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Determinacy and Identification with Taylor Rules
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ABSTRACT

The new-Keynesian, Taylor-rule theory of inflation determination relies on explosive dynamics. By raising interest rates in response to inflation, the Fed induces ever-larger inflation or deflation, unless inflation jumps to one particular value on each date. However, economics does not rule out inflationary or deflationary equilibria. Attempts to fix this problem assume that people believe the government will choose to blow up the economy if alternative equilibria emerge, by following policies we usually consider impossible. Therefore, inflation is just as indeterminate under “active” interest rate targets as it is under fixed interest rate targets.

If one accepts the new-Keynesian solution, the parameters of the Taylor rule relating interest rates to inflation and other variables are not identified without unrealistic assumptions. Thus, Taylor rule regressions cannot be used to argue that the Fed conquered inflation by moving from a “passive” to an “active” policy in the early 1980s.

The updated version presented here merges the content of two companion papers, w13409 and w13410, into a single current version. Thus, as of September 2010, w13409 and w13410 are identical.

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An online appendix is available at:
<http://www.nber.org/data-appendix/w13409>

1 Introduction

How is the price level determined? The new-Keynesian, Taylor-rule theory (sometimes also called “DSGE”) provides the current standard answer to this, perhaps the most fundamental, question of macroeconomics. In this theory, inflation is determined because the central bank systematically raises nominal interest rates more than one-for-one with inflation. This “active” interest rate target is thought to eliminate the indeterminacy that results from fixed interest rate targets, and thus to provide the missing “nominal anchor.”

Theories ultimately rise and fall on their ability to organize facts. Keynes had the *General Theory* of the great depression. Friedman and Schwartz had the *Monetary History*. The central new-Keynesian story is that U. S. inflation was conquered in the early 1980s, by a change from a “passive” policy in which interest rates did not respond sufficiently to inflation to an “active” policy in which they do so. Most famously, Clarida, Galí and Gertler (2000) ran regressions of federal funds rates on inflation and output. They found inflation coefficients below one up to 1980, and above one since then. (Complex new-Keynesian models also “fit the data” well, but so do other models. While important reassurance for applying models, this observation is not a useful test of a model’s basic structure.)

I challenge this theory and its interpretation of the data. First, I conclude that the Taylor rule, in the context of a new-Keynesian model, leaves the same inflation indeterminacy as with fixed interest rate targets. Second, if the model is true, the parameters of the Fed’s reaction function are not identified, so regression evidence does not say anything about determinacy in a new-Keynesian model.

The same key point drives both observations: New-Keynesian models *do not* say that higher inflation causes the Fed to raise real interest rates, which in turn lowers “demand,” which reduces future inflation. That’s “old-Keynesian,” stabilizing logic. In new-Keynesian models, higher inflation leads the Fed to set interest rates in a way that leads to even *higher* future inflation. For only one value of inflation today will inflation fail to explode, or, more generally, eventually leave a local region. Ruling out non-local equilibria, new-Keynesian modelers conclude that inflation today must jump to the unique value that leads to a locally-bounded equilibrium path.

But there is no reason to rule out nominal explosions or “non-local” nominal paths. Transversality conditions can rule out *real* explosions, but not nominal explosions. I conclude that the multiple non-local equilibria are perfectly valid. Furthermore, if we do rule out the non-local paths, interest rates that generate explosive inflation are an outcome that is never realized in the observed equilibrium, so that response cannot be measured.

Responses: determinacy and dilemma

These points are known, if perhaps not fully appreciated. Many authors have advanced proposals to trim new-Keynesian multiple equilibria by adding to the policy description. One can always add such assumptions; for example if one assumes that the government will blow up the world in any but one desired equilibrium, then alternative equilibria are ruled out. The question is whether the assumptions made to rule out multiple equilibria are at all reasonable. In particular, to bring a model to data, the assumptions must be vaguely plausible descriptions of what people *currently* believe our government would do, and not wildly at odds with what governments actually do in similar circumstances.

Most proposals to trim equilibria sound superficially like sensible descriptions of what govern-

ments do to stop extreme inflation or deflation – switch to a commodity standard, exchange-rate peg, money growth rule, or undertake a fiscal expansion. However, *stopping an inflation* or deflation is a completely different act than *disallowing an equilibrium*. If an inflation-stopping policy still describes how an equilibrium forms at each date, then the inflation or deflation and its end remain an equilibrium path and we have ruled nothing out. To rule out equilibria, somewhere along the path the government must specify policy so that it is *impossible* for an equilibrium to form. Some proposals specify a commodity standard, which implies zero inflation, but *also* a very high interest rate. Others specify a commodity standard with a tight and inconsistent limit on money supply. Still others specify inconsistent fiscal and monetary policies, introduce arbitrage opportunities, or set infinite inflation. It is these inconsistent or overdetermined policies, not the inflation stabilization, which trims equilibria.

Such assumptions seem wildly implausible, as descriptions of government behavior or as descriptions of people’s current beliefs about government behavior. A policy configuration for which “no equilibrium can form” or “private first-order conditions cannot hold” means a threat to blow up the economy. Why would a government choose to blow up the economy, when tested policies that stop inflation or deflation, while allowing equilibria to form at each date, are sitting on the shelf? Governments that stop inflations do not also insist that the real quantity of money remain at a low level, do not try to target hyperinflationary interest rates, do not introduce arbitrage opportunities, and do coordinate fiscal and monetary policies.

In fact, in most (Ramsey) analyses of policy choices, we label such policy configurations as “impossible,” not just “implausible.” We think of governments choosing policy configurations, while taking private first-order conditions as constraints; we think of governments acting in markets. We don’t think governments *can* follow policies for which private first-order conditions don’t hold. For example, to operate a commodity standard, we would say that a government must offer to exchange currency for the commodity freely at the stated price; it simply *cannot* also maintain a low limit on money supply or a very high interest rate target.

