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Arnaud Costinot
Ivana Komunjer

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ABSTRACT

Though one of the pillars of the theory of international trade, the extreme predictions of the Ricardian model have made it unsuitable for empirical purposes. A seminal contribution of Eaton and Kortum (2002) is to demonstrate that random productivity shocks are sufficient to make the Ricardian model empirically relevant. While successful at explaining trade volumes, their model remains silent with regards to one important question: What goods do countries trade? Our main contribution is to generalize their approach and provide an empirically meaningful answer to this question.

Arnaud Costinot
Department of Economics
University of California, San Diego
9500 Gilman Drive
La Jolla, CA 92093-0508
and NBER
costinot@ucsd.edu

Ivana Komunjer
Department of Economics
University of California, San Diego
9500 Gilman Drive
La Jolla, CA 92093-0508
komunjer@ucsd.edu

1. Introduction

Though one of the pillars of the theory of international trade, the extreme predictions of the Ricardian model have made it unsuitable for empirical purposes. As Leamer and Levinsohn (1995) point out: "The Ricardian link between trade patterns and relative labor costs is much too sharp to be found in any real data set."

A seminal contribution of Eaton and Kortum (2002) is to demonstrate that random productivity shocks are sufficient to make the Ricardian model empirically relevant. When drawn from an extreme value distribution, these shocks imply a gravity-like equation in a Ricardian framework with a continuum of goods, transport costs, and more than two countries. While successful at explaining trade volumes, their model remains silent with regards to one important question: What goods do countries trade? Our main contribution is to generalize their approach and provide an empirically meaningful answer to this question.

Section 2 describes the model. We consider an economy with one factor of production, labor, and multiple goods, each available in many varieties. There are constant returns to scale in the production of each variety. The key assumption of our model is that labor productivity may be separated into: a deterministic component, which is country and industry specific; and a stochastic component, randomly drawn across countries, industries, and varieties. The former, to which we refer as "fundamental productivity", captures factors such as climate, infrastructure, and institutions that affect the productivity of all producers in a given country and industry.[1] The latter, by contrast, reflects idiosyncratic differences in technological know-how across varieties.

Section 3 derives our predictions on the pattern of trade. Because of random productivity shocks, we can no longer predict trade flows in each variety. Yet, by assuming that each good comes in a large number of varieties, we generate sharp predictions at the industry level. In particular, we show that, for any pair of exporters, the ranking by industry of the ratios of their fundamental productivity levels determines the ranking of the ratios of

[1] Acemoglu, Antras, and Helpman (2007), Costinot (2006), Cuñat and Melitz (2006), Levchenko (2007), Matsuyama (2005), Nunn (2007), and Vogel (2007) explicitly model the impact of various institutional features—e.g. labor market flexibility, the quality of contract enforcement, or credit market imperfections—on labor productivity across countries and industries.

their exports towards any importing country. Compared to the standard Ricardian model—see e.g. Dornbusch, Fischer, and Samuelson (1977)—our predictions hold under general assumptions on transport costs, the number of industries, and the number of countries.[2] Moreover, they do not imply the full specialization of countries in a given set of industries.

Section 4 investigates how well our model squares with the empirical evidence. Our empirical results are based on linear regressions tightly connected to the theory. They offer strong support for our new Ricardian predictions: countries do tend to export relatively more—towards any importing country—in industries where they are relatively more productive.

Our paper contributes to the previous trade literature in two ways. First, it contributes to the theory of comparative advantage. Our model generates clear predictions on the pattern of trade in environments—with both multiple countries and industries—where the standard Ricardian model loses most of its intuitive content; see e.g. Jones (1961) and Wilson (1980). Our approach mirrors Deardorff (1980) who shows how the law of comparative advantage may remain valid, under standard assumptions, when stated in terms of correlations between vectors of trade and autarky prices. In this paper, we weaken the standard Ricardian assumptions—the "chain of comparative advantage" will only hold in terms of first-order stochastic dominance—and derive a deterministic relationship between exports and labor productivity across industries.

Second, our paper contributes to the empirical literature on international specialization, including previous "tests" of the Ricardian model; see e.g. MacDougall (1951), Stern (1962), Balassa (1963), and more recently Golub and Hsieh (2000). While empirically successful, these tests have long been criticized for their lack of theoretical foundations; see Bhagwati (1964). Our model provides such foundations. Since it does not predict full international specialization, we do not have to focus on ad-hoc measures of export performance. Instead, we may use the theory to pin down explicitly what the dependent variable in cross-industry regressions ought to be.

As we discuss later in the paper, our model also provides an alternative theoretical underpinning of cross-industry regressions when labor is not the only factor of production; see

[2]Deardorff (2005) reviews the failures of simple models of comparative advantage at predicting the pattern of trade in economies with more than two goods and two countries.

e.g. Baldwin (1971). The validity of these regressions usually depends on strong assumptions on either demand—see e.g. Petri (1980) and the voluminous gravity literature based on Armington's preferences—or the structure of transport costs—see e.g. Harrigan (1997), Romalis (2004), and Morrow (2006). One of the main messages of our paper is that many of these assumptions can be relaxed, as long as there are stochastic productivity differences within each industry.

2. The Model

We consider a world economy comprising $i = 1, ..., I$ countries and one factor of production—labor. There are $k = 1, ..., K$ goods and constant returns to scale in the production of each good. Labor is perfectly mobile across industries and immobile across countries. The wage of workers in country i is denoted w_i. Up to this point, this is a standard Ricardian model. We generalize this model by introducing random productivity shocks. Following Eaton and Kortum (2002), we assume that each good k may come in N^k varieties $\omega = 1, ..., N^k$, and denote $a_i^k(\omega)$ the constant unit labor requirements for the production of the ωth variety of good k in country i. Our first assumption is that:

A1. *For all countries i, goods k, and their varieties ω*

$$\ln a_i^k(\omega) = \ln a_i^k + u_i^k(\omega), \tag{1}$$

where $a_i^k > 0$ and $u_i^k(\omega)$ is a random variable drawn independently for each triplet (i, k, ω) from a continuous distribution $F(\cdot)$ such that: $E[u_i^k(\omega)] = 0$.

We interpret a_i^k as a measure of the fundamental productivity of country i in industry k and $u_i^k(\omega)$ as a random productivity shock. The former, which can be estimated using aggregate data, captures cross-country and cross-industry heterogeneity. It reflects factors such as climate, infrastructure, and institutions that affect the productivity of *all* producers in a given country and industry. Random productivity shocks, on the other hand, capture intra-industry heterogeneity. They reflect idiosyncratic differences in technological know-how across varieties, which are assumed to be drawn independently from a unique distribution $F(\cdot)$. In our setup, cross-country and cross-industry variations in the distribution of productivity levels derive from variations in a single parameter: a_i^k.

Assumption A1 generalizes Eaton and Kortum's (2002) approach along two dimensions. First, it introduces the existence of exogenous productivity differences across industries. This will allow us to shift the indeterminacy in trade in individual goods to indeterminacy in trade in varieties. Second, it does not impose any restriction on the distribution of random productivity shocks.

We assume that trade barriers take the form of "iceberg" transport costs:

A2. *For every unit of commodity k shipped from country i to country j, only $1/d_{ij}^k$ units arrive, where:*

$$\begin{cases} d_{ij}^k = d_{ij} \cdot d_j^k \geq 1, & \text{if } i \neq j, \\ d_{ij}^k = 1, & \text{otherwise.} \end{cases} \tag{2}$$

The indices i and j refer to the exporting and importing countries, respectively. The first parameter d_{ij} measures the trade barriers which are specific to countries i and j. It includes factors such as: physical distance, existence of colonial ties, use of a common language, or participation in a monetary union. The second parameter d_j^k measures the policy barriers imposed by country j on good k, such as import tariffs and standards. In line with "the most-favored-nation" clause of the World Trade Organization, these impediments may not vary by country of origin.

We assume that markets are perfectly competitive.[3] Together with constant returns to scale in production, perfect competition implies:

A3. *In any country j, the price $p_j^k(\omega)$ paid by buyers of variety ω of good k is*

$$p_j^k(\omega) = \min_{1 \leq i \leq I} \left[c_{ij}^k(\omega) \right], \tag{3}$$

where $c_{ij}^k(\omega) = d_{ij}^k \cdot \mathrm{w}_i \cdot a_i^k(\omega)$ is the cost of producing and delivering one unit of this variety from country i to country j.

For each variety ω of good k, buyers in country j are "shopping around the world" for the best price available. Here, random productivity shocks lead to random costs of production $c_{ij}^k(\omega)$ and in turn, to random prices $p_j^k(\omega)$. In what follows, we let $c_{ij}^k = d_{ij}^k \cdot \mathrm{w}_i \cdot a_i^k > 0$.

[3] The case of Bertrand competition is discussed in details in Appendix B.

On the demand side, we assume that consumers have a two-level utility function with CES preferences across varieties. This implies:

A4(i). *In any country j, the total spending on variety ω of good k is*

$$x_j^k(\omega) = \left[p_j^k(\omega)/p_j^k\right]^{1-\sigma} e_j^k, \tag{4}$$

where $e_j^k > 0$, $\sigma > 1$ and $p_j^k = [\sum_{\omega'=1}^{N^k} p_j^k(\omega')^{1-\sigma}]^{1/(1-\sigma)}$.

The above expenditure function is a standard feature of the "new trade" literature; see e.g. Helpman and Krugman (1985). e_j^k is an endogenous variable that represents total spending on good k in country j. It depends on the upper tier utility function in this country and the equilibrium prices. p_j^k is the CES price index, and σ is the elasticity of substitution between varieties. It is worth emphasizing that while the elasticity of substitution σ is assumed to be constant, total spending, and hence demand conditions, may vary across countries and industries: e_j^k is a function of j and k.

Finally, we assume that:

A4(ii). *In any country j, the elasticity of substitution σ between two varieties of good k is such that $E\left[p_j^k(\omega)^{1-\sigma}\right] < \infty$.*

Assumption A4(ii) is a technical assumption that guarantees the existence of a well defined price index. Whether or not A4(ii) is satisfied ultimately depends on the shape of the distribution $F(\cdot)$.[4]

[4]Suppose, for example, that $u_i^k(\omega)$'s are drawn from a (negative) exponential distribution with mean zero: $F(u) = \exp[\theta u - 1]$ for $-\infty < u \leq 1/\theta$ and $\theta > 0$. This corresponds to the case where labor productivity $z_i^k(\omega) \equiv 1/a_i^k(\omega)$ is drawn from a Pareto distribution: $G_i^k(z) = 1 - (b_i^k/z)^\theta$ for $0 < b_i^k \leq z$ and $b_i^k \equiv (1/a_i^k)\exp(-1/\theta)$, as assumed in various applications and extensions of Melitz's (2003) model; see e.g. Helpman, Melitz, and Yeaple (2004), Antras and Helpman (2004), Ghironi and Melitz (2005), Bernard, Redding, and Schott (2006), and Chaney (2007). Then, our assumption A4(ii) holds if the elasticity of substitution $\sigma < 1 + \theta$. Alternatively, suppose that $u_i^k(\omega)$'s are distributed as a (negative) Gumbel random variable with mean zero: $F(u) = 1 - \exp[-\exp(\theta u - \mathsf{e})]$ for $u \in \mathbb{R}$ and $\theta > 0$, where e is Euler's constant $\mathsf{e} \simeq 0.577$. This corresponds to the case where labor productivity $z_i^k(\omega)$ is drawn from a Fréchet distribution: $G_i^k(z) = \exp(-b_i^k z^{-\theta})$ for $z \geq 0$ and $b_i^k \equiv (1/a_i^k)^\theta \exp(-\mathsf{e})$, as assumed, for example, in Eaton and Kortum (2002) and Bernard, Eaton, Jensen, and Kortum (2003). Then, like in the Pareto case, A4(ii) holds if $\sigma < 1 + \theta$.

In the rest of the paper, we let $x_{ij}^k = \sum_{\omega=1}^{N^k} x_{ij}^k(\omega)$ denote the value of exports from country i to country j in industry k, where total spending on each variety $x_{ij}^k(\omega)$ is given by:

$$\begin{cases} x_{ij}^k(\omega) = x_j^k(\omega), & \textit{if } c_{ij}^k(\omega) = \min_{1 \le i' \le I} c_{i'j}^k(\omega), \\ x_{ij}^k(\omega) = 0, & \textit{otherwise.} \end{cases} \tag{5}$$

3. The Pattern of Trade

We now describe the restrictions that Assumptions A1–A4 impose on the pattern of trade; and how they relate to those of the standard Ricardian model.

3.1. **The Wonderful World of Eaton and Kortum (2002).**

For expositional purposes, we first derive predictions on the pattern of trade when the distribution of a random productivity shocks, $F(\cdot)$, is a Gumbel with mean zero, as assumed in Eaton and Kortum (2002). Hence, the only difference between the present model and theirs is the existence of multiple industries. This corresponds to the case where

$$F(u) = 1 - \exp[-\exp(\theta u - \mathtt{e})] \tag{6}$$

with $u \in \mathbb{R}$, $\theta > 0$, and $\mathtt{e}$ the Euler's constant $\mathtt{e} \simeq 0.577$.

