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ABSTRACT

This paper develops a graphical analysis and an analytical model that demonstrate how weak substitution can be used for non-market valuation. Both weak complementarity and weak substitution can be evaluated as restrictions that allow quantity or quality changes in non-market goods to be described as price changes that yield equivalent changes in individual well being. They are Hicksian equivalents in that the price changes yield the same utility changes as would the quantity or quality changes. After discussion of several potential applications of weak substitution, the paper develops the parallel between the restriction and recent strategies from modeling differentiated goods.

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Can Weak Substitution be Rehabilitated?

I. Introduction

Revealed preference (RP) methods for non-market valuation generally ignore weak substitution (WS), a form of demand interdependency introduced by Feenberg and Mills [1980] nearly thirty years ago. This restriction focuses on a relationship between a private good and a non-market good or service that arises when the relative price of the private good is *high*. That is, when consumption of the private good (the *weak substitute*) reaches some level, further increases in the non-market service have no value.

Numerous RP applications assume weak complementarity (WC), an alternative form of demand interdependency. Mäler [1974] credits Stevens [1966] with the original insights that motivate his demonstration of how to recover the demand for a public good from the demand for the private good serving as the weak complement. Since this early discussion, which began with the role of water quality and the demand for water-based recreation, environmental applications have provided a number of examples where a private good can be described as a weak complement to an environmental service. The service usually plays a role that is analogous to a quality attribute of a private good.[1]

In the same time span that the relevance of weak complementarity has grown, weak substitution has languished. Evidence of this trend can be found in the revised edition of Freeman's classic treatment of non-market valuation methods. After reviewing the concept in some detail in his 1992 book, he seems to have reflected the apparent professional consensus and

[1] The logic was used (without appreciating the connection to weak complementarity) in Bresnahan and Gordon's [1997] introduction to the issues posed in modeling the demand for new goods. It is also important to the restrictions required to recover the Hicksian demand for a non-market good from the Marshallian demand for a non-market good from the Marshallian demand for the private weak complement. See Bullock and Minot [2006] and Von Haefen [2007] for discussion.

excluded weak substitution from his revised edition (see Freeman [1992, 2003]). He maintains that an assumption of less than perfect substitution is not especially informative without including a more detailed preference specification. Following Bartik [1988], Freeman argues that, at best, substitution offers upper and lower bounds for Hicksian welfare measures.

The new set of questions facing environmental and security-related policy analysts may require re-thinking this judgment. In some applications *ex ante* mitigation activities are best described as weak substitutes for non-market "bads." The first area for potential applications is motivated by the recent wildfires near San Diego, California. The fires focused attention on a variety of private services, including concierge fire protection and remotely activated fire retardant systems, that can serve as weak substitutes for public fire protection. An example of the former is AIG's Wildfire Protection Unit for specialized protection offered to homeowners with large homes (generally valued at several million dollars) who were, at the time of the fires, paying premiums of $10,000 or more a year for homeowners' insurance. The services offered were widely publicized as involving *private* fire fighting experts who were contracted to protect specific homes. Efforts to protect neighboring homes were only undertaken if the activity complemented the protection of the enrolled home.

A second class of applications involves homeland security and the precautions households can take to purchase extra food, water, and emergency supplies as well as prescription drugs as a form of mitigation to avoid serious impacts from short term interruptions in supply chains due to natural disasters or terrorist activities[2]. These resources could be

[2] Using Knowledge Network's internet panel the first author on this paper, Carol Mansfield and Aaron Strong collected information about households' stated preferences for public activities to protect the food supply. Part of the survey included information on non-prescription and prescription medicines households had on hand to deal with food borne illness that might be associated with contamination of the food supply. A simple regression of the count of six medicines or related equipment (i.e. thermometer) study participants had on hand indicated that income, awareness of the potential for food-borne illnesses, experience with cases of food poisoning, and risk attitudes were significant determinants (with p-values of 0.10 or smaller) of the count of mitigating substances households had on

described as weak substitutes for resources devoted to public planning and activities to reduce the risk of short term disruptions.

A third class of applications involves private behaviors to avert and mitigate environmental health impacts due to exposures to environmental pollutants. For example, asthma medicine may represent a weak substitute for improvements in the air quality conditions households face. Under this interpretation when the medicine is sufficiently cheap, improvements in air quality are irrelevant.[3] A second home in a pristine environment, available for short term trips, may serve a similar role.

While it is difficult to gauge the importance of each of these types of responses (and many others that could be cited as examples), it is clear these activities are an increasingly popular response when households, particularly those with relatively higher incomes, demand higher levels of amenities or protection from disamenities. As such, they can be used to measure households' demands for some types of non-market and public goods.

This paper has three objectives. First, we develop a graphical approach introduced in Smith and Banzhaf [2004] to demonstrate how discrete changes in substitution effects influence the Hicksian price equivalents defined for either WC or WS. Second, we demonstrate how weak substitution would be used to derive Hicksian consumer surplus measures. This is accomplished by adapting one of the examples that Larson [1991] used to illustrate how weak complementarity revealed sufficient information to describe the role of the non-market good in quasi-expenditure functions. Finally, we discuss the relationship between weak substitution and weak

hand. This simple reduced form model is consistent with the observable sources of heterogeneity in consumer preferences for mitigation of environmental and other public bads that the weak substitution restriction can describe.

[3] This example of course assumes that the individual or household does not derive utility directly from improved air quality but only through its impact on the severity of asthma symptoms experienced.

complementarity and the role a more general characterization of weak complementarity and weak substitution could play for non-market valuation.

II. Hicksian Equivalent Price Changes

Much of the literature describing alternative surplus measures, comparing policy instruments, and analyzing changes in multiple, interrelated, public goods is based on the early literature in the economics of rationing. It is somewhat surprising that until recently, the descriptions of the preference restrictions used to recover measures of tradeoffs people would make for changes in non-market goods did not also follow this same line of reasoning. Recently Smith and Banzhaf [2004, 2007] adapt the rationing literature to demonstrate why weak complementarity and the Willig [1978] condition are sufficient to specify a quality adjusted price index. The index defines the price adjustment equivalent to the Hicksian consumer surplus (i.e., the income adjustment) for a change in a non-market good or a quality attribute. We extend their framework to describe weak substitution in more detail.