The dilemma is unavoidable. If we specify that the government will stop an inflation or deflation in such a way that equilibria can form on each date, we get quite sensible proposals and descriptions of what governments might do – can do, and have done – to end inflations or deflations, but we don’t rule out any equilibria. To rule out equilibria, people must believe that the government will blow up the economy. Whether the rest of the policy description resembles historically successful stabilizations is irrelevant. Whether the impossible policies occur on the date of stabilization or at any other point on the path is irrelevant. I survey the extensive literature, and do not find any successful escape from this dilemma.

Responses: identification

The literature also contains many attempts to rescue identification. One can also always add assumptions to produce identification. But we can and must ask whether identification assumptions are reasonable, as a description of Fed behavior, of people’s expectations of Fed behavior, and of the theory with which the regressions are interpreted.

The central identification problem is that the theory predicts there is no movement in the crucial right hand variable, the difference between actual inflation and inflation in the desired equilibrium. (The issue is not “in” vs. “out of” equilibrium, the issue is selection between multiple equilibria.) At a deep level, then, we must assume that the correlations between interest rates and inflation *in* the equilibrium are the same as the Fed’s response to movements *away* from that equilibrium. More deeply, the right “natural rate,” the behavior of interest rates in the desired equilibrium, is

a completely different issue from determinacy; how the interest rate should vary if inflation veers away from the desired equilibrium. To identify the latter from the former, we must assume they are the same.

But then a second classic problem arises. *In* the equilibrium, the Taylor-rule right hand variables (inflation, output) and all potential instruments are correlated with the monetary policy disturbance term. This correlation is not accidental; it is central to the theory: If a monetary policy shock occurs, then inflation and other right hand variables are supposed to jump to the unique values that lead to a locally-bounded equilibrium. Really, identification depends on the right hand variables, not the errors. The errors are structural shocks about which we can say little. The right hand variables are determined by the model. It takes restrictions on the *model* to produce right hand variables which are uncorrelated with the error terms.

Furthermore, new-Keynesian theory also advocates a “stochastic intercept,” that the central bank should vary the interest rate to follow variations in the “natural rate” induced by other shocks. These movements are part of the empirical monetary policy disturbance, so the theory predicts that these other shocks, and variables which depend on them, cannot be used as instruments. All of the disturbances are structural shocks, not forecast errors, so they are serially correlated, invalidating lags as instruments. And even if the disturbances were serially uncorrelated, then lags of endogenous variables would not be correlated with right hand variables.

In sum, New-Keynesian models specify policy rules that are a snake-pit for econometricians. There is no basis for all the obvious devices, such as excluding variables from the policy rule, the use of instruments, assuming monetary policy disturbances are orthogonal to other variables, and lag-length restrictions on shocks. (Lag length and exclusion restrictions as approximations are not a problem; restrictions to produce identification are.) Not only *might* these problems exist, but theory predicts that most of them *do* exist. Empiricists must throw out key elements of the theory to identify parameters. Finally, even if one could identify parameters from a determinate new-Keynesian equilibrium (1980s), no one has ever addressed what one measures if the world is *indeterminate*, as supposedly was the case in the 1970s. The *change* in coefficients is a crucial part of the story.

If not this then what?

If not this theory, what theory *can* account for price-level determination, in a modern fiat-money economy whose central bank follows an interest rate target? This paper is entirely negative, and long enough, so it does not exposit or test an alternative theory. But it is worth pointing out one logical conclusion that one can take from a negative result, and a glimmer of hope that a positive result exists.

The new-Keynesian Taylor rule model is an “active money, passive fiscal” regime, in Leeper’s (1991) terminology: It assumes that the valuation equation for government debt, which states that the real value of nominal debt equals the present value of real primary surpluses, holds by ex-post lump-sum taxation, and therefore plays no role in price level determination. The opposite possibility is the “active fiscal, passive money” regime, in which the valuation equation for government debt determines the price level, but the central bank follows an interest rate rule that does not destabilize the economy. Since this model of price-level determination relies on ruling out real, rather than nominal explosions, through the consumer’s transversality condition, it does uniquely determine the price level. It is not inconsistent with Taylor Rule regressions. It therefore does provide a coherent economic theory of the price level, that can address current institutions.

This paper and my conclusion are not a criticism of new-Keynesian economics in general. In particular I do not have anything to say here that criticizes its basic ingredients: an intertemporal, forward-looking “IS” curve, or an intertemporally-optimizing, forward-looking model of price-setting subject to frictions, as captured in the “new-Keynesian Phillips curve.” The “passive money, active fiscal” regime of such a model can determine inflation. Whether such a model can fully address the data is another as-yet unanswered question.

An acknowledgement

Indeterminacy, multiple equilibria, and identification in dynamic rational-expectations models are huge literatures that I cannot possibly adequately cite, acknowledge, or review in the space of one article. The body of the paper reviews specific important contributions in the context of new-Keynesian models. This is not a critique of those specific papers. I choose these papers as concrete and well-known examples of general points, repeated throughout the literature. The web Appendix contains a much more comprehensive review, both to properly acknowledge others’ efforts, and to establish that no, these problems have not been solved.

The equations in this paper are simple and not new. In this field, however, there is great debate over how one should read and interpret simple and fairly well-known equations. This paper’s novelty is a contribution to that worthwhile and subtle debate.

2 Simplest model

We can see the main points in a very simple model consisting only of a Fisher equation (consumer first order conditions) and a Taylor rule describing Fed policy,

$$i_t = r + E_t\pi_{t+1}, \tag{1}$$

$$i_t = r + \phi\pi_t + x_t, \tag{2}$$

where i_t = nominal interest rate, π_t = inflation, r = constant real rate.

The monetary policy disturbance x_t represents variables inevitably left out of any regression model of central bank behavior, such as responses to financial crises, exchange rates, time-varying rules, mismeasurement of potential output, and so on. It is not a forecast error, so it is serially correlated,

$$x_t = \rho x_{t-1} + \varepsilon_t. \tag{3}$$

(Equivalently, applying $(1 - \rho L)$ to both sides of (2), the target may be smoothed and react to past inflation.)

