the model has the potential to improve the "fit" to the data on this margin.

The average fraction of firms buying information is 3.5%. However, in the steady states, the average fraction is much smaller and equal to 0.2%. This shows that firms stay optimally ignorant if changes in the signals are relatively small ("business as usual"). Firms choose to free ride on other firms because public signal $\overline{P}$ is free while private signal z is costly. Note that the fraction of firms buying information is smaller than the fraction of firms changing prices. This pattern matches the stylized fact that firms change information less frequently than they do prices (Fabiani et al 2005).

Although inflation is persistent with a serial correlation of 0.919, there is a minute negative serial correlation of price changes at the firm level (-0.010) because of the Ss nature of the price setting: firms reset prices by large amounts and keep prices fixed until another large adjustment is needed. This result is consistent with the stylized fact that at the micro level price changes are not persistent.

3.3 Comparative statics

Now I consider how changes in the precision of signals x (σ_ε) and z (σ_ζ), the menu cost τ, cost of an additional endogenously acquired signal ξ, and persistence of nominal demand λ influence the policy function, optimal reset price, equilibrium conditional distribution of the price level ϕ_H^* and several statistics describing price setting.

Increasing the persistence of y_t from $\lambda = 0.95$ to $\lambda = 0.96$ raises the average duration of regimes from 20 quarters to 25 quarters. This greater duration makes the option of updating information more valuable and, hence, optimal on a greater number of occasions (see Panel B in Figure 11). Greater persistence of y_t implies that a set price is likely to be in place for a longer time. Hence, when firms set their prices, the effective horizon increases and the cost of buying additional information can be spread over a greater number of periods. Hence, the acquisition of costly information becomes more attractive.[24] Conversely, if λ is small, it may be optimal to never buy information because that information quickly becomes obsolete in a fast-changing environment.

The increase in the persistence affects the equilibrium conditional distribution of the price level (Figure 12). The mass is reallocated towards the endpoints of the admissible price range. The distribution becomes more concentrated and spiked than in the benchmark distribution. The reason is that firms buy additional information in a greater number of situations and, hence, have a more precise estimate of the current state of y_t. This better information at the firm level is further reinforced with the equilibrium effect that the price level becomes more informative; that is, the distribution of the price level has a much greater mass at the end points of the price range.

Finally, the optimal reset price, now crosses the 45° line only once at $\mu = ½$ due to the fact that the price level is now more informative. This makes firms more certain about nominal demand y_t and, hence, the motive to set a lower price to catch the possibility of being in another regime is weak.

[24] If the persistence is increased further, the "buy information" regions connect. In those situations, it does not pay to wait until more signals arrive and only then change price or buy information. Instead, waiting entails forgone profits greater than the cost of buying information.

Now consider the case with a more precise signal z. Specifically, I reduce the standard deviation of the noise ζ in the signal z from 0.25 to 0.225. A more informative signal z makes buying information more attractive. The consequences are similar to raising the persistence of y_t. The region of buying information increases (Panel C, Figure 11). The average fraction of firms buying information increases to 4.7% (Table 4). The equilibrium distribution of the price level conditional on the regime has greater spikes (Figure 12) because firms buy information more frequently and, hence, the price level becomes more informative. Inflation becomes less inertial. The optimal reset price crosses the 45° line only once and has a distinct s-shape.

Reducing the cost of signal z leads to similar results. Firms choose to buy information more frequently (Panel F, Figure 11). This makes the price distribution spikier, inflation less inertial and optimal reset price always above the 45° line for $\mu > ½$ and always below the 45° line for $\mu < ½$.

When I increase the standard deviation of the noise ε in the signal x from 0.7 to 0.8, the option of buying information becomes somewhat more attractive but the spikes in the equilibrium distribution of the price level are now shifted towards the center (Figure 12). Furthermore, the optimal reset price crosses the 45° line three times as in the benchmark case but in contrast to the benchmark case the optimal reset price bends more toward the center when the posterior is close to zero or one (Figure 13). There are two opposing effects in this case. On the one hand, a greater "buy information" region leads to the effects described above. On the other hand, a less precise signal x works in the other direction, that is, makes the price level less informative. In equilibrium the second effect dominates the first. Because the price level is less informative, inflation becomes more persistent (Table 4).

Increasing the menu cost τ from 0.035 to 0.045 makes the option of updating prices without buying information less favorable (Panel E, Figure 11). Clearly, the decision to buy information depends on the cost of changing prices because after buying information a firm can choose to update price. Why should a firm buy information if it is not going to change its price given a larger menu cost? After all, no firm wants to let information become obsolete. Indeed, the firm should buy information only if there is a good chance it is going to change its price. However, there is another force to be considered. Because price changes are more costly, firms should be more certain about the environment before they change their price. This increased caution induces firms to buy information more frequently. In the end, the second motive dominates and the region of "buy information" increases. A greater occurrence of endogenous purchases of information makes the price level more informative. On the other hand, greater menu costs exacerbate the information externality thus making price adjustment more sluggish and the price level less informative. In summary, the second effect dominates and the distribution of the price level is shifted towards the center. Predictably, a greater menu cost raises the size of the average absolute price change from 0.485 to 0.595.

One can draw several lessons from these comparative statics. First, the equilibrium distribution of the price level is highly sensitive to the frequency at which firms buy information. Specifically, the equilibrium distribution generally becomes concentrated in the regions close to the end points of the price range when firms buy information more frequently. Second, an increased concentration of the price distribution represented by spikes generally makes the price level more informative and, hence, reduces the persistence of inflation. Third, equilibrium effects play an important role in determining the optimal policy and optimal reset price. Also, because of the equilibrium effects, the responses are highly non-linear in parameter values.

3.4 Alternative information and pricing assumptions

Next I present the effects of different information and pricing assumptions on the response of the price level and inflation to changes in nominal demand. Specifically, I contrast the benchmark model with the following alternatives: 1) there is no signal z; 2) the price level $\overline{P}$ is not observed; 3) there is no signal z and the price level $\overline{P}$ is not observed; 4) the menu cost τ is zero; 5) time-dependent price setting; 6) firms are hit with idiosyncratic shocks to menu costs. Figure 14 presents the path of inflation for the benchmark and alternative models in response to nominal demand switching from low to high when the economy is in the low-demand steady state.

In the case when signal z is not available, the amplitude of the inflation response is greatly attenuated. Furthermore, inflation peaks much later than in the benchmark case and the decay after the peak is slow because there is a thick tail of firms that have not learned about the change and have not changed price. In contrast, in the baseline specification, inflation accelerates rapidly and the tail disappears quickly because the price level transmits more information about y_t from firms that bought additional signals. For this reason, the half-life of the price level adjustment increases from 12 periods in the benchmark model to 18 periods in the alternative model with no endogenously acquired signal. In addition, persistence of inflation increases (Table 5). Thus, the endogenous acquisition of information has a sizable impact on the overall dynamics of inflation. The qualitative pattern of leads and lags in inflation and adjustment margins is similar to the benchmark case (Table 6). However, the autocorrelations and cross-correlations are larger at longer horizons, reflecting the thick tail of firms that learn slowly about the change.[25]

[25] Note that this and other alternative scenarios have different cumulative inflation because alternative assumptions and parameterizations affect "steady-state" price levels.

When the price level is not observed, firms' price setting is governed by private signals only. At the time of the regime change, a firm receives a private signal about the change. There is no incentive to wait until more information is revealed by other firms because the price level is not observed by firms. Consequently, inflation jumps on impact and peaks shortly after the change. The amplitude of the inflation response is much larger than in the benchmark case. Autocorrelations and cross-correlations decay faster than in the benchmark case (Table 6). Furthermore, the intensive margin now does not lead inflation and the extensive margin. The half-life of the price level adjustment falls to 3 periods and serial correlation of inflation falls from 0.919 to 0.802 (Table 5). The observation that elimination of an additional signal (the price level) accelerates prices adjustment may seem striking as the reduction in the number of signals might be expected to slow down learning about the state of y_t. However, this result highlights the central point of the paper. If the price level signal is available to firms, there is a strong incentive to postpone price adjustment until more information is revealed by other firms and then delayed adjustment at the micro level is propagated through general equilibrium effects described in section 2.7 to the macro level. In turn, slow adjustment at the macro level delays adjustment at the micro level and thus further delays subsequent adjustment at the macro level. These general equilibrium effects are so strong that the speed of price adjustment falls when firms can observe the price level. If the price level signal is not available to firms, firms cannot learn from other firms, there is no general equilibrium feedback between micro and macro levels of price adjustment and, thus, price adjustment is completed quickly. The differences from the benchmark case show the importance of the endogenous price level signal for generating rigidity in price setting.

In the case when neither z nor $\overline{P}$ is observed, inflation jumps at the time of the regime change and peaks early but the decay in inflation is slow. An early peak is determined by the lack of coordination; that is, there is no transmission of information via the price level across firms. A slow decay reflects that firms cannot buy information or learn it from other firms to alleviate uncertainty and, thus, there is a thick tail of firms that have received signals more likely to be observed in the low regime. In the end, however, the persistence of inflation falls to 0.862 (Table 5), the half-life of the price level adjustment decreases to 5 periods, and the intensive margin does not lead inflation and the extensive margin (Table 6). This scenario is the closest imperfect information analogue to the setup considered in Golosov and Lucas (2007). The qualitative conclusions are similar to Golosov-Lucas's results in the sense that inflation persistence is relatively small, there is no gradual, delayed hump-shaped response in inflation, and prices adjust quickly.

If there is no menu cost, there is no information externality and no incentive to postpone price adjustment.[26] Consequently, all firms change prices and reveal their private information. The price level aggregates information and transmits a strong public signal to firms about a change in nominal demand. This public signal coordinates actions of firms and stimulates further price adjustments. For this reason, inflation does not have significant inertia. Relative to the benchmark case, the half-life of the price level adjustment decreases to only 4 periods and the persistence of inflation is much smaller (Table 5). To emphasize, the scenario with zero menu costs shows that the delayed response of inflation is not an artifact of bounded rationality of firms but a genuine result stemming from the wait-and-see game discussed above. Note that in my parameterization it is never optimal to buy information because the price is always set to the optimal given the posterior and the gains from additional information are not large enough to compensate for the cost of additional information. Because signal x is noisy, firms reset their prices every period (with probability 1) and, hence, the extensive margin does not vary with changes in the money regime. Here, inflation and the average price change are equivalent.

Now consider a model with a Calvo-type price setting and perfect information. In this model, each firm has a 25% probability of having an opportunity to reset its price (Calvo parameter). By construction the Calvo model does not have an extensive margin because the fraction of adjusting firms is fixed. Furthermore, because there is no strategic interaction in price setting across firms, the price change at the firm level takes only two values $p^*_{H,Calvo} - p^*_{L,Calvo}$ or $p^*_{L,Calvo} - p^*_{H,Calvo}$, where $p^*_{H,Calvo}$ and $p^*_{L,Calvo}$ are optimal reset prices in high and low regimes. Therefore, the only reason why the price level does not adjust instantaneously is that only a fraction of firms is allowed to change prices. Indeed, inflation jumps at the time of the regime change and then gradually falls to zero at the Calvo rate. The half-life of price adjustment in the Calvo-type model is only 3 periods and serial correlation of inflation is only 0.658. This lack of inertia sharply contrasts with the slow-moving response in my model.[27]

Bils and Klenow (2004) and others report the firms appear to be hit with large idiosyncratic shocks that lead to frequent price adjustments. In my model, firms receive firm-specific informational shocks which can push firms into price adjustment even when aggregate factors do not change. However, one might be interested in the effects of non-informational shocks (e.g., productivity)

[26] Even if the menu cost is zero, the price level is not fully revealing in my model and, thus, firms continue to use both current and past signals about the state of nominal demand. By using past signals, firms improve the precision of the posterior and thus it is never optimal to use only current signals.
[27] If firms are free to choose the timing of their price changes and information is perfect, price adjustment is completed instantaneously. Hence, nominal shocks have no real effects.

inducing price adjustments. Since my model does not have an explicit production function, markup or the like, I follow Dotsey, King and Wolman (1999) and model these firm-specific innovations as shocks to menu cost. To simplify computation, I set that each period there is a 25 percent chance for any given firm to get a shock that enables this firm to reset its price for free. This probability roughly matches magnitudes reported in micro level studies.

One might have expected that idiosyncratic shocks are likely to make nominal shocks neutral in the sense that aggregate price level adjustment is completed faster than in the absence of idiosyncratic shocks because a fraction of firms is not "waiting" for other firms to adjust prices. Indeed, the impulse response of inflation with idiosyncratic shocks has a jump on impact and inflation accelerates faster than in the benchmark case. Yet, the qualitative shape of the inflation response is similar to the response in the benchmark case. There are several factors that slow down the response of inflation. First and most importantly, although it is true that idiosyncratic shocks make price level more informative, at the same time a more informative price level carries a greater weight in filtering and therefore reaction to firm-specific information is smaller. This important equilibrium effect induces firms to adjust prices by small amounts at the time of a nominal shock and hence inflation jumps only by a small amount. Because the price level is not fully revealing, small changes in the price level lead to small price adjustments at the micro level and consequently gradual price adjustment at the macro level. However, a more informative price level makes learning quicker and, hence, inflation accelerates faster than in the benchmark case. Second, optimal reset price is concave (convex) in the posterior when the posterior is close to unity (zero). Hence, price changes are smaller than changes in the posterior when firms are relatively certain about the current state of y_t. Furthermore, by Jensen's inequality, the change in the aggregate price level is smaller than the change in the average posterior. Thus, even when firms are free to reset prices, the price level changes only by small increments at the time of a nominal shock.

Note that although idiosyncratic shocks significantly affect the fraction of firms adjusting prices, they only marginally affect the persistence of inflation relative to the benchmark case. At the same time, intensive and extensive margins are more strongly correlated with inflation than in the benchmark case (Table 5). In this case, neither intensive nor extensive margins lead inflation. Although the half-life falls from 12 periods to 9 periods, the change is not dramatic given that 25% of firms are free to reset prices every period.

In summary, the benchmark model strikes a balance between inertia and the timing of the response. The availability of additional costly information does not change the qualitative response of inflation per se (i.e., hump shape), but it does change the persistence of inflation. Importantly, it is the combination of menu costs and the endogenous price level signal that delivers the main results. To reiterate, menu costs trap private information so that the price level does not aggregate information completely. Furthermore, the price level informs firms about y_t and creates incentives to wait (not change price) until more information is revealed by other firms via the price level. But if individual prices change by a little, the price level changes by a little and induces firms to postpone price changes. Thus, the information externality and menu costs reinforce each other and make price adjustment sluggish. If the menu cost is zero or the price level is not observed, inflation becomes much less inertial because there is no incentive to delay price change (no information externality). Specifically, inflation peaks immediately or shortly after the change in nominal demand and then it quickly decays to zero. It is also clear that large idiosyncratic shocks do not destroy the delayed response of inflation.

