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ABSTRACT

This paper studies policy in a class of economies in which information about commonly-relevant fundamentals -- such as aggregate productivity and demand conditions -- is dispersed and can not be centralized by the government. In these economies, the decentralized use of information can fail to be efficient either because of discrepancies between private and social payoffs, or because of informational externalities. In the first case, inefficiency manifests itself in excessive non-fundamental volatility (overreaction to common noise) or excessive cross-sectional dispersion (overreaction to idiosyncratic noise). In the second case, inefficiency manifests itself in suboptimal social learning (low quality of information contained in macroeconomic data, financial prices, and other indicators of economic activity). In either case, a novel role for policy is identified: the government can improve welfare by manipulating the incentives agents face when deciding how to use their available sources of information. Our key result is that this can be achieved by appropriately designing the contingency of marginal taxes on aggregate activity. This contingency permits the government to control the reaction of equilibrium to different types of noise, to improve the quality of information in prices and macro data, and, in overall, to restore efficiency in the decentralized use of information.

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"*The peculiar character of the problem of a rational economic order is determined precisely by the fact that the knowledge of the circumstances of which we must make use never exists in concentrated or integrated form but solely as the dispersed bits of incomplete and frequently contradictory knowledge which all the separate individuals possess. The economic problem of society is ... a problem of the utilization of knowledge which is not given to anyone in its totality.* " (Friedreich A. Hayek, 1945)

1 Introduction

The dispersion of information is an essential part of the economic problem faced by society. This concerns not only the idiosyncratic needs and means of different households and firms, but also commonly-relevant fundamentals. For example, think of the business cycle. Information about aggregate productivity and demand conditions is crucial for individual consumption, production, and pricing decisions. Yet, such information is widely dispersed and it is only imperfectly aggregated through markets, the media, or other channels of communication in society.

As emphasized by Hayek (1945), such information can not be centralized within a single institution, such as the government. Instead, society must rely on decentralized mechanisms for the utilization of such information. One can then be assured that rational agents will always use their available information in the most privately-efficient way. This, however, need not coincide with what best serves social interests. For example, complementarities in investment or pricing decisions may induce firms to overreact to public information, because public information helps forecast the decisions of others; this can crowd out valuable private information and can also amplify the impact of common noise, resulting in higher non-fundamental volatility. Furthermore, individuals may fail to internalize how their own choices affect the information contained in financial prices or other indicators of aggregate activity; this can lead to inefficient social learning about the underlying economic fundamentals.

A novel role for policy then emerges: even if the government cannot centralize the information that is dispersed in society and can not otherwise collect and communicate information, there may exist policies that improve efficiency in the decentralized use of information by appropriately manipulating the incentives faced by individual agents. Identifying such policies is the objective of this paper. Our key result is to show how efficiency is achieved by appropriately designing the contingency of marginal taxes on realized aggregate activity.

Preview. Rather than focusing on a specific application, we seek to highlight a more general policy lesson. We thus conduct our analysis within an abstract, but tractable, class of games that allow for two sources of inefficiency in the decentralized use of information: *payoff externalities* and *informational externalities*. The former are short-cuts for a variety of strategic and other external effects featured in applications, such as those originating in production spillovers, investment

or pricing complementarities, monopoly power, social networking, and the like; they summarize potential discrepancies between private and social payoffs. The latter reflect the (imperfect) aggregation of information obtained through financial prices, the publication of data on macroeconomic activity, or other forms of social learning.

Our analysis then proceeds in two steps. First, we compare the equilibrium in the absence of policy intervention with the allocation that maximizes welfare subject to the sole constraint that information can not be directly transferred from one agent to another; this helps detect the potential inefficiencies in the decentralized use of information. Next, we identify tax schemes that implement the efficient strategy as an equilibrium; this gives the key policy result of the paper.

The symptoms of inefficiency that we detect by comparing the equilibrium strategy with the aforementioned socially optimal strategy depend on whether the inefficiency originates in payoff or informational externalities. In the first case, inefficiency manifests itself in excessive non-fundamental volatility (overreaction to common noise) or excessive cross-sectional dispersion (overreaction to idiosyncratic noise). In the second case, inefficiency manifests itself in suboptimal social learning (too much noise in macroeconomic indicators, financial prices, or other channels of information aggregation).

Yet, the same policy prescription works for either case, as well as for economies that combine the two sources of inefficiency. Our key result is that the government can control how agents use different sources of information in equilibrium by making the marginal tax rate contingent on *aggregate* activity. An appropriate design of the tax system then restores efficiency in the decentralized use of information, irrespective of the specific source of inefficiency.

The logic behind this result is simple. When individuals expect marginal taxes to decrease with realized aggregate activity, they also expect the realized net-of-taxes return on their own activity to increase with aggregate activity. It follows that a negative dependence of marginal taxes on aggregate activity imputes strategic complementarity in individual choices: agents have an incentive to align their choices with those of others. Symmetrically, a positive dependence imputes strategic substitutability: agents have an incentive to differentiate their choices.

Next note that a better alignment of individual decisions obtains when agents rely more on common sources of information, whereas more differentiation obtains when agents rely more on idiosyncratic sources of information. It follows that the government can use the contingency of marginal taxes on aggregate activity to fashion how agents respond to different sources of information. In particular, when marginal taxes decrease with aggregate activity, by inducing strategic complementarity the policy also induces higher *relative* sensitivity of equilibrium actions to common information. Symmetrically, when marginal taxes increase with aggregate activity, by inducing strategic substitutability the policy also induces higher relative sensitivity to idiosyncratic information.

It follows that, by appropriately designing the aforementioned contingency, the government can control how agents use their available information. In so doing, the government can control, not only the impact of noise on equilibrium activity, but also the quality of information contained in prices and macroeconomic data. The government can thereby improve welfare even without itself centralizing or communicating information to the market.

Discussion. It is often argued that financial markets overreact to public news, causing inefficient fluctuations in both asset prices and real investment. This idea goes back at least to Keynes (1936), who argued that professional investors, instead of focusing on the long-run fundamental value of the assets, try to second-guess the demands of one another, thus causing inefficient fluctuations in asset prices and investment.[1] In this paper, although we are partly motivated by the broader theme that markets may react inefficiently to available information, we do not limit attention to any specific application, nor do we examine the deeper origins of such inefficiency (which, unavoidably, will be specific to the application of interest). Rather, our goal is to identify policy remedies that need not be sensitive to the details of the origin of the inefficiency. This explains our choice to work with a theoretical framework that is abstract and flexible enough to allow for a variety of distortions in the decentralized use of information.

Our analysis also takes as exogenous the limits society faces in aggregating dispersed information. Investigating the foundations for these limits, and their potential implications for policy and institutional design, is a challenging topic beyond the scope of this paper. Nevertheless, our analysis offers some relevant insights. In our class of economies, equilibrium welfare may *decrease* with additional information because of possible inefficiencies in the equilibrium use of information. However, once these inefficiencies have been removed, more information can only improve welfare. Therefore, policies that provide the market with the right incentives for how to use available information also complement policies, or other institutions, that provide the market with more information.

Furthermore, while our analysis focuses on how the contingency of taxes on aggregate activity can improve the decentralized use of information, in practice, the contingency of monetary policy on aggregate activity might also help in the same direction. Indeed, the more general insight that comes out of our analysis is how the *anticipation* of such contingencies affects the incentives agents face in using their available information, and how this in turn affects efficiency.

Methodological remarks. The policy exercise conducted in this paper strikes a balance between two dominant paradigms: the Ramsey tradition to optimal policy (e.g., Barro, 1979, Lucas and Stokey, 1983, Chari, Christiano and Kehoe, 1994); and the Mirrlees tradition, or the "new public finance" paradigm (e.g., Kocherlakota, 2005; Golosov, Tsyvinski and Werning, 2006). We deviate from the Ramsey tradition by introducing heterogeneous information and by avoiding

[1] This point was epitomized in Keynes' famous beauty-contest metaphor for financial markets, which highlighted the potential role of higher-order expectations. Elements of this role have recently been formalized in Allen, Morris and Shin (2003), Bacchetta and Wincoop (2005), and Angeletos, Lorenzoni and Pavan (2007).

ad hoc restrictions on the set of available policy instruments. In this respect, our policy exercise is closer to the new public finance literature. At the same time, we deviate from the Mirrlees tradition by abstracting from redistributive taxation (or social insurance) and focusing instead on policies that correct inefficiencies in the response of equilibrium outcomes to aggregate shocks. In this respect, our policy exercise is closer to the Ramsey tradition.

Furthermore, the Mirrlees literature studies environments in which agents have private information regarding purely *idiosyncratic* shocks, such as an agent's own tastes, talent, or labor productivity; whenever aggregate shocks are featured in this literature, information regarding these shocks is assumed to be common. In contrast, we study environments in which agents have dispersed information regarding aggregate shocks; these shocks could be about either the underlying fundamentals or the distribution of information in society. The key difference then is that the absence of common knowledge regarding these shocks generates strategic uncertainty: agents face uncertainty regarding aggregate activity. It is precisely this uncertainty that makes the contingency of taxes on aggregate activity essential for restoring efficiency—which also explains why the results presented here are, to the best of our knowledge, completely novel to the policy literature.

Finally, note that the policies we identify resemble Pigou-like taxes in the sense that they make agents internalize externalities. However, they are different with regard to both the nature of the underlying distortion and the way they restore efficiency. In standard Pigou-like contexts, the market produces/consumes too much or too little of a certain commodity. The Pigou remedy is then to impose a tax or subsidy on this commodity. In our context, instead, the market reacts too much or too little to certain sources of information. The analogue of the Pigou remedy would then consist in imposing a tax or subsidy *directly* on the use of these sources of information. However, this seems practically impossible. Our contribution is to show how the same goal can be achieved *indirectly* by appropriately designing the contingency of taxes on aggregate activity.

Other related literature. The literature that studies dispersed information in macroeconomic contexts goes back to Phelps (1970), Lucas (1972), Townsend (1983), and the rational-expectations revolution of the 70's and early 80's. More recently, Mankiw and Reis (2000) and Woodford (2001) have raised interest on the business cycle implications of combining information heterogeneity with strategic complementarity in pricing decisions.[2] We complement this line of work by studying policy in environments that share this key combination.

Another line of work, following Morris and Shin (2002), has examined whether welfare increases with more precise public information within specific models.[3] Some of these papers have found a negative result, which has then been used to make a case against central-bank transparency; others have found the opposite result. In Angeletos and Pavan (2007) we showed how these apparently

[2]See also Amato and Shin (2006), Hellwig (2005), Lorenzoni (2006), and Mackowiak and Wiederholt (2006).

[3]See also Amador and Weil (2007), Angeletos and Pavan (2004), Baeriswyl and Conrand (2007), Heinemann and Cornand (2006), Hellwig (2005), Roca (2006), and Svensson (2005).

conflicting results can be explained by understanding the underlying inefficiencies in the equilibrium use of information. For that purpose, we considered an abstract framework that restricted information to be exogenous but was flexible enough to nest most of the applications examined in the literature; we then used this framework to study the social value of information (i.e., the comparative statics of equilibrium welfare with respect to the information structure). In the first part of the present paper, we use a generalized version of that framework for a different purpose: to study how efficiency in the decentralized use of information can be restored through an appropriately designed tax system. In the second part of the paper, we extend the analysis to dynamic environments in which information is partially endogenous; we then show how similar policies also correct inefficiencies that originate from informational externalities.

In this last respect, our analysis complements that in Vives (1993, 1997) and Amador and Weill (2007). These papers study the speed of social learning and the social value of information in a dynamic economy where agents learn from noisy observations of past aggregate activity. Our results identify policies that can control the speed of social learning and also guarantee that any exogenous information is socially valuable.

Layout. Section 2 introduces the baseline framework. Section 3 studies inefficiencies in the decentralized use of information due to payoff externalities. Section 4 identifies tax systems that remove such inefficiencies. Section 5 extends the analysis to dynamic settings and Section 6 to settings with informational externalities. Section 7 discusses implications for the social value of information. Section 8 concludes. All proofs are in the Appendix.

2 The baseline static framework

In this section we outline our baseline framework: a game that abstracts from the institutional details of any specific application but is flexible enough to capture the role of strategic interactions, external payoff effects, and dispersed information in a variety of applications.

Actions and payoffs. The economy is populated by a continuum of agents of measure one, indexed by $i \in [0,1]$, each choosing an action $k_i \in \mathbb{R}$. In addition, there is a government, which imposes a tax $\tau_i \in \mathbb{R}$ on each agent i, subject to the constraint that the budget is balanced.

Let ψ denote the cumulative distribution function of individual actions in the cross-section of the population and let $K \equiv \int k d\psi(k)$ and $\sigma_k \equiv [\int (k-K)^2 d\psi(k)]^{1/2}$ denote, respectively, the mean and the dispersion of individual actions. The (reduced-form) payoff of agent i is given by

$$u_i = U(k_i, K, \sigma_k, \theta_i) - \tau_i, \tag{1}$$

for some $U : \mathbb{R}^2 \times \mathbb{R}_+ \times \Theta \to \mathbb{R}$. The variable $\theta_i \in \Theta \subseteq \mathbb{R}$ represents a shock to agent i's payoff. For concreteness, in what follows we often think of k_i as "investment" and θ_i as a "productivity shock." The external and strategic effects exhibited in U may originate, not only from preferences

and technologies, but also from pecuniary externalities, monopoly power, oligopolistic competition, social networks, and the like. What is crucial, though, is that payoffs can be reduced to the specification assumed above without missing any channels of endogenous information aggregation; the analysis of the latter is postponed to Section 6. Finally note that, by assuming linearity of payoffs in transfers, we are ruling out any redistributive (or insurance) role for taxation.

Payoff restrictions. To maintain tractability, we assume that U is a concave quadratic polynomial and that the external effect of dispersion depends only on its own level (i.e., $U_\sigma(k, K, \sigma_k, \theta) = U_{\sigma\sigma}\sigma_k$ for all (k, K, σ_k, θ)).[4] To guarantee existence and uniqueness of both the equilibrium and the efficient allocation, we assume the following: $U_{kk} < 0$, which imposes concavity at the individual's decision problem; $-U_{kK}/U_{kk} < 1$, which ensures that the slope of the individual's best response with respect to aggregate activity is less than one; and $U_{kk} + 2U_{kK} + U_{KK} < 0$ and $U_{kk} + U_{\sigma\sigma} < 0$, which imposes concavity at the planner's problem.[5]

Timing and information. There are three stages. In stage 1, the government announces a policy rule T that specifies how taxes will be collected in stage 3 as a function of information that will be public at that stage.[6] In stage 2, agents simultaneously choose their actions under dispersed information (described below). Finally, in stage 3, actions and aggregate productivity are publicly revealed, taxes are paid, payoffs are realized, and the game ends.[7]

The information structure is as follows. Let $\omega_i \in \Omega$ denote the information (also the "type") of agent i. Next, let $f \in F$ denote a joint distribution for (θ_i, ω_i), with marginal distributions for θ_i and ω_i given by $h \in H$ and $\phi \in \Phi$, respectively. The distribution f describes the joint distribution of (θ, ω) in the cross-section of the population; we refer to f as the "state of the world."[8] First, Nature draws f from a set of possible distributions F according to the probability measure $\mathcal{F}$ which is common knowledge among the agents. Nature then uses f to draw a pair (θ_i, ω_i) for each agent i, with the pairs $(\theta_i, \omega_i)_{i\in[0,1]}$ drawn independently from f. Each agent i then observes his own ω_i, but does not observe either θ_i or the distribution f. Note, though, that ω_i encodes information, not only about the agent's own productivity θ_i, but also about the distribution f of productivities and information in the population.

Although most of the analysis does not require any restriction on the information structure,

[4]The external effect of dispersion is relevant for certain applications: in new-keynesian models, e.g., dispersion in relative prices has a negative welfare effect.

[5]Although the restriction $-U_{kK}/U_{kk} < 1$ is necessary for the equilibrium to be unique in the absence of government intervention, our main policy result does not rely on this restriction: the policies identified in Section 4 implement the efficient allocation as the unique equilibrium even in economies that feature multiple equilibria in the absence of policy intervention.

[6]Because the government has no private information, the announcement of T does not convey any information about the fundamentals; its only role is to affect the agents' incentives.

[7]Throughout, we do not require idiosyncratic productivities to be revealed at stage 3. Also, in Section 4.3 we consider an extension that adds noise to the observation of actions and aggregate productivity at stage 3.

[8]This is with a slight abuse of terminology because f does not describe the specific (θ_i, ω_i) for each single agent.

we find it useful to impose a certain "regularity" condition. Fix an information structure $(\Omega, F, \mathcal{F})$ and consider any two payoff structures, U^1 and U^2. Let $k^1 : \Omega \to \mathbb{R}$ be an equilibrium strategy for the economy $\mathcal{E}^1 = (U^1; \Omega, F, \mathcal{F})$ and $k^2 : \Omega \to \mathbb{R}$ an equilibrium strategy for the economy $\mathcal{E}^2 = (U^2; \Omega, F, \mathcal{F})$. We say that the information structure $(\Omega, F, \mathcal{F})$ is "regular" if and only if, whenever $U^1_{k\theta}/U^1_{kk} \neq U^2_{k\theta}/U^2_{kk}$ or $U^1_{kK}/U^1_{kk} \neq U^2_{kK}/U^2_{kk}$, there exists a positive-measure set $\tilde{\Omega} \subseteq \Omega$ such that $k^1(\omega) \neq k^2(\omega)$ for any $\omega \in \tilde{\Omega}$. This restriction can be fully appreciated (in terms of primitives of the environment) only once we characterize the equilibrium. However it has a simple economic meaning: it requires that different sensitivities of individual best responses to either the fundamentals or others' activity result to different equilibrium actions for a positive measure of types. The role of this condition is to rule out trivial cases in which changes in incentives do not lead to any change in the use of information.[9]

To illustrate the type of shocks and information structures that we allow, consider the following Gaussian example. Agent i's productivity is given by $\theta_i = \bar{\theta} + \varsigma_i$, where $\bar{\theta}$ is an aggregate productivity shock while ς_i is an idiosyncratic productivity shock. The former is Normally distributed with mean μ_θ and variance σ^2_θ; the latter is i.i.d. across agents and independent of $\bar{\theta}$, Normally distributed with zero mean and variance σ^2_ς. Each agent's information ω_i consists of a private signal $x_i = \theta_i + \xi_i$ about own productivity and a public signal $y = \bar{\theta} + \varepsilon$ about the average productivity in the market. The variable ξ_i is idiosyncratic noise, i.i.d. across agents, Normally distributed with zero mean and variance σ^2_x, whereas the variable ε is public noise, Normally distributed with zero mean and variance σ^2_y. The noises ξ_i and ε are independent of one another as well as of $\bar{\theta}$ and of ς_i. In this example, f is a distribution whose marginal h over θ_i is Normal with mean $\bar{\theta}$ and variance σ^2_θ and whose marginal ϕ over $\omega_i = (x_i, y)$ assigns measure one to $y = \bar{\theta} + \varepsilon$ and is Normal in x_i with mean $\bar{\theta}$ and variance $\sigma^2_\varsigma + \sigma^2_x$. As it will become clear in Section 3, imposing the "regularity" condition in this example is equivalent to imposing that the variances σ^2_θ, σ^2_x and σ^2_y are positive and finite.