To develop this idea graphically, consider first the Marshallian consumer surplus for a price change using indifference curves. With a decrease in the price of private good *X* from P_0 to P_1, the area P_1TSRP_0 in panel A of figure 1 represents the change in Marshallian consumer surplus (MCS) using a conventional (linear) demand function. Equation (1) provides an algebraic description of P_1TSRP_0 for a linear demand function.

$$MCS = (P_0 - P_1)X_0 + \tfrac{1}{2}(P_0 - P_1)(X_1 - X_0) \quad (1)$$

Rearranging terms, we have equation (2):

$$MCS = \tfrac{1}{2}(P_0 - P_1)(X_1 + X_0) \quad (2)$$

Panel B of figure 1 provides a representation of this relationship using indifference curves. Assume the interior budget constraint corresponds to P_0 for *X* and the exterior to P_1. The tangency at A corresponds to X_1 and at C to X_0. The Marshallian consumer surplus associated with the price change for this case of indifference curves corresponding to linear demand is equal to (1/2)($CD + AB$). To see this, note that with the numeraire good, *z*, priced at unity and income equal to m_0 the following relationships hold:

$$\begin{aligned} m_0 L &= P_0 X_0 \\ m_0 N &= P_1 X_0 \\ m_0 M &= P_1 X_1 \\ m_0 J &= P_0 X_1 \end{aligned} \tag{3}$$

Moreover, $CD = m_0 L - m_0 N$ and $AB = m_0 J - m_0 M$. Substituting from equation (3) yields the relationship given in equation (4).

$$\begin{aligned} &\tfrac{1}{2}(CD + AB) = \tfrac{1}{2}\left[(P_0 - P_1)X_0 + (P_0 - P_1)X_1\right] \\ &MCS = \tfrac{1}{2}(P_0 - P_1)(X_1 + X_0) = \tfrac{1}{2}(CD + AB) \end{aligned} \tag{4}$$

A similar logic illustrates how changes in a public good, denoted *q*, can be described using equivalent (in welfare terms) price changes. Consider first the case of weak complementarity (with *X* and *q* as weak complements) depicted in figure 2. Weak complementarity implies increases in *q* have no value to the consumer when there is zero consumption of *X*. Therefore in figure 2 the indifference curves for a given utility level, denoted *V*, with different levels of *q*, will intersect at the same point (R) on the vertical axis (i.e., where $X = 0$).

Begin at point A in figure 2 where quality is q_0, the price of good *X* is P_0, utility is *V*, and income is $m = T$. Now consider an improvement in quality from q_0 to q_1. At this new level of *q*, the indifference curve in *x*-*z* space is lower everywhere but at point *R*; in figure 2 it appears to fan inward. Quality improvements imply an inward fanning because the amounts of the weak complement (*X*) and the numeraire (*z*) required to maintain the same level of utility, *V*, decrease as q increases. The WTP for this quality change, holding both utility and price constant is given by:

$$WTP^* = e(P_0, q_0, V) - e(P_0, q_1, V).^4 \quad (5)$$

The quality change and reduction in income by WTP^* is equivalent to a parallel shift in the budget constraint from *TN* to *SO* (dashed line) in figure 2. The new point of tangency is at A', where quality is q_1, the price of good *X* is P_0, utility is *V*, and income is $S = T - WTP^*$.

Now consider the definition of a price change for *X* equivalent to the quality change in *q*. Instead of reducing the individual's income by WTP^*, increase the price of *X* sufficiently to return the individual to her original utility level, *V*, given the quality improvement. Let the new budget constraint be TP and the new point of tangency be C. The pivot from TN to TP represents this price change from P_0 to $P_1 > P_0$. The price change is defined so that individual is indifferent between getting the improvement at the original price along with the reduction in income given in equation (5) (point A'), or getting the improvement and an increase in price but without the subsequent income reduction (point C). Because the price change is constructed to exactly offset the quality change and hold utility constant, we label it as the *Hicksian equivalent price change* for the quality change. In other words, points A and C would fall along the same Hicksian demand curve for *X*, constructed for the utility level *V*.

The income adjustment and the price adjustment are equivalent in the sense that they bring the consumer to the same indifference curve. An alternative way to define equivalence

[4] The Hicksian consumer surplus measure associated with the quality change is given by the difference in the Hicksian surplus for the market good at the two quality levels:

$$\int_{P_0}^{P^C(q_1)} h(P, q_1, V)dP - \int_{P_0}^{P^C(q_0)} h(P, q_0, V)dP$$

where $P^C(\cdot)$ represents the choke price. Integration yields $e(P^C(q_1), q_1, V) - e(P_0, q_1, V) - [e(P^C(q_0), q_0, V) - e(P_0, q_0, V)]$. By the definition of weak complementarity, the first and third terms cancel and we have the expression given in equation (5).

would be the price adjustment that would "un-do" the income adjustment. To construct such a price decrease, we first decrease income by WTP^* (from TN to SO). The pivot from SO to SJ represents the price decrease that is equivalent in the sense that it brings us back to the starting indifference curve. This adjustment is equivalent to the pivot from TN to TP only if the so-called Willig condition is satisfied (Willig [1978].).

Bullock and Minot [2006] have demonstrated that the Willig condition is not needed to recover the Hicksian consumer surplus for a quality change. They extend Vartia's [1983] numerical solution to include two additional equations and use numerical integration to measure the Hicksian surplus for non-market goods. The first of these equations is the difference in the two integrals that defines the Hicksian surplus for a change in quality (or a non-market good) with weak complementarity (i.e., equation (8) in Bullock and Minot and given in our footnote #3). The second uses the envelope condition to define the relationship between Marshallian and Hicksian demand under weak complementarity. Solving them simultaneously completes the logic needed for their proposal. One could also solve numerically for the Hicksian equivalent price by recognizing that the value of the quantity change defined in terms of the change in the Hicksian price adjustment has to equal the value defined in terms of the price change. We consider how this same logic can be applied in the case of weak substitution in section IV. In the next section, we describe some examples of weak substitution.