4 Extensions

4.1 Large shocks

To study the effects of big changes in monetary policy, one could introduce more states of the money regime into the model. However, even the present two-state version of the model can provide intuition of what one can get with more money regimes. Suppose one can rank regimes from low to high. If monetary policy switches from one regime to another, adjacent regime, it may be difficult for agents to discern the change because the likelihood ratio of the signals is likely to be close to one. Hence, price adjustment is gradual. On the other hand, if the change in the money regime is greater (that is, regime switches to a more "distant" one), agents can learn more quickly about the change because the likelihood ratio is going to be significantly different from one. It follows that the real impact of a large nominal shock is likely to be small (less than proportional) because price adjustment in this case is faster than in the case with a "small" nominal shock.

4.2 Hyperinflation

Although my model does not have a regime consistent with hyperinflation (high inflation), the intuition behind my main results suggests that the cost of disinflation in a high inflation economy should be smaller than the proportionally rescaled cost of disinflation in a low inflation economy. Indeed, the previous sub-section suggests that large changes are easier to discern for agents and, hence, the real cost of stopping hyperinflation can be small when compared with the cost of a small to moderate disinflation. In principle, the cost of reducing inflation from 100% to 10% may have smaller real effects than reducing inflation from 2% to 1%. Therefore, "shock therapy" may be a good policy prescription for disinflations.

There can be, however, another channel that diminishes the real effects of stopping hyperinflation. Recall that information is a non-rival good and, hence, when demand for information is high, the cost of producing information can be spread over many customers and the price of information falls. As a result, information is cheaper and more abundant and precise. Indeed, Carroll (2003) and Veldkamp (2006b) report that the frequency of words associated with inflation in newspapers increases with the level of inflation. One can find inflation reports, exchange rate quotations, and macroeconomic forecasts in economies with high inflation more frequently than in economies with low inflation. In my model, less expensive additional signals z or more precise free signals x result in lower inflation inertia. In the limit when information is perfect, price adjustment is instantaneous. Thus, the real effects of money shocks are likely to be small.

4.3 Inflation forecasts

In the present model, agents have heterogeneous information sets because agents receive private signals and public signals that are not fully revealing. This heterogeneity leads to different forecasts for all variables including inflation. This feature of the model is consistent with the observed dispersion of inflation forecasts in the data (e.g., Mankiw, Reis and Wolfers 2004). Clearly, standard models with perfect information cannot capture this important stylized fact. In addition, sticky information models have difficulty with matching the level of heterogeneity in forecasts because the variation in information sets over time is small (e.g., Gorodnichenko 2006). In contrast, the present model can in principle match the level of heterogeneity in forecasts if private signals are sufficiently noisy.

4.4 Strategic complementarity

If there is strategic complementarity, firms may have an incentive to conceal their information to convince other firms that demand is different from what it actually is. In other words, firms may intentionally distort signals they send to other firms (see, e.g., Andersen and Hviid 1994, Caplin and Leahy 1994). Because communication between firms becomes "noisier", the degree of rigidity induced by the information externality increases and, hence, price adjustment become more sluggish.[28]

Strategic complementarity affects the response of inflation in another way. Given strategic complementarity in demand, adjusting firms choose to under-react to any shock because otherwise they would lose their markets to competitors. But if adjusting firms change their prices by a little, the price level changes slightly. A small change in the price level can indicate a small probability of a regime change. Given the information content of the price level, firms will have a weaker incentive to change price in subsequent periods. Hence, the information externality and strategic complementarity

[28] If there is a non-anonymous firm with better information (informational leader), price adjustment will be completed faster since the price of the informational leader works like an additional signal provided that the leader does not use its price to profit from confusion of other firms. If the informational leader is anonymous (i.e., firms do not observe the price of the informational leader), then the implications for aggregate dynamics are similar to the case when the precision of signal x is increased.

can reinforce each other. These two mechanisms combined can make price adjustment even more sluggish.

5 Concluding remarks

This paper develops a model with state-dependent pricing and acquisition of information. The model generates endogenous rigidity in price setting and a sluggish, hump-shaped response of inflation to nominal shocks. This result is in sharp contrast with previous findings for models with state-dependent pricing. The model also matches qualitatively a number of stylized facts about micro level price setting such as low synchronization in price changes, large average price changes, a sizable fraction of firms changing price, a downward-sloping hazard of price adjustment, use of imperfect information in price setting, and serially uncorrelated price changes.

The paper emphasizes the key role of the information externality in generating the results. Specifically, private information is not communicated to other firms because firms may find it suboptimal to change prices in the presence of menu costs. Furthermore, privately-borne menu costs create incentives for firms to postpone price adjustment until more information is revealed by the actions of other firms via the endogenous price level which aggregates private information about nominal demand by aggregating firm-level prices. If firms delay their price adjustment in anticipation of more information being revealed by the price level, the price level adjustment is delayed, further delaying the subsequent response of firms' prices and, consequently, the price level to nominal shocks. In summary, there are important equilibrium effects in how firms respond to shocks; that is, pricing policies at the firm level affect the behavior of aggregate variables and vice versa.

I make a number of simplifying assumptions to highlight the interplay between the information externality, menu costs, and price adjustment. These assumptions make the model abstract and not appropriate for quantitative exercises. The way to approach this paper is the way economists think about, for example, Caplin and Spulber (1987). Few economists believe that the model in Caplin and Spulber (1987) is the true description of the world but Caplin and Spulber (1987) show an important *qualitative* result that under certain circumstances state-dependent pricing can lead to neutrality of nominal shocks. Likewise, this paper presents *qualitative* results that may change the way we approach models with state-dependent pricing, i.e., with imperfect information and endogenous public signals state-dependent pricing can entail significant non-neutralities of nominal shocks. Future work should be directed toward incorporating the main ideas of the paper into a fully articulated, more realistic, dynamic stochastic general equilibrium model to assess the quantitative importance of these ideas.

References

Akerlof, George A., and Janet L. Yellen, 1985. "A Near-rational Model of the Business Cycle with Wage and Price Inertia," *Quarterly Journal of Economics* 100(5): 823-838.

Alvarez, Luis J., Pablo Burriel, and Ignacio Hernando, 2005a. "Do Decreasing Hazard Functions for Price Changes Make Any Sense?" European Central Bank, Working paper 461.

Alvarez, Luis J.; Emmanuel Dhyne, Marco M. Hoeberichts, Claudia Kwapil, Herve Le-Bihan, Patrick Lunnemann, Roberto Sabbatini, Fernando Martins, Harald Stahl, Philip Vermeulen, and Jouko Vilmunen, 2005b. "Sticky Prices in the Euro Area — A Summary of New Micro Evidence," European Central Bank, Working paper 563.

Andersen, Torben M., and Morten Hviid, 1994. "Acquisition and Dissemination of Information in Imperfectly Competitive Markets," *Economic Inquiry* 32(3): 498-510.

Ball, Laurence, and David Romer, 1990. "Real Rigidities and the Non-neutrality of Money," *Review of Economic Studies* 57(2): 183-203.

Ball, Laurence, and N. Gregory Mankiw, 1994. "Asymmetric Price Adjustment and Economic Fluctuations," *Economics Journal* 104 (423): 247-261.

Ball, Laurence, and Stephen G. Cecchetti, 1988. "Imperfect Information and Staggered Price Setting," *American Economic Review* 78(5): 999-1018.

Ball, Laurence, N. Gregory Mankiw, and Ricardo Reis, 2005. "Monetary Policy for Inattentive Economies," *Journal of Monetary Economics* 52(4): 703-725.

Bertsekas, Dimitri P., and Steven E. Shreve, 1996. *Stochastic Optimal Control: The Discrete-Time Case*. Boston: Athena Scientific.

Bils, Mark, and Peter J. Klenow, 2004. "Some Evidence on the Importance of Sticky Prices," *Journal of Political Economy* 112(5): 947-985.

Blanchard, Olivier Jean, and Nobuhiro Kiyotaki, 1987. "Monopolistic Competition and the Effects of Aggregate Demand," *American Economic Review* 77(4): 647-666.

Bonomo, Marco, and Carlos Carvalho, 2004. "Endogenous Time-Dependent Rules and Inflation Inertia," *Journal of Money, Banking and Credit* 36(6): 1015-1041.

Burnstein, Ariel T., 2006. "Inflation and Output Dynamics with State-Dependent Pricing Decisions," *Journal of Monetary Economics* 53(7): 1235-1257.

Caballero, R., 1989. "Time Dependent Rules, Aggregate Stickiness and Information Externalities." Mimeo.

Calvo, Guillermo A., 1983. "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics* 12(3): 383-398.

Caplin, Andrew S., and Daniel F. Spulber, 1987. "Menu Costs and the Neutrality of Money," *Quarterly Journal of Economics* 102(4): 703-725.

Caplin, Andrew, and John Leahy, 1991. "State-Dependent Pricing and the Dynamics of Money and Output," *Quarterly Journal of Economics* 106(3): 683-708.

Caplin, Andrew, and John Leahy, 1994. "Business as Usual, Market Crashes, and Wisdom after the Fact," *American Economic Review* 84(3): 548-565.

Caplin, Andrew, and John Leahy, 1997. "Aggregation and Optimization with State-Dependent Pricing," *Econometrica* 65(3): 601-625.

Carroll, Christopher D., 2003. "Macroeconomic Expectations of Households and Professional Forecasters," *Quarterly Journal of Economics* 118(1): 269-298.

Chamley, Christophe P., 2004. *Rational Herds*. Cambridge University Press.

Chamley, Christophe P., and Douglas Gale, 1994. "Information Revelation and Strategic Delay in a Model of Investment," *Econometrica* 62(5): 1065-1085.

Chari, V.V., and Patrick J. Kehoe, 2003. "Hot Money," *Journal of Political Economy* 111(6): 1262-1292.

Christiano, L., M. Eichenbaum, and C. Evans, 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy* 113(1), 1-45.

Danziger, Leif, 1999. "A Dynamic Economy with Costly Price Adjustments," *American Economic Review* 89(4): 878-901.

Dhyne, Emmanuel, Luis J.Alvarez, Herve Le-Bihan, Giovanni Veronese, Daniel Dias, Johannes Hoffmann, Nicole Jonker, Patrick Lunnemann, Fabio Rumler, and Jouko Vilmunen, 2005. "Price Setting in the Euro Area: Some Stylized Facts from Individual Consumer Price Data," European Central Bank, Working paper 524.

Dotsey, Michael, and Robert G. King, 1986. "Informational Implications of Interest-Rate Rules," *American Economic Review* 76(1): 33-42.

Dotsey, Michael, and Robert G. King, 2005. "Implications of State-Dependent Pricing for Dynamic Macroeconomic Models," *Journal of Monetary Economics* 52(1): 213-242.

Dotsey, Michael, Robert G. King, and Alexander L. Wolman, 1999. "State-Dependent Pricing and the General Equilibrium Dynamics of Money and Output," *Quarterly Journal of Economics* 114(2): 655-690.

Erceg, Christopher J., and Andrew T. Levin, 2003. "Imperfect credibility and inflation persistence," *Journal of Monetary Economics* 50(4): 915-944.

Fabiani, S., C. Kwapil, M. Druant, I. Hernando, B. Landau, C. Loupias, F. Martins, T. Matha, R. Sabbatini, H. Stahl, and A.C.J. Stokman, 2005. "The Pricing Behaviour of Firms in the Euro Area - New Survey Evidence," European Central Bank, Working paper 535.

Fischer, Stanley, 1977. "Long-Term Contracts, Rational Expectations, and the Optimal Money Supply Rule," *Journal of Political Economy* 85(1): 191-205.

Fuhrer, J., and G. Moore, 1995. "Inflation Persistence," *Quarterly Journal of Economics* 110(1): 127-159.

Gertler, M., and J. Leahy, 2006. "A Phillips Curve with an Ss Foundation," NBER Working Papers: 11971.

Golosov, Mikhail, and Robert E. Lucas, 2007. "Menu Costs and Phillips Curves" *Journal of Political Economy* 115(2): 171-199.

Gordon, Robert J., 1982. "Why Stopping Inflations May Be Costly: Evidence From Fourteen Historical Episodes," in R.E. Hall, ed., *Inflation: Causes and Effects*, University of Chicago Press.

Gorodnichenko, Yuriy, 2006. Monetary Policy and Forecast Dispersion: A Test of the Sticky Information Model. Mimeo.

Grossman, Sanford J., and Joseph E. Stiglitz, 1976. "Information and Competitive Price Systems," *American Economic Review* 66(2): 246-253.

Grossman, Sanford J., and Joseph E. Stiglitz, 1980. "On the Impossibility of Informationally Efficient Markets," *American Economic Review* 70(3): 393-408.

Hellwig, Martin, 1982. "Rational Expectations Equilibrium with Conditioning on Past Prices: A Mean-Variance Example," *Journal of Economic Theory* 26, 279-312.

Jackson, Matthew O., 1991. "Equilibrium, Price Formation and the Value of Private Information," *Review of Financial Studies* 4(1): 1-16.

Jonung, Lars, 1981. "Are Perceptions of Inflation Rational? Some Evidence for Sweden", *American Economic Review* 78(5), 1080-1087.

Kiley, Michael T., 2000. "Endogenous Price Stickiness and Business Cycle Persistence," *Journal of Money, Credit, and Banking* 32(1): 28-53.

Kimball, Miles S., 1995. "The Quantitative Analytics of the Basic Neomonetarist Model," *Journal of Money, Credit, and Banking* 27(4): 1241-1277.

Klenow, Peter J., and Oleksiy Kryvtsov, 2005. "State-Dependent or Time-Dependent Pricing: Does It Matter for Recent U.S. Inflation?" NBER Working Papers 11043.

Lach, Saul, and Daniel Tsiddon, 1992. "The Behavior of Prices and Inflation: An Empirical Analysis of Disaggregated Price Data," *Journal of Political Economy* 100(2): 349-389.

Lorenzoni, Guido. 2005. "Imperfect Information, Consumers' Expectations and Business Cycles" mimeo.

Lucas, Robert E., 1972. "Expectations and the Neutrality of Money," *Journal of Economic Theory* 4(2): 103-124.

Mackowiak, Bartosz, and Mirko Wiederholt, 2006. "Optimal Sticky Price under Rational Inattention" mimeo.

Mankiw, N. Gregory, 1985. "Small Menu Costs and Large Business Cycles: A Macroeconomic Model," *Quarterly Journal of Economics* 100(2): 529-538.

