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HOLD-UP, ASSET OWNERSHIP, AND REFERENCE POINTS

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Working Paper 13540

<http://www.nber.org/papers/w13540>

NATIONAL BUREAU OF ECONOMIC RESEARCH

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October 2007

I am grateful to John Moore for discussions on some of the elements of this paper, and to Bob Gibbons and Birger Wernerfelt for many helpful conversations. I would also like to thank Mathias Dewatripont, Florian Englmaier, Rob Gertner, Louis Kaplow, Josh Lerner, Bentley MacLeod, and Jeremy Stein for useful comments, and to Georgy Egorov for excellent research assistance. Financial support from the U. S. National Science Foundation through the National Bureau of Economic Research is gratefully acknowledged. The views expressed herein are those of the author(s) and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 13540
October 2007
JEL No. D23,D86,K12

ABSTRACT

We study two parties who desire a smooth trading relationship under conditions of value and cost uncertainty. A rigid contract fixing price works well in normal times since there is nothing to argue about. However, when value or cost is exceptional, one party will hold up the other, damaging the relationship and causing deadweight losses as parties withhold cooperation. We show that a judicious allocation of asset ownership can help by reducing the incentives to engage in hold up. In contrast to the literature, the driving force in our model is payoff uncertainty rather than noncontractible investments.

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1. Introduction

This paper reexamines some of the themes of the incomplete contracts literature – in particular, the hold-up problem and asset ownership – through a new theoretical lens, the idea that contracts serve as reference points (see Hart and Moore (2008)). We consider a buyer and seller who are involved in a (long-term) economic relationship where the buyer’s value and seller’s cost are initially uncertain. For the relationship to work out the parties need to cooperate in ways that cannot be specified in an initial contract. The buyer and seller face the following trade-off. On the one hand they can write a flexible contract that attempts to index the terms of trade – price – to the state of the world. However, to the extent that value and cost are not objective, such a contract will lead to argument, aggrievement and shading; this in turn creates deadweight losses. On the other hand the parties can write a (relatively) rigid contract, e.g., a fixed price contract. A rigid contract has the advantage that there is less to argue about in “normal” times, but the disadvantage that, if value or cost falls outside the normal range, one party will have an incentive to engage in hold-up, i.e., to threaten to withhold cooperation unless the contract is renegotiated. We suppose that hold-up transforms a friendly relationship into a hostile one. The consequence is that the parties operate within the letter rather than the spirit of their (renegotiated) contract, causing deadweight losses that are at least as great as those from shading. However, even a hostile relationship is assumed to create more surplus than no trade, and so, if value or cost has moved sufficiently far outside the normal range, hold-up will occur.

We show that an appropriate allocation of asset ownership can mitigate hold-up (and shading) costs. The idea is that, if the buyer (resp., the seller) owns key assets, then this improves his outside opportunities, and so in states of the world where his value inside the relationship is high, his value outside the relationship is also high. But this reduces the seller’s

(resp., the buyer's) ability to engage in hold-up, which increases efficiency. One feature of our approach is that, in contrast to much of the literature, it focuses on ex post rather than ex ante inefficiencies. Indeed (noncontractible) ex ante investments play no role.

It is useful to compare the current paper with the existing literature on asset ownership and vertical integration: in particular, transaction cost economics (see, e.g., Williamson (1971) and Klein et al. (1978)) and property rights theory (see, e.g., Grossman and Hart (1986) and Hart and Moore (1990)). According to transaction cost economics, contracts between independent parties are problematic because, given contractual incompleteness, deadweight losses will occur as parties haggle over the ex post division of the quasi-rents. A key factor in determining vertical integration decisions is the size of these quasi-rents. According to the property rights literature, parties will bargain around the deadweight losses from haggling, but ex ante investments will be distorted. A key factor determining vertical integration is the marginal product of quasi-rents with respect to (noncontractible) ex ante investments. This paper emphasizes a third factor: the variability of quasi-rents with respect to the state of the world; that is, payoff uncertainty.

We have both a theoretical and an empirical motivation for introducing a new factor into the analysis. First, existing models, such as Grossman and Hart (1986) and Hart and Moore (1990), have some foundational weaknesses. Second, these models seem to apply better to small entrepreneurial firms than to large companies. Third, as the recent empirical survey by Lafontaine and Slade (2007) shows, payoff uncertainty does seem to be an important determinant of vertical integration (see particularly their Tables 1 and 14). Yet neither transaction cost economics nor (present versions of) property rights theory stress payoff uncertainty per se. Our hope is that the current model will not only aid in the interpretation of existing studies, but will

also stimulate new empirical work.

Although most of the literature does not stress uncertainty, there is a small part that does. Goldberg and Erickson (1987) in their study of long-term contracts between buyers and sellers of petroleum coke find that these contracts are often indexed, and are shorter-term if the environment is more volatile. In Sections 2 and 4 we argue that our model is consistent with these observations. Our paper is close in spirit to Klein (1996).¹ Klein, using the Fisher Body – General Motors and Alcoa – Essex cases as motivating examples, argues that “hold-ups occur when market conditions change sufficiently to place the relationship outside the self-enforcing range.” Klein goes on to argue that integration can ameliorate this problem. However, Klein does not formalize the costs of hold-up and does not explain what limits the self-enforcing range of contracts.²

While Klein’s analysis is mainly informal, Baker et al. (2002) provide a formal analysis of (relational) contracts and hold-up in a world of uncertainty (see also Halonen (2002)). Baker et al. use a standard property rights model and focus on ex ante rather than ex post inefficiencies. They are concerned mainly with how relational contracts and asset ownership can help to mitigate the underinvestment problem. In contrast our model stresses the role of non-relational contracts and asset ownership in reducing the ex post inefficiency losses of hold-up (and shading).

The paper is organized as follows. In Sections 2 and 3 we lay out the model, focusing on rigid or “simple” contracts. The model is very much in the spirit of Hart and Moore (2008) (particularly Section III), but with the novelty that it includes the possibility of hold-up and

¹ See also Klein and Murphy (1997) and Klein (2007).

² As another example of hold-up occurring under extreme conditions, consider the case of a company that incentivizes its employees by promising high rewards if they are successful, but then is tempted to renege on its promise when the rewards turn out to be excessive. For a discussion of how Xerox Corporation dealt with this problem, see Harvard Business School cases 9-295-127, 9-298-109,5-298-152.

explores the role of asset ownership. (In Hart and Moore (2008) there are no assets, renegotiation or hold-up but the parties can quit.) In Section 4 we consider a more general class of contracts. Section 5 contains concluding remarks.

2. The Model

We consider a buyer B and a seller S who are engaged in a long-term relationship. The parties meet at date 0 and can trade a widget at date 1. There is uncertainty at date 0, but this is resolved shortly before date 1, at date 1^- , say. There is symmetric information throughout and the parties are risk neutral and face no wealth constraints. Each party has an outside option that he (or she) earns if trade does not occur. Let v , c denote B's value and S's cost if trade proceeds smoothly (i.e., the parties cooperate at date 1), and let r_b , r_s denote B and S's outside options. We assume that v , c , r_b , r_s , although observable, are not verifiable.

We follow Hart and Moore (2008) in supposing that for the gains from trade to be fully realized each party must take a number of "helpful" or "cooperative" actions at date 1. These actions cannot be specified in a date 0 contract because they are too complicated to describe in advance. When the uncertainty is resolved at date 1^- , some of these actions become describable and so can be contracted on, while others are never contractible.³ Thus some modification or renegotiation of the contract is possible at date 1^- . We assume that all the helpful actions are chosen simultaneously by B and S at date 1. See Figure 1 for a time line.

³ An example of a contractible helpful action would be where one party allows the other party to modify the characteristics of the good to be traded, e.g., the delivery time. An example of a noncontractible (or less contractible) helpful action would be where one party provides useful information to the other party, consults with them before making a decision, or responds to their phone calls or emails. For evidence that the same contractible action can lead to different performance outcomes (depending on ownership structure), suggesting that noncontractible actions are important, see Januszewski, Forbes and Lederman (2007).