III. Weak Substitution – Introduction and Examples

In contrast to weak complementarity, weak substitution assumes the existence of a level of consumption for the private good (the weak substitute) above which improvements in the non-market good have no value. When consumption of the weak substitute is below this threshold, denoted X_a, quality improvements are valuable. Equivalently, using the links implied by the economic descriptions of demand in the presence of quantity restrictions, quality improvements are valuable when the price of the weak substitute exceeds the price that induces the threshold consumption level, X_a. Let P_a denote the price that implies the threshold consumption level:[5]

$$-\left.\frac{V_P}{V_m}\right|_{P=P_a} = X_a . \tag{6}$$

Then weak substitution is defined in equation (7):

$$\left.\frac{\partial V}{\partial q}\right|_{P=\hat{P}} = 0 \;\; \forall \;\hat{P} \le P_a . \tag{7}$$

Figure 3 illustrates the fanning indifference curves associated with weak substitution. As in Smith and Banzhaf's [2004] proposal for illustrating weak complementarity, we define indifference curves for varying levels of *q* but the <u>same</u> utility level. The inward fanning represents the effects of quality improvements. The shape of the indifference curve to the right of X_a will vary with each application. As drawn, the indifference curve continues to slope downward for levels of the weak substitute above X_a representing the case where *X*—but not *q*—continues to have value beyond the threshold consumption level. In any case, to the right of X_a, changes in *q* do not shift the indifference curve in *x*-*z* space.

Feenberg and Mills' original example illustrating the concept of WS focused on education. They assumed that private education is preferred to public education provided the price of private school is sufficiently low. In this case (and for their story) improvements in the

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[5] P_a is a function defined by the specified level of X_a and the prices of other goods and income. We illustrate how this relationship influences the direction of the Hicksian surplus below.

quality of public education have no value to the parent who sends her children to private schools.[6]

Examples of weak substitution usually involve cases where the private good serving as a weak substitute for some non-market service mitigates low levels of the non-market service or an undesirable outcome[7]. If we consider cases in which consumption of the private good and provision of non-market services are not contemporaneous, we can introduce other potential examples. In what follows we describe three situations where weak substitution offers a basis for explaining the link between the private goods and non-market services.

A. *"Concierge" Private Protection*

The private fire protection offered to high-end property owners in Malibu, Beverly Hills, Newport Beach and Menlo Park as well as in a dozen Colorado resort communities by AIG's Wildfire Protection Unit is an example of a weak substitute for public fire protection. According to news accounts, the service, involving private firefighters, was effective in saving a number of homes in areas of Orange and San Diego Counties in California that were hardest hit by wildfires. A Los Angeles Times story on the fire protection services indicate that the AIG Private Client Group Insurance began in 2000 and has grown to nearly 1 billion dollars in gross

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[6] Actually, Feenberg and Mills did not specifically discuss the quality of public education, but this interpretation is consistent with the overall logic of their discussion.

[7] Bockstael and Mc Connell's [2007] new book on revealed preference approaches to non-market valuation uses weak substitution to introduce a discussion of how mitigating expenditures can provide an upper bound for the willingness to pay for improvements in environmental services. They are skeptical of finding plausible examples where weak substitution can be useful. For example they suggest:

"..cases in which such a threshold (as implied by weak substitution) makes sense are not easily identified. It might seem reasonable to assume that at $r=0$ (the price of the weak substitute in their notation) the condition would hold automatically, because at that point as much x (the weak substitute) as is needed can be acquired costlessly. And as long as increases in x can completely mitigate any changes in q (the non-market resource), then at $r=0$, changes in q would not matter. However at $r=0$ the household may more than mitigate the decline in q" (pp. 256-257). (parenthetical comments added). This comment misses the point. The weak substitute is not a perfect substitute for q with sufficient consumption of x. Rather it eliminates the need to consider q. The individual can choose to consume higher levels of x so long as there is a price where further changes in q no longer matter.

premiums.[8] The firm also offers a similar service for hurricanes which involves dispatching pre-disaster consultants to assess shutter protection, property storage, and landscaping. Restoration teams help with restoration and repairs after storms. This first example illustrates how heterogeneity in preferences and abilities to pay are likely to imply differences in the virtual price thresholds for a weak substitute for different income groups.

B. *Health Production*

Most discussions of mitigation rely on some specified health production function and use the relationship between measures of environmental quality and purchased inputs to derive measures of the willingness to pay for improvements in quality (see Freeman [2003] for an overview and Agee and Crocker [1996] as an example). Epidemiological research suggests high concentrations of ground level ozone can affect the frequency and severity of asthma attacks if asthmatic children are outside during these episodes (see Friedman et. al. [2001]). Under these conditions the services of a high-quality day care (or the time of one household member) supervising young children during periods of high ozone similarly could be treated as a weak substitute. To implement the model empirically, one would require a detailed record of the time profile of ozone conditions and an especially extensive record of how households with young asthmatic children respond by allocating their time or resources to assure their children avoid these exposures.[9]

C. *Disasters and Self Protection*

Durable and non-durable goods can play a mitigating role in the presence of natural and man-made disasters. Some households maintain inventories of food, water and medicines to

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[8] See Yoshimo [2007].

[9] With the collaboration of Carol Mansfield of RTI International, we investigated the feasibility of using a panel data set she had collected on ozone and children's time outdoors. However, the data included insufficient detail on prices (actual and virtual), expenditures incurred, and time use records to allow a test of this proposed example of weak substitution..

respond to short-term disasters. Others have private wells, electric generators, wood burning stoves, etc. in part to respond to short term outages in public sources for water or power.[10] Purchases of auxiliary power or water supplies may, in some situations, be treated as weak substitutes for services to respond to disruptions in public systems. In other words, if the price of household, point-of-use water treatment were low enough (including any shadow costs associated with non-price rationing), then some public water treatment services would not be missed.[11] Likewise if the price of a generator (at the extensive margin) and /or the price of fuel (at the intensive margin) were low enough, then electricity would not be missed during a power failure.

Each of these examples offers an opportunity to observe the tradeoffs a specific group of households would be willing to make for a change (or to avoid a change) in one or more non-market goods. Private expenditures (or insurance payments) are generally not available with sufficient spatial resolution to undertake a test without significant data collection. However, often analysts do not consider looking for the required supplementary data when there is not a clear theoretical basis to describe how the data would be used to estimate these incremental values. In the next section we develop the weak substitution argument graphically and illustrate how Larson's [1991] early example using weak complementarity can be adapted for application to weak substitution.