Our first result can be stated as follows:

Theorem 1. *Suppose that Assumptions A1-A4 hold. In addition, assume that the number of varieties N^k of any good k is large, and that $F(\cdot)$ satisfies Equation* (6)*. Then, for any exporter i, any importer j, and any good k,*

$$\ln x_{ij}^k \simeq \theta_{ij} + \theta_j^k - \theta \ln a_i^k \tag{7}$$

The proof of Theorem 1 mainly is a matter of algebra. First, we relate total exports x_{ij}^k to the expected value of exports coming from country i, using the law of large numbers.[5] Second, we compute the expected value explicitly using Equation (6); see Appendix A.

The first term $\theta_{ij} \equiv -\theta \ln(d_{ij} \cdot \mathrm{w}_i)$ is importer and exporter specific; it reflects wages w_i in the exporting country and trade barriers d_{ij} between countries i and j. The second term

[5] Alternatively, we could have assumed the existence a continuum of varieties and argued, like many before us, that total exports were equal to their expected value. By assuming that the number of varieties is large but finite, we avoid, however, the technical difficulties of invoking the law of large numbers with a continuum of i.i.d. variables; see e.g. Al-Najjar (2004). Nothing substantial hinges on this particular modeling choice.

$\theta_j^k \equiv \ln e_j^k - \theta \ln d_j^k - \ln \left(\sum_{i'=1}^{I} (c_{i'j}^k)^{-\theta} \right)$ is importer and industry specific; it reflects the policy barriers d_j^k imposed by country j on good k and demand differences e_j^k across countries and industries. The main insight of Theorem 1 comes from the third term $\theta \ln a_i^k$. Since $\theta > 0$, $\ln x_{ij}^k$ should be decreasing in $\ln a_i^k$: *ceteris paribus*, countries should export less in industries where their firms are, on average, less efficient.

It is worth emphasizing that Theorem 1 *cannot* be used for comparative static analysis. If the fundamental productivity level goes up in a given country and industry, this will affect wages, demand, and, in turn, exports in other countries and industries through general equilibrium effects. In other words, changes in a_i^k also lead to changes in the country and industry fixed effects, θ_{ij} and θ_j^k. By contrast, Theorem 1 *can* be used to analyze the cross-sectional variations of bilateral exports, as we shall further explore in Section 4.

Finally, note that the two fixed-effects, θ_{ij} and θ_j^k, do not depend on the elasticity of substitution σ. Thus, the predictions of Theorem 1 still hold if we relax Assumption A4(i), so that the elasticity of substitution may vary across countries and industries, $\sigma \equiv \sigma_j^k$. We come back to this intriguing result when discussing the general case.

3.2. **The General Case.** We now relax the assumption that $F(\cdot)$ is a Gumbel distribution. In this situation, we can no longer obtain a closed form solution, but we can still derive a log-linear relationship between total exports and the fundamental productivity level a_i^k, using a first-order Taylor series development around a symmetric situation where costs are identical across exporters, $\left(c_{1j}^k = ... = c_{Ij}^k\right)$.

Theorem 2. *Suppose that Assumptions A1-A4 hold. In addition, assume that the number of varieties N^k of any good k is large, and that cost differences across exporters are small: $c_{1j}^k \simeq ... \simeq c_{Ij}^k$. Then, for any exporter i, any importer $j \neq i$, and any good k,*

$$\ln x_{ij}^k \simeq \gamma_{ij} + \gamma_j^k - \gamma \ln a_i^k. \tag{8}$$

where $\gamma > 0$.

The proof as well as the exact expressions for γ_{ij}, γ_j^k, and γ are given in Appendix A. Theorem 2 predicts that, like in the Gumbel case, total exports can be decomposed into an importer-exporter specific term, γ_{ij}; an importer-industry specific term, γ_j^k; and a third term, $\gamma \ln a_i^k$, which captures the impact of productivity differences. Since $\gamma > 0$, Theorem

2 also predicts that: *ceteris paribus*, countries should export less in industries where their firms are, on average, less efficient.

The predictive power of Theorem 2 crucially relies on the fact that γ is constant across countries and industries. To understand this result, it is convenient to think about total exports in terms of their extensive and intensive margins, that is how many and how much of each variety are being exported, respectively. The unique distribution of random productivity shocks $F(\cdot)$ makes sure that marginal changes in the costs of production c_{ij}^k have the same impact on the extensive margin across countries and industries. Similarly, the constant elasticity of substitution σ guarantees that they have the same impact on the intensive margin. This is the basic idea behind Theorem 2. The other assumptions simply allow us to identify the effect of labor productivity by bundling the impact of changes in wages, demand, and transport costs into fixed effects.

Relaxing Eaton and Kortum's (2002) distributional assumption in Theorem 2 comes at the cost of two new restrictions. First, we must move from global predictions—which hold for any $\left(c_{1j}^k, ..., c_{Ij}^k\right)$—to local predictions—which only hold if costs differences across all exporters are small.[6] Second, we must assume that the elasticity of substitution is constant across countries and industries. This assumption was not necessary in Theorem 1 because of one key property of the Gumbel distribution: conditional on exporting a given variety to country j, the expected value of exports was identical across countries. Hence, transport costs, wages and fundamental productivity levels only affected the extensive margin, not the intensive margin. Unfortunately, this attractive property of the Gumbel does not generalize to other standard distributions, as we show in Appendix C.

In order to prepare the comparison between our results and those of the standard Ricardian model, we conclude by offering a Corollary to Theorems 1 and 2. Consider an arbitrary pair of exporters, i_1 and i_2, an importer $j \neq i_1, i_2$, and an arbitrary pair of goods, k_1 and k_2.

[6] Although this requirement may seem unreasonably strong, the predictions of Theorem 2 hold more generally if, for each industry and each importing country, exporters can be separated into two groups: small exporters, whose costs are very large (formally, close to infinity), and large exporters, whose costs of production are small and of similar magnitude. Then, small exporters export with probability close to zero and the results of Theorem 2 apply to the group of large exporters. In other words, Theorem 2 does not require Gambia and Japan to have similar costs of producing and delivering cars in the United States. It simply requires that Japan and Germany do.

Taking the differences-in-differences in Equation (7) we get

$$(\ln x_{i_1 j}^{k_1} - \ln x_{i_1 j}^{k_2}) - (\ln x_{i_2 j}^{k_1} - \ln x_{i_2 j}^{k_2}) \simeq -\theta \left[\left(\ln a_{i_1}^{k_1} - \ln a_{i_1}^{k_2} \right) - \left(\ln a_{i_2}^{k_1} - \ln a_{i_2}^{k_2} \right) \right]$$

A similar manipulation of Equation (8) implies

$$(\ln x_{i_1 j}^{k_1} - \ln x_{i_1 j}^{k_2}) - (\ln x_{i_2 j}^{k_1} - \ln x_{i_2 j}^{k_2}) \simeq -\gamma \left[\left(\ln a_{i_1}^{k_1} - \ln a_{i_1}^{k_2} \right) - \left(\ln a_{i_2}^{k_1} - \ln a_{i_2}^{k_2} \right) \right]$$

Since $\theta > 0$ and $\gamma > 0$, we obtain

$$\frac{a_{i_1}^{k_1}}{a_{i_2}^{k_1}} > \frac{a_{i_1}^{k_2}}{a_{i_2}^{k_2}} \Leftrightarrow \frac{x_{i_1 j}^{k_1}}{x_{i_2 j}^{k_1}} < \frac{x_{i_1 j}^{k_2}}{x_{i_2 j}^{k_2}}, \tag{9}$$

under the assumptions of Theorems 1 or 2. Still considering the pair of exporters i_1 and i_2 and generalizing the above reasoning to all K goods, we derive the following Corollary:

Corollary 3. *Suppose that the assumptions of Theorem 1 or 2 hold. Then, the ranking of relative unit labor requirements determines the ranking of relative exports:*

$$\left\{ \frac{a_{i_1}^1}{a_{i_2}^1} > .... > \frac{a_{i_1}^k}{a_{i_2}^k} > ... > \frac{a_{i_1}^K}{a_{i_2}^K} \right\} \Leftrightarrow \left\{ \frac{x_{i_1 j}^1}{x_{i_2 j}^1} < .... < \frac{x_{i_1 j}^k}{x_{i_2 j}^k} < ... < \frac{x_{i_1 j}^K}{x_{i_2 j}^K} \right\}.$$

3.3. **Relation to the Standard Ricardian Model.** Note that we can always index the K goods so that:

$$\frac{a_{i_1}^1}{a_{i_2}^1} > .... > \frac{a_{i_1}^k}{a_{i_2}^k} > ... > \frac{a_{i_1}^K}{a_{i_2}^K}. \tag{10}$$

Ranking (10) is at the heart of the standard Ricardian model; see e.g. Dornbusch, Fischer, and Samuelson (1977). When there are no random productivity shocks, Ranking (10) merely states that country i_1 has a comparative advantage in (all varieties of) the high k goods. If there only are two countries, the pattern of trade follows: i_1 produces and exports the high k goods, while i_2 produces and exports the low k goods. If there are more than two countries, however, the pattern of pairwise comparative advantage no longer determines the pattern of trade. In this case, the standard Ricardian model loses most of its intuitive content; see e.g. Jones (1961) and Wilson (1980).

When there are stochastic productivity differences within each industry, Assumption A1 and Ranking (10) further imply:

$$\frac{a_{i_1}^1(\omega)}{a_{i_2}^1(\omega)} \succ .... \succ \frac{a_{i_1}^k(\omega)}{a_{i_2}^k(\omega)} \succ ... \succ \frac{a_{i_1}^K(\omega)}{a_{i_2}^K(\omega)}, \tag{11}$$

where $\succ$ denotes the first-order stochastic dominance order among distributions.[7] In other words, Ranking (11) is just a stochastic—hence weaker—version of the ordering of labor requirements a_i^k, which is at the heart of the Ricardian theory. Like its deterministic counterpart in (10), Ranking (11) captures the idea that country i_1 is relatively better at producing the high k goods. But whatever k is, country i_2 may still have lower labor requirements on some of its varieties.

According to Corollary 3, Ranking (11) does not imply that country i_1 should only produce and export the high k goods, but instead that it should produce and export relatively more of these goods. This is true irrespective of the number of countries in the economy. Unlike the standard Ricardian model, our stochastic theory of comparative advantage generates a clear and intuitive correspondence between labor productivity and exports. In our model, the pattern of comparative advantage for any pair of exporters fully determines their relative export performance across industries.

Another perspective on Corollary 3 is that, for any pair of exporters, the ranking of their relative exports towards any importing country fully *reveals* their comparative advantage. By observing exports across countries and industries, one can directly infer—according to our model—the ranking of relative productivity levels. Thus, our results also provide theoretical foundations to measures of revealed comparative advantage à la Balassa (1965).[8] We explore that idea in details in Appendix D.

The previous discussion may seem paradoxical. As we have just mentioned, Ranking (11) is a weaker version of the ordering at the heart of the standard theory. If so, how does our stochastic theory lead to finer predictions? The answer is simple: it does not. While the standard Ricardian model is concerned with trade flows in each variety of each good, we only are concerned with the total trade flows in each good. Unlike the standard model, we recognize that random shocks—whose origins remain outside the scope of our

[7]To see this, note that for any $A \in \mathbb{R}^+$ we have $\Pr\left\{a_{i_1}^k(\omega)/a_{i_2}^k(\omega) \leq A\right\} = \Pr\{u_{i_1}^k(\omega) - u_{i_2}^k(\omega) \leq \ln A - \ln a_{i_1}^k + \ln a_{i_2}^k\}$. Since for any $k < k'$, $u_{i_1}^k(\omega) - u_{i_2}^k(\omega)$ and $u_{i_1}^{k'}(\omega) - u_{i_2}^{k'}(\omega)$ are drawn from the same distribution by A1, Ranking (10) implies:

$$\Pr\left\{\frac{a_{i_1}^k(\omega)}{a_{i_2}^k(\omega)} \leq A\right\} < \Pr\left\{\frac{a_{i_1}^{k'}(\omega)}{a_{i_2}^{k'}(\omega)} \leq A\right\} \Leftrightarrow \frac{a_{i_1}^k(\omega)}{a_{i_2}^k(\omega)} \succ \frac{a_{i_1}^{k'}(\omega)}{a_{i_2}^{k'}(\omega)}$$

[8]We thank Kei-Mu Yi for bringing this to our attention.

Table 1: Data Set Description

Sources: OECD STAN Bilateral Trade Database; International Comparisons of Output and Productivity Industrial Database

Years 1988-2001

Exporters:	Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, Spain, Sweden, United Kingdom, United States
Importers:	Exporters + Argentina, Australia, Brazil, Canada, Chile, China, Cyprus, Czech Republic, Estonia, Hong Kong, Hungary, Iceland, India, Indonesia, Japan, Korea, Latvia, Lithuania, Luxembourg, Malaysia, Malta, Mexico, New Zealand, Norway, Phillipines, Poland, Russian Federation, Singapore, Slovak Republic, Slovenia, South Africa, Switzerland, Taipei, Thailand, Turkey

Product Classification System: The industrial breakdown presented for the STAN indicators database is based upon the International Standard Industrial Classification (ISIC) Revision 3.