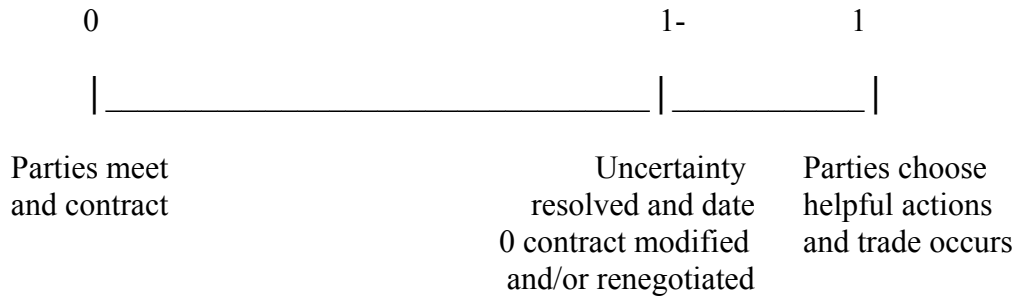


Figure 1

We make the following assumptions:

- (A1) If at date 1 all helpful actions are taken, the value of the widget to B is v and the cost to S is c , where $v > c$. Hence net surplus = $v - c$ in this case.
- (A2) If at date 1 all the contractible, but none of the noncontractible, helpful actions are taken, the value of the widget to B is $v - \frac{1}{2} \lambda (v - c)$ and the cost to S is $c + \frac{1}{2} \lambda (v - c)$, where $0 < \lambda < 1$. Hence net surplus = $(1 - \lambda)(v - c)$ in this case.
- (A3) If at date 1 none of the helpful actions (contractible or otherwise) is taken, B's value is very low (approximately, $-\infty$) and S's cost is very high (approximately, $+\infty$). In this case each party walks away from the contract (neither party has an incentive to enforce it) and no trade occurs; that is, the parties earn their outside options.

Note that the import of (A2) is that withholding noncontractible helpful actions moves v and c in the direction of $\frac{1}{2}(v + c)$.

What determines whether a party is helpful? As in Hart and Moore (2008), we suppose that being helpful does not cost significantly more than not being helpful: either it costs slightly more or it costs slightly less, that is, a party may actually enjoy being helpful. To simplify matters, we assume that a party is completely indifferent between being helpful and not.

Given this indifference we take the view that a party will be willing to be helpful if he is “well treated” by the other party, but not otherwise (negative reciprocity)⁴. Importantly, as in Hart and Moore (2008), a party is “well treated” if he receives what he feels entitled to, where the date 0 contract is a reference point for date 1 entitlements. To be more precise, neither party feels entitled to an outcome outside those permitted by the contract. However, within the contract there can be disagreement about the appropriate outcome.⁵

It is useful to begin with the case where the parties write a “simple” contract at date 0 that specifies a single trading price p (we normalize so that the no trade price is zero). Consider what happens at date 1-, once the uncertainty is resolved. Each party has a choice. He can stick to the contract. Or he can try to force the other party to renegotiate the contract – we interpret this as “hold-up.”

Let’s consider the scenario where the parties agree to stick to the contract. Under these conditions, each party feels well-treated by the other party since he is getting exactly what the

⁴ There is a large amount of empirical evidence in support of negative reciprocity. Many references can be found in Hart and Moore (2008). For a recent paper, see Anderson and Simester (2007), which shows that customers who have bought a product from a retailer, and later observe the same retailer selling it for less, reduce their demand for other products sold by the retailer.

⁵ To motivate the idea that the date 0 contract is a reference point for date 1 entitlements, Hart and Moore (2008) make the companion assumption that the date 0 contract is negotiated under competitive conditions. The competitive date 0 market provides objective measures of what B and S bring to the relationship, and it is supposed that the parties accept these measures as “fair”. In contrast there are no objective measures or competitive market at date 1 (or 1-), e.g., because the parties have made (contractible) specific investments between dates 0 and 1. For a more detailed discussion, see Hart and Moore (2008).

contract said he would: the contract specifies a single trading price p . Thus each party is willing to be helpful, and all cooperative actions are undertaken. The buyer and seller's payoff are, respectively,

$$(2.1) U_b = v - p,$$

$$(2.2) U_s = p - c.$$

In the second scenario, one party engages in hold-up: that is, he tries to force the other party to renegotiate the contract. He does this by threatening not to undertake any helpful actions unless he receives a sidepayment. We assume that such behavior is viewed as outrageous by the victim— it is a breach of the spirit of the date 0 contract – and leads, in the first instance, to the end of cooperation. The result is a Nash equilibrium where neither party cooperates. This yields the no-trade outcome described in (A3), with payoffs r_b, r_s for B, S, respectively.

However, renegotiation is possible.⁶ Even if the relationship is soured the parties can and will agree to undertake the helpful contractible actions at date 1. At the same time, it is supposed that neither party will provide noncontractible cooperation (again this is a Nash equilibrium). In effect the parties have a cold but correct relationship.⁷ Renegotiation therefore yields surplus $(1 - \lambda)(v - c)$ by (A2). Thus if

⁶ In fact we suppose that the parties cannot commit ex ante not to renegotiate. Below we discuss whether such a commitment would be desirable.

⁷ Note that we suppose that the parties cannot negotiate around this coldness. One could imagine that the buyer, anticipating that the seller is about to hold him up, would make a sidepayment to the seller to deflect the hold-up and preserve the relationship. We take the view that, given that there is a perceived threat in the background, the relationship is poisoned nonetheless. For some evidence supporting this view, and suggesting that even “big” players act in this way, see New York Times, August 27, 2007, p.A1 (reporting on the fact that the renegotiation of a large buy-out deal of a Home Depot business, triggered by the 2007 subprime mortgage crisis, created considerable bad feeling).

$$(A4) \quad (1 - \lambda)(v - c) > r_b + r_s,$$

the parties will renegotiate away from the no-trade outcome. We will assume that (A4) holds in what follows, but we will discuss in Section 5 what happens if (A4) is relaxed.⁸

In summary, hold-up leads to a loss of surplus of $\lambda(v-c)$.

We suppose a 50:50 split of the surplus from renegotiation, so after hold-up the parties' payoffs are

$$(2.3) \quad U_b = r_b + \frac{1}{2} G,$$

$$(2.4) \quad U_s = r_s + \frac{1}{2} G,$$

where

$$(2.5) \quad G = (1 - \lambda)(v - c) - r_b - r_s.$$

It is easy to determine when hold-up occurs. Define p_L to be the price p such that S is indifferent between receiving p and holding B up, and p_H to be the price such that B is indifferent between paying p and holding S up. Then from (2.1) – (2.4)

$$(2.6) \quad p_L - c = r_s + \frac{1}{2} G,$$

$$(2.7) \quad v - p_H = r_b + \frac{1}{2} G,$$

and so

⁸ Hart and Moore (2008) implicitly assume the opposite of (A4), with the consequence that hold-up and renegotiation do not occur. Instead the parties quit if their outside options exceed what they earn in the relationship. See Section 5 for more on the difference between the models.

$$(2.8) p_L = c + r_s + \frac{1}{2} G,$$

$$(2.9) p_H = v - r_b - \frac{1}{2} G.$$

Note that

$$(2.10) p_H - p_L = \lambda (v-c) > 0.$$

(2.10) reflects the fact that there is some friction in the renegotiation process. If hold-up did not lead to the souring of the relationship, λ would be zero, and $p_H = p_L$. However, since hold-up causes some dissipation of surplus, $p_H > p_L$: the price at which B is just ready to hold up S is strictly greater than the price at which S is just ready to hold up B.

Since S is indifferent between holding B up and not at $p = p_L$, S will strictly prefer to hold up B when $p < p_L$. Similarly, B will strictly prefer to hold up S when $p > p_H$. Thus hold-up is avoided if and only if

$$(2.11) p_L < p < p_H.$$

Note that p_H, p_L varies with the state of the world, whereas p is chosen ex ante. The situation is illustrated in Figure 2⁹.

⁹ The interval $[p_L, p_H]$ can be thought of as corresponding to Klein's self-enforcing contractual range (see Klein (1996)). However, in our model, this range is derived without the need to introduce relational contracts.

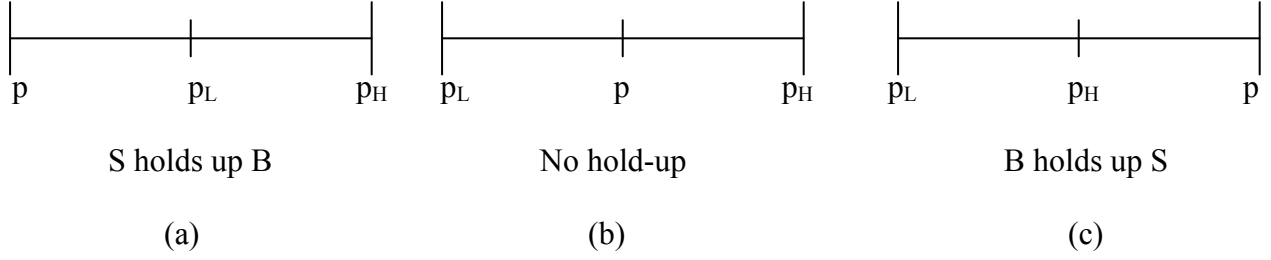


Figure 2

To make progress, we put more structure on the random variables r_b, r_s . We suppose

$$(2.12) \quad r_b = \alpha_b + \beta_b v + \varphi + \gamma_b \varepsilon,$$

$$(2.13) \quad r_s = \alpha_s - \beta_s c + \gamma_s \eta,$$

where

$$(2.14) \quad 1 - \lambda > \beta_b > 0, 1 - \lambda > \beta_s > 0, \gamma_b > 0, \gamma_s > 0.$$

Here $\alpha_b, \beta_b, \gamma_b, \alpha_s, \beta_s$ and γ_s are constants (later they will depend on the assets the parties own), and $\varphi, \varepsilon, \eta$ are independent random variables with mean zero. (2.12) – (2.14) capture the idea that B and S's outside options co-vary with v, c , respectively, but not too strongly, and are also subject to exogenous noise (ε, η). The noise term φ is a smoothing device: its rationale will become clear in Section 3.