IV. Weak Substitution: Graphical and Algebraic Analysis

A. *Describing Weak Substitution with Graphs*

[10] Often these decisions serve multiple objectives. As a result, it is not possible to associate the tradeoffs they imply *exclusively* with a specific non-market good (or bad) as a substitute. However, this jointness in objectives is not always present and we raise them here as an opportunity for future research.

[11] In the north eastern corner of Maricopa County in Arizona, public water supplies are becoming more uncertain for private homeowners. Wells are also unreliable sources for water with persistent droughts. As a result a number of the homeowners have installed large private water tanks to ensure a reliable water supply independent of the decisions of water providers for the general public.

Figure 4 uses the Smith-Banzhaf format to describe how quality change can be translated into a Hicksian price change for the case of weak substitution. The two panels in figure 4 illustrate two roles for the relative prices of *X* and *z*. In panel A the budget constraint pivots at X_a. As drawn, an individual who consumes the threshold level of the weak substitute, X_a, is unable to consume *z* (e.g., $P(X_a)\cdot X_a = \text{income}$). Panel B places the *X* intercept to the right of the threshold.[12]

In both panels, the pivot in the budget constraint from the outer budget line, tangent to $V(q_0)$ at A, to the inner constraint and corresponding tangency at C, defines a change in the relative price of *z*. The budget constraint tangent at A depicts the lower price for the numeraire good *z*, denoted $P_z(q_0)$. The inner budget constraint, tangent at C, relates to the higher price, $P_z(q_1)$. By construction, points A and C will lie along the same Hicksian demand curve. In this case, the weighted average of *AB* and *CD* (measured along the X-axis) corresponds to the Hicksian surplus for the price (and quality) change as detailed in equations (8a) through (8c).

$$AB = \frac{1}{P_a}\left(P_z(q_1) - P_z(q_0)\right)\cdot z_0 \tag{8a}$$

$$CD = \frac{1}{P_a}\left(P_z(q_1) - P_z(q_0)\right)\cdot z_1 \tag{8b}$$

$$\frac{1}{2}(AB + CD) = \frac{1}{2}\left(\frac{1}{P_a}\right)\cdot\left(P_z(q_1) - P_z(q_0)\right)\cdot(z_1 + z_0) \tag{8c}$$

When P_a is normalized to unity, this relationship is comparable to our earlier description of the Hicksian willingness to pay for a quality change with weak complementarity. However, with weak substitution the most direct graphical approach to illustrate the connection to Hicksian

[12] Alternatively, one could consider a third figure where the *X* intercept was placed to the left of the threshold consumption level. As we illustrate below each characterization implies a different set of Marshallian demands for *z* that provide the basis for recovering measures of the economic value of changes in *q*.

equivalent prices uses the price of our *numeraire*. This is because weak substitution implies a discontinuity in the demand for z at the threshold level of consumption of the weak substitute. We can introduce a third indifference curve (labeled $\overline{V}(q_1)$ in panel A of figure 4), corresponding to a quality level of q_1 and a higher utility level, to define the Marshallian measure of the quality change (or price change in z) as $\frac{1}{2} \cdot P_a \cdot (CD + EF)$.

It may seem surprising that the best price to use for changes in q, when q and X are linked as weak substitutes, is the price of z (the numeraire good in our graph). However, closer inspection of the figure reveals that z, in fact, serves as a weak complement to q. That is, if we pivot panel A in figure 4 and place X on the vertical axis with z on the horizontal, then the level of z corresponding to X_a (labeled z_a in the figure) defines what Smith and Banzhaf [2004] characterize as weak complementarity "at a point." Thus, following the same logic as with weak complementarity, this interpretation provides an alternative explanation for why P_z serves as a price index for the amenity change. To put it in other terms, in the two good case, a level of X (coupled with a given income) defines a level of z. Weak substitution identifies the level of X, X_a, <u>above</u> which q has no value thus implicitly defining a level of z, $z_a = \frac{m - P_a X_a}{P_z}$, <u>below</u> which q has no value.

This suggests a dual interpretation of the WC and WS preference restrictions, which is straightforward once we realize that the properties of the indirect utility function allow another interpretation of the restrictions. $V(\cdot)$ is homogenous of degree zero in prices and income. This property implies, for our two good example, that an adjustment to the price of X is equivalent to an adjustment to the price of *z* plus an income adjustment.[13] To illustrate how this parallel relates to applications, consider a case where weak complementarity is frequently used—a discrete choice over communities.[14] Suppose in a two community world, a household is indifferent among different levels of air quality in all communities except the community in which the household resides. That is, for any given price of housing in Community 2, the household is indifferent to air quality in Community 1 unless the price in Community 1 is low enough. In this case, we can say that housing in Community 1 is a weak complement to air quality in Community 1. Alternatively, we could say that for any given price of housing in Community 1, the household is indifferent to air quality in Community 1 unless the real price in Community 2 is high enough. This example illustrates the mathematical connection, but the similarity of the two goods disguises the fact that the interpretation can be quite different. In practical applications, it will not always be possible to model *z* as a weak complement to *q*; the weak substitution relationship with a private good *X* may be more intuitive and more empirically tractable.

Figure 5 illustrates the discrete change implied by WS as a discontinuous jump in the Hicksian demands for *z*, labeled here as $D_z(\cdot)$ for utility at V, that occurs at z_a. With two market goods, quantities of *z* below z_a are equivalent to quantities of *X* above X_a, a region in which changes in *q* have no value. Hence, the Hicksian demand for *z*, at a constant level of utility, does not change over this range with changes in *q*. Weak substitution provides the information that allows the value of the quality change from q_0 to q_1 to be expressed in terms of the change in the area under this demand in a way that is comparable to weak complementarity.

[13] Our formulation actually parallels Friedman's [1949] interpretation of the Marshallian demand curve, in which money incomes are constant but the prices of all other goods are adjusted to maintain utility levels.
[14] We could have equivalently used the example of modeling the choice between different recreation sites.