We can solve this model by substituting out the nominal interest rate, leaving the equilibrium condition

$$E_t\pi_{t+1} = \phi\pi_t + x_t. \tag{4}$$

2.1 Determinacy

Equation (4) has many solutions. This observation forms the classic doctrine (Sargent and Wallace (1975)) that inflation, to say nothing of the price level, is indeterminate with an interest rate target. We can write the equilibria of this model as

$$\pi_{t+1} = \phi\pi_t + x_t + \delta_{t+1}; E_t(\delta_{t+1}) = 0 \tag{5}$$

where δ_{t+1} is any conditionally mean-zero random variable. Multiple equilibria are indexed by arbitrary initial inflation π_0 , and by the arbitrary random variables or “sunspots” δ_{t+1} .

If $\|\phi\| > 1$, all of these equilibria except one eventually explode, i.e. $E_t(\pi_{t+j})$ grows without bound. If we disallow such solutions, then a unique locally-bounded solution remains. We can find this solution by solving the difference Equation (4) forward, or by undetermined coefficients (which assumes a bounded solution, depending only on x_t),

$$\pi_t = - \sum_{j=0}^{\infty} \frac{1}{\phi^{j+1}} E_t(x_{t+j}) = - \frac{x_t}{\phi - \rho}. \quad (6)$$

Equivalently, by this criterion we select the variables $\pi_0, \{\delta_{t+1}\}$, which index multiple equilibria, as

$$\pi_0 = - \frac{x_0}{\phi - \rho}; \quad \delta_{t+1} = - \frac{\varepsilon_{t+1}}{\phi - \rho}. \quad (7)$$

Thus we have it: if the central bank’s interest rate target reacts sufficiently to inflation – if $\|\phi\| > 1$ – then it seems that a pure interest rate target, with no control of monetary aggregates, no commodity standard or peg, and no “backing” beyond pure fiat, can determine at least the inflation rate, if not quite the price level. It seems that making the peg react to economic conditions overturns the classic doctrine that inflation is indeterminate under an interest-rate peg.

This simple example makes it crystal-clear that inflation determination does not come from the stabilization of inflation in old-Keynesian stories. Real rates are constant. Nominal rate rises cannot raise real rates and so dampen “demand.” If the Fed increases the nominal rate, it must allow expected inflation to increase, thus de-stabilizing inflation. As King (2000) emphasizes, $\phi < -1$, oscillating hyper-inflation and deflation, works just as well as $\phi > 1$ to ensure determinacy. That example is hard to describe by “stabilizing” intuition.

But what’s wrong with non-local equilibria? Transversality conditions can rule out real explosions, but not nominal explosions. Hyperinflations are historic realities. This condition didn’t come from any economics of the model. I conclude there’s nothing wrong with them, and this model does not determine inflation.

This is an example, which needs fleshing out. First, I need to write down a fully-specified model, to show there truly is nothing wrong with non-local equilibria. Second, I need to examine the standard three-equation model, including varying real rates and price stickiness, to verify that this simple frictionless model indeed captures the same issues. Third, haven’t the legions of people who have addressed these issues solved all these problems? I review the literature to verify they have not done so.

2.2 Identification

Now, suppose the solution (6) is in fact correct, what are its observable implications? Since π_t is proportional to x_t , the dynamics of equilibrium inflation are simply those of the disturbance x_t ,

$$\pi_t = \rho\pi_{t-1} + w_t. \quad (8)$$

($w_t \equiv -\varepsilon_t/(\phi - \rho)$, but ε_t and x_t are not directly observed, so we can summarize observable dynamics with the new error w_t .) Using (1) and (8), we can find the equilibrium interest rate,

$$i_t = r + \rho\pi_t. \quad (9)$$

Equation (9) shows that a *Taylor-rule regression of i_t on π_t will estimate the disturbance serial correlation parameter ρ rather than the Taylor rule parameter ϕ .*

What happened to the Fed policy rule, Equation (2)? The solution (6) shows that the right hand variable π_t and the disturbance x_t are correlated – perfectly correlated in fact. That correlation is no accident or statistical assumption, it is central to how the model behaves. The whole *point* of the model, the whole way it generates responses to shocks, is that endogenous variables (π_t) “jump” in response to shocks (ε_t), so as to head off expected explosions.

Perhaps we can run the regression by instrumental variables? Alas, the only instruments at hand are lags of π_t and i_t , themselves endogenous, and thus invalid. For example, if we use all available lagged variables as instruments, we have from (8) and (9)

$$\begin{aligned} E(\pi_t | \pi_{t-1}, i_{t-1}, \pi_{t-2}, i_{t-2}, \dots) &= \rho \pi_{t-1} \\ E(i_t | \pi_{t-1}, i_{t-1}, \pi_{t-2}, i_{t-2}, \dots) &= r + \rho^2 \pi_{t-1}. \end{aligned}$$

Thus the instrumental variables regression gives exactly the same estimate

$$E(i_t | \pi_{t-1}, i_{t-1}, \pi_{t-2}, i_{t-2}, \dots) = r + \rho E(\pi_t | \pi_{t-1}, i_{t-1}, \pi_{t-2}, i_{t-2}, \dots).$$

If the disturbance x_t were i.i.d., then the correlation of instruments with errors would be removed, but so would the correlation of instruments with right hand variables.

Is there nothing clever we can do? No. The equilibrium dynamics of the observable variables $\{i_t, \pi_t\}$ are completely described by equations (8) and (9). The equilibrium dynamics, and the resulting likelihood function, *do not involve ϕ* . *ϕ is not identified from data on $\{i_t, \pi_t\}$ in the equilibrium of this model.* Inflation is supposed to jump to the one value for which accelerating inflation at rate ϕ is not observed. If it does so, there is no way to measure how fast the inflation would accelerate if it did not.

Absence of ϕ from equilibrium dynamics and the likelihood function means that we can’t even test whether the data are generated from the *region* of determinacy $\|\phi\| > 1$, abandoning hope of measuring the precise value of ϕ , as Lubik and Schorfheide (2004) try to do. For every equilibrium generated by a ϕ^* with $\|\phi^*\| > 1$, the same equilibrium dynamics (8) and (9) can be generated by a different ϕ with $\|\phi\| < 1$. The web Appendix elaborates this point.

Again, this is the beginning. I need to show that the same problems occur in more complex models, including the standard three-equation new-Keynesian model that Clarida, Galí and Gertler (2000) and other authors use, and that the many attempts at identification don’t convincingly surmount them.