Applications. The following examples are directly nested in our framework:

- $u_i = A_i k_i - \frac{1}{2} k_i^2$, with $A_i = \theta_i + aK$, $K = \int k_i di$, $\theta_i = \bar{\theta} + \varsigma_i$, and $0 < a < 1$. This example can be interpreted as a stylized version of models with production or network externalities: the private return to investment (A) increases with aggregate investment (K) . The scalar a then parametrizes the strength of the spillover effect, while $\bar{\theta}$ is a common productivity shock and ς_i is an idiosyncratic productivity shock.

- $\pi_i = \pi^\star - (p_i - p^\star)^2$, with $p^\star = a\bar{\theta} + (1-a)\, P$, $P = \int p_i di$, $a \in (0,1)$, and $\pi^\star \in \mathbb{R}$. This example captures the incomplete-information new-Keynesian business-cycle models of Woodford (2002), Mackowiak and Wiederholt (2006), Baeriswyl and Conrand (2007), and others: firms suffer a loss whenever their price (p_i) deviates from some target level $(p^\star)$, which in turn

[9] As it will become clear in Sections 3 and 4, the only result that is affected if we relax this condition is the uniqueness of the optimal policy (not its existence).

depends on the aggregate price level (P). In this context, $\bar{\theta}$ represents exogenous aggregate nominal demand conditions, while $1-a$ determines the degree of strategic complementarity in pricing decisions (a.k.a. "real rigidities").

- $\pi_i = (a + b\theta_i + cK)\, k_i - \frac{1}{2}k_i^2$, with $K = \int k_i di$, and $(a, b, c) \in \mathbb{R}^3$. This example nests the large Cournot and Bertrand games studied in Vives (1984, 1998), Raith (1996), and various other IO papers. In this context, θ_i represents a common demand or cost shock, k_i the quantity (for Cournot) or the price (for Bertrand) set by firm i, and c the degree of strategic substitutability (for Cournot) or complementarity (for Bertrand) among firms' decisions.

- $u_i = -(1-r)\left(k_i - \bar{\theta}\right)^2 - r(L_i - \bar{L})$, with $L_i = \int (k_j - k_i)^2\, dj$, $\bar{L} = \int L_i di$, and $r \in (0, 1)$. This example is the beauty-contest game studied in Morris and Shin (2002), Svensson (2005) and Heinemann and Cornand (2006): an agent faces a cost whenever the distance of his own action from the actions of others (L_i) is higher than the average distance in the population ($\bar{L}$). This game is supposed to capture, in a stylized fashion, Keynes' idea that financial markets involve a zero-sum race between professional investors for who will second-guess the demands of others.

More generally, as it will become clear in Section 3.2, our framework nests—at least as far as equilibrium is concerned—any model in which the agents' interaction can be summarized in the following best-response structure:

$$k_i = \mathbb{E}_i[\ \Lambda(\theta_i, \bar{\theta}, K)\ ]$$

for some linear function Λ. Clearly, the institutional details and deeper micro-foundations behind this structure vary from one application to another. Nevertheless, by conducting our exercise within an abstract framework that does not take any particular stand on these "details," the lessons we will provide in the subsequent sections are likely to hold across all these applications.

Remarks. Because the primary goal of this paper is to study policies that improve welfare *without* centralizing the information that is dispersed in the population, throughout the analysis we restrict attention to tax systems that utilize only information that is in the public domain. In so doing, we rule out direct mechanisms in which the agents send reports about their types to the government and then the government collects taxes on the basis of such reports. As we will show in Section 4, this is actually without any loss of optimality within our framework as long as the government does not use such reports to transfer information from one agent to another *before* individual actions have been committed.

Note, however, that this does not mean that we rule out all forms of aggregation and exchange of information in society; it only means that the government is not itself a channel of communication. Indeed, we could readily reinterpret some of the exogenous information as the result of certain types of information aggregation; for example, some or all of the agents may observe a signal about

the underlying fundamentals that is the outcome of an opinion poll or the price of an unmodeled financial market. What is crucial for the baseline analysis is that the information available to any given agent does not depend on other agents' strategies, nor is it affected by government policies; these alternative cases are considered in Sections 6 and 7.

Finally, note that, while our prior work on the social value of information (Angeletos and Pavan, 2007) and virtually all the related literature (Morris and Shin, 2002, etc.) have limited attention to the case of perfectly correlated shocks and to a specific Gaussian information structure, here we allow the shocks to be imperfectly correlated and we consider more general information structures. This will permit us to highlight that our main policy results need not be sensitive to the specific details of the information structure. Nevertheless, we will still illustrate certain insights in the Gaussian example considered above, while restricting, for simplicity, the shocks to be perfectly correlated ($\sigma_\varsigma^2 = 0$).

3 Decentralized use of information

In this section we show how the equilibrium and the efficient use of information depend on the payoff structure U. This permits us to identify inefficiencies in the equilibrium use of information that originate from payoff effects.

3.1 Common-information benchmarks

Before we proceed to the analysis of equilibrium and efficiency with dispersed information, it is useful to review the case of *common* information; this will help isolate the inefficiencies that emerge only under dispersed information.

To start, suppose that information were complete, so that each agent knows both his own productivity and the cross-sectional distribution h of productivities in the population, and this fact is common knowledge. We can then show that the complete-information equilibrium strategy exists, is unique, and is given by

$$k_i = \kappa\left(\theta_i, \bar{\theta}\right) \equiv \kappa_0 + \kappa_1\theta_i + \kappa_2\bar{\theta},$$

where $\bar{\theta} \equiv \int \theta dh(\theta)$ denotes aggregate productivity and where the coefficients $(\kappa_0, \kappa_1, \kappa_2)$ are determined by the payoff structure.[10] Note that, by definition, an agent's payoff depends only on his *own* productivity; that an agent's equilibrium action depends also on *average* productivity is

[10] The characterization of the coefficients $(\kappa_0, \kappa_1, \kappa_2)$, as well as of the coefficients $(\kappa_0^*, \kappa_1^*, \kappa_2^*)$ for the first-best allocation, is in the proof of Proposition 1. These coefficients depend on the reduced-form payoff structure U, which in turn depends on the particular application under consideration. For the purposes of our analysis, however, understanding the specific values of these coefficients is not essential.

because the latter impacts the average action of others, which in turn affects the incentives of the individual agent.

We can further show that the first-best allocation exists, is unique, and is given by

$$k_i = \kappa^* \left(\theta_i, \bar{\theta}\right) \equiv \kappa_0^* + \kappa_1^* \theta_i + \kappa_2^* \bar{\theta},$$

where the coefficients $(\kappa_0^*, \kappa_1^*, \kappa_2^*)$ are again determined by the payoff structure. Note that, as with equilibrium, the first-best action prescribed to an agent depends both on his own productivity and on aggregate productivity; however, the dependence on aggregate productivity now emerges only because of external effects.

Now suppose that information is incomplete but common across all agents. Because of the quadratic specification of payoffs, a form of certainty equivalence holds: the equilibrium and efficient actions under incomplete but common information are the best predictors of their complete-information counterparts.

Proposition 1 *Suppose all information is common. The unique equilibrium strategy is given by* $k_i = \mathbb{E}[\kappa\left(\theta_i, \bar{\theta}\right) | \mathcal{P}]$, *while the strategy that maximizes welfare is given by* $k_i = \mathbb{E}[\kappa^*\left(\theta_i, \bar{\theta}\right) | \mathcal{P}]$, *where* $\mathcal{P}$ *denotes the common information set.*

Clearly, in economies in which $\kappa = \kappa^*$, there is no room for policy intervention as long as information remains common. However, as we will show in the next few sections, even in these economies there can be room for policy intervention once information is dispersed.[11]

3.2 Equilibrium use of information

We now turn to the analysis of equilibrium allocations when information is dispersed and when there is no policy intervention. We define an equilibrium in the standard Bayes-Nash fashion.

Definition 1 *An equilibrium is a (measurable) strategy* $k : \Omega \to \mathbb{R}$ *such that, for all* $\omega \in \Omega$,

$$k(\omega) = \arg\max_k \ \mathbb{E}[\ U(k, K(\phi), \sigma_k(\phi), \theta) \mid \omega\ ], \tag{2}$$

with $K(\phi) = \int_\Omega k(\omega')\, d\phi(\omega')$ *and* $\sigma_k(\phi) = [\int_\Omega [k(\omega') - K(\phi)]^2 d\phi(\omega')]^{1/2}$ *for all* $\phi \in \Phi$.

Consider the following coefficient, which is the slope of an individual's best response with respect to aggregate activity:

$$\alpha \equiv \frac{U_{kK}}{-U_{kk}}. \tag{3}$$

This coefficient measures the degree of strategic complementarity or substitutability among individual actions. The equilibrium use of information can then be characterized as follows.

[11] None of our results requires $\kappa \neq \kappa^*$. Indeed, the reader may henceforth assume $\kappa = \kappa^*$ if he/she wishes to focus on economies in which inefficiency emerges *only* under dispersed information.

Proposition 2 *The equilibrium strategy exists, is unique, and satisfies*

$$k(\omega) \;=\; \mathbb{E}[\; \kappa\left(\theta,\bar{\theta}\right) + \alpha \cdot \left(K\left(\phi\right) - \kappa\left(\bar{\theta},\bar{\theta}\right)\right) \;\mid \omega \;] \tag{4}$$

for all $\omega \in \Omega$, *with* $K(\phi) = \int_{\Omega} k\left(\omega'\right) d\phi(\omega')$ *for all* $\phi \in \Phi$.

Although Proposition 2 does not provide a closed-form solution for the equilibrium strategy, it is an insightful representation of it. To see this, recall that $\kappa(\theta_i, \bar{\theta})$ is the action agent i would have taken had information been complete. Next, note that $K(\phi) - \kappa(\bar{\theta}, \bar{\theta})$ is the average deviation in the actions other agents are taking relative to what they would have done under complete information. When actions are strategically independent ($\alpha = 0$), a form of certainty equivalence continues to hold: an agent's equilibrium action under incomplete information is simply his expectation of the action he would have taken under complete information. When instead actions are interdependent ($\alpha \neq 0$), the agent adjusts his action away from the aforementioned certainty-equivalence benchmark on the basis of his expectation of the average deviation in the population. In particular, if actions are strategic complements ($\alpha > 0$), the agent adjusts his action upwards whenever he expects aggregate activity to be higher than what it would have been under complete information, while he does the opposite if actions are strategic substitutes ($\alpha < 0$). In other words, equilibrium behavior is tilted to permit more alignment of actions when $\alpha > 0$ and more differentiation when $\alpha < 0$. The coefficient α thus also captures how much agents value aligning their choices with one another.

Proposition 2 has direct implications for how information is used in equilibrium. Because relying on common sources of information facilitates alignment of individual choices, while relying on idiosyncratic sources inhibits it, strategic complementarity increases the sensitivity of equilibrium actions to the former and reduces the sensitivity to the latter, while the converse is true for strategic substitutability.

To see this more clearly, consider the case where the agents' shocks are perfectly correlated and information is Gaussian. In this example, $\theta_i = \bar{\theta}$ for all i, where $\bar{\theta} \sim N\left(\mu, \sigma_\theta^2\right)$, and the information of agent i consists of a private signal $x_i = \bar{\theta} + \xi_i$ and a public signal $y = \bar{\theta} + \varepsilon$, where $\xi_i \sim N\left(\mu, \sigma_y^2\right)$ is idiosyncratic noise and $\varepsilon \sim N\left(0, \sigma_y^2\right)$. The unique equilibrium is then

$$k(x, y) \;=\; \kappa_0 + (\kappa_1 + \kappa_2)\left[\gamma_\mu \mu + \gamma_y y + \gamma_x x\right], \tag{5}$$

where the coefficients $(\gamma_\mu, \gamma_y, \gamma_x)$ are given by

$$\gamma_\mu = \frac{\pi_\theta}{\pi_\theta + \pi_y + (1-\alpha)\,\pi_x}, \quad \gamma_y = \frac{\pi_y}{\pi_\theta + \pi_y + (1-\alpha)\,\pi_x}, \quad \gamma_x = \frac{(1-\alpha)\,\pi_x}{\pi_\theta + \pi_y + (1-\alpha)\,\pi_x}, \tag{6}$$

and where $\pi_\theta \equiv \sigma_\theta^{-2}$, $\pi_y \equiv \sigma_y^{-2}$, and $\pi_x \equiv \sigma_x^{-2}$ denote the precisions of, respectively, the prior, the public signal, and the private signal.

A higher α reduces γ_x and raises γ_μ and γ_y: stronger complementarity tilts the use of information towards the prior and the public signal because they are relatively better predictors of others' activity. Next, note that $\gamma_x + \gamma_y = 1 - \gamma_\mu$, so that the equilibrium action of agent i can also be expressed in terms of the underlying fundamentals and noises as follows:

$$k_i \;=\; \kappa_0 + (\kappa_1 + \kappa_2)\left[\gamma_\mu \mu + (1-\gamma_\mu)\bar{\theta} + \gamma_y \varepsilon + \gamma_x \xi_i\right],$$

By strengthening the anchoring effect of the prior, stronger complementarity dampens the overall sensitivity of individual actions to changes in the underlying fundamentals; in other words, it increases "inertia." By increasing the reliance to noisy public information, it also amplifies the impact of common noise and hence increases the non-fundamental volatility of aggregate activity. Finally, by reducing the reliance on noisy private information it mitigates the impact of idiosyncratic noise and hence reduces the non-fundamental cross-sectional dispersion in activity.

Beyond the Gaussian example, a closed-form solution of the equilibrium strategy can be obtained in terms of the hierarchy of beliefs regarding the underlying fundamentals.

Corollary 1 *Let $\bar{E}^1 \equiv \int \mathbb{E}[\theta|\omega]d\phi(\omega)$ denote the average of the agents' expectations of their own shocks and, for any $n \geq 2$, let $\bar{E}^n \equiv \int \mathbb{E}[\bar{E}^{n-1}|\omega]d\phi(\omega)$ denote the corresponding n-th order average expectation. The equilibrium strategy is given by*

$$k(\omega) = \mathbb{E}[\kappa(\theta, \hat{\theta})|\omega] \quad \forall\ \omega, \tag{7}$$

where $\hat{\theta} \equiv \sum_{n=1}^{\infty}\left((1-\alpha)\alpha^{n-1}\right)\bar{E}^n$.

The equilibrium action under incomplete information thus has the same structure as the one under common information replacing $\bar{\theta}$ with $\hat{\theta}$. The latter is simply a weighted sum of the entire hierarchy of expectations about the underlying shocks, with the weights depending on the degree of complementarity: the stronger the complementarity, the higher the relative weight on higher order expectations. Since higher order expectations tend to be more anchored to common sources of information, this suggests that the intuitions provided by the Gaussian example are more general.

3.3 Efficient use of information

We now turn to the following question. Suppose the government can not centralize information, or otherwise transfer information from one agent to another, but can manipulate the way agents use their available information. Can the government then improve upon the equilibrium use of information?

In this section we address this question by bypassing the details of specific policy instruments that may permit such manipulation and instead characterizing directly the strategy that maximizes welfare under the sole restriction that information can not be centralized. We henceforth call this strategy the *efficient strategy*, or the *efficient use of information.*

Definition 2 *An efficient strategy is a mapping $k^* : \Omega \to \mathbb{R}$ that maximizes ex-ante utility.*

Because payoffs are linear in transfers, the latter impact welfare only through incentives. Hence, the combination of the efficient strategy with any transfer scheme that makes this strategy incentive compatible defines the very best the government can do without centralizing information. That is, when implementable, the efficient strategy is welfare-equivalent to the best incentive-compatible direct mechanism among the ones that restrict the actions the planner recommends to each agent to depend only on his report and not on the reports made by other agents. We will show how the efficient strategy can be implemented in the next section; here, we focus on its characterization.

Towards this goal, consider any arbitrary strategy $k : \Omega \to \mathbb{R}$ and let

$$\hat{k}(\theta; h) \equiv \mathbb{E}[k(\omega)|\theta, h]$$

denote the component of individual activity that is "explained" by the fundamentals. Similarly, let

$$\hat{K}(h) \equiv \mathbb{E}[\hat{k}|h] = \int \hat{k}\,(\theta; h)\, dh(\theta) \quad \text{and} \quad \hat{\sigma}_k^2(h) \equiv \text{Var}(\hat{k} - \hat{K}|h) = \int [\hat{k}\,(\theta; h) - \hat{K}(h)]^2 dh(\theta)$$

be the fundamental components of the mean and the dispersion of activity. The action of any given agent i can be decomposed in three components:

$$k_i = \hat{k}_i + \epsilon + v_i$$

The term $\hat{k}_i$ captures the variation in individual activity that reflects variation in fundamentals. The term $\epsilon \equiv (K - \hat{K})$ captures the non-fundamental variation in individual activity that is common across agents; that is, ϵ captures the impact of common noise in information. Finally, the term $v_i \equiv (k - K) - (\hat{k} - \hat{K})$ captures the non-fundamental variation in individual activity that is idiosyncratic to the agent; that is, v_i captures the impact of idiosyncratic noise.[12]

The following result then shows that a similar decomposition applies to ex-ante welfare.

Lemma 1 *Given any strategy $k : \Omega \to \mathbb{R}$, ex-ante utility (welfare) is given by*

$$\mathbb{E}u = \mathbb{E}[U(\hat{k}, \hat{K}, \hat{\sigma}_k, \theta)] + \frac{1}{2} W_{vol} \cdot vol + \frac{1}{2} W_{dis} \cdot dis \tag{8}$$

where $W_{vol} \equiv U_{kk} + 2U_{2kK} + U_{KK} < 0$ and $W_{dis} \equiv U_{kk} + U_{\sigma\sigma} < 0$, and where

$$vol \equiv \text{Var}\,(\epsilon) = \text{Var}\,(K) - \text{Var}(\hat{K}) \quad \textit{and} \quad dis \equiv \text{Var}\,(v_i) = \text{Var}\,(k - K) - \text{Var}(\hat{k} - \hat{K}).$$

The first term in (8) captures the welfare effects of the fundamental-driven variation in activity. The other two terms capture the welfare effects of the residual variation in activity: *vol* measures *non-fundamental aggregate volatility*, which originates in common noise, while *dis* measures *non-fundamental cross-sectional dispersion*, which originates in idiosyncratic noise. The coefficients W_{vol}

[12] Note that, by construction, $\hat{k}_i$, ϵ and v_i are orthogonal one to the other.

and W_{dis} then summarize the sensitivity of welfare to these two types of noise: W_{vol} measures social aversion to non-fundamental volatility, while W_{dis} measures social aversion to non-fundamental dispersion.