Given (2.12) – (2.13), we can represent the state of the world by the 5-tuple $\omega = (v, c, \varphi, \varepsilon, \eta)$. Both parties observe ω at date 1-. Recall that a simple contract consists of a single price p , where p is chosen before the state of the world ω is realized. It is useful to rewrite p_L, p_H as functions of ω . From (2.8) – (2.9), we have

$$(2.15) \quad p_L(\omega) = \frac{1}{2} [\alpha_s + \gamma_s \eta - \alpha_b - \varphi - \gamma_b \varepsilon + ((1 - \lambda) - \beta_b) v + ((1 + \lambda) - \beta_s) c],$$

$$(2.16) \quad p_H(\omega) = \frac{1}{2} [\alpha_s + \gamma_s \eta - \alpha_b - \varphi - \gamma_b \varepsilon + ((1 + \lambda) - \beta_b) v + ((1 - \lambda) - \beta_s) c].$$

Clearly p_L, p_H are monotonic in v, c . Since hold-up occurs when $p_L(\omega) > p$ or $p_H(\omega) < p$, that is, when $p_L(\omega)$ is high or $p_H(\omega)$ is low, it follows that hold-up occurs, *ceteris paribus*, if v is exceptionally high or low or c is exceptionally high or low. This is intuitive: if v or c is high, S does relatively well in the renegotiation process and so has an incentive to hold B up; similarly, if v or c is low, B does relatively well in the renegotiation process and so has an incentive to hold S up. In addition it is clear from (2.15) – (2.16) that the effect of exceptional values of v, c is less pronounced if β_b and β_s are large since p_L, p_H are less sensitive to v, c under these conditions.

We turn now to an optimal simple contract. Since date 0 lump-sum transfers can be used to allocate surplus, an optimal contract maximizes expected net surplus. Thus an optimal simple contract solves:

$$(2.17) \quad \left\{ \begin{array}{ll} \text{Max} & \int (v - c) dF(\omega) + \int (1 - \lambda)(v - c) dF(\omega) \\ p & p_L(\omega) \leq p \leq p_H(\omega) \quad \begin{array}{l} p < p_L(\omega) \\ \text{or } p > p_H(\omega) \end{array} \end{array} \right\},$$

where F is the distribution function of ω .

It is clear that the first-best can be achieved in the case of certainty: just pick any price p in the interval $[p_L(\omega_0), p_H(\omega_0)]$, where ω_0 is the state of the world. However, the first-best typically cannot be achieved under uncertainty since it is not generally possible to find a single price that lies in the intersection of a number of different $[p_L(\omega), p_H(\omega)]$ intervals.

Our analysis so far has an obvious shortcoming. Suppose that the parties write a simple contract at date 0. Then, as we have just observed, with uncertainty it is very likely that p will lie outside the $[p_L(\omega), p_H(\omega)]$ range for some ω , and so one party will hold up the other to get a better price. Why don't the parties anticipate this and build the renegotiated price into the original contract? This leads to a consideration of more flexible contracts. In Section 4 we therefore broaden the analysis to allow the parties to specify a range of possible trading prices $[\underline{p}, \bar{p}]$ in their date 0 contract, along the lines of Hart and Moore (2008). As we shall show, the main insights of our analysis do not change and so for most of the paper we will confine our attention to simple contracts.

We close this section by noting that our model is consistent with Goldberg and Erickson's (1987) finding that price indexation is a common feature of contracts between suppliers and purchasers of petroleum coke. Although risk aversion is a possible explanation, Goldberg and Erickson argue that it is more likely that price indexation is used to reduce opportunistic behavior. Our model is consistent with this. By indexing the trading price p on a verifiable signal σ that is correlated with ω , the parties can more easily ensure that $p \in [p_L(\omega), p_H(\omega)]$. This reduces hold-up.

3. Asset Ownership

In this section we explore the idea that asset ownership can improve the parties' trading relationship. We take a simple view of asset ownership. Asset ownership matters because it determines which assets each party can walk away with if trade does not occur.¹⁰ This in turn affects parties' outside options and their incentives to engage in hold-up.

Denote by A the set of all assets at B and S 's disposal; we assume A is fixed and finite.¹¹

¹⁰ As in Hart and Moore (1990). See also Grossman and Hart (1986).

¹¹ We suppose that the assets in A are already specialized, and so cannot be bought or sold on the open market.

Let A_b be the set of assets B owns and A_s the set of assets S owns. We suppose

$$(3.1) \quad A_b \cap A_s = \emptyset, A_b \cup A_s \subseteq A.$$

The first part of (3.1) says that B and S can't walk away with the same asset. The inclusion in the second part reflects the possibility that if an asset is jointly owned neither party can walk away with it: that is, joint ownership gives each party a veto right on its use.¹²

We now suppose that the coefficients $\alpha_b, \beta_b, \gamma_b, \alpha_s, \beta_s, \gamma_s$ depend on asset ownership. In particular, $\alpha_b = \alpha_b(A_b), \beta_b = \beta_b(A_b), \gamma_b = \gamma_b(A_b), \alpha_s = \alpha_s(A_s), \beta_s = \beta_s(A_s), \gamma_s = \gamma_s(A_s)$. We also make assumptions similar to those in the property rights literature (see Grossman and Hart (1986) and Hart and Moore (1990)) about how these coefficients vary with asset ownership. In particular, we assume that owning more assets increases the marginal payoffs of r_b, r_s , with respect to v and c . That is,

$$(3.2) \quad \beta_b \text{ is nondecreasing in } A_b,$$

$$(3.3) \quad \beta_s \text{ is nondecreasing in } A_s.$$

We also assume that (2.14) and (A4) hold for all ownership structures.

We suppose that assets can be traded at date 0. Thus a contract is now a 3-tuple (A_b, A_s, p) , specifying an asset ownership allocation (A_b, A_s) and a price p , where A_b, A_s satisfy (3.1).

An optimal contract solves:

¹² We confine attention to simple ownership structures. It would not be difficult to generalize the analysis to allow for options to own, etc.

$$(3.4) \quad \text{Max}_{(A_b, A_s, p)} \left\{ \begin{array}{l} \int (v - c) dF(\omega) + \int (1 - \lambda)(v - c) dF(\omega) \\ p_L(\omega; A_b, A_s) \leq p \leq p_H(\omega; A_b, A_s) \quad \begin{array}{l} p < p_L(\omega; A_b, A_s) \\ \text{or } p > p_H(\omega; A_b, A_s) \end{array} \end{array} \right\},$$

where p_L , p_H are now indexed by the asset ownership allocation (A_b, A_s) , as well as by ω .

We begin our analysis of asset ownership by considering what happens if, *ceteris paribus*, assets are transferred at date 0 from S to B. Then, given (3.2) – (3.3), β_b rises and β_s falls. As is clear from (2.15) – (2.16), this makes p_L and p_H less sensitive to v than before since

$$(3.5) \quad \frac{\partial p_L}{\partial v} = \frac{1}{2} ((1 - \lambda) - \beta_b),$$

$$(3.6) \quad \frac{\partial p_H}{\partial v} = \frac{1}{2} ((1 + \lambda) - \beta_b),$$

and these both decrease. On the other hand, $p_L(\omega)$ and $p_H(\omega)$ become more sensitive to c since

$$(3.7) \quad \frac{\partial p_L}{\partial c} = \frac{1}{2} ((1 + \lambda) - \beta_s),$$

$$(3.8) \quad \frac{\partial p_H}{\partial c} = \frac{1}{2} ((1 - \lambda) - \beta_s),$$

and these both increase.