3 An explicit frictionless model

3.1 The model

To keep the discussion compact and consistent with the literature, I simplify standard sources, Benhabib Schmitt-Grohé and Uribe (2002) and Woodford (2003). Consumers maximize a utility function

$$\max E_t \sum_{j=0}^{\infty} \beta^j u(C_{t+j}).$$

Consumers receive a constant nonstorable endowment $Y_t = Y$; markets clear when $C_t = Y$. Consumers trade in complete financial markets described by real contingent claims prices m_{t+1} and hence nominal contingent claims prices

$$Q_{t,t+1} = \frac{P_t}{P_{t+1}} m_{t+1}.$$

The nominal interest rate is related to contingent claim prices by

$$\frac{1}{1+i_t} = E_t [Q_{t,t+1}].$$

The government issues one-period nominal debt; B_{t-1} denotes the face value issued at time $t-1$ and coming due at date t . The government levies lump-sum taxes S_t , net of transfers. S_t denotes the real primary surplus. I follow Benhabib Schmitt-Grohé and Uribe (2002), Woodford (2003), Cochrane (2005) and many others in describing a frictionless economy. The dollar can be a unit of account even if, in equilibrium, nobody chooses to hold any dollars overnight.

The consumer faces a present-value budget constraint

$$E_t \sum_{j=0}^{\infty} Q_{t,t+j} P_{t+j} C_{t+j} = B_{t-1} + E_t \sum_{j=0}^{\infty} Q_{t,t+j} P_{t+j} (Y_{t+j} - S_{t+j}). \quad (10)$$

or, in real terms,

$$E_t \sum_{j=0}^{\infty} m_{t,t+j} C_{t+j} = \frac{B_{t-1}}{P_t} + E_t \sum_{j=0}^{\infty} m_{t,t+j} (Y_{t+j} - S_{t+j}). \quad (11)$$

3.2 Equilibria

The consumer's first order conditions state that marginal rates of substitution equal contingent claims price ratios, and equilibrium $C_t = Y$ implies a constant real discount factor,

$$\beta \frac{u_c(C_{t+1})}{u_c(C_t)} = m_{t+1} = \beta \frac{u_c(Y)}{u_c(Y)} = \beta. \quad (12)$$

Therefore, the real interest rate is constant,

$$\frac{1}{1+r} = E_t (m_{t+1}) = \beta$$

and the nominal discount factor is

$$Q_{t,t+1} = \frac{P_t}{P_{t+1}} m_{t+1} = \beta \frac{P_t}{P_{t+1}}. \quad (13)$$

The interest rate follows a Fisher relation,

$$\frac{1}{1+i_t} = E_t(Q_{t,t+1}) = \beta E_t \left(\frac{P_t}{P_{t+1}} \right) = \frac{1}{1+r} E_t \left(\frac{1}{\Pi_{t+1}} \right) \quad (14)$$

The usual relation (1) follows by linearization.

From the consumer’s present value budget constraint (10), and using contingent claim prices from (13), equilibrium $C_t = Y$ also requires

$$\frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} E_t(S_{t+j}). \quad (15)$$

The value of government debt is the present value of future tax payments. This is not a “government budget constraint,” it is an equilibrium condition, an implication of supply = demand or $C_t = Y_t$ in goods markets, as you can see directly by looking at (11). Section 1 of the web Technical Appendix and Cochrane (2005) discuss this issue in more detail. I assume that the present value of future primary surpluses is positive and finite, $0 < \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} E_t(S_{t+j}) < \infty$.

The Fisher equation (14) and the government debt valuation equation (15) are the only two conditions that need to be satisfied for the price sequence $\{P_t\}$ to represent an equilibrium. If they hold, then the allocation $C_t = Y$ and the resulting contingent claims prices (13) imply that markets clear and the consumer has maximized subject to his budget constraint. The equilibrium is not yet *unique*, in that many different price or inflation paths will work. Unsurprisingly, we need some specification of monetary and fiscal policy to determine the price level.

3.3 New - Keynesian policy and multiple equilibria

The new-Keynesian/Taylor-rule analysis maintains a “Ricardian” fiscal regime; net taxes S_{t+j} are assumed to adjust so that the government debt valuation equation (15) holds given any price level. (Woodford (2003) p. 124.) It also specifies a Taylor rule for monetary policy.

We have answered the first question needed from this explicit model: yes, solutions of the simple model consisting of a Fisher equation and a Taylor rule (1)-(2), as I studied above, do in fact represent the full set of (linearized) equilibrium conditions of this explicit model, if we assume a Ricardian fiscal regime. My simple model didn’t leave anything out.

Are the non-local equilibria really globally valid? Here I follow the standard sources, in part to emphasize agreement that they are. (Woodford (2003) Ch. 2.4, starting p. 123, and Ch. 4.4 starting on p. 311, with a review; Benhabib Schmitt-Grohé and Uribe (2002).)

Restrict attention to perfect foresight equilibria. Adding uncertainty (sunspots) can only increase the number of equilibria. Consider an interest rate (Taylor) rule

$$1 + i_t = (1 + r)\Phi(\Pi_t); \quad \Pi_t \equiv P_t/P_{t-1}. \quad (16)$$

$\Phi(\cdot)$ is a function allowing nonlinear policy rules. The consumer’s first order condition (14) reduces to

$$\Pi_{t+1} = \beta(1 + i_t). \quad (17)$$

We are looking for solutions to the pair (16) and (17). As before, we substitute out the interest rate and study the equation

$$\Pi_{t+1} = \Phi(\Pi_t). \quad (18)$$

We have a nonlinear, global, perfect-foresight version of the analysis in Section 2.

