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PENSION PLAN INTEGRATION AS  
INSURANCE AGAINST SOCIAL SECURITY RISK

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ABSTRACT

The manifest purposes of integrating an employer-provided pension plan with social security are: (1) to ensure retirement income adequacy for all covered employees; and (2) to ensure retirement income equity, defined as equal total replacement rates for all employees regardless of salary level. The focus of this paper, however, is on an equally important (and perhaps latent) consequence of integration: the alteration of the risk-bearing relationships between employees, employers and the government vis-a-vis social security benefits. The main alteration is that the employer in effect insures his covered employees against adverse changes in their social security retirement benefit. Using the option-pricing methodology of modern contingent claims analysis, we develop a formal model to explore the quantitative aspects of this change.

While the focus of the analysis is on full integration, we do explicitly deal with various degrees of partial integration as is currently practiced. We also analyze the effects of a switch from a non-integrated to an equivalent-cost integrated plan when private benefits are fixed in nominal terms and when they are indexed. In this connection we examine how integrated plans are affected when the sponsor makes ad hoc post-retirement benefit increases. We also consider the incentive effects on worker mobility of the adoption of integrated plans. The analysis is also used to highlight what we believe to be important unintended consequences of integrating pension plans with social security.

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# Pension Plan Integration As Insurance Against Social Security Risk

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## I. Introduction

According to recent surveys, more than half of private pension plans and a significant fraction of public plans in the United States today are explicitly integrated with social security.<sup>1</sup> The manifest purposes of this integration are: (1) to ensure retirement income adequacy for all covered employees; and (2) to ensure retirement income equity, defined as equal total replacement rates for all employees regardless of salary level. Integrated plans seek to achieve these goals by taking into account the amount that the retiree will be receiving from social security and then providing a benefit from the plan sufficient to produce a combined plan-plus-social security benefit that constitutes approximately the same percentage of the employee's preretirement compensation independent of his position on the pay scale.

Virtually all of the existing literature on integration and integrated plans has been concerned with the issues of adequacy and equity of integrated plans versus nonintegrated plans. The focus of this study is quite different. One of the primary side effects of plan integration is the alteration or the change in the risk-bearing relationships between employees, employers, and the government vis-a-vis social security benefits. In effect, an integrated plan causes the employer to insure his covered employees against adverse changes in

the social security benefit to which they will be entitled. Specifically, the employer provides a contingent liability against the firm in return for the claim which the employee currently has on the social security system, and thus substitutes in part the risks inherent in holding liabilities of the firm for the risks inherent in holding the claim on the social security system.

There exists in the United States today considerable uncertainty surrounding the future structure of the social security system and the level of benefits which that system will provide. The issue of social security risk and schemes for providing insurance against that risk is therefore of substantive importance from a policy perspective. Prior analyses of integration have exclusively addressed the issues of retirement-income adequacy and equity of integrated plans and thereby left the risk-sharing implications of integration as "unintended consequences" of those schemes. We therefore have chosen to focus on these risk-sharing aspects.

In two previous papers, one of us [Merton (1983a,b)] addressed the issues of retirement income risk and adequacy and the role of social security. The specific normative questions analyzed in those papers were whether social security should be a mandatory or voluntary system, how it should be funded, and what form contributions and/or benefits should take. This paper, while related to the previous ones in its general perspective and methodology, focuses on the positive questions about integration surrounding the interaction between employer-provided pensions and social security.

The plan of the paper is as follows: in section II, we briefly explain

how integration works. In section III we present a stylized formal model of an integrated plan which seeks to explore and highlight the insurance and risk-sharing aspects of integration and to determine its costs and benefits. The model uses the tools and the analytical framework of contingent claims analysis in order to quantify the tradeoffs involved. In section IV we extend the formal model in several directions in order to add greater realism. Finally, the concluding section summarizes our main results and presents our agenda for future research on the integration issue.

## II. How Integration Works

As noted, the general purpose of integration is to provide a retiree with a combined benefit that will constitute approximately the same percentage of the employee's preretirement compensation independent of his position on the pay scale. Since the social security benefit formula is highly progressive or tilted towards the lower end of the pay scale, the effect of integration is to provide a benefit from the employer which is tilted in the opposite direction. There are two main approaches that can be and are used to produce this result. One is the "offset" approach and the other the "excess" approach.

In offset plans, a portion of an individual's social security benefit is subtracted from the benefit to which he is entitled according to some defined benefit formula to determine the amount the employer will have to provide. Thus a typical defined benefit plan might provide for a benefit which is equal to 2 percent of the worker's final average salary per year of service. For a worker

with 25 years of service and a final average salary of \$24,000, this plan leads to an annual benefit of \$12,000 per year. If the social security benefit which that worker is entitled to comes to \$7,000 and if there is a full 100 percent offset under the plan, the employer would have to pay the worker only (\$12,000 - 7,000 or) \$5,000 per year. The Internal Revenue Service however does not currently permit a full 100 percent offset. The maximum allowed offset presently is 83 and one-third percent of an employee's primary insurance amount (PIA). Whatever the offset percentage is, once the benefit payable by the employer is determined, it is then frozen at that level throughout the retirement period and will not be lowered if there are subsequent increases in social security. The effect of an offset plan is illustrated in Table 1, which is taken from Schulz and Leavitt (1983).

The table illustrates the effect on total replacement rates of an integrated plan with an 83 and one-third percent offset. The last column of Table 1A illustrates the "progressivity" of the tilt associated with social security replacement rates, falling from 70 percent for the lowest paid worker to 9 percent for the highest paid. Column (5) in Table 1B illustrates the impact of the social security offset. Through the offset, the lowest paid workers in effect lose all of their private pension, while the highest paid retain almost all. The ultimate impact of integration is to make the total replacement rates shown in column (8) more equal across salary levels than they otherwise would be.

The other form of integration is the so-called excess approach. Unlike

Table 1: Effect of an Offset Plan on Replacement Rates

A. Social Security Benefits and Replacement Rates for Workers

Retiring at Age 65 in 1982<sup>a</sup>

	Average Annual Earnings <sup>b</sup>	Final Year's Earnings	Social Security Benefit	Social Security Replacement Rate <sup>c</sup>
Worker 1 <sup>d</sup>	\$ 6,000	\$ 6,599	\$ 4,611	70%
Worker 2 <sup>e</sup>	12,000	13,198	7,149	54
Worker 3 <sup>f</sup>	22,540	29,700	8,148	27
Worker 4 <sup>g</sup>	67,620	89,100	8,148	9

a. Assumed to retire at the beginning of 1982.

b. Average of highest five years of earnings, which in these hypothetical example are the last five years.

c. Benefit divided by final year's earnings.

d. Annual earnings are assumed to be \$5,429 in the fourth year before retirement. Earnings are assumed to change at a rate of 5 percent per year.

e. Annual earnings are assumed to be \$10,858 in the fourth year before retirement. Earnings are assumed to change at a rate of 5 percent per year.

f. Worker earns the taxable wage base in all years.

g. Worker earns three times the taxable wage base in all years.

