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INTANGIBLE CAPITAL AND PRODUCTIVITY: AN EXPLORATION ON A PANEL OF ITALIAN MANUFACTURING FIRMS

Maria Elena Bontempi
Jacques Mairesse

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ABSTRACT

The paper examines the size and productivity of total intangible capital relative to total tangible capital for a large panel of Italian Manufacturing firms. In the analysis, we decompose total intangibles in two different ways: in intangibles expensed in firms' current accounts (as usually considered in empirical studies) versus intangible capitalized in firms' balance sheets (usually not considered); and in "intellectual capital" (i.e. R&D expenditures, and patenting and related costs) versus "customer capital" (i.e., advertising expenditure, and trademarks and related costs). We systematically assess the robustness of our results by using different specifications of the production functions implying different elasticities of substitution between tangible and intangible capital, and comparing different panel data estimates. Our results underscore that firms' accounting information on intangible investments is genuinely informative, showing that intangible capital and its different components are at least as productive as tangible capital.

Maria Elena Bontempi
Department of Economics, University of Ferrara
Palazzo Bevilacqua Costabili, via Voltapaletto 11
44100 Ferrara Italy
bntmln@unife.it

Jacques Mairesse
INSEE, CREST
15, Boulevard Gabriel PERI
92245 MALAKOFF CEDEX
FRANCE
and NBER
mairesse@ensae.fr

1. Introduction

Inventing new products, improving existing products, refining the techniques used to produce goods and services, and creating a unique public image of a product's quality, are all aspects of a complex process, the end results of which may be termed "intangibles".

Although "intangibles", more broadly speaking, are considered of vital importance in determining the growth of productivity, the link between "intangibles" and productivity is poorly understood.

The stream of new ideas in arrival is neither predictable, nor steady, nor continuous. Studies designed to estimate the connection between spending on intangibles and productivity growth encounter considerable problems. Which data level (line of business, firm, industry, country) would be best used? Do we have sufficient information regarding all types of intangible (human capital, R&D, patents, etc.)? If yes, what are the "right" methods to be adopted when measuring output and inputs, and what is the functional link between output and input? How can account be taken of individuals' heterogeneity when purchasing and/or internally generating intangibles? What is the best way of dealing with the presence of simultaneity (individuals undertaking intangible costs are motivated by the expected profits on such investment, and these profits, in turn, encourage further investment in intangibles)?

Hence, empirical estimates may differ substantially, ranging from no effect on productivity to a very sizeable effect exceeding that of all other types of investment. A survey of empirical studies and their results is provided, for example, in Mairesse and Sassenou (1991).

The present paper aims to assess the productivity of total intangibles and their components, both in relation to that of tangible capital, and in absolute terms. We have attempted to provide answers to some previous questions, as well as furnishing certain innovative information regarding this issue. The sample is a large unbalanced panel of Italian manufacturing companies; data are provided at company level, as is often the case in productivity studies, whereas the country in question, Italy, is a relatively unusual choice for empirical studies in this field. In particular, the definition we provide of intangibles is a new one, compared to the definition usually adopted by both English-language and European studies. This new definition extends our understanding of the most appropriate measure of intangibles, and offers us the chance to join the debate on the expensing and capitalising methods of measuring intangible costs, which is currently of interest from the point of view of the new System of National Accounts (SNA), as well as for the definition of International Financial Reporting Standards (IFRS). We have taken advantage of detailed (and rarely available) accounting information on balance sheets, current accounts and investment flows for the various different components of intangible capital. In fact, we can compare: (a) total intangibles; (b) what we term "intangibles capitalised by us", i.e. intangible stock computed by capitalising the costs recorded in a firm's current accounts (this is the definition usually employed by empirical studies of this issue); (c) intangible capital as recorded in a firm's detailed balance sheets (a new definition of intangibles, in line with Italy's Generally Accepted Accounting Principles, GAAP); (d) what we term "intellectual capital" (i.e. R&D costs; patenting and related costs); (e) what we term "customer capital" (i.e., advertising costs; trademarks and related costs).

As far as the functional link between output and input is concerned, we estimate three different specifications of the production function. The first, which is the accepted standard in the literature on this issue, is an extended Cobb-Douglas function into which intangible and tangible assets enter multiplicatively. It implies that the output elasticities of intangibles and tangibles are constant, while their elasticity of substitution is one. The technical rate of substitution - i.e. the marginal productivity of intangibles compared to that of tangibles – changes according to the tangibles/intangibles ratio. The second Cobb-Douglas has an additive form of total capital, expressed as a weighted sum of its intangible and tangible components. It implies a constant technical rate of substitution, but varying output elasticities and an infinite elasticity of substitution. This formulation leads to estimates that can be difficult to reconcile with the estimates obtained using the more characteristic fully multiplicative Cobb-Douglas formulation (partly because they are usually less robust, depending more importantly on the measurement of intangible and tangible assets,– see, for example, Mairesse and Sassenou, 1991). We therefore experimented with the third encompassing formulation, in which total capital is expressed as a CES function of its tangible and intangible components; this implies constant elasticity of substitution, but possibly different from one or infinite of the particular nested cases (multiplicative and additive).

Besides comparing different functional forms, we also systematically analyse a variety of specifications of the production functions, from the least constrained one – i.e. non-constant returns to scale - to the most constrained one – i.e. total factor productivity. Moreover, we check the robustness of our results to different measures of intangibles and tangibles (for example, at book value or at replacement value) as well as to different samples (for example, unbalanced and balanced panels, the presence or otherwise of null values for intangibles).

Finally, we devote particular attention to the application of panel data estimation techniques to various different specifications. We systematically investigate the empirical evidence regarding the cross-sectional and time-series dimensions of our data (in this latter case the estimates are usually much more fragile). We do so mainly by considering the traditional standard pooled total, within-firm, and one-year and five-year difference estimates; however we also consider various GMM estimators, in an attempt to control for heterogeneity, potential simultaneity and errors in variables-specification bias. These estimates also assess the robustness of our results and attempt to deal with those unsatisfactory results - such as low and insignificant capital coefficients or unreasonably low estimates of returns to scale – which often arise when applying panel methods to micro-data (Griliches and Mairesse, 1998).

The paper is organised as follows. Section 2 outlines Italian reporting rules on intangibles, and presents the accounting information available at the firm-level for both intangible and tangible capital stocks, together with the main figures for those variables used in our analysis. Section 3 illustrates our framework of analysis: the Cobb-Douglas production function with either multiplicative or additive specifications of total capital, and the encompassing CES production function. We also adopt a variety of specifications (conventional production function, total factor productivity production function) and a number of estimation techniques (pooled OLS, within, first- and long-differences). In Section 4 we explore the robustness of our results to the endogeneity issue, as well as to alternative measurements of both intangibles and tangibles or to different samples. In Section 5 we disentangle the contribution made to productivity by each intangible component: intangibles capitalised by firms or by us, and intellectual and customer capital. Section 6 presents our principal conclusions. Further details on data and on the estimating methods employed are provided in the Appendices.

2. The data

Company-data are taken from the Company Accounts Data Service CADS. Centrale dei Bilanci is a company - set up jointly by the Bank of Italy, the ABI-Italian Banking Association and other leading Italian banks - which has been collecting CADS data since 1982. Appendix A1 describes the variables in our analysis, as well as the cleaning rules applied to the original CADS data-set. The final sample we selected is an unbalanced panel of 14,254 Italian manufacturing firms with an average of 6.7 years over the 1982-1999 period (94,968 observations).

2.1. Capitalising or expensing the different categories of intangible investments?

The question of whether it is better to capitalise or to expense intangibles is one of the most controversial issues to emerge recently in the literature, as is clear from the debate tackled by the International Accounting Standards Committee (IASC) when developing the

International Financial Reporting Standards (IFRS, or International Accounting Standards, IAS, until 2002) designed to be universally adopted. Other analyses of this issue are provided by the works of Lev (see e.g. Lev, 2001). This debate is also of interest from the point of view of the new System of National Accounts (SNA).

In Italy at present, the reporting of intangibles is subject to a combination of national Generally Accepted Accounting Principles (GAAP, based on art. 2424 of the Italian Civil Code, on Legislative Decree no. 127/91[1], and on principle no. 24 of the *Commissione per la Statuizione dei Principi Contabili* of the *Consiglio Nazionale dei Dottori Commercialisti e Ragionieri*) and IAS 38 plus IFRS 3 standards (which supersedes IAS 22). This combination implies that, notwithstanding the fact that the criteria employed in defining intangible assets are similar to those of the IAS/IFRS, more intangible assets than those provided for by IAS/IFRS are allowed. In fact, Italian GAAP present a specific classification of intangible assets; at the same time, they require that certain other specific intangibles (or those intangibles that do not qualify for capitalisation as assets) be recognised as costs when incurred. For an international comparison of intangibles accounting principles, see Stolowy and Jany-Cazavan (2001); for a focus on Italy see Bontempi (2005).

Deferred charges – such as start-up and formation costs - and applied research spending, are just some examples of the intangibles capitalised as assets in Italy but not considered for such treatment by the IAS/IFRS. Contrary to Italy's GAAP, and applying the prudence principle, the IAS/IFRS establish that such costs can only be expended. The justification is the uncertain, discontinuous nature of such intangibles: the amount of intangibles to be capitalised would be too subjective, thus offering managers a means by which to manipulate reported earnings and asset values.

It could be argued, however, that the expensing of intangibles also affords managers a powerful manipulation tool, arguably more damaging than manipulation-via-capitalisation. Furthermore, the level of uncertainty of specific intangibles is not notably higher than the uncertainty of other corporate investments, such as stocks or bonds. Finally, several descriptive studies report correlations between (a) current and past intangible expenditure and (b) the future growth in sales, earnings and stock prices (Lev, 2001), expected from assets. Statistical evidence suggests that, at least on average, the capitalised value of intangibles, such as software development and R&D costs, provides important information to investors when they are pricing securities (see, among others, Aboody and Lev, 1998, and Lev and Sougiannis, 1996).

The aforementioned issues are of central importance both to the current debate on capitalisation versus expensing of intangibles, as tackled by the IASC when defining the IAS/IFRS, and to the discussion about the new SNA. In order to try to lighten the effect of both capitalised and expensed intangibles on company productivity, and thus to disentangle the informative content of this distinction as allowed by Italy's GAAP, we propose a new definition of intangible capital stock. This new definition shall take account not only o the stock constructed from expensed intangibles - as is the practice in English-language studies - but also the intangible stocks originally reported as assets in Italian companies' balance sheets. The exclusion of the latter, in fact, implies downwards biased estimates.

2.2. The measurement of intangible and tangible capital: the information provided by company accounts

Table 1 shows how we define intangible (panel A) and tangible (panel B) stocks in accounting terms. Bontempi (2005) illustrates the procedures we followed in order to link the reporting rules of Italy's GAAP with the accounting information available for our sample of Italian companies, and the empirical variables suitable for productivity analysis. A few important points are briefly mentioned here.

Table 1 here

We define total intangible capital (*K*) in two different ways. The rows in Table 1, panel A, show the two components constituting our first definition of total intangible capital, $K = IKBS + IKCA$, where: *IKBS* denotes intangibles capitalised as assets and reported in company balance sheets; *IKCA* indicates non-capitalised intangible capital that is directly expensed by firms in their current accounts. We estimated the stock of *IKCA* by capitalising direct expenses (see Appendix A1 for details) and hereinafter we refer to such as "intangibles capitalised by us".

As well as distinguishing between intangible assets and expensed intangibles, we also disentangle the contribution made by different types of intangibles on the basis of the nature of their productive resources. Thus, the second and third columns in Table 1, panel A, contain the two addendum contributing towards our second definition of total intangible capital, $K=IK+CK$, where *IK* and *CK* stand for "intellectual capital" (R&D and patents) and "customer capital" (trade marks and advertising), respectively. The last column in Table 1, panel A, shows the accounting information available for other intangible categories that we have either excluded from our analysis, or have reallocated to those categories under investigation.[2]

Each cell of Table 1, panel A, contains details of the accounting categories of intangibles that contribute towards the definition of intangible assets, intangible capital constructed by us, intellectual capital, customer capital, and unconsidered or reallocated intangibles. The categories from I1. to I7. represent those intangible assets provided for by Italy's GAAP. We have labelled these assets $IKBS^j$, indicating intangible stock reported in the balance sheets at book values and net of depreciation fund (*j=start, rd, pat, mark, god, fin*). The following categories are not taken into consideration in the present analysis: formation-expansion assets (*start*) and goodwill (*good*), given their miscellaneous or peculiar natures which require further, specific study; deferred financial charges (*fin*), which have to be excluded from the analysis of company productivity. Note that no label is reported for the I6. category of intangible assets: given their specific nature, we decided to reallocate them to a series of other categories, from I1. to I5. Thus, total intangible assets, *IKBS*, are given by $\sum_j IKBS^j$, where j=*rd* (applied R&D), *pat* (patents), and *mark* (marks and brands).

For the sake of homogeneity, we decided to label the categories of intangibles that have to be expensed from I8. to I10. The label $IKCA^h$ is used to denote intangible stock created by capitalising the corresponding direct expenses, DE^h, reported by firms in their current accounts (details on the capitalising procedure are given in Appendix A1). Intangibles capitalised by us are indicated by $IKCA = \sum_h IKCA^h$, where basic R&D is h=*rd*; patents not respecting recognition-as-an-asset criteria are h=*pat*; advertising is h=*adv*. According to the second method of disaggregating total intangibles, intellectual capital is given by $IK = \sum_j IKBS^j + \sum_h IKCA^h$, where j=*rd, pat* and h=*rd, pat*; customer capital is given by $IC = \sum_j IKBS^j + \sum_h IKCA^h$, where j=*mark* and h=*adv*.

Table 1, panel B, also illustrates our definition of tangible stock. As with intangibles, the cells in Table 1, panel B, show the six tangible assets, from T1. to T6., enumerated by the Italian GAAP. We label these categories $TKBS^c$, where c=*bui, pla, equ, oth, unc, lea*, to indicate tangible stock reported in the balance sheets at book values and net of depreciation. We define the total tangible capital used as $C = \sum_c TKBS^c$, where c=*bui* (buildings), *pla* (plants), and *equ* (equipment). Hence, we exclude: leasing (c=*lea*) because it is basically irrelevant; dismissed and uncompleted tangibles (c=*oth, unc*) because of their not-yet and/or no-longer productive nature (for details, see Bontempi, 2005).[3]

2.3. Intangible capital components: occurrence and magnitude compared with tangible capital

In this section we proceed to analyse the magnitude and occurrence of intangible capital. Each cell of Table 2 shows the mean (in the first row), the weighted average (in the second row) and the median (in squared brackets, third row) of the intangibles / tangibles ratio. Tangibles are given by the sum of buildings, plants and equipment; it should be said that we have excluded companies with zero *C*, because of unreliable accounting information (see Appendix A1). On the contrary, we allow total intangible assets to equal zero. The figures in columns (1) refer to the book values of both intangibles capitalised by firms and tangible assets; columns (2) indicate replacement values for intangibles capitalised by firms and the book values of tangible assets; columns (3) indicate the replacement values of both intangibles capitalised by firms and of tangible assets (see Appendix A1).

Table 2 here

Table 3 shows the percentage composition of total intangible capital; as in Table 2, each cell reports the mean (in the first row), the weighted average (in the second row) and the median (in squared brackets, third row). Given robustness to the use of different measures (book or replacement values), both intangibles capitalised by firms (*IKBS*) and tangibles (*C*) are at book values, as was the case in columns (1) of Table 2.

Table 3 here

Sorting out the different forms of intangibles, customer capital (capitalised by us from current expenses) and intellectual capital (capitalised by the firms) are the first and second most important components of intangibles, regardless of the scale afforded to the phenomenon (total tangibles or total intangibles). Customer capital capitalised by firms, and intellectual capital capitalised by us, follow in third and fourth positions respectively. It should also be pointed out that customer capital mainly consists of the stock we estimated by capitalising current expenses. Intellectual capital, on the other hand, is mostly capitalised by firms.

As we have already mentioned, in previous tables total intangible capital has been allowed to equal zero, while tangible capital had to be other than zero. Hence, the rows in Table 4 reveal how the average intangibles-to-tangibles ratio changes in the sub-samples in which total intangible capital and various combinations of its components are never equal to zero. To make comparison easier, the first row, labelled "Full sample", shows the same results as those given in Table 2, columns (1).