As Benhabib Schmitt-Grohé and Uribe emphasize, a Taylor rule with slope greater than one should not apply globally to an economy in which consumers can hold money, because nominal interest rates cannot be negative. Thus, if we want to specify a Taylor rule with $\Phi_\pi > 1$ at

some point, we should think about the situation as illustrated in Figure 1. The equilibrium at Π^* satisfies the Taylor principle, and is a unique *locally bounded* equilibrium. Any value of Π_0 other than Π^* leads away from the neighborhood of Π^* as shown. With a lower bound on nominal interest rates, however, the function $\Phi(\Pi)$ must also have another stationary point, labeled Π_L . This stationary point must violate the Taylor principle. Therefore, many paths lead to Π_L and there are “multiple local equilibria” near this point. In addition, the equilibria descending from Π^* to Π_L are “bounded” though not “locally bounded.”

(Yes, Π^* is the “good” equilibrium and Π_L is the “bad” equilibrium. The point is to find determinacy by ruling out multiple equilibria. Π^* is a unique locally-bounded equilibrium. “Stability” near Π_L comes with “indeterminacy.”)

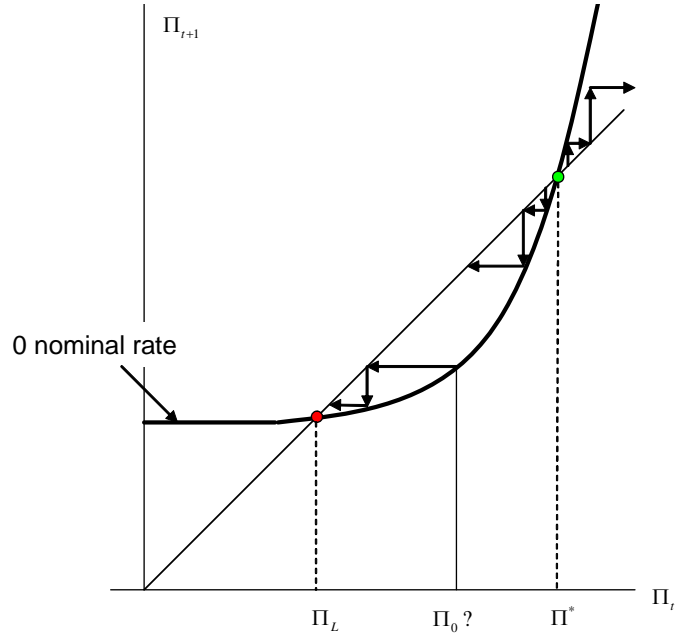


Figure 1: Dynamics in a perfect foresight Taylor-rule model.

All of these paths are equilibria. Since these paths satisfy the policy rule and the consumer’s first-order conditions by construction, all that remains is to check that they satisfy the government debt valuation formula (15), i.e. that there is a set of ex-post lump-sum taxes that can validate them and hence ensure the consumer’s transversality condition is satisfied. There are lots of ways the government can implement such a policy. We only need to exhibit one. If the government simply sets net taxes in response to the price level as

$$S_t = \frac{r}{1+r} \frac{B_{t-1}}{P_t} \quad (19)$$

then the real value of government debt will be constant, and the valuation formula will always hold.

To see why this is true, start with the flow constraint, proceeds of new debt sales + taxes = old debt redemption,

$$\frac{B_t}{1+i_t} + P_t S_t = B_{t-1}.$$

With $1+i_t = (1+r)P_{t+1}/P_t$, this can be rearranged to express the evolution of the real value of

the debt,

$$\frac{B_t}{P_{t+1}} = (1+r) \left(\frac{B_{t-1}}{P_t} - S_t \right). \quad (20)$$

Substituting the rule (19) we obtain

$$\frac{B_t}{P_{t+1}} = \frac{B_{t-1}}{P_t}.$$

We're done. With constant real debt and the flow relation (20) the transversality condition holds, and (20) implies (15). All the “explosive” equilibria of the simple model of Section 2 are, in fact, perfectly valid. Deflationary equilibria that approach Π_L are also valid equilibria, as is Π_L itself. If we write the Taylor rule such that $i = 0$ at Π_L (for example, $i_t = \max(0, r + \phi\pi_t)$), the “liquidity trap” equilibrium $i_t = 0$, $\Pi_L = \beta$ (deflation at the rate r) is also a valid equilibrium.

3.4 Non-Ricardian Policy

The price level i_s is uniquely determined in this frictionless model if we strengthen, rather than throw out, the government valuation equation – if the government follows a “non-Ricardian” fiscal regime. This is a natural alternative theory to consider, it is the basis for a lot of equilibrium-trimming and related discussion that follows, and it clarifies the fundamental issue.

As the simplest example, suppose fiscal policy sets the path of real net taxes $\{S_t\}$ independently of the price level. (A proportional income tax achieves this result.) The initial stock of government debt B_{t-1} is predetermined. Then, (15) determines the price level P_t ,

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} S_{t+j}. \quad (21)$$

This is the same mechanism by which stock market prices are determined as the present value of dividends (Cochrane (2005)).

The government can still follow an interest rate rule. By varying the amount of nominal debt sold at each date, the government can control expected future prices and hence the interest rate. Multiplying (21) at $t+1$ by $\frac{1}{1+r}$ and taking expectations,

$$\frac{B_t}{P_t} E_t \left(\frac{1}{1+r} \frac{P_t}{P_{t+1}} \right) = \frac{B_t}{P_t} \frac{1}{1+i_t} = E_t \sum_{j=1}^{\infty} \frac{1}{(1+r)^j} S_{t+j}.$$

P_t is determined by (21). Then, by changing debt sold at time t , B_t , the government can determine i_t and $E_t(P_t/P_{t+1})$. Alternatively, the government can simply auction bonds at the interest rate i_t , and this equation tells us how many B_t it will sell.

Ex-post inflation is determined by the ex-post value of (21), which we can write in a pretty proportional form

$$\frac{(E_{t+1} - E_t) \left(\frac{1}{1+\pi_{t+1}} \right)}{E_t \frac{1}{1+\pi_{t+1}}} = \frac{(E_{t+1} - E_t) \sum_{j=1}^{\infty} \frac{1}{(1+r)^j} S_{t+j}}{E_t \sum_{j=1}^{\infty} \frac{1}{(1+r)^j} S_{t+j}} \quad (22)$$

Linearizing in the style of Section 2, innovations to the present value of surpluses determine ex-post inflation, the quantity $\delta_{t+1} = \pi_{t+1} - E_t \pi_{t+1}$ which indexed multiple equilibria in (5).