Table 1: continued

B. Pension Benefits, Total Benefits and Replacement Rates for Workers in a Pension Plan with an 83 1/3 Percent Offset<sup>h</sup>

	Gross Benefit Prior to Offset	Social Security Benefit	Offset= (2) x .833	Final Pension Benefit= max[0,(1)-(3)]	Pension Replacement Rate <sup>i</sup>	Total Benefit (2)+(4)	Total Replacement Rate <sup>j</sup>
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Worker 1	\$ 3,000	\$ 4,611	\$3,841	\$ 0	0%	\$ 4,611	70%
Worker 2	6,000	7,149	5,955	45	--j	7,194	55
Worker 3	11,268	8,148	6,787	4,481	15	12,629	43
Worker 4	33,804	8,148	6,787	27,017	30	35,165	40

Source: Schulz and Leavitt (1983), p.11.

h. Calculations assume workers retire at age sixty-five in 1982. Calculations also assume that plan provides a benefit equal to 50 percent of final average earnings, minus 83 1/3 percent of social security benefits. Final average earnings represent the average of the highest five years, which in these hypothetical cases are the last five years. See table 1A for the value of the high-five average and final year's earnings in each case.

i. Benefit divided by final year's earnings.

j. Less than 1 percent.



offset plans, excess plans do not directly use social security benefits in calculating pension benefits. Instead they use social security contributions, or to be more precise the taxable wage base for social security. Plan benefits are computed and paid only on earnings in excess of an "integration level," which is directly related to the social security taxable wage base (also called "covered compensation"). Under defined benefit plans, the pension benefit accrual rate is applied only to earnings in excess of the plan integration level. In defined contribution plans, the contribution rate is applied only to earnings in excess of the integration level. In the case of step-rate excess defined contribution plans, contributions on earnings below the integration level, while not zero, are lower than they are on the earnings above the integration level. Excess plans have a similar effect to offset plans on the profile of combined replacement rates.

It should be clear that offset plans are by their nature defined benefit plans while in the case of defined contribution plans the excess approach is the only one which can be used to perform integration. In the stylized model which we present in the next section of the paper, we assume for simplicity a defined benefit plan with a 100 percent social security offset. The same mode of analysis can be applied to examine the effects of an excess plan.

### III. A Formal Model of Pension Integration

To analyze the effects of integration, we first describe the equivalent nonintegrated plan to be used as a basis for comparison. In a nonintegrated pension plan, the firm's payments to retirees are independent of the payments made by the social security system. We will denote social security payments at time  $t$  as  $S_t$ .  $B$  will denote the firm's promised payments in the nonintegrated plan, the level of which, we will assume, is currently known. Once the individual retires, the stream of total income will be  $B + S_{T+\tau}$  where  $T$  is the date of retirement and  $\tau > 0$ .

Our stylized integrated plan involves an offset provision: once social security payments exceed a stipulated minimum level, further increases in those benefits entitle the firm to reduce benefits paid via the pension fund. The offset provisions of integrated plans thus shift a portion of the risk and return of uncertain future social security payments from workers to employers.  $S_t$  evolves stochastically over time since social security benefits are linked to uncertain future wage or price levels and are subject to unforeseen legislative changes.

In practice, the offset is less than one-for-one, so that total benefits (i.e., pension plus social security), increase with the level of social security payments. For analytic simplicity, we first compare the polar cases of fully integrated plans that incorporate one-for-one offset provisions with fully nonintegrated plans. In section IV, we show how the analysis is modified to accommodate partially integrated plans.

Fully integrated plans guarantee workers a minimum combined retirement income from social security and pension payments of  $F$  dollars per period. At the date of retirement,  $T$ , if social security payments fall short of  $F$ , the employer is obliged to pay retirees  $F - S_T$  dollars in each subsequent year of retirement. Therefore, when  $S_T < F$ , every dollar increase in the initial retirement year's social security payment,  $S_T$ , reduces the employer's required payment by an equal amount. In this regime, employers capture the entire benefit of increases in social security. Once  $S_T = F$ , however, the employer's obligation is reduced to zero, so that workers capture the benefits of further increases in social security. Total retirement income at  $T$  in the integrated plan equals the maximum of the guaranteed floor or current social security benefits, i.e.,  $\text{Max}(F, S_T)$ .

An important feature of integrated plans as currently implemented is that the employer's stream of pension obligations is fixed at time  $T$ . Future increases or decreases in social security benefits which occur after commencement of the retirement period, do not induce offsetting changes in employer-provided pension payments. Thus, as with a non-integrated plan, the employee receives a fixed life annuity from his employer at retirement. Unlike the non-integrated plan, the level of the fixed annuity payments in the integrated plan,  $\text{Max}[0, F - S_T]$ , depends upon the level of the social security payment in the year of retirement,  $S_T$ . The total retirement income from social security and private pension received by the employee in year  $\tau$  of his retirement is given by  $S_{T+\tau} + \text{Max}[0, F - S_T]$ .

This institutionally-established feature of integrated plans leads to a simplification of the analysis by permitting the transformation of what would

appear to be a dynamic multiperiod problem into a one-period problem. To see this and prepare for the analysis to follow, we develop the valuation equations for future social security payments and life annuities. If we denote by  $g$  the expected (real) rate of growth of social security payments, then it follows that

$$(1) \quad E_t(S_T) = S_t e^{g(T-t)},$$

where  $E_t$  is the conditional expectation operator, conditional on information available at time  $t$ . If there were a traded financial claim which paid its owner  $\$S_T$  at time  $T$ , then its market price at time  $t$  would be

$E_t(S_T) \exp[-\alpha(T-t)]$ , where  $\alpha$  is the market equilibrium expected rate of return for a security in this risk class. It follows from (1) that the present value of the social security payment at time  $T$  can be written as

$$(2) \quad V_0 = S_0 e^{-\delta T},$$

where  $\delta \equiv \alpha - g$ .