Mankiw, N. Gregory, and Ricardo Reis, 2002, "Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve," *Quarterly Journal of Economics* 117(4): 1295-1328.
Mankiw, N. Gregory, Ricardo Reis, and Justin Wolfers, 2004. "Disagreement about Inflation Expectations," *NBER Macroeconomics annual* 18: 209.
Midrigan, Virgiliu. 2006. Menu Costs, Multi-Product Firms and Aggregate Fluctuations. Mimeo.
Miranda, Mario J., and Paul L. Fackler, 2002. *Applied Computational Economics and Finance*. MIT Press.
Morris, Stephen, and Hyun Song Shin, 2003. "Global Games: Theory and Applications," in M. Dewatripont, L. Hansen and S. Turnovsky, eds., *Advances in Economics and Econometrics*, Cambridge: Cambridge University Press, 56-114.
Nakamura, Emi, and Jon Steinsson, 2007. Five Facts About Prices: A Reevaluation of Menu Cost Models. Mimeo.
Reis, Ricardo, 2006. "Inattentive Producers," *Review of Economic Studies* 73 (3): 793-821.
Rotemberg, Julio J., 1982. "Sticky prices in the United States," *Journal of Political Economy* 90(6): 1187-1211.
Rondina, G., 2007. Incomplete Information and Informative Pricing: Theory and Application. Mimeo.
Sargent, Thomas J., 1986. "The Ends of Four Big Inflations," in T.J. Sargent, *Rational Expectations and Inflation*, New York: Harper and Row, 40-190.
Sargent, Thomas J., 1991. "Equilibrium with Signal Extraction from Endogenous Variables," *Journal of Economic Dynamics and Control* 15(2): 245-273.
Schorfheide, F., 2005. "Learning and Monetary Policy Shifts," *Review of Economic Dynamics* 8: 392-419.
Schultze, Charles L., 1985. "Microeconomic Efficiency and Nominal Wage Stickiness," *American Economic Review* 75(1), 1-15.
Singleton, Kenneth, 1987. "Asset Prices in a Time-Series Model with Disparately Informed, Competitive Traders," in W.A. Barnett and K.J. Singleton, eds., *New Approaches to Monetary Economics*, Cambridge: Cambridge University Press.
Slade, Margaret E., 1998. "Optimal Pricing with Costly Adjustment: Evidence from Retail-Grocery Prices," *Review of Economic Studies* 65, 87–107.
Stock, James H., and Mark W. Watson, 1999. "Business Cycle Fluctuations in US Macroeconomic Time Series" in J.B. Taylor and M. Woodford, eds., *Handbook of macroeconomics*. Amsterdam, New York and Oxford: Elsevier Science, North-Holland, 1999; 3-64.
Striebel, Charlotte, 1965. "Sufficient Statistics in the Optimum Control of Stochastic Systems," *Journal of Mathematical Analysis and Applications* 12: 576-592.
Striebel, Charlotte, 1975. *Optimal Control of Discrete Time Stochastic Systems*. Springer-Verlag.
Taylor, J.B., 1980 "Aggregate Dynamics and Staggered Contracts," *Journal of Political Economy* 88(1): 1-23.
Townsend, R.M., 1983. "Forecasting the Forecasts of Others," *Journal of Political Economic* 91(4): 546-588.
Veldkamp, Laura, 2006a. "Information Markets and the Comovement of Asset Prices," *Review of Economic Studies* 73: 823-845.
Veldkamp, Laura, 2006b. "Media Frenzies in Markets for Financial Information," *American Economic Review* 96(3): 577-601.
Willis, J.L., 1999. "Estimation of Adjustment Costs in a Model of State-Dependent Pricing," Boston University PhD Dissertation.
Willis, J.L., 2003. "General Equilibrium of a Monetary Model with State-Dependent Pricing," mimeo.
Woodford, Michael, 2003. "Imperfect Common Knowledge and the Effects of Monetary Policy," Philippe Aghion et al, eds., *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, Princeton and Oxford: Princeton University Press, 2003; 25-58.
Woodford, Michael, 2008. Information-Constrained State-Dependent Pricing. Mimeo.
Zbaracki, Mark J., M. Ritson, D. Levy, S. Dutta, and M. Bergen, 2004. "Managerial and Customer Costs of Price Adjustment: Direct Evidence from Industrial Markets," *Review of Economics and Statistics* 86(2): 514-533.

Table 1. Classification of models

<table>
<tr><td rowspan="3">Information</td><td colspan="4">Prices</td></tr>
<tr><td colspan="3">Fixed</td><td rowspan="2">Flexible</td></tr>
<tr><td>Menu costs</td><td>Fixed duration</td><td>Endogenous duration</td></tr>
<tr><td>Perfect</td><td>Spulber-Caplin (1987)
Caplin-Leahy (1991, 1997)
Gertler-Leahy (2006)
Golosov-Lucas (2007)
Dotsey-King-Wolman (1999)
Burstein (2006)
Midrigan (2006)</td><td>Calvo (1983)
Taylor (1980)
Fischer (1977)</td><td>Ball and Mankiw (1994)

Kiley (2000)</td><td>Classical</td></tr>
<tr><td>Imperfect, exogenous</td><td>Present model</td><td>Ball-Cecchetti (1988)

Erceg-Levin (2003)</td><td>Bonomo-Carvalho (2001)

Caballero (1989)</td><td>Lucas (1972)

Mankiw-Reis (2002)

Reis (2006)

Woodford (2003)

Lorenzoni (2005)

Mackowiak-Wiederholt (2006)

Rondina (2007)</td></tr>
<tr><td>Imperfect, endogenous</td><td>Present model</td><td>--</td><td>--</td><td>Grossman and Stiglitz (1976, 1980)</td></tr>
</table>

Table 2. Correlations

	abs(Inflation)	Extensive margin	Fraction of firms that change price only	Fraction of firms that update info	Intensive margin
abs(Inflation)	1.000				
Extensive margin	0.766	1.000			
Fraction of firms that change price only	0.856	0.639	1.000		
Fraction of firms that update information	0.978	0.742	0.825	1.000	
Intensive margin	0.973	0.737	0.934	0.972	1.000

Note: Intensive margin is the average price change. Extensive margin is the fraction of firms that change price.

Table 3. Autocorrelation and cross correlations: Benchmark model

Lags	Cross correlation of inflation to					Intensive margin to extensive margin
	Inflation	Extensive margin	Fraction of firms that change price only	Fraction of firms that update information	Intensive margin	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
-10	0.124	0.295	0.071	0.145	0.122	0.118
-9	0.171	0.352	0.105	0.197	0.170	0.154
-8	0.232	0.409	0.147	0.264	0.230	0.198
-7	0.313	0.478	0.202	0.348	0.306	0.250
-6	0.408	0.553	0.269	0.446	0.396	0.307
-5	0.508	0.617	0.345	0.554	0.496	0.367
-4	0.610	0.660	0.432	0.665	0.601	0.432
-3	0.713	0.687	0.528	0.773	0.708	0.503
-2	0.822	0.716	0.633	0.873	0.815	0.580
-1	0.919	0.744	0.748	0.950	0.911	0.660
0	1.000	0.766	0.856	0.978	0.973	0.737
1		0.660	0.904	0.921	0.954	0.762
2		0.572	0.908	0.829	0.895	0.763
3		0.492	0.887	0.721	0.817	0.752
4		0.425	0.850	0.605	0.729	0.733
5		0.366	0.799	0.484	0.633	0.695
6		0.311	0.731	0.366	0.531	0.638
7		0.254	0.642	0.265	0.431	0.570
8		0.198	0.542	0.186	0.341	0.502
9		0.154	0.447	0.128	0.265	0.438
10		0.118	0.366	0.084	0.204	0.373

Note: Cross correlation of variable x to variable y is computed as $\rho(x_t, y_{t+k})$. Intensive margin is the average price change. Extensive margin is the fraction of firms that change price. In column (2) autocorrelation is for inflation while in columns (3)-(6) cross correlations are for the absolute value of inflation (price change).

Table 4. Summary statistics: Comparative statics

model	Serial correlation of inflation	Average fraction of firms changing price		Average absolute price change		Average fraction of firms buying information	
		Full sample	Steady state	Full sample	Steady state	Full sample	Steady state
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Benchmark	0.919	5.25%	0.54%	0.485	0.301	3.45%	0.16%
$\lambda = 0.96$	0.914	3.73%	0.45%	0.559	0.326	3.51%	0.23%
$\sigma_\zeta = 0.225$	0.824	4.78%	0.44%	0.623	0.310	4.65%	0.30%
$\sigma_\varepsilon = 0.800$	0.931	3.31%	0.28%	0.556	0.477	4.38%	0.42%
$\tau = 0.450$	0.933	3.39%	0.20%	0.595	0.532	4.43%	0.36%
$\xi = 0.125$	0.829	5.11%	0.42%	0.627	0.361	5.23%	0.35%

Note: Each row corresponds to the benchmark model with a structural parameter modified as shown in column (1). Steady state corresponds to the subsample with absolute price level change less than 0.001.

Table 5. Alternative models: Summary statistics

model	Serial correlation of inflation	fractions				correlation of abs(inflation) with		correlation between extensive and intensive margins
		keep price	change price only	update information				
				keep price	change price	intensive margin	extensive margin	
Benchmark	0.919	0.941	0.025	0.007	0.028	0.766	0.973	0.737
No endogenous information	0.935	0.918	0.082	-	-	0.694	0.982	0.643
No price level	0.802	0.737	0.030	0.171	0.062	0.911	0.982	0.896
No endogenous information, No price level	0.862	0.840	0.160	-	-	0.988	0.978	0.964
No menu cost	0.754	0.000	1.000	0.000	0.000	1.000	-	-
Calvo	0.658	0.750	0.250	-	-	1.000	-	-
Idiosyncratic shocks	0.894	0.689	0.266	0.022	0.024	0.994	0.962	0.950

Note: The table reports summary statistics for different information scenarios. The case of "no endogenous information" corresponds to the case where signal z_{it} is not available to firms. The case of "no price level" corresponds to the case where the price level is not observed by firms. "Idiosyncratic shocks" corresponds to the scenario where firms are randomly hit with firm-specific shocks to menu costs. See text for further details.

Table 6. Cross-correlations: Alternative models

Lags	No endogenous information				Price level is not observed				No endogenous information Price level is not observed			
	abs(inflation) to			Intensive margin to extensive margin	abs(inflation) to			Intensive margin to extensive margin	abs(inflation) to			Intensive margin to extensive margin
	Extensive margin	Fraction of firms that update information	Intensive margin		Extensive margin	Fraction of firms that update information	Intensive margin		Extensive margin	Fraction of firms that update information	Intensive margin	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
-8	0.430	0.008	0.570	0.505	-0.030	-0.021	-0.024	0.017	0.049	-0.013	0.047	0.118
-7	0.470	0.007	0.635	0.526	-0.016	-0.008	-0.015	0.061	0.090	-0.015	0.083	0.167
-6	0.509	0.014	0.700	0.545	0.002	0.015	0.005	0.131	0.145	-0.021	0.134	0.230
-5	0.544	0.019	0.764	0.562	0.042	0.065	0.037	0.221	0.219	-0.025	0.206	0.308
-4	0.572	0.016	0.824	0.578	0.109	0.150	0.105	0.339	0.318	-0.021	0.302	0.399
-3	0.593	0.017	0.878	0.592	0.240	0.310	0.220	0.465	0.459	-0.019	0.435	0.512
-2	0.615	0.022	0.924	0.606	0.470	0.577	0.433	0.615	0.643	-0.030	0.612	0.651
-1	0.637	0.024	0.958	0.620	0.778	0.909	0.744	0.772	0.846	-0.032	0.814	0.811
0	0.694	0.026	0.982	0.643	0.911	0.919	0.982	0.896	0.988	-0.029	0.978	0.964
1	0.622	0.017	0.965	0.634	0.763	0.641	0.797	0.822	0.831	-0.007	0.903	0.900
2	0.595	0.015	0.937	0.625	0.607	0.361	0.496	0.528	0.668	-0.027	0.729	0.728
3	0.575	0.020	0.895	0.609	0.458	0.182	0.278	0.298	0.527	-0.029	0.547	0.546
4	0.561	0.017	0.843	0.587	0.336	0.083	0.144	0.151	0.411	-0.024	0.398	0.397
5	0.545	0.014	0.784	0.560	0.221	0.032	0.065	0.066	0.319	-0.023	0.282	0.280
6	0.526	0.000	0.718	0.525	0.133	0.002	0.022	0.017	0.241	-0.009	0.192	0.191
7	0.505	0.001	0.651	0.486	0.064	-0.017	-0.004	-0.010	0.176	-0.009	0.127	0.126
8	0.486	0.001	0.583	0.444	0.018	-0.026	-0.019	-0.027	0.123	-0.015	0.076	0.075

Note: Cross correlation of variable x to variable y is computed as $\rho(x_t, y_{t+k})$. Intensive margin is the average price change. Extensive margin is the fraction of firms that change price. The case of "no endogenous information" corresponds to the case where signal z_{it} is not available to firms. The case of "no price level" corresponds to the case where the price level is not observed by firms.