In this regime, the price level (not just inflation) is determinate, even with a constant interest rate target $i_t = i$. This regime also overturns the doctrine that interest rate targets lead to indeterminacy. (Leeper (1991), Sims (1994), Woodford (1995).)

Since it is free to set interest rates, the government can follow a Taylor rule. Thus, the empirical finding that a Taylor rule seems to fit well is not inconsistent with this theory, nor is the observation that central banks can and do set interest rates. A Taylor rule with $\phi > 1$ will generically lead to equilibria that are not locally bounded, unless fiscal shocks happen to select the new-Keynesian equilibrium. Thus, we obtain the usual doctrine following Leeper (1991) that “active” fiscal policy should be paired with “passive” ($\phi < 1$) monetary policy.

Without direct observations of δ_{t+1} , similar identification problems remain, discussed in Section 3.1 of the web Appendix. However, estimates of ϕ are not particularly important in this regime, as price level determinacy or the control of inflation do not hinge on ϕ . In fact, problems in measuring ϕ are to some extent welcome. They mean we do not have to take regression estimates as strong evidence for a troublesome structural $\phi > 1$ despite stable inflation. Non-Ricardian models can generate spurious $\phi > 1$ as easily as new-Keynesian models can do.

At a minimum, the fiscal regime offers a way to understand U.S. history in periods that even new-Keynesians believe are characterized by passive ($\phi < 1$) monetary policy. This offers an improvement over “indeterminacy” or “sunspots” which give few restrictions on the data. Woodford (2001) applies this regime to the Fed’s interest rate peg in the late 1940s and early 1950s. Applying it to the 1970s is an obvious possibility.

3.5 Ricardian asymmetry, asset prices, and observational equivalence.

Equations (21) and (22) also describe an equilibrium in which a variable, the price level, is a forward-looking expectation, and jumps to avoid an explosive root. Recall the evolution of government debt (20) as

$$\frac{B_t}{P_{t+1}} = (1 + r) \left(\frac{B_{t-1}}{P_t} - S_t \right). \quad (23)$$

Again, we have an unstable root. If P_t is too low, then the real value of government debt explodes. In response to a shock, P_t jumps to the unique value that prevents such an explosion.

How do I accept explosive solutions in the new-Keynesian model, while I deny them in the non-Ricardian regime? Why do I solve asset pricing equations $p_{t+1} = R_{t+1}p_t - d_{t+1}$ forward, but not $\pi_{t+1} = \phi\pi_t - x_t$? There is a fundamental difference. There is a transversality condition forcing the consumer to avoid *real* explosions, explosions of B_{t-1}/P_t or the real value of assets. There is no corresponding condition forcing anyone to avoid nominal explosions, explosions of P_t itself.

Correspondingly, there is an economic mechanism forcing (21) to hold in a non-Ricardian regime. If the price level is below the value specified by (21), nominal government bonds appear as net wealth to consumers. They will try to increase consumption. Collectively, they can’t do so, therefore this increase in “demand” will push prices back to the equilibrium level. Supply equals demand and consumer optimization are satisfied only at the unique equilibrium. Stock prices are pushed to the present value of dividends by the same mechanism.

There is no corresponding mechanism to push inflation to the new-Keynesian value (6). In the new-Keynesian model we are choosing *among* equilibria; supply equals demand and consumer optimization hold for any of the alternative paths, any choice of δ_{t+1} ; we’re finding the unique *locally bounded* equilibrium, not the unique equilibrium itself. The economy is supposed simply to

jump to the right equilibrium.

In asset pricing equations such as (23) and $p_{t+1} = R_{t+1}p_t - d_{t+1}$ we can also measure the “explosive eigenvalue,” the rate of return, despite the forward-looking solution. This occurs because we can measure the dividend directly. In a deep sense, the reason we can’t measure ϕ is because we have no independent measure of the monetary policy shock.

Alas, new-Keynesian and fiscal regimes are observationally equivalent without further assumptions that are well beyond the scope of this paper. All the equilibrium conditions hold in each case. We cannot test whether inflation occurred, and this caused the government to “passively” change taxes ex-post, or whether people knew that taxes were going to change, and the price level jumped in their expectation. Canzoneri, Cumby and Diba’s (2001) fiscal test has the same flaw as Clarida, Galí and Gertler’s (2000) monetary test, shown in Cochrane (1998).

Really, the regimes are not so distinct. From a fiscal point of view, a commodity standard is just a device to communicate and enforce a fiscal commitment. If the government will always buy and sell a commodity at a given price, it must adjust lump-sum taxes ex post to have sufficient stocks of the commodity on hand. One could say that the government valuation equation (21) “really” determines the price level, and the commodity standard with “Ricardian regime” just communicates the fiscal commitment. Commodity standards and pegs fall apart precisely when the underlying fiscal commitment is no longer credible. Similarly, if the new-Keynesian equilibrium selection were successful, one could say from a fiscal point of view that the government valuation equation (21) “really” determines the price level, with interest rate policy merely a way to communicate and enforce that fiscal commitment. Alas, however, the new-Keynesian interest rate regime does not select a unique equilibrium, and thus a unique fiscal commitment. So the bottom line is that one must find expectations of future surpluses in (21) somewhere else.

4 New-Keynesian Solutions

Of course, the New Keynesian literature is aware of these issues. How do they pick the locally-bounded solution Π^* and get rid of the other ones?

4.1 Reasonable expectations?

Much of the approach is simply to argue about what is reasonable. For example, Woodford (p.128) argues that

“The equilibrium $..[\Pi^*]..$ is nonetheless *locally* unique, which may be enough to allow expectations to coordinate upon that equilibrium rather than on one of the others.”

Similarly, King (2000, p. 58-59) writes

“By specifying [$\phi > 1$] then, the monetary authority would be saying, ‘if inflation deviates from the neutral level, then the nominal interest rate will be increased relative to the level which it would be at under a neutral monetary policy.’ If this statement is believed, then it may be enough to convince the private sector that the inflation and output will actually take on its neutral level.