At retirement, the present value of the worker's lifetime social security benefits can be written as

$$(3) \quad PV(S_T) = \int_0^{\infty} E_T(S_{T+t}) e^{-\alpha t} Pr(t) dt,$$

where  $Pr(t)$  is the probability that the retiree is alive  $t$  years after retiring. If the mortality table remains stable over time, then from (1) we can rewrite (3) as

$$(3') \quad PV(S_T) = S_T h(\delta),$$

where  $h(\delta)$  does not depend on  $S_T$  or time. Similarly, at retirement, the present value of a riskless life annuity of  $\$A$  per year can be written as

$$(4) \quad PV(A) = Ah(r) \quad ,$$

where  $h(\ )$  is the identical function as in (3') and  $r$  is the riskless real rate of interest.

At the employee's retirement,  $S_T$  will be known, and hence, the value of employer-provided benefits at time  $T$  can be written as

$$(5) \quad PV = \text{Max}[0, F - S_T]h(r) \quad .$$

Thus, because there are no further adjustments to these payments as the result of subsequent post-retirement changes in social security benefits, the analysis of this type of integrated plan need only focus on a single date,  $T$ . The multiple period framework required to analyze alternative versions of integration is presented in Section IV.

Armed with these basic valuation relations, we turn now to the changes in risk-bearing caused by a change from a nonintegrated to an integrated plan. From the perspective of the employer, the firm changes from a commitment to pay  $\$B$  a year during the retirement period to a commitment to pay  $\$ \text{Max}[0, F - S_T]$ . When the worker retires, the firm knows precisely what the level of annuity payments will be in either plan. At that time, from (4), the value of the liability is  $Bh(r)$  for the nonintegrated plan and  $\text{Max}[0, F - S_T]h(r)$  for the integrated plan. However, when viewed from dates earlier than  $T$ , the level of annuity payments for the integrated plan is uncertain because  $S_T$  is unknown. A convenient interpretation of the provisions of the integrated plan can be used

to determine the value of the firm's pension liability prior to the worker's retirement. The structure of the contingent liability payment,  $\text{Max}[0, F - S_T]$ , is formally equivalent to a European put option of maturity date  $T$  with an exercise price  $F$  on a stock with a price at time  $T$  given by  $S_T$ . This equivalence permits the use of established results from the put option pricing literature to value the obligations of the employer under the provisions of the integrated plan.<sup>2</sup>

The employer's major policy variable under an integrated plan is the level of guaranteed combined retirement income,  $F$ . To focus on the risk-sharing aspects of integration, we impose the constraint that the present value or cost of (contingent) employer payments over the life of any retiree be equal for integrated and comparable nonintegrated plans. That is, the present value or cost of the two plans is the same.

In the nonintegrated plan, the worker receives from the firm a stream of payments in retirement of  $B$  per year. From (4), the present value of this liability to the firm at time  $T$  is  $Bh(r)$ . If today's calendar date is normalized to zero and we neglect pre-retirement mortality, then the current value of this liability is  $Bh(r)e^{-rT}$ .

In an integrated plan, the worker receives from the firm a stream of payments of  $\text{Max}[0, F - S_T]$  per year and the corresponding present value of this liability to the firm at time  $T$  is  $\text{Max}[0, F - S_T]h(r)$ . Neglecting pre-retirement mortality, the current value of this liability is  $P(F, S_0, T)h(r)$  where  $P$  denotes the current (time 0) value of a European put option that gives its "owner" (the employee) the right to sell the social security payment at  $T$  for  $F$ , when the

social security benefit level is currently at  $S_0$ .

Under the hypothesized condition that the current value of the pension cost to the employer is the same for the integrated and nonintegrated plans, it follows that  $F$  must be chosen so that

$$(6) \quad P(F, S_0, T) = Be^{-rT}.$$

Given a valuation formula for the put, (6) can be used to solve for the level of the floor on combined retirement income,  $F$ , that equates the present value of the firm's obligations in the integrated and nonintegrated plans.

From the viewpoint of the employee, the effect on risk bearing of changing from a nonintegrated to an integrated pension plan is to provide the employee with an implicit insurance scheme. To see this, we compare the value of the worker's combined social security and private pension benefits at retirement for the nonintegrated plan to the corresponding value at retirement for the integrated plan. From (3') and (4), the value at time  $T$  under the nonintegrated plan can be written as

$$(7) \quad S_T h(\delta) + Bh(r) = [h(\delta) - h(r)]S_T + h(r)\{S_T + B\}.$$

Similarly, the value at time  $T$  under the integrated plan can be written as

$$(8) \quad S_T h(\delta) + \text{Max}\{0, F - S_T\}h(r) = [h(\delta) - h(r)]S_T + h(r)\{S_T + \text{Max}\{0, F - S_T\}\} \\ = [h(\delta) - h(r)]S_T + h(r)\{\text{Max}(F, S_T)\}.$$

By inspection of (7) and (8), the difference in benefits to the employee between the two plans is the difference in the terms in curly brackets. For the

integrated plan, the worker receives the social security payment of  $S_T$ , plus contingent lifetime annuity payments from the firm equal to the shortfall, if any, between  $S_T$  and the guaranteed combined income,  $F$ . The worker, therefore, receives insurance (the put option) from the employer against low levels of the social security benefit. If  $S_T$  is below the "insured value,"  $F$ , the employer-provided insurance policy pays off and makes up the difference.<sup>3</sup>

As is evident from (7), the nonintegrated plan also provides a "floor" on combined retirement income: namely,  $B$ . However, if the floor  $F$  in the integrated plan is chosen so as to satisfy (6), then it is straightforward to show  $F > B$  whenever  $B > 0$  and  $S_0 > 0$ . Moreover, in practical cases,  $F \gg B$ . That is, the combined minimum guaranteed level of benefits in the integrated plan will be much higher than in the nonintegrated plan. By more formal measures of risk such as the variance of the employee's retirement benefit, it is straightforward to show that  $\text{Var}(S_T + B) > \text{Var}(\text{Max}[F, S_T])$ . Thus, it is appropriate to characterize the change from a nonintegrated to an integrated plan as providing the employee with insurance and reducing the uncertainty about his combined retirement income.