Table 4 here

The percentage of observations featuring zero intangibles is not relevant (about 17%), as is clear if we compare the numbers of observations in the "Full sample" and the "*K* never zero" rows of Table 4. At the parameter estimation stage, we have chosen to focus on the "*K* never zero" sample.[4]

The "Both *IK* and *CK* never zero" and "Both *IKBS* and *IKCA* never zero" observations represent 64% and 35%, respectively, of the "*K* never zero" sample. Advertising expenses are rarely characterised by continuous initial zeros; hence, the stock originated through the permanent inventory formula, which is the main component of *CK*, is almost not affected by zeros. On the contrary, the rare presence of initial non-zero observations in R&D and patent expenses affects the corresponding constructed stocks, included in *IKCA*. The two samples "*IK* and *CK* both never zero" and "*IKBS* and *IKCA* both never zero" will be used for estimates of the contribution of specific categories of intangibles, in Section 5.[5]

Given the definition of intangible capital presented in Table 1, the comparison of the percentages reported in the "*IK* never zero (and *CK* zero)" row shows that intellectual capital is mainly composed by applied R&D and patents (77%), which are recognised as an asset and thus included in *IKBS*. Basic research and patent royalties (mostly expensed out and included in the *IKCA* component) represent the 23% only of *IK*. Hence, if, ignoring the informative content of the Italian GAAP, we had merely measured intellectual capital with current expenses capitalised by us (as the Anglo-Saxon literature does), we would have obtained downwards biased total intangibles. On the contrary, advertising is the main component of *CK* (88%) and it is principally operative and recurrent, and thus expensed and reported in the *IKCA* column; this is shown by the "*CK* never zero (and *IK* zero)" row of Table 4. Marks account for just the 12% of customer capital, see the combination of "*CK* never zero (and *IK* zero)" row with the *IKBS* column.

The "*IKBS* never zero (and *IKCA* zero)" row shows that intangibles capitalised by firms are composed by applied R&D and patents (two-thirds) and marks (one-third). Conversely, basic R&D and patents-royalties components of *IKCA* are almost irrelevant, if compared to the advertising item (see the "*IKCA* never zero (and *IKBS* zero)" row).

2.4. Basic descriptive statistics

The main statistics of the variables of interest are reported along the columns of Table 5.

Table 5 here

Per-employee level statistics, measured in million of Italian Lira at 1995 prices (in the upper part of Table 5), suggest considerable departures from normality: means are always bigger than the corresponding medians; the effect of outliers in causing departures between parametric and non-parametric measures of spread (standard deviation, SD, and inter-quartile range, IQR) is evident; these results particularly characterise the number of employees and intangibles. Among the variables, intangibles represent the most extreme cases: for example, the parametric measures of centre and spread of the total intangible stock per-employee are about five times bigger than the corresponding non-parametric measures (in particular, the mean is well over the 3rd quartile). The same features are largely reproduced by the intangible over tangible ratio because of intangibles at numerator. These facts suggest that large intangible stocks are concentrated in relatively few companies, and that zeros are more prevailing here than for the other variables.[6]

The distribution of labour costs seems almost normal, with variability that is less than one-third of the average. In other terms, the share is quite well summarised by the measures of centre of the distribution; labour share on value added averages at about 65% of production[7].

As far as growth rates are concerned (in the lower part of Table 5), per capita figures of production, value added, intermediate inputs, and, to a lesser extent, tangible stock statistics are similar each other over the sample period. The employment growth is slightly more stable than previous productivity measures, while statistics for total intangibles suggest a 30-50% higher variability than previous variables. Variability of intangibles is emphasised when disaggregated components are considered, mainly due to the larger presence of zeros, as shown by the reduced number of observations and companies involved in the computations of growth rates (the numbers for *NT* and *N*, respectively, reported in the notes to Table 5). The variability is reduced when measured by robust statistics.

Table 5 also present the total variability decomposition in between (*i.e.* across firms) and within (*i.e.* due to time). Variables measured in levels have a between-firm variability that is always bigger than 70-80% of the total variability, with the only exception of the labour cost share. Between variability greatly loses its relevance when growth rates are considered and level information is lost: sample variability due to individual effects drops to about 15-20%. The higher between-variability for the intangible stock growth rate confirms the relevance of few individual companies, as outlined above. In general, time never shows significant role in explaining variability; this result, in line with the findings of other studies (see, among the others, Griliches and Mairesse, 1984), must be taken into consideration in interpreting estimation results.

The main features illustrated in Table 5 for the whole sample are qualitatively the same if we split the sample in the three sub-samples corresponding to the high-, medium- and low-technology sectors.

3. Assessing the overall productivity of intangible capital

3.1. Models specification and estimation.

We will compare the results from three specifications of the production function: a Cobb-Douglas with multiplicative total capital, $TC^m_{it}=(C^{\alpha}_{it}\,K_{it}^{\gamma})^{1/(\alpha+\gamma)}$; a Cobb-Douglas with additive total capital, $TC^a_{it}=(C_{it}+\zeta K_{it})$; and the CES in capital inputs production function, in which the total capital is $TC^c_{it}=(C^{-\rho}_{it}+\xi K^{-\rho}_{it})^{-1/\rho}$. The corresponding equations are:

(1) $Q_{it}=A_i\,B_t\,L^{\beta}_{it}\,C^{\alpha}_{it}\,K^{\gamma}_{it}\,e^{\varepsilon m_{it}}$,

(2) $Q_{it}=A_i\,B_t\,L^{\beta}_{it}\,(C_{it}+\zeta K_{it})^{\lambda}\,e^{\varepsilon a_{it}}$,

(3) $Q_{it}=A_i\,B_t\,L^{\beta}_{it}\,(C^{-\rho}_{it}+\xi\,K^{-\rho}_{it})^{-\lambda/\rho}\,e^{\varepsilon c_{it}}$.

Label Q indicates the value added. The terms A_i and B_t are efficiency parameters or indicators of the state of technology: A_i expresses non-measurable firm-specific characteristics; B_t expresses the macroeconomic events that affect all companies to the same degree. Labels C and K are tangible and intangible stocks, respectively; the related parameters α and γ are the elasticities of output with respect to each stock; hence, $\lambda=\alpha+\gamma$ measures the returns to scale to capital inputs. L is the labour input, and the associated parameter, β, is the elasticity of output with respect to L. The disturbance terms εm, εa, and εc are the usual idiosyncratic shocks. Note that in all the above cases, we assume a one-period gestation lag before intangible and tangible stocks (K and C) become fully productive. Beginning-of-period capital measures avoid the correlation between capital inputs and the disturbance terms.[8]

Equations (1) and (2) can be viewed as particular cases of the CES specification in (3) where: ξ is the distribution parameter for capital inputs (or input intensity parameter) associated with the relative factor shares in the product; -1 ≤ ρ ≤ ∞ is the substitution parameter that determines the value of elasticity of substitution, in other words, the extent of capital-input substitution. The technical rate of substitution (TRS), *i.e.* the marginal productivity of intangible capital over the marginal productivity of tangible capital, is $TRS = \zeta = \frac{\partial Q / \partial K}{\partial Q / \partial C} = \xi \left(\frac{C}{K} \right)^{\rho+1}$; the elasticity of substitution is $\sigma = \frac{d\log(C/K)}{d\log(|TRS|)} = \frac{1}{1+\rho}$. Finally, the elasticities of output with respect to capital inputs change according to each capital input/total capital ratio, and to the values of the parameters λ, ξ and ρ; for example, in the case of intangible capital, we have $\frac{\partial Q / \partial K}{Q / K} = \frac{Q}{C^{-\rho} + \xi K^{-\rho}} \lambda \xi K^{-\rho} \frac{K}{Q} = \lambda \xi \frac{K^{-\rho}}{TC^{c}}$.

The following can be shown:

- For ρ → 0, σ → 1 which is the elasticity of substitution in the Cobb-Douglas production function with multiplicative specification of the total capital, TC^{m}, reported in equation (1). In fact, $\lim_{\rho \to 0} \left[C^{-\rho} + \xi K^{-\rho} \right]^{-1/\rho} = (C_{it} K_{it}^{\xi})^{1/1+\xi}$ and, hence:

$$Q_{it} = A_i B_t L_{it}^{\beta} \left[\left(C_{it} K_{it}^{\xi} \right)^{1/1+\xi} \right]^{\lambda} e^{\varepsilon m_{it}},$$

where $\alpha = \lambda/(1+\xi)$, $\gamma = \lambda\xi/(1+\xi)$, and $TRS = \xi \left(\frac{C}{K} \right) = \frac{\gamma}{\alpha} \left(\frac{C}{K} \right)$.

The Cobb-Douglas production function with multiplicative specification of total capital assumes that output elasticities with respect to tangibles or intangibles are constant (equal to α and γ, respectively), while the technical rate of substitution (TRS) changes in accordance with the ratio *C/K*; the hypothesis whereby different types of capital are not fully substitutable could be unlikely and not supported by the data.

- For ρ → -1, σ → ∞ i.e. the capital inputs are perfect substitutes for each other, $TC^{c} \equiv TC^{a}$, and $\xi \equiv \zeta = TRS = \frac{\partial Q / \partial K}{\partial Q / \partial C}$. In other words, we have the additive production function of equation (2), in which the assumption of the unitary elasticity of substitution, as in equation (1), is relaxed in favour of fixed factor proportions (elasticity of substitution equal to infinity), which is met in the short run as a result of the indivisibility of tangibles, and the complementary nature of intangible and tangible capital. In equation (2), as in the CES production function, the elasticities of output with respect to capital inputs change according to each capital input/total capital ratio. For example, in the case of intangible capital, we get:

$$\frac{\partial Q / \partial K}{Q / K} = \frac{\lambda Q}{C + \zeta K} \zeta \frac{K}{Q} = \lambda \zeta \frac{K}{TC^{a}} .$$

The relationships between the parameters of the three production function specifications are summarised in Table 6.

Table 6 here

By taking the logarithms of the previous expressions (1)-(3), and defining all the variables per employee, the multiplicative, additive and CES production function specifications become, respectively:

(1') $(q-l)_{it} = a_i + b_t + (\mu -1)l_{it} + \alpha(c_{it} - l_{it}) + \gamma(k_{it} - l_{it}) + \varepsilon m_{it}$,

(2') $(q-l)_{it} = a_i + b_t + (\mu -1)l_{it} + \lambda(tc^{a}_{it} - l_{it}) + \varepsilon a_{it}$,

(3') $(q-l)_{it} = a_i + b_t + (\mu -1)l_{it} + (\lambda / -\rho)\,(tc^{c}_{it} - l_{it}) + \varepsilon c_{it}$,

where lower-case letters denote logarithms; $\mu = (\lambda+\beta)$; in (2') $tc^{a}_{it}-l_{it} = log[(C_{it}+\zeta K_{it})/L_{it}]$; and in (3') $tc^{c}_{it}-l_{it} = log[(C_{it}/L_{it})^{-\rho}+\xi(K_{it}/L_{it})^{-\rho})]$.

In estimating the parameters of (1'), (2') and (3'), we carry out a number of alternative options regarding: (a) specification of the individual and temporal heterogeneity (a_i and b_t), and of the error terms (εm_{it}, εa_{it}, εc_{it}); (b) the non-linearity of (2') and (3') in the ζ, ρ and ξ unknown parameters; (c) the endogeneity issue; (d) estimation of the TRS in equation (1'), of the elasticity of output with respect to intangibles (and tangibles) in equation (2'), and of the TRS and the elasticities in equation (3').

Point (a) concerns: non-measurable firm-specific advantages (like management ability); macro influences (like business cycle, “disembodied technical changes ” i.e. changes over time in the rates of productivity growth, and the use of common price deflators across firms); the assumption of parameter homogeneity (whereas companies may have different production functions and diverse rates of utilisation of the various categories of input). We assume the following four alternative specifications for the a_i and b_t terms.

- The absence of individual effects (pooled OLS estimation), but the presence of per-industry and temporal heterogeneity, which exploratory data analysis and accounting standards changes indicate as being important. This has been tackled by adding industry and temporal dummies to the model.
- Two-way fixed effects, both individual and temporal (within estimation).
- Estimates of growth rates (first-differences OLS).
- Estimates of rates of growth over 5 years (non-overlapping long-differences or five-year differences).

Furthermore, we also assume that the error terms, εm_{it}, εa_{it}, and εc_{it}, have zero-means and variances which vary by firm i and time t; we therefore estimated all models with the Eicker-Huber-White estimator, robust to the presence of general heteroschedasticity.[9]

In order to deal with point (b), we firstly performed grid-searches on the ζ, ρ and ξ unknown parameters of the additive and CES specifications, in order to obtain values minimising the residual sum of squares. Secondly, we used iterative procedures on first-order Taylor-series approximations of equations (2') and (3') around initial values of ζ, ρ and ξ parameters set to the values obtained by means of the aforesaid grid-searches. Details of the grid-searches and iterative procedures are given in Appendix A2.

The endogeneity issue (c) derives from: the simultaneous choice of inputs and output; the efficiency levels - known to companies but not to the researcher - which could lead to correlation between firm-effects and explanatory variables; the omission of labour and capital intensity-of-utilisation variables (such as hours of work per employee and hours of operation per machine); other measurement errors deriving from changes to accounting standards and requirements , lack of information on economic depreciation rates and prices at the firm-level. The endogeneity issue may be tackled using GMM estimation methods, which we shall in fact be employing in Section 4. The imposition of theoretical restrictions on parameters in equations (1')-(3') to specific values represents another way of tackling the endogeneity issue. In particular, the constant returns to scale hypothesis implies that $\mu = 1$ in equations (1'), (2') and (3') above; hence, the corresponding restricted models are obtained by dropping the l regressor, which is probably the one most significantly affected by measurement errors. A few additional assumptions enable us to move certain inputs to the left-hand side. In particular, under the assumption of perfect competition (price-taking firms in both the labour and output markets), the β parameter is not estimated but set at some reasonable value. In this case, still assuming constant returns to scale, we can obtain measures of total factor productivity (*tfpc*), and then use them to estimate the intangible parameters in the following equations:

(1'') $tfpc^{m}_{it} = a_i + b_t + \gamma(k\text{-}c)_{it} + \varepsilon m_{it}$,

(2'') $tfpc^{a}_{it} = a_i + b_t + (1\text{-}\beta_0)(\zeta\text{-}\zeta^{(0)})p_K{}^{a(0)}{}_{it} + \varepsilon a_{it}$,

(3'') $tfpc^{c}_{it} = a_i + b_t + [(1\ \beta_0)/\text{-}\rho](\xi\text{-}\xi^{(0)})p_K{}^{c(0)}{}_{it} + \varepsilon c_{it}$,

where $tfpc^{m}_{it} = q_{it} - \beta_0\, l_{it} \text{-}(1\text{-}\beta_0)c_{it}$; $tfpc^{a}_{it} = q_{it} - \beta_0 l_{it} \text{-}(1\text{-}\beta_0)tc^{a(0)}_{it}$ and $tc^{a(0)}_{it} = log(C_{it}+\zeta^{(0)}K_{it})$; $tfpc^{c}_{it} = q_{it} - \beta_0 l_{it} - [(1\text{-}\beta_0)/\text{-}\rho]tc^{c(0)}_{it}$ and $tc^{c(0)}_{it} = log(C^{-\rho}_{it} + \xi^{(0)}K^{-\rho}_{it})$; β_0 is set equal to *slmed*, the sample median of the share of labour cost in value added (*sl*)[10]. To facilitate presentation, in equations (2'') and (3'') we have employed the first-order Taylor-series approximations around certain initial values $\zeta^{(0)}$ and $\xi^{(0)}$ (see Appendix A2 for details).