Industry:	*STAN Description:*	*ISIC Rev. 3*
Food	Food products, beverages and tobacco	15-16
Textile	Textiles, textile products, leather and footwear	17-19
Wood	Wood and products of wood and cork	20
Paper	Pulp, paper, paper products, printing and publishing	21-22
Chemicals	Chemicals	24
Plastic	Rubber and plastics products	25
Minerals	Other non-metallic mineral products	26
Basic metals	Basic Metals	27
Metal products	Fabricated metal products, except machinery and equipment	28
Machinery	Machinery and equipment, n.e.c.	29
Office	Office, accounting and computing machinery	30
Electrical	Electrical machinery and apparatus, n.e.c.	31
Telecom	Radio, television and communication equipment	32
Medical	Medical, precision and optical instruments, watches and clocks	33
Automobile	Motor vehicles, trailers and semi-trailers	34
Shipbuilding	Building and repairing of ships and boats	351
Aircraft	Aircraft and spacecraft	353
Transport	Railroad equipment and transport equipment n.e.c.	352+359
Other	Manufacturing n.e.c.	36-37

model—may affect the costs of production of any variety. Yet, by assuming that these shocks are identically distributed across a large number of varieties, we manage to generate sharp predictions at the industry level.

4. Empirical Evidence

We now confront our theoretical predictions with the data.

4.1. **Data Description.** Table 1 describes our data set. It includes 15 exporters, 14 European countries plus the United States; 50 importers, both OECD and large non-OECD countries; and 19 manufacturing industries from 1988 to 2001. Sample selection was entirely dictated by the availability of *both* bilateral trade data *and* productivity data comparable across countries and industries. Trade data are from the OECD Structural Analysis (STAN) Bilateral Trade Database. The value of exports x_{ij}^k by exporting country i, importing country j, and industry k is directly available in thousands of US dollars, at current prices. Productivity data are from the International Comparisons of Output and Productivity (ICOP) Industrial Database developed by the University of Groningen.[9] A key characteristic of the ICOP is the use of relative producer prices, or "unit value ratios", to convert output by industry to a common currency. Throughout this section, we use "total hours worked" divided by "value added in 1997 $US at unit value ratios" as our measure of the unit labor requirement a_i^k in country i and industry k. We come back to the formal relationship between this proxy and our model in Section 4.4.

4.2. **A First Look at the Pattern of Exports.** Corollary 3 imposes a strong restriction on the pattern of exports. For any pair of exporters, i_1 and i_2, the ranking by industry of their relative exports should be constant across importers. Formally, we should observe that

$$\left(\frac{x_{i_1j_1}^{k_1}}{x_{i_2j_1}^{k_1}} - \frac{x_{i_1j_1}^{k_2}}{x_{i_2j_1}^{k_2}}\right) \cdot \left(\frac{x_{i_1j_2}^{k_1}}{x_{i_2j_2}^{k_1}} - \frac{x_{i_1j_2}^{k_2}}{x_{i_2j_2}^{k_1}}\right) > 0 \tag{12}$$

for all goods, k_1 and k_2, and importers, j_1 and j_2. Consider 2 exporters, the United States and Germany, and 2 goods, aircrafts and cars. According to Property (12), if Germany exports relatively more cars towards France than the United States, then it should also export relatively more cars towards Mexico. The absolute levels of German and US exports may vary between France and Mexico due to changes in demand and transport costs, but the relative export performance of Germany and the United States in these two industries may not.

A raw look at the data suffices to show that this restriction does not hold with certainty. Among the $17,955$ groups of exporters and industries included in our 1997 sample, the probability that the two terms in Equation (12) have the same sign for two distinct importers is equal to 69%. This fairly small number should not be too surprising. First, trade data are notoriously plagued with measurement errors; see Anderson and Wincoop (2004). Second,

[9]See http://www.ggdc.net/index-dseries.html for details

there exist trade barriers violating Assumption A2 in practice. For example, bilateral distance may have a differential impact on goods of different weights; see e.g. Harrigan (2005). With this in mind, we turn to linear regressions that incorporate explicitly the existence of measurement error in trade flows and/or transport costs not accounted by Assumption A2.

4.3. **Exports and Measured Productivity.** In line with Theorems 1 and 2, we consider the following linear regression model

$$\ln x_{ij}^k = \beta_{ij} + \beta_j^k + \beta \ln a_i^k + \varepsilon_{ij}^k, \tag{13}$$

where β_{ij} and β_j^k are treated as importer–exporter and importer–industry fixed effects, respectively, and ε_{ij}^k is an error term. Whether ε_{ij}^k is interpreted as measurement error in trade flows or unobserved transport costs, we shall assume that ε_{ij}^k is independent across countries i and j as well as across industries k; that ε_{ij}^k is heteroskedastic conditional on i, j and k; and that ε_{ij}^k is uncorrelated with $\ln a_i^k$.

The previous orthogonality condition rules out situations where country j tends to discriminate more against country i in industries where i is more productive. Were these situations prevalent in practice, due to endogenous trade protection, our OLS estimates of β would be biased towards zero.[10] Similarly, our orthogonality condition rules out any potential errors in the measurement of labor productivity at the industry level, which obviously is a strong assumption. If this measurement error is uncorrelated with $\ln a_i^k$, this should further bias our OLS estimates of β towards zero.

The main prediction of Theorems 1 and 2 is that the elasticity of exports with respect to the average unit labor requirement should be negative and constant across importers,

[10] Suppose that trade barriers, d_{ij}^k, and exports, x_{ij}^k, are simultaneously determined according to

$$\begin{cases} \ln d_{ij}^k = \ln d_{ij} + \ln d_j^k + \mu \ln x_{ij}^k \\ \ln x_{ij}^k = \widetilde{\beta}_{ij} + \widetilde{\beta}_j^k + \beta \ln a_i^k + \beta \ln d_{ij}^k \end{cases}$$

where $\mu > 0$ captures the fact that higher levels of import penetration lead to higher levels of protection. The previous system can be rearranged as

$$\begin{cases} \ln d_{ij}^k = (1-\mu\beta)^{-1}[\ln d_{ij} + \ln d_j^k + \mu\widetilde{\beta}_{ij} + \mu\widetilde{\beta}_j^k + \mu\eta \ln a_i^k] \\ \ln x_{ij}^k = \beta_{ij} + \beta_j^k + \beta \ln a_i^k + \varepsilon_{ij}^k \end{cases}$$

where $\delta_{ij} = (1-\mu\beta)^{-1}[\widetilde{\beta}_{ij} + \beta \ln d_{ij}]$, $\delta_j^k = (1-\mu\beta)^{-1}[\widetilde{\beta}_j^k + \beta \ln d_j^k]$, and $\varepsilon_{ij}^k = \mu\eta^2(1-\mu\beta)^{-1}\ln a_i^k$. This implies $E[\ln a_i^k \varepsilon_{ij}^k] = \mu\beta^2(1-\mu\beta)^{-1}E[(\ln a_i^k)^2] > 0$, and in turn, the upward bias in the OLS estimate of β.

Table 2: Year-by-Year OLS Regressions
(Dependent Variable: lnx)

Variable	**2001**	**2000**	**1999**	**1998**	**1997**	**1996**	**1995**
ln a	-0.98	-0.73	-0.97	-1.08	-1.42	-1.39	-1.27
	(-20.90)***	(-16.67)***	(-20.25)***	(-20.84)***	(-25.59)***	(-24.13)***	(-22.05)***
Exporter-Importer FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry-Importer FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	11858	11966	11967	11770	11748	11699	11638
R^2	0.83	0.83	0.83	0.83	0.83	0.83	0.82

Variable	**1994**	**1993**	**1992**	**1991**	**1990**	**1989**	**1988**
ln a	-1.31	-1.06	-1.20	-1.16	-0.99	-1.00	-0.74
	(-20.56)***	(-18.79)***	(-17.87)***	(-17.25)***	(-15.01)***	(-15.16)***	(-9.83)***
Exporter-Importer FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry-Importer FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	9316	9223	8731	8003	7807	7752	6938
R^2	0.83	0.83	0.84	0.83	0.83	0.83	0.83

Note: Values of t-statistics in parentheses, calculated from heteroskedasticity-consistent (White) standard errors
* Significant at 10% confidence level
** Significant at 5% confidence level
*** Significant at 1% confidence level

exporters, and industries:

$$(\partial \ln x_{ij}^k)/\left(\partial \ln a_i^k\right) = \beta < 0$$

with $\beta = -\theta$ or $-\gamma$ under the assumptions of Theorem 1 or 2, respectively. The OLS estimates of β are reported in Table 2 for each year 1988-2001. Overall, we view these results as strongly supportive of our new Ricardian predictions. In line with Theorems 1 and 2—and in spite of the potential biases towards zero discussed above—we find that β is negative and significant at the 1% level for every year in the sample. The largest regression estimate, in absolute value, is obtained in 1997, which is the year for which the ICOP's relative producer prices were collected.

Is the impact of measured productivity on the pattern of international specialization economically significant as well? As mentioned in Section 3, we cannot use our estimate of β to predict the changes in *levels* of exports associated with a given change in labor productivity. However, we can follow a difference-in-difference approach to predict *relative* changes in exports across countries and industries. Consider, for example, two exporters, i_1 and i_2, and two industries, k_1 and k_2, in 2001. If $a_{i_1}^{k_1}$ decreases by 10%, then our prediction is that:

$$\left(\Delta \ln x_{i_1 j}^{k_1} - \Delta \ln x_{i_1 j}^{k_2}\right) - \left(\Delta \ln x_{i_2 j}^{k_1} - \Delta \ln x_{i_2 j}^{k_2}\right) = \widehat{\beta}_{2001} \Delta \ln a_{i_1}^{k_1} \simeq 9.8\%.$$

This is consistent with a scenario where country i_1's exports of good k_1 go up by 7% and (because of the associated wage increase in country i_1) those of k_2 go down by 2.8%, while they remain unchanged in both industries in country i_2.

To help understand the size of the effects reported in Table 2, we can also use the standard deviations of $\ln a_i^k$ and $\ln x_{ij}^k$ in 2001, 0.74 and 2.72, respectively. Our estimates suggest that, *ceteris paribus*, a one standard deviation decrease in $\ln a_i^k$ should increase the dependent variable by 0.26 standard deviations.

4.4. **Selection bias.** There is one serious concern regarding the previous empirical results. In practice, statistical agencies do not observe the entire "universe" of varieties. Instead, they only observe the varieties that are actually produced in a given country and industry. This selection bias may, in principle, *lower* our OLS estimates. If better productivity draws are observed when a_i^k is high, then differences in measured productivity will be smaller than differences in fundamental productivity levels, which may artificially raise (in absolute value) the elasticity of exports with respect to the average unit labor requirement.

In Appendix E, we show how to control explicitly for selection bias under a mild restriction on the structure of transport costs: $d_{i_1 i_3} \leq d_{i_1 i_2} \cdot d_{i_2 i_3}$ for any three countries, i_1, i_2, and i_3. More specifically, we show that if the assumptions of Theorem 1 are satisfied, then

$$\ln E\left[a_i^k(\omega)\,|\omega \in \Omega_i^k\right] - \ln E\left[a_i^k(\omega)\right] = \frac{1}{\theta}\ln\left(1 - m_i^k\right) \tag{14}$$

where θ is the parameter of the Gumbel distribution; Ω_i^k is the set of varieties of good k actually produced by country i; and $m_i^k \equiv \sum_{i' \neq i} x_{i'i}^k \Big/ \sum_{i'=1}^{I} x_{i'i}^k$ is the import penetration ratio in country i and industry k. We derive a similar result under the assumptions of Theorem 2.

According to Equation (14), the import penetration ratio is a sufficient statistic for the extent of the selection bias. As the import penetration ratio goes down, more varieties are produced in country i, which decreases the measurement error associated with selection. In the extreme case where $m_i^k = 0$, country i is under autarky (in that particular industry), and the selection bias disappears.

Using Assumption A1 and Equation (14), we can rearrange Equation (13) as

$$\ln x_{ij}^k = \beta_{ij} + \beta_j^k + \beta \ln \widehat{a}_i^k - \beta' \ln\left(1 - m_i^k\right) + \varepsilon_{ij}^k$$

Table 3: Year-by-Year OLS Regressions with Selection
(Dependent Variable: lnx)

Variable	**2001**	**2000**	**1999**	**1998**	**1997**	**1996**	**1995**
ln a	-0.94	-0.68	-0.84	-0.99	-1.39	-1.17	-1.18
	(-20.05)***	(-12.09)***	(-15.29)***	(-17.31)***	(-22.59)***	(-19.84)***	(-19.07)***
ln (1-m)	0.33	0.39	0.43	0.31	0.17	0.30	0.21
	(14.90)***	(11.62)***	(14.39)***	(11.73)***	(6.54)***	(9.81)***	(9.12)***
Exporter-Importer FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry-Importer FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	10067	10173	10226	10147	10232	10357	10479
R^2	0.85	0.84	0.84	0.84	0.84	0.84	0.83
Variable	**1994**	**1993**	**1992**	**1991**	**1990**	**1989**	**1988**
ln a	-1.09	-0.90	-1.08	-1.12	-0.63	-0.83	-0.60
	(-15.42)***	(-14.71)***	(-15.23)***	(-16.20)***	(-9.52)***	(-11.74)***	(-7.12)***
ln (1-m)	0.14	0.41	0.27	0.09	0.91	0.31	0.25
	(4.85)***	(7.67)***	(7.79)***	(2.93)***	(15.62)***	(8.73)***	(6.06)***
Exporter-Importer FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry-Importer FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	8768	8711	8296	7615	7143	7088	6270
R^2	0.84	0.84	0.85	0.84	0.85	0.84	0.83

Note: Values of t-statistics in parentheses, calculated from heteroskedasticity-consistent (White) standard errors
* Significant at 10% confidence level
** Significant at 5% confidence level
*** Significant at 1% confidence level

with $\ln \widehat{a}_i^k \equiv \ln E\left[a_i^k(\omega)\,|\omega \in \Omega_i^k\right]$ and $\beta' = \beta/\theta$. Table 3 reports the OLS estimates of β when $\ln\left(1 - m_i^k\right)$ is included on the right hand side.[11] After controlling for selection, our estimates of β increase slightly, but remain negative and statistically significant. Similarly, the impact of $\ln\left(1 - m_i^k\right)$ has the right sign and is statistically significant.