The insurance provided by integration does not come to the employee for "free." The price paid is that the employee gives up his nonintegrated plan claim of  $B$  in return for the integrated plan's insurance on the value of  $S_T$ . By inspection of (7) and (8), the employee will, ex post, be worse off in an integrated plan if  $S_T > F - B$ . Thus, it cannot be claimed, as a normative matter, that all risk-averse employees would prefer an integrated plan over a comparable-in-value nonintegrated plan. From (6) and the well-known put option



price property that  $0 \leq \partial P / \partial F \leq e^{-rT}$ , it does follow, however, that  $d(F-B)/dB > 0$ . Hence, for a fixed probability distribution for  $S_T$ , the larger is  $B$ , the smaller is the probability that the worker will experience (ex post) regret for having chosen an integrated plan over a nonintegrated one.

To obtain solutions for  $F$  in (6) that are amenable to comparative-static analysis, we continue the examination of the properties of integrated plans under the simplifying assumption that  $S_t$  follows a geometric Brownian motion.<sup>4</sup> That is, the dynamics of  $S_t$  are assumed to be described by the stochastic differential equation

$$(9) \quad dS = gSdt + \sigma S dz \quad ,$$

where, as previously defined,  $g$  is the expected rate of growth of social security payments;  $\sigma^2$  is the instantaneous constant variance rate for the percentage change in  $S$ ; and  $dz$  denotes a Wiener process.

From (2) and (9), arguments along the lines presented in Constantinides (1978) can be used to show that the put option price can be expressed as

$$(10) \quad P(F, S_0, T) = Fe^{-rT} [1 - N(d_2)] - S_0 e^{-\delta T} [1 - N(d_1)]$$

where

$$d_1 = \frac{\ln(S_0/F) + (r - \delta + 1/2 \sigma^2)T}{\sigma \sqrt{T}}$$

$$d_2 = d_1 - \sigma \sqrt{T}$$

$N(\cdot)$  is the cumulative standard-normal distribution function.

Equation (10) is formally equivalent to the well-known Black-Scholes (1973) put option formula on a dividend-paying stock. The "dividend adjustment,"  $\delta$ , reflects the difference in the expected rate of "capital gains" on  $S$ ,  $g$ , and the total required rate of return,  $\alpha$ , given its risk characteristics. Some relevant comparative-static properties of  $P(F, S_0, T)$  are presented in table 2. Equation (10) can be used to determine the floor levels,  $F$ , that equate  $P(F, S_0, T)$  and  $Be^{-rT}$ .

There is considerable controversy over the issue of whether benefits accruing under a defined benefit plan ought to be viewed as fixed in real or nominal terms.<sup>5</sup> While this controversy has potentially significant implications for the magnitude of the effects we are examining, it is essentially unrelated to our main thrust. However, with this controversy in mind, we do present tables of analysis which reflect the two polar extremes: (1) the case in which employer-provided benefits are fixed in real terms (i.e., indexed to the price level), and (2) the case in which they are fixed in nominal terms, as argued by Bulow (1982). By analyzing the extremes we are in essence covering all the cases in between as well.

Table 3 presents floor levels corresponding to several possible combinations of social security and nonintegrated benefit levels. The table presents results for case (1), in which both employer-provided and social security benefits are interpreted as real obligations. Column (1) of Table 3 contains hypothetical employer-provided benefits of various amounts. Column (2) of Table 3 contains the expected real social security benefit, which we fix at

Table 2:

Change in Floor Income of Integrated Plan in Response to  
Increase in Various Parameters

<u>Variable Increasing</u>	<u>Response of Floor Income</u>
$S_0$	Increase
$\sigma$	Decrease
T	Indeterminate

Table 3: Integrated Floor-Benefit Levels  
(Real Contracting)

(1) Employer- Provided Pension Nonintegrated Benefit	(2) Social Security Benefit	(3) Total Nonintegrated Benefit [(1) + (2)]	(4) Floor Benefit for Corresponding Integrated Plan		
			$\sigma = .01$	$\sigma = .025$	$\sigma = .05$
A. Time to retirement 15 years.					
\$ 0	\$10,000	\$10,000	\$ 0	\$ 0	\$ 0
100	10,000	10,100	9,880	9,205	7,955
500	10,000	10,500	10,480	10,205	9,435
1,000	10,000	11,000	11,000*	10,900	10,400
5,000	10,000	15,000	15,000*	15,000*	14,990
10,000	10,000	20,000	20,000*	20,000*	20,000*
B. Time to retirement 25 years.					
0	10,000	10,000	0	0	0
100	10,000	10,100	9,765	8,840	7,260
500	10,000	10,500	10,450	10,005	8,930
1,000	10,000	11,000	11,000*	10,785	10,015
5,000	10,000	15,000	15,000*	15,000*	14,935
10,000	10,000	20,000	20,000*	20,000*	20,000*
C. Time to retirement 35 years.					
0	10,000	10,000	0	0	0
100	10,000	10,100	9,660	8,545	6,725
500	10,000	10,500	10,415	9,825	8,515
1,000	10,000	11,000	10,990	10,670	9,675
5,000	10,000	15,000	15,000*	15,000*	14,855
10,000	10,000	20,000	20,000*	20,000*	19,990

\* In these cases the value to the employee of receiving social security payments in excess of the floor, while always positive, has a present value of less than \$5.

\$10,000. Therefore, the different rows of Table 3 may be interpreted as corresponding to different scenarios in which private (nonintegrated) pension plan benefits as a fraction of social security benefits differ widely. These comparisons are of interest because (as demonstrated in Table 1), employer-provided pension payments for low-income individuals are small relative to social security, while for high-income individuals, private pension benefits exceed social security, at least under the assumption that they are real.

The third column of Table (3) is simply the sum of private plus expected social security benefits in the nonintegrated plan. This value is a useful benchmark against which to compare the guaranteed floor benefit of the integrated plan. Under certainty ( $\sigma = 0$ ), and with no expected real growth in social security benefits,  $\alpha = r = \delta$ , and the guaranteed floor would be exactly  $B + S_0$ , which is in fact column (3). Of course,  $S_T$  is uncertain; hence, with  $\delta = r$ , column (3) is interpreted as the expected level of total combined benefits in the nonintegrated plan.<sup>6</sup> Column (3) and columns (4)-(6) compare the guaranteed minimum incomes in the integrated plan with this combined expected benefit from the nonintegrated plan.

Columns (4)-(6) are the minimum real income levels that the employer would provide in an integrated plan with the same present value as the nonintegrated plan, computed using standard deviations for the real percentage change in  $S_t$  of 1, 2.5, and 5 percent per year. Panel A of the table uses a time to retirement of 15 years, while panels B and C use 25 and 35 years respectively.