Finally, point (d) above has to do with summarising measures which depend on the level of certain variables. The TRS in equation (1') may be estimated as:

$$\hat{\zeta}_q = \frac{\hat{\gamma}}{\hat{\alpha}}\left(\frac{C}{K}\right)_q ,$$

where sub-index q indicates that we adopt three measures of the tangibles to intangibles ratio, namely the 1st, 2nd and 3rd quartiles of the C/K ratio distribution. The elasticity of output with respect to intangible capital (and similarly to tangible capital) in equation (2') may be estimated by:

$$\hat{\gamma}_q = \frac{\hat{\lambda}Q}{C+\hat{\zeta}K}\hat{\zeta}\left(\frac{K}{Q}\right)_q = \hat{\lambda}\hat{\zeta}\left(\frac{K}{T\hat{C}^a}\right)_q ,$$

where, once again, the sub-index q shows that we estimate three $\hat{\gamma}$, corresponding to the 1st, 2nd and 3rd quartiles of distribution of the ratio of intangibles to estimated total capital. Finally, the same procedure is followed in the case of equation (3'), in order to compute the elasticities of output with respect to capital inputs, and the marginal productivity of intangibles over that of tangibles:

$$\hat{\gamma}_q = \hat{\lambda}\hat{\xi}\left(\frac{K^{-\hat{\rho}}}{C^{-\hat{\rho}}+\hat{\xi}K^{-\hat{\rho}}}\right)_q = \hat{\lambda}\hat{\xi}\left(\frac{K^{-\hat{\rho}}}{T\hat{C}^c}\right)_q \text{ and } \hat{\zeta}_q = \hat{\xi}\left[\left(\frac{C}{K}\right)^{\hat{\rho}+1}\right]_q .$$

3.2. Main results

Tables 7a-c report the results of estimations for the multiplicative, additive and CES specifications of total capital (equations (1'), (2') and (3') and their restricted versions, respectively).

Tables 7a-7c here

Tables 7a-c share the same structure: moving along the columns in a rightward direction, we have set a growing number of restrictions on the model parameters. The estimation of equations (1')-(3') requires predetermined inputs compared to output. By using beginning-of-period measures of capital, we minimise the effects of simultaneity between capital inputs and output. The simultaneity issue should be of less importance in the case of labour, which was measured for half of observations by the average number of employees; however, changes in accounting legislation together with a lack of information about the different categories of workers, may lead to measurement errors. One way of dealing with this issue, which may affect the model, involves using total factor productivity, as shown in the final three columns of Tables 7a-c.In order to do so, three steps must be taken. Firstly, we need to assume a situation of perfect competition (price taking in both labour and output markets), which implies that labour elasticity can be estimated by the share of value added accounted for by labour costs. Secondly, we have to calculate total factor productivity, by implicitly imposing constant returns to scale on the traditional factors (labour and tangible capital). Finally, we must regress total factor productivity against intangible capital. Note that this way of avoiding simultaneity is easy to implement and interpret.

Less restrictive hypotheses regarding the error term are assumed as we move along the rows in Tables 7a-c. "OLS" uses the OLS estimator (no individual effects are allowed, but industry and time dummies are added to the model specification). The disturbance terms in equations (1')-(3') include specification errors arising as a result of companies’ different production functions, different rates of utilisation of the various inputs, and different firm-specific advantages. These individual effects, which may be correlated with the regressors, are not taken into account by the pooled estimates: if the heterogeneity of companies in terms of their technologies, efficiency levels and utilisation of inputs, is not taken into account, then the OLS estimates will be affected by an omitted-variable bias. The "Within" and "First-difference" estimates, on the other hand, allow for additive firm-effects in two different ways:

the first uses demeaned data (by firm), while the second estimates growth rates. Apart from simultaneity and measurement-error biases in the parameter estimates (see Section 4), within and first-differences estimation methods should give fairly similar results. This is generally true, except in the labour coefficient case. The labour coefficient …] is lower in the growth rate estimates, suggesting the presence of simultaneity and/or measurement errors. Finally, the "five-year-differences" estimation method represents another way of estimating equations with individual effects. The advantage of long-differences over within and first-difference transformations is that the former preserves the cross-sectional dimension of variability. In panel data with a large *N* compared to *T*, this implies that greater variance between companies is used to identify the relevant coefficients, thus preventing other misspecifications from overwhelming the remaining signal in the data (Griliches and Mairesse, 1998).

In the multiplicative specification, intangible capital is more closely correlated to overall firm-effects than tangible capital is. This is shown by the decrease in the former's coefficient, which is more evident compared to that of the latter, when moving from OLS to within and first-differences estimates in the specification with no constant returns to scale imposed. The hypothesis of constant returns to scale, when imposed, does not significantly increase the standard error of the estimates. The most important finding here is that intangible capital coefficients remain significant even when we take account of firm-effects, and are particularly high in the total factor productivity specification (the median value of intangibles' share of value added is 2.4%). Long-differences estimates of equation (1") give reliable results.

In general, the magnitude of our capital estimates is comparable with that of Hall and Mairesse (1996) for the USA (6,521 observations for the 1981-1989 period)[11], and of Hall and Mairesse (1995) for France (2,670 observations for the 1980-1987 period)[12].

Tables 8a-d and Figures 1-2 enable direct comparisons to be made between multiplicative, additive and CES specifications for the least constrained (no constant returns to scale) and for the most constrained (total factor productivity) models.

The first three columns in Tables 8a-d show the elasticity of output with respect to intangible capital: it is constant and directly estimated in the multiplicative specification; it is computed from estimated coefficients and quartiles of data sample distribution in the additive and CES specifications. The last three columns of Tables 8a-d show the marginal productivity of intangibles over that of tangibles: it is constant and directly estimated in the additive specification case, while it is computed from the estimates and the sample quartiles of the data in the multiplicative and CES specifications. Elasticity, as directly estimated by the multiplicative specification, is similar to elasticity as computed from the additive (CES) specification in correspondence to the third (second) quartile of the distribution of intangible capital/estimated total capital ratio. Symmetrically, the TRS directly estimated by the additive specification resembles the ones obtained by the estimates of the multiplicative and CES specification in correspondence to the first quartile of the distribution of the tangibles/intangibles ratio. These facts may reflect the patterns evidenced in Table 5: the distribution of intangibles over tangibles is positively skewed; intangibles seem to be concentrated within a limited number of companies.

Tables 8a-d here

Figures 1 and 2 display the results obtained by different estimation methods and production functions. If we focus on the results computed at the sample medians, the within and five year-difference estimates appear somewhat similar. This result confirms the relevance of heterogeneity. As far as the specifications are concerned, the CES capital is a reasonable compromise between a point estimate (additive capital) and a range of values (multiplicative capital). Finally, a comparison of Figures 1 and 2 reveals that the total factor productivity specification (3'') produces higher estimates than those produced by the non constant returns–

to-scale specification (3). This result confirms what was previously shown, i.e. that the restricted model is better suited to dealing with endogeneity.

Figures 1 and 2 here

4. Confirmation of the robustness of results

4.1. GMM estimates

This section focuses more closely on the simultaneity issue and on the measurement-bias problems affecting the estimates of equations (1')-(3'). For example, our available labour input measure does not distinguish between blue- and white-collar workers. As far as our capital measure is concerned, it is worth remembering that both intangibles and tangibles required data-intensive construction procedures;[13] moreover, balance-sheet data may not represent the capital actually employed by companies. Changes in the parameter estimates when first differences are used - the method most affected by random and short-term measurement errors and by simultaneity problems - mean that an instrumental variables approach would be preferable.

Table 9 shows GMM estimates of first-differences and of levels and system equations (Blundell and Bond, 1998). In order to keep the presentation as simple as possible, we have focused on the multiplicative production function and on the two extreme specifications, namely non constant returns to scale (the least constrained), and total factor productivity (the most constrained).

Table 9 here

The GMM levels use moment conditions which are appropriate if the individual effects are not correlated with the explanatory variables. GMM first-differences and the GMM system, on the contrary, allow for individual effects to be potentially correlated to the explanatory variables. GMM estimates set all available moment conditions for each year of data separately. Thus the GMM approach should be more efficient than the IV approach, unless the excess of over-identifying restrictions emphasises the problem of weak instruments (Ziliak, 1997). The advantage of GMM-lev and GMM-sys over GMM-dif approaches, is that the former avoid the poor performance of first-difference transformations characterised by the predominance of measurement and timing errors (Griliches and Mairesse, 1998). Another cause of invalid instruments in levels for first-differenced equations is the almost random walk statistical behaviour of the variables (on this particular topic, see *e.g.* Bond, 2002).

In order to tackle both simultaneity and potential measurement problems, we instrumented all the inputs, checking for alternative instrument sets.[14] The choice of relevant (i.e. correlated with endogenous explanatory variables) instruments is a difficult one, as it implies both the bias of parameter estimates and the distribution to depart from asymptotic normal in finite-sample. Further, the "weak instruments" problem arises even in large-sized samples (see Bound, Jaeger and Baker, 1995). Due to the large number of observations in our data-set, we only use the lags from 2 to 3 of the variables as our instruments.[15] We use the "standard" GMM instrumenting technique, i.e. lags of the r.h.s. variables., as well as "external instruments", i.e. variables that do not belong to the explanatory variables in each equation. We prefer the external instrument approach, since lags of the explanatory variables may be affected by the same measurement error (possibly correlated over time) that we are trying to tackle. Our chosen external instruments are gross investments. First-differences of net capital stock would be a good alternative, as they take disinvestment into account as well. However, they may be more markedly affected by measurement errors than the levels, and in particular by the occasional de- and re-valuation. Moreover, they imply a larger loss of observations.

The GMM system results shown in Table 9 are very similar to the total factor productivity specification estimated by long-differences (see Table 7a), thus pointing to the simple effectiveness of this approach in tackling simultaneity problems.

4.2. Variants on intangible capital measures

This Section examines whether the use of replacement values for intangibles capitalised by firms or for tangibles, significantly affects the estimation results presented in Section 3.2. It also investigates the role played by the depreciation rate in relation to intangibles. Once again, in order to keep the presentation as simple as possible, we have chosen to focus on the multiplicative production function and the total factor productivity specification (which, as shown in Section 4.1, is the one least affected by the endogeneity issue). Our results are presented in Table 10.

Table 10 here

We present:

- the same estimates as those in Tables 8a-d (in the first row of each estimation method), obtained using intangibles capitalised by firms and tangibles at book values;
- the estimates from all intangibles (both those capitalised by firms and those capitalised by us), measured at replacement values subject to a depreciation rate of 30%, or alternatively of 20%, and from tangibles at book value (shown in the second and third rows);
- the estimates obtained by computing all intangibles (those capitalised by firms and those capitalised by us) at replacement value, subject to a depreciation rate of 30%, and tangibles, once again at replacement value, subject to depreciation rates of 5% for buildings and of 11% for machinery-equipment (in the fourth row).

The sample is the same as that featured in Tables 8a-d. We checked (results not reported) for the effect of using a greater number of observations (for example, 71,761 against 66,953 in the OLS and within cases) available when capital stock at replacement value is employed (we should not forget that the permanent inventory method fills more zero-cases). Moreover, we tested results in a sample cleaned on the basis of capital stocks at replacement values rather than book values (105,998 observations are available for the OLS and within estimates cases). Results are qualitatively the same; of course, in the latter case, the larger sample size leads to more efficient estimates.

5. An examination of the relative productivity of the major components of intangible capital

By using the same theoretical framework illustrated in Section 3.1 for the CES production function, we disentangled the contribution made to productivity by the different types of intangible shown in Table 1 (firstly down the columns, and secondly along the rows). We focused on CES capital because results in Section 3.2. suggest that this specification is a reasonable compromise between the multiplicative and the additive production functions.[16] In order to deal with the endogeneity issue in a simple manner, we limited our analysis to the total factor productivity model. Hence, the new definition of the total factor productivity is $tfpc^{d}_{it} = q_{it} – \beta_0 l_{it} –[(1-\beta_0)/-\rho]tc^{d}_{it}$, where: the up-index "$d$" stands for disaggregated intangibles; $tc^{d}_{it} = log(TC^{d}_{it}) = log(C^{-\rho}_{it} + \xi_1 K_1{}^{-\rho}{}_{it} + \xi_2 K_2{}^{-\rho}{}_{it})$; sub-indexes 1 and 2 indicate the different types of intangible; β_0, as before, is set equal to *slmed*, the sample median of the share of labour cost in value added (*sl*). See Appendix A2 for details of the estimating methodology.

Results regarding the elasticity of output with respect to the two types of intangible, and the marginal productivity of the two types of intangible with respect to that of tangibles, are presented in Tables 11a-b. For example, for intangible *1*:

$$\hat{\gamma}_{1q} = \hat{\lambda}\hat{\xi}_1\left(\frac{{K_1}^{-\hat{\rho}}}{C^{-\hat{\rho}} + \hat{\xi}_1{K_1}^{-\hat{\rho}} + \hat{\xi}_2{K_2}^{-\hat{\rho}}}\right)_q = \hat{\lambda}\hat{\xi}_1\left(\frac{{K_1}^{-\hat{\rho}}}{T\hat{C}^d}\right)_q \text{ and } \hat{\zeta}_{1q} = \hat{\xi}_1\left[\left(\frac{C}{K_1}\right)^{\hat{\rho}+1}\right]_q .$$

where, as before, the sub-index q indicates the 1st, 2nd and 3rd quartiles of the data sample distribution.

Tables 11a-b here

The first distinction we made, shown in Table 11a, is intellectual capital, *IK*, versus customer capital, *CK*. As has been pointed out in the literature (e. g. Hirschey, 1982 and, more recently, Lev, 2005), continuous advertising is important if consumers are not to forget the innovations developed by a company. Similarly, brand-names are essential for the economic development of businesses: they allow for the identification and distinction of one product from other products, creating a unique image of a product's quality among the buying public. Hence, brands and similar items represent key competitive factors which influence company sales.

The second distinction we make, as shown in Table 11b, is that between intangibles capitalised by firms, *IKBS*, and intangibles capitalised by us, *IKCA*. Hence we explore the informative content of the distinction between capitalised and expensed intangibles, established by Italy's GAAP. We could expect, for example, that R&D and intellectual property assets, when compared to the corresponding costs, play a predominant role in determining productivity levels. If this is true, the capitalisation options chosen by managers, albeit subjective and affected by uncertainty, reveal which part of intangibles (the advanced part rather than the basic one) drives company performance.

Figures 3 and 4 display the results obtained by different estimation methods.

Figures 3 and 4 here

The differences between the estimated marginal productivities of *IK* and of *CK* tend to be concentrated in the third quartile, and to lose statistical significance as we move from the OLS and first-differences estimation methods to the within and long-differences methods. On the contrary, the differences between the estimated marginal productivities of *IKBS* and of *IKCA* are significant regardless of the estimation method used; within and five years-differences show smoother results than those obtained by means of OLS and first-differences methods. The path of estimates achieved by the estimation method confirmed what had been previously found at the aggregate level, as shown in section 3.2. If our benchmark is the aggregated intangibles estimates obtained by long-differences applied to the total factor productivity specification of the CES production function (the final row of Table 8d), then the intangibles capitalised by us display a marginal productivity in line with that of total intangibles; this marginal productivity is smaller than that of those intangibles capitalised by firms and, in particular, than that of intellectual capital and customer capital (the highest of all).

Moreover, the *IK* and *IKBS* estimated coefficients do not statistically differ from each other: going back to the definitions given in Table 1, intellectual capital, in fact, consists mainly of intangibles assets (intangibles capitalised by firms). This is no longer true if we compare *CK* and *IKCA* components: customer capital displays higher coefficients than those displayed by the intangibles capitalised by us. This indicates that the productivity of *IKCA* is mainly driven by advertising, rather than by basic R&D and patent-royalties;[17] moreover, marks and brand – part of *CK* derived from intangibles capitalised by firms, *IKBS* – also play an important part in raising the productivity of intangibles over that of tangibles.

6. Concluding remarks

The present paper contains experimental analyses of the relationship between productivity and intangibles in Italy. Our definition of intangibles is a broad one which, according to the Italian GAAP, includes both intangible expenses (which is in line with the empirical studies of this issue), and intangible assets as reported in company balance sheets (and as such constitutes a new definition of the term). Furthermore, we have tried to separate the contribution to productivity made by different categories of intangibles: not only those intangibles capitalised by us from current expenses compared with intangibles capitalised by firms, but also intellectual capital (mainly R&D and patents) against customer capital (trademarks and advertising).

We use three different production function specifications. The first is the Cobb-Douglas production function with the multiplicative specification of total capital; it is mathematically simple, and its parameters are easy to interpret and to estimate using regression techniques. However, its simplicity implies that rigidity is imposed on the observed production processes. For example, the output elasticities are assumed to be constants, varying neither with time, nor with output levels, nor with the ratio of inputs, etc.; the elasticity of substitution is set at one, implying a uniform flexibility of the response of the input ratio to changes in relative input costs – while the varying economic circumstances faced by a company may lead it to change the degree to which an input is utilised.