In this case, weak substitution allows the analyst to define $P_z(q_1)$ and $P_z(q_0)$ and it also assures that equation (9) reduces to (10).

$$ABCD = \int_{P_z}^{P_z^C} D_z(t, P_a, q_1, V)dt - \int_{P_z}^{P_z^C} D_z(t, P_a, q_0, V)dt \tag{9}$$

$$ABCD = \int_{P_z}^{P_z(q_1)} D_z(t, P_a, q_1, V)dt - \int_{P_z}^{P_z(q_0)} D_z(t, P_a, q_0, V)dt \tag{10}$$

where P_z^c denotes the choke price for z. When $q = q_1$, the demand corresponds to the higher segment, BC, and for $q = q_0$ the lower segment AD represents demand. The integral of $D_z(\cdot)$ from $P_z(q_0)$ to P_z^c exactly offsets the integral from $P_z(q_1)$ to P_z^c implying the terms for this segment cancel and we are left with the two terms in (10).

B. *Describing Weak Substitution with an Algebraic Example*

In addition to the graphical interpretation in the last section, we can also illustrate how weak substitution works algebraically using the virtual price for the weak substitute. To do so we simply adapt the analysis Larson [1991] outlined for the case of a weak complement with a linear Marshallian demand.[15] Assume equation (11) describes the Marshallian demand for *X*, the weak substitute for *q*.

$$X = \alpha + \beta P + \gamma q + \delta m \tag{11}$$

where *P* denotes the price of the weak substitute. α, β, γ, and δ are parameters in the demand function. Using Hausman's [1981] logic we can apply Roy's identity and integrate back to obtain the quasi-expenditure function as in equation (12), with $b(q,V)$ the constant of integration.

[15] There is nothing that precludes more general cases or an analogue to the Bullock-Minot numerical approach for computing the Hicksian surplus.

$$e(P,q,b(q,V)) = b(q,V)\exp(\delta P) - \frac{1}{\delta}\left(\alpha + \beta P + \gamma q + \frac{\beta}{\delta}\right) \tag{12}$$

This model is well defined for $e_{PP} < 0$ and $X \geq 0$. Larson uses weak complementarity, together with the boundary conditions to derive an expression for $b(q,V)$. We follow his logic for the case of weak substitution with one change to simplify the algebra: we substitute $b(q,V) = \exp(a(q,V))$. Simply repeating the steps to derive the Hicksian demand from equation (12), we have equation (13) as the demand function and (14) as P_a, the price that induces the consumer to acquire X_a.

$$X = \delta\exp(a(q,V))\exp(\delta P) - \frac{\beta}{\delta} \tag{13}$$

$$P_a = -\frac{1}{\delta}a(q,V) - \frac{1}{\delta}\ln(\delta) + \frac{1}{\delta}\ln\left(X_a + \frac{\beta}{\delta}\right) \tag{14}$$

Substituting equation (14) into (12) and using weak substitution ($\frac{\partial e(P_a,\cdot,\cdot)}{\partial q} = 0$), we have a closed form expression for the constant of integration identified in equation (12). More specifically, $e(P_a)$ is given in equation (15) and $\frac{\partial e(P_a,\cdot,\cdot)}{\partial q}$ in (16)

$$e(P_a,\cdot,\cdot) = [-\ln(\delta)] + X_a + \frac{\beta}{\delta} - \frac{1}{\delta}\left\{\alpha + \beta\left[\frac{-a(q,V)}{\delta} - \frac{1}{\delta}\ln\left(X_a + \frac{\beta}{\delta}\right)\right] + \gamma q + \frac{\beta}{\delta}\right\} \tag{15}$$

$$\frac{\partial e(P_a,\cdot,\cdot)}{\partial q} = \frac{\beta}{\delta^2}\frac{\partial a(q,V)}{\partial q} - \frac{\gamma}{\delta} = 0. \tag{16}$$

Solving (16) and integrating we have in (17)

$$a(q,V^0) = \frac{\gamma\delta}{\beta}q + c(V^0) \tag{17}$$

where $c(V^0)$ denotes the constant of integration and V^0 indicates we use the initial or baseline conditions to define it. Substituting (17) into equation (12) we have the expenditure function that incorporates weak substitution for *X* at X_a.

$$e(P,q,V^0) = \exp\left[\delta P + \frac{\gamma\delta}{\beta} q + c(V^0)\right] - \frac{1}{\delta}\left(\alpha + \beta P + \gamma q + \frac{\beta}{\delta}\right) \tag{18}$$

This expression can be used to measure the Hicksian surplus for changes in *q* provided the condition defining *q* holds only for $X < X_a$. This threshold is essential because the analysis is based on the assumption that improvements in *q* have no value to an individual when his consumption of *X* exceeds X_a. As a result, equation (18) defines a conditional quasi-expenditure function that is relevant only for $P > P_a$ and $X < X_a$.

V. Summary and Extensions

The short answer to the rhetorical question in our title is a resounding "yes". Indeed, it would seem that weak substitution offers the potential for direct application in the analysis of a wide range of private mitigating activities, especially those currently undertaken primarily by high income households. A graphical interpretation of the restriction illustrates how to define and measure the Hicksian price equivalent of amenity changes under weak substitution. In addition to providing this graphical analysis, we offer some potential explanations that would motivate the use of weak substitution in applications. Both mitigation and a more general conception of the implications of different aggregation strategies for amenity services consumed over time provide prospects for applications involving weak substitution between an amenity service and one (or a composite of) private good(s).

Weak complementarity and weak substitution are preference restrictions that imply changes in substitution effects at specific threshold levels of consumption of the related private goods. Once we acknowledge the prospect for discrete changes in pure Hicksian substitution effects, there is no reason to stop with the assumption that the changes only take place once or at "corners". Instead, the possibilities are constrained only by analysts' ability to envision applications displaying the changes that are implicitly hypothesized by WC or WS.