From a quantitative standpoint, we need to acknowledge that our OLS estimates of β and β' appear to be too small. Under Eaton and Kortum's (2002) distributional assumptions, $\widehat{\beta}$ should be equal to the parameter of the Gumbel θ, which they have estimated at 8.27, and $\widehat{\beta}'$ should be equal to 1. This is not what we observe in the data. Interestingly, however, the ratio of our estimates, $\widehat{\beta}/\widehat{\beta}'$, is much more in line with Eaton and Kortum (2002). In 1997, the year for which relative producer prices have been collected, we get $\widehat{\beta}/\widehat{\beta}' = 8.21$. Taken together, these results are consistent with the idea that *both* productivity and import penetration ratios are poorly measured in practice.[12]

[11] Import penetration ratios are directly available in the OECD STAN database.

[12] In STAN, potential sources of error in the measurement of import penetration ratios include: (*i*) the existence of "transit trade"; (*ii*) the underreporting of secondary activities in industrial surveys; and (*iii*) the misclassifications due to conversions from product-based trade statistics to activity-based industry statistics.

4.5. **Relation to the previous empirical literature.** As mentioned in the introduction, previous Ricardian "tests" were remarkably successful. In light of this evidence, it is perhaps not too surprising to uncover, as we just did, a positive association between measured productivity and trade flows in the data.[13]

Nevertheless, we believe that the tight connection between the theory and the empirical analysis that our paper offers is a significant step beyond the existing literature. First, we do not have to rely on ad-hoc measures of export performance such as total exports towards the rest of the world (MacDougall, 1951; Stern, 1962); total exports to third markets (Balassa, 1963); or bilateral net exports (Golub and Hsieh, 2000). The theory tells us exactly what the dependent variable in the cross-industry regressions ought to be: ln(exports), disaggregated by exporting and importing countries. Second, the careful introduction of country and industry fixed effects allows us to move away from the bilateral comparisons inspired by the two-country model, and in turn, to take advantage of a much richer data set. Third, our clear theoretical foundations make it possible to discuss the economic origins of the error terms—measurement errors in trade flows or unobserved trade barriers—and as a result, the plausibility of our orthogonality conditions.

Of course, one might argue that the model developed in this paper—Assumptions A1-A4—is not the *only* way to bring the Ricardian model to the data. For example, we could also obtain Equation (13) by directly imposing Armington's preferences.[14] While this is certainly true, the attractiveness of our approach lies in the weakness of the assumptions under which Equation (13) is derived. As long as there are stochastic productivity differences within each industry, our analysis demonstrates that many of the assumptions usually invoked to rationalize cross-industry regressions—either on preferences or on transport costs—can be relaxed. Put simply, our paper may not offer researchers brand new regressions to run, but we hope it can make them more comfortable running them.

[13]In terms of magnitude, our estimates lie between those of the early Ricardian "tests"—MacDougall (1951), Stern (1962), and Balassa (1963)—and the more recent findings of Golub and Hsieh (2000). Using US and UK data, MacDougall (1951), Stern (1962), and Balassa (1963) find elasticities of exports with respect to average unit labor requirements around -1.6. By contrast, the highest elasticity estimated by Golub and Hsieh (2000) is equal to -0.37; see Table 2 p228.

[14]Deardorff (2004) analyzes the impact of production and trade costs on the net direction of countries' bilateral trade with a model developed along these lines.

In particular, we think that our theoretical approach may be fruitfully applied to more general environments, where labor is not the only factor of production. The basic idea, already suggested by Bhagwati (1964), is to reinterpret differences in a_i^k as differences in total factor productivity. With multiple factors of production, the volume of exports would be a function of both technological differences, captured by a_i^k, and differences in relative factor prices. The rest of our analysis would remain unchanged; see Appendix F.

5. Concluding Remarks

The Ricardian model has long been perceived has a useful pedagogical tool with, ultimately, little empirical content. Over the last twenty years, the Heckscher-Ohlin model, which emphasizes the role of cross-country differences in factor endowments, has generated a considerable amount of empirical work ; see e.g. Bowen, Leamer, and Sveikauskas (1987), Trefler (1993), Trefler (1995), Davis and Weinstein (2001), and Schott (2004). The Ricardian model, which emphasizes productivity differences, almost none.

The main reason behind this lack of popularity is not the existence of strong beliefs regarding the relative importance of factor endowments and technological considerations. Previous empirical work on the Heckscher-Ohlin model unambiguously shows that technology matters. It derives instead from the obvious mismatch between the real world and the extreme predictions (and assumptions) of the standard Ricardian model. In the words of Leamer and Levinsohn (1995), "[it] is just too simple."

Although the deficiencies of the Ricardian model have not lead to the disappearance of technological considerations from the empirical literature, they have had a strong influence on *how* the relationship between technology and trade has been studied. In the Heckscher-Ohlin-Vanek literature—with or without technological differences—the factor content of trade remains the main variable of interest. Building on the seminal work of Eaton and Kortum (2002), our paper develops a "robust" theoretical framework that puts back productivity differences at the forefront of the analysis.

References

Acemoglu, D., P. Antras, and E. Helpman (2007): "Contracts and Technology Adoption," *forthcoming American Economic Review*.

Al-Najjar, N. (2004): "Aggregation and the Law of Large Numbers in Large Economies," *Games and Economic Behavior*, pp. 1–35.

Anderson, J. A., and E. V. Wincoop (2004): "Trade Costs," *Journal of Economic Literature*, pp. 691–751.

Antras, P., and E. Helpman (2004): "Global Sourcing," *Journal of Political Economy*, pp. 552–580.

Balassa, B. (1963): "An Empirical Demonstration of Comparative Cost Theory," *Review of Economics and Statistics*, pp. 231–238.

——— (1965): "Trade Liberalization and Revealed Comparative Advantage," *The Manchester School of Economic and Social Studies*, 33, 99–123.

Baldwin, R. E. (1971): "Determinants of the Commodity Structure of U.S. Trade," *American Economic Review*, 61(1), 126–46.

Bernard, A. B., J. Eaton, J. B. Jensen, and S. Kortum (2003): "Plants and Productivity in International Trade," *The American Economic Review*, pp. 1268–1290.

Bernard, A. B., S. J. Redding, and P. K. Schott (2006): "Comparative Advantage and Heterogeneous Firms," *(forthcoming) The Review of Economic Studies.*

Bhagwati, J. (1964): "The Pure Theory of International Trade: A Survey," *The Economic Journal*, pp. 1–84.

Billingsley, P. (1995): *Probability and Measure*. John Wiley & Sons, Inc.

Bowen, H. P., E. E. Leamer, and L. Sveikauskas (1987): "Multicountry, Multifactor Tests of the Factor Abundance Theory," *The American Economic Review*, pp. 791–809.

Chaney, T. (2007): "Distorted Gravity: The Intensive and Extensive Margins of International Trade," *mimeo University of Chicago.*

Chor, D. (2006): "Unpacking Sources of Comparative Advantage: A Quantitative Approach," *mimeo Harvard University.*

Costinot, A. (2005): "Three Essays on Institutions and Trade," *Ph.D. Dissertation, Princeton University.*

——— (2006): "On the Origins of Comparative Advantage," *mimeo University of California, San Diego.*

Cuñat, A., and M. Melitz (2006): "Labor Market Flexibility and Comparative Advantage," *mimeo Harvard University.*

DAVIDSON, J. (1994): *Stochastic Limit Theory*. Oxford University Press, Oxford.

DAVIS, D. R., AND D. E. WEINSTEIN (2001): "An Account of Global Factor Trade," *The American Economic Review*, pp. 1423–1453.

DEARDORFF, A. V. (1980): "The General Validity of the Law of Comparative Advantage," *Journal of Political Economy*, pp. 941–957.

——— (2004): "Local Comparative Advantage: Trade Costs and the Pattern of Trade," *mimeo University of Michigan*.

——— (2005): "How Robust is Comparative Advantage," *mimeo University of Michigan*.

DORNBUSCH, R., S. FISCHER, AND P. SAMUELSON (1977): "Comparative Advantage, Trade, and Payments in a Ricardian Model with a Continuum of Goods," *American Economic Review*, 67, 823–839.

EATON, J., AND S. KORTUM (2002): "Technology, Geography, and Trade," *Econometrica*, 70, 1741–1779.

GHIRONI, F., AND M. MELITZ (2005): "International Trade and Macroeconomic Dynamics with Heterogeneous Firms," *Quarterly Journal of Economics*, pp. 865–915.

GOLUB, S. S., AND C.-T. HSIEH (2000): "Classical Ricardian Theory of Comparative Advantage Revisited," *Review of International Economics*, pp. 221–234.

HARRIGAN, J. (1997): "Technology, Factor Supplies, and International Specialization: Estimating the Neo-Classical Model," *The American Economic Review*, pp. 475–494.

——— (2005): "Airplanes and Comparative Advantage," *NBER working paper*.

HELPMAN, E., AND P. R. KRUGMAN (1985): *Market Structure and Foreign Trade*. MIT Press.

HELPMAN, E., M. MELITZ, AND S. YEAPLE (2004): "Export versus FDI with Heterogeneous Firms," *The American Economic Review*, pp. 300–316.

JONES, R. W. (1961): "Comparative Advantage and the Theory of Tariffs: A Multi-Country, Multi-Commodity Model," *The Review of Economic Studies*, 28(3), 161–175.

LEAMER, E., AND J. LEVINSOHN (1995): "International Trade Theory: The Evidence," in *Handbook of International Economics*, ed. by G. Grossman, and K. Rogoff, vol. 3, pp. 1139–1194. Elsevier Science, New York.

LEVCHENKO, A. (2007): "Institutional Quality and International Trade," *forthcoming Review of Economic Studies*.

MacDougall, G. (1951): "British and American Exports: A Study Suggested by the Theory of Comparative Costs, Part," *Economic Journal*, pp. 697–724.

Matsuyama, K. (2005): "Credit Market Imperfections and Patterns of International Trade and Capital Flows," *Journal of the European Economic Association*, pp. 714–723.

Melitz, M. (2003): "The Impact of Trade on Intra-Industry Reallocations and Aggregate Inudstry Productivity," *Econometrica*, pp. 1695–1725.

Morrow, P. (2006): "East is East and West is West: A Ricardian-Heckscher-Ohlin Model of Comparative Advantage," *mimeo Michigan University*.

Nunn, N. (2007): "Relationship Specificity, Incomplete Contracts and the Pattern of Trade," *forthcoming Quarterly Journal of Economics*.

Petri, P. A. (1980): "A Ricardian Model of Market Sharing," *Journal of International Economics*, pp. 201–211.

Romalis, J. (2004): "Factor Proportions and the Structure of Commodity Trade," *The American Economic Review*, pp. 67–97.

Schott, P. (2004): "Across-Product versus Within-Product Specialization in International Trade," *Quarterly Journal of Economics*, pp. 647–678.

Stern, R. M. (1962): "British and American Productivity and Comparative Costs in International Trade," *Oxford Economic Papers*, pp. 275–303.

Trefler, D. (1993): "International Factor Price Differences: Leontief was Right!," *Journal of Political Economy*, pp. 961–987.

——— (1995): "The Case of the Missing Trade and Other Mysteries," *The American Economic Review*, pp. 1029–1046.

Vogel, J. (2007): "Institutions and Moral Hazard in Open Economies," *forthcoming Journal of International Economics*.

Wilson, C. A. (1980): "On the General Structure of Ricardian Models with a Continuum of Goods: Applications to Growth, Tariff Theory, and Technical Change," *Econometrica*, pp. 1675–1702.