Figure 1. Flow of information, actions and states

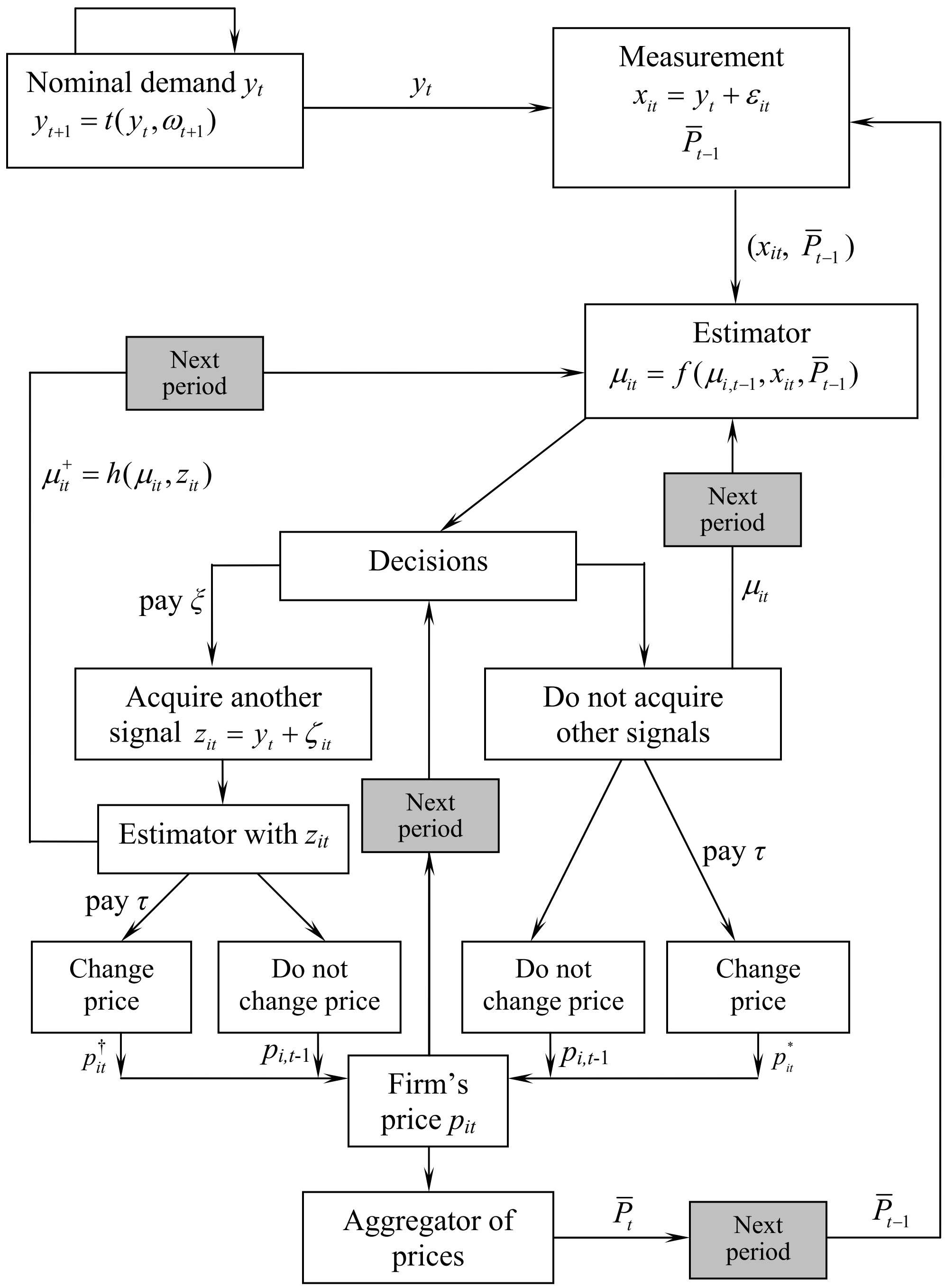
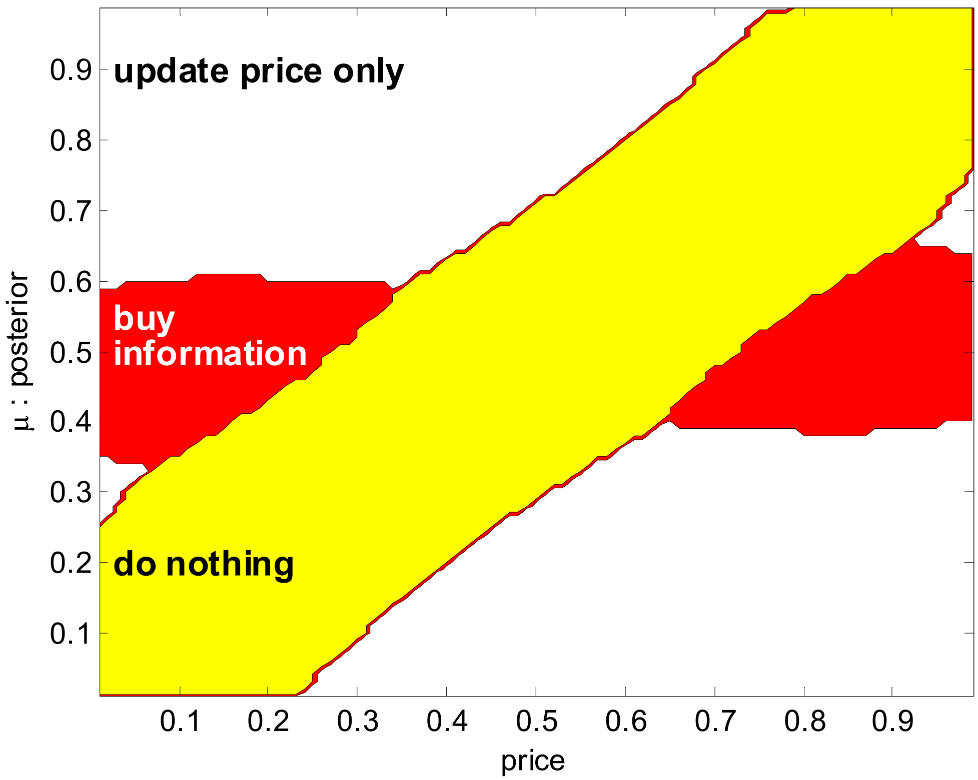
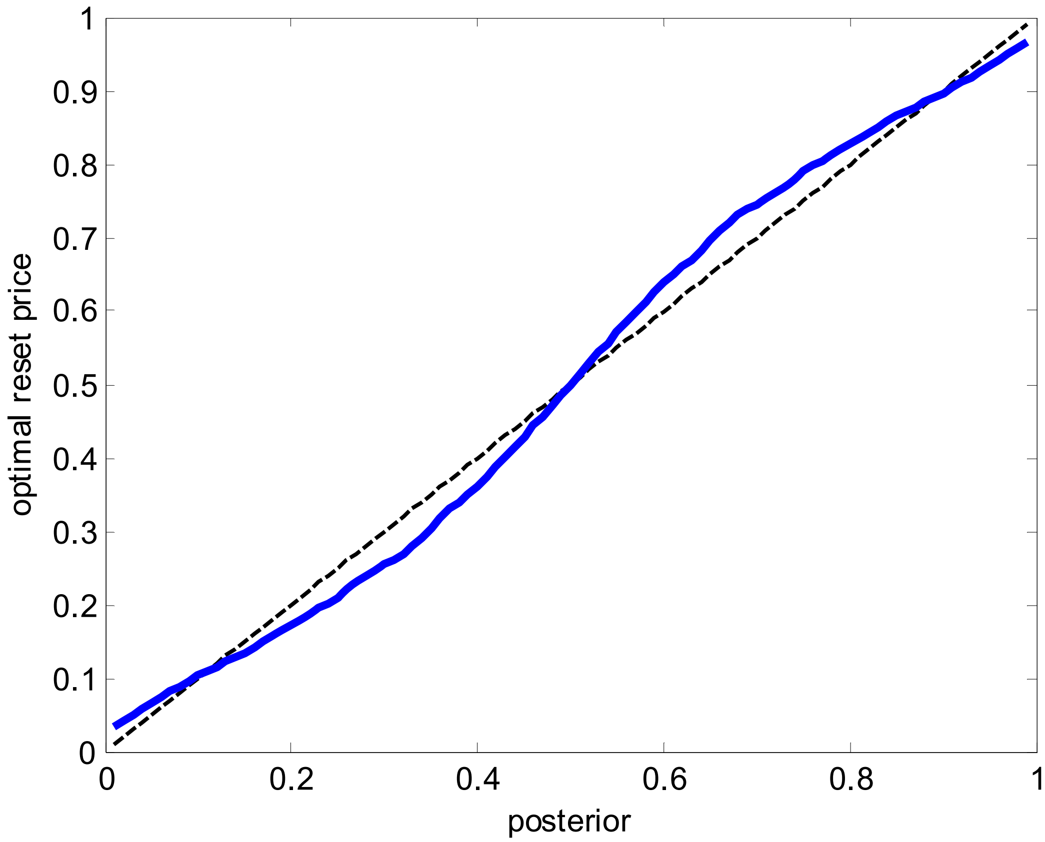


Figure 2. Policy function



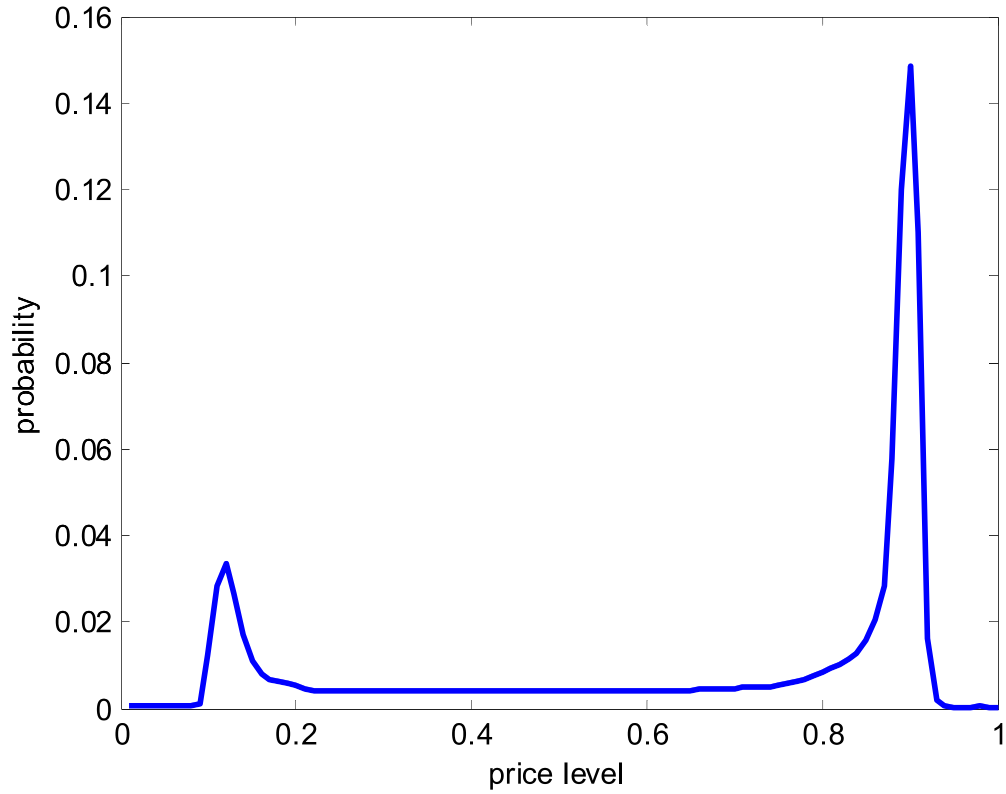
Note: This figure plots regions of actions as a function of two state variables: price and posterior probability of nominal demand being high. Baseline calibration.

Figure 3. Optimal reset price



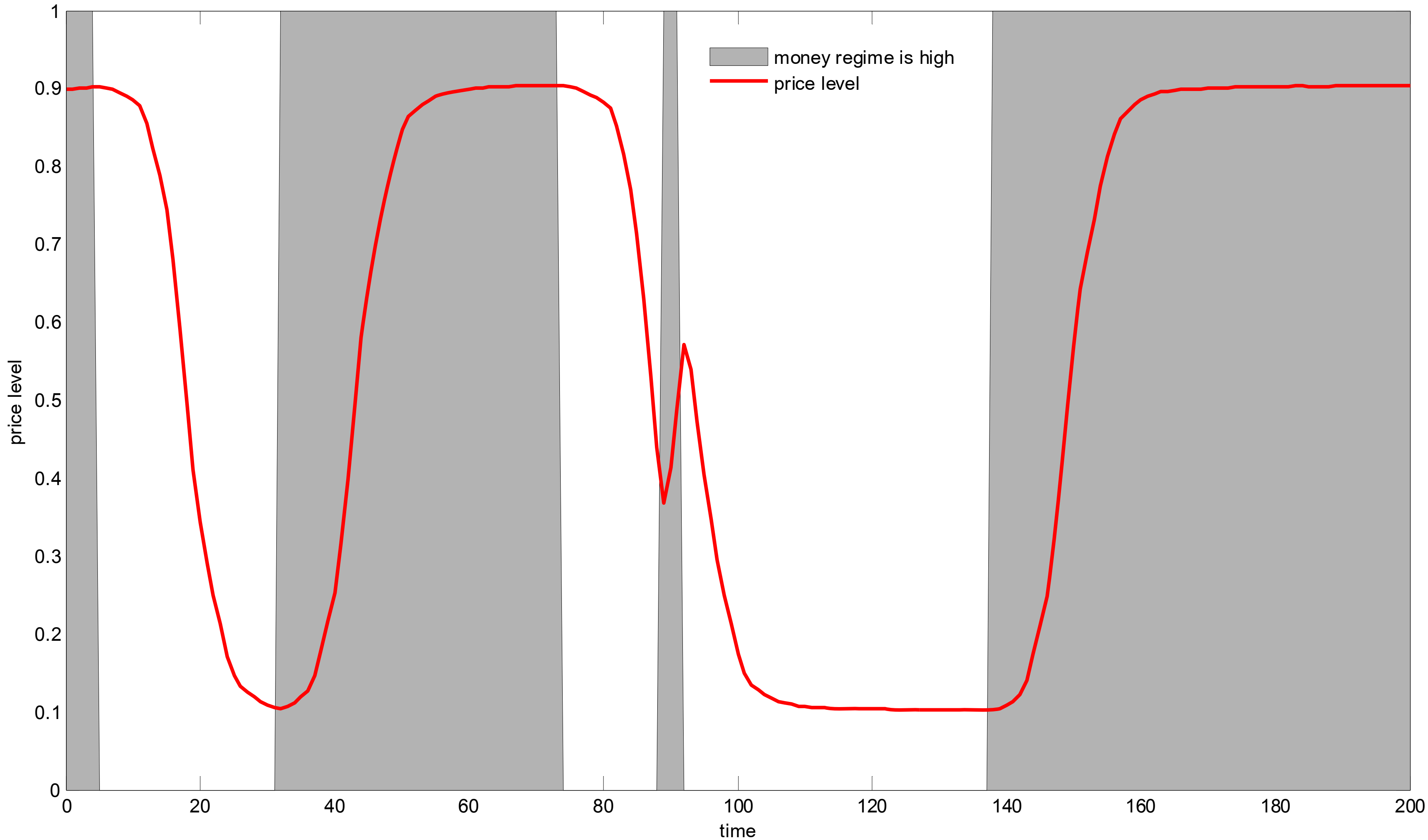
Note: The figure plots the optimal reset price as a function of the posterior probability of nominal demand being high. The broken (45°) line is the optimal frictionless reset price $p_t^{\#}$. Baseline calibration.