Thus, in principle, we could have a set of discrete changes in a WC or WS relationship, provided there was a discrete set of choice alternatives available, as in our two-community choice example above. Suppose for the case of weak substitution that the threshold where the private good ceases to serve as a substitute for non-market quality changes with the level of the non-market good or even with the levels of other goods consumed. The concierge fire protection could be differentiated based on size, location or other characteristics of the structure being protected. Or, in the health context, at relatively low ambient lead exposure, a single course of chelation treatment may be sufficient to remove all the accumulated lead. After that single course of treatment, reductions in environmental lead would have no further value. At higher levels of ambient exposure, multiple courses of treatment may be required before exhausting the weak substitution relationship. Figure 6 illustrates such a case with the indifference curves associated with each level of q "spiraling" from a common envelop.[16] The relevant domain for each curve begins with its respective quality-defined threshold. It is easy to see that even in this

[16] Figure 6 is constructed using the following preference specification:,

$$U = \sqrt{X} - \overline{X} + \sqrt{(Z-c)*q} \text{ for } X^{1/2} \le \overline{X},$$

$$U = Z - c \text{ for } X^{1/2} > \overline{X},$$

and

$$\overline{X} = a - bq.$$

q both enters as a repackaging factor for Z and lowers the threshold $\overline{X}$. Figure 6 has been constructed assuming a=10, b=1, and c=10 at q={1,2,3}.

case a pivoting of the budget constraint around its X-intercept – that is, an adjustment to the price of z – can also define a compensating adjustment for the quality change. Indeed, this would be true even if q only affected the threshold point and not the marginal rate of substitution conditional on the distance from the threshold point (i.e., when q does not repackage z). The reason for this result is that, in all of these cases, the marginal rates of substitution for each fan (or spiral), fixing X, are ordered by q. When weak complementarity and weak substitution are used to motivate orderings for the marginal rates of substitution between z and X with changes in q, they can be seen as illustrations of how insights from the use of montone comparative statics can be considered for revealed preference approaches to non-market valuation.

Heterogeneity in mitigation services for natural hazards or private services to supplement public services that reduce the risks of harm due to these hazards would be consistent with these structures. They are also potentially useful in offering a specific structural restriction for alternative plans presented in a conjoint survey framework. In this context, they are usually designed to measure the economic tradeoffs people would be willing to make for different aspects of non-market services. The stated choice analysis usually formulates a set of policies that are intended to serve as substitutes for losses in environmental services or as mechanisms for restoring or enhancing non-market services. When the questions involve substitutes for non-market services then the different levels of attributes can capture how features of a plan affect the price threshold for the weak substitute.

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Figure 1. Marshallian consumer surplus for a price decrease

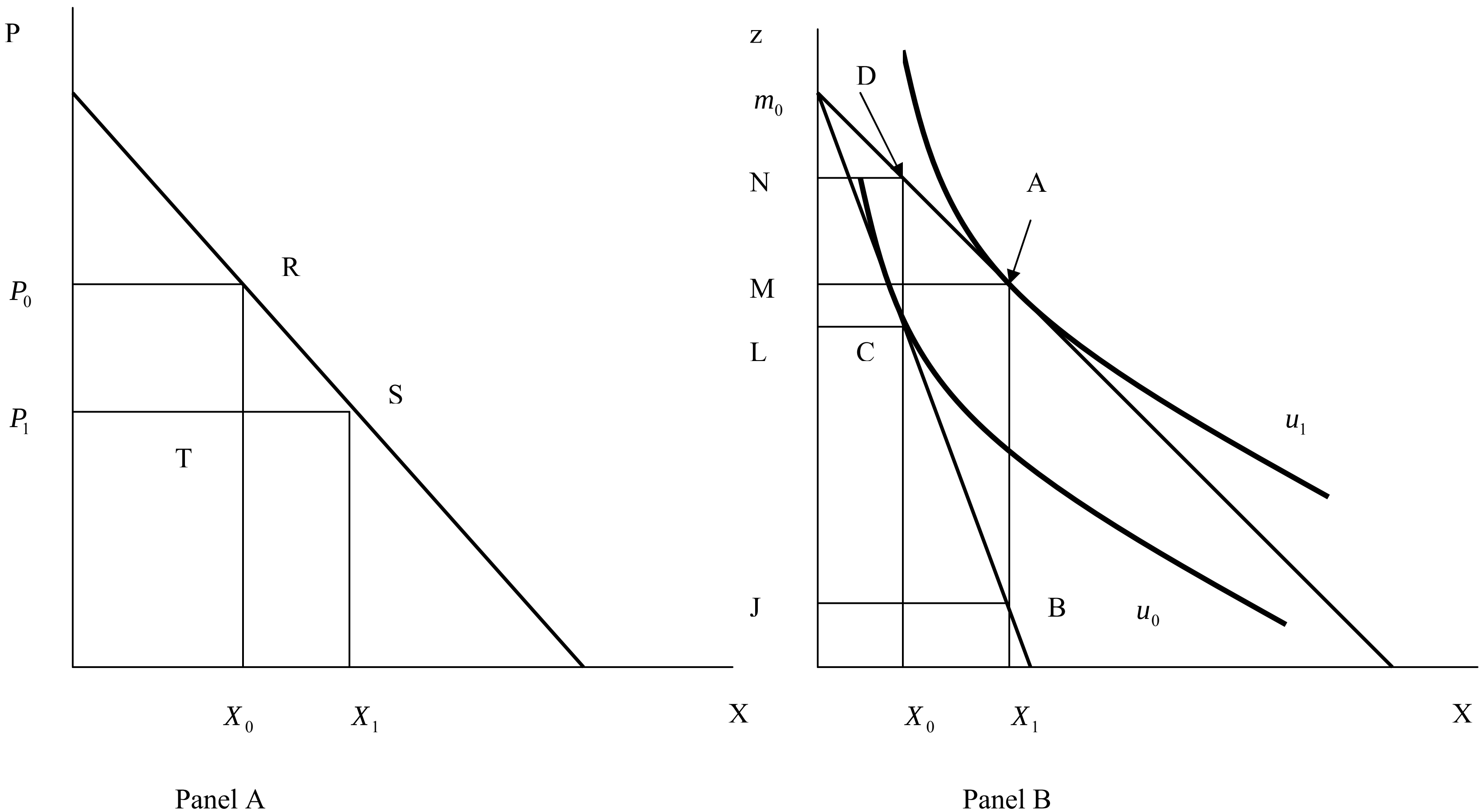

Figure 2. Deriving the price change equivalent to the quality change with weak complementarity