Both non-fundamental volatility and non-fundamental dispersion contribute to a reduction in welfare because of the concavity of payoffs. How much each of them contributes to welfare losses depends on the details of the application under examinations: different primitive preferences, technologies and market structures induce different social preferences over volatility and dispersion. For our purposes, however, it suffices to summarize these social preferences in the coefficients W_{vol} and W_{dis}. Their relative contribution can then be measured by the following coefficient:

$$\alpha^* \equiv 1 - \frac{W_{vol}}{W_{dis}}. \tag{9}$$

Since W_{vol} captures social aversion to volatility, while W_{dis} captures social aversion to dispersion, higher α^* can be interpreted as higher aversion to dispersion relative to volatility.

Strategies that share the same fundamental-driven variation in activity may differ in the levels of non-fundamental volatility and dispersion that they induce. Intuitively, the higher the sensitivity of actions to common sources of information relative to idiosyncratic sources, the higher the exposure to common noise relative to idiosyncratic noise, and hence the higher the non-fundamental volatility of activity relative to its dispersion. One should thus expect the efficient strategy to depend on social preferences over volatility and dispersion. This insight is formalized in the following proposition.

Proposition 3 *The efficient strategy exists, is unique,*[13] *and satisfies*

$$k(\omega) \;=\; \mathbb{E}[\; \kappa^*(\theta,\bar{\theta}) + \alpha^* \left(K(\phi) - \kappa^*(\theta,\bar{\theta})\right) \;\mid \omega \;] \tag{10}$$

for almost all ω*, with* $K(\phi) = \int_\Omega k(\omega')\, d\phi(\omega')$ *for all* ϕ*.*

In equilibrium, an agent's action was anchored to his expectation of κ, the complete-information equilibrium action; however, it was also adjusted on the basis of his expectation of aggregate activity, K, with the weight on the latter given by α. A similar result holds for the efficient strategy once we replace κ with κ^* and α with α^*. It follows that, just as α summarized the *private* value of aligning actions across agents, α^* summarizes the *social* value of such alignment.

That the efficient strategy is anchored to κ^*, the first-best action, is quite intuitive. That α^* in turn is inversely related to the ratio W_{vol}/W_{dis} reflects our preceding discussion about volatility and dispersion: the degree of alignment associated with the efficient strategy increases with social aversion to dispersion and decreases with social aversion to volatility.

[13] Hereafter, when we say unique, we mean up to a zero-measure subset of Ω; this is a standard qualification that one has to make with a continuum of types.

Furthermore, just as α pinned down the relative sensitivity of equilibrium actions to different sources of information, α^* pins down the corresponding relative sensitivity of efficient actions. To see this more clearly, consider again the Gaussian example with perfectly correlated shocks. The efficient strategy is then given by

$$k(x,y) = \kappa_0^* + (\kappa_1^* + \kappa_2^*)\left[\gamma_\mu^* \mu + \gamma_y^* y + \gamma_x^* x\right], \tag{11}$$

for almost all (x,y), where the coefficients $(\gamma_\mu^*, \gamma_y^*, \gamma_x^*)$ are given by

$$\gamma_\mu^* = \frac{\pi_\theta}{\pi_\theta + \pi_y + (1-\alpha^*)\,\pi_x}, \quad \gamma_y^* = \frac{\pi_y}{\pi_\theta + \pi_y + (1-\alpha^*)\,\pi_x}, \quad \gamma_x^* = \frac{(1-\alpha^*)\,\pi_x}{\pi_\theta + \pi_y + (1-\alpha^*)\,\pi_x}. \tag{12}$$

It follows that, just as α determines the equilibrium levels of inertia, non-fundamental volatility and dispersion, α^* determines the levels that are optimal from a social perspective. As for the case of more general information structures, the analogue of Corollary 1 applies for the efficient strategy once we replace κ with κ^* and α with α^*.

3.4 Inefficiency only under dispersed information

To further appreciate the inefficiencies that can emerge due to the dispersion of information, consider economies in which $\kappa = \kappa^*$. In these economies, the equilibrium is (first-best) efficient whenever information is common, leaving no room for policy intervention. Nevertheless, whenever $\alpha \neq \alpha^*$, inefficiency emerges under dispersed information, opening the door to policy intervention. The following is then an immediate implication of the results in the preceding sections.

Corollary 2 *Consider an economy that is efficient under common information* ($\kappa = \kappa^*$).

(**i**) *The equilibrium is efficient under dispersed information if and only if* $\alpha = \alpha^*$;

(**ii**) *When* $\alpha > \alpha^*$ *and information is Gaussian, the equilibrium exhibits overreaction to public information and excessive non-fundamental volatility.*

(**iii**) *When* $\alpha < \alpha^*$ *and information is Gaussian, the equilibrium exhibits overreaction to private information and excessive non-fundamental dispersion.*

4 Optimal policy

We now turn to the core contribution of the paper. We first explain how different tax schemes affect the decentralized use of information. We then identify the tax schemes that implement the efficient use of information as an equilibrium.

4.1 Equilibrium with taxes

Suppose that the information that is publicly available at stage 3 includes all individual actions $(k_i)_{i\in[0,1]}$ as well as aggregate productivity $\bar{\theta}$.[14] Then, without any loss of optimality (as it will become clear in the next subsection), consider policies defined by

$$\tau_i = T\left(k_i, K, \sigma_k, \bar{\theta}\right),$$

where the function $T : \mathbb{R}^2 \times \mathbb{R}_+ \times \Theta \to \mathbb{R}$ is a quadratic polynomial in $(k, K, \bar{\theta})$, it is linear in σ_k^2, and satisfies the following properties: $U_{kk} - T_{kk} < 0$, $-\frac{U_{kK}-T_{kK}}{U_{kk}-T_{kk}} < 1$, $T\left(K, K, 0, \bar{\theta}\right) = 0$ for all $(K, \bar{\theta})$, and $T_{kk} + T_{\sigma\sigma} = 0$. These properties preserve existence and uniqueness of equilibrium, while also guaranteeing budget balance state-by-state.[15] We denote the class of policies that satisfy these properties by $\mathcal{T}$. Finally, note that this class includes both progressive tax schemes (i.e., with $T_{kk} > 0$) and regressive tax schemes (i.e., with $T_{kk} < 0$).[16] One should thus think of this as a non-linear tax-schedule that does not directly depend on individual productivity (θ_i), but is contingent on aggregate outcomes $\left(K, \sigma_k, \bar{\theta}\right)$.

We then have the following result (to simplify the formulas, we henceforth normalize the payoff structure by setting $U_{kk} = -1$; the general case is in the appendix).

Proposition 4 *Given any tax scheme $T \in \mathcal{T}$, let*

$$\tilde{\alpha} \equiv \frac{\alpha - T_{kK}}{1 + T_{kk}}$$

$$\tilde{\kappa}_0 \equiv \frac{(1-\alpha)\kappa_0 - T_k(0,0,0)}{1 - \alpha + T_{kk} + T_{kK}} \qquad \tilde{\kappa}_1 \equiv \frac{1}{1 + T_{kk}}\kappa_1 \qquad \tilde{\kappa}_2 \equiv \frac{(1-\alpha)(\kappa_1 + \kappa_2) - T_{k\bar{\theta}}}{1 - \alpha + T_{kk} + T_{kK}} - \tilde{\kappa}_1$$

The equilibrium strategy exists, is unique, and satisfies

$$k(\omega) = \mathbb{E}\left[\ \tilde{\kappa}\left(\theta, \bar{\theta}\right) + \tilde{\alpha}\left(K(\phi) - \tilde{\kappa}\left(\bar{\theta}, \bar{\theta}\right)\right)\ \mid \omega\ \right] \tag{13}$$

for all $\omega \in \Omega$, where $\tilde{\kappa}(\theta, \bar{\theta}) \equiv \tilde{\kappa}_0 + \tilde{\kappa}_1\theta + \tilde{\kappa}_2\bar{\theta}$ and $K(\phi) = \int_\Omega k(\omega')\, d\phi(\omega')$ for all ϕ.

There are three instruments that permit the government to influence the agents' activity: T_{kk}, the progressivity of the tax system; T_{kK}, the contingency of marginal taxes on aggregate activity; and $T_{k\bar{\theta}}$, the contingency of marginal taxes on aggregate productivity. While all these instruments matter for equilibrium outcomes, each of them has a distinctive role. The progressivity T_{kk} is the only instrument that permits the government to control $\tilde{\kappa}_1$, the sensitivity of the agents' actions to their information about their *own* productivity shocks. For given T_{kk}, the instrument that

[14] We relax this assumption in Section 4.3.

[15] To see the latter property, note that, for any cross-sectional distribution ψ of individual activity, $\int T\left(k, K, \sigma_k, \bar{\theta}\right) d\psi(k) = T\left(K, K, 0, \bar{\theta}\right) + \frac{1}{2}(T_{kk} + T_{\sigma\sigma})\sigma_k^2$.

[16] In our environment, the progressivity of the tax system will turn out to affect the decentralized use of information, but it does not interfere with redistributive concerns; this is because of the linearity of payoffs in transfers.

permits the government to control the degree of complementarity ($\tilde{\alpha}$) and thereby the sensitivity of actions to common noise relative to idiosyncratic noise is the contingency T_{kK} of taxes on aggregate activity. Finally, for given T_{kk} and T_{kK}, the instrument that permits the government to control the sensitivity of individual actions to variations in aggregate productivity is the contingency $T_{k\bar{\theta}}$ of marginal taxes on $\bar{\theta}$.

4.2 Implementation of the efficient strategy

We now turn to the question of whether there exists a policy $T^* \in \mathcal{T}$ that implements the efficient strategy as an equilibrium. Whenever this is the case, by the very definition of the efficient strategy, this also guarantees that there is no other transfer scheme that can improve upon T^*. This is true even for transfer schemes that violate budget balance and/or anonymity, and even if one allows the agents to send arbitrary messages to the planner and the planner to make the transfers contingent on these messages. What is essential is only that the planner does not send informative messages to the agents before they commit their choices. The next result establishes existence and uniqueness of a policy $T^* \in \mathcal{T}$ that implements the efficient allocation.

Proposition 5 *A policy in $\mathcal{T}$ that implements the efficient strategy always exists, is unique, and has the property that, for given (κ, κ^*), the optimal T_{kK} increases with α and decreases with α^*.*

The proof of this result is follows from Proposition 4. First, note that there exists a unique T_{kk} such that $\tilde{\kappa}_1 = \kappa_1^*$. But then there also exists a unique T_{kK} such that $\tilde{\alpha} = \alpha^*$, a unique $T_{k\bar{\theta}}$ such that $\tilde{\kappa}_2 = \kappa_2^*$, and a unique $T_k(0,0,0)$ such that $\tilde{\kappa}_0 = \kappa_0^*$. The rest of the parameters of the policy function T are then pinned down by budget balance, establishing that there exists a unique policy that implements the efficient strategy as an equilibrium.[17]

The optimal policy has the property that, keeping κ and κ^* constant, the optimal T_{kK} increases with α and decreases with α^*. This is because a higher T_{kK}, by reducing the degree of complementarity perceived by the agents, it reduces the sensitivity of individual decisions to common noise in information. If we thus look across economies that share the same equilibrium and efficiency properties under complete information (i.e., they feature the same κ and κ^*) but differ in these properties under incomplete information (i.e., they feature different α and α^*), we then find that the optimal T_{kK} is higher in economies that exhibit a larger discrepancy between the private and the social value of aligning choices; equivalently, the optimal T_{kK} is higher the more excessive the non-fundamental volatility of the equilibrium relative to its non-fundamental dispersion.

[17] The uniqueness result holds for "regular" information structures. For "non-regular" information structures, the degree of complementarity is irrelevant, leaving one degree of indeterminacy. See Section 4.5 for such a case.

4.3 Implementation with measurement error

The policies considered so far do not require that the government have superior information than the agents at any time: the taxes are conditioned on information that is public at the time taxes are levied. However, the preceding analysis has assumed that actions are perfectly revealed at that time. In contrast, for many applications it may be more appropriate to assume that actions, as well as aggregate fundamentals, are only imperfectly observed.

To accommodate this possibility, we now consider a variant that allows actions and average productivity to be observed with noise in stage 3. In particular, if agent i chooses k_i in stage 2, then in stage 3 the government—and all other agents—observes $\tilde{k}_i = k_i + \eta + \nu_i$, where η is a common noise while ν_i is an idiosyncratic noise, with respective variances σ_η^2 and σ_ν^2. Similarly, instead of the true aggregate productivity $\bar{\theta}$, the government—as well as any other agent—observes a signal $\tilde{\theta} = \bar{\theta} + \varsigma$, where ς is noise with variance σ_ς^2. All these noises could be interpreted as measurement errors and are assumed to be independent of the fundamentals and of the information that agents have in stage 2. We then let $\tilde{K} = K + \eta$ and $\tilde{\sigma}_k^2 = \sigma_k^2 + \sigma_\nu^2$ denote, respectively, the cross-sectional average and dispersion of $\tilde{k}$, and consider tax schedules of the form

$$\tau_i = T(\tilde{k}_i, \tilde{K}, \tilde{\sigma}_k, \tilde{\theta}),$$

where the function T is assumed to satisfy the same properties as in the previous section (i.e. $T \in \mathcal{T}$). The tax an agent expects to pay is then given by:

$$\begin{aligned} \mathbb{E}[T(\tilde{k}_i, \tilde{K}, \tilde{\sigma}_k, \tilde{\theta})|\omega_i] &= \mathbb{E}[T(k_i, K, \sigma_k, \bar{\theta})|\omega_i] \\ &\quad + \tfrac{1}{2}\left(T_{kk} + 2T_{kK} + T_{KK}\right)\sigma_\eta^2 + \tfrac{1}{2}(T_{kk} + T_{\sigma\sigma})\sigma_v^2 + T_{\theta\theta}\sigma_\varsigma^2. \end{aligned} \tag{14}$$

The last three terms in (14) capture the impact of measurement errors on the expected tax. Because these terms are independent of the agents' actions, they have no impact on individual incentives and hence they do not interfere with the incentives provided by the tax system. It follows that, not only the noise does not interfere with the ability to implement the efficient strategy, but also it does not affect the properties of the optimal tax system.

Of course, the last property relies on the noise being additive (i.e., separable from the agents' actions). When, instead, the noise is multiplicative, it does impact incentives. However, by appropriately adjusting the policy, the government can fully undo the incentive effects of the noise. It follows that measurement error once again does not interfere with the ability to implementable the efficient strategy, although it now may affect the details of the optimal tax system.

Proposition 6 *The efficient strategy can be implemented regardless of whether activity and fundamentals are observed with measurement error.*

Given this result, for the remainder of the analysis we can abstract from measurement error without any significant loss of generality.

4.4 Inefficiency only under dispersed information

As mentioned in Section 1, financial-market observers—at both the academic and the policy front—are often concerned about how information is processed in financial markets. In particular, while many believe that financial markets work well on average, many also feel that financial markets often overreact to noisy public news, causing excessive non-fundamental volatility in both asset prices and real investment. At some level, this possibility can be captured in our framework by the restriction that $\kappa = \kappa^*$ and $\alpha > \alpha^*$. More generally, economies in which $\kappa = \kappa^*$ but $\alpha \neq \alpha^*$ offer an interesting benchmark because in these economies policy intervention becomes desirable *only* when information is dispersed. We now thus examine the properties of optimal policy for this special class economies.

Because $\alpha \neq \alpha^*$ means that the equilibrium is inefficient in its relative response to different sources of information and hence to different types of noise, it is necessary that the marginal tax co-varies with the components of activity that are not explained by the fundamentals. In particular, letting $\epsilon = K - \hat{K}$ denote the component of activity that is due to common noise, we have the following result.

Corollary 3 *Consider economies in which inefficiency emerges only under dispersed information. When $\alpha > \alpha^*$, the optimal policy has $T_{kK} > 0$ and the marginal tax co-varies positively with ϵ. The converse is true when $\alpha < \alpha^*$.*

This result is intuitive. In situations in which, if it were not for policy intervention, the equilibrium would exhibit overreaction to common sources of information and excessive non-fundamental volatility, the optimal marginal tax co-varies positively with the common noise that drives this non-fundamental volatility. It is then precisely this property of the tax system that discourages agents from overreacting to common sources of information and dampens non-fundamental volatility.

Note, however, that this appealing property of the tax system is achieved only in an indirect way, without any need to monitor and quantify the various sources of information available to the agents. This is done by making marginal taxes contingent on observable aggregate outcomes. In particular, when $\alpha > \alpha^*$, the optimal policy reduces the complementarity in individual actions, and thereby dampens the reaction of the equilibrium to common noise, by making the contingency of marginal taxes on realized aggregate activity positive ($T_{kK} > 0$). The optimal policy then guarantees that the overall response of the equilibrium to aggregate productivity is not distorted by making the marginal tax also a decreasing function of realized aggregate productivity ($T_{k\bar{\theta}} < 0$).

4.5 Idiosyncratic vs aggregate shocks

Another special case of interest is the case of independent private values typically considered in the new public finance literature. Studying this particular case helps appreciate how our policy

results depend on the dispersion of information regarding aggregate shocks, as opposed to purely idiosyncratic shocks.

Within our framework, this case can be captured as follows: first, suppose that $\omega_i = \theta_i$, so that θ_i is the private information of agent i and $\phi = h$; second, suppose that the cross-sectional distribution h is common knowledge, so that there is no uncertainty about the cross-sectional distribution of types in society. Because ϕ, the cross-sectional distribution of information, is common knowledge, in equilibrium aggregate investment is also common knowledge. Together with the fact that h, and hence $\bar{\theta}$, is commonly known and the fact that θ_i is known to agent i, this implies that the unique equilibrium is given by $k(\omega_i) = \kappa\left(\theta_i, \bar{\theta}\right)$. Similarly, the efficient allocation is given by $k(\omega_i) = \kappa^*\left(\theta_i, \bar{\theta}\right)$. As a result, neither α matters for equilibrium behavior, nor α^* matters for efficient allocations. We conclude that in the case of independent private values it is not necessary to condition the tax system on realized aggregate activity.[18]

What renders α and α^*, and hence also the contingency T_{kK}, irrelevant in the aforementioned environment is not per se the fact that private values are independent but rather that there is no strategic uncertainty: no agent faces uncertainty regarding the distribution of actions in the population. Indeed, if we relax either the assumption that θ_i is known to agent i or the assumption that h is common knowledge but maintain the assumption that ϕ is common knowledge, then the distribution of actions is also common knowledge in equilibrium. This in turn eliminates the problem of forecasting aggregate activity, once again guaranteeing that neither α matters for equilibrium behavior nor α^* matters for the efficient allocation—and therefore nor T_{kK} is essential for achieving efficiency. We conclude that the key distinctive property of the correlated-value environments we consider in this paper is the strategic uncertainty created by the dispersion of information regarding *aggregate* shocks.

Corollary 4 *When the cross-sectional distribution of information in society* (ϕ) *is common knowledge, the contingency of taxes on aggregate activity* (T_{kK}) *is not essential for implementing the efficient strategy.*

5 A dynamic economy

The analysis so far has been confined to a static game. We now show how this static game can be embedded in a dynamic setting with a more macro flavor. This serves two goals. First, it helps further appreciate how our results can be relevant for applications. Second, it accommodates

[18] In particular, the efficient allocation can be implemented with a tax schedule that depends only on own activity and $\bar{\theta}$. Moreover, should the tax be made contingent on K, this contingency would matter for equilibrium behavior only through κ_2 (the sensitivity of the complete-information equilibrium to $\bar{\theta}$), not through α (the degree of alignment); the contingency on $\bar{\theta}$ should then be adjusted to perfectly offset the effect of T_{kK} on κ_2.

the possibility that interesting dynamics in actions originate in the dynamics of information. In this section, we start by taking the dynamics of information as entirely exogenous; we turn to the analysis of endogenous information in Section 6.