Needless to say, these are rather weak arguments on which to hang the foundational question of macroeconomics. They are especially weak in ruling out equilibria between Π_L and Π^* . One might think that expectations of accelerating inflation are not reasonable, but if 2% inflation expectations are reasonable, is 1% – which sends us off on a spiral to Π_L – really so unreasonable?

Importantly for judging the reasonableness of alternative equilibria, Woodford argues that we should not think of an economy or Fed making a small “mistake” and therefore slipping from Π^* into an explosive equilibrium; instead we should think of expectations of future inflation driving inflation today: (p. 128)

Indeed it is often said that .. the steady state with inflation rate Π^* is “unstable” implying that an economy should be expected almost inevitably to experience either a self-fulfilling inflation or a self-fulfilling deflation under such a regime.

Such reasoning involves a serious misunderstanding of the causal logic of the difference equation [(18)]. This equation does not indicate how the equilibrium inflation rate in period $t + 1$ is determined by the inflation that happens to have occurred in the previous period. If it did it would be correct to call Π^* an unstable fixed point of the dynamics—even if that point were fortuitously reached, any small perturbation would result in divergence from it. But instead, the equation indicates how the equilibrium inflation rate in period t is determined by *expectations* regarding inflation in the following period... The equilibria that involve initial inflation rates near (but not equal to) Π^* can only occur as a result of expectations of *future* inflation rates (at least in some states) that are even *further* from the target inflation rate. Thus the economy can only move to one of these alternative paths if expectations about the future change significantly, something that one may suppose would not easily occur.”

A “serious misunderstanding of causal logic” is a strong charge, and I think unwarranted here. The equations of the model do not specify a causal ordering. They are just equilibrium conditions. And I think Woodford has slipped in a small misunderstanding of his own, by thinking of expectations that are formed exogenously, or in a prior causal chain. If you are observing an unstable dynamic system, and you see a small change today, the fact of that change causes a large change in your expectations of the future. If you see the waiter trip, it’s a good bet the stack of plates he is carrying will crash. In these models, agents might well see a disturbance, know the Fed will feed back on its past mistakes, think “oh no, here we go,” and radically change their expectations of the future. They don’t need to wake up and think “gee, I think there will be a hyperinflation” before reading the morning paper. The forward-looking solutions rely exactly on endogenous expectations: They rely on a fortuitous jump in near-term expectations in response to a shock, to put the economy back on the saddle path that has no change in asymptotic expectations.

Still, there is some appeal to the argument that expectations of hyperinflations or liquidity traps seem far-fetched. But expectations that are far-fetched in our intuitive understanding of our world are not necessarily so far-fetched for agents in this model, once we recognize that this model may not represent our world. *In this model*, the Fed is absolutely committed to raising interest rates more than one for one with inflation, forever, no matter what. In this model, real rates are constant, so the rise in nominal rates must correspond to a rise in inflation – precisely the opposite of the stabilizing language that pours out of the real-world Federal Reserve’s account of its actions. If we really lived in such a world, I, for one, would confidently expect hyperinflation. If we think that forecast is “unreasonable,” it means we don’t believe the model describes the world in which we live.

4.2 Solutions and dilemma

Recognizing, I think, the weakness of these arguments—if not, they would not need to go on—New-Keynesian theorists have explored a variety of ways to trim multiple equilibria. Alas, these fall in the logical conundrum explained in the introduction: To trim equilibria, we must assume that the government will blow up the world – to set policy in such a way that private first order conditions cannot hold – even though policies exist that would allow the government to stop inflation or deflation while letting the economy operate.

4.2.1 Stabilizations; shift to commodity standard or money growth rule

Woodford’s (2003) section 4.3 studies proposals to cut off inflationary equilibria to the right of Π^* . Woodford’s main suggestion is (p. 138):

...self-fulfilling inflations may be excluded through the addition of policy provisions that apply only in the case of hyperinflation. For example, Obstfeld and Rogoff (1986) propose that the central bank commit itself to peg the value of the monetary unit in terms of some real commodity by standing ready to exchange the commodity for money in event that the real value of the total money supply ever shrinks to a certain very low level. If it is assumed that this level of real balances is one that would never be reached except in the case of a self-fulfilling inflation, the commitment has no effect except to exclude such paths as possible equilibria. ...[This proposal could] well be added as a hyperinflation provision in a regime that otherwise follows a Taylor rule.

In real life governments often stop inflations by a firm peg to a foreign currency (with a fiscal reform, to make credible the Ricardian fiscal policy commitment), which is the modern equivalent of a commodity standard. Atkeson, Chari and Kehoe (2009) advocate a similar idea, but specify that the government switches to a money growth rule in a model with non-interest-elastic money demand. Switching to a non-Ricardian regime to enforce a fixed price level would have the same effect.

However, this quote and the surrounding discussion does not explain how *stabilizing an inflation* serves to *rule out an equilibrium path*. First-order conditions can hold throughout the inflation and stabilization, and then the path is not ruled out.

The answer is that each of these proposals implicitly or pairs the stabilization with another, superfluous, inconsistent policy specification, in such a way that equilibrium cannot form. *Inconsistent policy* rules out the equilibrium path, not inflation stabilization. The key assumption in Woodford’s quote is “otherwise follows a Taylor rule.” His government continues to follow the Taylor rule even after it has switched to a commodity standard! You can’t have two monetary policies at once; if you do, no equilibrium can form.

To be precise, suppose that at inflation past some level Π_U the government changes to a commodity standard (a peg), switches to a money growth rule with interest-inelastic demand, or switches a non-Ricardian regime. At date $T - 1$, $\Pi_{T-1} < \Pi_U$, so the consumer obeys his first order conditions, the Fed follows the Taylor rule, and equilibrium inflation still follows

$$\Pi_T = \beta(1 + i_{T-1}) = \Phi(\Pi_{T-1}).$$

Now, suppose $\Pi_T > \Pi_U$ so at date T , the government freezes this price level at P_T by one of the above policies, and $P_{T+1} = P_T$. Equilibrium at date T therefore requires $i_T = r$, from the

consumer's first order conditions

$$\Pi_{T+1} = 1 = \beta(1 + i_T).$$

The hyperinflation had ended, but this fact does not “exclude such paths as possible equilibria.” The key to “excluding equilibria” is that Woodford, Atkeson, Chari and Kehoe, etc., assume that the Fed *also* continues to follow the Taylor rule, $1 + i_T = (1 + r)\Phi(\Pi_T)$ which is a huge number, and inconsistent with $i_T = r$ demanded by first-order conditions.