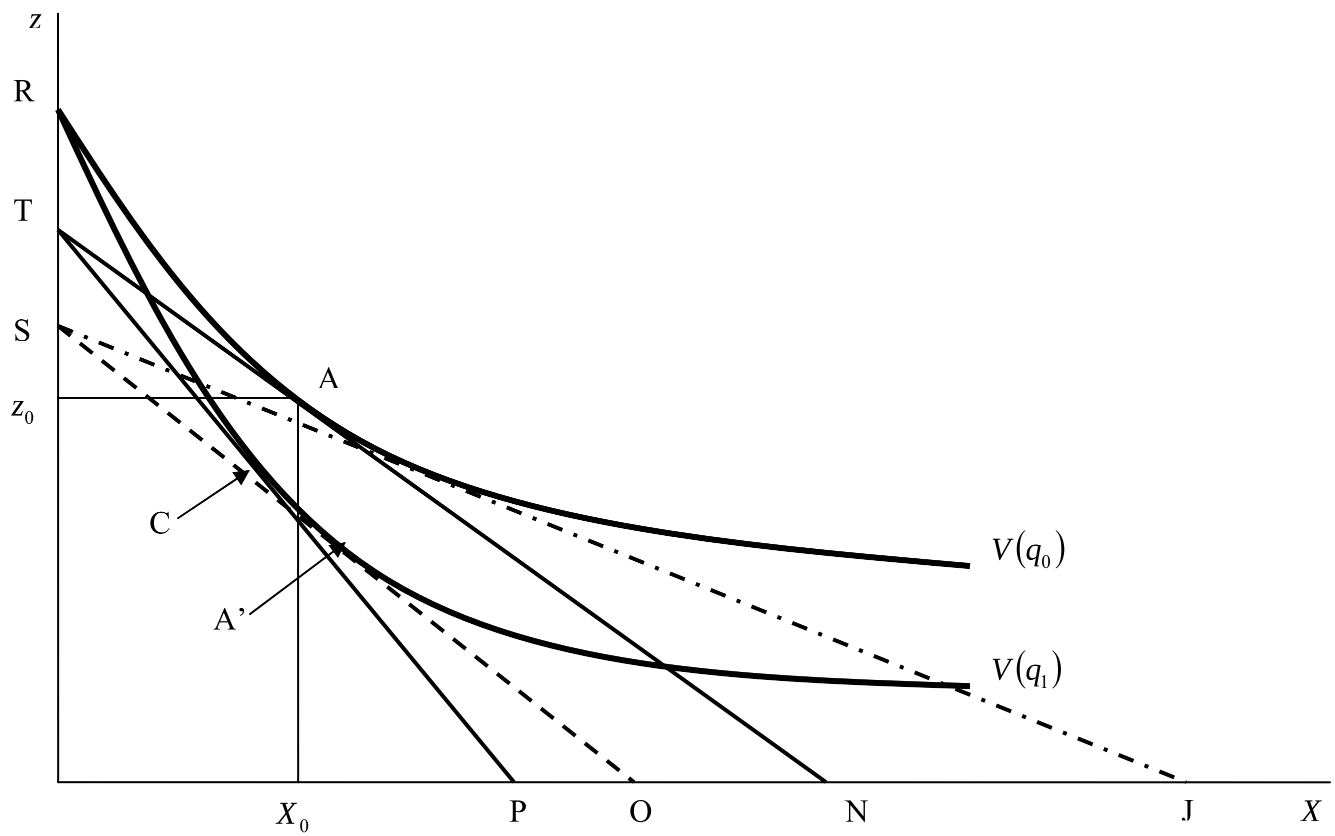

Figure 3. Fanning indifference curves with weak substitution

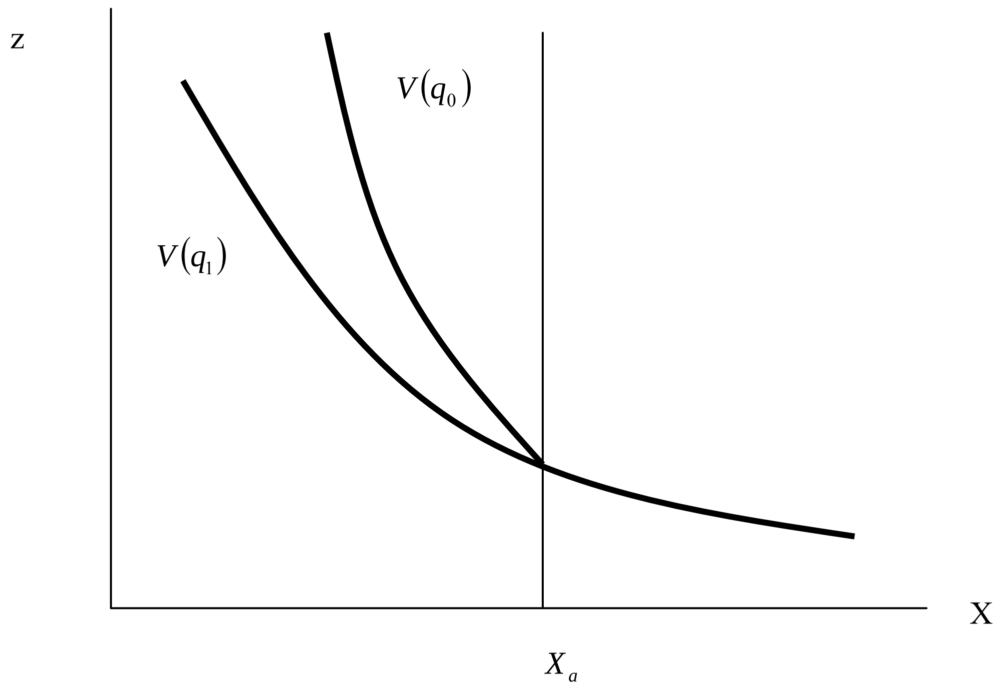

Figure 4. Weak substitution and Hicksian demand