Figure 4. Distribution of the price level in the high nominal demand regime, ϕ_H



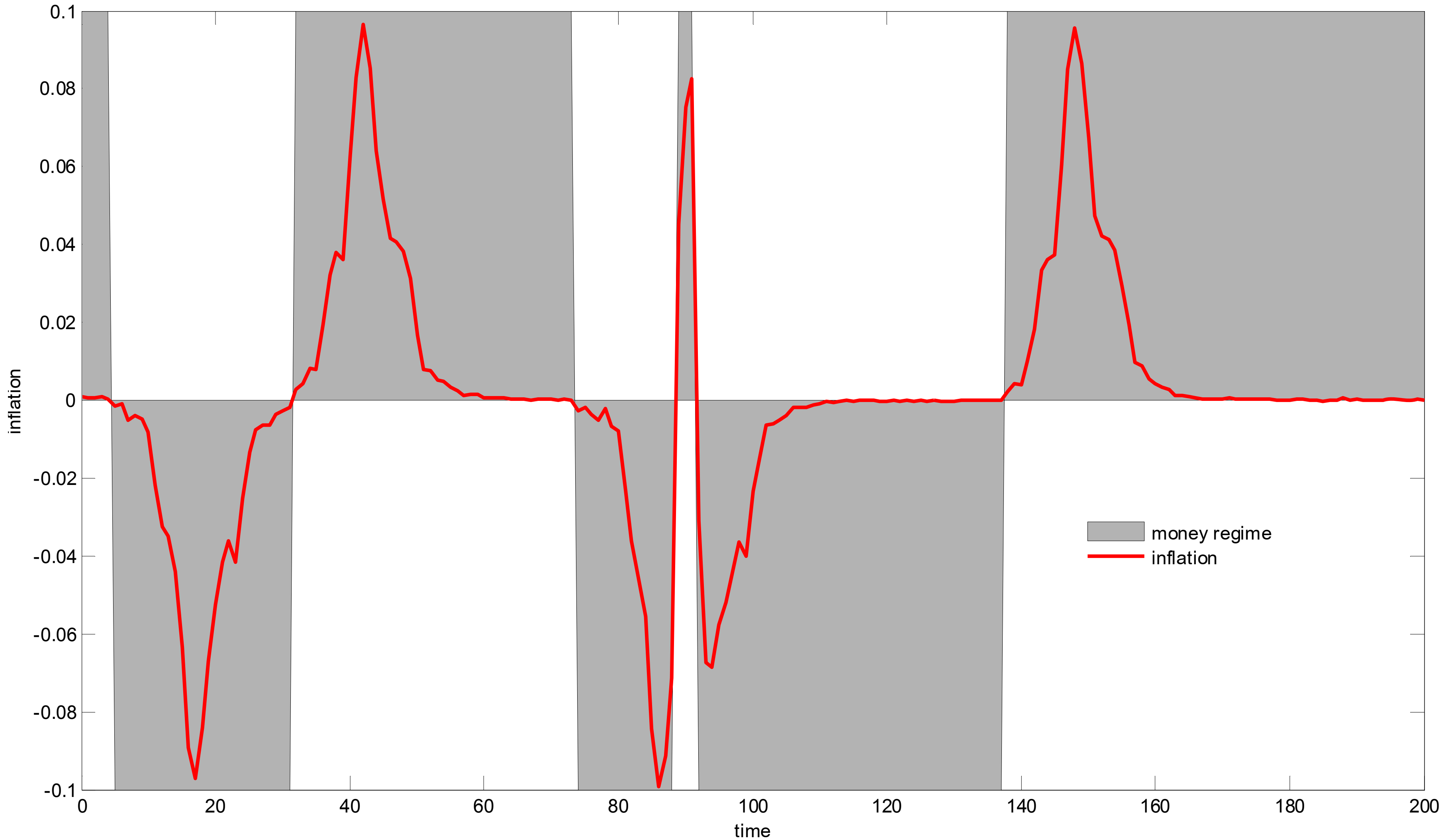
Note: The figure plots the equilibrium distribution of the price level $\overline{P}$ conditional on nominal demand being high. The conditional distribution of $\overline{P}$ given the low regime is symmetric to the presented distribution around 0.5.

Figure 5. Price level and nominal demand



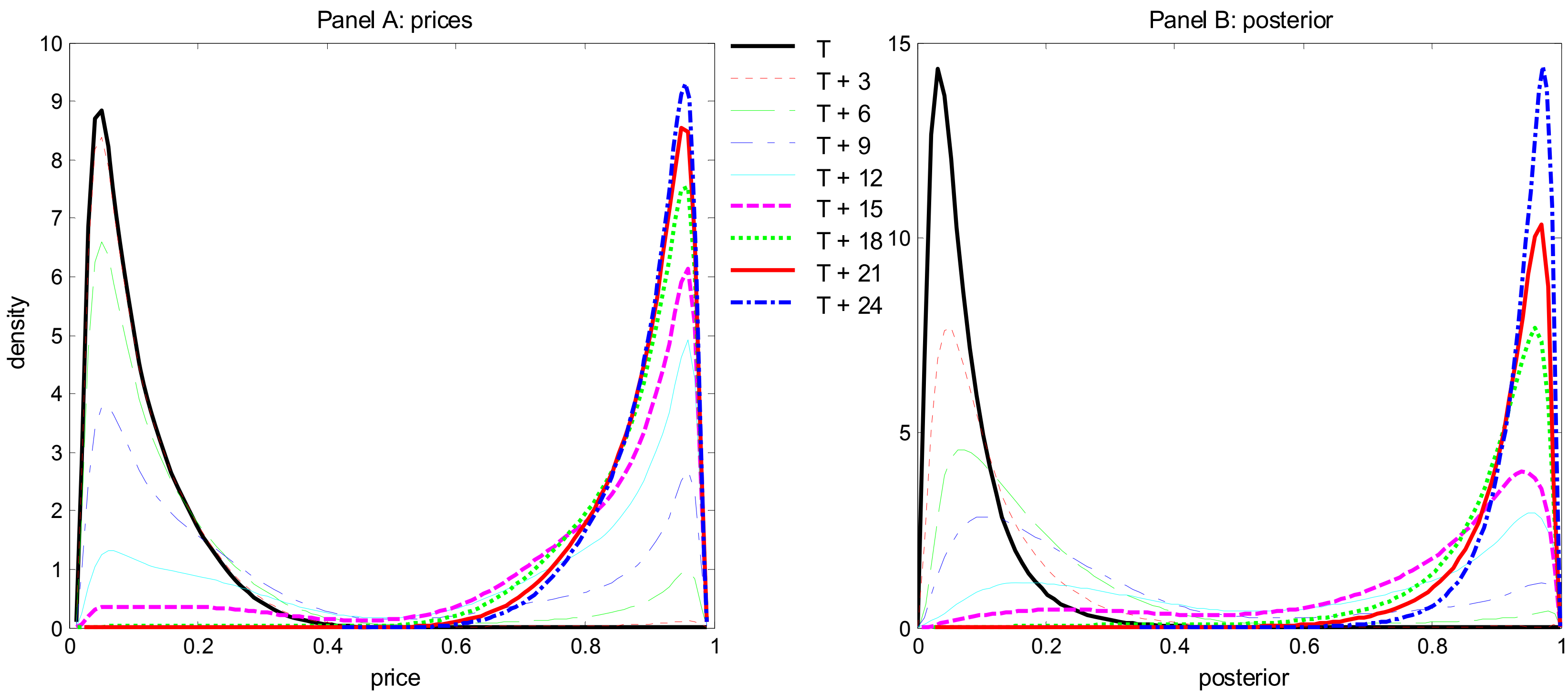
Note: This figure plots a typical path of the price level. The model is simulated for 4,200 periods and the first 4,000 periods are discarded. Shaded area indicates times when nominal demand is high.

Figure 6. Inflation and nominal demand



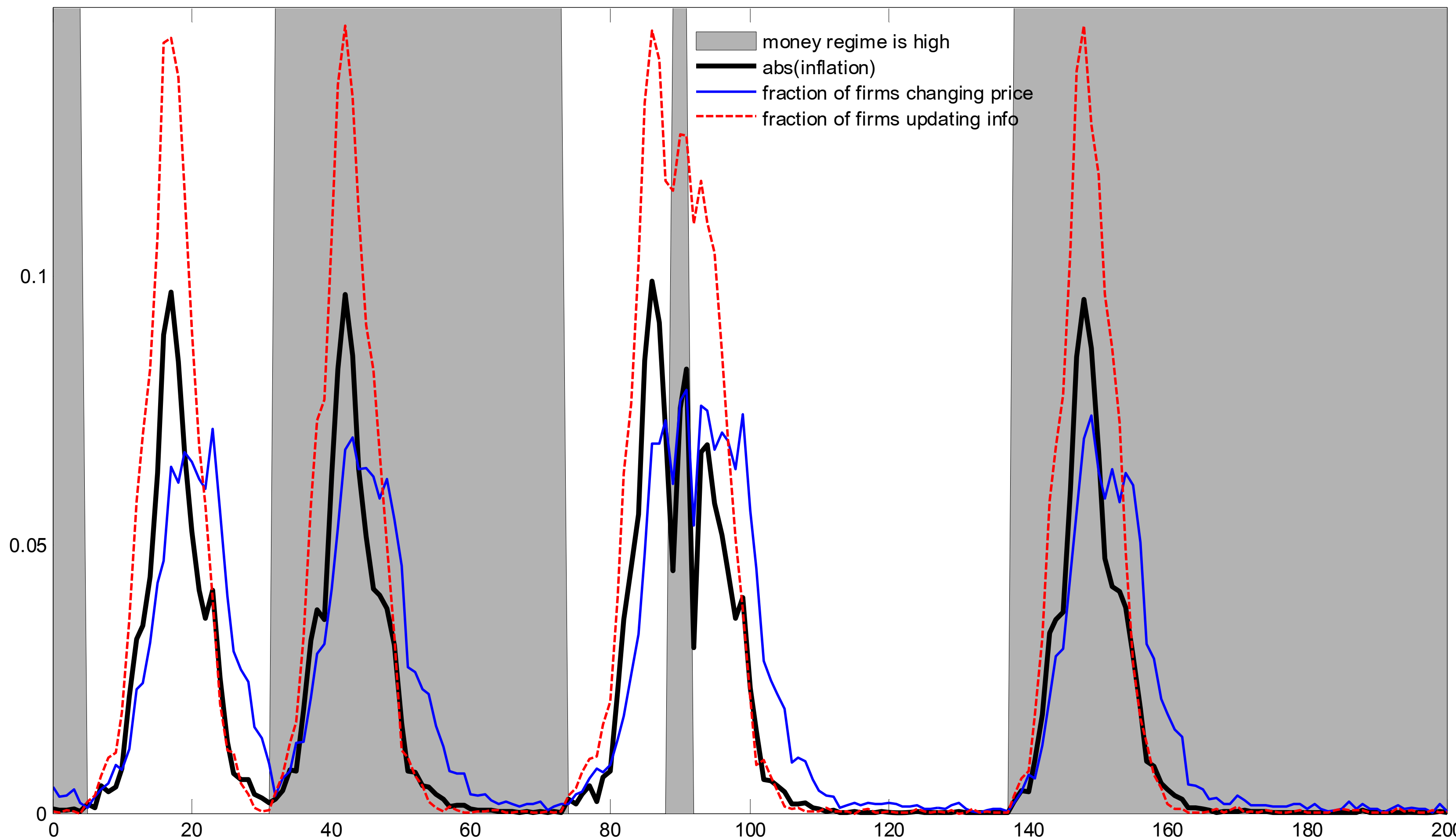
Note: This figure plots a typical path of inflation. The model is simulated for 4,200 periods and the first 4,000 periods are discarded. Shaded area above (below) horizontal axis indicates times when nominal demand is high (low).

Figure 7. Evolution of the cross-sectional density of firm-level prices and posteriors



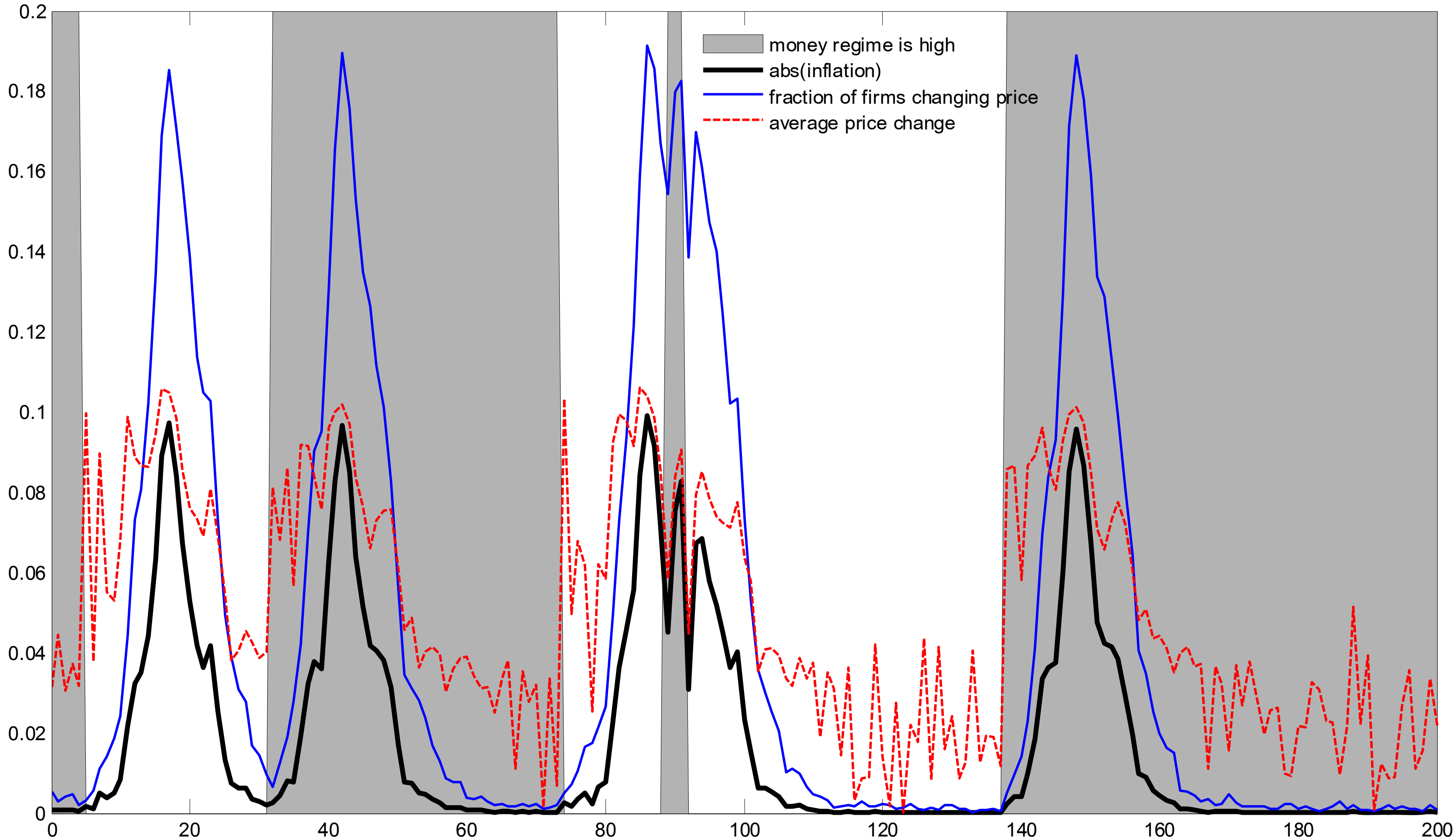
Note: Panel A plots the evolution of the distribution of firm-level prices. Panel B plots the evolution of the distribution of firm-level posteriors. T=138 is the time of change in the money regime, which corresponds to the regime change in Figure 9.