Intuitively, a reduction in sensitivity of p_L, p_H is good since, if the interval $[p_L, p_H]$ does not vary much, it is easier to find a price p that lies in $[p_L, p_H]$ for many ω . That is, hold-up is less likely. This suggests that it is optimal for B to own all the assets if only v varies, since this minimizes the sensitivity of p_L and p_H with respect to the state of the world; while, it is optimal for S to own all the assets if only c varies. Proposition 1 confirms this.

Proposition 1. (1) Suppose that $\phi = \varepsilon = \eta \equiv 0$, and $c \equiv c_0$ where c_0 is a constant. Then there exists an optimal contract in which B owns all the assets, i.e., $A_b = A, A_s = \phi$.

(2) Suppose that $\phi = \varepsilon = \eta \equiv 0$ and $v \equiv v_0$, where v_0 is a constant. Then there exists an optimal contract in which S owns all the assets, i.e., $A_s = A, A_b = \phi$.

In the Appendix we prove a more general version of Proposition 1, and of all the other propositions in this section. In these more general versions, a range of possible trading prices is allowed in the date 0 contract.

Note that it would not be difficult to establish uniqueness in Proposition 1 under slightly stronger stochastic assumptions.

Proposition 1 is reminiscent of the result in the property rights literature that one party should own all the assets if his investment is important. Here the conclusion is that one party should own all the assets if his payoff is uncertain. Proposition 1 is also similar to results that have been obtained by Simon (1951) and Wernerfelt (1997), among others, showing that the party with a more variable payoff should be the boss.

Of course, in general, both v and c vary. Proposition 1 is not very helpful here since it tells us only when one party should own everything. However, some progress can be made if we introduce the idea of an idiosyncratic asset.

Define an asset to be idiosyncratic to B if B's owning it increases the sensitivity of r_b to v and S's not owning it has no effect on the sensitivity of r_s to c . Define an asset to be idiosyncratic to S similarly.

Definition (i) Asset a is idiosyncratic to B if $\beta_b(A_b \cup \{a\}) > \beta_b(A_b)$ for all $A_b \subseteq A$, $A_b \cap \{a\} = \emptyset$, and $\beta_s(A_s \cup \{a\}) = \beta_s(A_s)$ for all $A_s \subseteq A$.

(ii) Asset a is idiosyncratic to S if $\beta_s(A_s \cup \{a\}) > \beta_s(A_s)$ for all $A_s \subseteq A$, $A_s \cap \{a\} = \emptyset$, and $\beta_b(A_b \cup \{a\}) = \beta_b(A_b)$ for all $A_b \subseteq A$.

Note that one reason an asset may be idiosyncratic to a party is if that party has human capital that is complementary to the asset, e.g., he is the only person who knows how to operate it.

Allocating an asset to the party to whom it is idiosyncratic would seem desirable since it reduces payoff variability. However, it turns out that to establish this one must make strong assumptions about the stochastic structure. In the next proposition we suppose that with high probability v, c take on "normal" values $v = v_0, c = c_0$, while with small probability v, c can each take on an "exceptional" value. Since exceptional values are unusual, we ignore the possibility that v and c can take on an exceptional value at the same time. We also suppose that there is a small amount of exogenous noise through the random variable ϕ , but we set $\varepsilon = \eta = 0$.

Proposition 2. Assume that $\varepsilon = \eta = 0$ and φ is uniformly distributed on $[-k, k]$. Suppose that with probability $0 < \pi < 1$ event 1 occurs: $v = v_0, c = c_0$; with probability $(1 - \pi) \alpha_v$ event 2 occurs: $c = c_0, v$ has support $[v_L, v_H]$, where $v_L \leq v_0$ and $v_H \geq \frac{-\beta_b(A)v_0 + (1 + \lambda)v_0 - 2\lambda c_0}{1 - \lambda - \beta_b(A)}$; with probability $(1 - \pi) \alpha_c$ event 3 occurs: $v = v_0, c$ has support $[c_L, c_H]$, where $c_H \geq c_0$ and $c_L \leq \frac{-\beta_s(A)c_0 - 2\lambda v_0 + (1 + \lambda)c_0}{1 - \lambda - \beta_s(A)}$. Here $\alpha_v > 0, \alpha_c > 0, \alpha_v + \alpha_c = 1, k > 0$, and φ is independent of v and c in events 2 and 3 respectively. Then for small enough k the following is true: if π is close to 1 it is uniquely optimal for B to own asset a if a is idiosyncratic to B and for S to own asset a if a is idiosyncratic to S.

Proof. See Appendix

It is useful to understand why Proposition 2 requires such strong assumptions about the probability distribution of v and c . The reason is the following. Let p be the optimal price for the general case where v, c are uncertain. Suppose that we transfer an asset that is idiosyncratic to B from S to B. (In what follows we suppress assets in the notation.) We know that this will reduce the variability of $p_L(\omega), p_H(\omega)$ with respect to v . But this might reduce the probability that p lies in $[p_L(\omega), p_H(\omega)]$ if, say, $p \in [p_L(\omega_1), p_H(\omega_1)], p \notin [p_L(\omega_2), p_H(\omega_2)]$ and $[p_L(\omega_1), p_H(\omega_1)]$ moves closer to $[p_L(\omega_2), p_H(\omega_2)]$. The stochastic structure in Proposition 2 avoids this kind of situation.

A simple application of Proposition 2 is to the case of strictly complementary assets. Suppose assets a_1 and a_2 are strictly complementary. Then a_2 by itself is of no use to S, while a_1 and a_2 together may be very useful to B. Assume B owns a_1 . Then we can define a new economy in which a_1 is inalienable, i.e., B always owns a_1 , and the effective set of (alienable) assets is $A \setminus \{a_1\}$. For this economy, a_2 is idiosyncratic to B in the sense of Definition (i).

Hence, according to Proposition 2, it is better for B to own a_1 and a_2 . The same argument shows that if S owns a_1 , it is better for S to own both. The conclusion is that strictly complementary assets should be owned together (by B or S – without further information we cannot say which). A similar argument shows that joint ownership is suboptimal under the conditions of Proposition 2.

Of course, these results are very reminiscent of those obtained in the property rights literature (see particularly Hart and Moore (1990)). However, the driving force is different: uncertainty rather than ex ante investments.

So far we have emphasized the idea that ownership of an asset is good for one party because it reduces the variability of their payoff relative to their outside option. However, there is also a class of cases where ownership can increase variability and it may be better to take assets away from people. The next proposition describes a situation where it is better to take assets away from both parties, i.e., joint ownership is optimal. In this proposition v and c are constant while ϕ and ε or η vary.

Proposition 3. Assume γ_b, γ_s are strictly increasing in A_b, A_s respectively, and ϕ is uniformly distributed on $[-k, k]$. Suppose that with probability $0 < \pi < 1$ event 1 occurs: $v = v_0, c = c_0, \varepsilon = 0, \eta = 0$; with probability $(1 - \pi) \alpha_\varepsilon$ event 2 occurs: $v = v_0, c = c_0, \eta = 0, \varepsilon$ has support $[\varepsilon_L, \varepsilon_H]$, where $\varepsilon_L \leq \frac{2\lambda(c_0 - v_0)}{\gamma_b(\phi)}$ and $\varepsilon_H > 0$; with probability $(1 - \pi) \alpha_\eta$ event 3 occurs: $v = v_0, c = c_0, \varepsilon = 0, \eta$ has support $[\eta_L, \eta_H]$, where $\eta_L \leq \frac{2\lambda(c_0 - v_0)}{\gamma_s(\phi)}$ and $\eta_H > 0$. Here $\alpha_\varepsilon \geq 0, \alpha_\eta \geq 0, \alpha_\varepsilon + \alpha_\eta = 1, k > 0$, and ϕ is independent of ε and η in events 2 and 3 respectively. Then for small enough k the

following is true: if π is close to 1 it is uniquely optimal for all assets to be jointly owned by B and S.

Proof. See Appendix

Again similar results have been obtained in the property rights literature (see, e.g., Halonen (2002) and Rajan and Zingales (1998)). Note that Proposition 3 depends on the assumption that owning more assets increases the variance of the outside option. Although plausible, one can certainly imagine other possibilities. For this reason we are inclined to put less weight on Proposition 3 than on Propositions 1 and 2.

4. Flexible Contracts

As we noted in Section 2, anticipating the possibility of hold-up, the parties can build some price flexibility into their contract. Following Hart and Moore (2008), we therefore now allow the parties to specify a range of possible trading prices $[\underline{p}, \bar{p}]$ in their date 0 contract; the idea is that the parties will agree on a price in $[\underline{p}, \bar{p}]$ once the uncertainty is resolved at date 1-. At the end of the section we also briefly discuss the possibility of state-contingent contracts or mechanisms.