To facilitate the comparison of integrated and nonintegrated benefits,

note that the annuity levels in column (3) are equal to the guaranteed annuity the employer would provide if the employee would assign all his rights to future social security benefits to the employer. That is, an extreme form of risk shifting would be that the worker transfers all of his social security benefits to the employer in return for a guaranteed annuity. This sale causes the employer to bear all social security risk and to receive all of its benefits.

What level annuity would the employer offer in return for the social security benefit? From (2) and (3'), the current value of the employee's stream of social security benefits is given by  $S_0 h(\delta) e^{-\delta T}$ . From (4), the current value of a life annuity of \$A beginning at time T is  $Ah(r)e^{-rT}$ . Under the assumed condition of Table 3 that  $\delta = r$ , it follows, therefore, that the level of annuity payments, A, which the firm would exchange in return for the employee's social security benefits is given by  $A = S_0$ . Thus, under the posited conditions, the number reported in column (3),  $B + S_0$ , is the guaranteed annuity level associated with the market value of the combined benefits in the nonintegrated plan.

In actual integrated plans, of course, the worker does not transfer all rights to social security benefits: if  $S_T$  exceeds F, the worker collects the additional amount  $S_T - F$ . In effect, the worker retains rights to the upper tail of the social security distribution. Whereas the worker would receive a guaranteed annuity level of payments  $F' \equiv B + S_0$  in the hypothetical extreme case in which social security benefits are actually sold to the employer, in the integrated plan, the worker receives  $F + \max(0, S_T - F)$  as his annuity at

retirement. Thus, unlike the hypothetical sale in which the employer receives  $S_T+B$  in exchange for the guaranteed floor,  $F$ , the employer actually receives  $\text{Min}(F, S_T+B)$ , which is always less than or equal to  $S_T+B$ . For the nonintegrated and corresponding integrated plans in Table 3 to have equal present value of costs it must therefore be the case that the floor promised under the integrated plan not exceed the guaranteed annuity in the case of an outright sale, i.e.,  $F \leq F' = S_0+B$ . Thus column (3) provides an upper bound on the guaranteed benefit levels in columns (4) - (6). If there is no chance that  $S_T$  will exceed  $S_0+B$  then  $F=S_0+B$ , otherwise  $F$  will be less than  $S_0+B$ .

As Table 3 demonstrates, individuals who would receive small private pension benefits relative to social security in nonintegrated plans, will be offered a guaranteed combined benefit that is significantly less than the current combined benefit. This effect is more pronounced for large uncertainty rates (high  $\sigma$ ) and for longer times to retirement. At the limit of zero private pension benefits, the floor integrated replacement benefit is zero. In this extreme case, the employer has no obligations in the nonintegrated scenario and thus the value of the insurance (the put) provided by the employer must also be zero. The floor benefit guarantee with equivalent present value in the integrated plan is zero, and the employer provides no insurance against declines in social security benefit levels. As employer-provided nonintegrated benefits increase, the corresponding floor benefit level rises. For private nonintegrated pension benefit levels of \$100, the employer offers a floor level that is significantly below the current (and expected future) level for social

security of \$10,000. The \$100 nonintegrated benefit given up by the employee to the employer can buy only "disaster" insurance which will pay off only if social security falls significantly below its current level.

For higher employer-provided pension levels, the minimum benefit guarantee rises and indeed can exceed the current level of social security of \$10,000. For the highest employer-provided nonintegrated benefit considered in Table 3 (\$10,000), the employer offers a corresponding benefit floor in the integrated plan of \$20,000.<sup>7</sup> Under the posited dynamic process for social security, there is virtually no chance that  $S_T$  will exceed the \$20,000 floor. Thus, almost surely the employer will end up paying at T the floor benefit equal to \$20,000 and will receive the social security benefit,  $S_T$ . In effect, the employer has purchased the employee's social security benefit.

The differences between the combined nonintegrated benefit levels and the floor income thus have a straightforward interpretation. For large floors, say greater than twice  $S_0$ , the social security benefit level must double in real terms before the employer fails to capture all the benefits from social security. Thus, the employer will almost certainly end up receiving the employee's social security benefit. In this regime, the employee has simply sold his rights to social security to the employer, who will pay  $F - S_T$  in pension benefits at time T. In order to provide the employee with an integrated benefit level equal to the obligation B in the nonintegrated plan, the floor level must approximately satisfy  $F - S_0 = B$ , or  $F = B + S_0$ . Therefore, the benefit guarantees in columns (4)-(6) approach the values in column (3). As B



declines relative to  $S_0$  there is a significant chance that  $S_T$  will be less than  $S_0+B$  and therefore as we have seen, the guaranteed minimum benefit,  $F$ , must be strictly less than  $S_0+B$ .

All of these conclusions assumed that employer-provided benefits are fixed in real terms. Table 4 provides the same analysis as in Table 3, but computed under the assumption suggested by Bulow (1982) that promised employer-provided benefits are fixed in nominal terms. Thus, for the same level of nominal benefits,  $B$ , the real level of benefits must be deflated by the rate of inflation. An inflation rate of 6 percent is assumed in Table 4.

Columns (1) and (2) give the nominal and associated real employer-provided benefit levels corresponding to column (1) of Table 3. For the same nominal benefits, the real benefit levels will, of course, fall as one considers longer times to retirement. Column (3) of Table 4 presents the sum of the \$10,000 real social security benefit plus the real employer-provided benefit, while column (4) presents real floor benefits for the nonintegrated plan. (Our analysis ignores price level risk; hence, the only source of uncertainty is social security risk.)

The floor benefit levels in table 4 are, as expected, lower than those in Table 3. This pattern results from the decreased real value of employer-provided benefits when those benefits are nominally fixed. As is perhaps not surprising, the difference in floor levels is most pronounced for high values of  $\sigma$  and for panel C, in which time to retirement equals 35 years. In these cases, the floor benefits range from approximately 50 to 85 percent of

Table 4: Integrated Floor Levels

(Nominal Contracting)

(1) Employer- Provided Nominal Benefit	(2) Employer- Provided Real Benefit**	(3) Total Real Nonintegrated Benefit	(4) Real Floor Benefit for Corresponding Integrated Plan		
			$\sigma=.01$	$\sigma=.025$	$\sigma=.05$
A. Time to retirement 15 years.					
\$ 0	0	10,000	0	0	0
100	41	10,041	9,671	8,811	7,371
500	203	10,203	10,092	9,577	8,517
1,000	473	10,407	10,374	10,040	9,200
5,000	2,033	12,033	12,033*	12,026	11,806
10,000	4,066	14,066	14,066*	14,066*	14,032
B. Time to retirement 25 years.					
0	0	10,000	0	0	0
100	22	10,022	9,374	8,124	6,239
500	112	10,112	9,799	8,904	7,349
1,000	223	10,223	10,049	9,344	7,989
5,000	1,116	11,116	11,113	10,938	10,218
10,000	2,231	12,231	12,231*	12,202	11,827
C. Time to retirement 35 years.					
0	0	10,000	0	0	0
100	12	10,012	9,086	7,491	5,306
500	61	10,061	9,501	8,257	6,327
1,000	122	10,122	9,734	8,674	6,905
5,000	612	10,612	10,533	10,044	8,819
10,000	1,225	11,225	11,225*	10,978	10,089

## Notes:

\* Present value of social security payments in excess of the floor is less than \$5.