5.1 Set up

There are $N+1$ periods, with $N \geq 2$. In each period $t = 1, ..., N$, each agent i chooses his level of investment in a riskless discount bond, $b_{i,t}$, and his level of consumption, $c_{i,t}$. The agent also chooses an action $k_{i,t} \in \mathbb{R}$, which we henceforth interpret as capital invested in a risky technology. Investing $k_{i,t}$ costs $G(k_{i,t})$ in period t and delivers $F(k_{i,t}, K_t, \sigma_t, A_{i,t+1})$ in period $t+1$, where K_t and σ_t are the mean and the dispersion of activities in period t, $A_{i,t+1}$ is an exogenous productivity shock, and G and F are real-valued functions. To simplify the exposition, we henceforth impose that the productivity shocks are perfectly correlated across agents, so that $A_{i,t+1} = \theta_t$ for all i, t. (The timing convention we adopt here is that θ_t denotes the common shock that is relevant for period-t decisions.)

The agent's period-t budget is given by

$$c_{i,t} + G(k_{i,t}) + q_t b_{i,t} = F(k_{i,t-1}, K_{t-1}, \sigma_{t-1}, \theta_{t-1}) + b_{i,t-1} - \tau_{i,t},$$

where q_t denotes the period-t price of discount bonds (the reciprocal of the period$-t$ risk-free rate) and $\tau_{i,t}$ denotes the period-t taxes the agent pays to (or the transfers he receives from) the government.[19] Finally, the agent's intertemporal preferences are given by

$$\mathcal{U}_i = \sum_{t=1}^{N+1} \beta^{t-1} U(c_{i,t}, k_{i,t}).$$

where U_t is a real-valued function.

This framework is quite flexible. All the applications we considered in the static benchmark can be nested by setting $U(c,k) = c$ and then letting $-G(k) + \beta F(k, K, \sigma, \theta)$ equal the payoffs assumed in those statics examples (e.g., $-G(k) + \beta F(k, K, \sigma, \theta)$ can be interpreted as the profit of a firm). Alternatively, a stylized version of the neoclassical growth model with convex investment costs is nested by letting $U(c,k) = c$, $F(k, K, \sigma, \theta) = \theta k$, and $G(k) = k + \chi k^2$, for some constant $\chi > 0$. One could also interpret k as individual effort, in which case it would be natural to let $G(k) = 0$ and $U(c,k) = c - H(k)$, with $H(k) = k^2$ representing the disutility of effort. Finally, one could further allow U and G to depend on (K, σ, θ), as to capture externalities in leisure, pecuniary externalities in the costs of investment, and so on.

[19] For $t = N+1$, we impose that $k_{i,t} = b_{i,t} = 0$ for all i, in which case the last-period budget constraint reduces to $c_{i,N+1} = F(k_{i,N}, K_N, \sigma_N, A_{N+1}) - G(0) + b_{i,N} - \tau_{i,N+1}$. Furthermore, for $t = 1$, without loss, we normalize each agent's endowment to zero so that $F(k_{i,0}, K_0, \sigma_0, A_1) = b_{i,0} = 0$ for all i.

If agents had common information about the shocks in all periods, then the analysis could proceed essentially without any further restrictions on the functions F, G and U. Here, however, we are interested in cases where agents have heterogeneous information. To keep the analysis tractable, we impose two restrictions. First, we assume that U is linear in consumption: $U(c,k) = c - H(k)$, for some function H. Second, we let

$$V(k, K, \sigma, \theta) \equiv -[G(k) + H(k)] + \beta F(k, K, \sigma, \theta)$$

and assume that V satisfies the same properties with respect to (k, K, σ, θ) as the function U in the static model. The first restriction ensures that, in all periods and states, the bond market clears if and only if $q_t = \beta$ (in which case the demand for the risk-free bond is indeterminate) and that the life-time utility of agent i (in the absence of taxes) reduces to

$$\mathcal{U}_i = \sum_{t=1}^{N} \beta^{t-1} V(k_{i,t}, K_t, \sigma_t, \theta_t).$$

The second restriction then permits to extend our previous static analysis to the dynamic model.

The key here is to rule out informational externalities—the possibility that what an agent knows in period t about θ_t depends on the actions other agents took in periods $s < t$. To ensure this, we model the dynamics of the information structure as follows. The (exogenous) information of an agent in period t is represented by $\omega_{i,t} \in \Omega_t$. Let $f \in F$ denote a joint distribution for $\{\theta_t, \omega_{i,t}\}_{t=1}^N$, with marginal distributions for $\omega_{i,t}$ given by $\phi_t \in \Phi_t$. The distribution f also describes the cross-sectional distribution of $\{\theta_t, \omega_{i,t}\}_{t=1}^N$ in the population. First, Nature draws f from a set of possible distributions F according to the probability measure $\mathcal{F}$. Nature then uses f to draw a sequence $\{\theta_t, \omega_{i,t}\}_{t=1}^N$ for each agent i, with $\{\theta_t, \omega_{i,t}\}_{t=1}^N$ drawn independently from f. Finally, we assume that $(\omega_{i,t-1}, \phi_{t-1}, \theta_{t-1})$ belongs to $\omega_{i,t}$, for all i and t. This ensures that there is nothing to learn about $(\theta_s, \phi_s)_{s=t}^N$ from the observation of other agents' (past) actions—whether such actions are observable is then irrelevant.[20] It is then without loss of generality, for either equilibrium or efficiency, to restrict attention to strategies that depend only on $\omega_{i,t}$.

5.2 Equilibrium, efficiency and policy

Given that information is exogenous in all dates and states, the analysis of both the equilibrium and efficient allocations parallels that in the static benchmark. Let $\kappa(\theta)$ denote the (unique) solution to $V_k(\kappa, \kappa, 0, \theta) = 0$ and let $\kappa^*(\theta) \equiv \arg\max_\kappa V(\kappa, \kappa, 0, \theta)$; if information were complete, the equilibrium action in period t would be $\kappa(\theta_t)$, while the first-best action would be $\kappa^*(\theta_t)$.[21] Next,

[20] An alternative that would also guarantee that agents do not learn anything about $(\theta_s, \phi_s)_{s=t}^N$ from the observation of past actions is to assume that for all $t > 1$, $\omega_{i,t}$ is a sufficient statistic for $\left(\omega_{i,t}, (\omega_{i,s}, \phi_s, \theta_s)_{s=1}^{t-1}\right)$ with respect to $(\theta_s, \phi_s)_{s=t}^N$.

[21] Both κ and κ^* are linear functions of θ. In particular, $\kappa(\theta) = \kappa_0 + (\kappa_1 + \kappa_2)\theta$ and $\kappa^*(\theta) = \kappa_0^* + (\kappa_1^* + \kappa_2^*)\theta$ with the coefficients $(\kappa_0, \kappa_1, \kappa_2, \kappa_0^*, \kappa_1^*, \kappa_2^*)$ determined as in the proof of Proposition 1 replacing U with V.

let

$$\alpha \equiv -\frac{V_{kK}}{V_{kk}} \qquad \text{and} \qquad \alpha^* \equiv 1 - \frac{W_{vol}}{W_{dis}},$$

with $W_{vol} \equiv V_{kk} + 2V_{kK} + V_{KK}$ and $W_{dis} \equiv V_{kk} + V_{\sigma\sigma}$; once again, α summarizes the private value of aligning choices (the equilibrium degree of complementarity), while α^* summarizes the social value of such alignment (the relative social aversion to dispersion and volatility). The equilibrium and efficient allocations under incomplete information are then characterized in the following two propositions, which are direct extensions of Propositions 2 and 3.

Proposition 7 *The equilibrium strategy exists, is unique, and satisfies, for all periods t and all $\omega_t \in \Omega_t$,*

$$k_t(\omega_t) \;=\; \mathbb{E}[\; \kappa\left(\theta_t\right) + \alpha \cdot \left(K_t\left(\phi_t\right) - \kappa_t\left(\theta_t\right)\right) \;\mid \omega_t \;],$$

with $K_t(\phi_t) = \int k_t\left(\omega'\right) d\phi_t(\omega')$.

Proposition 8 *The efficient strategy exists, is unique, and satisfies, for all periods t and almost all $\omega_t \in \Omega_t$,*

$$k_t(\omega_t) \;=\; \mathbb{E}[\; \kappa^*\left(\theta_t\right) + \alpha^* \cdot \left(K_t\left(\phi_t\right) - \kappa^*\left(\theta_t\right)\right) \;\mid \omega_t \;],$$

with $K_t(\phi_t) = \int_\Omega k_t\left(\omega'\right) d\phi_t(\omega')$.

The efficient strategy can be implemented in a similar fashion as in Section 4. In particular, efficiency can be induced in period t by making taxes in period $t+1$ contingent on information about K_t and θ_t that becomes publicly available at $t+1$. As in the static model, the optimal policy does not require any informational advantage on the side of the government. It merely depends on the agents anticipating when they make their decision that the marginal tax they will pay in the future will be contingent on public information about aggregate economic conditions.

6 Informational externalities

A key functioning of modern economies that is missed in our preceding analysis is the aggregation of dispersed information in various indicators of aggregate activity, such as financial prices, trade volume, aggregate employment, output and investment data. What is crucial for our purposes is that the informational content of these indicators depends on the way agents use their available information in the first place: the more individuals rely on their private information, the more informative aggregate activity is of the underlying fundamentals. The various channels of information aggregation and social learning thus introduce informational externalities that have to be taken into account when determining the socially optimal use of information.

We study this issue, and its policy implications, within a variant of the dynamic framework introduced in the previous section. Past shocks are no longer directly revealed to the agents. Rather,

the agents learn about these shocks from the observation of a noisy signal of past aggregate activity. This signal is a proxy for the informational role of macroeconomic data, financial prices, and other channels of information aggregation and social learning.

6.1 Set up

To be able to analyze the endogenous dynamics of information in a tractable way, we must sacrifice some of the generality we have permitted in the preceding analysis: we henceforth restrict all exogenous information to be Gaussian. In particular, we assume that the component of the fundamentals about which the agents have heterogeneous information is constant over time, we denote this component by θ, and we assume that it is drawn from a Normal distribution with mean μ and variance σ_θ^2. The realization of θ is never revealed. Instead, at any date t, agents observe a public signal $y_t = \theta + \varepsilon_t$ and private signals $x_{i,t} = \theta + \xi_{i,t}$, where $\varepsilon_t \sim \mathcal{N}(0, \sigma_{y,t}^2)$ and $\xi_{i,t} \sim \mathcal{N}(0, \sigma_{x,t}^2)$ are noises, independent of one another, independent across time, and independent of θ, with $\xi_{i,t}$ also independently and identically distributed across agents. In addition, at any date $t \geq 2$, agents observe the following three random variables, which affect payoffs and convey information about θ: $\tilde{K}_{t-1} = K_{t-1} + \eta_t$, $\tilde{\sigma}_{t-1} = \sigma_{t-1} + \upsilon_t$, and $\tilde{A}_t = \theta + a_t$, where $\eta_t \sim \mathcal{N}(0, \sigma_{\eta,t}^2)$, $\upsilon_t \sim \mathcal{N}(0, \sigma_{\upsilon,t}^2)$, and $a_t \sim \mathcal{N}(0, \sigma_{a,t}^2)$ are shocks, common across agents, independent across time, and independent of any other random variable. The period-t budget of the agent is given by

$$c_{i,t} + G\left(k_{i,t}\right) + q_t b_{i,t} = F(k_{i,t-1}, \tilde{K}_{t-1}, \tilde{\sigma}_{t-1}, \tilde{A}_t) + b_{i,t-1} - \tau_{i,t}.$$

The variable $\tilde{A}_t$ can be interpreted as the period$-t$ productivity shock, while θ is the underlying mean (trend) productivity. That the variables $\tilde{K}_{t-1}$ and $\tilde{\sigma}_{t-1}$ that enter period$-t$ income (through F) coincide with the signals about past activity is not essential. What is essential is that the observation of income does not perfectly reveal either θ or K_{t-1}.[22]

The rest of the model is as in Section 5. The intertemporal payoff of an agent is given by $\sum_{t=1}^{N+1} \beta^{t-1} U\left(c_{i,t}, k_{i,t}\right)$, where $U\left(c_{i,t}, k_{i,t}\right) = c_{i,t} - h(k_{i,t})$. That preferences are linear in consumption ensures once again that $q_t = \beta$, that the trades of riskless bonds and the timing of consumption are indeterminate, and that the intertemporal payoff of an agent (in the absence of taxes) reduces to

$$\sum_{t=1}^{N+1} \beta^{t-1} V\left(k_{i,t}, \tilde{K}_t, \tilde{\sigma}_t, \tilde{A}_{t+1}\right),$$

where $V\left(k, K, \sigma, A\right) \equiv -[G\left(k\right) + H(k)] + \beta F\left(k, K, \sigma, A\right)$. The function V is quadratic and satisfies the same restrictions as in the previous sections.

[22] Also, the results that follow do not depend on whether the signals about past actions are public or private. In particular, we could allow the agents to receive private signals $\tilde{K}_{i,t-1} = K_{t-1} + \eta_{i,t}$, $\tilde{\sigma}_{t-1} = \sigma_{t-1} + \upsilon_{i,t}$, and $\tilde{A}_{i,t} = \theta + a_{i,t}$, in addition to, or in substitution for, the aforementioned public signals.

6.2 Equilibrium

The essential difference between the economy of this section and the one examined in Section 5 is the endogeneity of information: the strategy agents follow in period t determines how much information about θ is contained in $(\tilde{K}_t, \tilde{\sigma}_t)$ and hence affects the agents' behavior in periods $t+1$ on. In the absence of policy, this informational externality is not internalized by the agents: the fact that the use of information in the present affects the information available in the future does not alter private incentives. Thus, letting once again $\kappa(\theta) = \kappa_0 + (\kappa_1 + \kappa_2)\theta$ denote the unique solution to $V_k(\kappa, \kappa, 0, \theta) = 0$ and $\alpha \equiv -V_{kK}/V_{kk}$, we have the following result.

Proposition 9 *The equilibrium strategy exists and is unique. Let $\{\Omega_t, \Phi_t\}_{t=1}^N$ denote the (unique) information structure generated by the equilibrium strategy. Then, for all t, the strategy and the information structure jointly satisfy*

$$k_t(\omega_t) \;=\; \mathbb{E}[\; \kappa(\theta) + \alpha \cdot (K_t(\phi_t) - \kappa(\theta)) \;\mid \omega_t \;] \tag{15}$$

for all $\omega_t \in \Omega_t$, with $K_t(\phi_t) = \int_\Omega k_t(\omega')\, d\phi_t(\omega')$ for all $\phi_t \in \Phi_t$.

This result does not require the information structure to be Gaussian. However, once we restrict θ and the exogenous noises to be Gaussian, this result ensures that the information contained in the signals of past activity is also Gaussian. All the information—exogenous and endogenous—that is available in any given period can then be summarized in two sufficient statistics, one for the private and the other for the public signals; the dynamics of these two statistics admit a simple recursive structure and the equilibrium strategy reduces to an affine combination of the two.

Proposition 10 *The equilibrium strategy is given by*

$$k_{i,t}(\omega_{i,t}) = \kappa(\gamma_t X_{i,t} + (1-\gamma_t) Y_t),$$

with

$$\gamma_t = \frac{(1-\alpha)\pi_t^x}{(1-\alpha)\pi_t^x + \pi_t^y}. \tag{16}$$

The variables $X_{i,t}$ and Y_t are sufficient statistics for all the private and public information about θ that is available to agent i in period t, while π_t^x and π_t^y are their respective precisions. The sufficient statistics are given recursively by

$$X_{i,t} = \frac{\pi_{t-1}^x}{\pi_t^x} X_{i,t-1} + \frac{\sigma_{x,t}^{-2}}{\pi_t^x} x_{i,t} \quad \textit{and} \quad Y_t = \frac{\pi_{t-1}^y}{\pi_t^y} Y_{t-1} + \frac{\sigma_{y,t}^{-2}}{\pi_t^y} y_t + \frac{\sigma_a^{-2}}{\pi_t^y} A_t + \frac{\gamma_{t-1}^2 (\kappa_1+\kappa_2)^2 \sigma_{\eta,t}^{-2}}{\pi_t^y} \tilde{y}_t \tag{17}$$

where

$$\tilde{y}_t \equiv \frac{\tilde{K}_{t-1} - \kappa_0 - (\kappa_1+\kappa_2)(1-\gamma_{t-1}) Y_{t-1}}{(\kappa_1+\kappa_2)\gamma_{t-1}} \tag{18}$$

is a linear transformation of the signal of past activity. Similarly, the precisions π_t^x *and* π_t^y *are given recursively by*

$$\pi_t^x = \pi_{t-1}^x + \sigma_{x,t}^{-2} \quad \textit{and} \quad \pi_t^y = \pi_{t-1}^y + \sigma_{y,t}^{-2} + \sigma_{a,t}^{-2} + (\kappa_1 + \kappa_2)^2 \gamma_{t-1}^2 \sigma_{\eta,t}^{-2}. \tag{19}$$

Finally, the initial conditions are $X_{i,0} = 0$, $Y_{i,0} = \mu$, $\gamma_0 = 0$, $\pi_0^x = 0$ *and* $\pi_0^y = \sigma_\theta^{-2}$.

The logic behind condition (16) is the same as the one we encountered in the static benchmark. For given degree of complementarity α, the relative sensitivity γ_t of the equilibrium strategy to private information increases with the precision of private information and decreases with the precision of public information. At the same, for given precisions, a higher α tilts the equilibrium strategy away from private information and towards public information, as agents find it optimal to better align their choices.

Conditions (17)-(19), on the other hand, describe the endogenous evolution of the information, which can be understood as follows. First note that, because $X_{i,t-1}$ equals θ plus idiosyncratic noise, aggregate activity in period $t-1$ is given by

$$K_{t-1} = \kappa_0 + (\kappa_1 + \kappa_2)(\gamma_{t-1}\theta + (1 - \gamma_{t-1}) Y_{t-1}).$$

Because Y_{t-1} is publicly known (and so are the coefficients κ_0, κ_1, κ_2 and γ_{t-1}), observing $\tilde{K}_{t-1} = K_{t-1} + \eta_t$ in period t is informationally-equivalent to observing the variable $\tilde{y}_t$ defined in (18). But now note that

$$\tilde{y}_t = \theta + \frac{1}{(\kappa_1 + \kappa_2)\gamma_{t-1}} \eta_t,$$

which is simply a Gaussian signal with precision $\tilde{\pi}_t = \gamma_{t-1}^2 (\kappa_1 + \kappa_2)^2 \sigma_{\eta,t}^{-2}$. It follows that all private signals can be combined in the sufficient statistic $X_{i,t}$, while all public signals can be combined in the sufficient statistics Y_t. Condition (17) then states that these statistics are simply weighted averages of all the available signals, with the weights dictated by the respective precisions of these signals, while condition (19) states that the precisions of these statistics are simply the sums of the precisions of the component signals.