Suppose that the parties pick the range $[\underline{p}, \bar{p}]$. What happens at date 1-, once ω is realized? There are two cases. Define

$$H(\omega, \underline{p}, \bar{p}; A_b, A_s) = [p_L(\omega; A_b, A_s), p_H(\omega; A_b, A_s)] \cap [\underline{p}, \bar{p}].$$

If $H \neq \emptyset$ the parties can avoid hold up by choosing a price in H: such a price is consistent with the date 0 contract and gives neither party an incentive to hold up the other. However, if $H = \emptyset$ hold-up cannot be avoided.

Start with the first case. Even though no hold-up occurs, there will be disagreement about the appropriate outcome within the contract. In the spirit of Hart and Moore (2008), we

suppose that each party is aggrieved to the extent that he does not receive what he feels entitled to, and will respond by shading, i.e., cutting back on helpful actions. We assume that each party feels entitled to the best outcome permitted by the contract. However, each party recognizes that he faces the feasibility constraint that the other party can trigger hold-up, i.e., B doesn't expect to pay less than p_L or S to receive more than p_H . In other words B feels entitled to the lowest price in H , $\text{Max}(p_L, \bar{p})$, and S feels entitled to the highest price in H , $\text{Min}(p_H, \underline{p})$. Note that the assumption that entitlements are constrained by what is feasible simplifies the analysis, but is not crucial.

To simplify matters we assume that the parties split the difference and set

$$(4.1) \hat{p} = \frac{1}{2} \{ \text{Max}(p_L(\omega; A_b, A_s), \underline{p}) + \text{Min}(p_H(\omega; A_b, A_s), \bar{p}) \}.$$

As part of this deal the parties agree to undertake the contractible helpful actions. However, each party cuts back on the noncontractible helpful actions, in proportion to his aggrievement. B is aggrieved by

$$(4.2) a_b = \hat{p} - \text{Max}(p_L(\omega; A_b, A_s), \underline{p}),$$

and shades to the point where S 's payoff falls by θa_b . S is aggrieved by

$$(4.3) a_s = \text{Min}(p_H(\omega; A_b, A_s), \bar{p}) - \hat{p},$$

and shades to the point where B's payoff falls by θa_s . The parameter θ is taken to be exogenous and the same for B and S, and $0 < \theta \leq 1$.

Thus in Case 1, where $H \neq \emptyset$, net surplus is

$$(4.4) \quad W_1(\omega, \underline{p}, \bar{p}; A_b, A_s) = v - c - \theta(a_b + a_s) = v - c - \theta \{ \text{Min}(p_H(\omega; A_b, A_s), \bar{p}) - \text{Max}(p_L(\omega; A_b, A_s), \underline{p}) \}.$$

In contrast, in Case 2, where $H = \emptyset$, hold-up occurs, followed by renegotiation, and net surplus is given by

$$(4.5) \quad W_2(\omega) = (1 - \lambda)(v - c).$$

Note that since hold-up leads to the withdrawal of all noncontractible helpful actions, while shading leads to the withdrawal of only some of them, there is an implicit constraint that total shading costs cannot exceed total hold-up costs, i.e.,

$$(4.6) \quad W_1(\omega, \underline{p}, \bar{p}; A_b, A_s) \geq W_2(\omega).$$

Fortunately, this constraint is automatically satisfied, given $0 < \theta \leq 1$, since

$$(4.7) \quad \begin{aligned} & \text{Min}(p_H(\omega; A_b, A_s), \bar{p}) - \text{Max}(p_L(\omega; A_b, A_s), \underline{p}) \\ & \leq p_H(\omega; A_b, A_s) - p_L(\omega; A_b, A_s) \\ & = \lambda(v - c), \end{aligned}$$

where we are using (2.10). In other words, however large the price range $[\underline{p}, \bar{p}]$ is, net surplus is higher if hold-up is avoided than if it occurs.¹³

An optimal contract maximizes expected net surplus. Thus an optimal contract solves:

$$(4.8) \quad \text{Max}_{(A_b, A_s, \underline{p}, \bar{p})} \left\{ \begin{array}{l} \int W_1(\omega, \underline{p}, \bar{p}; A_b, A_s) dF(\omega) + \int W_2(\omega) dF(\omega) \\ H(\omega, \underline{p}, \bar{p}; A_b, A_s) \neq \emptyset \quad H(\omega, \underline{p}, \bar{p}; A_b, A_s) = \emptyset \end{array} \right\},$$

where F is the distribution function of ω . (We assume that F has bounded support.)¹⁴ The trade-off is the following. As \underline{p} falls or \bar{p} rises, the set H becomes larger and so hold-up is less likely. This is good given that hold-up reduces surplus, i.e., $W_1 \geq W_2$. However, shading represented by $\theta \{ \text{Min}(p_H, \bar{p}) - \text{Max}(p_L, \underline{p}) \}$ rises, which means that surplus in the absence of hold-up falls, i.e., W_1 is lower. This is bad.

It is worth noting that this model provides a rationale for a long-term contract different from the traditional one based on noncontractible investments. No long-term contract corresponds to the case where $\underline{p} = -\infty$, $\bar{p} = \infty$ (everything is left to date 1). The advantage of this is that hold-up is avoided (there is no contract to force renegotiation of). The disadvantage is that shading costs are high on average.

¹³ In fact (A2) implies the further constraint that each party's shading costs cannot exceed $\frac{1}{2} \lambda(v - c)$. This is automatically satisfied given (4.1).

¹⁴ It is easy to show that an optimal contract exists since the objective function is upper semicontinuous in $[\underline{p}, \bar{p}]$.

It is useful to analyze the optimal contract in some simple cases. We have already seen in Section 2 that a simple contract can achieve the first-best if there is no uncertainty. It turns out that a non-simple contract can achieve the first-best if there are just two states: $\omega = \omega_1$ or ω_2 . (In what follows we suppress assets.) To see why, note that there are two possibilities. Either $[p_L(\omega_1), p_H(\omega_1)] \cap [p_L(\omega_2), p_H(\omega_2)]$ is non-empty or it is empty. In the first case, choose any price \hat{p} in the intersection and set $\underline{p} = \hat{p} = \bar{p}$. In the second case, suppose without loss of generality that

$$p_L(\omega_1) < p_H(\omega_1) < p_L(\omega_2) < p_H(\omega_2).$$

Then set $\underline{p} = p_H(\omega_1)$, $\bar{p} = p_L(\omega_2)$. In state ω_1 , $p = p_H(\omega_1)$ and in state ω_2 , $p = p_L(\omega_2)$. Hold-up is avoided and there is no grievement since the set H is a singleton in both states.

Once there are three states the first-best typically cannot be achieved even with a non-simple contract. An example is presented in the Appendix.

We close this section with some interpretations and observations. One obvious question that can be asked about the model of this section is, why can't the parties do better than specify a price range? In fact, given that they will both learn the state of the world at date 1-, even though it is not verifiable, why can't they write an informal state contingent contract? Or why can't they agree to play a message game that will reveal the state of the world? Of course, if they could do either of these things, they would arrange that the trading price $p(\omega) \in [p_L(\omega; A_b, A_s), p_H(\omega; A_b, A_s)]$ for all ω , and the first-best is achieved.

Informal understandings about contingencies are not uncommon. However, they are likely to work better when the contingency is relatively objective (i.e., verifiable). It may be

reasonable to agree in advance that price should rise if industry costs increase, but it seems problematic, and also less common, to let price vary with the seller's own observable but unverifiable cost (or the buyer's observable but unverifiable value). The latter arrangement is likely to lead to argument ex post. The reason may be that in practice there is always at least a small amount of asymmetric information. The seller's cost may appear to be high but there is always the chance that it is low. Suppose that the buyer and seller are subject to self-serving biases of the kind emphasized in the behavioral economics and psychology literatures, and they exaggerate the probabilities of events favorable to them. Under these conditions specifying that price should vary with cost will lead to disagreement about what the cost is. There will be aggrievement and shading, just as there is with a price range. Similarly, a mechanism to reveal the state may lead to annoyance about how the mechanism is played; again there will be aggrievement and shading. For further discussion see Hart and Moore (2008).

The above discussion also helps us to answer a second question. Since price ranges don't seem that common in practice, how should we interpret them? As we have just seen, one interpretation of a price range is that it represents an attempt by the parties to write a state-contingent contract. However, there are at least two other possible interpretations. First, the date 0 contract could be an "agreement to agree"; that is, a non-legally binding framework for future negotiations. (In contrast, trades of assets at date 0 would be legally binding.) As noted in Hart and Moore (2008), price ranges are sometimes observed in the case of agreements to agree. Moreover, agreements to agree may not be that uncommon in the context we are studying. For example, Goldberg (2007) argues that the famous contract between Fisher Body and General Motors, which we mentioned in the Introduction, was not legally binding, i.e., it was actually an agreement to agree.