As we move toward a less restrictive form, such as the Cobb-Douglas production function with the additive specification of total capital, we can remove the assumption of constant output elasticity in relation to intangibles. However, we assume that the elasticity of substitution is zero, i.e. that the intangibles-tangibles ratio cannot be altered, no matter what happens to the marginal product ratio; this fixed-factor proportions assumption is justified in the short-run by the indivisibility of tangible capital and the complementary nature of tangibles and intangibles. The greater flexibility of this latter production function is counterbalanced by its more complex mathematical form and the greater difficulty encountered in applying estimating techniques.

In order to furnish more proof of the goodness of the two Cobb-Douglas production functions' fit-to-our-data, we also employed a third, more general (and sophisticated) specification, i.e. the constant elasticity of substitution (CES); it nests the two previous multiplicative and additive production functions; within this specification, the elasticity of substitution is estimated rather than being taken as equal to 1 or 0.

We also perform various checks on the model, involving: different specifications of the production functions (from the least constrained ones – non-constant returns to scale – to the most constrained ones – total factor productivity); alternative measures of both tangible and intangible stocks (for example, excluding advertising from the intangibles definition; replacement values instead of book values); alternative measures of the dependent variable (sales, with materials among inputs); different samples (the balanced panel for the 1994-1999 sub-period; the sample with zeros in intangibles). All such checks confirm the robustness of our results.

The model specifications and estimation panel techniques employed tackle a number of problems, such as simultaneity, measurement errors, unknown individual and temporal effects. Measurement errors seem to affect labour input in particular; consequently, the production function that uses total factor productivity as its dependent variable is the least biased specification. The general predominance of between-firms variability over temporal variability, led to a preference for estimates in long-term growth rates. In fact, we discovered that: in the cross-sectional dimension of the data there is a not negligible relationship between firm productivity and intangibles; in the time dimension of the data (using deviations from firm-means or short-term growth rates as our observations), this relationship becomes weaker, especially in the unconstrained model specifications. Overall, estimates are quite robust and comparable with those obtained for other countries, such as the USA.

Focusing on intangible components, the highest marginal productivity is that of intellectual capital, customer capital and intangibles capitalised by firms. Intangibles constructed by capitalising direct expenditures display the lowest level of productivity; hence, if by ignoring the informative content of the Italian GAAP, we had merely measured intangibles with current expenses capitalised by us (as English-language studies do), we would have obtained downwards biased results.

Despite the fact that, as we would have expected, there are certain discrepancies between some of our results, they nevertheless convey a rather consistent, acceptable overall picture. In particular, the time-series type results agree with the cross-sectional results, to provide economically and statistically significant estimates of the order of magnitude of the elasticity of intangible capital and the range of its marginal productivity relative to tangible capital. In spite of the substantial measuring difficulties encountered, and contrary to the scepticism shared by many company analysts and economists, the abovementioned findings confirm that companies' accounting figures for intangible capital stocks are of a genuinely informative nature. This should provide some comfort to those who advocate the development of reporting and accounting requirements for intangibles and for their capitalisation in company accounts (as well as in national accounts), whereby they are treated as a form of investment (not only in theory but in practice).

Appendix A1: Data

The source of our data is the CADS (Company Accounts Data Service), a large database with detailed accounting information from more than 50,000 Italian companies operating in a wide range of industrial sectors, covering the 1982-1999 period. The CADS is highly representative of the population of Italian companies, covering over 50% of the value-added produced by those companies included in the Italian Central Statistical Office's Census. Further details of this data-set can be found in Bontempi (2005).

A list of labels and the definitions of the corresponding variables

- k is the logarithm of total intangible stock (K), computed at the beginning of year at net book values. Total intangibles may be defined as $K = IKBS + IKCA = IK + CK$, where: $IKBS = \sum_j IKBS^j$ (with j=*rd, pat, mark*) represents intangibles capitalised by firms and reported in their balance sheets (*rd* is applied R&D; *pat* is patents; *mark* is trademarks and brands); $IKCA = \sum_h IKCA^h$ (with h = *rd, pat, adv*) represents intangibles capitalised by us from intangible expenses reported by firms in their current accounts (*rd* now indicates basic R&D; *pat* is patent-royalties; *adv* is advertising); $IK = \sum_j IKBS^j + \sum_h IKCA^h$ (with j, h = *rd, pat*) represents intellectual capital; $CK = \sum_j IKBS^j + \sum_h IKCA^h$ (with j = *mark* and h = *adv*) represents customer capital. Intangibles capitalised by us are constructed according to the formula $IKCA_{it}^h = (1-\bar{\delta})IKCA_{it-1}^h + DE_{it-1}^h$, where: $\bar{\delta}$ is the depreciation rate, taken to be equal to 30%; $IKCA_{i0}^h = DE_{i0}^h/(\bar{\delta}+g)$ is the initial value of the stock; g is the pre-sample growth rate, taken to be equal to 3%; h indicates the category of intangible expenses. Note that the effect of the initial value disappears as time goes by. Estimates of intangibles capitalised by firms at replacement costs and at the beginning of the year (*IKBSR*) are obtained on the basis of the series of corresponding gross investments (new purchases gross of disinvestments), I^j, by using the formula $IKBSR_{it}^j = (1-\bar{\delta})IKBSR_{it-1}^j + I_{it-1}^j$ where: j indicates the intangible categories; $\bar{\delta}$ is the depreciation rate for all intangibles, taken to be equal to 30% (the same value used in the case of direct expenses capitalising) or to 20%. The assets at book values are used as starting values.
- c is the logarithm of the total tangible stock (C), computed at the beginning of the year at net book values. Total tangibles are defined as the sum of all buildings, plant and equipment, that is $C = \sum_c TK^c$, c = *bui, pla, equ*. Estimates of tangibles at replacement cost and at the beginning of year (*CR*) are obtained on the basis of the series of

corresponding gross investments (new purchases gross of disinvestments), I^c, by using the formula $CR_{it}^c = (1-\bar{\delta}^c)CR_{it-1}^c + I_{it-1}^c$, where: c indicates the tangible categories; $\bar{\delta}^c$ indicates the depreciation rates for tangibles, taken to be equal to 5% for buildings, and to 11% for plant and equipment (by-industry averages from the National Accounts of the Italian Statistical Office, ISTAT). The assets at book values are used as starting values.

- l is the logarithm of the number of employees (L).[18]
- q is the logarithms of value added (Q) at the end of year.
- WL indicates labour costs (wages, social security contributions and various other provisions) at the end of year.

Nominal variables were transformed into real terms. Our chosen deflators were: the value-added deflator for Q and WL; the investment-in-buildings deflator for TK^{bui}; the investment-in-machinery-transport-equipment-and-other-tangibles deflator for TK^{pla} and TK^{equ}; the GDP deflator for all the intangible stocks included in the definition of K[19].

Cleaning rules and sample descriptive statistics

We selected limited liability manufacturing companies (about 35% of the total CADS data-set) complying with basic accounting standards and possessing information about the variables in question (206,538 observations for 22,387 companies). We then defined our cleaned sample (94,968 observations for 14,254 firms) according to the following criteria (see Hall and Mairesse, 1995).

a) Those observations for which value added, labour costs, production and intermediate costs were zero or negative were removed (1.0% of our initial sample), as they clearly create problems when it comes to performing logarithmic transformations. Moreover, any observations with tangible net stock (defined as total buildings, plants and equipment) equal to zero were also removed (4.7% of observations); in fact, we deemed those firms lacking tangible capital to be unreliable.

b) Researchers usually base their empirical analyses on companies with a minimum of 20, or even 50, employees. In Italy, small firms (with fewer than 20 workers) predominate. Hence, in order to preserve the representativeness of our sample while maintaining the meaningfulness of accounting data, we removed those firms with fewer than 5 employees in the first year of the sample (0.3% of all observations).

c) Observations for which value added per worker, tangible capital stock per worker, or intangible capital stock per worker, lay outside the range median ± three times the inter-quartile range, were removed (5.6% of all observations). Such outliers could have affected the distribution of variables, since in a Gaussian variable they should only represent 0.0002% of all observations (see Hoaglin-Iglewicz-Tukey, 1986).

d) Observations for which the growth rate of value added lay outside the [–90%, +300%] range, or for which the growth rates of employees, tangible and intangible capitals lay outside the [–50%, +200%] range, were removed (16.5% of all observations).

e) We also removed those observations for which the mean of labour cost's share of sales at t and t+1, and the mean of intermediate costs' share of sales at t and t+1, were lower than the 1st quartile of the corresponding per-industry Törnqvist indicator, or greater than 1 (1.5% of all observations).

f) Previous selection criteria created further gaps in the temporal per-firm data, in addition to the ones originally present in the sample (for a discussion of this point, see Bontempi, 2005). Hence, we selected only those companies with data available for at least 4 consecutive years, and we chose the longest or the most recent sub-period if an interruption in the temporal pattern was present (37.1% of all observations).

Overall, 46% of total observations were excluded; this percentage is lower than the one resulting from the selection rules a) to f) (66.6%), because some annual company data are wrong according to several criteria at the same time.

Table A1 illustrates the composition of our cleaned sample by industry and by size. Manufacturing industries are listed according to their global technological intensity (ISIC Revision 2, see Hatzichronoglou, 1997), using the 4-digit industry code. Note the low size of the HT macro-industry compared to that of the others.

Table A1 here

Our data, which mirrored the nature of Italian industry during the 1982-1999 period, mainly covered non-listed companies: only 0.51% of the chosen firms were listed on the stock exchange (compared to 0.13% of Italian manufacturing companies listed on the Stock Exchange in 1995); 22.23% of companies belonged to a business group (mainly of a pyramidal nature).

Table A2 clearly indicates the inclusion in our dataset of a large number of small and medium-sized firms. Such firms tend to be predominant in Italy: in fact, the average Italian limited liability company employs 44 workers. The average number of employees in our sample is 132, and 47.28% of those companies in our sample have fewer than 50 employees.

Table A2 here

Table A3 compares average intangible/tangible ratios for the total sample, and for three manufacturing macro-industries: high-medium technology (HT+MHT); medium-low technology (MLT); low technology (LT). We also disaggregated the information reported in Table 4 according to the taxonomy suggested by Hatzichronoglou (1997).

Table A3 here

The “Full sample” row and the “*K*” column of Table A3 display a high value of the ratio of total intangible capital (*K*) to total tangible capital (*C*) in the LT industry. This result can be explained by looking at the other columns on the right-hand side of Table A3: it is evidently driven by the component consisting of expensed-out intangibles (*IKCA*) and, in particular, of customer capital (*CK*). Advertising and trademarks are also important to the HT+HMT industry; nevertheless, as we expected, applied R&D and patents (included in the *IKBS* category) played an important role compared to other branches. These results are confirmed by the “*K* never zero” row, and are further emphasised by the “*IK* never zero (and *CK* zero)”, “*CK* never zero (and *IK* zero)”, “*IKBS* never zero (and *IKCA* zero)” and “*IKCA* never zero (and *IKBS* zero)” rows.

Appendix A2: Estimation methods for the non linear specifications

By using a first-order Taylor-series approximation around any assumed initial value for the unknown TRS, $\zeta^{(0)}$, the production function with additive capital can be written as:

(A2') $(q-l)_{it} = a_i + b_t + (\mu -1)l_{it} + \lambda(tc^{a(0)}_{it}-l_{it}) + \lambda\,(\zeta-\zeta^{(0)})p_K{}^{a(0)}{}_{it} + \varepsilon a_{it}$,

where lower-case letters denote logarithms; $\mu = (\lambda+\beta)$; $tc^{a(0)}_{it}-l_{it} = log(TC^{a(0)}_{it}/L_{it}) = log[(C_{it}+\zeta^{(0)}K_{it})/L_{it}]$; and $p_K{}^{a(0)}{}_{it} = K_{it}/TC^{a(0)}_{it} = K_{it}/[exp(tc^{a(0)}_{it}-l_{it})L_{it}]$.

The starting value, $\zeta^{(0)}$, is selected by a single grid-search on the ζ parameter; the grid-search uses equation (2'), it sets ζ equal to all the values in the range 0-2 with step 0.1, and it looks for the value of ζ that minimises the residual sum of squares (min-RSS). Setting this min-RSS as the initial value, $\zeta^{(0)}$, equation (A2') is estimated by an iterative procedure on:

(A2a') $(q-l)_{it}= a_i + b_t + (\mu-1)l_{it} + \lambda\,(tc^{a(n)}_{it}-l_{it}) + \lambda\,(\zeta^{(n+1)}-\zeta^{(n)})p_K{}^{a(n)}{}_{it} + \varepsilon a_{it}$,

where the exponent *(n)* is for the n^{th} iteration; $\zeta^{(n)}=\zeta^{(0)}$ and $tc^{a(n)}_{it} = tc^{a(0)}_{it}$ for the initial iteration $n=0$; $tc^{(n)}_{it} = tc^{(n-1)}_{it} + log[1+p(\zeta^{(n)}-\zeta^{(n-1)})p_K{}^{a(n-1)}{}_{it}]$ for $n>0$, where p is a smoothing parameter.[20]

The iterative procedure stops when $\left|\frac{\zeta^{(n+1)}-\zeta^{(n)}}{\zeta^{(n)}}\right|<0.0001$ and the estimated parameter associated with $p_K{}^{a(n)}{}_{it}$ is not significantly different from zero (because in this case $\zeta^{(n+1)}$ is not significantly different from $\zeta^{(n)}$). Of course, this method of estimating the non-linear equation (A2') converges in one iteration if the final estimate of the ζ parameter falls within the 0-2 range. In this case, in fact, the grid-search step delivers an initial value, $\zeta^{(0)}$, that is very close to the final estimate of ζ, and the linear approximation is very precise, since $(\zeta-\zeta^{(0)})p_K{}^{a(0)}{}_{it} \cong 0$ in equation (A2').[21] Similar procedures, modified in order to take constraints into account, are used to estimate the constant-returns-to-scale version of (2'), as well as equation (2").

As with the additive specification, estimates of the production function with CES capital are obtained by an iterative procedure on a first-order Taylor-series approximation around an initial value $\xi^{(0)}$ and with a fixed ρ of -0.5:

(A3') $(q-l)_{it} = a_i + b_t + (\mu -1)l_{it} + (\lambda / -\rho)(tc^{c(0)}{}_{it}-l_{it}) + (\lambda / -\rho)(\xi-\xi^{(0)})p_K{}^{c(0)}{}_{it} + \varepsilon c_{it}$,

where lower-case letters denote logarithms; $\mu = (\lambda+\beta)$; $tc^{c(0)}{}_{it}-l_{it} = log(TC^{c(0)}{}_{it}/L^{-\rho}{}_{it}) = log[(C^{-\rho}{}_{it}+\xi^{(0)}K^{-\rho}{}_{it})/L^{-\rho}{}_{it}]$; $p_K{}^{c(0)}{}_{it} = K^{-\rho}{}_{it}/TC^{c(0)}{}_{it} = K^{-\rho}{}_{it}/[exp(tc^{c(0)}{}_{it}-l_{it})L^{-\rho}{}_{it}]$; and $\xi^{(0)}$ is any assumed starting value for the distribution parameter for capital inputs. The starting value $\xi^{(0)}$ is selected by a single-grid search on the ξ parameter, with ρ set at -0.5. The results obtained by following this approach are reported in Tables 8a-8d.

The choice of a fixed value for ρ, equal to -0.5, was made by comparing the results given in Table 7c, and obtained by a double-grid search on the unknown ξ and ρ parameters of the CES specification in equation (3'). Once the values of $\xi^{(0)}$ and $\rho^{(0)}$ that minimise the residual sum of squares were attained, we used the Gauss-Newton regression to obtain the standard errors of the ξ and ρ parameters. The Gauss-Newton regression is derived from a first-order Taylor-series approximation around the obtained values $\xi^{(0)}$ and $\rho^{(0)}$:

(A3a') $ec_{it} = a^*{}_i + b^*{}_t + (\mu^* -1)l_{it} + (\lambda^* / -\rho^*)(tc^{c(0)}{}_{it}-l_{it}) + \xi^*d\_\xi_{it} + \rho^*d\_\rho_{it} + uc_{it}$, where ec_{it} is the residual of (3') estimated by fixing the ξ and ρ parameters equal to the values found by the double grid-search; d_ξ_{it} and d_ρ_{it} are the derivatives of (3') with respect to ξ and ρ; and uc_{it} is a zero-mean error term with variance varying by firm i and by time t.[22]

The double-grid search on the ξ_1 and ξ_2 parameters fixing ρ at -0.5, together with the Gauss-Newton regression to obtain standard errors for the ξ_1 and ξ_2 parameters, was the same approach adopted in order to estimate the production function, with CES capital and different types of intangibles, presented in Section 5.