The key property to notice is that the precision of information available in one period depends on the strategy followed in previous periods. In particular, for all t, $\tilde{\pi}_t$ and thereby π_t^y is increasing in γ_{t-1}.[23] This is because the informative content of the signals of aggregate activity is higher the more sensitive the strategies of the agents to their private information.[24] This is an important informational externality that the equilibrium fails to internalize in the absence of policy intervention.

[23] When θ changes over time, the period-t precision of information regarding the period-t fundamental need be monotonic over time; but it remains an increasing function of the sensitivities of past strategies to private information.

[24] A similar property typically holds in rational-expectation-equilibria models: the information contained in the price increases with the sensitivity of individual asset demands to private information.

6.3 Efficiency and policy

We now turn to the policy implications of the aforementioned informational externality by characterizing the strategy that maximizes ex-ante utility taking into account this externality. However, unlike the cases of exogenous information examined in the previous section, we now restrict attention to strategies that are linear in the available private signals. Without this restriction, the endogenous signals are no longer Gaussian and the analysis becomes intractable.

Suppose, for a moment, that the government fails to recognize that the strategy the agents follow in period t affects the information available in subsequent periods. Suppose further that the period-t private and public information were summarized in sufficient statistics $X_{i,t}$ and Y_t with respective precisions π_t^x and π_t^y, so that

$$\mathbb{E}\left[\theta|\omega_{i,t}\right] = \tfrac{\pi_t^x}{\pi_t^x+\pi_t^y} X_{i,t} + \tfrac{\pi_t^y}{\pi_t^x+\pi_t^y} Y_t,$$

and let $\kappa^*(\theta) \equiv \arg\max_\kappa V(\kappa,\kappa,0,\theta) = \kappa_0^* + (\kappa_1^* + \kappa_2^*)\theta$ and $\alpha^* \equiv 1 - W_{vol}/W_{dis} \equiv 1 - (V_{kk} + 2V_{kK} + V_{KK})/(V_{kk} + V_{\sigma\sigma})$. Proposition 8 would then imply that the efficient strategy is given by

$$k_{i,t}(\omega_{i,t}) = \kappa^*\left(\gamma_t^* X_{i,t} + (1-\gamma_t^*) Y_t\right),$$

with

$$\gamma_t^* = \frac{(1-\alpha^*)\pi_t^x}{(1-\alpha^*)\pi_t^x + \pi_t^y}.$$

Now suppose the government takes into account the endogeneity of the information. As long as welfare in the subsequent periods is increasing in the precision of available information, it should be desirable to adjust the current use of information so as to induce more learning in subsequent periods. Because more learning is achieved only by the aggregation of private information, this suggests that the informational externality raises the sensitivity of efficient strategies to private information. The following result verifies this intuition.

Proposition 11 *The linear strategy that maximizes ex-ante utility is given by*

$$k_{i,t}(\omega_{i,t}) = \kappa^*\left(\gamma_t^{**} X_{i,t} + (1-\gamma_t^{**}) Y_t\right),$$

where

$$\gamma_t^{**} = \frac{(1-\alpha^*)\pi_t^x}{(1-\alpha^*)\pi_t^x + \pi_t^y - \beta(1-\alpha^*)\left(1-\gamma_{t+1}^{**}\right)^2 \pi_t^x \pi_t^y \left(\pi_{t+1}^y\right)^{-2} (\kappa_1^* + \kappa_2^*)^2 \sigma_{\eta,t+1}^{-2}}$$

for all $t < N$, while $\gamma_N^{**} = (1-\alpha^*)\pi_N^x / \left[(1-\alpha^*)\pi_N^x + \pi_N^y\right]$. $X_{i,t}$ and Y_t are sufficient statistics for all the private and public information about θ available to agent i in period t, while π_t^x and π_t^y are their respective precisions; they are obtained recursively using (17)-(19), replacing $(\gamma_t, \kappa_0, \kappa_1, \kappa_2)$ with $(\gamma_t^{**}, \kappa_0^*, \kappa_1^*, \kappa_2^*)$

The key result here is that, holding constant the current precisions of private and public information, the optimal weight on private information is higher than what it would have been had information in the subsequent periods been exogenous:

$$\gamma_t^{**} > \frac{(1-\alpha^*)\,\pi_t^x}{(1-\alpha^*)\,\pi_t^x + \pi_t^y}.$$

As anticipated, this follows directly from the internalization of the informational externality: by raising the reliance on private information in one period, society achieves higher precision of information and hence higher welfare in subsequent periods.[25]

The following alternative representation of the optimal strategy helps translate the result here in terms of the degree of complementarity that the policy must induce in equilibrium.

Proposition 12 *Consider the efficient linear strategy and let* $\{\Omega_t, \Phi_t\}_{t=1}^N$ *be the associated information structure. There exists a unique sequence* $\{\alpha_t^{**}\}_{t=1}^N$, *with* $\alpha_t^{**} < \alpha^*$ *for all* $t < N$ *and* $\alpha_N^{**} = \alpha^*$, *such that the efficient strategy and the information structure jointly satisfy, for all* t,

$$k_t^*(\omega_t) \;=\; \mathbb{E}[\; \kappa^*(\theta) + \alpha_t^{**} \cdot (K_t(\phi_t) - \kappa^*(\theta_t)) \;\mid \omega_t \;] \tag{20}$$

for almost all $\omega_t \in \Omega_t$, *with* $K_t(\phi_t) = \int_\Omega k_t^*(\omega')\, d\phi_t(\omega')$ *for all* $\phi_t \in \Phi_t$.

As in the case without informational externalities, the weight α_t^{**} in condition (20) summarizes how much society would like the agents to factor their expectations of other agents' choices in their own choices. Unlike the case without informational externalities, this weight now depends on the information structure. Nevertheless, condition (20) remains a valid and insightful representation of the optimal strategy: the result that $\alpha_t^{**} < \alpha^*$ highlights that having the agents internalize the informational externality is isomorphic to having them perceive a lower complementarity in their actions than the one they should have perceived had information been exogenous. This in turn guides policy analysis: the optimal linear strategy can be implemented with similar tax schemes as in the benchmark model, but now the sensitivity T_{kK} of the marginal tax to aggregate activity must be *higher* than what it would have been with exogenous information.

Corollary 5 *Informational externalities unambiguously contribute to a higher optimal sensitivity of the marginal tax to aggregate activity.*

This result is true irrespective of the specific payoff interdependencies and irrespective of whether the equilibrium would have been efficient had information been exogenous. Moreover, it easily extends to richer Gaussian information structures, with multiple private and public signals

[25] Of course, this informational externality is absent in the very last period, which explains why the result does not hold at $t = N$.

of aggregate activity. It relies merely on two properties: (i) that a higher T_{kK} induces more learning by increasing the sensitivity of actions to private information; and (ii) that the social value of such learning is positive because the optimal policy removes any inefficiencies in the use of information.

We further discuss the importance of this last point in the next section, where we study how the policies we have identified affect the social value of information. Before turning to this issue, however, we study the properties of optimal policy for a special case of interest: economies in which inefficiency emerges *only* because of informational externalities.

This special case is motivated by the following observations. In certain settings (e.g., Walrasian economies with no externalities), one may expect that competitive market forces achieve a perfect, or near perfect, coincidence of private and social payoffs. Although such a coincidence may fail to obtain in the absence of complete markets or in the presence of other market distortions, this case still represents an important benchmark. At the same time, when information is dispersed, one may expect that individual market participants fail to internalize how their choices affect the quality of information contained in financial prices, macroeconomic indicators, and other endogenous signals of the underlying fundamentals. In our set up, these settings correspond to economies in which information is endogenous but where $(\kappa, \alpha) = (\kappa^*, \alpha^*)$, so that private and social payoffs coincide and inefficiency emerges only because of informational externalities.

The following is then an immediate implication of Corollary 5 along with the fact that, for these economies, the optimal tax would be zero had information been exogenous.

Corollary 6 *In economies in which inefficiency emerges only because of informational externalities, the optimal policy is such that* $T_{kK} > 0$.

This result may be relevant for understanding optimal policy over the business cycle. Consider standard real-business-cycle models in which all firms and households share the same information regarding aggregate productivity and taste shocks. The assumption of frictionless competitive markets along with the absence of direct payoff externalities then guarantees that the equilibrium business cycle is efficient. Now consider a small, yet realistic, modification of these models: let information be dispersed and only imperfectly aggregated through prices and macro data. This modification is likely to render the business cycle inefficient as agents fail to internalize how their choices affect the information of others. Our results then suggest the following policy remedy: by having marginal taxes increase with realized aggregate macroeconomic activity and decrease with realized productivity, the government can improve the information contained in prices and macro data, and can thereby reduce the non-fundamental component of the business cycle (i.e., the fluctuations that are driven by noise in information regarding aggregate demand and productivity conditions).

7 Implications for the social value of information

Throughout our analysis, we have ruled out policies that convey information to the agents. However, if the government possess information that is not directly available to the market (e.g. macroeconomic data collected by government agencies such as the Bureau of Labor Statistics, the US Census Bureau, or the Federal Reserve Banks), then it is important to understand whether it is socially desirable to communicate such information to the market. A positive answer is not obvious.

Indeed, as long the equilibrium use of information is inefficient, an increase in the precision of available information can have a detrimental effect on welfare. For example, if $\alpha > \alpha^*$, agents may overreact to any additional public information, exacerbating the already excessive non-fundamental volatility. However, this can not be the case if the equilibrium use of information is efficient. This is because the equilibrium strategy then coincides with the solution to a planning problem where the planner directly controls how agents use their available information. An argument analogous to Blackwell's theorem then guarantees that additional information can not reduce welfare.[26] The following result is then a direct implication of these observations.

Corollary 7 *In general, more precise information can reduce welfare. However, policies that restore efficiency in the decentralized use of information also guarantee a positive social value for any information disseminated by policy makers or other institutions.*

In an influential paper, Morris and Shin (2002) used an elegant example to illustrate the possibility that more precise public information can reduce welfare: a "beauty-contest" game, where the strategic complementarity perceived by the agents is not warranted from a social perspective, causing overreaction to public news.[27] This example has lead to a renewed debate on the merits of transparency in central bank communications.[28] Whereas this question has been studied largely in isolation from other aspects of policy making, our results indicate that a central bank's optimal communication policy is far from orthogonal to the corrective roles of monetary and fiscal policies.[29]

In a related but different line of reasoning, Amador and Weill (2007) argue that, by crowding out private information, an increase in the precision of exogenous public information can reduce the precision of the endogenous information contained in prices and other indicators of economic

[26] For further details on how the welfare effect of additional information depends on the inefficiencies, if any, of the equilibrium use of information, see Angeletos and Pavan (2007).

[27] Their example is nested in our baseline static framework with $\kappa = \kappa^*$ and $\alpha > \alpha^*$; it is an economy where inefficiency emerges only under dispersed information and manifests itself in excessive non-fundamental volatility.

[28] See, for example, Amato and Shin (2006), Angeletos and Pavan (2004, 2007), Baeriswyl and Conrand (2007), Heinemann and Conrand (2006), Hellwig (2005), Morris and Shin (2002, 2005), Roca (2006), Svensson (2005), Woodford (2005).

[29] An exception is Baeriswyl and Conrand (2007), which focuses on the signaling effects of monetary policy.

activity and can thereby slow down social learning.[30] A similar theme is explored in Morris and Shin (2005) and Amato and Shin (2006). Our results imply that the government can improve the informational content of prices, can raise the speed of social learning, and can guarantee that any public information it disseminates is welfare improving, once it sets in place policies that correct the underlying inefficiency in the decentralized use of information.

8 Concluding remarks

Information about commonly-relevant fundamentals—such aggregate productivity and demand conditions or the profitability of available technologies—is highly dispersed in society, is only imperfectly aggregated through markets, and can not be centralized by the government or any other institution. As first emphasized by Hayek (1945), this means that society has to rely on decentralized market mechanisms for an effective utilization of such information. However, this does not necessarily mean that the government should not interfere with the decentralized use of information: to the extent that private and social incentives in the use of such information do not coincide, the equilibrium's response to certain sources of information may be inefficient, leading to excessive non-fundamental volatility, excessive dispersion, or suboptimal social learning.

The key contribution of the paper was to identify policies that correct such inefficiencies: by appropriately designing the contingency of marginal taxes on *aggregate* activity, along with other properties of the tax system, the government can manipulate the incentives the agents face in using different sources of information and can thereby improve welfare *even if* it cannot itself collect and disseminate information or create new channels through which information is aggregated in society.

We established this result within an abstract but flexible framework in order to highlight the potential generality of the insight. Of course, the details of the optimal contingency will depend on the details of the application under examination. If the key inefficiencies are overreaction to public news and excessive non-fundamental volatility, as it is often argued to be the case for financial markets, then marginal taxes must increase with aggregate activity. The same is true if the key inefficiency is the failure of markets to internalize the endogeneity of the information contained in financial prices and macroeconomic data. In both cases, it is desirable to provide incentives so that agents rely less on common sources of information; this can be achieved by introducing a positive contingency of marginal taxes on signals of aggregate activity. The opposite policy prescription applies to markets exhibiting overreaction to private information and excessive cross-sectional dispersion. Nevertheless, the key principle—the optimality of marginal taxes contingent on aggregate economic conditions—remains valid for any economy featuring dispersed information regarding *commonly-relevant* fundamentals.

[30] Amador and Weill (2007) extend Vives (1993, 1997) to situations with both private and public learning. Both models are nested in our analysis of Section 6 as special cases that rule out payoff externalities.

Examining the practical, political-economy, considerations that might complicate the introduction of explicit aggregate contingencies in the tax code is clearly beyond the scope of this paper. However, it is important to note that there are various direct and indirect ways through which such contingencies can obtain in practice. For example, the government could first collect non-contingent taxes and subsequently make rebates whose magnitude depends on realized aggregate activity. Alternatively, the tax code could be revised over time on the basis of past macroeconomic performance. To the extent that such rebates or revisions are systematic, they can have similar incentive effects as the type of contingencies envisioned in this paper.

Moreover, the contingency of monetary policy to macroeconomic performance could serve a similar role: how firms use different sources of information when making their pricing and production choices depends on how they anticipate monetary policy to respond to the information about aggregate employment, output and prices that arrives over time. For example, to the extent that higher interest rates have similar incentive effects as higher taxes, the Central Bank can use the contingency of its interest-rate policy on aggregate economic conditions, not only for the familiar stabilization purposes, but also to improve the information that is contained in prices and macro data. Further exploring how the policy objectives we have studied in this paper filter in the optimal design of monetary policy is a fruitful direction for future research.

9 Appendix

Proof of Proposition 1. We prove the result in three steps. Steps 1 and 2 characterize, respectively, the complete-information equilibrium and the first-best allocation. Step 3 extends the results to the case of incomplete but common information.

Step 1. Consider the complete-information equilibrium. Because payoffs are concave in k, for any given K the optimal action for agent i is pinned down by the first-order condition $U_k(k_i, K, \theta_i) = 0$. Because U is quadratic in (k, K, θ), the first-order condition is equivalent to

$$U_k(0,0,0) + U_{kk}k_i + U_{kK}K + U_{k\theta}\theta_i = 0. \tag{21}$$

Aggregating across agents (i.e., integrating over θ_i), we thus have that

$$U_k(0,0,0) + [U_{kk} + U_{kK}]K + U_{k\theta}\bar{\theta} = 0. \tag{22}$$

Combining (21) with (22), and letting

$$\kappa_0 \equiv \frac{U_k(0,0,0)}{-(U_{kk} + U_{kK})}, \quad \kappa_1 \equiv \frac{U_{k\theta}}{-U_{kk}}, \quad \kappa_2 \equiv \frac{U_{k\theta}}{-(U_{kk} + U_{kK})} - \kappa_1, \tag{23}$$

gives the result.

Step 2. Next, consider the first-best allocation. A feasible allocation consists of a combination of a strategy $k : \Theta \times H \to \mathbb{R}$ and a system of budget-balanced transfers across agents. For any given cross-sectional distribution of productivities $h \in H$ and any given strategy $k : \Theta \times H \to \mathbb{R}$, let $K(h) \equiv \int k(\theta; h)\, dh(\theta)$ and $\sigma_k(h) \equiv (\int [k(\theta; h) - K(h)]^2\, dh(\theta))^{1/2}$ be the corresponding mean and dispersion of activity in the cross section of the population. Next, let

$$w(k; h) \equiv \int U(k(\theta; h), K(h), \sigma_k(h), \theta) dh(\theta) \tag{24}$$

denote ex-ante utility behind the veil of ignorance (equivalently, welfare under an utilitarian aggregator). Because of the quasi-linearity of payoffs in transfers, ex-ante utility depends only on the strategy k. An allocation is thus efficient if and only if the strategy $k : \Theta \times H \longrightarrow \mathbb{R}$ maximizes $w(k; h)$. Because U is quadratic, and $U_\sigma(k, K, \sigma_k, \theta) = U_{\sigma\sigma}\sigma_k$, we then have that

$$w = U(K, K, 0, \bar{\theta}) + \frac{1}{2}(U_{\sigma\sigma} + U_{kk})\sigma_k^2 + \frac{1}{2}U_{\theta\theta}\sigma_\theta^2 + U_{\theta k}\left\{\int [k(\theta)\theta]\, dh(\theta) - K\bar{\theta}\right\}$$

where for simplicity we dropped the dependence of k, K and σ_k on h. Then let

$$L \equiv w - \lambda\left(\int k(\theta) dh(\theta) - K\right) - \mu\left(\int [k(\theta) - K]^2\, dh(\theta) - \sigma_k^2\right)$$

denote the Lagrangian for this problem. Optimizing L with respect to K, σ_k^2 and $k(\theta)$ we have that

$$U_{k\theta}\theta - \lambda - 2\mu[k(\theta) - K] = 0$$
$$U_k(K, K, \bar{\theta}) + U_K(K, K, \bar{\theta}) - U_{\theta k}\bar{\theta} + \lambda = 0$$
$$\frac{1}{2}(U_{\sigma\sigma} + U_{kk}) + \mu = 0$$

Combining, we have that

$$U_{k\theta}\theta + U_k(K,K,\bar{\theta}) + U_K(K,K,\bar{\theta}) - U_{k\theta}\bar{\theta} + (U_{\sigma\sigma} + U_{kk})\left[k(\theta) - K\right] = 0 \tag{25}$$

or equivalently

$$U_{k\theta}\theta + U_k(0,0,0) + U_K(0,0,0) + \left[U_{kK} + 2U_{kK} + U_{KK}\right]K + U_{K\theta}\bar{\theta} + (U_{\sigma\sigma} + U_{kk})\left[k(\theta) - K(h)\right] \tag{26}$$

Integrating over θ we then have that

$$K = \frac{U_k(0,0,0) + U_K(0,0,0)}{-\left[U_{kK} + 2U_{kK} + U_{KK}\right]} + \frac{(U_{K\theta} + U_{k\theta})\,\bar{\theta}}{-\left[U_{kK} + 2U_{kK} + U_{KK}\right]} \tag{27}$$

Substituting (27) into (26), and letting

$$\kappa_0^* \equiv \frac{U_k(0,0,0) + U_K(0,0,0)}{-(U_{kk} + 2U_{kK} + U_{KK})}, \quad \kappa_1^* \equiv \frac{U_{k\theta}}{-(U_{kk} + U_{\sigma\sigma})}, \quad \kappa_2^* \equiv \frac{U_{k\theta} + U_{K\theta}}{-(U_{kk} + 2U_{kK} + U_{KK})} - \kappa_1^*, \tag{28}$$

gives the result.