Another interpretation of a price range is the following. Suppose date 1 trade is not instantaneous: rather the parties trade over a period of time, e.g., between dates 1 and 2. (For simplicity, suppose uncertainty is still resolved at date 1-) Then the parties might agree at date 0 on the length of their contract. For example, they might agree on a trading price that will operate for a fraction τ of the period between dates 1 and 2. The parties recognize that when they get to date 1-, they will renegotiate the price for the remaining fraction $(1 - \tau)$ of the period. A small value of τ then corresponds to a flexible contract -- there is a wide range of possible (average) prices over the period -- while a large value of τ corresponds to a rigid contract.

This last interpretation can throw light on the work of Goldberg and Erickson (1987). Goldberg and Erickson find that parties in the petroleum coke industry tend to write shorter-term, i.e., more flexible, contracts in a more volatile environment. The parallel result in our model is that an increase in uncertainty leads to a larger price range $[\underline{p}, \bar{p}]$. Although further assumptions would be required to prove a general result along these lines, the result is at least true at the extremes. With no uncertainty the optimal contract is a single price $p \in [p_L(\omega; A_b, A_s), p_H(\omega; A_b, A_s)]$; while with sufficiently large uncertainty the optimal contract will be a non-degenerate price interval since the chance that a single price will lie in $[p_L(\omega; A_b, A_s), p_H(\omega; A_b, A_s)]$ becomes vanishingly small. Thus in a broad sense our model seems consistent with Goldberg and Erickson (1987).

5. Conclusions

In this paper we have studied the trade-off between contractual flexibility and rigidity in a buyer-seller relationship. A flexible contract is good because it allows the parties to adjust the terms of trade to uncertain events, but bad because it leads to argument, grievance, and

shading. A rigid contract avoids argument in normal times, but has the undesirable feature that if value or cost falls outside the normal range one party will attempt to force renegotiation, damaging the relationship and causing deadweight losses. We have shown that a judicious allocation of asset ownership can improve this trade-off. For example, if the buyer's value varies a lot relative to the seller's cost, allocating assets to the buyer will cause the value of the buyer's trade inside the relationship and his value outside the relationship to move together in such a way that the seller's ability to engage in hold-up is reduced. This increases efficiency.

It is useful to review how our analysis differs from that found in the transactions cost and property rights literatures. The transaction cost literature has emphasized that parties will vertically integrate when the quasi-rents from their relationship are large. The property rights literature has emphasized that asset ownership is determined by the marginal product of quasi-rents with respect to noncontractible investments. In this paper we have suggested that the driver of integration and asset ownership decisions might be a third factor: payoff uncertainty. Although several of our results are similar to those obtained in the property rights literature, the fact that we emphasize payoff uncertainty should, we hope, allow the theories to be distinguished empirically.

There are several assumptions of our analysis that could usefully be relaxed. First, we have assumed that, even though hold-up causes deadweight losses, the parties prefer to continue trading with each other after hold-up rather than to go their separate ways (assumption (A4)). Relaxing this assumption opens up interesting new possibilities. Two cases need to be distinguished. In the first, although the trading relationship is terminated after hold-up, the parties can renegotiate asset ownership (this is similar to the assumption made in Baker et al. (2002)). Under these conditions, the surplus obtained after hold-up is independent of the date 0

assignment of assets, which is the critical feature of (A4). Thus our results are unlikely to change much in this case. The second case is where there are frictions in the asset renegotiation process (perhaps because it is difficult even to trade assets after a relationship is poisoned), in which case the date 0 ownership structure will affect the final outcome. Under these conditions the initial assignment of assets matters not only in determining when hold-up occurs, but also in affecting ex post surplus when it does. This opens the door to a richer theory of ownership.

It would also be worth relaxing our assumption that hold-up sours the parties' relationship by the same amount regardless of the reasons for it. Consider two cases of seller hold-up. In one the seller holds the buyer up because the buyer's value has increased. In the other the seller holds up the buyer because the seller's cost has increased. It is arguable that the buyer will be less angry in the second case since the seller's behavior seems less opportunistic. We have taken the contrary view that the buyer will view the cases as the same on the grounds that if the seller wanted the price to rise under some conditions she should have built this into the initial contract. However, it seems useful to develop a model where the two cases are treated differently.

In most models of incomplete contracts, if the parties can commit not to renegotiate their contract, the first-best is achieved. It is worth noting that this is not true here. In the absence of renegotiation hold-up will never occur since it is impossible to change the terms of trade. However, the parties will have an incentive to quit, by refusing to cooperate, whenever their outside option exceeds their payoff inside the relationship. (This is similar to the case analyzed in Hart and Moore (2008).) There will still be inefficiency—in fact in some cases it will be greater. Asset ownership will matter although the effects will be somewhat different. In fact the

model is likely to resemble the second case discussed above in which (A4) doesn't hold, since, as in that case, asset ownership will affect ex post surplus in the event that the parties quit.¹⁵

An obvious question to ask is : to what kind of firm does the model apply? One point of view is that the feelings of hostility and aggrievement that we have emphasized are plausible in the case of small owner-managed firms, but less so in the case of large corporations. We are not sure that this is entirely true. Large corporations are run by individuals who have big egos and presumably therefore can have strong emotions. In fact the Home Depot case, described in footnote 7, where contract renegotiation caused coldness, is one involving a large company. Still it is undoubtedly the case that the effects we describe are likely to be different in a company where many decisions are delegated. In fact one interesting trade-off is that it might be good for a subordinate to make contract (re)negotiation decisions given that he does not care about hold-up (e.g., because he is on low-powered incentives), and hence this will reduce ex post deadweight losses, but it might be bad because he will negotiate an unfavorable deal in the first place.

As we have emphasized, our theory ignores ex ante considerations. In future work it would be interesting to reintroduce these. For example, suppose that the seller can take an action—an ex ante relationship-specific investment, say—that reduces her cost. To the extent that this moves cost outside the “normal” range, this will give the buyer an incentive to hold the seller up. Anticipating this, the seller will have less incentive to engage in such an investment. A model that includes ex ante as well as ex post inefficiencies is likely to provide a richer understanding

¹⁵ An interesting possibility is that, rather than committing not to renegotiate, the parties might try to constrain the renegotiation process in their date 0 contract. For example, they could put in place an information revelation game to be played if the relationship breaks down. Note, however, that one point of view is that incorporating a renegotiation process in the initial contract enlarges the set of feasible outcomes and hence increases parties' feelings of entitlement in the event that the relationship does not break down, thus raising aggrievement and shading costs.

of the costs and benefits of asset ownership and more generally of vertical integration.

Developing a model along these lines is an interesting and challenging goal for future research.

Appendix

We first present an example where the first-best cannot be achieved even with flexible contracts. We then prove Propositions 1-3.

Example 1

Suppose that there are three states, $\omega_1, \omega_2, \omega_3$, with probabilities π_1, π_2, π_3 , respectively. Only v varies across the states: $v(\omega_1) = 20, v(\omega_2) = 60, v(\omega_3) = 80$. Assume $c = 10, r_b = r_s = 0, \lambda = \frac{1}{2}$.

Then

$$p_L(\omega) = \frac{1}{4}v + \frac{3}{4}c,$$

$$p_H(\omega) = \frac{3}{4}v + \frac{1}{4}c.$$

The $[p_L(\omega), p_H(\omega)]$ intervals are illustrated in Figure 3.

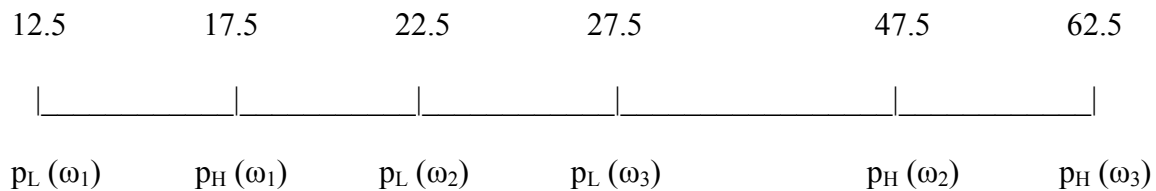


Figure 3

We see that the $[p_L(\omega_2), p_H(\omega_2)]$, $[p_L(\omega_3), p_H(\omega_3)]$ intervals overlap, but neither overlaps with $[p_L(\omega_1), p_H(\omega_1)]$. There are three candidates for an optimal contract: one can avoid hold-up in all states with some aggrievement; one can avoid hold-up in ω_1 and ω_2 without aggrievement; or one can avoid hold-up in ω_2 and ω_3 without aggrievement.