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Table 1- Definition of intangible capital ($K = IKBS+IKCA = IK+CK$) and tangible capital (C) based on company current and capital accounts

PANEL A: Intangible assets categories	**Intellectual capital: *IK***	**Customer capital: *CK***	**Not considered**
Intangible assets: *IKBS*	I2. **($IKBS^{rd}$)** Applied research and development costs; advertising costs functional and essential to the start-up phase. I3. **($IKBS^{pat}$)** Purchased patents, intellectual property rights and applied software (included unlimited licences to use the said software). Internally developed patents, intellectual property rights, software (protected by law).	I4. **($IKBS^{mark}$)** Trademarks and similar rights, public concessions and licences.	I1. **($IKBS^{start}$)** Formation-expansion-start-up expenses (not considered). I5. **($IKBS^{good}$)** Goodwill (not considered). I6. Being evaluated and payments on account (reallocated to I1.-I5. categories). I7. **($IKBS^{fin}$)** Others, largely deferred financial charges (not considered).
Intangible capital constructed from direct expenses: *IKCA*	I8. **($IKCA^{rd}$ computed from DE^{rd})** Basic R&D, and applied R&D not complying with recognition-as-an-asset criteria. I10. **($IKCA^{pat}$ computed from DE^{pat})** Patents, intellectual property rights and software purchased subject to a limited user's licence obtained against payment of regular fees, or obtained free of charge, or not complying with recognition-as-an-asset criteria.	I9. **($IKCA^{adv}$ computed from DE^{adv})** Advertising not related to I1., but operative and recurrent.	
PANEL B: Tangible assets categories	**Tangible assets: *C***		**Not considered**
	T1. **($TKBS^{bui}$)** Lands and buildings. T2. **($TKBS^{pla}$)** Plant and machinery. T3. and T4. **($TKBS^{equ}$)** Equipment, furniture and hardware.		T5. **($TKBS^{oth}$ + $TKBS^{unc}$ + $TKBS^{lea}$)** Other tangibles (mainly divested, fully depreciated or no longer utilised) plus incomplete tangibles (mainly under construction or being purchased) plus leased tangibles (for building societies).

Notes: $K = IK+CK = IKBS+IKCA$ is the total intangible stock; $IK=\sum_j IKBS^j + \sum_h IKCA^h$, $j, h=rd, pat$ is intellectual capital; $CK=\sum_j IKBS^j + \sum_h IKCA^h$, $j=mark, h=adv$ is customer capital; $IKBS=\sum_j IKBS^j$, $j=rd, pat, mark$ is intangible capital as capitalised by firms in their balance sheets; $IKCA=\sum_h IKCA^h$, $h=rd, pat, adv$ is intangible capital not capitalised by firms and directly expensed. *IKCA* is capitalised by the authors based on expenditures, using a constant depreciation rate of 30%. $C=\sum_c TKBS^c$, $c=bui, pla, equ$ is tangible capital. See Appendix for details.

Table 2- Magnitude of different forms of intangible capital compared to total tangible capital: simple and weighted averages, and median (in %)

		Intellectual capital *IK/C*			**Customer capital** *CK/C*			**Total**		
		(1)	**(2)**	**(3)**	**(1)**	**(2)**	**(3)**	**(1)**	**(2)**	**(3)**
Intangible capital capitalised by the firm *IKBS/C*	Average	5.22	6.13	4.44	2.56	2.58	1.85	7.78	8.71	6.30
	Weighted average	3.48	4.12	3.35	1.84	1.34	1.09	5.32	5.46	4.44
	Median	[0.44]	[0.69]	[0.54]	[0.06]	[0.12]	[0.10]	[0.69]	[1.04]	[0.82]
Intangible capital capitalised by us *IKCA/C*	Average	5.53	5.53	3.80	19.32	19.32	14.05	24.85	24.85	17.84
	Weighted average	1.88	1.88	1.53	7.18	7.18	5.83	9.05	9.05	7.36
	Median	[0.00]	[0.00]	[0.00]	[0.45]	[0.45]	[0.36]	[0.72]	[0.72]	[0.58]
Total	Average	10.75	11.66	8.24	21.88	21.91	15.90	32.63	33.56	24.14
	Weighted average	5.36	5.99	4.87	9.02	8.51	6.92			
	Median	[0.64]	[0.90]	[0.71]	[1.36]	[1.49]	[1.18]			

Notes: Total Sample: *NT*= 94988. *K=IK+IC=IKBS+IKCA*; see Table 1 for definitions of the various different forms of intangible capital and for total tangible capital.
In columns (1) intangibles capitalised by us (*IKCA*) are at replacement values; intangibles capitalised by firms (*IKBS*) and tangibles (*C*) are at book values. In columns (2), both those intangibles capitalised by us and those capitalised by firms (*IKCA* and *IKBS* respectively) are estimated at replacement values; while tangibles (*C*) are at book values. In columns (3), both intangibles capitalised by us and by firms (*IKCA* and *IKBS*) are estimated at replacement values; and tangibles (*C*) are also estimated at replacement cost. All the stocks are measured at the beginning of the year and in millions of Italian Lire at 1995 prices. Intangibles capitalised by us (*IKCA*) are constructed from expenditure by using a constant depreciation rate of 30%. The estimates of intangibles capitalised by firms (*IKBS*) and of tangibles (*C*) at replacements costs are obtained on the basis of the series of corresponding gross investments; a constant depreciation rate of 30% is used for all intangibles; depreciation rates of 5% for buildings and of 11% for plant and equipment are employed. See Appendix A1 for details.
Inside each cell: the estimates reported in the first and third rows are the simple averages and medians (in squared brackets) of the sample distribution of the firm's ratios of the different forms of intangible capital to total tangible capital. The averages reported in the second row are weighted by tangible capital (i.e. they are computed as the ratio of the average values of the different forms of intangible capital to the average value of tangible capital).

Table 3- Magnitude of different forms of intangible capital compared to total intangible capital: simple and weighted averages, and median (in %)

		Intellectual capital **_IK/K_**	**Customer capital** **_CK/K_**	**Total**
Intangible capital capitalised by the firm ***IKBS/K***	Average Weighted average Median	37.01 24.20 [19.96]	11.07 12.83 [2.47]	48.08 37.03 [36.03]
Intangible capital capitalised by us ***IKCA/K***	Average Weighted average Median	5.68 13.05 [0.00]	46.25 62.97 [44.65]	51.92 62.97 [63.97]
Total	Average Weighted average Median	42.69 37.26 [34.48]	57.31 62.74 [65.52]	100 100 [100]

Notes: Total Sample: *NT*= 94988. *K=IK+IC=IKBS+IKCA*; *K* = total intangible stock; *IK* = intellectual capital; *CK* customer capital; *IKBS* = intangibles capitalised by firms; *IKCA* = intangibles capitalised by us; see Table 1 for a definition of the various different forms of intangible capital. Intangibles capitalised by us (*IKCA*) are estimated at replacement values; intangibles capitalised by firms (*IKBS*) are at book values, as in columns (1) of Table 2. All the stocks are measured at the beginning of the year and in millions of Italian Lire at 1995 prices.

Inside each cell: the estimates reported in the first and third rows are the simple averages and medians (in squared brackets) of the sample distribution of the firm's ratios of the different forms of intangible capital to total tangible capital. The averages reported in the second row are weighted by tangible capital (i.e. they are computed as the ratio of the average values of the different forms of intangible capital to the average value of tangible capital).

Table 4- Occurrence and relative magnitude of intangible capital for different samples

				Averages of intangible over tangible (*C*) ratios (% values)				
	NT	***N***	$\overline{T}$	***K***	***IK***	***CK***	***IKBS***	***IKCA***
Full sample	94968	14254	6.66	32.63	10.75	21.88	7.78	24.85
***K* never zero**	78481	11528	6.81	38.92	12.73	26.19	9.11	29.81
Both *IK* and *CK* never zero	50317	7646	6.58	46.69	17.11	29.58	11.71	34.98
Both *IKBS* and *IKCA* never zero	27483	4028	6.82	63.87	19.4	44.47	14.15	49.72
***IK* never zero (and *CK* zero)**	1446	299	4.84	11.73	11.73	0	8.99	2.74
***CK* never zero (and *IK* zero)**	3573	643	5.56	23.4	0	23.4	2.83	20.56
***IKBS* never zero (and *IKCA* zero)**	21656	3759	5.76	10.65	7.85	2.8	10.65	0
***IKCA* never zero (and *IKBS* zero)**	3211	564	5.69	25.44	2.87	22.57	0	25.44

Notes: *NT* = total number of observations; *N* = total number of firms; $\overline{T}$ = average number of per-firm years. *K=IK+IC=IKBS+IKCA*; *K* = total intangible stock; *IK* = intellectual capital; *CK* customer capital; *IKBS* = intangibles capitalised by firms; *IKCA* = intangibles capitalised by us; see Table 1 for a definition of the various different forms of intangible capital and for total tangible capital. Intangibles capitalised by us (*IKCA*) are estimated at replacement values. Intangibles capitalised by firms (*IKBS*) and tangibles (*C*) are at book values, as in columns (1) of Table 2. All the stocks are measured at the beginning of the year and in millions of Italian Lire at 1995 prices. The "Full sample" row shows the same results as those shown in Table 2, so as to facilitate comparison. Details of firms' classification according to their global technological intensity at the 4-digit level are given in Appendix A1.

Table 5- Descriptive statistics for main variables

	1st Q	Median	3rd Q	Mean	SD	IQR	% variability	
							Between	**Within**
Levels								
L [1]	29.0	52.0	105.0	131.8	737.7	75.95	99.0%	1.0%
Q/L [2]	53.0	70.2	94.3	79.1	41.1	41.28	76.9%	13.1%
C/L [2]	22.8	42.8	76.4	62.7	72.8	53.69	88.5%	11.5%
K/L [2]	0.3	1.7	6.0	7.9	28.7	5.67	88.5%	11.5%
WL/Q share [3]	51.3%	63.5%	75.0%	64.5%	26.6%	23.74%	52.4%	47.6%
K/C ratio [2]	0.7%	3.8%	16.5%	32.6%	266.5%	15.92%	88.7%	11.3%
IK/C ratio [2]	0.0%	0.6%	3.5%	10.8%	122.2%	3.51%	89.2%	10.8%
CK/C ratio [2]	0.1%	1.4%	9.3%	21.9%	183.0%	9.17%	83.4%	16.6%
$IKBS/C$ ratio [2]	0.0%	0.7%	3.4%	7.8%	151.0%	3.37%	93.6%	6.4%
$IKCA/C$ ratio [2]	0.0%	0.7%	9.4%	24.9%	176.0%	9.31%	80.0%	20.0%
Growth rates (%)								
L [1]	-4.0	0.0	6.3	2.2	15.8	10.25	23.0%	76.5%
Q/L [2]	-8.2	2.8	14.7	5.3	26.7	22.93	13.7%	88.5%
C/L [2]	-13.6	-3.2	12.8	4.0	31.6	26.44	18.4%	79.8%
K/L [2]	-30.5	-10.4	15.9	0.6	44.4	46.54	29.3%	70.1%
WL/Q share [3]	-7.7	0.8	10.3	3.7	28.9	17.94	22.3%	77.1%
K/C ratio [2]	-29.4	-10.1	19.3	1.7	47.9	48.70	28.6%	70.6%
IK/C ratio [2]	-35.6	-13.1	25.3	91.33	3795.5	60.97	33.3%	76.7%
CK/C ratio [2]	-31.5	-14.5	16.8	30.2	1952.0	48.29	9.9%	90.0%
$IKBS/C$ ratio [2]	-36.3	-11.1	28.6	65.6	1142.4	64.89	26.3%	73.7%
$IKCA/C$ ratio [2]	-32.7	-20.6	11.1	0.5	258.9	43.84	22.0%	87.9%

Notes: L = number of employees; Q = value added; C = total tangible stock; WL = labour cost; $K=IK+IC=IKBS+IKCA$; K = total intangible stock; IK = intellectual capital; CK customer capital; $IKBS$ = intangibles capitalised by firms; $IKCA$ = intangibles capitalised by us; see Table 1 for a definition of the various different forms of intangible capital and for total tangible capital. Intangibles capitalised by us ($IKCA$) are estimated at replacement values. Intangibles capitalised by firms ($IKBS$) and tangibles (C) are at book values, as in columns (1) of Table 2. All the stocks are measured at the beginning of the year.

Decomposition of variability is between firm and within firm and year (as obtained in a two-way fixed-effects model).

All the level-statistics are computed on NT=94,968 (total number of observations), N=14,254 (total number of firms), $\bar{T}$ =6.7 (average number of years): zeros in intangible capital stocks are included in the computation. Growth rates are calculated as the first differences of logarithms. When only the first year of each firm is lost, as in L, Q/L, , C/L, and WL/Q share cases, we have NT=80,714, N=14,254 and $\bar{T}$ =5.7. Growth rates of intangibles are affected to a greater extent by the presence of zeros: NT=70,567, N=12,748 and $\bar{T}$ =5.5 for K/L and K/C ratios; NT=67,446, N=12,232 and $\bar{T}$ =5.5 for CK/C ratio; NT=60,535, N=11,775 and $\bar{T}$ =5.1 for IK/C ratio; NT=60,201, N=11,919 and $\bar{T}$ =5.1 for $IKBS/C$ ratio; NT=48,339, N=8,034 and $\bar{T}$ =6.0 for $IKCA/C$ ratio.

[1] The number of employees is the average number of workers during the accounting year when this information is available (42% of total observations); when it is not available, we use the number of employees reported by the firms at the end of the accounting year (58% of total observations). When both definitions are available (20% of total observations) the difference is usually small.

[2] In millions of Italian Lire at 1995 prices.

[3] Shares are computed by using variables at current prices.

Table 6- Production function parameters

Functional form	**Returns to scale to all inputs**	**Elasticity of output with respect to labour**	**Returns to scale to capital inputs**	**Substitution parameter for capital inputs**	**Distribution parameter for capital inputs**	**Elasticity of output with respect to tangibles**	**Elasticity of output with respect to intangibles**	**Marginal productivity of intangibles over that of tangibles (TRS)**
	μ	β	λ	ρ	ξ	α	γ	ζ
Multiplicative (1)	$\mu=\lambda+\beta$	β	$\lambda=\alpha+\gamma$	$= 0$	$\xi = \frac{\gamma}{\alpha}$	α	γ	$\zeta = \frac{\gamma}{\alpha}\left(\frac{C}{K}\right)$
Additive (2)	$\mu=\lambda+\beta$	β	$\lambda=\alpha+\gamma$	$= -1$	$\xi = \zeta$	$\alpha=\lambda\frac{C}{C+\zeta K}$	$\gamma = \lambda\zeta\frac{K}{C+\zeta K}$	ζ
CES (3)	$\mu=\lambda+\beta$	β	$\lambda=\alpha+\gamma$	ρ	ξ	$\alpha=\lambda\frac{C^{-\rho}}{C^{-\rho}+\xi K^{-\rho}}$	$\gamma = \lambda\xi\frac{K^{-\rho}}{C^{-\rho}+\xi K^{-\rho}}$	$\zeta = \xi\left(\frac{C}{K}\right)^{\rho+1}$

Notes:

(1) Multiplicative capital specification: $Q_{it}=A_i B_t L^{\beta}_{it} C^{\alpha}_{it} K^{\gamma}_{it} e^{\varepsilon m_{it}}$,

(2) Additive capital specification: $Q_{it}=A_i B_t L^{\beta}_{it} (C_{it}+ \zeta K_{it})^{\lambda} e^{\varepsilon a_{it}}$,

(3) CES capital specification: $Q_{it}=A_i B_t L^{\beta}_{it} (C^{-\rho}_{it} + \xi K^{-\rho}_{it})^{-\lambda/\rho} e^{\varepsilon c_{it}}$.