Step 3. Now suppose that information is incomplete but common. Since all information is common, the aggregate activity K is also commonly known in equilibrium. The first-order condition for agent i is thus given by

$$U_k(0,0,0) + U_{kk}k_i + U_{kK}K + U_{k\theta}\mathbb{E}[\theta_i|\mathcal{P}] = 0,$$

where $\mathcal{P}$ denotes the commonly-available information set (whatever this is). Aggregating this across agents, and noting that the cross-sectional average of $\mathbb{E}[\theta_i|\mathcal{P}]$ is simply $\mathbb{E}[\bar{\theta}|\mathcal{P}]$, we get

$$U_k(0,0,0) + \left[U_{kk} + U_{kK}\right]K + U_{k\theta}\mathbb{E}[\theta_i|\mathcal{P}] = 0.$$

Note that the above two conditions are identical to conditions (21) and (22), except for the fact that θ_i and $\bar{\theta}$ have been replaced by $\mathbb{E}[\theta_i|\mathcal{P}]$ and $\mathbb{E}[\bar{\theta}|\mathcal{P}]$. It is thus immediate that the equilibrium action for agent i is given by

$$k_i = \mathbb{E}\left[\kappa(\theta_i,\bar{\theta})|\mathcal{P}\right].$$

A similar argument implies that the efficient action is given by

$$k_i = \mathbb{E}\left[\kappa^*(\theta_i,\bar{\theta})|\mathcal{P}\right],$$

which completes the proof of the result. ■

Proof of Proposition 2. We prove the result in two steps. Step 1 proves that condition (4) characterizes *any* equilibrium. Step 2 proves existence and uniqueness.

Step 1. Take any strategy $k : \Omega \to \mathbb{R}$ and let $K(\phi) = \int k(\omega)\,d\phi(\omega)$. A best-response is a strategy $k' : \Omega \to \mathbb{R}$ such that, for all ω, $k'(\omega)$ solves the first-order condition

$$\mathbb{E}[U_k(k', K(\phi), \theta) \mid \omega] = 0. \tag{29}$$

Using the fact that $U_k(k', K, \theta) = U_k(\kappa(\theta, \bar{\theta}), \kappa(\bar{\theta}, \bar{\theta}), \theta) + U_{kk} \cdot (k' - \kappa(\theta, \bar{\theta})) + U_{kK} \cdot (K - \kappa(\bar{\theta}, \bar{\theta}))$, where κ stands for the complete-information equilibrium allocation and the fact that κ solves $U_k(\kappa(\theta, \bar{\theta}), \kappa(\bar{\theta}, \bar{\theta}), \theta) = 0$ for all $\left(\theta, \bar{\theta}\right)$, (29) reduces to

$$\mathbb{E}[U_{kk} \cdot (k' - \kappa(\theta, \bar{\theta})) + U_{kK} \cdot (K(\phi) - \kappa\left(\bar{\theta}, \bar{\theta}\right)) \mid \omega] = 0,$$

or equivalently $k'(\omega) = \mathbb{E}[\kappa(\theta, \bar{\theta}) + \alpha\left(K(\phi) - \kappa\left(\bar{\theta}, \bar{\theta}\right)\right) \mid \omega]$. In equilibrium, $k'(\omega) = k(\omega)$ for all ω, which gives (4).

Step 2. What remains to prove is that the equilibrium exists and is unique; this can be done with the help of Proposition 3, which characterizes the efficient use of information. Let $\mathcal{U}$ denote the class of payoff functions U that satisfy the properties specified in Section 2. An economy is given by $\mathbf{e} \equiv (U, \Theta, \Omega, F)$. By comparing conditions (4) and (10), it is immediate that the set of equilibrium strategies for the economy $\mathbf{e} = (U, \Theta, \Omega, F)$ coincides with the set of efficient strategies for an economy $\mathbf{e}' = (U', \Theta, \Omega, F)$ such that the κ^* and α^* corresponding to $\mathbf{e}'$ coincide with the κ and α corresponding to $\mathbf{e}$. Moreover, $U' \in \mathcal{U}$ as long as $U \in \mathcal{U}$. As shown in Proposition 3, the efficient strategy for *any* economy $\mathbf{e}'$ with $U' \in \mathcal{U}$ exists and is uniquely determined for all but a measure-zero set of ω. It follows that an equilibrium for the economy $\mathbf{e}$ exists and is unique. ■

Proof of Corollary 1. From (4), we have that aggregate investment satisfies

$$K(\phi) = \int \mathbb{E}[\ \kappa(\theta, \bar{\theta}) - \alpha\kappa(\bar{\theta}, \bar{\theta}) \mid \omega\ ]d\phi(\omega) + \alpha \int \mathbb{E}[\ K(\phi)\ \mid \omega]d\phi(\omega).$$

Using the fact that $\kappa_1 = \kappa_2(1-\alpha)/\alpha$,

$$\kappa(\theta, \bar{\theta}) - \alpha\kappa(\bar{\theta}, \bar{\theta}) = (1-\alpha)\kappa_0 + \kappa_1\theta.$$

It follows that

$$K(\phi) = (1-\alpha)\kappa_0 + \kappa_1\bar{E}^1 + \alpha \int \mathbb{E}[\ K(\phi)\ \mid \omega]d\phi(\omega)$$

and hence that

$$\mathbb{E}[K(\phi)\ \mid \omega] = (1-\alpha)\kappa_0 + \kappa_1\mathbb{E}[\bar{E}^1|\omega] + \alpha\mathbb{E}[\int \mathbb{E}[\ K(\phi)\ \mid \omega']d\phi(\omega') \mid \omega\ ].$$

Iterating and then substituting into (4) gives

$$k(\omega) = \mathbb{E}[\ \kappa_0 + \kappa_1\theta + \kappa_1 \sum_{n=1}^{\infty} \alpha^n \bar{E}^n \mid \omega\ ], \tag{30}$$

which together with $\kappa_1 = \kappa_2(1-\alpha)/\alpha$ and the definition of $\hat{\theta}$ gives the result. ■

Proof of Lemma 1. A Taylor expansion of $U(k, K, \sigma_k, \theta)$ around $(\hat{k}, \hat{K}, \hat{\sigma}_k, \theta)$ gives

$$\begin{aligned}\mathbb{E}u \;=\; & \mathbb{E}[\ U(\hat{k}, \hat{K}, \hat{\sigma}_k, \theta) + U_k(\hat{k}, \hat{K}, \hat{\sigma}_k, \theta)(k - \hat{k}) + U_K(\hat{k}, \hat{K}, \hat{\sigma}_k, \theta)(K - \hat{K}) + U_\sigma(\hat{k}, \hat{K}, \hat{\sigma}_k, \theta)(\sigma_k - \hat{\sigma}_k) \\ & + \frac{1}{2}U_{kk}(k - \hat{k})^2 + \frac{1}{2}U_{KK}(K - \hat{K})^2 + \frac{1}{2}U_{\sigma\sigma}(\sigma_k - \hat{\sigma}_k)^2 + U_{kK}(k - \hat{K})(K - \hat{K})\ ].\end{aligned}$$

By the law of iterated expectations,

$$\begin{aligned}\mathbb{E}[K|h] &= \mathbb{E}\big[\mathbb{E}[K|\theta,h]|h\big] = \mathbb{E}\big[\mathbb{E}\big[\mathbb{E}[k(\omega)|\phi]|\theta,h\big]|h\big] \\ &= \mathbb{E}\big[\mathbb{E}[k(\omega)|\theta,h]|h\big] = \mathbb{E}[\hat{k}(\theta;h)|h] = \hat{K}(h).\end{aligned}$$

It follows that $\mathbb{E}[K] = \mathbb{E}[\hat{K}]$ and that

$$\mathbb{E}[U_k(\hat{k},\hat{K},\hat{\sigma}_k,\theta)(k-\hat{k})] = \mathbb{E}[U_K(\hat{k},\hat{K},\hat{\sigma}_k,\theta)(K-\hat{K})] = 0.$$

Furthermore, because U is linear in σ_k^2, $U_\sigma(\hat{k},\hat{K},\hat{\sigma}_k,\theta) = U_{\sigma\sigma}\hat{\sigma}_k(\sigma_k-\hat{\sigma}_k)$. It follows that

$$U_\sigma(\hat{k},\hat{K},\hat{\sigma}_k,\theta)(\sigma_k-\hat{\sigma}_k) + \frac{1}{2}U_{\sigma\sigma}(\sigma_k-\hat{\sigma}_k)^2 = \frac{1}{2}U_{\sigma\sigma}\left[\sigma_k^2-\hat{\sigma}_k^2\right]$$

Next, note that

$$(k-\hat{k})^2 = [(k-K)-(\hat{k}-\hat{K})]^2 + (K-\hat{K})^2 + 2(K-\hat{K})[(k-K)-(\hat{k}-\hat{K})].$$

Because $\mathbb{E}[k] = \mathbb{E}[K] = \mathbb{E}[\hat{k}] = \mathbb{E}[\hat{K}]$ and because $(K-\hat{K})$ is orthogonal to $[(k-K)-(\hat{k}-\hat{K})]$, we then have that

$$\mathbb{E}[(k-\hat{k})^2] = \mathrm{Var}[(k-K)-(\hat{k}-\hat{K})] + \mathrm{Var}[K-\hat{K}].$$

Finally, note that

$$\mathbb{E}[(k-\hat{K})(K-\hat{K})] = \mathbb{E}[(K-\hat{K})^2] + \mathbb{E}[(k-K)(K-\hat{K})] = \mathbb{E}[(K-\hat{K})^2]$$

because $(k-K)$ is orthogonal to $(K-\hat{K})$.

Combining all the above results, we thus have that

$$\begin{aligned}\mathbb{E}u &= \mathbb{E}[U(\hat{k},\hat{K},\hat{\sigma}_k,\theta)] + \frac{1}{2}U_{\sigma\sigma}\left[\sigma_k^2-\hat{\sigma}_k^2\right] + \frac{1}{2}U_{kk}\mathrm{Var}[(k-K)-(\hat{k}-\hat{K})] \\ &\quad + \frac{1}{2}U_{kk}\mathrm{Var}[K-\hat{K}] + \frac{1}{2}U_{KK}\mathrm{Var}(K-\hat{K}) + U_{kK}\mathrm{Var}(K-\hat{K}).\end{aligned}$$

Using the fact that

$$\mathrm{Var}[(k-K)-(\hat{k}-\hat{K})] = \mathrm{Var}[k-K] - \mathrm{Var}[\hat{k}-\hat{K}] = \sigma_k^2-\hat{\sigma}_k^2$$

and rearranging, then gives the expression in (8). ■

Proof of Proposition 3. A strategy is efficient if and only if it maximizes

$$\mathbb{E}u = \int_F \int_{\Omega,\Theta} U(k(\omega),K(\phi),\sigma_k(\phi),\theta)df(\omega,\theta)d\mathcal{F}(f),$$

with $K(\phi) = \int_\Omega k(\omega)d\phi(\omega)$ and $\sigma_k(\phi) = \left[\int_\Omega [k(\omega)-K(\phi)]^2 d\phi(\omega)\right]^{1/2}$ with ϕ denoting the marginal distribution over Ω generated by f. The strict concavity and the quadratic specification of U ensures

that a solution to this problem exists and is unique for almost all ω. Let $G(\phi)$ denote the marginal distribution of ϕ and $Z(\theta|\phi)$ denote the distribution of θ conditional on ϕ, as implied by their joint distribution $\mathcal{F}$. The Lagrangian for this problem can be written as

$$\begin{aligned}\Lambda = \; & \int_\Phi \int_\Theta \int_\Omega U(k(\omega), K(\phi), \sigma_k(\phi), \theta) d\phi(\omega) dZ(\theta|\phi) dG(\phi) \\ & + \int_\Phi \lambda(\phi) \left[K(\phi) - \int_\Omega k(\omega) d\phi(\omega) \right] dG(\phi) \\ & + \int_\Phi \eta(\phi) \left[\sigma_k^2(\phi) - \int_\Omega [k(\omega) - K(\phi)]^2 d\phi(\omega) \right] dG(\phi)\end{aligned}$$

Therefore, the first order conditions with respect to $K(\phi)$, $\sigma_k(\phi)$, and $k(\omega)$, which are necessary and sufficient for optimality, are given by the following:

$$\int_\Theta \int_\Omega U_K(k(\omega), K(\phi), \theta) d\phi(\omega) dZ(\theta|\phi) + \lambda(\phi) = 0 \quad \text{for almost all } \phi \tag{31}$$

$$\int_\Theta \int_\Omega U_\sigma(k(\omega), K(\phi), \sigma_k(\phi), \theta) d\phi(\omega) dZ(\theta|\phi) + 2\eta(\phi)\sigma_k(\phi) = 0 \quad \text{for almost all } \phi \tag{32}$$

$$\int_{\Theta\times\Phi} [U_k(k(\omega), K(\phi), \theta) - \lambda(\phi) - 2\eta(\phi)(k(\omega) - K(\phi))] dP(\theta, \phi|\omega) = 0 \quad \text{for almost all } \omega \tag{33}$$

where $P(\theta, \phi|\omega)$ denotes the cumulative distribution function of (θ, ϕ) conditional on ω.

Using the facts that $U_K(k, K, \theta)$ is linear in its arguments, that $K(\phi) = \int_\Omega k(\omega) d\phi(\omega)$, and that $U_\sigma(k, K, \sigma_k, \theta) = U_{\sigma\sigma}\sigma_k$, conditions (31) and (32) reduce to

$$\begin{aligned}\lambda(\phi) &= -\int_\Theta U_K(K(\phi), K(\phi), \theta) dZ(\theta|\phi) = -U_K(K(\phi), K(\phi), \bar{\theta}) \\ \eta(\phi) &= -\tfrac{1}{2} U_{\sigma\sigma}.\end{aligned}$$

Substituting the above into (33), we conclude that the strategy $k : \Omega \to \mathbb{R}$ is efficient if and only if it satisfies the following condition for almost all $\omega \in \Omega$:

$$\mathbb{E}[\, U_k(k, K, \theta) + U_K(K, K, \bar{\theta}) + U_{\sigma\sigma}[k - K] \mid \omega \,] \;=\; 0 \tag{34}$$

where, for simplicity, we have dropped the dependence of k on ω and of K on ϕ. Because both $U_k(k, K, \theta)$ and $U_K(k, K, \theta)$ are linear, condition (34) can be rewritten as

$$\begin{aligned}&\mathbb{E}[\, U_k(\kappa^*(\bar{\theta}, \bar{\theta}), \kappa^*(\bar{\theta}, \bar{\theta}), \bar{\theta}) + U_{kk} \cdot (k - \kappa^*(\bar{\theta}, \bar{\theta})) + U_{kK} \cdot (K - \kappa^*(\bar{\theta}, \bar{\theta})) + U_{k\theta}(\theta - \bar{\theta}) \\ &+ U_K(\kappa^*(\bar{\theta}, \bar{\theta}), \kappa^*(\bar{\theta}, \bar{\theta}), \bar{\theta}) + (U_{kK} + U_{KK}) \cdot (K - \kappa^*(\bar{\theta}, \bar{\theta})) + U_{\sigma\sigma}(k - K) \mid \omega \,] \;=\; 0.\end{aligned} \tag{35}$$

Now note that, when all agents follow the first-best allocation, then in each state aggregate investment is given by $K = \kappa^*(\bar{\theta}, \bar{\theta})$. Replacing $\kappa^*(\theta, \bar{\theta})$ and $\kappa^*(\bar{\theta}, \bar{\theta})$ into condition (25) in the proof of Proposition 1, we thus have that the first best strategy solves

$$U_k(\kappa^*(\bar{\theta}, \bar{\theta}), \kappa^*(\bar{\theta}, \bar{\theta}), \bar{\theta}) + U_K(\kappa^*(\bar{\theta}, \bar{\theta}), \kappa^*(\bar{\theta}, \bar{\theta}), \bar{\theta}) + U_{k\theta}(\theta - \bar{\theta}) + (U_{\sigma\sigma} + U_{kk}) \left[\kappa^*(\theta, \bar{\theta}) - \kappa^*(\bar{\theta}, \bar{\theta}) \right] = 0 \tag{36}$$

Substituting (36) into (35) and rearranging gives (10). ■

Proof of Corollary 2. Part (i) follows from Propositions (2) and (3): for any "non-degenerate" information structure $\mathcal{F}$, given $\kappa = \kappa^*$, the unique solution to (4) coincides with the unique solution to (10) if and only if $\alpha = \alpha^*$. Next, consider parts (ii) and (iii). When information is Gaussian and shocks are perfectly correlated

$$\begin{aligned} k &= \kappa_0 + (\kappa_1 + \kappa_2)\left[\gamma_\mu \mu + \gamma_y y + \gamma_x x\right], \\ K &= \kappa_0 + (\kappa_1 + \kappa_2)\left[\gamma_\mu \mu + (\gamma_y + \gamma_x)\,\theta + \gamma_y \varepsilon\right], \\ \hat{k} &= \hat{K} = \kappa_0 + (\kappa_1 + \kappa_2)\left[\gamma_\mu \mu + (\gamma_y + \gamma_x)\,\theta\right]. \end{aligned}$$

It follows that

$$vol = \left[(\kappa_1 + \kappa_2)\,\gamma_y\right]^2 \sigma_y^2 \text{ and } dis = \left[(\kappa_1 + \kappa_2)\,\gamma_x\right]^2 \sigma_\xi^2$$

The results then follow directly from (6) and (12). ■

Proof of Proposition 4. Given any policy $T \in \mathcal{T}$, let

$$\tilde{U}\left(k, K, \sigma_k, \theta, \bar{\theta}\right) \equiv U\left(k, K, \sigma_k, \theta\right) - T(k, K, \sigma_k, \bar{\theta})$$

denote an agent's payoff, net of taxes.