Figure 8. Frequency of actions



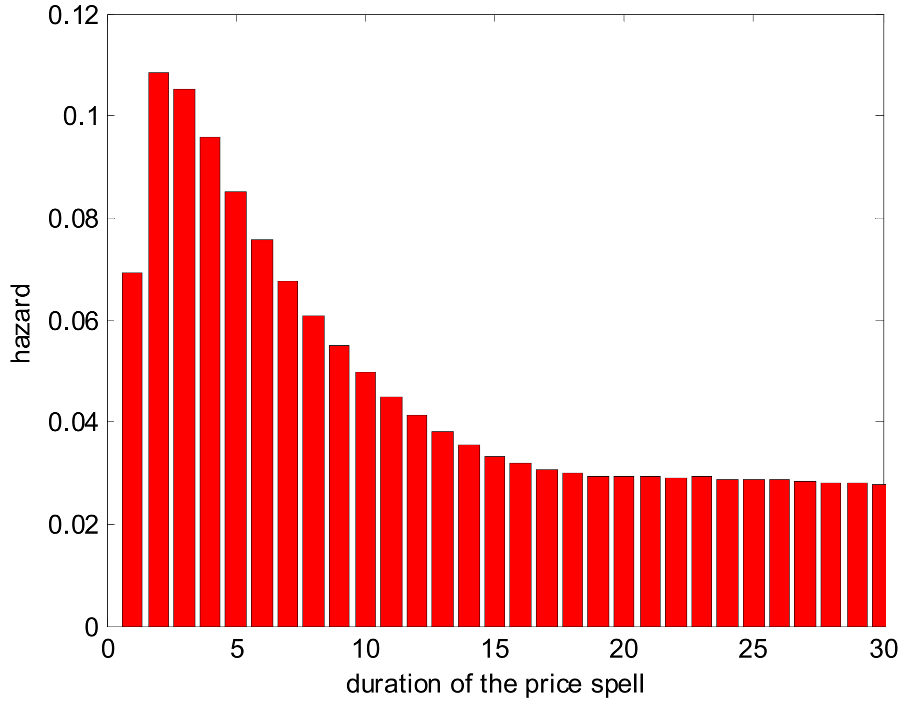
Note: This figure plots a typical path of inflation vis-à-vis the frequency of two actions: 1) change price without updating information and 2) update information. The model is simulated for 4,200 periods and the first 4,000 periods are discarded. Shaded area indicates times when nominal demand is high.

Figure 9. Intensive and extensive margins of inflation



Note: This figure plots a typical path of inflation vis-à-vis the intensive (average price change) and extensive (fraction of firms changing price) margins of inflation. The model is simulated for 4,200 periods and the first 4,000 periods are discarded. Shaded area indicates times when nominal demand is high.

Figure 10. Hazard function for price adjustment



Note: The figure plots the probability of price adjustment given that price remained fixed for a given number of periods (duration of the price spell). Benchmark model and calibration.

Figure 11. Comparative statistics: Regions of actions

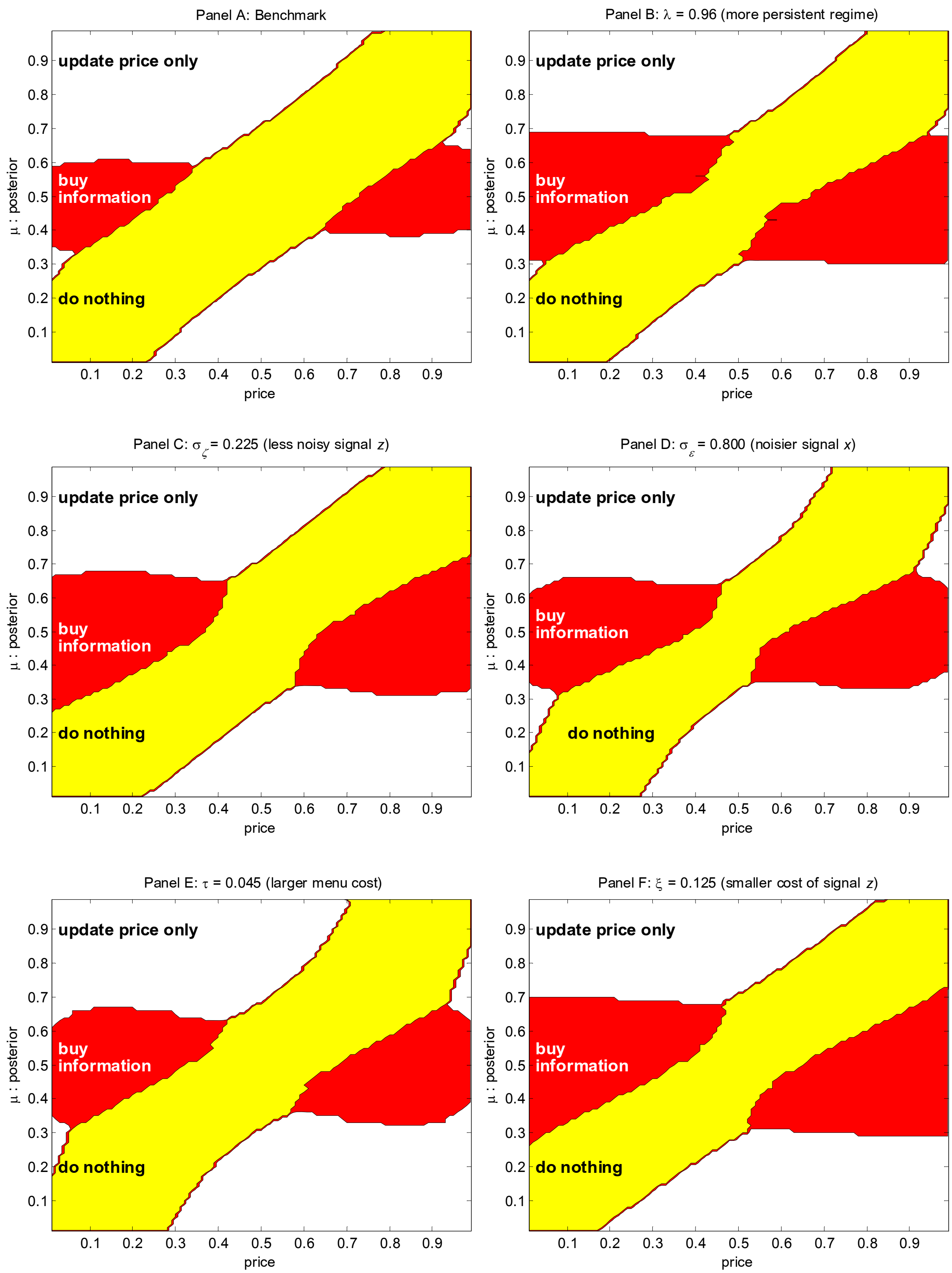
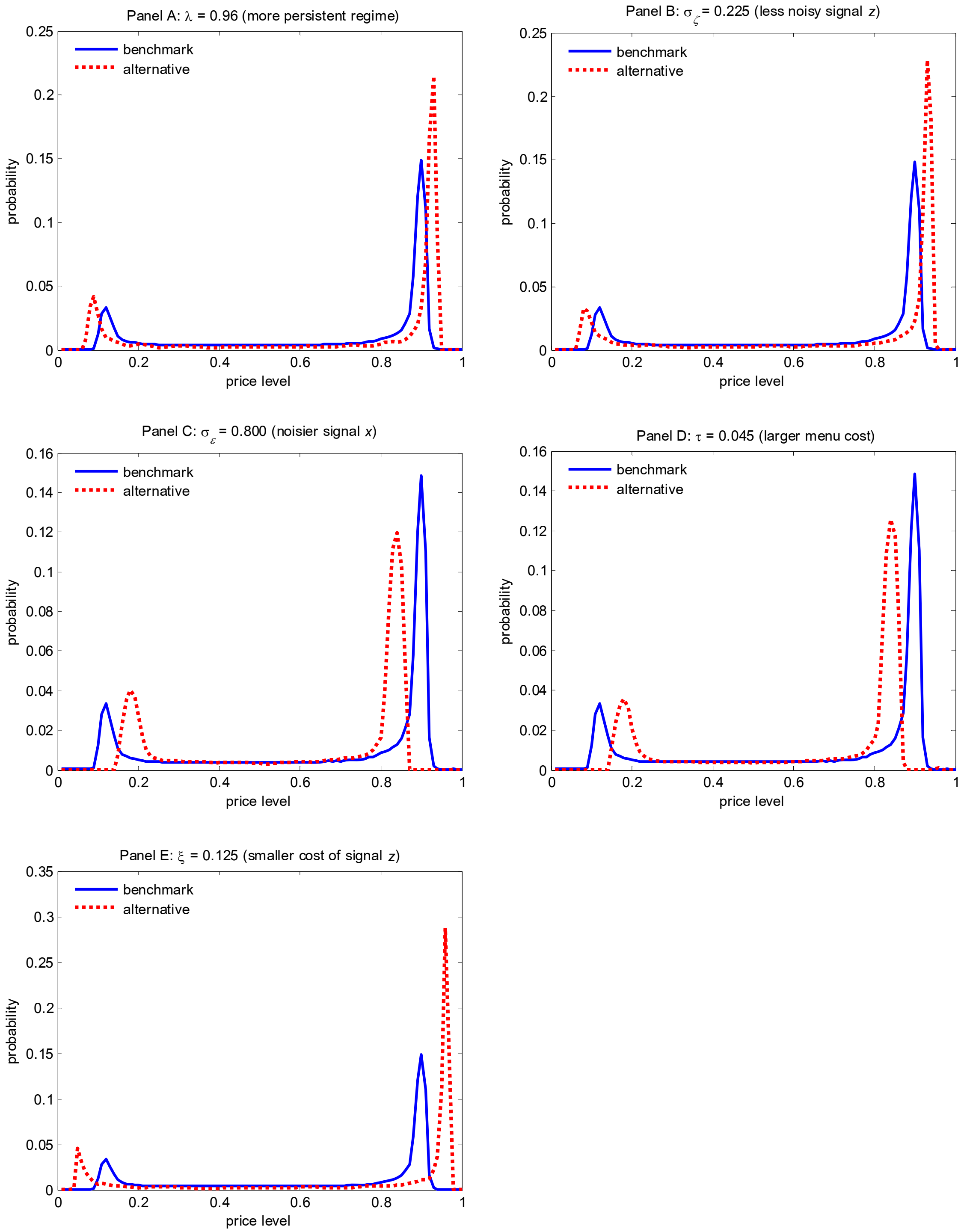
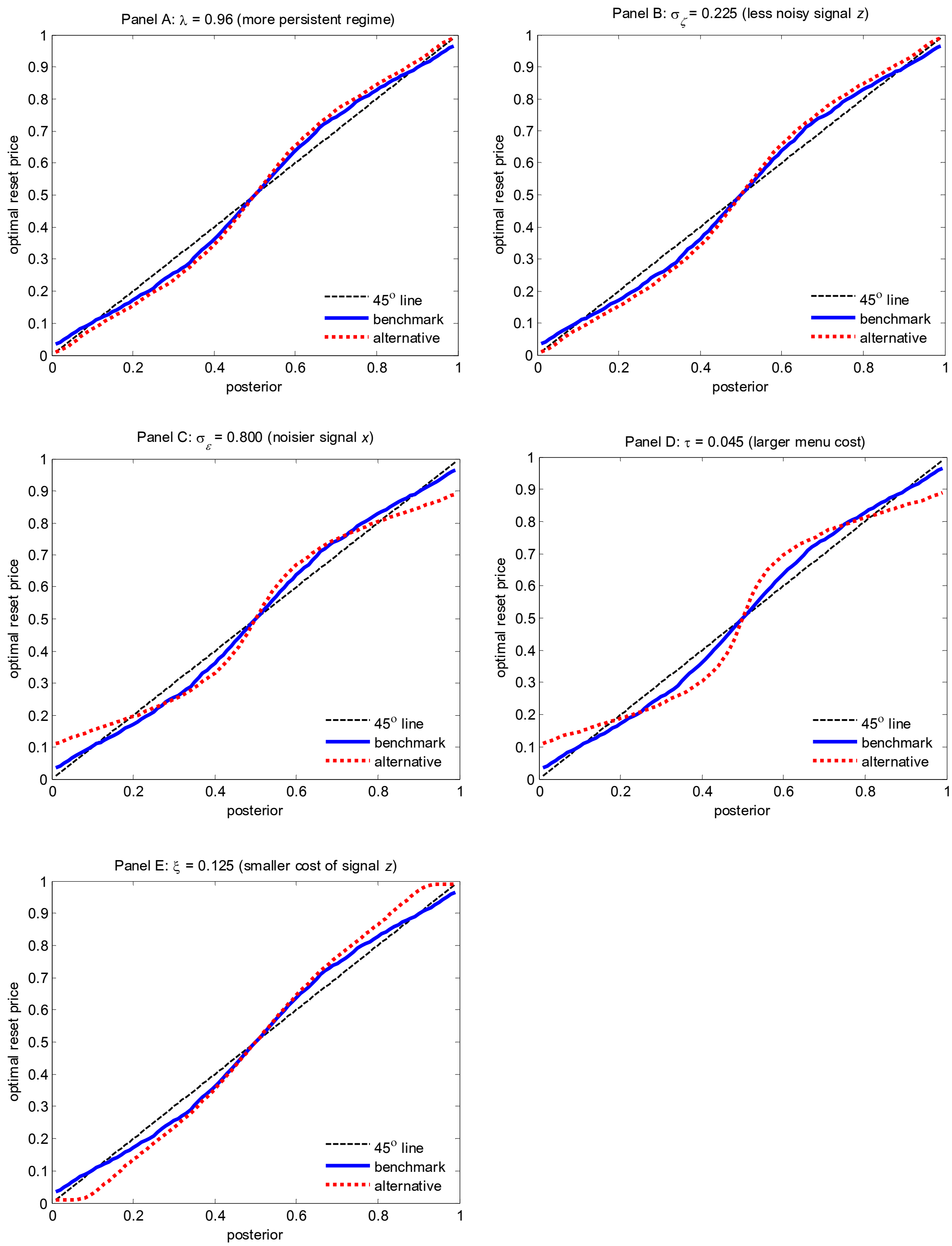


Figure 12. Comparative statistics: Equilibrium distribution of P_t



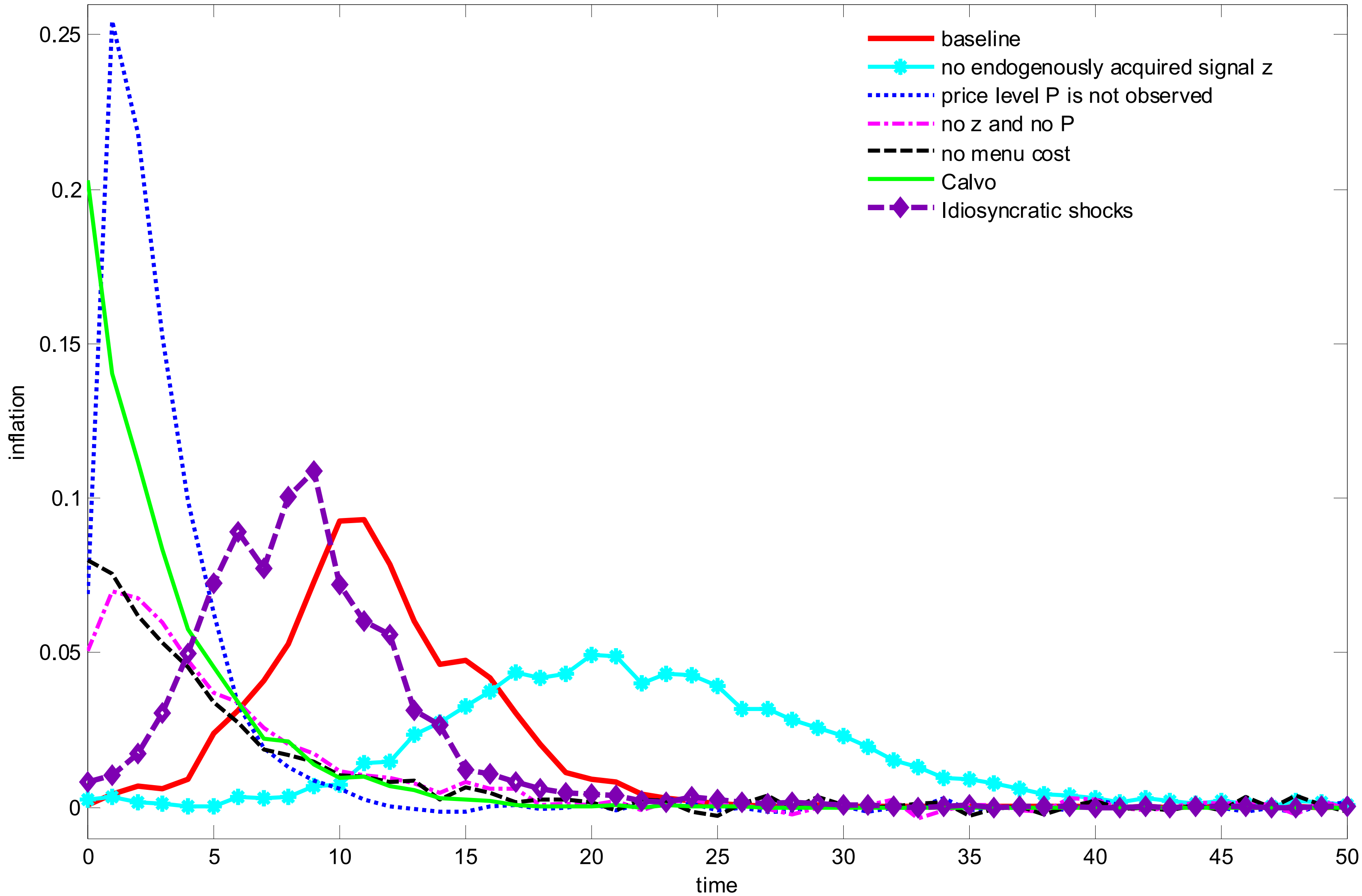
Note: "Benchmark" corresponds the benchmark calibration of the model. "Alternative" corresponds to the calibration with the indicated modification to the benchmark calibration and holding other parameters unchanged.