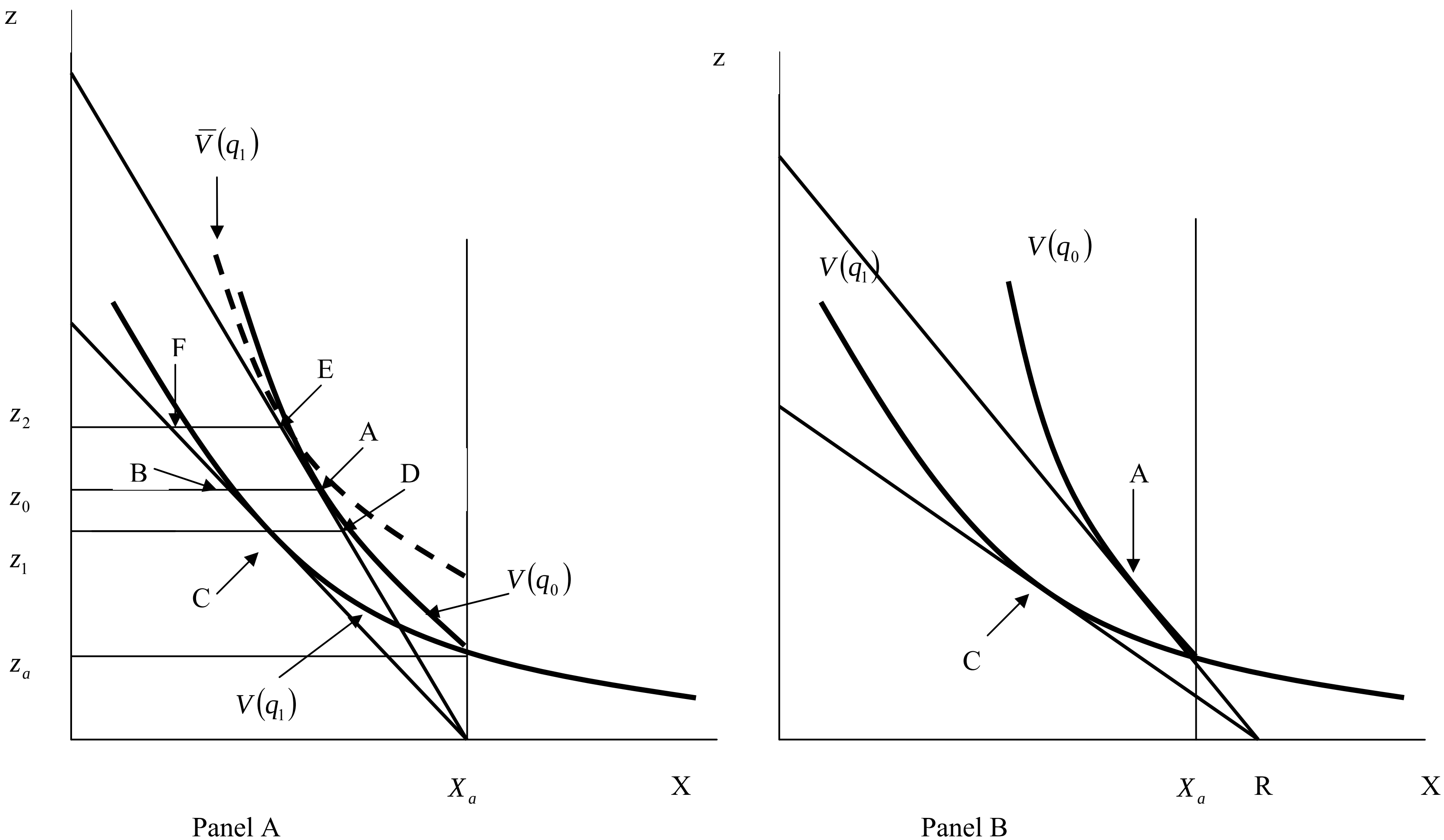

Figure 5. Hicksian demand for *z* with *q* and *X* weak substitutes

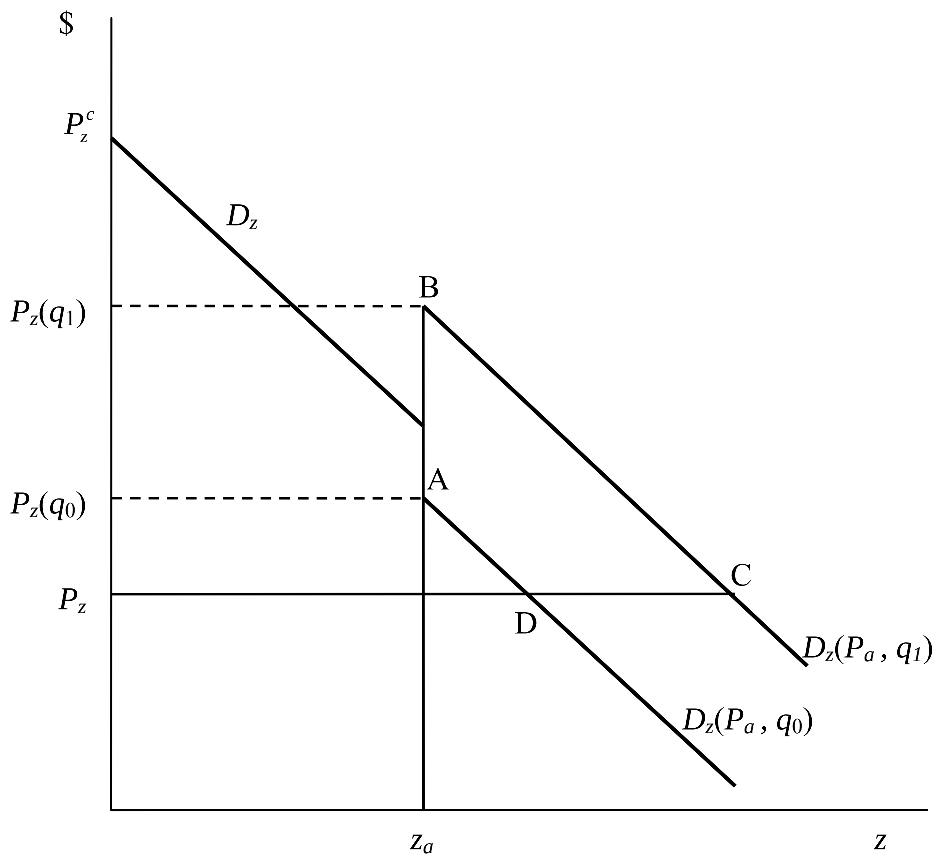

Figure 6. Cascading thresholds with weak substitution