Table 7 a- Production function estimates with multiplicative capital

	No constant returns to scale imposed (equation (1'))					*Constant returns to scale imposed on equation (1')* (μ=1)				*Total factor productivity (equation (1''))* (μ=1 and β_0=slmed[1])		
Type of estimates	γ **(k-l)**	α **(c-l)**	μ**-1 (l)**	β	**MSE**	γ **(k-l)**	α **(c-l)**	β	**MSE**	γ **(k-l)**	α	**MSE**
OLS (NT=66953)	0.025 (0.001)	0.130 (0.002)	-0.013 (0.002)	0.832 (0.002)	0.3656	0.026 (0.001)	0.131 (0.002)	0.843 (0.002)	0.3658	0.070 (0.001)	0.297 (0.001)	0.4020
Within (NT=66953)	0.008 (0.002)	0.091 (0.003)	-0.219 (0.007)	0.682 (0.007)	0.1860	0.024 (0.002)	0.127 (0.004)	0.849 (0.004)	0.1887	0.071 (0.002)	0.297 (0.002)	0.1962
First-differences (NT=55425)	0.012 (0.003)	0.070 (0.004)	-0.511 (0.010)	0.407 (0.009)	0.2210	0.062 (0.003)	0.179 (0.004)	0.758 (0.005)	0.2293	0.101 (0.003)	0.267 (0.003)	0.2312
Five-year-differences (NT=5518)	0.009 (0.005)	0.101 (0.009)	-0.132 (0.017)	0.758 (0.018)	0.3415	0.018 (0.005)	0.120 (0.010)	0.861 (0.010)	0.3466	0.064 (0.005)	0.303 (0.005)	0.3622

Table 7 b- Production function estimates with additive capital[2]

	No constant returns to scale imposed (equation (2'))					*Constant returns to scale imposed on equation (2')* (μ=1)				*Total factor productivity (equation (2''))* (μ=1 and β_0=slmed[1])		
Type of estimates	ζ **(p_K^a)**	λ **(tc^a-l)**	μ **-1 (l)**	β	**MSE**	ζ **(p_K^a)**	λ **(tc^a-l)**	β	**MSE**	ζ **(p_K^a)**	λ	**MSE**
OLS (NT=66953)	1.365 (0.063)	0.164 (0.002)	-0.012 (0.002)	0.824 (0.002)	0.3629	1.396 (0.062)	0.165 (0.002)	0.835 (0.002)	0.3631	1.395 (0.031)	0.367 (-)	0.3977
Within (NT=66953)	0.467 (0.104)	0.101 (0.004)	-0.218 (0.007)	0.680 (0.007)	0.1859	0.690 (0.088)	0.148 (0.004)	0.852 (0.004)	0.1886	0.810 (0.042)	0.367 (-)	0.1970
First-differences (NT=55425)	0.536 (0.180)	0.079 (0.005)	-0.513 (0.010)	0.407 (0.009)	0.2210	1.627 (0.139)	0.228 (0.005)	0.772 (0.005)	0.2296	1.951 (0.100)	0.367 (-)	0.2322
Five-year-differences (NT=5518)	0.504 (0.226)	0.112 (0.010)	-0.131 (0.017)	0.756 (0.018)	0.3412	0.613 (0.209)	0.137 (0.010)	0.863 (0.010)	0.3436	0.672 (0.093)	0.367 (-)	0.3635

Table 7 c- Production function estimates with CES capital[3]

	No constant returns to scale imposed (equation (3'))						***Constant returns to scale imposed on equation (3')*** (μ=1)					***Total factor productivity (equation (3''))*** (μ=1 and β_0=slmed[1])			
Type of estimates	ρ	ξ	λ **(tc^c-l)**	**μ-1 (l)**	β	**MSE**	ρ	ξ	λ **(tc^c-l)**	β	**MSE**	ρ	ξ	λ **(tc^c-l)**	**MSE**
OLS (NT=66953)	-1 (0.064)	1.4 (0.125)	0.164 (0.002)	-0.012 (0.002)	0.824 (0.002)	0.3629	-1 (0.064)	1.4 (0.125)	0.165 (0.002)	0.835 (0.002)	0.3631	-0.4 (0.014)	0.6 (0.147)	0.367 (-)	0.3964
Within (NT=66953)	-0.7 (0.160)	0.4 (0.085)	0.103 (0.004)	-0.217 (0.007)	0.680 (0.007)	0.1859	-0.4 (0.057)	0.5 (0.061)	0.154 (0.004)	0.846 (0.004)	0.1886	-0.4 (0.023)	0.6 (0.031)	0.367 (-)	0.1959
First-differences (NT=55425)	-0.4 (0.155)	0.4 (0.131)	0.084 (0.005)	-0.509 (0.010)	0.407 (0.009)	0.2210	-0.4 (0.038)	0.9 (0.083)	0.245 (0.005)	0.755 (0.005)	0.2292	-0.1 (0.019)	0.5 (0.031)	0.367 (-)	0.2310
Five-year-differences (NT=5518)	-1 (0.478)	0.5 (0.241)	0.112 (0.010)	-0.132 (0.017)	0.756 (0.018)	0.3412	-0.7 (0.252)	0.5 (0.178)	0.1397 (0.010)	0.860 (0.010)	0.3435	-0.4 (0.060)	0.5 (0.064)	0.367 (-)	0.3614

Notes:

OLS estimates include industry and temporal dummies; within, first-differences and five-year differences include individual and temporal effects. Robust standard errors are shown in brackets.

[1] *slmed=0.633* is the sample median of labour cost's share of value added.

[2] Estimates of the production function with additive capital are obtained by an iterative procedure on a first-order Taylor-series approximation around an initial value $\zeta^{(0)}$. The starting value $\zeta^{(0)}$ is selected by a grid search on the ζ parameter (see Appendix A2).

[3] Estimates of the production function with CES capital are obtained by using a grid search on the ρ and ξ parameters. Standard errors of ρ and ξ parameters are obtained by using the Gauss-Newton regression derived by a first-order Taylor-series approximation around the minimum residual sum of squares estimates of the ρ and ξ parameters. Estimates and standard errors of the λ, μ-1 and β parameters correspond to the minimum residual sum of squares estimates of the ρ and ξ parameters (see Appendix A2).

Table 8 a- Pooled estimates of the γ and ζ parameters

	Elasticity of output with respect to intangible capital			*Marginal productivity of intangibles over that of tangibles*		
OLS (NT=66953)	**γ(Q1)**	**γ(med)**	**γ(Q3)**	**ζ(Q1)**	**ζ(med)**	**ζ(Q3)**
No constant returns to scale:						
Multiplicative	0.025 (0.001)	0.025 (0.001)	0.025 (0.001)	0.875 (0.034)	3.097 (0.120)	11.422 (0.442)
Additive[1]	0.004 (0.000)	0.013 (0.001)	0.038 (0.002)	1.365 (0.063)	1.365 (0.063)	1.365 (0.063)
CES (ρ = -0.5)[2]	0.014 (0.001)	0.025 (0.001)	0.041 (0.002)	1.480 (0.053)	2.783 (0.099)	5.345 (0.190)
Total factor productivity:[3]						
Multiplicative	0.070 (0.001)	0.070 (0.001)	0.070 (0.001)	1.052 (0.017)	3.722 (0.060)	13.729 (0.219)
Additive[1]	0.009 (0.000)	0.030 (0.001)	0.087 (0.002)	1.395 (0.031)	1.395 (0.031)	1.395 (0.031)
CES (ρ = -0.5)[2]	0.035 (0.001)	0.062 (0.001)	0.101 (0.002)	1.693 (0.026)	3.184 (0.049)	6.115 (0.095)

Table 8 b- First-differences estimates of γ and ζ parameters

	Elasticity of output with respect to intangible capital			*Marginal productivity of intangibles over that of tangibles*		
First-differences (NT=55425)	**γ(Q1)**	**γ(med)**	**γ(Q3)**	**ζ(Q1)**	**ζ(med)**	**ζ(Q3)**
No constant returns to scale:						
Multiplicative	0.012 (0.003)	0.012 (0.003)	0.012 (0.003)	0.752 (0.185)	2.661 (0.653)	9.815 (2.408)
Additive[1]	0.001 (0.000)	0.003 (0.001)	0.009 (0.003)	0.536 (0.180)	0.536 (0.180)	0.536 (0.180)
CES (ρ = -0.5)[2]	0.005 (0.001)	0.009 (0.003)	0.015 (0.004)	1.013 (0.263)	1.906 (0.495)	3.661 (0.952)
Total factor productivity:[3]						
Multiplicative	0.101 (0.003)	0.101 (0.003)	0.101 (0.003)	1.686 (0.059)	5.964 (0.208)	21.997 (0.768)
Additive[1]	0.012 (0.001)	0.040 (0.002)	0112 (0.006)	1.951 (0.100)	1.951 (0.100)	1.951 (0.100)
CES (ρ = -0.5)[2]	0.049 (0.002)	0.084 (0.003)	0.131 (0.005)	2.472 (0.101)	4.650 (0.190)	8.929 (0.365)

Table 8 c- Within estimates of γ and ζ parameters

	Elasticity of output with respect to intangible capital			*Marginal productivity of intangibles over that of tangibles*		
Within (NT=66953)	**γ(Q1)**	**γ(med)**	**γ(Q3)**	**ζ(Q1)**	**ζ(med)**	**ζ(Q3)**
No constant returns to scale:						
Multiplicative	0.008	0.008	0.008	0.400	1.414	5.214
	(0.002)	(0.002)	(0.002)	(0.101)	(0.357)	(1.316)
Additive[1]	0.001	0.003	0.010	0.467	0.467	0.467
	(0.000)	(0.001)	(0.002)	(0.104)	(0.104)	(0.104)
CES (ρ = -0.5)[2]	0.005	0.008	0.015	0.744	1.400	2.688
	(0.001)	(0.002)	(0.003)	(0.140)	(0.262)	(0.504)
Total factor productivity:[3]						
Multiplicative	0.071	0.071	0.071	1.062	3.757	13.856
	(0.002)	(0.002)	(0.002)	(0.036)	(0.127)	(0.467)
Additive[1]	0.005	0.018	0.056	0.810	0.810	0.810
	(0.000)	(0.001)	(0.003)	(0.042)	(0.042)	(0.042)
CES (ρ = -0.5)[2]	0.029	0.052	0.087	1.381	2.598	4.989
	(0.001)	(0.002)	(0.003)	(0.054)	(0.102)	(0.196)

Table 8 d- Five-year-differences estimates of γ and ζ parameters

	Elasticity of output with respect to intangible capital			*Marginal productivity of intangibles over that of tangibles*		
Five-year-differences (NT=5518)	**γ(Q1)**	**γ(med)**	**γ(Q3)**	**ζ(Q1)**	**ζ(med)**	**ζ(Q3)**
No constant returns to scale:						
Multiplicative	0.009	0.009	0.009	0.413	1.461	5.388
	(0.005)	(0.005)	(0.005)	(0.223)	(0.790)	(2.915)
Additive[1]	0.001	0.003	0.011	0.504	0.504	0.504
	(0.000)	(0.002)	(0.005)	(0.226)	(0.226)	(0.226)
CES (ρ = -0.5)[2]	0.005	0.009	0.016	0.748	1.407	2.701
	(0.002)	(0.004)	(0.007)	(0.300)	(0.564)	(1.084)
Total factor productivity:[3]						
Multiplicative	0.064	0.064	0.064	0.944	3.339	12.314
	(0.005)	(0.005)	(0.005)	(0.084)	(0.298)	(1.098)
Additive[1]	0.004	0.015	0.048	0.672	0.672	0.672
	(0.001)	(0.002)	(0.007)	(0.093)	(0.093)	(0.093)
CES (ρ = -0.5)[2]	0.025	0.046	0.078	1.194	2.247	4.314
	(0.003)	(0.005)	(0.008)	(0.120)	(0.226)	(0.434)

Notes:

OLS estimates include industry and temporal dummies; within, first-differences and five year-differences include individual and temporal effects. Robust standard errors are shown in brackets.

[1] Estimates of the production function with additive capital are obtained by an iterative procedure on a first-order Taylor-series approximation around an initial value $\zeta^{(0)}$. The starting value $\zeta^{(0)}$ is selected by a grid search on the ζ parameter (see Appendix A2).

[2] Estimates of the production function with CES capital are obtained by an iterative procedure on a first-order Taylor-series approximation around an initial value $\xi^{(0)}$ with a fixed ρ equal to -0.5. The starting value $\xi^{(0)}$ is selected by a grid search on the ξ parameter fixing ρ equal to -0.5 (see Appendix A2).

[3] *slmed=0.633* is the sample median of labour cost's share of value added.

Table 9- GMM estimates of production function with multiplicative capital

	No constant returns to scale imposed (equation (1'))					***Total factor productivity (equation (1''))*** (μ=1 and β_0=slmed[1])			
Type of estimate and instrument list	**γ (k-l)**	**α (c-l)**	**μ-1 (l)**	**R^2 (Wald Test)**	**χ^2 (d.f.)**	**γ (k-l)**	**α (c-l)**	**R^2 (Wald Test)**	**χ^2 (d.f.)**
GMM first-differences: first-difference equations instrumented by lags t-2 and t-3 of the three explanatory variables in levels. (NT=55425)	0.021 (0.019)	0.029 (0.034)	-0.496 (0.081)	0.0142 (1866.1)	195.9 (86)	0.104 (0.016)	0.263 (0.016)	0.0194 (1954.1)	230.2 (88)
GMM first-differences: first-difference equations instrumented by lags t-2 and t-3 of the investment-labour-ratios in levels, and by the corresponding dummies capturing null values. (NT=55425)	0.087 (0.018)	-0.074 (0.078)	-0.292 (0.124)	0.0152 (1595.4)	126.8 (113)	0.100 (0.019)	0.267 (0.019)	0.0198 (1822.0)	140.3 (115)
GMM levels: level equations instrumented by lags t-2 and t-3 of the three explanatory variables in first-differences. (NT=66953)	-0.009 (0.008)	0.101 (0.023)	0.112 (0.027)	0.2385 (6494.6)	179.1\ (80)	0.020 (0.007)	0.348 (0.007)	0.1443 (3986.2)	272.6 (82)
GMM levels: level equations instrumented by lags t-2 and t-3 of the investment-labour-ratios in first-differences, and by the corresponding dummies capturing null values. (NT=66953)	0.004 (0.006)	0.193 (0.039)	0.053 (0.019)	0.2996 (7304.0)	171.6 (109)	0.012 (0.006)	0.356 (0.006)	0.1501 (3981.1)	180.1 (111)
GMM system: level and first-difference equations instrumented by lags t-2 and t-3 of the three explanatory variables respectively in first-differences and in levels. (NT=66953)	0.011 (0.006)	0.132 (0.015)	0.117 (0.015)	0.2569 (7390.0)	446.9 (132)	0.035 (0.006)	0.332 (0.006)	0.1294 (4735.6)	622.8 (134)
GMM system: level and first-difference equations instrumented by lags t-2 and t-3 of the investment-labour-ratios respectively in first-differences and in levels, and by the corresponding dummies capturing null values. (NT=66953)	0.029 (0.005)	0.235 (0.030)	0.063 (0.014)	0.2936 (7299.2)	288.4 (175)	0.035 (0.005)	0.332 (0.005)	0.1288 (4480.1)	298.5 (177)

Notes:

Estimates include industry and temporal dummies. The reported estimates and their heteroskedasticity robust standard errors (shown in brackets) are the one-step estimates. The R^2 are computed as the squared (sample) correlation coefficient between actual and fitted values. The Wald statistics test the joint significance of the explanatory variables. The chi-2 test statistics of over-identification are the second-step estimates.

[1] *slmed=0.633* is the sample median of labour cost's share of value added.