Appendix A: Proofs of Theorems 1 and 2

Proof of Theorem 1. Fix $i \neq j$; by the definition of total exports x_{ij}^k and Assumption A4(i), we have

$$\begin{aligned} x_{ij}^k &= \sum_{\omega=1}^{N^k} x_{ij}(\omega) \cdot \mathbb{1}\left\{c_{ij}^k(\omega) = \min_{1\leq i'\leq I} c_{i'j}^k(\omega)\right\} \\ &= \frac{e_j^k}{\left(p_j^k\right)^{1-\sigma}} \sum_{\omega=1}^{N^k} p_j^k(\omega)^{1-\sigma} \cdot \mathbb{1}\left\{c_{ij}^k(\omega) = \min_{1\leq i'\leq I} c_{i'j}^k(\omega)\right\} \\ &= e_j^k \left[\frac{1}{N^k}\sum_{\omega'=1}^{N^k} p_j^k(\omega')^{1-\sigma}\right]^{-1} \left[\frac{1}{N^k}\sum_{\omega=1}^{N^k} p_j^k(\omega)^{1-\sigma} \cdot \mathbb{1}\left\{c_{ij}^k(\omega) = \min_{1\leq i'\leq I} c_{i'j}^k(\omega)\right\}\right], \end{aligned}$$

where the function $\mathbb{1}\{\cdot\}$ is the standard indicator function, i.e. for any event A, we have $\mathbb{1}\{A\} = 1$ if A true, and $\mathbb{1}\{A\} = 0$ otherwise. By Assumption A1, $u_i^k(\omega)$ is independent and identically distributed (i.i.d.) across varieties so same holds for $c_{ij}^k(\omega)$. In addition, $u_i^k(\omega)$ is i.i.d. across countries so $\mathbb{1}\left\{c_{ij}^k(\omega) = \min_{1\leq i'\leq I} c_{i'j}^k(\omega)\right\}$ is i.i.d. across varieties as well. This implies that $p_j^k(\omega)^{1-\sigma}$ and $p_j^k(\omega)^{1-\sigma} \cdot \mathbb{1}\left\{c_{ij}^k(\omega) = \min_{1\leq i'\leq I} c_{i'j}^k(\omega)\right\}$ are i.i.d. across varieties. Moreover, by Assumption A4(ii), $E\left[p_j^k(\omega)^{1-\sigma}\right] < \infty$ so we can use the strong law of large numbers for i.i.d. random variables (e.g. Theorem 22.1 in Billingsley (1995)) to show that

$$\frac{1}{N^k}\sum_{\omega'=1}^{N^k} \left[p_j^k(\omega')\right]^{1-\sigma} \overset{a.s.}{\to} E\left[p_j^k(\omega)^{1-\sigma}\right], \tag{15}$$

as $N^k \to \infty$. Note that $a_i^k > 0$, $d_{ij}^k \geq 1$ ensure that $c_{ij}^k > 0$ whenever $\mathrm{w}_i > 0$; hence $E\left[p_j^k(\omega)^{1-\sigma}\right] > 0$. Similarly, Assumption A4(ii) implies that

$$E\left[p_j^k(\omega)^{1-\sigma} \cdot \mathbb{1}\left\{c_{ij}^k(\omega) = \min_{1\leq i'\leq I} c_{i'j}^k(\omega)\right\}\right] < \infty,$$

so we can again use the strong law of large numbers for i.i.d. random variables (e.g. Theorem 22.1 in Billingsley (1995)) to show that

$$\begin{aligned} &\frac{1}{N^k}\sum_{\omega=1}^{N^k} p_j^k(\omega)^{1-\sigma} \cdot \mathbb{1}\left\{c_{ij}^k(\omega) = \min_{1\leq i'\leq I} c_{i'j}^k(\omega)\right\} \\ &\qquad \overset{a.s.}{\to} E\left[p_j^k(\omega)^{1-\sigma} \cdot \mathbb{1}\left\{c_{ij}^k(\omega) = \min_{1\leq i'\leq I} c_{i'j}^k(\omega)\right\}\right], \end{aligned} \tag{16}$$

as $N^k \to \infty$. Combining Equations (16) and (15) together with the continuity of the inverse function $x \mapsto x^{-1}$ away from 0, yields by continuous mapping theorem (e.g. Theorem 18.10

(i) in Davidson (1994))

$$\left[\frac{1}{N^k}\sum_{\omega'=1}^{N^k} p_j^k(\omega')^{1-\sigma}\right]^{-1}\left[\frac{1}{N^k}\sum_{\omega=1}^{N^k} p_j^k(\omega)^{1-\sigma}\cdot \mathbb{I}\left\{c_{ij}^k(\omega)=\min_{1\le i'\le I} c_{i'j}^k(\omega)\right\}\right] \tag{17}$$
$$\xrightarrow{a.s.} \left\{E\left[p_j^k(\omega)^{1-\sigma}\right]\right\}^{-1}\left\{E\left[p_j^k(\omega)^{1-\sigma}\cdot \mathbb{I}\left\{c_{ij}^k(\omega)=\min_{1\le i'\le I} c_{i'j}^k(\omega)\right\}\right]\right\},$$

as $N^k\to\infty$. Note that the quantities in Equation (17) are positive; hence, applying again the continuous mapping theorem (e.g. Theorem 18.10 (i) in Davidson (1994)) to their logarithm we get, with probability one,

$$\ln x_{ij}^k \to \ln e_j^k + \ln E\left[p_j^k(\omega)^{1-\sigma}\cdot \mathbb{I}\left\{c_{ij}^k(\omega)=\min_{1\le i'\le I} c_{i'j}^k(\omega)\right\}\right] - \ln E\left[p_j^k(\omega)^{1-\sigma}\right], \tag{18}$$

as $N^k\to\infty$.

Consider $H_i(c_{1j}^k,...,c_{Ij}^k)\equiv E\left[p_j^k(\omega)^{1-\sigma}\cdot \mathbb{I}\left\{c_{ij}^k(\omega)=\min_{1\le i'\le I} c_{i'j}^k(\omega)\right\}\right]$. Assumptions A1, A3 and straightforward computations yield

$$H_i(c_{1j}^k,...,c_{Ij}^k)=\left(c_{ij}^k\right)^{1-\sigma}\int_{-\infty}^{+\infty}\exp\left[(1-\sigma)u\right]f(u)\prod_{n\neq i}\left[1-F(\ln c_{ij}^k-\ln c_{nj}^k+u)\right]du. \tag{19}$$

where we let $f(u)\equiv F'(u)$.

Using Equation (19) together with the expressions for the (negative) Gumbel distribution and density, we then have

$$\begin{aligned}
&E\left[p_j^k(\omega)^{1-\sigma}\cdot \mathbb{I}\left\{c_{ij}^k(\omega)=\min_{1\le i'\le I} c_{i'j}^k(\omega)\right\}\right]\\
&=(c_{ij}^k)^{1-\sigma}\int_{-\infty}^{+\infty}\theta\exp\left\{(\theta+1-\sigma)u-\mathsf{e}-\left[1+\sum\nolimits_{i'\neq i}(c_{ij}^k/c_{i'j}^k)^\theta\right]\exp\left(\theta u-\mathsf{e}\right)\right\}du\\
&=(c_{ij}^k)^{1-\sigma}\exp\left(-\mathsf{e}\frac{\sigma-1}{\theta}\right)\Gamma(\frac{\theta+1-\sigma}{\theta})\left[1+\sum\nolimits_{i'\neq i}(c_{ij}^k/c_{i'j}^k)^\theta\right]^{-(\theta+1-\sigma)/\theta}\\
&=\exp\left(-\mathsf{e}\frac{\sigma-1}{\theta}\right)\Gamma(\frac{\theta+1-\sigma}{\theta})\frac{(c_{ij}^k)^{-\theta}}{\left[\sum\nolimits_{i'=1}^{I}(c_{i'j}^k)^{-\theta}\right]^{(\theta+1-\sigma)/\theta}},
\end{aligned} \tag{20}$$

where the second equality uses the change of variable $v\equiv\left(1+\sum\nolimits_{i'\neq i}(c_{ij}^k/c_{i'j}^k)^\theta\right)\exp(\theta u-\mathsf{e})$, and where $\Gamma(\cdot)$ denotes the Gamma function, $\Gamma(t)=\int_0^{+\infty}v^{t-1}\exp(-v)dv$ for any $t>0$. Note that

$$E\left[p_j^k(\omega)^{1-\sigma}\right] = \sum_{i=1}^{I} E\left[p_j^k(\omega)^{1-\sigma} \cdot \mathbb{I}\left\{c_{ij}^k(\omega) = \min_{1\le i'\le I} c_{i'j}^k(\omega)\right\}\right],$$

so that by using Equation (20) we get

$$E\left[p_j^k(\omega)^{1-\sigma}\right] = \exp\left(-\mathsf{e}\frac{\sigma-1}{\theta}\right)\Gamma(\frac{\theta+1-\sigma}{\theta})\frac{1}{\left[\sum_{i'=1}^{I}(c_{i'j}^k)^{-\theta}\right]^{(1-\sigma)/\theta}},$$

and hence

$$\ln x_{ij}^k \simeq \ln e_j^k - \theta \ln c_{ij}^k - \ln\left(\sum_{i'=1}^{I}(c_{i'j}^k)^{-\theta}\right). \tag{21}$$

for N^k large. Combining the above with the definition of $c_{ij}^k = d_{ij}^k \cdot \mathrm{w}_i \cdot a_i^k$ and Assumption A2, then gives

$$\ln x_{ij}^k \simeq \theta_{ij} + \theta_j^k - \theta \ln a_i^k,$$

for N^k large, where we have let $\theta_{ij} \equiv -\theta \ln(d_{ij} \cdot \mathrm{w}_i)$ and $\theta_j^k \equiv \ln e_j^k - \theta \ln d_j^k - \ln\left(\sum_{i'=1}^{I}(c_{i'j}^k)^{-\theta}\right)$. This completes the proof of Theorem 1. □

Proof of Theorem 2. Since Assumption A1-A4 hold, the results of Theorem 1 apply. In particular, we know that, with probability one

$$\ln x_{ij}^k \to \ln e_j^k + \ln E\left[p_j^k(\omega)^{1-\sigma} \cdot \mathbb{I}\left\{c_{ij}^k(\omega) = \min_{1\le i'\le I} c_{i'j}^k(\omega)\right\}\right] - \ln E\left[p_j^k(\omega)^{1-\sigma}\right],$$

as $N^k \to \infty$. We now approximate $\ln \tilde{H}_i(c_{1j}^k, ..., c_{Ij}^k) \equiv \ln H_i(c_{1j}^k, ..., c_{Ij}^k) - (1-\sigma)\ln c_{ij}^k$ obtained from Equation (19) by its first order Taylor series around the symmetric case $\ln c_{1j}^k = ... = \ln c_{Ij}^k = \ln c$. Without loss of generality, we choose units of account in each industry k such that $\ln c = 0$. We have

$$\tilde{H}_i(c_{1j}^k, ..., c_{Ij}^k)\Big|_{(0,...,0)} = \int_{-\infty}^{+\infty} \exp\left[(1-\sigma)u\right] f(u)\left[1-F(u)\right]^{I-1} du, \tag{22}$$

$$\left.\frac{\partial \tilde{H}_i(c_{1j}^k, ..., c_{Ij}^k)}{\partial \ln c_{ij}^k}\right|_{(0,...,0)} = -(I-1)\int_{-\infty}^{+\infty} \exp\left[(1-\sigma)u\right] f^2(u)\left[1-F(u)\right]^{I-2} du, \tag{23}$$

and, for $i' \neq i$,

$$\left.\frac{\partial \tilde{H}_i(c_{1j}^k, ..., c_{Ij}^k)}{\partial \ln c_{i'j}^k}\right|_{(0,...,0)} = \int_{-\infty}^{+\infty} \exp\left[(1-\sigma)u\right] f^2(u)\left[1-F(u)\right]^{I-2} du. \tag{24}$$

Let

$$\kappa \equiv \int_{-\infty}^{+\infty} \exp\left[(1-\sigma)u\right] f(u) \left[1-F(u)\right]^{I-1} du,$$

and

$$\delta \equiv \kappa^{-1} \left[\int_{-\infty}^{+\infty} \exp\left[(1-\sigma)u\right] f^2(u) \left[1-F(u)\right]^{I-2} du \right].$$

Combining Equations (22), (23), and (24), we then get

$$\begin{aligned} \ln H_i(c_{1j}^k, ..., c_{Ij}^k) &= \ln \kappa + (1-\sigma) \ln c_{ij}^k - (I-1)\,\delta \ln c_{ij}^k + \delta \sum_{i' \neq i} \ln c_{i'j}^k + o\left(\left\|\ln c_j^k\right\|\right) \\ &= \ln \kappa - (\delta I + \sigma - 1) \ln c_{ij}^k + \delta \sum_{i'=1}^{I} \ln c_{i'j}^k + o\left(\left\|\ln c_j^k\right\|\right), \end{aligned} \tag{25}$$

where $\left\|\ln c_j^k\right\|^2 = \sum_{i'=1}^{I} \left[\ln c_{i'j}^k\right]^2$ denotes the usual L_2-norm, and $\delta > 0$ only depends on $f(\cdot)$, $F(\cdot)$, σ and I. Combining Equation (25) with the definition of $c_{ij}^k = d_{ij}^k \cdot \mathrm{w}_i \cdot a_i^k$ and Assumption A2, then gives

$$\ln H_i(c_{1j}^k, ..., c_{Ij}^k) \simeq \gamma_{ij} + g_j^k - \gamma \ln a_i^k, \tag{26}$$

where

$$\begin{aligned} \gamma_{ij} &\equiv \ln \kappa - (\delta I + \sigma - 1) \ln(d_{ij} \cdot \mathrm{w}_i) \\ g_j^k &\equiv -(\delta I + \sigma - 1) \ln d_j^k + \delta \sum_{i'=1}^{I} \ln c_{i'j}^k \\ \gamma &\equiv \delta I + \sigma - 1 \end{aligned}$$

Note that γ_{ij} does not depend on the good index k, g_j^k does not depend on the country index i, and $\gamma > 0$ is a positive constant which only depends on $f(\cdot)$, $F(\cdot)$, σ and I. Combining Equations (18) and (26) then yields

$$\ln x_{ij}^k \simeq \gamma_{ij} + \gamma_j^k - \gamma \ln a_i^k,$$

for N^k large, where we have let $\gamma_j^k \equiv \ln e_j^k + g_j^k - \ln E\left[p_j^k(\omega)^{1-\sigma}\right]$. This completes the proof of Theorem 2. □

Appendix B: Bertrand Competition

Instead of Assumption A3, we now consider:

A3'. *In any country j, the price $p_j^k(\omega)$ paid by buyers of variety ω of good k is*

$$p_j^k(\omega) = \min\left\{\min_{i' \neq i^*}\left[c_{i'j}^k(\omega)\right], \overline{m} c_{i^*j}^k(\omega)\right\},$$

where $c_{i^*j}^k(\omega) = \min_{1 \leq i' \leq I} c_{ij}^k(\omega)$ and $\overline{m} = \sigma/(\sigma - 1)$ is the monopoly markup.