Figure 13. Comparative statics: Optimal reset price



Note: "Benchmark" corresponds the benchmark calibration of the model. "Alternative" corresponds to the calibration with the indicated modification to the benchmark calibration and holding other parameters unchanged.

Figure 14. Impulse response of inflation under alternative information and pricing scenarios.



Note: This figure plots the impulse response of inflation when nominal demand changes from low to high and the economy starts from the low-demand steady state. These impulse responses correspond to the event of the regime change at T=138 in Figure 6 and Figure 8. The case of "no z and no P" corresponds to the scenario where the price level is not observable to firms and signal z_{it} is not available to firms. Line "Idiosyncratic shocks" corresponds to the scenario where firms are randomly hit with firm-specific shocks to menu costs. See text for further details.

Appendix 1. Proofs

Proof of Proposition 1:

Without loss of generality, one can drop firm index *i*.

Step 1:

Using the Markov property of the money regime chain and Bayes formula, one can show that

$$
\begin{aligned}
\Pr(\overline{P}_{t-1} \cap y_t = y_H \cap I_{t-1}) &= \Pr(\overline{P}_{t-1} \cap y_t = y_H \cap [(y_{t-1} = y_H) \cup (y_{t-1} = y_L)] \cap I_{t-1}) \\
&= \Pr(\overline{P}_{t-1} \cap y_t = y_H \cap y_{t-1} = y_H \cap I_{t-1}) + \Pr(\overline{P}_{t-1} \cap y_t = y_H \cap y_{t-1} = y_L \cap I_{t-1}) \\
&= \Pr(y_t = y_H \mid \overline{P}_{t-1} \cap y_{t-1} = y_H \cap I_{t-1}) \Pr(\overline{P}_{t-1} \mid y_{t-1} = y_H \cap I_{t-1}) \Pr(y_{t-1} = y_H \cap I_{t-1}) + \\
&\qquad \Pr(y_t = y_H \mid \overline{P}_{t-1} \cap y_{t-1} = y_L \cap I_{t-1}) \Pr(\overline{P}_{t-1} \mid y_{t-1} = y_L \cap I_{t-1}) \Pr(y_{t-1} = y_L \cap I_{t-1}) \\
&\overset{(A)}{=} \Pr(\overline{P}_{t-1} \mid y_{t-1} = y_H \cap I_{t-1}) \Pr(y_t = y_H \mid y_{t-1} = y_H) \Pr(y_{t-1} = y_H \cap I_{t-1}) + \\
&\qquad + \Pr(\overline{P}_{t-1} \mid y_{t-1} = y_L \cap I_{t-1}) \Pr(y_t = y_H \mid y_{t-1} = y_L) \Pr(y_{t-1} = y_L \cap I_{t-1}) \\
&\overset{(B)}{=} \Pr(\overline{P}_{t-1} \mid y_{t-1} = y_H \cap I_{t-1}) \lambda \Pr(y_{t-1} = y_H \cap I_{t-1}) + \\
&\qquad \Pr(\overline{P}_{t-1} \mid y_{t-1} = y_L \cap I_{t-1})(1-\lambda) \Pr(y_{t-1} = y_L \cap I_{t-1}) \\
&= \Pr(\overline{P}_{t-1} \mid y_{t-1} = y_H \cap I_{t-1}) \lambda \Pr(y_{t-1} = y_H \mid I_{t-1}) \Pr(I_{t-1}) + \\
&\qquad \Pr(\overline{P}_{t-1} \mid y_{t-1} = y_L \cap I_{t-1})(1-\lambda) \Pr(y_{t-1} = y_L \mid I_{t-1}) \Pr(I_{t-1}) \\
&\overset{(C)}{=} \phi_H(\overline{P}_{t-1}; I_{t-1}) \lambda \Pr(y_{t-1} = y_H \mid I_{t-1}) \Pr(I_{t-1}) + \phi_L(\overline{P}_{t-1}; I_{t-1})(1-\lambda) \Pr(y_{t-1} = y_L \mid I_{t-1}) \Pr(I_{t-1}) \\
&\overset{(D)}{=} \phi_H(\overline{P}_{t-1}; I_{t-1}) \lambda \mu_{t-1} \Pr(I_{t-1}) + \phi_L(\overline{P}_{t-1}; I_{t-1})(1-\lambda)(1-\mu_{t-1}) \Pr(I_{t-1})
\end{aligned}
$$

where equality (A) follows from the Markov property of the chain y_t; equality (B) follows from the definition of the transition probability (transition matrix); equality (C) follows from the definition of the conditional distribution of the signal $\overline{P}$; equality (D) follows from the definition of the μ_t.

Since agents believe that aggregate endogenous signals $\overline{P}$ are conditionally independent (i.e., $\Pr(P_{t-1} \mid y_{t-1}, I_{t-1}) = \Pr(P_{t-1} \mid y_{t-1})$), $\phi_H(\overline{P}_{t-1}; I_{t-1}) = \phi_H(\overline{P}_{t-1})$ and, hence,

$$\Pr(\overline{P}_{t-1} \cap y_t = y_H \cap I_{t-1}) = \phi_H(\overline{P}_{t-1}) \lambda \mu_{t-1} \Pr(I_{t-1}) + \phi_L(\overline{P}_{t-1})(1-\lambda)(1-\mu_{t-1}) \Pr(I_{t-1}).$$

Likewise, $\Pr(\overline{P}_{t-1} \cap y_t = y_L \cap I_{t-1}) = \phi_H(\overline{P}_{t-1})(1-\lambda) \mu_{t-1} \Pr(I_{t-1}) + \phi_L(\overline{P}_{t-1}) \lambda (1-\mu_{t-1}) \Pr(I_{t-1})$.

Step 2:

One can also show that signals $\overline{P}$ and *x* are conditionally independent, that is,

$\Pr(\overline{P}_{t-1} \cap x_t \mid y_t = y_j \cap I_{t-1}) = \Pr(\overline{P}_{t-1} \mid y_t = y_j \cap I_{t-1}) \Pr(x_t \mid y_t = y_j)$ for $j = \{H, L\}$.

This property follows from signal x being conditionally independent from the rest of the model, i.e., $\Pr(x_t = x \mid y_t, y_{t-1}, ..., I_t) = \Pr(x_t = x \mid y_t)$ because by assumption $x_t = y_t + \varepsilon_t$ and ε_t is independently distributed across time and firms.

Step 3:

$$\mu_t \equiv \Pr(y_t = y_H \mid I_t) = \frac{\Pr(y_t = y_H \cap I_t)}{\Pr(I_t)} = \frac{\Pr(y_t = y_H \cap \overline{P}_{t-1} \cap x_t \cap I_{t-1})}{\Pr(\overline{P}_{t-1} \cap x_t \cap I_{t-1})}$$

$$= \frac{\Pr(\overline{P}_{t-1} \cap x_t \mid y_t = y_H \cap I_{t-1}) \Pr(y_t = y_H \cap I_{t-1})}{\left\{\Pr(\overline{P}_{t-1} \cap x_t \mid y_t = y_H \cap I_{t-1}) \Pr(y_t = y_H \cap I_{t-1}) + \Pr(\overline{P}_{t-1} \cap x_t \mid y_t = y_L \cap I_{t-1}) \Pr(y_t = y_L \cap I_{t-1})\right\}}$$

$$\overset{(A)}{=} \frac{\Pr(\overline{P}_{t-1} \mid y_t = y_H \cap I_{t-1}) \Pr(x_t \mid y_t = y_H) \Pr(y_t = y_H \cap I_{t-1})}{\left\{\begin{array}{l} \Pr(\overline{P}_{t-1} \mid y_t = y_H \cap I_{t-1}) \Pr(x_t \mid y_t = y_H) \Pr(y_t = y_H \cap I_{t-1}) + \\ \qquad + \Pr(\overline{P}_{t-1} \mid y_t = y_L \cap I_{t-1}) \Pr(x_t \mid y_t = y_L) \Pr(y_t = y_L \cap I_{t-1}) \end{array}\right\}}$$

$$= \frac{\Pr(\overline{P}_{t-1} \cap y_t = y_H \cap I_{t-1}) \Pr(x_t \mid y_t = y_H)}{\left\{\Pr(\overline{P}_{t-1} \cap y_t = y_H \cap I_{t-1}) \Pr(x_t \mid y_t = y_H) + \Pr(\overline{P}_{t-1} \cap y_t = y_L \cap I_{t-1}) \Pr(x_t \mid y_t = y_L)\right\}}$$

$$\overset{(B)}{=} \frac{\Pr(\overline{P}_{t-1} \cap y_t = y_H \cap I_{t-1}) \psi_H(x_t)}{\Pr(\overline{P}_{t-1} \cap y_t = y_H \cap I_{t-1}) \psi_H(x_t) + \Pr(\overline{P}_{t-1} \cap y_t = y_L \cap I_{t-1}) \psi_L(x_t)} =$$

$$\overset{(C)}{=} \frac{\psi_H(x_t)[\phi_H(\overline{P}_{t-1})\lambda\mu_{t-1} + \phi_L(\overline{P}_{t-1})(1-\lambda)(1-\mu_{t-1})]}{\left\{\psi_H(x_t)[\phi_H(\overline{P}_{t-1})\lambda\mu_{t-1} + \phi_L(\overline{P}_{t-1})(1-\lambda)(1-\mu_{t-1})] + \psi_L(x_t)[\phi_H(\overline{P}_{t-1})(1-\lambda)\mu_{t-1} + \phi_L(\overline{P}_{t-1})\lambda(1-\mu_{t-1})]\right\}},$$

where equality (A) follows from step 2; equality (B) definition of the conditional distribution for signal x; equality (C) follows from step 1. ■

Derivation of the joint density $\Theta(x_{i,t+1}, \overline{P}_t, \mu_{it})$:

Observe that

$$\begin{aligned} &\Pr(x = x_{i,t+1}, \overline{P} = \overline{P}_t \mid I_{it}) = \\ &\quad = \Pr(x = x_{i,t+1}, \overline{P} = \overline{P}_t, y_{t+1} = y_H, y_t = y_H \mid I_{it}) \\ &\qquad + \Pr(x = x_{i,t+1}, \overline{P} = \overline{P}_t, y_{t+1} = y_L, y_t = y_H \mid I_{it}) \\ &\qquad + \Pr(x = x_{i,t+1}, \overline{P} = \overline{P}_t, y_{t+1} = y_H, y_t = y_L \mid I_{it}) \\ &\qquad + \Pr(x = x_{i,t+1}, \overline{P} = \overline{P}_t, y_{t+1} = y_L, y_t = y_L \mid I_{it}) \end{aligned}$$

$$
\begin{aligned}
&= \Pr(x = x_{i,t+1}, \bar{P} = \bar{P}_t \mid y_{t+1} = y_H, y_t = y_H, I_{it}) \Pr(y_{t+1} = y_H \mid y_t = y_H, I_{it}) \Pr(y_t = y_H \mid I_{it}) \\
&\quad + \Pr(x = x_{i,t+1}, \bar{P} = \bar{P}_t \mid y_{t+1} = y_L, y_t = y_H, I_{it}) \Pr(y_{t+1} = y_L \mid y_t = y_H, I_{it}) \Pr(y_t = y_H \mid I_{it}) \\
&\quad + \Pr(x = x_{i,t+1}, \bar{P} = \bar{P}_t \mid y_{t+1} = y_H, y_t = y_L, I_{it}) \Pr(y_{t+1} = y_H \mid y_t = y_L, I_{it}) \Pr(y_t = y_L \mid I_{it}) \\
&\quad + \Pr(x = x_{i,t+1}, \bar{P} = \bar{P}_t \mid y_{t+1} = y_L, y_t = y_L, I_{it}) \Pr(y_{t+1} = y_L \mid y_t = y_L, I_{it}) \Pr(y_t = y_L \mid I_{it}) \\
&\overset{(A)}{=} \Pr(x = x_{i,t+1} \mid y_{t+1} = y_H) \Pr(\bar{P} = \bar{P}_t \mid y_t = y_H) \Pr(y_{t+1} = y_H \mid y_t = y_H, I_{it}) \Pr(y_t = y_H \mid I_{it}) \\
&\quad + \Pr(x = x_{i,t+1} \mid y_{t+1} = y_L) \Pr(\bar{P} = \bar{P}_t \mid y_t = y_H) \Pr(y_{t+1} = y_L \mid y_t = y_H, I_{it}) \Pr(y_t = y_H \mid I_{it}) \\
&\quad + \Pr(x = x_{i,t+1} \mid y_{t+1} = y_H) \Pr(\bar{P} = \bar{P}_t \mid y_t = y_L) \Pr(y_{t+1} = y_H \mid y_t = y_L, I_{it}) \Pr(y_t = y_L \mid I_{it}) \\
&\quad + \Pr(x = x_{i,t+1} \mid y_{t+1} = y_L) \Pr(\bar{P} = \bar{P}_t \mid y_t = y_L) \Pr(y_{t+1} = y_L \mid y_t = y_L, I_{it}) \Pr(y_t = y_L \mid I_{it}) \\
&\overset{(B)}{=} \Pr(x = x_{i,t+1} \mid y_{t+1} = y_H) \Pr(\bar{P} = \bar{P}_t \mid y_t = y_H) \Pr(y_{t+1} = y_H \mid y_t = y_H) \Pr(y_t = y_H \mid I_{it}) \\
&\quad + \Pr(x = x_{i,t+1} \mid y_{t+1} = y_L) \Pr(\bar{P} = \bar{P}_t \mid y_t = y_H) \Pr(y_{t+1} = y_L \mid y_t = y_H) \Pr(y_t = y_H \mid I_{it}) \\
&\quad + \Pr(x = x_{i,t+1} \mid y_{t+1} = y_H) \Pr(\bar{P} = \bar{P}_t \mid y_t = y_L) \Pr(y_{t+1} = y_H \mid y_t = y_L) \Pr(y_t = y_L \mid I_{it}) \\
&\quad + \Pr(x = x_{i,t+1} \mid y_{t+1} = y_L) \Pr(\bar{P} = \bar{P}_t \mid y_t = y_L) \Pr(y_{t+1} = y_L \mid y_t = y_L) \Pr(y_t = y_L \mid I_{it}) \\
&\overset{(C)}{=} \psi_H(x_{i,t+1}) \phi_H(\bar{P}_t) \lambda \mu_{it} + \psi_H(x_{i,t+1}) \phi_L(\bar{P}_t)(1-\lambda)\mu_{it} + \\
&\quad \psi_L(x_{i,t+1}) \phi_H(\bar{P}_t)(1-\lambda)(1-\mu_{it}) + \psi_L(x_{i,t+1}) \phi_L(\bar{P}_t) \lambda (1-\mu_{it}) \\
&\equiv \Theta(x_{i,t+1}, \bar{P}_t, \mu_{it}),
\end{aligned}
$$