Table 10- Estimates of the ζ parameters, multiplicative capital and total factor productivity[1]

	Marginal productivity of intangibles over that of tangibles		
	ζ(Q1)	**ζ(med)**	**ζ(Q3)**
OLS (N=66953)			
Intangibles non capitalised by firms at replacement values (30%); intangibles capitalised by firms and tangibles at book values	1.052 (0.017)	3.722 (0.060)	13.729 (0.219)
All intangibles at replacement values (30%) and tangibles at book values	0.868 (0.015)	3.179 (0.054)	12.540 (0.211)
All intangibles at replacement values (20%) and tangibles at book values	0.747 (0.012)	2.678 (0.044)	10.211 (0.168)
All intangibles at replacement values (30%) and tangibles at replacement values	0.877 (0.016)	3.145 (0.059)	12.413 (0.233)
First-differences (N=55425)			
Intangibles non capitalised by firms at replacement values (30%); intangibles capitalised by firms and tangibles at book values	1.686 (0.059)	5.964 (0.208)	21.997 (0.768)
All intangibles at replacement values (30%) and tangibles at book values	1.131 (0.050)	4.141 (0.182)	16.334 (0.719)
All intangibles at replacement values (20%) and tangibles at book values	1.307 (0.057)	4.688 (0.206)	17.873 (0.785)
All intangibles at replacement values (30%) and tangibles at replacement values	1.175 (0.062)	4.210 (0.223)	16.620 (0.880)
Within (N=66953)			
Intangibles non capitalised by firms at replacement values (30%); intangibles capitalised by firms and tangibles at book values	1.062 (0.036)	3.757 (0.127)	13.856 (0.467)
All intangibles at replacement values (30%) and tangibles at book values	0.875 (0.031)	3.204 (0.115)	12.637 (0.453)
All intangibles at replacement values (20%) and tangibles at book values	0.981 (0.033)	3.519 (0.119)	13.414 (0.453)
All intangibles at replacement values (30%) and tangibles at replacement values	0.971 (0.040)	3.481 (0.142)	13.739 (0.561)
Five-year-differences (N=5518)			
Intangibles non capitalised by firms at replacement values (30%); intangibles capitalised by firms and tangibles at book values	0.944 (0.084)	3.339 (0.298)	12.314 (1.098)
All intangibles at replacement values (30%) and tangibles at book values	0.810 (0.078)	2.964 (0.285)	11.690 (1.125)
All intangibles at replacement values (20%) and tangibles at book values	0.922 (0.084)	3.309 (0.301)	12.614 (1.149)
All intangibles at replacement values (30%) and tangibles at replacement values	0.808 (0.093)	2.897 (0.333)	11.434 (1.313)

Notes:

To facilitate comparison, the first row of each estimation method shows the same estimates as those given in Tables 8 a-d. OLS estimates include industry and temporal dummies; within, first-differences and five-year-differences include individual and temporal effects. Robust standard errors are shown in brackets.

Intangibles capitalised by us (*IKCA*) are constructed from expenditure, at a constant depreciation rate of 30%. Estimation of intangibles capitalised by firms (*IKBS*) and of tangibles (*C*) at replacements costs are obtained on the basis of the series of corresponding gross investments, by using a depreciation rate for all intangibles taken to be 30% or 20%, and depreciation rates of 5% for buildings and of 11% for plant and equipment. See Appendix A1 for details.

[1] *slmed=0.633* is the sample median of labour cost's share of value added.

Table 11 a- γ and ζ estimates for intellectual capital (*IK*) and for customer capital (*CK*). CES capital[1], total factor productivity.[2]

	Elasticity of output with respect to the two types of intangible capital						*Marginal productivity of the two types of intangibles over that of tangibles*						
	Intellectual capital *IK*			**Customer capital *CK***			**Intellectual capital *IK***			**Customer capital *CK***			
Type of estimates	**γ(Q1)**	**γ(med)**	**γ(Q3)**	**γ(Q1)**	**γ(med)**	**γ(Q3)**	**ζ(Q1)**	**ζ(med)**	**ζ(Q3)**	**ζ(Q1)**	**ζ(med)**	**ζ(Q3)**	**MSE**
OLS (NT=44096)	0.016 (0.001)	0.029 (0.001)	0.049 (0.002)	0.010 (0.000)	0.024 (0.001)	0.050 (0.001)	2.709 (0.094)	5.053 (0.175)	9.637 (0.333)	1.316 (0.038)	3.044 (0.087)	7.694 (0.221)	0.3982
First-differences (NT=36166)	0.017 (0.001)	0.030 (0.002)	0.051 (0.003)	0.018 (0.001)	0.042 (0.003)	0.080 (0.005)	3.083 (0.209)	5.760 (0.391)	11.004 (0.747)	2.368 (0.148)	5.390 (0.337)	13.472 (0.842)	0.2334
Within (NT=44096)	0.011 (0.001)	0.021 (0.001)	0.035 (0.002)	0.013 (0.001)	0.030 (0.002)	0.061 (0.003)	1.935 (0.124)	3.609 (0.249)	6.884 (0.475)	1.579 (0.086)	3.652 (0.199)	9.233 (0.504)	0.1948
Five-year-differences (NT=3452)	0.011 (0.002)	0.020 (0.004)	0.035 (0.006)	0.013 (0.002)	0.029 (0.004)	0.056 (0.008)	1.993 (0.355)	3.726 (0.664)	7.011 (1.249)	1.739 (0.237)	3.708 (0.506)	8.685 (1.186)	0.3636

Table 11 b- γ and ζ estimates for intangibles capitalised by firms (*IKBS*) and for intangibles not-capitalised by firms (*IKCA*). CES capital[1], total factor productivity.[2]

	Elasticity of output with respect to the two types of intangible capital						*Marginal productivity of the two types of intangibles over that of tangibles*						
	Intangibles capitalised by firms *IKBS*			**Intangibles not-capitalised by firms *IKCA***			**Intangibles capitalised by firms *IKBS***			**Intangibles not-capitalised by firms *IKCA***			
Type of estimates	**γ(Q1)**	**γ(med)**	**γ(Q3)**	**γ(Q1)**	**γ(med)**	**γ(Q3)**	**ζ(Q1)**	**ζ(med)**	**ζ(Q3)**	**ζ(Q1)**	**ζ(med)**	**ζ(Q3)**	**MSE**
OLS (NT=24395)	0.015 (0.001)	0.026 (0.001)	0.043 (0.002)	0.035 (0.001)	0.063 (0.002)	0.103 (0.003)	2.682 (0.147)	5.007 (0.274)	9.341 (0.511)	1.412 (0.037)	2.720 (0.072)	5.451 (0.144)	0.3839
First-differences (NT=20184)	0.012 (0.001)	0.022 (0.002)	0.036 (0.004)	0.039 (0.003)	0.070 (0.005)	0.112 (0.008)	2.295 (0.230)	4.264 (0.427)	7.971 (0.797)	1.617 (0.115)	3.125 (0.223)	6.259 (0.446)	0.2350
Within (NT=24395)	0.011 (0.001)	0.020 (0.002)	0.033 (0.003)	0.028 (0.002)	0.051 (0.003)	0.086 (0.005)	1.916 (0.167)	3.576 (0.312)	6.672 (0.583)	1.059 (0.065)	2.040 (0.125)	4.088 (0.251)	0.1988
Five-year-differences (NT=2026)	0.013 (0.003)	0.021 (0.005)	0.036 (0.008)	0.018 (0.003)	0.035 (0.006)	0.062 (0.011)	1.877 (0.433)	3.503 (0.807)	6.206 (1.431)	1.092 (0.187)	2.133 (0.366)	4.551 (0.781)	0.3720

Notes:

OLS estimates include industry and temporal dummies; within, first-differences and five-year-differences include individual and temporal effects. Robust standard errors are shown in brackets.

[1] Estimates of the production function with CES capital are obtained using a grid search on the ξ_1 and ξ_2 parameters by setting ρ equal to -0.5. Standard errors of the ξ_1 and ξ_2 parameters are obtained by using the Gauss-Newton regression derived by a first-order Taylor-series approximation around the minimum residual sum of squares estimates of the ξ_1 and ξ_2 parameters (see Appendix A2).

[2] *slmed=0.638* and *slmed=0.633* are the sample medians of labour costs' share of value added in parts a and b of Table 11, respectively.

Figure 1 – Different estimates of relative productivity of intangibles,
Non constant returns to scale specification

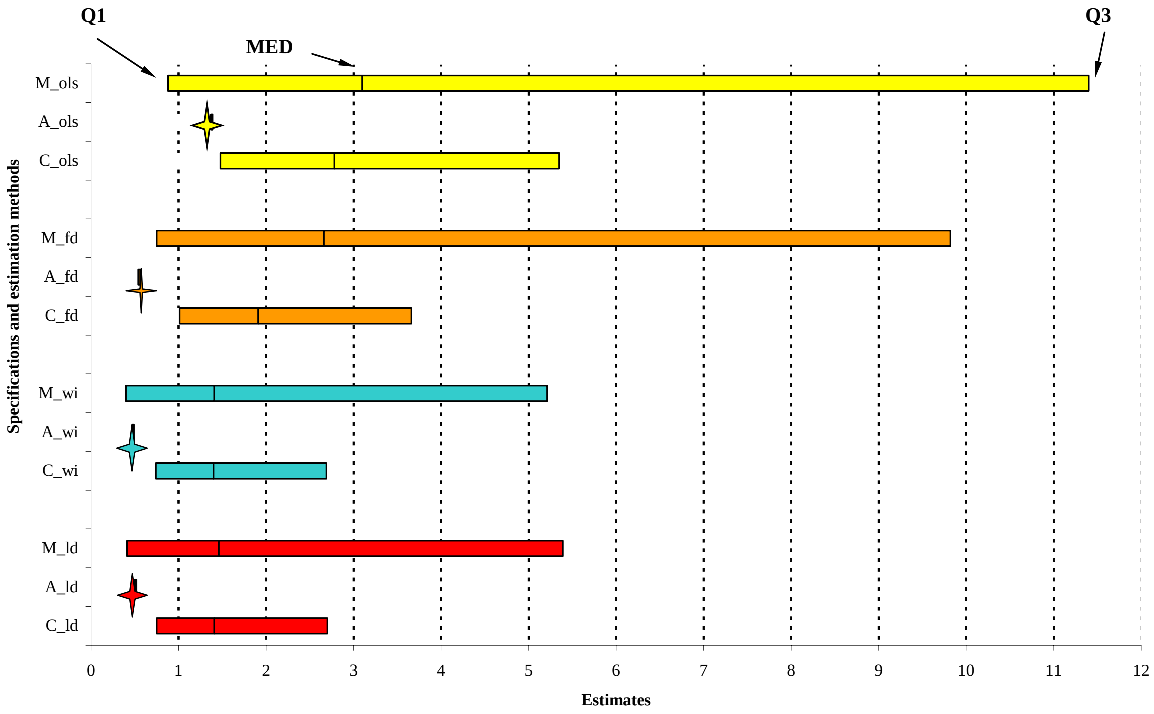
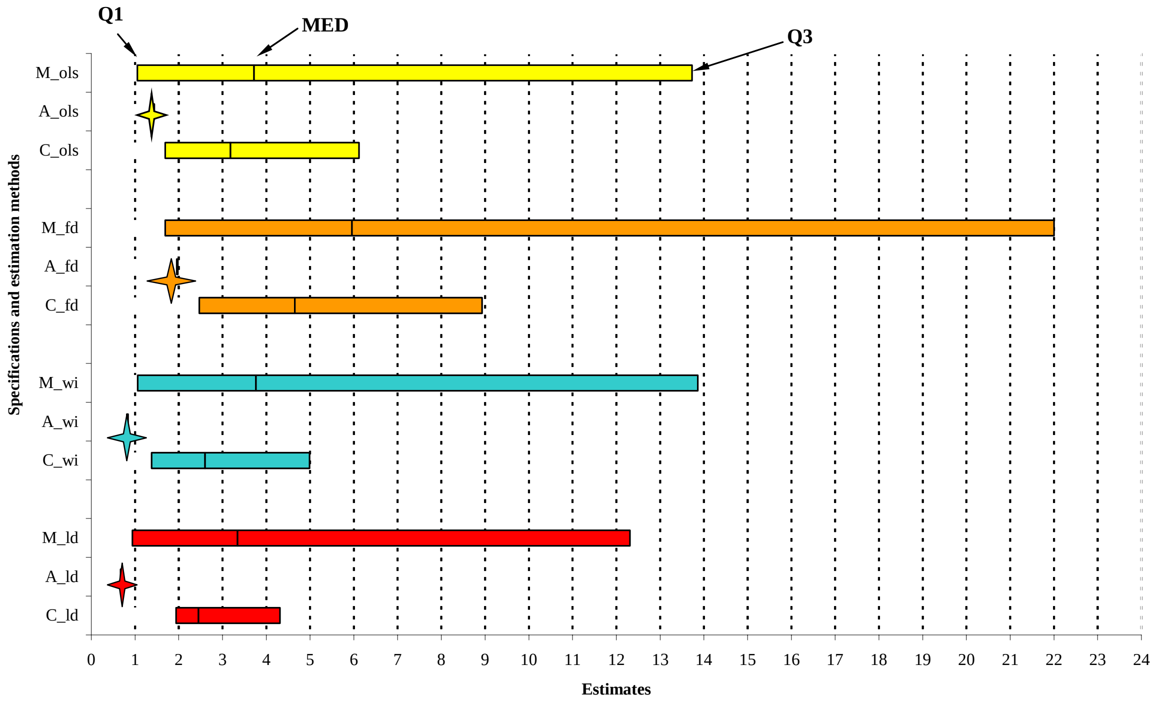


Figure 2 – Different estimates of relative productivity of intangibles,
Total factor productivity specification



Note:
By definition, the additive estimates have no range.

Figure 3 – Different estimates of relative productivity of intellectual and customer capital,
CES capital, total factor productivity specification

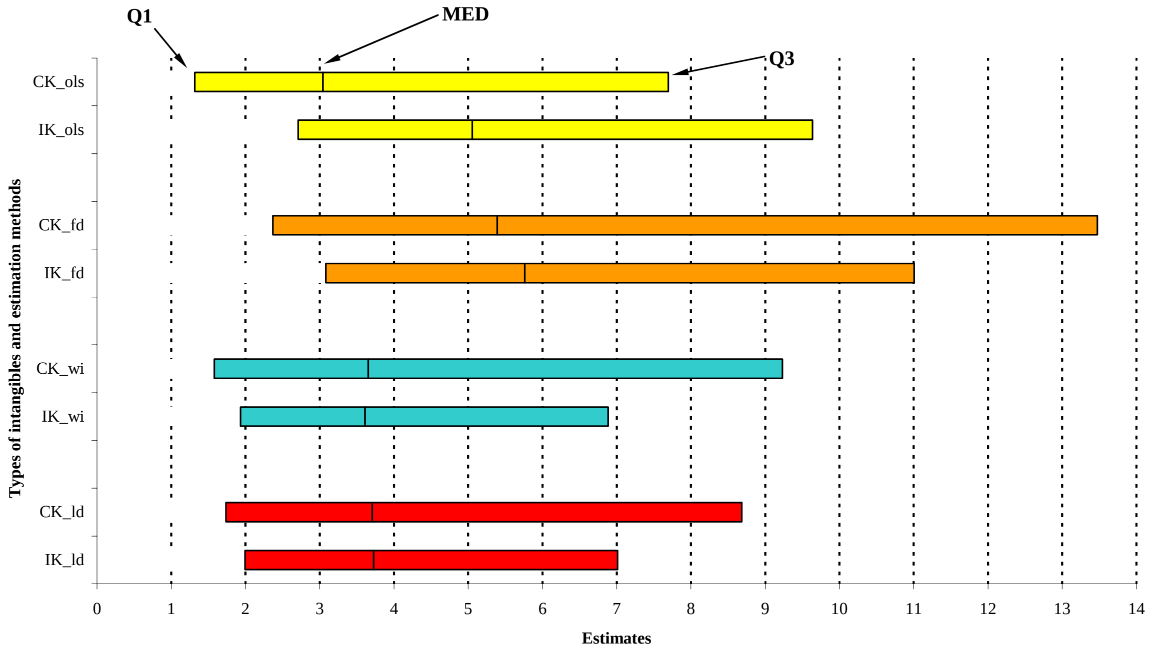
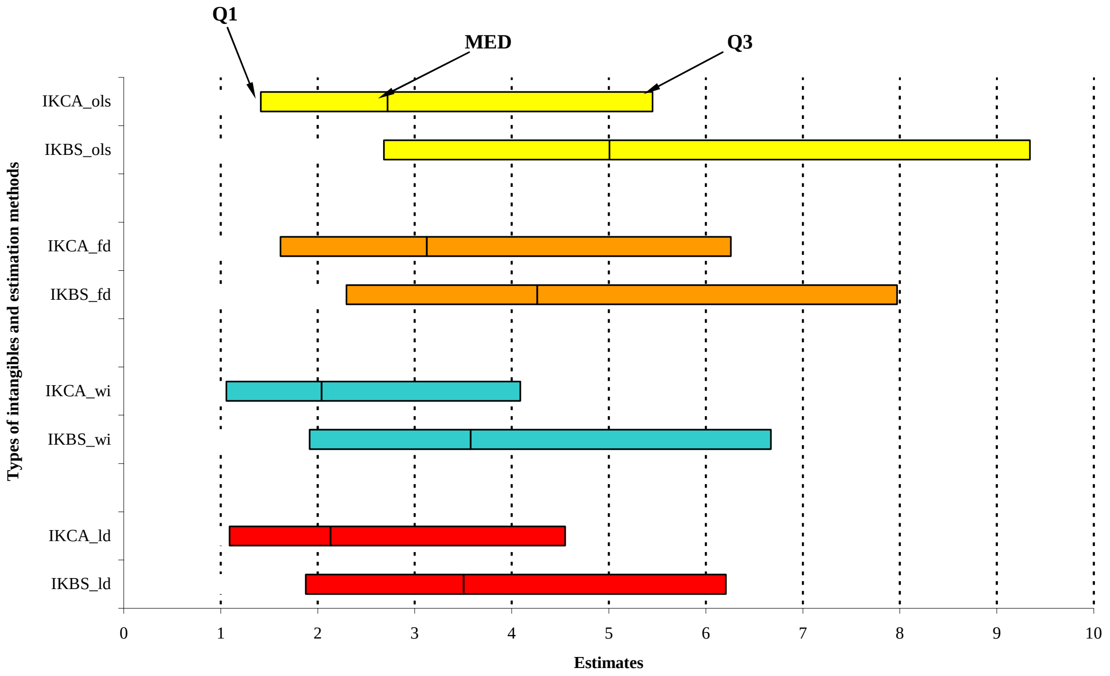


Figure 4 – Different estimates of relative productivity of intangible capital capitalised and non-capitalised by the firm,
CES capital, total factor productivity specification



APPENDIX Tables

Table A1: Industry classification.