This is in the spirit of Bernard, Eaton, Jensen, and Kortum (2003): the producer with the minimum cost may either charge the cost of its closest competitor or the monopoly price. We then have the following result:

Theorem 4. *Suppose that Assumptions A1, A2, A3', and A4 hold. In addition, assume that the number of varieties N^k of any good k is large, and that technological differences across exporters are small: $c_{1j}^k \simeq ... \simeq c_{Ij}^k$. Then, for any exporter i, any importer $j \neq i$, and any good k,*

$$\ln x_{ij}^k \simeq \widetilde{\gamma}_{ij} + \widetilde{\gamma}_j^k - \widetilde{\gamma} \ln a_i^k. \tag{27}$$

where $\widetilde{\gamma} > -(\sigma - 1)/(I - 1)$.

Under Bertrand competition, the qualitative insights of Theorem 2 remain valid, albeit in a weaker form. We obtain new importer–exporter and importer–industry fixed effects, $\widetilde{\gamma}_{ij}$ and $\widetilde{\gamma}_j^k$, and a new parameter $\widetilde{\gamma}$ constant across countries and industries. However, the restriction $\widetilde{\gamma} > -(\sigma - 1)/(I - 1)$ is less stringent than in the case of perfect competition. When $\sigma \to 1$, that is when varieties become perfect substitutes, or when $I \to +\infty$, that is when the number of exporters is very large, this collapses to: $\widetilde{\gamma} \geq 0$.

Proof of Theorem 4. Compared to the proof of Theorem 2, the only difference comes from the expression of $H_i(c_{1j}^k, ..., c_{Ij}^k) = E\left[p_j^k(\omega)^{1-\sigma} \cdot \mathbb{1}\left\{c_{ij}^k(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^k(\omega)\right\}\right]$. Assumptions

A1, A3' and straightforward computations now yield

$$H_i(c_{1j}^k,...,c_{Ij}^k) = \left(c_{ij}^k\right)^{1-\sigma} \int_{-\infty}^{+\infty} f(u_1)du_1 \int_{u_1}^{+\infty} \left[\min\left(\exp u_2, \overline{m}\exp u_1\right)\right]^{1-\sigma} \cdot \sum_{i'\neq i} \left\{ \prod_{i''\neq i,i'} \left[1-F(\ln c_{ij}^k - \ln c_{i''j}^k + u_2)\right] f(\ln c_{ij}^k - \ln c_{i'j}^k + u_2) \right\} du_2. \tag{28}$$

where we let $f(u) \equiv F'(u)$.

As previously, we approximate $\ln \tilde{H}_i(c_{1j}^k,...,c_{Ij}^k) \equiv \ln H_i(c_{1j}^k,...,c_{Ij}^k) - (1-\sigma)\ln c_{ij}^k$, obtained from Equation (28), by its first order Taylor series around the symmetric case $\ln c_{1j}^k = ... = \ln c_{Ij}^k = 0$. We have

$$\tilde{H}_i(c_{1j}^k,...,c_{Ij}^k)\Big|_{(0,...,0)} = \int_{-\infty}^{+\infty} f(u_1)\,du_1 \int_{u_1}^{+\infty} \left[\min\left(\exp u_2, \overline{m}\exp u_1\right)\right]^{1-\sigma} \cdot (I-1)\left[1-F(u_2)\right]^{I-2} f(u_2)du_2, \tag{29}$$

$$\left.\frac{\partial \tilde{H}_i(c_{1j}^k,...,c_{Ij}^k)}{\partial \ln c_{ij}^k}\right|_{(0,...,0)} = -(I-1)\int_{-\infty}^{+\infty} f(u_1)\,du_1 \int_{u_1}^{+\infty} \left[\min\left(\exp u_2, \overline{m}\exp u_1\right)\right]^{1-\sigma} \cdot \left\{-f'(u_2)\left[1-F(u_2)\right]^{I-2} + (I-2)\,f^2(u_2)\left[1-F(u_2)\right]^{I-3}\right\} du_2, \tag{30}$$

and, for $i' \neq i$,

$$\left.\frac{\partial H_i(c_{1j}^k,...,c_{Ij}^k)}{\partial \ln c_{i'j}^k}\right|_{(0,...,0)} = \int_{-\infty}^{+\infty} f(u_1)\,du_1 \int_{u_1}^{+\infty} \left[\min\left(\exp u_2, \overline{m}\exp u_1\right)\right]^{1-\sigma} \cdot \left\{-f'(u_2)\left[1-F(u_2)\right]^{I-2} + (I-2)\,f^2(u_2)\left[1-F(u_2)\right]^{I-3}\right\} du_2. \tag{31}$$

Let then

$$\kappa \equiv (I-1)\int_{-\infty}^{+\infty} f(u_1)\,du_1 \int_{u_1}^{+\infty} \left[\min\left(\exp u_2, \overline{m}\exp u_1\right)\right]^{1-\sigma} \left[1-F(u_2)\right]^{I-2} f(u_2)du_2, \tag{32}$$

and

$$\delta \equiv \kappa^{-1} \int_{-\infty}^{+\infty} f(u_1)\, du_1 \int_{u_1}^{+\infty} \left[\min\left(\exp u_2, \overline{m} \exp u_1\right)\right]^{1-\sigma} \cdot$$

$$\left\{ -f'(u_2)\left[1-F(u_2)\right]^{I-2} + (I-2)\, f^2(u_2)\left[1-F(u_2)\right]^{I-3} \right\} du_2. \tag{33}$$

Combining Equations (29), (30), and (31), we get

$$\begin{aligned}
\ln H_i(c_{1j}^k, ..., c_{Ij}^k) &= \ln \kappa + (1-\sigma) \ln c_{ij}^k - (I-1)\,\delta \ln c_{ij}^k + \delta \sum_{i' \neq i} \ln c_{i'j}^k + o\left(\left\|\ln c_j^k\right\|\right) \\
&= \ln \kappa - (\delta I + \sigma - 1) \ln c_{ij}^k + \delta \sum_{i'=1}^{I} \ln c_{i'j}^k + o\left(\left\|\ln c_j^k\right\|\right),
\end{aligned}$$

where $\left\|\ln c_j^k\right\|^2 = \sum_{i'=1}^{I}\left[\ln c_{i'j}^k\right]^2$ as previously, and δ only depends on $f(\cdot)$, $F(\cdot)$, σ and I. Let

$$\widetilde{\gamma} \equiv \delta I + \sigma - 1$$

It remains to be shown that $\widetilde{\gamma} > -(\sigma-1)/(I-1)$.

For this, let $I(u_1) \equiv \int_{u_1}^{+\infty} \left[\min\left(\exp u_2, \overline{m} \exp u_1\right)\right]^{1-\sigma} f'(u_2)\left[1-F(u_2)\right]^{I-2} du_2$. We can rearrange $I(u_1)$ as

$$\begin{aligned}
I(u_1) = &\int_{u_1}^{u1+\ln \overline{m}} \left[\exp u_2\right]^{1-\sigma} f'(u_2)\left[1-F(u_2)\right]^{I-2} du_2 \\
&+ \left[\overline{m} \exp u_1\right]^{1-\sigma} \int_{u1+\ln \overline{m}}^{+\infty} f'(u_2)\left[1-F(u_2)\right]^{I-2} du_2 \\
&= -\left[\exp u_1\right]^{1-\sigma} f(u_1)\left[1-F(u_1)\right]^{I-2} \\
&- (1-\sigma) \int_{u_1}^{u1+\ln \overline{m}} \left[\exp u_2\right]^{1-\sigma} f(u_2)\left[1-F(u_2)\right]^{I-2} du_2 \\
&+ (I-2) \int_{u_1}^{+\infty} \left[\min\left(\exp u_2, \overline{m} \exp u_1\right)\right]^{1-\sigma} f^2(u_2)\left[1-F(u_2)\right]^{I-3} du_2
\end{aligned} \tag{34}$$

where the second equality uses a simple integration by parts. Combining Equations (33) and (34), we then get

$$\delta = \kappa^{-1} \int_{-\infty}^{+\infty} f(u_1)\, du_1 \left\{ [\exp u_1]^{1-\sigma} f(u_1) [1 - F(u_1)]^{I-2} \right.$$

$$\left. - (\sigma - 1) \int_{u_1}^{u1+\ln \overline{m}} [\exp u_2]^{1-\sigma} f(u_2) [1 - F(u_2)]^{I-2} du_2 \right\}. \tag{35}$$

Using Equations (32) and (35), we then have

$$\begin{aligned}
&(I-1)\delta + \sigma - 1 \\
&= (I-1)\kappa^{-1} \int_{-\infty}^{+\infty} [\exp u_1]^{1-\sigma} f^2(u_1) [1 - F(u_1)]^{I-2} du_1 \\
&- (I-1)(\sigma-1)\kappa^{-1} \int_{-\infty}^{+\infty} f(u_1)\, du_1 \int_{u_1}^{u1+\ln \overline{m}} [\exp u_2]^{1-\sigma} f(u_2) [1 - F(u_2)]^{I-2} du_2 \\
&+ (I-1)(\sigma-1)\kappa^{-1} \int_{-\infty}^{+\infty} f(u_1)\, du_1 \int_{u_1}^{+\infty} [\min(\exp u_2, \overline{m} \exp u_1)]^{1-\sigma} [1 - F(u_2)]^{I-2} f(u_2) du_2 \\
&= (I-1)\kappa^{-1} \int_{-\infty}^{+\infty} [\exp u_1]^{1-\sigma} f^2(u_1) [1 - F(u_1)]^{I-2} du_1 \\
&+ (I-1)(\sigma-1)\kappa^{-1} \int_{-\infty}^{+\infty} f(u_1)\, du_1 \int_{u_1+\ln \overline{m}}^{+\infty} [\overline{m} \exp u_1]^{1-\sigma} [1 - F(u_2)]^{I-2} f(u_2) du_2,
\end{aligned}$$

which is positive by inspection. Hence, writing $\widetilde{\gamma} = I(I-1)^{-1}[(I-1)\delta + \sigma - 1] - (I-1)^{-1}(\sigma - 1)$ and using $(I-1)\delta + \sigma - 1 > 0$ yields the desired result: $\widetilde{\gamma} > -(I-1)^{-1}(\sigma - 1)$. $\square$

Appendix C: The Gumbel Distribution

In the main text, we showed that under Eaton and Kortum's (2002) distributional assumption, the elasticity of exports with respect to the average unit labor requirement is equal to the shape parameter of the Gumbel θ. Hence, changes in the elasticity of substitution σ across countries and industries do not affect the predictions of Theorem 1.

This result crucially relies on the following property of the Gumbel distribution:

$$\Pr\left\{p_j^k(\omega) \leq p\right\} = \Pr\left\{p_j^k(\omega) \leq p \left| c_{ij}^k(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^k(\omega) \right.\right\}, \tag{36}$$

for any $p > 0$ and any $1 \leq i \leq I$. Property (36) states that the distribution of the price $p_j^k(\omega)$ of a given variety ω of good k in country j is independent of the country of origin i; see Eaton and Kortum (2002) p1748 for a detailed discussion. Unfortunately, this property does not generalize to other standard distributions, as we show in the following Theorem.

Theorem 5. *Suppose that Assumptions A1-A4 hold and that $f(u) \equiv F'(u) > 0$ for any u in $\mathbb{R}$. Then, for any $p > 0$ and any $1 \leq i \leq I$, we have:*

$$\Pr\left\{p_j^k(\omega) \leq p\right\} = \Pr\left\{p_j^k(\omega) \leq p \left| c_{ij}^k(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^k(\omega) \right.\right\} \Leftrightarrow F(\cdot) \textit{ satisfies Equation } (6)$$

Put simply, the *only* distribution with full support satisfying Property (36) is the Gumbel.