Now let $\tilde{k} : \Theta \times H \longrightarrow \mathbb{R}$ denote the complete-information equilibrium strategy when payoffs are given by $\tilde{U}$. Because $\tilde{U}$ is concave in k, $\tilde{k}\left(\theta; h\right)$ must solve the first-order condition $\tilde{U}_k(\tilde{k}\left(\theta; h\right), \tilde{K}\left(h\right), \theta, \bar{\theta}) = 0$, with $\tilde{K}(h) = \int \tilde{k}\left(\theta'; h\right) dh\left(\theta'\right)$. Because $\tilde{U}$ is quadratic in $(k, K, \theta, \bar{\theta})$, the first-order condition can be rewritten as

$$\tilde{U}_k\left(0,0,0,0\right) + \tilde{U}_{kk}\tilde{k}\left(\theta; h\right) + \tilde{U}_{kK}\tilde{K}\left(h\right) + \tilde{U}_{k\theta}\theta + \tilde{U}_{k\bar{\theta}}\bar{\theta} = 0. \tag{37}$$

Integrating over θ, we then have that

$$\tilde{U}_k\left(0,0,0,0\right) + \left[\tilde{U}_{kk} + \tilde{U}_{kK}\right]\tilde{K}\left(h\right) + \left[\tilde{U}_{k\theta} + \tilde{U}_{k\bar{\theta}}\right]\bar{\theta} = 0. \tag{38}$$

Combining (37) with (38) then gives $\tilde{k}\left(\theta; h\right) = \tilde{\kappa}(\theta, \bar{\theta}) \equiv \tilde{\kappa}_0 + \tilde{\kappa}_1\theta + \tilde{\kappa}_2\bar{\theta}$ with

$$\begin{aligned} \tilde{\kappa}_0 &= \frac{\tilde{U}_k\left(0,0,0,0\right)}{-\left[\tilde{U}_{kk} + \tilde{U}_{kK}\right]} = \frac{U_k\left(0,0,0\right) - T_k\left(0,0,0\right)}{-U_{kk} - U_{kK} + T_{kk} + T_{kK}} \\ \tilde{\kappa}_1 &= \frac{\tilde{U}_{k\theta}}{-\tilde{U}_{kk}} = \frac{U_{k\theta}}{-U_{kk} + T_{kk}} \\ \tilde{\kappa}_2 &= \frac{\tilde{U}_{k\theta} + \tilde{U}_{k\bar{\theta}}}{-\left[\tilde{U}_{kk} + \tilde{U}_{kK}\right]} - \tilde{\kappa}_1 = \frac{U_{k\theta} - T_{k\bar{\theta}}}{-U_{kk} - U_{kK} + T_{kk} + T_{kK}} - \tilde{\kappa}_1 \end{aligned}$$

Using (23) the coefficients $(\tilde{\kappa}_0, \tilde{\kappa}_1, \tilde{\kappa}_2)$ can then be conveniently rewritten as follows:

$$\tilde{\kappa}_0 \equiv \frac{(1-\alpha)\kappa_0 + \frac{1}{U_{kk}} T_k(0,0,0)}{1-\alpha - \frac{1}{U_{kk}} T_{kk} - \frac{1}{U_{kk}} T_{kK}} \tag{39}$$

$$\tilde{\kappa}_1 \equiv \frac{1}{1 - \frac{1}{U_{kk}} T_{kk}} \kappa_1 \tag{40}$$

$$\tilde{\kappa}_2 \equiv \frac{(1-\alpha)(\kappa_1 + \kappa_2) + \frac{1}{U_{kk}} T_{k\bar{\theta}}}{1-\alpha - \frac{1}{U_{kk}} T_{kk} - \frac{1}{U_{kk}} T_{kK}} - \tilde{\kappa}_1 \tag{41}$$

Normalizing $U_{kk} = -1$ then gives the formulas in the proposition.

Next, consider the game under incomplete information. Take any strategy $k : \Omega \to \mathbb{R}$ and let $K(\phi) = \int k(\omega)\, d\phi(\omega)$. A best-response is a strategy $k' : \Omega \to \mathbb{R}$ such that, for all ω, $k'(\omega)$ solves the first-order condition

$$\mathbb{E}[\tilde{U}_k(k', K(\phi), \theta, \bar{\theta}) \mid \omega] = 0. \tag{42}$$

Using the fact that $\tilde{U}_k(k', K, \theta, \bar{\theta}) = \tilde{U}_k(\tilde{\kappa}(\theta, \bar{\theta}), \tilde{\kappa}(\bar{\theta}, \bar{\theta}), \theta, \bar{\theta}) + \tilde{U}_{kk} \cdot (k' - \tilde{\kappa}(\theta, \bar{\theta})) + \tilde{U}_{kK} \cdot (K - \tilde{\kappa}(\bar{\theta}, \bar{\theta}))$ and the fact that $\tilde{\kappa}$ solves $\tilde{U}_k(\tilde{\kappa}(\theta, \bar{\theta}), \tilde{\kappa}(\bar{\theta}, \bar{\theta}), \theta, \bar{\theta}) = 0$ for all $(\theta, \bar{\theta})$, (42) reduces to

$$k'(\omega) = \mathbb{E}[\tilde{\kappa}(\theta, \bar{\theta}) + \tilde{\alpha}\left(K(\phi) - \tilde{\kappa}\left(\bar{\theta}, \bar{\theta}\right)\right) \mid \omega]$$

with

$$\tilde{\alpha} \equiv \frac{\tilde{U}_{kK}}{-\tilde{U}_{kk}} = \frac{U_{kK} - T_{kK}}{-U_{kk} + T_{kk}} = \frac{\alpha + \frac{1}{U_{kk}} T_{kK}}{1 - \frac{1}{U_{kk}} T_{kk}}. \tag{43}$$

In equilibrium, $k'(\omega) = k(\omega)$ for all ω, which gives (13). That a solution to (13) exists and is unique follows from the same arguments as in step 2 in the proof of Proposition 2. ■

Proof of Proposition 5. Take any generic information structure $\mathcal{F}$. For the equilibrium with policy to coincide with the efficient strategy, it is necessary and sufficient that

$$\tilde{\alpha} = \alpha^*, \quad \tilde{\kappa}_0 = \kappa_0^*, \quad \tilde{\kappa}_1 = \kappa_1^*, \quad \tilde{\kappa}_2 = \kappa_2^*. \tag{44}$$

It thus suffices to prove that there exists a policy $T^* \in \mathcal{T}$ that satisfies (44) and that this policy is unique. This is easily shown from conditions (39)-(41) and (43). First, note that $\tilde{\kappa}_1 = \kappa_1^*$ if and only if

$$T_{kk} = U_{kk}(1 - \kappa_1/\kappa_1^*) = -U_{\sigma\sigma}. \tag{45}$$

Because $U_{kk} + U_{\sigma\sigma} < 0$ this also guarantees that $U_{kk} - T_{kk} < 0$. Next, note that, given $T_{kk} = -U_{\sigma\sigma}$, $\tilde{\alpha} = \alpha^*$ if and only

$$T_{kK} = -U_{kk}(\alpha - \alpha^*) - T_{kk}\alpha^* = -U_{kk}\alpha + (U_{kk} + U_{\sigma\sigma})\alpha^* = U_{\sigma\sigma} - U_{kK} - U_{KK}. \tag{46}$$

That T_{kk} and T_{kK} satisfy $-\frac{U_{kK} - T_{kK}}{U_{kk} - T_{kk}} < 1$ then follows from the fact that $W_{vol} \equiv U_{kk} + 2U_{kK} + U_{KK} < 0$. With (T_{kk}, T_{kK}) thus determined, and with both $T_{k\bar{\theta}}$ and $T_k(0,0,0)$ being unconstrained, it is

then immediate that there exist a unique $T_{k\bar{\theta}}$ such that $\tilde{\kappa}_2 = \kappa_2^*$ and a unique $T_k(0,0,0)$ such that $\tilde{\kappa}_0 = \kappa_0^*$; these are given by

$$T_{k\bar{\theta}} = U_{kk}(1-\alpha)\left[(\kappa_1^* + \kappa_2^*) - (\kappa_1 + \kappa_2)\right] - (T_{kk} + T_{kK})(\kappa_1^* + \kappa_2^*) = -U_{K\theta}, \tag{47}$$

$$T_k(0,0,0) = U_{kk}(1-\alpha)(\kappa_0^* - \kappa_0) - (T_{kk} + T_{kK})\kappa_0^* = -U_K(0,0,0).$$

Finally, for T to balance the budget (state by state) it must be that $T\left(K,K,0,\bar{\theta}\right) = 0$ for all $\left(K,\bar{\theta}\right)$ and that $T_{kk} + T_{\sigma\sigma} = 0$. Along with the other properties identified above, it is then easy to verify that this is equivalent to imposing the following:

$$\begin{aligned}
T(0,0,0,0) &= T_{\bar{\theta}}(0,0,0) \;=\; T_{\bar{\theta}\bar{\theta}} \;=\; 0 \\
T_K(0,0,0) &= -T_k(0,0,0) \;=\; U_K(0,0,0) \\
T_{KK} &= -2T_{kK} - T_{kk} \;=\; -U_{\sigma\sigma} + 2U_{kK} + 2U_{KK} \\
T_{K\bar{\theta}} &= -T_{k\bar{\theta}} \;=\; U_{K\theta} \\
T_{\sigma\sigma} &= -T_{kk}
\end{aligned}$$

This also implies that the policy T is unique. Finally, that T_{kK} increases with α and decreases with α^* follows directly from (46) along with the facts that $U_{kk} < 0$ and $W_{dis} \equiv U_{kk} + U_{\sigma\sigma} < 0$. ■

Proof of Proposition 6. For the case of additive measurement error, the result follows directly from (14) noting that the last three terms in (14) do not affect individual decisions. For the case of multiplicative measurement error, let $\tilde{k}_i = k_i(1+\eta+\nu_i)$, $\tilde{K} = K(1+\eta)$, $\tilde{\sigma}_k^2 = \sigma_k^2(1+\eta)^2$ and $\tilde{\theta} = \bar{\theta}(1+\varsigma)$. A Taylor expansion of $T(\tilde{k}_i, \tilde{K}, \tilde{\sigma}_k, \tilde{\theta})$ around $T(k_i, K, \sigma_k, \bar{\theta})$ then gives

$$\mathbb{E}[T(\tilde{k}_i, \tilde{K}, \tilde{\sigma}_k, \tilde{\theta})|\omega_i] = \mathbb{E}[T(k_i, K, \sigma_k, \bar{\theta}) + \tfrac{1}{2}T_{kk}\left(\sigma_\eta^2 + \sigma_v^2\right)k_i^2 + T_{kK}\sigma_\eta^2 k_i K + \tfrac{1}{2}T_{KK}\sigma_\eta^2 K^2 + \tfrac{1}{2}T_{\sigma\sigma}\sigma_\eta^2\sigma_k^2|\ \omega_i]$$

Proposition 4 thus continues to hold with $\tilde{\kappa}$ and $\tilde{\alpha}$ redefined as follows:

$$\begin{aligned}
\tilde{\kappa}_0 &= \frac{U_k(0,0,0) - T_k(0,0,0)}{-U_{kk} - U_{kK} + T_{kK}(1+\sigma_\eta^2) + T_{kk}\left(1+\sigma_\eta^2+\sigma_v^2\right)} \\
\tilde{\kappa}_1 &= \frac{U_{k\theta}}{-U_{kk} + T_{kk}\left(1+\sigma_\eta^2+\sigma_v^2\right)} \\
\tilde{\kappa}_2 &= \frac{U_{k\theta} - T_{k\bar{\theta}}}{-U_{kk} - U_{kK} + T_{kK}(1+\sigma_\eta^2) + T_{kk}\left(1+\sigma_\eta^2+\sigma_v^2\right)} - \tilde{\kappa}_1 \\
\tilde{\alpha} &= \frac{U_{kK} - T_{kK}(1+\sigma_\eta^2)}{-U_{kk} + T_{kk}\left(1+\sigma_\eta^2+\sigma_v^2\right)}
\end{aligned}$$

By implication, Proposition 5 also continues to hold, although now the optimal tax contingencies depend on σ_η^2 and σ_v^2. ■

Proof of Corollary 3. From (45), (46) and (47), we have that the restriction $\kappa = \kappa^*$ implies that

$$T_{kk} = 0,\ T_{kK} = -U_{kk}\left(\alpha - \alpha^*\right), \text{ and } T_{k\bar{\theta}} = -T_{kK}\left(\kappa_1 + \kappa_2\right).$$

The tax system is thus linear in k, with a marginal tax rate given by

$$T_k(K, \bar{\theta}) = T_k\left(0, 0\right) + T_{kK}K + T_{k\bar{\theta}}\bar{\theta}.$$

Next, recall that $\hat{K} \equiv \mathbb{E}[K|h]$ and hence $\epsilon \equiv K - \hat{K}$ is orthogonal to any function of h, including $\bar{\theta}$ and $\hat{K}$. It follows that $Cov(T_k(K, \bar{\theta}), \epsilon) = T_{kK}Var(\epsilon)$, which is positive if and only if T_{kK} is positive, which in turn is true if and only if $\alpha > \alpha^*$. ■

Proof of Corollary 4. Because ϕ is common knowledge, the n-th order average expectation $\bar{E}^n$ of θ (which is measurable in ϕ) is also common knowledge and equal to $\bar{E}^1$, for all n. Moreover, because ϕ is common knowledge and because $\bar{\theta} \equiv \mathbb{E}\left[\theta|h\right] = \mathbb{E}\left[\theta|h, \phi\right]$, we have that

$$\bar{E}^1 \equiv \mathbb{E}\left[\mathbb{E}[\theta|\omega]|\phi\right] = \mathbb{E}\left[\mathbb{E}[\theta|\omega, \phi]|\phi\right] = \mathbb{E}\left[\theta|\phi\right] = \mathbb{E}\left[\mathbb{E}\left[\theta|h, \phi\right]|\phi\right] = \mathbb{E}\left[\bar{\theta}|\phi\right],$$

Combining these results, we have that

$$\hat{\theta} \equiv \sum_{n=1}^{\infty}\left((1-\alpha)\alpha^{n-1}\right)\bar{E}^n = \mathbb{E}\left[\bar{\theta}|\phi\right] = \mathbb{E}\left[\bar{\theta}|\omega\right].$$

From Corollary 1 it then follows that the unique equilibrium is given by

$$k\left(\omega\right) = \mathbb{E}\left[\kappa\left(\theta, \bar{\theta}\right)|\omega\right],$$

in which case α is irrelevant. Similar arguments imply that the efficient allocation is given by

$$k\left(\omega\right) = \mathbb{E}\left[\kappa^*\left(\theta, \bar{\theta}\right)|\omega\right],$$

so that α^* is also irrelevant. Along with the result in Proposition 4, we then have that the efficient strategy can always be implemented with a tax scheme for which $T_{kK} = 0$. ■

Proof of Propositions 7 and 8. These are direct extensions of Propositions 2 and 3. ■

Proof of Proposition 9. Start with $t = 1$. Because information is exogenous in the first period, that the equilibrium strategy at $t = 1$ is unique and solves (15) follows directly from Proposition 2. Now consider $t = 2$. The information structure is now endogenous but uniquely determined by the unique equilibrium strategy for $t = 1$. That the equilibrium strategy at $t = 2$ is unique and solves (15) thus follows again from Proposition 2. Repeating the same argument for all $t > 2$ establishes the result. ■

Proof of Proposition 10. Start with $t = 1$. In the first period, information is exogenous with $\omega_{i,1} = (x_{i,1}, y_1, A_1)$. Standard Gaussian updating then gives

$$\mathbb{E}\left[\theta|\omega_{i,1}\right] = \tfrac{\pi_1^x}{\pi_1^x+\pi_1^y} X_{i,1} + \tfrac{\pi_1^y}{\pi_1^x+\pi_1^y} Y_1, \tag{48}$$

where $X_{i,1} = x_{i,1}$, $\pi_1^x = \sigma_{x,1}^{-2}$, $Y_1 = \frac{\sigma_\theta^{-2}}{\pi_1^y}\mu + \frac{\sigma_{y,1}^{-2}}{\pi_1^y} y_1 + \frac{\sigma_{a,1}^{-2}}{\pi_1^y} A_1$ and $\pi_1^y = \sigma_\theta^{-2} + \sigma_{y,1}^{-2} + \sigma_{a,1}^{-2}$. Using $\kappa(\theta) = \kappa_0 + (\kappa_1 + \kappa_2)\theta$, we then have that the unique solution to (15) is given by

$$k_1\left(\omega_{i,1}\right) = \kappa_0 + \left(\kappa_1 + \kappa_2\right)\left(\gamma_1 X_{i,1} + \left(1 - \gamma_1\right) Y_1\right), \tag{49}$$

with

$$\gamma_1 \equiv \left[\left(1 - \alpha\right)\pi_1^x\right] / \left[\left(1 - \alpha\right)\pi_1^x + \pi_1^y\right].$$

To see this, start by guessing that the equilibrium strategy satisfies (49) for *some* coefficient γ_1. Next, use this guess to compute aggregate activity as $K_1 = \kappa_0 + (\kappa_1 + \kappa_2)(\gamma_1\theta + (1 - \gamma_1) Y_1)$. Finally, use the latter along with (15) and (48) to derive the equilibrium γ_1.

Next, consider $t = 2$. In the second period, $\omega_{i,2} = \omega_{i,1} \cup (x_{i,2}, y_2, A_2, \tilde{K}_1, \tilde{\sigma}_1)$. The endogenous signal is given by

$$\tilde{K}_1 = \kappa_0 + \left(\kappa_1 + \kappa_2\right)\left(\gamma_1\theta + \left(1 - \gamma_1\right) Y_1\right) + \eta_2$$

The information about θ contained in $\tilde{K}_1$ is thus the same as that contained in

$$\tilde{y}_2 \equiv \frac{\tilde{K}_1 - \kappa_0 - \left(\kappa_1 + \kappa_2\right)\left(1 - \gamma_1\right) Y_1}{\left(\kappa_1 + \kappa_2\right)\gamma_1} = \theta + \tilde{\eta}_2,$$

where $\tilde{\eta}_2 = \eta_2/[(\kappa_1+\kappa_2)\gamma_1]$ is Gaussian noise with variance $\sigma_{\tilde{\eta},2}^2 = \sigma_{\eta,2}^2 / \left(\kappa_1 + \kappa_2\right)^2 \gamma_1^2$. The signal $\tilde{\sigma}_1$, on the other hand, conveys no information about θ, because (49) implies that $\sigma_1 = \left(\kappa_1 + \kappa_2\right)^2 \gamma_1^2 \sigma_{x,1}^2$, which is common knowledge. It follows that the period-2 public information about θ can be summarized in a sufficient statistic Y_2 such that the posterior about θ conditional on $(y_1, A_1, \tilde{K}_1, \tilde{\sigma}_1, y_2, A_2)$ is Gaussian with mean

$$Y_2 = \frac{\pi_1^y}{\pi_2^y} Y_1 + \frac{\sigma_{y,2}^{-2}}{\pi_2^y} y_2 + \frac{\sigma_{a,2}^{-2}}{\pi_2^y} A_2 + \frac{\gamma_1^2\left(\kappa_1 + \kappa_2\right)^2 \sigma_{\eta,2}^{-2}}{\pi_2^y}\tilde{y}_2$$

and precision $\pi_2^y = \pi_1^y + \sigma_{y,2}^{-2} + \sigma_{a,2}^{-2} + \gamma_1^2\left(\kappa_1 + \kappa_2\right)^2 \sigma_{\eta,2}^{-2}$. Similarly, the private information can be summarized in the sufficient statistic $X_{i,2}$ such that the posterior about θ conditional on $(x_{i,1}, x_{i,2})$ is Gaussian with mean

$$X_{i,2} = \frac{\pi_1^x}{\pi_2^x} X_{i,1} + \frac{\sigma_{x,2}^{-2}}{\pi_2^x} x_{i,2}$$

and precision $\pi_2^x = \pi_1^x + \sigma_{x,2}^{-2}$. The unique solution to (15) is then given by

$$k_2\left(\omega_{i,2}\right) = \kappa_0 + \left(\kappa_1 + \kappa_2\right)\left(\gamma_2 X_{i,2} + \left(1 - \gamma_2\right) Y_2\right),$$

with $\gamma_2 \equiv \left[\left(1 - \alpha\right)\pi_2^x\right] / \left[\left(1 - \alpha\right)\pi_2^x + \pi_2^y\right]$.