where equality (A) follows from conditional independence of signals from each other (see proof of Proposition 1) and over time (by construction for x_{it} and by assumption for $\bar{P}_t$); equality (B) follows from the Markov property of the chain y_t; equality (C) follows from the definitions of conditional distributions, transition probabilities, and the posterior probability of being in a given money regime. ■

Derivation of the expected value of the posterior:

It follows from (1) that

$$
\begin{aligned}
E\mu_{i,t+1} &= \iint f(x_{i,t+1}, \bar{P}_t, \mu_{it}) \Theta(x_{i,t+1}, \bar{P}_t, \mu_{it}) dx_{i,t+1} d\bar{P}_t \\
&= \iint \left[\psi_H(x_{i,t+1}) \phi_H(\bar{P}_t) \lambda \mu_{it} + \psi_H(x_{i,t+1}) \phi_L(\bar{P}_t)(1-\lambda)(1-\mu_{it}) \right] dx_{i,t+1} d\bar{P}_t \\
&= \lambda \mu_{it} \underbrace{\iint \psi_H(x_{i,t+1}) \phi_H(\bar{P}_t) dx_{i,t+1} d\bar{P}_t}_{=1} + (1-\lambda)(1-\mu_{it}) \underbrace{\iint \psi_H(x_{i,t+1}) \phi_L(\bar{P}_t) dx_{i,t+1} d\bar{P}_t}_{=1} \\
&= \lambda \mu_{it} + (1-\lambda)(1-\mu_{it})
\end{aligned}
$$

Because $\lambda \in (0,1)$, the eigenroot of the above difference equation is strictly less than unity in absolute value. The unique solution to $\mu = \lambda\mu + (1-\lambda)(1-\mu)$ is $\mu = 1/2$. ■

Appendix 2. Three period model

To give the reader a better sense of how incentives to wait work and to show that the wait-and-see game survives and generates sluggish price adjustment and hump shaped inflation response even if agents are fully rational, I consider a simplified version of the baseline model with the following assumptions being modified:

1) firms are fully rational;
2) the game continues for three periods: $t = 1, 2, 3$ plus initial condition $t = 0$;
3) nominal demand regime permanently shifts from low to high at time $t = 1$ but firms to do observe the change;
4) firms have non-informative priors in period $t = 0$ about the state of the regime;
5) firms can choose only two prices $p_t = p_H^* \equiv 1$ or $p_t = p_L^* \equiv 0$ (this assumption makes intra-temporal optimization very simple);
6) initial prices at $t = 0$ are set equal to $p_0 = 0$;
7) no endogenous acquisition of information.

Since firm level prices can take only two values, the price level in period t is $\overline{P}_t = \int p_t(i)di = p_L^* + s_t(p_H^* - p_L^*)$ where s_t is the fraction of firms with high prices. Because the regime change in nominal demand is permanent and firms observe only noisy signals about the state of nominal demand, the posterior probability evolves as follows

$$\mu_t = \frac{\phi_H(x)\mu_{t-1}}{\phi_H(x)\mu_{t-1} + \phi_L(x)(1-\mu_{t-1})} = \mu(\mu_{t-1}, x),$$

where $\phi_H(x)$ and $\phi_L(x)$ is the p.d.f. of the signal x in high and low states. The expected loss function is $E(-\pi_t) \equiv (p_t - p_H^*)^2 \mu_t^H + (p_t - p_L^*)^2 (1-\mu_t^H) + \mathbf{1}\{p_t \neq p_{t-1}\} \times \tau$. The problem is solved by backward induction:

Period *t* = 3.

Price inherited from period $t = 2$ is high $p_2 = p_H^*$:

- keep the price fixed at p_H^*: payoff = $1 - \mu_3$
- change the price to p_L^*: payoff = $\mu_3 + \tau$

Optimized value is $V_{3,H} = \min\{1-\mu_3, \mu_3 + \tau\}$

Price inherited from period $t = 2$ is low $p_2 = p_L^*$:

keep the price fixed at p_L^*: payoff = μ_3

change the price to p_H^*: payoff = $1 - \mu_3 + \tau$

Optimized value is $V_{3,L} = \min\{\mu_3, 1-\mu_3+\tau\}$

Period $t = 2$.

Define continuation values of choosing high and low prices in period $t = 2$ as follows:

$$V^*_{3,H} \equiv EV_{3,H} = E_{\mu_3} \min\{1-\mu_3, \mu_3+\tau\} = \int \min\{1-\underbrace{\mu_3(\mu_2,x)}_{posterior}, \mu_3(\mu_2,x)+\tau\}\Theta(x,\mu_2)dx = V^*_{3,H}(\mu_2)$$

$$V^*_{3,L} \equiv EV_{3,L} = E_{\mu_3} \min\{\mu_3, 1-\mu_3+\tau\} = \int \min\{\mu_3(\mu_2,x), 1-\mu_3(\mu_2,x)+\tau\}\Theta(x,\mu_2)dx = V^*_{3,L}(\mu_2)$$

where $\Theta(x,\mu_2) = \phi_H(x)\mu_2 + \phi_L(x)(1-\mu_2)$ is the density. This integration makes the solution algebraically complicated.

Price inherited from period $t = 1$ is high $p_1 = p^*_H$:

- keep the price fixed at p^*_H: payoff = $1-\mu_2+\beta V^*_{3,H}(\mu_2)$
- change the price to p^*_L: payoff = $\mu_2+\tau+\beta V^*_{3,L}(\mu_2)$

Optimized value is $V_{2,H} = \min\left\{1-\mu_2+\beta V^*_{3,H}(\mu_2), \mu_2+\tau+\beta V^*_{3,L}(\mu_2)\right\}$

Price inherited from period $t = 1$ is low $p_1 = p^*_L$:

- keep the price fixed at p^*_L: payoff = $\mu_2+\beta V^*_{3,L}(\mu_2)$
- change the price to p^*_H: payoff = $1-\mu_2+\tau+\beta V^*_{3,H}(\mu_2)$

Optimized value is $V_{2,L} = \min\left\{\mu_2+\beta V^*_{3,L}(\mu_2), 1-\mu_2+\tau+\beta V^*_{3,H}(\mu_2)\right\}$

Period $t = 1$.

Define the expected value of choosing high and low prices in period $t = 1$ as follows:

$V^*_{2,H} \equiv E_{\mu_3} \min\left\{1-\mu_2+\beta V^*_{3,H}(\mu_2), \mu_2+\tau+\beta V^*_{3,L}(\mu_2)\right\} = V^*_{2,H}(\mu_1)$ and

$V^*_{2,L} \equiv E_{\mu_3} \min\left\{\mu_2+\beta V^*_{3,L}(\mu_2), 1-\mu_2+\tau+\beta V^*_{3,H}(\mu_2)\right\} = V^*_{2,L}(\mu_1)$

Since the price at $t = 0$ is equal to $p_0 = 0$, only one case is considered $p_0 = p^*_L$:

- keep the price fixed at p^*_L: payoff = $\mu_1+\beta V^*_{2,L}(\mu_1)$
- change the price to p^*_H: payoff = $1-\mu_1+\tau+\beta V^*_{2,H}(\mu_1)$

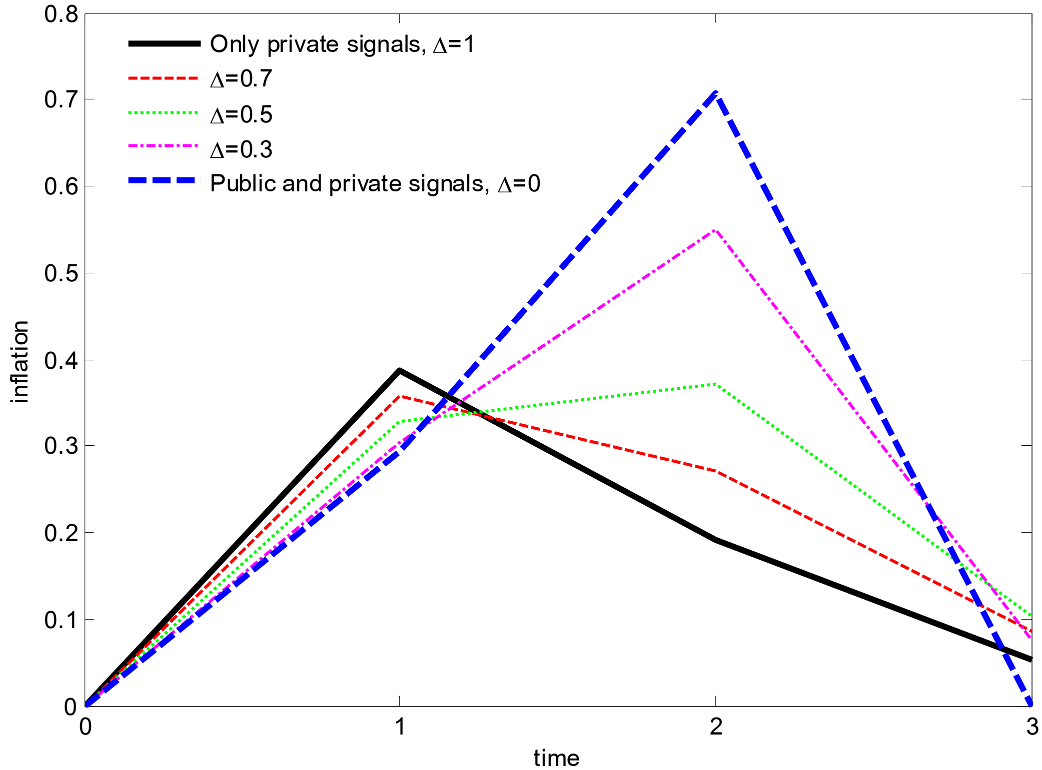
The value functions are approximated on the grid with 100 equally spaced points. Parameterization of the model is discussed in Section 3.1.

Suppose that firms can observe only private signals. The response of inflation for this scenario is represented by the thick solid line in Appendix Figure 1. The qualitative pattern of the response is simple: inflation increases sharply at the time of the shock and then gradually declines. This shape is determined by the fact that the change in the posterior probability of being in the high regime is concave in the prior and the strength of the received signal so that the change in beliefs is the greatest with the first signal. In other words, every additional signal brings less information than the preceding signal and therefore the response is strongest to the most informative piece of news, which arrives in the first period.

Now consider an alternative scenario where firms can observe the price level in previous periods. Since firms are fully rational (i.e., fit the right model to interpret public signals) and there is a continuum of firms, the price level is fully revealing. Thus, in period t = 2, firms know the state of nominal demand with certainty. In this case, firms have two options in period t = 1: A) change the price now; B) wait another period, observe the price level at t = 1, infer the state of nominal demand and then change the price. Since every firm knows the state of nominal demand in period t = 2, there is no price adjustment in t = 3. The response of inflation is represented by the thick broken line in Appendix Figure 1. Note that in this scenario the peak response of inflation is delayed. Most firms choose to postpone price adjustment until full information is revealed precisely for the reasons described above, i.e., the possibility to observe actions of other firms creates incentives to engage in the "wait-and-see" game and to delay price adjustment to free ride on other firms. Note that a fraction of firms chooses to change their prices in the first period because they receive strong private signals (recall that signals are drawn from normal distribution so that signals have unbounded support).

To show the sensitivity of the inflation response to information available in t = 2 or t = 3, it is convenient to work with private signals whose precision increases with time, that is, the standard deviation of the noise in the signals at t = 2 or t = 3 is a fraction of the standard deviation of the noise in the signal at t = 1. Denote the standard deviation of the noise in periods 1, 2 and 3 with $\sigma_1, \sigma_2, \sigma_3$ and assume that $\sigma_2 = \Delta\sigma_1, \sigma_3 = \Delta\sigma_1$ with $\Delta \in [0,1]$. The two scenarios I describe above are the limiting cases. The scenario with only private signals corresponds to the case when $\Delta = 1$. The scenario with public signals corresponds to the case where $\Delta = 0$. The intermediate cases $\Delta \in (0,1)$ correspond to the situation where public signals are contaminated with noise and thus public signals are informative but not fully revealing about the state of nominal demand. As one moves from $\Delta = 1$ to $\Delta = 0$, one can trace the incentive to delay price adjustment in period t = 1 due to the prospect of receiving better information in the future. Inflation response for selected values of Δ is presented in Appendix Figure 1. This comparative statics exercise shows that transition from $\Delta = 1$ to $\Delta = 0$ is monotonic for fractions of firms adjusting their prices in periods t = 1 and t = 2 but the change is nonlinear.

Appendix Figure 1. Inflation response in the three period model



Note: This figure shows the response of inflation in the three period model with perfectly rational firms (see Appendix 2 for description). The solid thick line corresponds to the case where firms observe only private signals (Δ=1). The broken thick line corresponds to the case where firms observe public and private signals (Δ=0). The case intermediate cases (Δ=0.7, 0.5, 0.3) correspond to the cases where firms have signals with precision that increases over time (smaller values of Δ mean greater improvements in the precision of the signals). Parameterization is described in section 3.1. Solution method is described in Appendix 2.