Proof of Theorem 5. That Equation (6) is sufficient for Property (36) to hold is a matter of simple algebra. We now show that it is also necessary: if Equation (36) is satisfied, then $F(\cdot)$ is Gumbel. First, note that Property (36) is equivalent to

$$\frac{\Pr\left\{p_j^k(\omega) \leq p, c_{ij}^k(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^k(\omega)\right\}}{\Pr\left\{c_{ij}^k(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^k(\omega)\right\}} = \Pr\left\{p_j^k(\omega) \leq p\right\},$$

for all $p > 0$ and any $1 \leq i \leq I$, which in turn is equivalent to having

$$\frac{\Pr\left\{p_j^k(\omega) \leq p, c_{i_1j}^k(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^k(\omega)\right\}}{\Pr\left\{c_{i_1j}^k(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^k(\omega)\right\}} = \frac{\Pr\left\{p_j^k(\omega) \leq p, c_{i_2j}^k(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^k(\omega)\right\}}{\Pr\left\{c_{i_2j}^k(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^k(\omega)\right\}}, \tag{37}$$

for all $p > 0$ and any $1 \leq i_1, i_2 \leq I$. Using Assumptions A1 and A3, we have

$$\Pr\left\{c_{i_1j}^k(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^k(\omega)\right\} = \int_{-\infty}^{+\infty} f(u) \prod_{i' \neq i_1} \left[1 - F(\ln c_{i_1j}^k - \ln c_{i'j}^k + u)\right] du \tag{38}$$

and

$$\Pr\left\{p_j^k(\omega) \le p, c_{i_1 j}^k(\omega) = \min_{1\le i'\le I} c_{i'j}^k(\omega)\right\}$$

$$= \int_{-\infty}^{\ln p - \ln c_{i_1 j}^k} f(u) \prod_{i'\neq i_1} \left[1 - F(\ln c_{i_1 j}^k - \ln c_{i'j}^k + u)\right] du, \tag{39}$$

with similar expressions for i_2. So the condition in Equation (37) is equivalent to

$$\frac{\int_{-\infty}^{\ln p - \ln c_{i_1 j}^k} f(u) \prod_{i'\neq i_1} \left[1 - F(\ln c_{i_1 j}^k - \ln c_{i'j}^k + u)\right] du}{\int_{-\infty}^{+\infty} f(u) \prod_{i'\neq i_1} \left[1 - F(\ln c_{i_1 j}^k - \ln c_{i'j}^k + u)\right] du}$$

$$= \frac{\int_{-\infty}^{\ln p - \ln c_{i_2 j}^k} f(u) \prod_{i'\neq i_2} \left[1 - F(\ln c_{i_2 j}^k - \ln c_{i'j}^k + u)\right] du}{\int_{-\infty}^{+\infty} f(u) \prod_{i'\neq i_2} \left[1 - F(\ln c_{i_2 j}^k - \ln c_{i'j}^k + u)\right] du},$$

for all $p > 0$ and any $1 \le i_1, i_2 \le I$. Differentiating the above equality with respect to $\ln p$ and using the fact that $f(x) > 0$ and hence $F(x) < 1$ for all $x \in \mathbb{R}$, this in turn implies

$$\frac{f(\ln p - \ln c_{i_1 j}^k)\left[1 - F(\ln p - \ln c_{i_2 j}^k)\right]}{f(\ln p - \ln c_{i_2 j}^k)\left[1 - F(\ln p - \ln c_{i_1 j}^k)\right]} = \frac{\int_{-\infty}^{+\infty} f(u) \prod_{i'\neq i_1} \left[1 - F(\ln c_{i_1 j}^k - \ln c_{i'j}^k + u)\right] du}{\int_{-\infty}^{+\infty} f(u) \prod_{i'\neq i_2} \left[1 - F(\ln c_{i_2 j}^k - \ln c_{i'j}^k + u)\right] du},$$

for all $p > 0$ and any $1 \le i_1, i_2 \le I$. Since the right-hand side of the above equality does not depend on p, we necessarily have that

$$\frac{h_F(p/c_{i_1 j}^k)}{h_F(p/c_{i_2 j}^k)} \text{ only depends on } c_{i_1 j}^k, c_{i_2 j}^k, \tag{40}$$

where $h_F(\cdot)$ is a modified hazard function of $F(\cdot)$, i.e. $h_F(x) \equiv [1 - F(\ln x)]^{-1} f(\ln x)$ for any $x > 0$. We now make use of the following Lemma:

Lemma 6. *If for any positive constants c_1 and c_2, $h_F(x/c_1)/h_F(x/c_2)$ only depends on c_1, c_2, then necessarily $h_F(x)$ is of the form $h_F(x) = \mu x^\theta$ where $\mu > 0$ and θ real.*

Proof of Lemma 6. Let $U(t,x) \equiv h_F(tx)/h_F(x)$ for any $x > 0$ and any $t > 0$. Consider $t_1, t_2 > 0$: we have

$$\begin{aligned} U(t_1 t_2, x) &= \frac{h_F(t_1 t_2 x)}{h_F(x)} \\ &= \frac{h_F(t_1 t_2 x)}{h_F(t_1 x)} \cdot \frac{h_F(t_1 x)}{h_F(x)} \\ &= U(t_2, t_1 x) \cdot U(t_1, x). \end{aligned} \tag{41}$$

If the assumption of Lemma (6) holds then $U(t,x)$ only depends on its first argument t and we can write it $U(t)$. Hence the Equation (41) becomes

$$U(t_1 t_2) = U(t_2) \cdot U(t_1).$$

So, $U(\cdot)$ solves the Hamel equation on $\mathbb{R}_*^+$ and is of the form $U(t) = t^\theta$ for some real θ. This implies that

$$h_F(xt) = x^\theta h_F(t). \tag{42}$$

Consider $t = 1$ and let $\mu \equiv h_F(1) > 0$; Equation (42) then gives

$$h_X(x) = \mu x^\theta,$$

which completes the proof of Lemma 6. □

(Proof of Theorem 5 continued). The result of Lemma 6 allows us to characterize the class of distribution functions $F(\cdot)$ that satisfy Property (36). For any $u \in \mathbb{R}$, we have

$$\frac{f(u)}{1 - F(u)} = \mu \exp(\theta u). \tag{43}$$

Note that when $u \to -\infty$ we have $f(u), F(u) \to 0$ so that necessarily $\theta > 0$. We can now integrate Equation (43) to obtain for any $u \in \mathbb{R}$

$$F(u) = 1 - \exp\left[-\exp\left(\theta u + \ln(\frac{\mu}{\theta})\right)\right] \text{ with } \mu > 0 \text{ and } \theta > 0, \tag{44}$$

which belongs to the (negative) Gumbel family. Noting the expected value of the (negative) Gumbel distribution in Equation (44) equals $-\theta^{-1}(\ln(\mu/\theta) + \mathsf{e})$, where e is the Euler's constant, we necessarily have, by Assumptions A1 and A4(ii),

$$F(u) = 1 - \exp\left[-\exp\left(\theta u - \mathsf{e}\right)\right] \text{ with } \theta > \sigma - 1 \text{ for any } u \in \mathbb{R},$$

which completes the proof of Theorem 5. □

Appendix D: Revealed Comparative Advantage

This Appendix illustrates how our theoretical framework may be used to reveal the pattern of comparative advantage. The basic idea is to follow the three-term decomposition offered by Theorems 1 and 2 and consider a panel model of the form

$$\ln x_{ij}^k = \delta_{ij} + \delta_j^k + \delta_i^k + \varepsilon_{ij}^k, \tag{45}$$

where δ_{ij}, δ_j^k, and δ_i^k are treated as importer–exporter, importer–industry, and exporter-industry fixed effects, respectively, and ε_{ij}^k is an error term.[15] In the absence of ε_{ij}^k, there would be, for any pair of exporters, a unique ranking of relative exports by industry, as suggested in Corollary 3. Furthermore, this ranking would be entirely determined by the cross-industry and cross-country variation of the third term, δ_i^k. If $\varepsilon_{ij}^k = 0$, then $\frac{x_{i_1 j}^{k_1}}{x_{i_2 j}^{k_1}} > \frac{x_{i_1 j}^{k_2}}{x_{i_2 j}^{k_2}}$ if and only if $\delta_{i_1}^{k_1} - \delta_{i_2}^{k_1} > \delta_{i_1}^{k_2} - \delta_{i_2}^{k_2}$. Hence, the estimates of δ_i^k can be interpreted as a *revealed measure*—up to a monotonic transformation—of the fundamental productivity levels, a_i^k, that determine the Ricardian chain of of comparative advantage.

Table 4 reports the ranking of the OLS estimates of $\left(\delta_{US}^k - \delta_i^k\right)$ across industries for all exporters $i \neq \textit{United States}$ in 1997, from the highest to the lowest value. According to our estimates, "Aircraft" always is the first industry in the chain of comparative advantage of the United States. Compared to any other country in our sample, the United States tend to export more in the aircraft industry than in any other industry. The industries at the bottom of the US chain of comparative advantage tend to be "Basic Metals" and "Textile", depending on the identity of the other exporter. A notable exception is Germany for which "Automobile" is the bottom industry.

Note that there is a close connection between Balassa (1965) and the present paper. Like Balassa (1965), we offer a methodology that uses data on relative exports to infer the pattern of comparative advantage across countries and industries. In his well-known paper, the revealed comparative advantage of country i in industry k is defined as

$$\left(\frac{x_{iWorld}^k}{\sum_{k'=1}^K x_{iWorld}^{k'}}\right) \Bigg/ \left(\frac{\sum_{i'=1}^I x_{i'World}^k}{\sum_{i'=1}^I \sum_{k'=1}^K x_{i'World}^{k'}}\right)$$

[15] We thank Stephen Redding for suggesting that approach.

Table 4: Ranking of Industries in the Chain of Comparative Advantage of the United States

Industry	Other Exporting Country														
	AUT	BEL	DEU	DNK	ESP	FIN	FRA	GBR	GRC	IRL	ITA	NLD	PRT	SWE	Mean	Balassa
Aircraft	.	1	1	.	1	1	1	.	.	.	1	1	.	1	1	1
Office	1	2	2	1	2	3	2	.	1	14	2	7	.	2	2	3
Medical	4	4	6	7	3	7	3	.	4	10	5	5	.	6	3	4
Telecom	3	6	4	3	4	15	6	.	2	11	4	4	.	16	4	2
Shipbuilding	2	3	3	8	8	14	5	.	.	.	3	6	.	4	5	18
Transport	11	5	9	2	5	2	10	.	.	2	12	3	.	5	6	8
Wood	10	7	7	12	7	17	4	1	5	3	8	2	7	17	7	9
Other	6	13	8	10	12	5	7	3	10	6	17	9	1	7	8	12
Machinery	8	9	12	13	6	12	9	2	7	5	14	8	2	10	9	6
Automobile	12	15	19	4	17	9	16	.	3	1	11	14	.	14	10	7
Electrical	7	10	13	11	10	16	14	.	8	8	9	10	.	12	11	5
Paper	14	8	10	5	9	19	8	5	9	15	7	11	3	18	12	10
Food	5	12	5	18	13	4	11	4	13	13	6	19	4	3	13	16
Chemicals	9	18	11	17	11	6	15	.	6	16	10	17	.	15	14	15
Plastic	13	14	14	14	14	10	13	6	12	9	13	13	5	11	15	13
Metal products	16	11	17	16	15	13	12	.	11	7	15	15	.	13	16	11
Minerals	15	16	15	6	19	11	19	7	14	4	18	12	6	9	17	17
Textile	18	17	16	15	18	8	17	8	15	12	19	16	8	8	18	19
Basic metals	17	19	18	9	16	18	18	.	.	.	16	18	.	19	19	14

where x^k_{iWorld} are the total exports of country i in industry k. The ranking of industries in terms of their Balassa's (1965) revealed comparative advantage for $i = United\ States$ is reported in the last column of Table 4.

There are, however, two important differences between Balassa's (1965) approach and ours. First, our empirical strategy is theoretically grounded. The ranking of the OLS estimates of $\left(\delta^k_{US} - \delta^k_i\right)$ is the empirical counterpart to the ranking of $\left(\ln a^k_i - \ln a^k_{US}\right)$ in our model. Second, our approach fundamentally is about pairwise comparisons. Our fixed effects aim to uncover which of Portugal and England is the country relatively better at producing wine than cloth. They do not try to find out whether Portugal is good at producing wine compared to an intuitive but ad-hoc benchmark. Unlike Balassa (1965), we do not aggregate exports across countries and industries, which—according to our model—allows us to separate the impact of technological differences from transport costs and demand differences.

Appendix E: Selection Bias

The objective of this Appendix is to relate the latent right hand side variable, $\ln a_i^k$, to its observed counterpart, $\ln \widehat{a}_i^k$, when selection is the only source of measurement error. By Assumption A1, we know that

$$\ln a_i^k = \ln E\left[a_i^k(\omega)\right] - \ln E\left[\exp(u)\right] \tag{46}$$

We assume that $\ln \widehat{a}_i^k$ is such that

$$\ln \widehat{a}_i^k = \ln E\left[a_i^k(\omega)\left|\omega \in \Omega_i^k\right.\right] \tag{47}$$

where $\Omega_i^k \equiv \left\{\omega \left| \exists j = 1, ..., N,\, c_{ij}^k(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^k(\omega)\right.\right\}$ is the set of varieties of good k produced by country i. We denote Δ_i^k the measurement error associated with selection

$$\Delta_i^k = \ln E\left[a_i^k(\omega)\left|\omega \in \Omega_i^k\right.\right] - \ln E\left[a_i^k(\omega)\right] \tag{48}$$

We make use of the following lemma.

Lemma 7. *Suppose that for any three countries, i_1, i_2, and i_3, we have $d_{i_1 i_3} \leq d_{i_1 i_2} \cdot d_{i_2 i_3}$. Then, for all countries i and goods k,*

$$\Omega_i^k = \left\{\omega \left| c_{ii}^k(\omega) = \min_{1 \leq i' \leq I} c_{i'i}^k(\omega)\right.\right\} \tag{49}$$

Proof of Lemma 7. We proceed by contradiction. Fix an exporter i, and suppose there exists a variety ω_0 of good k and a country $j_0 \neq i$ such that:

$$\begin{cases} c_{ij_0}^k(\omega_0) = \min_{1 \leq i' \leq I} c_{i'j_0}^k(\omega_0) \\ c_{ii}^k(\omega_0) \neq \min_{1 \leq i' \leq I} c_{i'i}^k(\omega_0) \end{cases}$$

Then, there must be an exporter $i_0 \neq i$ such that

$$\begin{cases} d_{ij_0} \cdot a_i^k(\omega_0) \leq d_{i_0 j_0} \cdot a_{i_0}^k(\omega_0) \\ d_{ii} \cdot a_i^k(\omega_0) > d_{i_0 i} \cdot a_{i_0}^k(\omega_0) \end{cases}$$

Since $d_{ii} = 1$, the two previous inequalities imply

$$d_{i_0 j_0} > d_{i_0 i} \cdot d_{ij_0}$$

which contradicts $d_{i_1 i_3} \leq d_{i_1 i_2} \cdot d_{i_2 i_3}$ for any three countries, i_1, i_2, and i_3. $\square$

In the reminder of this Appendix, we assume that iceberg transport costs satisfy the previous triangle inequality. Hence we can use Lemma 7 to compute the extent of selection bias under the assumptions of Theorems 1 and 2. Let us denote m_i^k the import penetration ratio in country i and industry k

$$m_i^k \equiv \frac{\sum_{i' \neq i} x_{i'i}^k}{\sum_{i'=1}^{I} x_{i'i}^k} \tag{50}$$

Our first result can be stated as follows.