Contract 1: Avoiding hold-up in all states

To avoid hold-up in all states, we need $\underline{p} \leq p_H(\omega_1)$, $\bar{p} \geq p_L(\omega_3)$. To minimize shading costs, we want the highest \underline{p} and lowest \bar{p} . Hence set $\underline{p} = 17.5$, $\bar{p} = 27.5$. In ω_1 , $p = 17.5$ and there is no aggrievement. In ω_2 there is aggrievement of 5 since B would like $p = 22.5$ and S would like $p = 27.5$. In ω_3 , $p = 27.5$ and there is no aggrievement. Net surplus is given by

$$W = 10 \pi_1 + (50 - 5\theta) \pi_2 + 70 \pi_3.$$

Contract 2: Avoiding hold-up in ω_1, ω_2

In this case it is best to set $\underline{p} = p_H(\omega_1) = 17.5$, $\bar{p} = p_L(\omega_2) = 22.5$. This avoids aggrievement in states ω_1, ω_2 (see the above discussion of the two-state example). However, hold-up occurs in ω_3 . Net surplus is given by

$$W = 10 \pi_1 + 50 \pi_2 + 35 \pi_3.$$

Contract 3: Avoiding hold-up in ω_2, ω_3

In this case it is best to set $\underline{p} = \bar{p} = \hat{p}$, where $27.5 \leq \hat{p} \leq 47.5$. This avoids aggrievement in ω_2 , ω_3 . However, hold-up occurs in ω_1 . Net surplus is given by

$$W = 5 \pi_1 + 50 \pi_2 + 70 \pi_3.$$

Which of the above contracts is best depends on the parameters π_1 , π_2 , π_3 , and θ .

Contract 1 is optimal if θ or π_2 is small. Contract 2 is optimal if π_3 is small. Contract 3 is optimal if π_1 is small. None of these contracts achieves the first-best.

Proof of Proposition 1. We prove (1). Let $(A_b, A_s, [\underline{p}, \bar{p}])$ be an optimal contract. The proof proceeds in two steps. We first replace $[\underline{p}, \bar{p}]$ by another price interval, and show that shading costs fall (weakly). We then allocate all the assets to B and make another change in the price interval, and show that shading costs fall again and that the hold-up region becomes (weakly) smaller. Thus the new contract in which B owns everything must also be optimal.

Index the state by v . Let \underline{v} be the smallest value of v , and \bar{v} the largest of v , in the support of F such that no hold-up occurs under contract $(A_b, A_s, [\underline{p}, \bar{p}])$. Then

$$(A.1) \quad [p_L(v), p_H(v)] \cap [\underline{p}, \bar{p}] \neq \emptyset$$

for $v = \underline{v}$ and $v = \bar{v}$. Since $p_L(v)$, $p_H(v)$ are increasing in v , (A.1) must also hold for $\underline{v} \leq v \leq \bar{v}$, i.e., hold-up does not occur for intermediate v 's. Note that (A.1) implies that $p_H(\underline{v}) \geq \underline{p}$, $p_L(\bar{v}) \leq \bar{p}$.

Now define a new price interval $[\underline{p}', \bar{p}']$, where

$$\underline{p}' = p_H(\underline{v}) \text{ and } \bar{p}' = \text{Max}(p_L(\bar{v}), p_H(\underline{v})).$$

Clearly $\underline{p}' \geq \underline{p}$. Also either $\bar{p}' \leq \bar{p}$ or $\bar{p}' = \underline{p}'$. In the first case the new price interval is a subset of the previous price interval. In the second case it is a singleton. In both cases

$$[p_L(v), p_H(v)] \cap [\underline{p}', \bar{p}'] \neq \emptyset$$

for $\underline{v} \leq v \leq \bar{v}$. Hence the new price interval avoids hold-up for $\underline{v} \leq v \leq \bar{v}$ just like the old one.

In addition aggrivement and shading costs are lower under the new price interval given that either the new price interval is a subset of the previous price interval or it is a singleton (in which case shading costs are zero).

Now assign all the assets to B, i.e., set $A_b = A$, $A_s = \emptyset$. Call this the new ownership structure. Define a new price interval $[\underline{p}'', \bar{p}'']$, given by

$$(A.2) \quad \underline{p}'' = p_H^N(\underline{v}), \quad \bar{p}'' = \text{Max}(p_L^N(\bar{v}), p_H^N(\underline{v})),$$

where p_L^N, p_H^N represent the values of p_L, p_H under the new ownership structure. The price interval $[\underline{p}'', \bar{p}'']$ avoids hold-up under the new ownership structure when $\underline{v} \leq v \leq \bar{v}$.

We show next that shading costs are lower for each $\underline{v} \leq v \leq \bar{v}$ under the new ownership structure and price interval $[\underline{p}'', \bar{p}'']$ than under the old ownership structure and $[\underline{p}', \bar{p}']$ (which in

turn are lower than those under the old ownership structure and $[\underline{p}, \bar{p}]$). That is, we demonstrate that

$$(A.3) \quad \begin{aligned} & \text{Min} (p_H^N(v), \bar{p}'') - \text{Max} (p_L^N(v), \underline{p}'') \\ & \leq \text{Min} (p_H(v), \bar{p}') - \text{Max} (p_L(v), \underline{p}'). \end{aligned}$$

There are several cases to consider. Note first that if $\underline{p}', \bar{p}' = p_H(\underline{v}) \geq p_L(\bar{v})$, i.e., the right-hand side (RHS) of (A.3) is zero, then

$$\begin{aligned} p_H^N(\underline{v}) - p_L^N(\bar{v}) &= - (p_H^N(\bar{v}) - p_H^N(\underline{v})) \\ &\quad + p_H^N(\bar{v}) - p_L^N(\bar{v}) \\ &\geq - (p_H(\bar{v}) - p_H(\underline{v})) \\ &\quad + p_H(\bar{v}) - p_L(\bar{v}) \\ &= p_H(\underline{v}) - p_L(\bar{v}) \\ &\geq 0, \end{aligned}$$

where we are using the fact that $p_H^N(\bar{v}) - p_H^N(\underline{v}) \leq p_H(\bar{v}) - p_H(\underline{v})$ since $\frac{\partial p_H}{\partial v}$ falls the more assets B owns (by (3.6)), and $p_H^N(\bar{v}) - p_L^N(\bar{v}) = p_H(\bar{v}) - p_L(\bar{v})$, i.e., $p_H - p_L$ is independent of the ownership structure (see (2.10)). Hence $p_H^N(\underline{v}) \geq p_L^N(\bar{v})$. It follows from (A.2) that $\underline{p}'' = \bar{p}'' = p_H^N(\underline{v})$ and so the left-hand side (LHS) of (A.3) is zero. Therefore (A.3) holds.

Consider next the case where $\underline{p}' = p_H(\underline{v}) < \bar{p}' = p_L(\bar{v})$. If $\underline{p}'' = \bar{p}'' = p_H^N(\underline{v}) \geq p_L^N(\bar{v})$, (A.3) again holds. So suppose $\underline{p}'' = p_H^N(\underline{v}) < p_L^N(\bar{v}) = \bar{p}''$. We must show that

$$(A.4) \quad \text{Min} (p_H^N(v), p_L^N(\bar{v})) - \text{Max} (p_L^N(v), p_H^N(\underline{v})) \\ \leq \text{Min} (p_H(v), p_L(\bar{v})) - \text{Max} (p_L(v), p_H(\underline{v})).$$

We can rewrite (A.4) as

$$(A.5) \quad \text{Min} \{p_H^N(v) - p_H^N(\underline{v}), p_L^N(\bar{v}) - p_H^N(\underline{v}), \\ p_H^N(v) - p_L^N(v), p_L^N(\bar{v}) - p_L^N(v)\} \leq \text{Min} \{p_H(v) - p_H(\underline{v}), p_L(\bar{v}) - p_H(\underline{v}), \\ p_H(v) - p_L(v), p_L(\bar{v}) - p_L(v)\}.$$

To establish (A.5) one shows that each component in the min formula on the LHS of (A.5) is no greater than the corresponding component on the RHS of (A.5). This follows from the fact that

$\frac{\partial p_H}{\partial v}$, $\frac{\partial p_L}{\partial v}$ are nonincreasing in the assets that B owns, and that $p_H - p_L$ is independent of

ownership structure for a given v . Hence (A.4) holds and so does (A.3).