It is immediate that the construction of the equilibrium strategy for $t = 2$ applies also to any $t \geq 3$ with the statistics $X_{i,t}$ an Y_t defined recursively as in the proposition. We conclude that the unique equilibrium strategy is

$$k_{i,t}(\omega_{i,t}) = \kappa_0 + (\kappa_1 + \kappa_2)(\gamma_t X_{i,t} + (1 - \gamma_t) Y_t),$$

with $\gamma_t \equiv [(1-\alpha)\pi_t^x] / [(1-\alpha)\pi_t^x + \pi_t^y]$. ■

Proof of Proposition 11. We prove the result in two steps. Part (i) characterizes the efficient linear strategy in the absence of payoff externalities; this helps isolate the role of informational externalities. Part (ii) then extends the result to general payoff structures.

Part (i). Suppose $V(k, K, \sigma, \theta)$ does not depend on (K, σ) and, without any further loss of generality, let

$$V(k, K, \sigma, \theta) = -(k - \theta)^2.$$

Let $h_t = \{y_1, A_1, \tilde{K}_1, ..., y_{t-1}, A_{t-1}, \tilde{K}_{t-1}, y_t, A_t\}$ denote the public history in period t and suppose agents follow a strategy $\mathbf{k} = \{k_t\}_{t=1}^N$ such that

$$k_t(\omega_{i,t}) = P_t(h_t) + \sum_{\tau=1}^{t} Q_{t,\tau} x_{i,\tau},$$

where $P_t(h_t)$ is a deterministic function of h_t and $Q_{t,\tau}$ are deterministic coefficients. It follows that

$$k_{i,t} = P_t + \gamma_t \theta + \sum_{\tau=1}^{t} Q_{t,\tau} \xi_{i,\tau},$$

and hence $\tilde{K}_t = P_t + \gamma_t \theta + \eta_{t+1}$, where P_t is a shortcut for $P_t(h_t)$ and γ_t is defined as

$$\gamma_t \equiv \sum_{\tau=1}^{t} Q_{t,\tau}.$$

Next consider welfare. Given any linear strategy, ex-ante utility is $\mathbb{E}u = \sum_{t=1}^N w_t$, where

$$w_t \equiv \mathbb{E}[v(k_{i,t}, A_{t+1})] = \mathbb{E}[v(k_{i,t}, \theta)] - \sigma_{a,t+1}^2$$

and where

$$\begin{aligned} \mathbb{E}[v(k_{i,t}, \theta)] &= \mathbb{E}\left[\mathbb{E}\left[-\left\{\left(P_t + \gamma_t\theta + \sum\nolimits_{\tau=1}^{t} Q_{t,\tau}\xi_{i,\tau}\right) - \theta\right\}^2 \;\middle|\; \theta, h_t\right]\right] \\ &= \mathbb{E}\left[-(P_t + \gamma_t\theta - \theta)^2 - \sum\nolimits_{\tau=1}^{t} Q_{t,\tau}^2 \sigma_{\xi,\tau}^2\right] \end{aligned}$$

Now consider a strategy $\hat{\mathbf{k}} = \{\hat{k}_t\}_{t=1}^N$ that is a variation of the initial strategy $\mathbf{k} = \{k_t\}_{t=1}^N$ constructed as follows. First, pick an arbitrary t and let $\hat{k}_{i,s}(\omega_{i,s}) = k_{i,s}(\omega_{i,s})$ for all $s < t$. Next, in period t, pick an arbitrary function $\hat{P}_t$ and any coefficients $\hat{Q}_{t,\tau}$ such that $\sum_{\tau=1}^t \hat{Q}_{t,\tau} = \gamma_t$, and let

$$\hat{k}_t(\omega_{i,t}) = \hat{P}_t(h_t) + \sum_{\tau=1}^{t} \hat{Q}_{t,\tau} x_{i,\tau}.$$

Finally, for all $s > t$, let

$$\hat{k}_s(\omega_{i,s}) = \hat{P}_s(h_s) + \sum_{\tau=1}^{s} Q_{s,\tau} x_{i,\tau},$$

where the functions $\hat{P}_s$ are such that

$$\hat{P}_s\left(..., \tilde{K}_t, ...\right) = P_s\left(..., \tilde{K}_t - \hat{P}_t(h_t) + P_t(h_t), ...\right).$$

By construction, at any period $s \neq t$, the strategy $\hat{\mathbf{k}}$ induces the same outcomes, and by implication the same per-period welfare level w_t, as the initial strategy $\mathbf{k}$. It follows that a necessary condition for the strategy $\mathbf{k}$ to be efficient is that, for all t and all h_t,

$$\begin{aligned} \left(P_t, (Q_{t,\tau})_{\tau=1}^t\right) \quad \in \quad & \arg\max_{\hat{P}_t, \hat{Q}_{t,\tau}} = \mathbb{E}\left[-\left(\hat{P}_t + \gamma_t\theta - \theta\right)^2 - \sum\nolimits_{\tau=1}^{t} \hat{Q}_{t,\tau}^2 \sigma_{\xi,\tau}^2 \;\middle|\; h_t\right] \\ & \text{s.t.} \qquad \textstyle\sum_{\tau=1}^{t} \hat{Q}_{t,\tau} = \gamma_t \end{aligned}$$

This in turn is the case if and only if, for all t and all h_t

$$P_t(h_t) = (1-\gamma_t)\mathbb{E}[\theta|h_t] \quad \text{and} \quad Q_{t,\tau} = \gamma_t \frac{\sigma_{\xi,\tau}^{-2}}{\sum_{j=1}^{t} \sigma_{\xi,j}^{-2}} \; \forall \tau. \tag{50}$$

Next note that, because P_t is public information, the observation in period $t+1$ of $\tilde{K}_t = K_t + \eta_t = P_t + \gamma_t\theta + \eta_t$ is informationally equivalent to the observation of a signal

$$\tilde{y}_{t+1} \equiv \frac{\tilde{K}_t - P_t}{\gamma_t} = \theta + \tilde{\eta}_{t+1} \tag{51}$$

where $\eta_{t+1} = \eta_{t+1}/\gamma_t$ is Gaussian noise with precision $\sigma_{\tilde{\eta},t+1}^{-2} = \gamma_t^2 \sigma_{\eta,t+1}^{-2}$. It follows that, given any linear strategy, the common posterior about θ in period t is Gaussian with mean $\mathbb{E}[\theta|h_t] = Y_t$ and precision π_t^y, where Y_t and π_t^y are defined recursively by

$$\begin{aligned} Y_t &= \frac{\pi_{t-1}^y}{\pi_t^y} Y_{t-1} + \frac{\sigma_{y,t}^{-2}}{\pi_t^y} y_t + \frac{\sigma_a^{-2}}{\pi_t^y} A_t + \frac{\gamma_{t-1}^2 \sigma_{\eta,t}^{-2}}{\pi_t^y} \tilde{y}_t \\ \pi_t^y &= \pi_{t-1}^y + \sigma_{y,t}^{-2} + \sigma_{a,t}^{-2} + \gamma_{t-1}^2 \sigma_{\eta,t}^{-2}, \end{aligned}$$

with initial conditions $Y_1 = \mu_0$ and $\pi_1^y = \sigma_0^{-2}$. Similarly, the private posteriors are Gaussian with mean

$$\mathbb{E}[\theta|\omega_{i,t}] = \frac{\pi_t^x}{\pi_t^x + \pi_t^y} X_{i,t} + \frac{\pi_t^y}{\pi_t^x + \pi_t^y} Y_t,$$

and precision $\pi_t = \pi_t^x + \pi_t^y$, where

$$X_{i,t} = \frac{\pi_{t-1}^x}{\pi_t^x} X_{i,t-1} + \frac{\sigma_{x,t}^{-2}}{\pi_t^x} x_{i,t} \quad \text{and} \quad \pi_t^x = \pi_{t-1}^x + \sigma_{\xi,t}^{-2},$$

with initial conditions $X_{i,1} = x_{i,1}$ and $\pi_1^x = \sigma_{\xi,1}^{-2}$.

Now note that

$$X_{i,t} = \sum_{\tau=1}^{s} \frac{\sigma_{x,\tau}^{-2}}{\sum_{j=1}^{t} \sigma_{x,j}^{-2}} x_{i,t},$$

which together with (50) gives

$$\sum_{\tau=1}^{s} Q_{t,\tau} x_{i\tau} = \gamma_t X_{i,t}.$$

We conclude that a linear strategy $\mathbf{k}$ maximizes ex-ante utility only if, for all t and all $\omega_{i,t}$,

$$k_{i,t}(\omega_{i,t}) = (1-\gamma_t) Y_t + \gamma_t X_{i,t} \tag{52}$$

for some coefficient $\gamma_t \in \mathbb{R}$.

To determine the optimal γ's, note that, when agents follow a strategy as in (52),

$$\mathbb{E}\left[v\left(k_{i,t},\theta\right)\right] = -\left\{\gamma_t^2 \left(\pi_t^x\right)^{-1} + \left(1-\gamma_t\right)^2 \left(\pi_t^y\right)^{-1}\right\}$$

and hence

$$\mathbb{E}u = W_{FB} - \sum\nolimits_{t=1}^{N} \beta^{t-1} \left\{\gamma_t^2 \left(\pi_t^x\right)^{-1} + \left(1-\gamma_t\right)^2 \left(\pi_t^y\right)^{-1}\right\},$$

where W_{FB} is the first-best level of welfare (the one obtained under complete information about θ). Because the evolution of π_t^x does not depend on γ_t, the choice of the optimal linear strategy reduces to the following problem:

$$\begin{aligned} &\min_{\{\gamma_t\}} \sum\nolimits_{t=1}^{N} \beta^{t-1} \left\{\gamma_t^2 \left(\pi_t^x\right)^{-1} + \left(1-\gamma_t\right)^2 \left(\pi_t^y\right)^{-1}\right\} \\ &\text{s.t.} \quad \pi_{t+1}^y = \pi_t^y + \Delta_t + \sigma_{\eta,t}^{-2} \gamma_t^2 \quad \forall t \end{aligned}$$

with initial condition $\pi_1^y = \sigma_\theta^{-2}$, where $\Delta_t \equiv \sigma_{\varepsilon,t}^{-2} + \sigma_{a,t}^{-2}$ is the exogenous change in the precision of public information.

Consider the value functions $L_t : \mathbb{R}_+^2 \to \mathbb{R}_+$ defined by

$$\begin{aligned} L_t\left(\pi_t^x, \pi_t^y\right) \quad \equiv \quad & \min_{\{\gamma_s\}_{s=t}^N} \sum\nolimits_{s=t}^{N} \beta^{s-t} \left\{\gamma_s^2 \left(\pi_s^x\right)^{-1} + \left(1-\gamma_s\right)^2 \left(\pi_s^y\right)^{-1}\right\} \\ & \text{s.t.} \quad \pi_{s+1}^y = \pi_s^y + \Delta_s + \sigma_{\eta,s}^{-2} \gamma_s^2 \quad \forall s \geq t \end{aligned}$$

For all $t \leq N$, $L_t\left(\pi_t^x, \pi_t^y\right)$ must satisfy

$$\begin{aligned} L_t\left(\pi_t^x, \pi_t^y\right) \quad = \quad & \min_{\gamma_t} \left\{\gamma_t^2 \left(\pi_t^x\right)^{-1} + \left(1-\gamma_t\right)^2 \left(\pi_t^y\right)^{-1} + \beta L_{t+1}\left(\pi_{t+1}^x, \pi_{t+1}^y\right)\right\} \\ & \text{s.t.} \quad \pi_{t+1}^y = \pi_t^y + \Delta_t + \psi_t \gamma_t^2 \end{aligned}$$

and hence the optimal γ_t is the solution to the following FOC:

$$\gamma_t \left(\pi_t^x\right)^{-1} - \left(1-\gamma_t\right) \left(\pi_t^y\right)^{-1} + \frac{1}{2} \beta \frac{\partial L_{t+1}}{\partial \pi_{t+1}^y} \frac{\partial \pi_{t+1}^y}{\partial \gamma_t} = 0.$$

¿From the envelope theorem,

$$\frac{\partial L_{t+1}}{\partial \pi_{t+1}^{y}} = -\left(1-\gamma_{t+1}\right)^{2}\left(\pi_{t+1}^{y}\right)^{-2}.$$

Finally, from the low of motion for π_t^y,

$$\frac{\partial \pi_{t+1}^{y}}{\partial \gamma_t} = 2\sigma_{\eta,t+1}^{-2}\gamma_t.$$

It follows that, for all $t \leq N-1$, the optimal γ_t satisfies

$$\gamma_t^{**} = \frac{\pi_t^x}{\pi_t^x + \pi_t^y - \beta\left(1-\gamma_{t+1}^{**}\right)^2 \pi_t^x \pi_t^y \left(\pi_{t+1}^y\right)^{-2} \sigma_{\eta,t+1}^{-2}} > \frac{\pi_t^x}{\pi_t^y + \pi_t^x}$$

Finally, for $t = N$, $\gamma_N^* = \pi_N^x / \left(\pi_N^x + \pi_N^y\right)$, simply because this is the last period and hence there is no more an informational externality.

Part (ii). Consider now the more general payoffs V and let

$$\kappa^*(\theta) \equiv \arg\max_{\kappa} V(\kappa, \kappa, 0, \theta) = \kappa_0^* + (\kappa_1^* + \kappa_2^*)\theta$$

with $(\kappa_0^*, \kappa_1^*, \kappa_2^*)$ as in (28) replacing U with V. A similar argument as in part (i) ensures that the efficient linear strategy must satisfy

$$k_t(\omega_{it}) = \kappa_0^* + (\kappa_1^* + \kappa_2^*)(\gamma_t X_{it} + (1-\gamma_t) Y_t),$$

for some γ_t, with X_{it} and Y_t are the relevant sufficient statistics of available private and public information. Ex ante utility is then given by

$$\mathbb{E}u = W_{FB} + \sum\nolimits_{t=1}^{N} \beta^{t-1}(\kappa_1^* + \kappa_2^*)\left\{\frac{W_{\sigma\sigma}}{2}(\gamma_t)^2(\pi_t^x)^{-1} + \frac{W_{KK}}{2}(1-\gamma_t)^2(\pi_t^y)^{-1}\right\},$$

where $W_{FB} \equiv \sum_{t=1}^{N} \beta^{t-1} W(\kappa^*(\theta), 0, \theta)$ is the first-best level of welfare. Using the fact that $W_{\sigma\sigma} < 0$, $W_{KK} < 0$, and $W_{KK}/W_{\sigma\sigma} = 1-\alpha^*$, we conclude that the optimal γ's must solve the following problem:

$$\min_{\{\gamma_t\}_{t=1}^{N}} \sum\nolimits_{t=1}^{N} \beta^{t-1}\left\{\gamma_t^2(\pi_t^x)^{-1} + (1-\alpha^*)(1-\gamma_t)^2(\pi_t^y)^{-1}\right\}$$
$$\text{s.t.} \quad \pi_{t+1}^y = \pi_t^y + \Delta_t + (\kappa_1^* + \kappa_2^*)^2 \sigma_{\eta,t}^{-2}\gamma_t^2 \quad \forall t$$

Letting $L_t(\pi_t^x, \pi_t^y)$ denote the associated value function in period t, we have

$$\begin{aligned} L_t(\pi_t^x, \pi_t^y) &= \min_{\gamma_t}\left\{\gamma_t^2(\pi_t^x)^{-1} + (1-\alpha^*)(1-\gamma_t)^2(\pi_t^y)^{-1} + \beta L_{t+1}\left(\pi_{t+1}^x, \pi_{t+1}^y\right)\right\} \\ &\quad \text{s.t.} \quad \pi_{t+1}^y = \pi_{t+1}^y = \pi_t^y + \Delta_t + (\kappa_1^* + \kappa_2^*)^2 \sigma_{\eta,t}^{-2}\gamma_t^2 \end{aligned}$$

The FOC for γ_t gives

$$\gamma_t(\pi_t^x)^{-1} - (1-\alpha^*)(1-\gamma_t)(\pi_t^y)^{-1} + \frac{1}{2}\beta\frac{\partial L_{t+1}}{\partial \pi_{t+1}^y}\frac{\partial \pi_{t+1}^y}{\partial \gamma_t} = 0.$$

The envelope condition for π_{t+1}^y gives

$$\frac{\partial L_t}{\partial \pi_{t+1}^y} = -\left(1-\alpha^*\right)\left(1-\gamma_{t+1}\right)^2\left(\pi_{t+1}^y\right)^{-2},$$

while the law of motion for π_{t+1}^y gives

$$\frac{\partial \pi_{t+1}^y}{\partial \gamma_t} = 2\left(\kappa_1^* + \kappa_2^*\right)^2 \sigma_{\eta,t+1}^{-2}\gamma_t.$$

It follows that the optimal γ's satisfy

$$\gamma_t^{**} = \frac{\left(1-\alpha^*\right)\pi_t^x}{\pi_t^y + \left(1-\alpha^*\right)\pi_t^x - \beta\left(1-\alpha^*\right)\left(1-\gamma_{t+1}^{**}\right)^2 \pi_t^x \pi_t^y \left(\pi_{t+1}^y\right)^{-2}\left(\kappa_1^* + \kappa_2^*\right)^2 \sigma_{\eta,t+1}^{-2}},$$

which completes the proof. ■

Proof of Proposition 12. Let $\{\gamma_t^{**}\}_{t=1}^N$ be the coefficients that characterize the efficient linear strategy as in Proposition 11 and let $\{\pi_t^x, \pi_t^y\}_{t=1}^N$ be the corresponding precisions of private and public information generated by the efficient linear strategy. The result then follows from letting α_t^{**} be the unique solution to

$$\frac{\left(1-\alpha_t^{**}\right)\pi_t^x}{\left(1-\alpha_t^{**}\right)\pi_t^x + \pi_t^y} = \gamma_t^{**}.$$

In fact, it is then and only then the unique solution to (20) coincides with the strategy obtained in Proposition 11. ■

Proof of Corollary 7. Consider the environments with both exogenous and endogenous Gaussian signals studied in Section 6. The result follows directly from the proof of Proposition 12, where it is shown that, for all periods t, the present-value welfare losses L_t obtained along the efficient linear strategy are decreasing functions of π_t^x and π_t^y, the precisions of private and public information available in the beginning of period t. Putting aside informational externalities, the result can also be established for non-Gaussian signals using a Blackwell-like argument for the planner's problem that characterizes the efficient strategy. ■

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