		NT	% *NT*	*C/Q*	*K/Q*	*IK/Q*	*CK/Q*	*IKBS/Q*	*IKCA/Q*
HT	Aerospace	146	(0.15)	67.96	12.85	10.45	2.40	11.35	1.50
HT	Computer	240	(0.25)	60.11	11.57	5.71	5.86	6.16	5.41
HT	Electronics	884	(0.93)	51.28	14.95	7.92	7.04	6.33	8.62
HT	Pharmaceutical	1379	(1.45)	54.74	21.67	10.55	11.12	7.80	13.87
Total HT		2649	(2.79)	54.80	18.03	9.23	8.80	7.36	10.67
MHT	Scientific instruments	1590	(1.67)	53.81	15.09	6.86	8.23	5.49	9.60
MHT	Motor vehicles	2124	(2.24)	72.40	9.75	3.20	6.55	3.31	6.44
MHT	Electric machinery	4603	(4.85)	64.64	8.56	3.77	4.79	3.47	5.09
MHT	Chemicals	4174	(4.40)	76.99	15.15	3.26	11.88	2.94	12.21
MHT	Other transport equipment	72	(0.08)	65.34	7.66	0.89	6.78	0.84	6.83
MHT	Non-electric machinery	12984	(13.67)	57.87	9.57	4.00	5.57	3.60	5.97
Total MHT		25547	(26.9)	63.19	10.65	3.94	6.71	3.56	7.10
MLT	Rubber-plastic	5512	(5.80)	85.00	5.91	2.27	3.63	2.08	3.83
MLT	Shipbuilding	328	(0.35)	106.1	13.73	6.49	7.24	5.65	8.08
MLT	Other manufacturing	359	(0.38)	68.53	14.73	2.55	12.18	2.14	12.59
MLT	Non-ferrous metal	777	(0.82)	94.93	4.25	1.15	3.10	1.07	3.18
MLT	Non-metallic mineral	7352	(7.74)	115.07	7.14	1.83	5.32	1.86	5.29
MLT	Fabricated metal	13173	(13.87)	81.40	5.78	1.94	3.83	1.85	3.93
MLT	Petroleum	319	(0.34)	114.43	6.58	3.12	3.45	4.46	2.11
MLT	Ferrous metal	723	(0.76)	122.57	2.07	1.32	0.76	1.34	0.73
Total MLT		28543	(30.06)	92.67	6.23	2.01	4.22	1.94	4.30
LT	Paper-printing	4815	(5.07)	81.75	10.30	4.98	5.32	2.45	7.85
LT	Textile-clothing	17275	(18.19)	67.69	11.07	3.04	8.03	1.69	9.38
LT	Food-tobacco	9455	(9.96)	145.61	15.15	2.27	12.89	3.29	11.87
LT	Wood	6684	(7.04)	99.04	12.83	3.95	8.89	3.51	9.32
Total LT		38229	(40.25)	94.21	12.29	3.25	9.04	2.50	9.79
Total		94968	(100)	84.3	10.19	3.23	6.96	2.75	7.44

Notes:

Firms are classified along the rows according to their global technological intensity at the 4-digit level (HT = high intensity; MHT = medium-high intensity; MLT = medium-low intensity; LT = low intensity).

The columns are as follows: *NT* is the total number of observations; *Q* = value added; *C* = total tangible stock; *K* = total intangible stock; *IK* = intellectual capital; *CK* customer capital; *IKBS* = intangibles capitalised by firms; *IKCA* = intangibles capitalised by us. See Table 1 for a definition of the different forms of intangible capital and for total tangible capital. Intangibles capitalised by us (*IKCA*) are estimated at replacement values; intangibles capitalised by firms (*IKBS*) and tangibles (*C*) are at book values. Value added and all stocks are in millions of Italian Lire at 1995 prices. All stocks are measured at the beginning of the year.

Table A2: Classification of observations by size and industry.

	HT+MHT		MLT		LT		Total	
Numbers of employees								
5-19	2511	(2.64%)	3600	(3.79%)	6130	(6.45%)	12241	(12.89%)
20-49	8641	(9.10%)	10635	(11.20%)	13384	(14.09%)	32660	(34.39%)
50-249	13376	(14.08%)	12537	(13.20%)	16464	(17.34%)	42377	(44.62%)
≥ 250	3668	(3.86%)	1771	(1.86%)	2251	(2.37%)	7690	(8.10%)
Total	28196	(29.69%)	28543	(30.06%)	38229	(40.25%)	94968	(100%)

Notes:

Firms are classified along the rows according to their global technological intensity at the 4-digits level (HT+MHT = high and medium-high intensity; MLT = medium-low intensity; LT = low intensity).

Along the columns, firms are classified according to the number of employees.

The number of employees is the average number of workers employed during the accounting year when this information is available (42% of total observations); otherwise, we use the number of employees reported by the firms at the end of the accounting year (58% of total observations). The difference between the two definitions (when both available, 20% of total observations) is usually small.

Table A3- Occurrence of intangible capital stocks and intangible capital stocks to total tangible capital stocks ratios

Total sample and sub-samples				**Averages of intangible to tangible (*C*) ratios (% values)**				
	NT	*N*	$\overline{T}$	***K***	***IK***	***CK***	***IKBS***	***IKCA***
Full sample								
Total	94968	14254	6.66	32.63	10.75	21.88	7.78	24.85
HT+MHT	28196	4327	6.52	38.91	14.35	24.55	12.24	26.66
MLT	28543	4408	6.48	13.46	3.9	9.56	3.25	10.21
LT	38229	5778	6.62	42.31	13.2	29.11	7.87	34.44
***K* never zero**								
Total	78481	11528	6.81	38.92	12.73	26.19	9.11	29.81
HT+MHT	23929	3608	6.63	45.01	16.41	28.6	13.89	31.12
MLT	23378	3528	6.63	16.22	4.67	11.55	3.86	12.36
LT	31174	4606	6.77	51.27	15.96	35.32	9.39	41.88
Both *IK* and *CK* never zero								
Total	50317	7646	6.58	46.69	17.11	29.58	11.71	34.98
HT+MHT	16028	2461	6.51	53.63	20.41	33.22	17.14	36.48
MLT	14495	2267	6.39	17.95	6.19	11.76	4.77	13.19
LT	19794	3003	6.59	62.12	22.43	39.69	12.39	49.73
Both *IKBS* and *IKCA* never zero								
Total	27483	4028	6.82	63.87	19.4	44.47	14.15	49.72
HT+MHT	9074	1354	6.70	69.21	22.32	46.89	16.68	52.53
MLT	7295	1109	6.58	27.46	8	19.47	5.56	21.9
LT	11114	1624	6.84	83.41	24.5	58.91	17.72	65.69
***IK* never zero (and *CK* zero)**								
Total	1446	299	4.84	11.73	11.73	0	8.99	2.74
HT+MHT	493	104	4.74	23.17	23.17	0	17.05	6.11
MLT	495	104	4.76	5.64	5.64	0	5.55	0.09
LT	458	99	4.63	6.01	6.01	0	4.04	1.97
***CK* never zero (and *IK* zero)**								
Total	3573	643	5.56	23.4	0	23.4	2.83	20.56
HT+MHT	829	152	5.45	20.83	0	20.83	3.87	16.96
MLT	1055	188	5.61	13.78	0	13.78	0.81	12.97
LT	1689	311	5.43	30.66	0	30.66	3.59	27.07
***IKBS* never zero (and *IKCA* zero)**								
Total	21656	3759	5.76	10.65	7.85	2.8	10.65	0
HT+MHT	7007	1232	5.69	18.88	15.2	3.69	18.88	0
MLT	6782	1199	5.66	5.09	3.88	1.2	5.09	0
LT	7867	1377	5.71	8.1	4.72	3.38	8.1	0
***IKCA* never zero (and *IKBS* zero)**								
Total	3211	564	5.69	25.44	2.87	22.57	0	25.44
HT+MHT	752	136	5.53	27.53	8.22	19.31	0	27.53
MLT	1019	174	5.86	12.79	1.11	11.68	0	12.79
LT	1440	261	5.52	33.3	1.32	31.97	0	33.3

Notes:

Firms are classified along rows according to their global technological intensity at the 4-digit level (HT+MHT = high and medium-high intensity; MLT = medium-low intensity; LT = low intensity).

Along columns *NT* = total number of observations; *N* = total number of firms; $\overline{T}$ = average number of per-firm years. *K* = total intangible stock; *IK* = intellectual capital; *CK* customer capital; *IKBS* = intangibles capitalised by firms; *IKCA* = intangibles capitalised by us. See Table 1 for a definition of the different forms of intangible capital and for total tangible capital. Intangibles capitalised by us (*IKCA*) are estimated at replacement values; intangibles capitalised by firms (*IKBS*) and tangibles (*C*) are at book values. All stocks are measured at the beginning of the year and in millions of Italian Lire at 1995 prices.

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[1] This implemented the Fourth European Commission Directive which modified some accounting standards.

[2] The reallocation procedures also take into account those legislative changes introduced since 1992, when the fourth European Commission Directive was implemented by statutory law (Legislative Decree no. 127/91). For details, see Bontempi (2005).

[3] When we use the broader definition of tangibles, estimation results do not change significantly. All non- reported results of the present paper are available upon request.

[4] Nevertheless, we also checked the robustness of the results by using the "Full sample" and by including a specific dummy indicating observations with zero intangibles, among the explanatory variables. The results were the same from the qualitative point of view.

[5] Again, results in the total sample with specific dummies - capturing null observations in the types of intangibles - as explanatory variables show robustness.

[6] These facts suggest the use of the 1st, 2nd and 3rd quartiles in computing the technical rate of substitution (TRS) in the multiplicative specification, and the elasticity of output with respect to intangible capital stock in the additive specification. See Section 3.1.

[7] Confirming that median values in the total factor productivity approaches do not bias results (see Section 3.1).

[8] We also estimated using end-of-period capital measures; results were qualitatively the same.

[9] The estimates are based on the STATA command developed in Baum, Schaffer and Stillman (2003), which constitutes a very flexible tool.

[10] We also experimented with different measures of the share of labour cost in value-added, namely the by-industry median, the by-company median, and the Törnqvist $1/2\Delta sl_{it}$. The results are robust.

[11] For tangibles: 0.289 (total), 0.126 (within), 0.148 (first-differences). For intangibles: 0.035 (total), 0.041 (within), 0.010 (first-differences). In the same paper, the estimates for France were: 0.295 (total), -0.046 (within) and -0.001 not significant (first-differences) for tangibles; 0.90 (total), 0.08 and -0.003 not significant (within and first-differences, respectively) for intangibles.

[12] For tangibles: 0.167 (total), 0.183 (within), 0.225 (first differences), 0.113 not significant (long differences). For intangibles: 0.198 (total), 0.070 (within), 0.067 and 0.077 not significant (first and long differences, respectively).

[13] A possible measurement problem arises from the changes made to accounting standards since 1992 (under the EC's Fourth Directive). However, if we limit our sample to the 1994-1999 sub-period, estimation results do not change significantly.

[14] Estimates obtained by instrumenting labour input only, do not produce significantly different estimates for both forms of capital stock. Thus measurement errors seem to affect labour to a greater degree.

[15] A highly computing-intensive check using lag 2 and higher lags shows that adding longer lags as instruments produces no efficiency gain and no significant changes in parameter estimates: these additional lags are highly correlated to those instruments already present.

[16] We also estimated various multiplicative specifications: in the non-zero sample; in the sample with null values in the intangible categories (accounted for by specific dummies); with intangible components capitalised by firms and recalculated at replacement values. Results generally display robustness.

[17] Estimates, not reported, of the productivity of total intangibles computed by excluding advertising show results qualitatively similar to the ones in Tables 7 and 8.

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[18] The reporting rule for the number of employees changes according to the accounting scheme: the number of employees at the end of the accounting year during the 1982-1991 period; the average number of workers during the accounting year since 1992, when the IV European Directive was due to be applied. Despite the fact that only 29% of all observations were made under the IV European Directive , the average number of workers is also available for the 20% of observations covering the 1982-1991 period (which more or less coincides with the number of employees reported by the firms at the end of the accounting year). Hence, we measure the number of workers as the average during the accounting year when this information is available (in 42% of all cases); otherwise, we use the number of employees reported by the firms at the end of the accounting year (in the remaining 58% of cases).

[19] All the deflators (base 1995=1) were taken from the National Accounts of the Italian Statistical Office, ISTAT. Apart from GDP deflator, they are disaggregated at 2-digit industry level.

[20] We also try the approximations $tc^{a(n)}_{it}=tc^{a(n-1)}_{it}+p(\zeta^{(n)}-\zeta^{(n-1)})p_K{}^{a(n-1)}_{it}$ and $tc^{a(n)}_{it}=tc^{a(n-1)}_{it}+p(\zeta^{(n)}-\zeta^{(n-1)})DEVp_K{}^{a(n-1)}_{it}$, where $DEVp_K{}^{a(n-1)}_{it}$ is the deviation of $p_K{}^{a(n-1)}_{it}$ from its sample median. Results do not change significantly. In the estimates reported in Tables 7 and 8, we set p = 1. Alternatively, we also set p=0.8 and p=2 without any significant changes in the convergence estimates.

[21] We also attempted the second-order Taylor-series approximation for the non-linear term of equation (2'), without any substantial changes in the results. The grid-search step for the initial values of the unknown ζ parameter is the same as above, while the iterated estimation procedure is applied to the equation $(q-l)_{it}= a_i + b_t + (\mu-1)l_{it} + \lambda(tc^{a(n)}_{it}-l_{it}) + \lambda(\zeta^{(n+1)}-\zeta^{(n)})p_K{}^{a(n)}_{it} - \tfrac{1}{2}\lambda(\zeta^{(n+1)}-\zeta^{(n)})^2(p_K{}^{a(n)})^2_{it} +\varepsilon a_{it}$, where $(p_K{}^{a(n)})^2_{it}$ is measured by the squared deviation of $p_K{}^{a(n)}_{it}$ from its sample median. If neither of the estimated parameters associated with the $p_K{}^{a(n)}$ and $(p_K{}^{a(n)})^2$ regressors are significantly different from zero, then the $\zeta^{(n+1)}$ and $\zeta^{(n)}$ estimates are not statistically different, and correspond to the unknown ζ estimate.

[22] Note that the grid plus Gauss-Newton regression approach gives the same standard errors as the iterative procedure; coefficients estimates are extremely close, differing only at the sixth decimal point.