In summary, the new ownership structure (in which B owns all the assets) and price range $[\underline{p}'', \bar{p}'']$ yields (weakly) lower shading costs than the original ownership structure and price range $[\underline{p}, \bar{p}]$. Also the hold-up region is no larger (hold-up does not occur for $\underline{v} \leq v \leq \bar{v}$). This shows that allocating all the assets to B is optimal. Q.E.D.

Proof of Proposition 2. Suppose a is idiosyncratic to B. We show that B should own a . The proof is by contradiction. If the proposition is false, then, however small k is, we can construct a sequence of optimal contracts $(A_{br}, A_{sr}, \underline{p}_r, \bar{p}_r)$ such that $a \in A_{sr}$ for all r , i.e., S owns asset a , and $\pi_r \rightarrow 1$ as $r \rightarrow \infty$. Without loss of generality (wlog) suppose that $A_{br} \rightarrow A_b(k)$, $A_{sr} \rightarrow A_s(k)$, \underline{p}_r

$\rightarrow \underline{p}(k), \bar{p}_r \rightarrow \bar{p}(k)$. Then $A_b(k), A_s(k), \underline{p}(k), \bar{p}(k)$ must be optimal for the case where event 1 occurs with probability 1. For small k the first-best can be achieved (exactly) in event 1 since there is almost no uncertainty. A necessary condition for this is that there is a single trading price $p(k)$ in the limit, i.e., $\underline{p}(k) = p(k) = \bar{p}(k)$ (so that shading costs are zero), and

$$(A.6) \quad p_L(\omega, A_b(k), A_s(k)) \leq \underline{p}(k) = p(k) = \bar{p}(k) \leq p_H(\omega, A_b(k), A_s(k))$$

for all $-k \leq \varphi \leq k$, where $\omega = (v_0, c_0, \varphi)$ and we now suppress $\varepsilon = 0, \eta = 0$. Here p_L, p_H are as in (2.15) – (2.16) and are indexed by the limiting ownership structure.

Consider a new sequence of contracts $(A_{br}', A_{sr}', \underline{p}_r', \bar{p}_r')$, where the only difference between A_{br}', A_{sr}' and A_{br}, A_{sr} is that asset a is transferred to B, and

$$(A.7) \quad \begin{aligned} \underline{p}_r' - \underline{p}_r &= \bar{p}_r' - \bar{p}_r \\ &= \frac{1}{2} [\alpha_s(A_{sr} \setminus \{a\}) - \alpha_s(A_{sr}) \\ &\quad + \alpha_b(A_{br}) - \alpha_b(A_{br} \cup \{a\}) \\ &\quad + \beta_b(A_{br}) v_0 - \beta_b(A_{br} \cup \{a\}) v_0] \\ &= \Delta_r(k). \end{aligned}$$

In other words we adjust $\underline{p}_r, \bar{p}_r$ by an amount equal to the change $\Delta p_L, \Delta p_H$ in p_L, p_H that occurs as a result of the shift in ownership structure, where $\Delta p_L, \Delta p_H$ are evaluated at $v = v_0$. Note that, given the assumption that a is idiosyncratic to B, $\Delta p_L, \Delta p_H$ depend on v , but not on c (or φ).

What happens to expected net surplus as a result of this change? Expected net surplus is a weighted average of surplus in the three events 1, 2, 3. Given that $\underline{p}_r, \bar{p}_r, p_{Lr}(\omega), p_{Hr}(\omega)$ all shift by Δ_r when $v = v_0$, nothing changes in events 1 and 3 for all r . That is, for each state ω , hold-up occurs if and only if it did before, and the level of shading costs, if hold-up doesn't occur, remains constant. Thus net surplus is unchanged in events 1 and 3.

Since the new contract cannot deliver higher expected net surplus than the original contract, given that the original contract is optimal, it follows that net surplus must be (weakly) lower in event 2. Let $r \rightarrow \infty$. Wlog $(A_{br}', A_{sr}', \underline{p}_r', \bar{p}_r') \rightarrow (A_b'(k), A_s'(k), \underline{p}'(k), \bar{p}'(k))$ and $\Delta_r(k) \rightarrow \Delta(k)$. Given (A.6) – (A.7), we must have

$$(A.8) \quad p_L(\omega, A_b'(k), A_s'(k)) \leq \underline{p}'(k) = p'(k) = \bar{p}'(k) \leq p_H(\omega, A_b'(k), A_s'(k))$$

for all $-k \leq \varphi \leq k$, where $\omega = (v_0, c_0, \varphi)$. By the above arguments the contract $(\underline{p}'(k), \bar{p}'(k), A_b'(k), A_s'(k))$ delivers surplus no higher in event 2 than the contract $(\underline{p}(k), \bar{p}(k), A_b(k), A_s(k))$.

We show that this conclusion is false. Since the primed and unprimed contracts both have a single trading price ($p'(k), p(k)$, respectively), shading costs are zero in both contracts. We demonstrate that there is less hold-up in the primed contract. Since we are in event 2 index the state by (v, φ) . Then, from (A.7),

$$(A.9) \quad \begin{aligned} p'(k) - p(k) &= p_L((v_0, \varphi), A_b'(k), A_s'(k)) - p_L((v_0, \varphi), A_b(k), A_s(k)) \\ &= p_H((v_0, \varphi), A_b'(k), A_s'(k)) - p_H((v_0, \varphi), A_b(k), A_s(k)) \\ &= \Delta(k) \end{aligned}$$

for all φ . Now hold-up occurs in the primed contract in state (v, φ) if and only if either $p'(k) < p_L((v, \varphi), A_b'(k), A_s'(k))$ or $p'(k) > p_H((v, \varphi), A_b'(k), A_s'(k))$. Consider the first. Given (A.8) and the fact that p_L is increasing in v , $p'(k) < p_L((v, \varphi), A_b'(k), A_s'(k))$ only if $v > v_0$. But, if $v > v_0$,

$$\begin{aligned}
\text{(A.10)} \quad & p_L((v, \varphi), A_b'(k), A_s'(k)) - p_L((v_0, \varphi), A_b'(k), A_s'(k)) \\
&= \frac{1}{2} [(1 - \lambda) - \beta_b (A_b \cup \{a\})] (v - v_0) \\
&< \frac{1}{2} [(1 - \lambda) - \beta_b (A_b)] (v - v_0) \\
&= p_L((v, \varphi), A_b(k), A_s(k)) \\
&\quad - p_L((v_0, \varphi), A_b(k), A_s(k)),
\end{aligned}$$

since a is idiosyncratic to B . From (A.9) – (A.10), we may conclude that $p'(k) < p_L((v, \varphi), A_b'(k), A_s'(k)) \Rightarrow p(k) < p_L((v, \varphi), A_b(k), A_s(k))$, i.e., hold-up occurs in the unprimed contract if it occurs in the primed contract. A similar argument shows that $p'(k) > p_H((v, \varphi), A_b'(k), A_s'(k)) \Rightarrow p(k) > p_H((v, \varphi), A_b(k), A_s(k))$. Putting the two arguments together, we may conclude that hold-up costs are weakly lower in the primed contract than the unprimed one. In fact, they are strictly lower: this follows from the assumption about the support of v in Proposition 2, which ensures that $p_L((v, \varphi), A_b(k), A_s(k)) > p_H((v, \varphi), A_b(k), A_s(k))$ for large v and φ close to zero (i.e., hold-up does occur sometimes), but not for v close to v_0 (i.e., hold-up does not always occur). Contradiction. Q.E.D.

Proof of Proposition 3. We sketch the proof since the argument is very similar to that of Proposition 2. Suppose joint ownership is not optimal. For small k choose a sequence of optimal contracts as $\pi \rightarrow 1$.

The limiting contract is optimal for event 1. Hence (A.6) is satisfied. Consider a new sequence of contracts where all assets are jointly owned and $\underline{p}_r, \bar{p}_r$ are adjusted to reflect the new ownership structure, i.e.,

$$\begin{aligned} \underline{p}'_r - \underline{p}_r &= \bar{p}'_r - \bar{p}_r \\ &= \frac{1}{2} [\alpha_s(\phi) - \alpha_s(A_{sr}) - \alpha_b(\phi) + \alpha_b(A_{br}) \\ &\quad - \beta_s(\phi) c_0 + \beta_s(A_{sr}) c_0 - \beta_b(\phi) v_0 + \beta_b(A_{br}) v_0]. \end{aligned}$$

Then surplus does not change in event 1. Since the initial contract is optimal surplus must weakly fall in events 2 or 3. Wlog suppose it falls in event 2. Take limits as $r \rightarrow \infty$. The limiting joint ownership contract has the property that p_L, p_H vary less with ε than under the original contract. But this makes hold-up less likely. Hence the joint ownership contract creates higher net surplus. Contradiction. Q.E.D

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