Theorem 8. *Under assumptions of Theorem 1, the measurement error equals*

$$\Delta_i^k = \frac{1}{\theta} \ln(1 - m_i^k)$$

where the constant θ is the parameter of the Gumbel distribution.

Proof of Theorem 8. Without loss of generality, we focus on the case $i = 1$. By definition, we know that $c_{11}^k(\omega) = a_1^k(\omega) \cdot \mathtt{w}_1 \cdot d_{11}^k$. Using Lemma 7 then yields

$$\ln E\left[a_1^k(\omega) \,|\omega \in \Omega_1^k\right] = \ln E\left[c_{11}^k(\omega) \mathbb{I}\left\{c_{11}^k(\omega) = \min_{1 \leq i' \leq I} c_{i'1}^k(\omega)\right\}\right] - \ln(d_{11}^k \cdot \mathtt{w}_1) - \ln \pi_{11}^k \tag{51}$$

where we have let $\pi_{11}^k \equiv \Pr\left\{c_{11}^k(\omega) = \min_{1 \leq i' \leq I} c_{i'1}^k(\omega)\right\}$. Now, consider

$$G_1(c_{11}^k, ..., c_{I1}^k) \equiv E\left[c_{11}^k(\omega) \mathbb{I}\left\{c_{11}^k(\omega) = \min_{1 \leq i' \leq I} c_{i'1}^k(\omega)\right\}\right]$$

The expressions for $G_1(c_{11}^k, ..., c_{I1}^k)$ and π_{11}^k are readily available from the proof of Theorem 1 when the result in Equation (20) is evaluated at $\sigma = 0$ and $\sigma = 1$, respectively. Hence,

$$\ln \widehat{a}_1^k = \frac{\mathsf{e}}{\theta} + \ln \Gamma\left(\frac{\theta + 1}{\theta}\right) - \frac{1}{\theta} \ln \sum_{i=1}^{I} (c_{i1}^k)^{-\theta} - \ln(d_{11}^k \cdot \mathtt{w}_1).$$

In addition, from assumption A1 and $c_{11}^k = a_{11}^k \cdot d_{11}^k \cdot \mathtt{w}_1$ we know that

$$\ln E\left[a_1^k(\omega)\right] = \ln c_{11}^k - \ln(d_{11}^k \cdot \mathtt{w}_1) + \ln E[\exp u] \tag{52}$$

so that

$$\Delta_1^k = \frac{\mathsf{e}}{\theta} + \ln \Gamma\left(\frac{\theta + 1}{\theta}\right) - \ln E[\exp u] + \frac{1}{\theta} \ln\left(\frac{(c_{11}^k)^{-\theta}}{\sum_{i=1}^{I} (c_{i1}^k)^{-\theta}}\right) \tag{53}$$

Now, from Equation (21), we know that for every i,

$$x_{i1}^k = e_1^k \cdot \frac{(c_{i1}^k)^{-\theta}}{\sum_{i'=1}^{I} (c_{i'1}^k)^{-\theta}}.$$

Using Equation (50), we then get

$$m_1^k = \frac{\sum_{i \neq 1} (c_{i1}^k)^{-\theta}}{\sum_{i=1}^I (c_{i1}^k)^{-\theta}} \tag{54}$$

In addition, under the Gumbel assumption, we have that

$$E[\exp u] = \exp\left[\frac{\mathtt{e}}{\theta}\Gamma\left(\frac{\theta+1}{\theta}\right)\right] \tag{55}$$

so combining Equations (53), (54), and (55) yields

$$\Delta_1^k = \frac{1}{\theta}\ln(1 - m_1^k),$$

□

We now derive a similar result in the general case.

Theorem 9. *Under assumptions of Theorem 2, the measurement error equals:*

$$\Delta_1^k \simeq \chi + \frac{\nu}{\gamma}\ln\left(1 - m_1^k\right),$$

with γ being the constant from Theorem 2, and χ and $\nu \in \mathbb{R}$.

Proof of Theorem 9. Without loss of generality, we again focus on the case $i = 1$. As in the proof of Theorem 8, we have

$$\ln E\left[a_1^k(\omega)\,|\omega \in \Omega_1^k\right] = \ln E\left[c_{11}^k(\omega)\mathbb{1}\left\{c_{11}^k(\omega) = \min_{1\le i'\le I} c_{i'1}^k(\omega)\right\}\right] - \ln(d_{11}^k \cdot \mathtt{w}_1) - \ln \pi_{11}^k \tag{56}$$

where $\pi_{11}^k \equiv \Pr\left\{c_{11}^k(\omega) = \min_{1\le i'\le I} c_{i'1}^k(\omega)\right\}$, as before. As previously, we let $G_1(c_{11}^k, ..., c_{I1}^k) \equiv E\left[c_{11}^k(\omega)\mathbb{1}\left\{c_{11}^k(\omega) = \min_{1\le i'\le I} c_{i'1}^k(\omega)\right\}\right]$. Now, consider a first order Taylor development of $\ln G_1(c_{11}^k, ..., c_{I1}^k)$ around $\ln c_{11}^k = ... = \ln c_{I1}^k = 0$. The latter is readily available from the proof of Theorem 2: we only need to set $\sigma = 0$ in Equations (22)-(24). By letting

$$\lambda \equiv \int_{-\infty}^{+\infty} [\exp u] f(u)\left[1 - F(u)\right]^{I-1} du$$

and

$$\mu \equiv \lambda^{-1}\left[\int_{-\infty}^{+\infty} [\exp u] f^2(u)\left[1 - F(u)\right]^{I-2} du\right]$$

we get

$$\ln G_1(c_{11}^k, ..., c_{I1}^k) = \ln \lambda - (\mu I - 1) \cdot \ln c_{11}^k + \mu \cdot \sum_{i=1}^{I} \ln c_{i1}^k + o\left(\left\|\ln c_1^k\right\|\right), \tag{57}$$

where $\|\cdot\|^2$ is the L_2-norm as previously. We can follow the same approach for $\ln \pi_{11}^k$. By setting $\sigma = 1$ in Equations (22)-(24), we get

$$\ln \pi_{11}^k = -\ln I - \tau \cdot I \cdot \ln c_{11}^k + \tau \cdot \sum_{i=1}^{I} \ln c_{i1}^k + o\left(\left\|\ln c_1^k\right\|\right), \tag{58}$$

where

$$\tau \equiv I \cdot \left[\int_{-\infty}^{+\infty} f^2(u) \left[1 - F(u)\right]^{I-2} du \right].$$

Combining Equations (56) with (57) – (58), we then obtain

(59)

$$\ln E\left[a_i^k(\omega) \,|\omega \in \Omega_i^k\right] = \ln(\lambda I) + [(\tau - \mu)I + 1] \cdot \ln c_{11}^k - \ln(d_{11}^k \cdot \mathtt{w}_1) + (\mu - \tau) \sum_{i=1}^{I} \ln c_{i1}^k + o\left(\left\|\ln c_1^k\right\|\right).$$

Now, combining Assumption A1 with $c_{11}^k = a_{11}^k \cdot d_{11}^k \cdot \mathtt{w}_1$ gives

$$\ln E\left[a_1^k(\omega)\right] = \ln c_{11}^k - \ln(d_{11}^k \cdot \mathtt{w}_1) + \ln E[\exp u] \tag{60}$$

Combining Equations (59) – (60) yields the following expression for the bias Δ_1^k defined in (48):

$$\Delta_1^k = \ln(\lambda I) - \ln E[\exp u] + (\mu - \tau) \sum_{i=1}^{I} \ln\left(\frac{c_{i1}^k}{c_{11}^k}\right) + o\left(\left\|\ln c_1^k\right\|\right). \tag{61}$$

Now, fix any constant γ, and note that in the neighborhood of $\ln c_{11}^k = ... = \ln c_{I1}^k = 0$ we have:

$$\ln\left(\sum_{i=1}^{I}\left(\frac{c_{i1}^k}{c_{11}^k}\right)^{-\gamma}\right) = \ln I + \frac{1}{I}\sum_{i=1}^{I}\left(\frac{c_{i1}^k}{c_{11}^k}\right)^{-\gamma} - 1 + o\left(\left\|\ln c_1^k\right\|\right),$$

and, in addition, for any i,

$$\left(\frac{c_{i1}^k}{c_{11}^k}\right)^{-\gamma} = 1 - \gamma \ln\left(\frac{c_{i1}^k}{c_{11}^k}\right) + o\left(\left\|\ln c_1^k\right\|\right).$$

Hence,

$$\sum_{i=1}^{I} \ln\left(\frac{c_{i1}^k}{c_{11}^k}\right) = \frac{I}{\gamma}\left[\ln I - \ln\left(\sum_{i=1}^{I}\left(\frac{c_{i1}^k}{c_{11}^k}\right)^{-\gamma}\right)\right] + o\left(\left\|\ln c_1^k\right\|\right).$$

From the proof of Theorem 2 and its Equation (25) recall that

$$\sum_{i=1}^{I}\left(\frac{c_{i1}^k}{c_{11}^k}\right)^{-\gamma}=\left[\sum_{i=1}^{I}\frac{x_{i1}^k}{x_{11}^k}\right]\left[1+o\left(\left\|\ln c_1^k\right\|\right)\right] \tag{62}$$

where we let γ be as defined in the proof of Theorem 2. From Equations (50), (61), and (62) we then get

$$\Delta_1^k=\ln(\lambda I)-\ln E[\exp u]+(\mu-\tau)\frac{I}{\gamma}\left[\ln I+\ln\left(1-m_1^k\right)\right]+o\left(\left\|\ln c_1^k\right\|\right),$$

where m_1^k is the import penetration ratio in country i and industry k defined in Equation (50) above. Let then $\nu\equiv(\mu-\tau)I$, so

$$\nu=I\left(\frac{\int_{-\infty}^{+\infty}[\exp u]f^2(u)\left[1-F(u)\right]^{I-2}du}{\int_{-\infty}^{+\infty}[\exp u]f(u)\left[1-F(u)\right]^{I-1}du}-\frac{\int_{-\infty}^{+\infty}f^2(u)\left[1-F(u)\right]^{I-2}du}{\int_{-\infty}^{+\infty}f(u)\left[1-F(u)\right]^{I-1}du}\right)$$

and $\chi\equiv\ln(\lambda I)+\gamma^{-1}(\mu-\tau)I\ln I-\ln E[\exp u]$, i.e.

$$\chi=\ln(\lambda I)+\frac{\nu}{\gamma}\ln I-\ln E[\exp u]$$

Then,

$$\Delta_1^k\simeq\chi+\frac{\nu}{\gamma}\ln\left(1-m_1^k\right),$$

as desired. □

Note that under the assumptions of Theorem 1, the parameter γ equals θ and we have the following equalities:

$$\nu=1\text{ and }\chi=0$$

so the results of Theorem 9 reduce to those of Theorem 8.

Appendix F: Multiple Factors of Production

In order to introduce multiple factors of production into the Eaton and Kortum's (2002) model, we follow Costinot's (2005) chapter 3.[16] Suppose that there are $f = 1, ..., F$ factors of production, which are perfectly mobile across industries and immobile across countries. Further, assume that the production technology is Cobb-Douglas in all sectors and countries so that the constant unit cost of variety ω of good k in country i equals

$$c_i^k(\omega) = a_i^k(\omega) \cdot \prod_{f=1}^{F} w_{if}^{\alpha_f^k}, \tag{63}$$

where w_{if} is factor f's reward in country i; and $0 < \alpha_f^k < 1$ is the intensity of factor f in the production of good k. Compared to Section 2, $a_i^k(\omega)$ now is the inverse of *total factor* productivity in the production of variety ω of good j in country i. Combining Assumption A1 and Equation (63), we get

$$\ln c_i^k(\omega) = \ln a_i^k + \sum_{f=1}^{F} \alpha_f^k \ln w_{if} + u_i^k(\omega).$$

Following the same reasoning as in Section 3, we may now generalize Theorems 1 and 2:

Theorem 10. *Suppose that the assumptions of Theorem 1 or 2 hold. Then, for any importer j, any exporter $i \neq j$, and any product k,*

$$\ln x_{ij}^k \simeq \eta_{ij} + \eta_j^k - \eta \ln a_i^k - \eta \sum_{f=1}^{F} \alpha_f^k \ln(w_{if}/w_{i1}). \tag{64}$$

where $\eta > 0$.

The interpretation of the two fixed effects is the same as in Section 3. The additional term $\eta \sum_{f=1}^{F} \alpha_f^k \ln w_{if}$ captures the impact of cross-country differences in relative factor prices—and therefore, cross-country differences in factor endowments—on the pattern of trade.

[16] Independently, Chor (2006) also incorporates multiple factors of production—together with institutional differences—into the Eaton and Kortum's (2002) model.