\*\* Nonstochastic inflation rate of 6 percent used to deflate nominal quantities.

their corresponding values in Table 3, in which the employer-provided pension benefit is fixed in real terms.

To perhaps provide further intuition for the comparative statics results presented in Tables 3 and 4, we note that the key expression in curly brackets in (8),  $\text{Max}(F, S_T)$ , can be rewritten as  $F + \text{Max}[0, S_T - F]$ .  $\text{Max}[0, S_T - F]$  is the functional form of the payoff to a call option of maturity date  $T$  with an exercise price of  $F$  on a security whose price at time  $T$  is given by  $S_T$ . In this formulation, the employee's claim in the integrated plan is equivalent to a risk-free payment of  $F$  plus an implicit call option to buy back from the employer the social security benefit at time  $T$  for exercise price  $F$ . For large floor levels relative to the expected level of social security benefits, the employee's call will be significantly out of the money, and  $F$  must be near  $F'$ ; since the call is unlikely to be exercised, the floor benefit must approach the combined nonintegrated benefit.

#### IV. Extensions of the Model

In the previous section we used contingent-claims analysis to value guaranteed replacement rates in a simple one-period model induced by the current institutional form of integrated plans. The contingent-claims approach, and the insights it yields are quite flexible, however, and are easily extended to handle both more realistic models of the current system as well as alternative types of integrated plans. In this section we illustrate that flexibility with a few important extensions to the basic model.

As was described in the Introduction, the current practice for

integrated plans is to provide only a partial offset for social security payments with a maximum of an 83 1/3 percent offset. It is, however, straightforward to modify the 100 percent offset model of the previous section to accommodate this partial offset feature. If  $\gamma$  denotes the fraction of offset provided by a specific plan, then the level of life annuity payments provided by the employer is given by  $\text{Max}\{0, F - \gamma S_T\}$ . Thus, as with the full-offset plan, the structure of the firm's liability in a partial-offset plan is equivalent to a put option. Therefore, the same formal analysis which led to the determination of the minimum guaranteed combined income,  $F$ , for the full offset plan can be applied to determine the floor for the partial offset one. If  $F(\gamma)$  denotes the floor for a plan with a  $\gamma$  offset, then from (6),  $F(\gamma)$ , will satisfy

$$(11) \quad P(F(\gamma), \gamma S_0, T) = Be^{-rT} .$$

Because the value of a put option is an increasing function of its exercise price and a decreasing function of the price of its underlying security, it follows from (11) that  $dF(\gamma)/d\gamma > 0$ . Therefore, a partial offset plan ( $\gamma < 1$ ) will have a lower guaranteed retirement income level,  $F(\gamma)$ , than a full offset plan ( $\gamma = 1$ ). A general property of put option prices is that they are first-degree homogeneous in these two variables. That is,  $P(F(\gamma), \gamma S_0, T) = \gamma P(F(\gamma)/\gamma, S_0, T)$ . It follows from (11) that the value of the put in all comparable integrated plans must equal  $Be^{-rT}$ ; therefore  $\gamma P(\frac{F(\gamma)}{\gamma}, S_0, T) = Be^{-rT} = P(F(1), S_0, T)$ . Because the value of a put is an increasing function of its exercise price, for  $\gamma < 1$  this equality can be maintained only if  $\frac{F(\gamma)}{\gamma} \geq F(1)$ , or  $F(\gamma) \geq \gamma F(1)$ . Hence, although the partial offset plan has a lower income floor than a full-offset plan, it is less than proportionately lower.

In summary, we can bound the guaranteed retirement income in a partial offset plan in terms of the floor level in a corresponding full offset plan by

$$(12) \quad \gamma F(1) \leq F(\gamma) \leq F(1) \quad , \quad \gamma \leq 1 \quad .$$

In the previous section, we also assumed that the employer-provided benefit is riskless and that the only source of uncertainty is the level of social security payments received in retirement. A more realistic model would take into account that the employer-provided benefit (in either the nonintegrated or integrated plan) is also uncertain. However, because the payoff structure to the employee in an integrated plan is still given by  $\text{Max}[F, S_T]$ , the same basic methodology of Section III can be used to extend the model to this more general case. Fischer (1978) has derived a valuation formula for the price of a contingent claim whose terminal value is  $\text{Max}[F, S_T]$  when both  $F$  and  $S_T$  are stochastic. Hence, by replacing  $P(F, S_0, T)$  in equation (6) by this more general valuation formula and reinterpreting  $Be^{-rT}$  in (6) as the present value of the uncertain benefit provided in the corresponding nonintegrated plan, one could proceed to analyze the impact on risk-bearing of integration when both private and social security benefits are uncertain.

As a third illustration of the flexibility of the approach presented here, consider the case of an integrated plan in which the employer-provided benefit is not fixed after retirement but is adjusted each period to reflect post-retirement changes in social security benefits. Despite the fact that integrated plans in the U.S. do not currently work this way, this case is of

interest for at least two reasons.

First, many employers do provide post-retirement benefit increases even though they are not contractually bound to do so. These increases are typically made on an ad hoc basis, and employers explain their rationale as stemming from a concern for maintaining a floor beneath the retirement income of their former employees. Indeed, some researchers view these ad hoc increases as part of an implicit contract between employer and employees. Given their expressed purpose, there can be little doubt that the magnitude and frequency of these ad hoc increases depend on the magnitude and frequency of changes in social security benefits. The second reason for examining this case is that while formal integration may not work this way right now, it is possible that it might at some point in the future or in some other national setting. This is especially relevant since the normative implications of integrated plans have not yet received a full review.

In this version of an integrated plan, the firm's obligation at each date  $t$  during the retirement period equals  $\text{Max}(0, F - S_t)$  so that the present value of contingent payments as of time 0 equals

$$(13) \quad \int_T^{\infty} \text{Pr}(t) P(F, S_0, t) dt$$

Given mortality tables for  $\text{Pr}(t)$ , and a formula for  $P$ , we can compute the level of  $F$  by equating the value in (13) to  $Bh(r)e^{-rT}$  in an analogous fashion to (6) in the previous section. By way of example, however, we compute the firm's reservation level for  $F$ , given  $B$ , for a particularly simple pattern for  $\text{Pr}(t)$ . Suppose, for example, as described in Merton (1983b), that the

probability of dying at  $t$  is determined by a Poisson-distributed random variable with characteristic parameter  $\lambda$ . Under this assumption,  $\Pr(t) = \lambda e^{-\lambda t}$ ; the expected time until death is  $1/\lambda$  and  $h(r) = 1/(r + \lambda)$ . Expression (13) can be written as

$$(14) \quad \int_T^{\infty} \{F e^{-(r + \lambda)t} [1 - N(d_2)] - S_0 e^{-(\delta + \lambda)t} [1 - N(d_1)]\} dt$$

where  $d_1$  and  $d_2$  were defined in (10). The integral can be approximated numerically by setting the upper limit of integration equal to a large positive value. One then can search over  $F$  for the benefit-floor guarantee that equates (14) to  $e^{-rT} B / (r + \lambda)$ .

Guaranteed combined benefit rates corresponding to combined income rates in the nonintegrated plan were computed using (14). For values of  $\lambda = .0667$ , and  $\sigma = .025$ , and times to retirement of 15 and 35 years, we found that benefit floors were virtually identical to those in Table 3.9

Another issue surrounding integrated plans that requires further study and clarification is the procedure for aggregating the worker's total private pension benefits when he has worked for more than one employer. For nonintegrated plans, the worker's total private pension annuity benefit,  $B'$ , is the sum of the annuity benefits earned from all plans,  $\sum_1^n B_i$ , where  $B_i$  is the annuity benefit from employer  $i$ ,  $i = 1, \dots, n$ . As noted in the Introduction, the typical nonintegrated plan determines the retirement benefit in terms of the number of years of service to the firm and some type of average salary during

that service. Hence, as has been widely discussed in the pension literature, for the same wage profile, the total private retirement benefit received by a worker who participates in more than one nonintegrated plan will in general be different than if he had participated in only one plan for his entire work life.

With integrated plans, the issue of aggregating benefits is considerably more complex. In addition to the effect on the level of benefits found in nonintegrated plans, the same aggregation procedure when applied to integrated plans has a substantial impact on the risk characteristics of the worker's total retirement income.

To illustrate this point, consider two workers both who earn the same constant wage throughout their work life. Worker 1 has a single employer and worker 2 works an equal number of years for each of  $n$  firms. Under these specialized conditions, worker 1 and worker 2 would have the same total retirement income if the plans were nonintegrated. That is, worker 1 would receive  $B'$  and worker 2 would receive  $B_i = B'/n$  from each firm  $i$ ,  $i = 1, \dots, n$ . If however, each of the firms' plans is integrated with social security, then the private pension benefits to the two workers will be quite different.

Worker 1 with a single lifetime employer fits the assumed conditions of our model in Section III. His private pension annuity is given by  $\text{Max}[0, F - S_T]$  where  $F$  is determined from the solution of equation (6) with  $B = B'$ . This implicit put option insures him against low levels of social security payments by compensating him dollar for dollar for payments below  $F$ . Hence, he has a total retirement income floor of  $F$ . If the minimum guaranteed income floor for each plan  $i$ ,  $F_i$ , is determined separately according to (6) with



$B = B_i = B'/n$ , ( $i = 1, \dots, n$ ), then the aggregate private pension benefit for worker 2 is given by  $\sum_1^n \text{Max}[0, F_i - S_T] = n \text{Max}[0, F' - S_T]$  where  $F' = F_i$  ( $i = 1, \dots, n$ ) is the common solution to (6) with  $B_i = B'/n$ .

In effect, worker 2 has been given a put option on his social security benefit by each of his employers and therefore, has an aggregate of  $n$  put options on his single social security benefit. Thus, unlike worker 1's single put option, once worker 2's options are "in-the-money" (i.e.,  $S_T < F'$ ), he receives  $n$  dollars in private annuity benefits for each dollar decline in  $S_T$  below  $F'$ . He will, therefore, receive a larger total retirement income if  $S_T < F'$  than if  $S_T = F'$  (which corresponds to his minimum retirement income).

Worker 2, of course, pays for this "extra" benefit received for very low levels of social security. By analysis similar to that used to derive (12),  $F/n < F' < F$  where, in general,  $F' \ll F$  for  $n > 2$ . Hence, worker 2 has no protection against declines in the level of social security payments for  $F' < S_T < F$  whereas worker 1 is "fully insured" in this regime. Thus, even for a worker with a large total nonintegrated private pension benefit  $B$ , the amount of "useful" insurance provided by integrated plans may be rather modest if the worker has had many employers and each  $F_i \ll F$ .

In summary, for a single-employer worker under an integrated plan, the schedule of total first-year retirement income as a function of the social security benefit,  $\text{Max}[F, S_T]$ , exhibits the standard insurance pattern of a "protective put" strategy. In contrast, the corresponding schedule of total income for an  $n$ -employer worker,  $\text{Max}[nF' - (n - 1)S_T, S_T]$ , is a piecewise linear function of  $S_T$  which is decreasing with slope  $-(n - 1)$  for  $S_T < F'$ ; reaches a

minimum at  $S_{\pi} = F'$ ; and is increasing with slope 1 for  $S_{\pi} > F'$ .

It is difficult to believe that this "vee-shaped" schedule of retirement income for multiple-employer workers is an intended consequence of integrated pension plans. Although the normative aspects of integrated plans is not the focus of this paper, our brief analysis here surely suggests that a widespread change from nonintegrated to integrated plans under current aggregation rules could have a significant and largely unintended effect on worker mobility.

#### V. Summary, Conclusions, and Agenda for Future Research

Our most robust finding in the previous section can be stated simply as follows. For extremely low values of  $B/S$ , that is, the ratio of employer benefits to social security in the nonintegrated scenario, the value of  $F$  in the integrated scenario is very low, indicating that integration would not in that situation provide much insurance. At the other extreme, for high ratios of employer-provided benefits to social security benefits in the nonintegrated scenario, integration results in virtually complete elimination of social security risk through employer insurance.

One's position on whether accruing benefits under a defined benefit plan are real or nominal thus has a significant impact on the degree of risk-shedding achieved through integration. If the benefit is real, then all but those with virtually no private benefit in the nonintegrated scenario will by switching to an integrated plan in effect sell all their rights to social

security. If the benefit is nominal then a greater proportion of individuals will retain a claim to at least some meaningful part of the distribution of social security benefits after integration.

Our analysis does not address the issue of whether or not integration under the offset plan examined here is desirable. Indeed, under the usual assumption of continuously-differentiable preference functions, one would not expect that a "kinked" schedule of income (e.g.,  $\text{Max}[F, S_T]$ ) would be an unconstrained optimum. Such schedules can, however, be optimal if there are constraints such as that the worker cannot sell his human capital. For example, under just this constraint, Diamond and Mirrlees (forthcoming) have examined the role of transferable private pensions in improving the risk-sharing opportunities for workers when they are mobile. As shown in Merton (forthcoming), under certain conditions, the Diamond-Mirrlees optimal transferable pension schedule is formally identical in structure to the one derived here for an integrated pension plan. Hence, neglecting the problems associated with worker mobility, a normative study may well find that integrated pension plans like those analyzed in sections III and IV do have optimal risk-bearing properties. If however, worker mobility is taken into account, then based on the analysis in Section IV, we conjecture that the optimal pension policy will be to integrate all pension plans, both private and public.

Thus, while the focus here has been to highlight what we believe to be some of the unintended consequences of integration in its current setting, the analysis also provides a footlight on the tradeoffs that are likely to be

encountered in a normative evaluation of integration.

One, presumably, unintended consequence of integration is that it allows for a de facto sale of social security benefits by participants in even moderately generous private pension plans. Our tables suggest that for typical profiles this sale is effectively complete despite the de jure prohibition against such assignment embodied elsewhere in the law. A related consequence is that the risk-shedding available to those with low employer-provided benefits is inferior to that of more generously provided-for retirees. Since low income individuals generally also have the lowest pension benefits relative to social security, this risk-sharing pattern would appear to be somewhat regressive. Finally, we note our finding that integrated plans have unintended consequences for worker mobility beyond those already identified for nonintegrated plans.

The analysis in this paper is our first step in exploring the issue of integration of employer-provided pensions as a means of insuring workers against retirement income risk. In addition to the normative analysis already noted, there are a number of extensions of the analysis which are on our agenda for future research.

First of these is to perform a study similar to the one presented here for excess plans, and in particular for defined contribution excess plans. Second, we plan to examine in greater depth the nature of social security risk and how it affects the value of the insurance provided through plan integration. For example, uncertainty regarding social security benefits, which are determined in large part through the political process, is not likely to be the same

across all income levels.

As described briefly in Section IV, a third obvious extension is to deal explicitly with other sources of retirement income risk in addition to social security and to see how they interact under plan integration. One major factor is inflation risk. Since the employer-provided benefit is usually fixed in nominal terms at least after retirement, its real value is risky because of price-level uncertainty. The latter risk can be reduced and indeed entirely eliminated through indexation, and a considerable literature on this issue already exists.<sup>10</sup> We therefore have chosen to ignore this issue in this paper, focusing exclusively on social security risk and integration. However, there clearly is an interaction between inflation risk and social security risk and any full analysis of the issues of integration and indexation would have to consider the interaction between the two.

Fourth, we have considered only social security benefits and the risk associated with them and have ignored social security taxes or contributions. Clearly, changes in social security benefits in the future imply changes in social security contributions under the pay-as-you-go funding system currently in place. In that sense our model is partial equilibrium in its analysis of the changes in risk-sharing between employer and worker. Future research will take account of the feedback between benefit changes and contribution changes in the future in assessing the risk profiles resulting from integration.

Finally, our model and the option pricing methodology which we have applied have clear implications for the actuarial methods used to cost

integrated pension plans. To our knowledge the actuarial profession does not currently employ this methodology, and we plan to explore the implications of its use in a more detailed setting than the one used in this paper.

Footnotes

1. See Schulz and Leavitt (1983), p. 26. According to the 1980 Bankers Trust Survey as many as 87 percent of private defined benefit plans with pay-related formulas were integrated.

2. For an explanation of options and how they work, see the seminal paper by Black and Scholes (1973). For a survey of the options literature and its application to nontraded assets, see Mason and Merton (1984). Because social security benefits change over time, while employer-provided benefits in integrated plans are linked to the level of social security at the time of retirement and are not thereafter adjusted, employees might engage in strategic retirement behavior. For example, it might pay to retire immediately prior to a large increase in social security benefits, so as to obtain larger private pension benefits. This gaming issue is absent from our analysis, because we set the retirement date exogenously. However, strategic behavior could easily be incorporated into the analysis. If retirement dates are chosen by optimizing employees, then the implicit option conferred to employees is simply American rather than European. While closed-form solutions for the values of these options are generally unavailable, the exercise decision is well understood and several numerical valuation algorithms are available to value such options.

3. For a further discussion of the analogy between put options and insurance schemes, see Merton, Scholes, and Gladstein (1982).

4. The quantitative properties of integrated plans can be sensitive to the particular stochastic process assumed for  $S$ . However, the important

qualitative properties of integration are independent of the particular process postulated. Geometric Brownian motion is the prototype process examined in the finance literature, and has the benefits of familiarity and simplicity.

5. For a full presentation of the view of defined benefit pension accruals as a nominal asset see Bulow (1982). For a good discussion of why they might best be viewed as real see Cohn and Modigliani (1983).

6. If the uncertainty surrounding the real value of future social security payments is diversifiable, then  $\alpha$  also equals  $r$ , and the actual expected growth rate  $g$  is zero. If  $\alpha$  exceeds  $r$  because of a risk premium associated with social security uncertainty, then  $g = \alpha - r > 0$  and column (3) is interpreted as the "risk-corrected" or "certainty-equivalent" expected level of total benefits.

7. The entries in column (4) are accurate to \$5. Floor levels equal to column (3) thus result from rounding error. Actual floor levels must be somewhat less than the corresponding entry in column (3).

8. See for example Clark, Allen and Sumner (1983).

9. We set the upper limit of the integral in (14) equal to 40 years. The value of the sum of the integrand using yearly increments for  $dt$  was no longer increasing noticeably at this point.

10. See for example Feldstein (1983), Summers (1983), and Bodie and Pesando (1983).



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