We would normally say it's impossible to *both* run a commodity standard which requires $i_T = r$ *and* set interest rates to hyperinflationary levels which requires i_T to be a huge number. As Woodford explains, the government implements a commodity standard by “standing ready to exchange the commodity for money.” It can't both do that *and* control the quantity of money to follow an interest rate target. If the government really could commit to such a thing there would be “no equilibrium.” But does it really make any sense that the government would try to do such a thing, that people would believe that it would try to do such a thing, that a government even *can* do such a thing, and persist long enough to “rule out equilibrium,” whatever that means?

At a minimum, we see that *stabilizing accelerating inflation* has nothing to do with *ruling out the equilibrium path*. One period of inconsistent policy anywhere along the path would have been enough to accomplish the latter.

Atkeson, Chari and Kehoe (2009) recognize the problem, are explicit about their assumptions, and carefully set up policy so that equilibrium can form on every date past T . However, they also assume that the Taylor rule requiring high interest rates coexist for one period with a money growth rule that demands low interest rates, in order to rule out equilibrium. Blowing up the world for one period is enough. The web Appendix reviews their proposal in detail.

What about Obstfeld and Rogoff (1983) (1986) and the related large literature that tries to trim indeterminacies in models with fixed money supply and interest-elastic money demand? Didn't they solve all these problems years ago, as Woodford seems to suggest? Since it requires setting up a different model, I review Obstfeld and Rogoff in detail in the web Appendix. The story is the same, however. Obstfeld and Rogoff also specify that the government switches to a commodity standard at a very high level of inflation. This stops the inflation, but it allows an equilibrium at each date, so the accelerating-inflation equilibrium path is not ruled out. To prevent the formation of an equilibrium, Obstfeld and Rogoff *also* specify that the nominal money supply after the reform must be no higher than it was before the inflation started, or before the reform. They thus disallow the recovery in real money balances which accompanies the end of hyperinflations. Their government specifies *both* a low quantity M *and* a large price level. Again, however, what government would do this? *How* would a government even do this? How could a government freely trade currency for the commodity at a given price, *and* impose an upper limit on the money supply? I conclude that models with $MV(i) = PY$ and fixed M have exactly the same unsolved indeterminacies as the Taylor rule models.

4.3 Fiscal equilibrium-trimming

Benhabib, Schmitt-Grohé and Uribe (2002), mirrored in Woodford (2003) section 4.2, try to trim equilibria by adding fiscal commitments to the Taylor rule. Their ideas are aimed at trimming deflationary equilibria, but either set of ideas could apply to both inflations and deflations. They specify that in low-inflation states, the government will lower taxes so much that real debt grows explosively, the consumer's transversality condition is violated, and the government debt valuation

equation no longer holds. Ergo, this region and all the equilibria below Π^* in Figure 1 that lead to it are ruled out. Specifically, (their equations (18)-(20)) they specify that in a neighborhood of Π_L , the government will commit to surpluses $S_t = \alpha(\Pi_t) (B_{t-1}/P_t)$ with $\alpha(\Pi_L) < 0$ in place of (19).

They also show that the same result can be implemented by a target for the growth rate of nominal liabilities, a “4% rule” for nominal debt. If deflation breaks out with such a commitment, real debt will then explode; to keep nominal debt on target, the government would need to start borrowing and spending as above. Woodford suggests this implementation as well (p. 132): “let total nominal government liabilities D_t be specified to grow at a constant rate $\bar{\mu} > 1$ while monetary policy is described by the Taylor rule ...” “Thus, in the case of an appropriate fiscal policy rule, a deflationary trap is not a possible rational expectations equilibrium.”

As the above proposals are grounded in sensible policies to stabilize hyperinflations, these proposals sound like sensible and time-honored prescriptions to inflate the economy, i.e., to head back to the desired equilibrium Π^* . Benhabib, Schmitt-Grohé and Uribe describe them this way (p. 548):

Interestingly, this type of policy prescription is what the U.S. Treasury and a large number of academic and professional economists are advocating as a way for Japan to lift itself out of its current deflationary trap...A decline in taxes increases the household’s after-tax wealth, which induces an aggregate excess demand for goods. With aggregate supply fixed, price level must increase in order to reestablish equilibrium in the goods market.

(They didn’t know that zero interest rates and \$1.5 trillion deficits would so soon follow!) And this is, indeed, how a coordinated fiscal-dominant regime works, it is good intuition for operation of the fiscal theory of the price level, and undoubtedly what real-world proponents of these policies have in mind.

But that’s *not* their proposal. The proposal does *not* “lift the economy out of a deflationary trap” back to Π^* . Their proposal sits at Π_L with an uncoordinated policy and lets government debt explode. If their proposal *did* successfully steer the economy back to Π^* then the whole path to Π_L and back would have been an equilibrium. As in the last section, they change tax policy *while also maintaining the Taylor rule* $\Phi(\Pi)$ and the dynamics of Figure 1. In Woodford’s quote “*while monetary policy is described by the Taylor rule*” is the key. We are switching to a Ricardian regime, which demands higher inflation, while simultaneously keeping the Taylor rule in place, which demands continued low inflation. The transversality condition is a consumer first-order condition. We are setting policy parameters for which consumer first order conditions cannot hold.

Once we see that central point, we can think of many monetary-fiscal policies that preclude deflationary equilibria equivalently and more transparently. If inflation gets to an undesired level, tax everything. Burn the money stock. Introduce an arbitrage opportunity. Best of all, specify a $\Phi(\Pi)$ function that includes negative nominal interest rates – just eliminate the Π_L equilibrium in the first place! Bassetto (2004) suggests this option. Since there can be no equilibrium at negative nominal rates, such a $\Phi(\Pi)$ function works exactly the same way to rule out equilibria: In a deflationary state, the government commits to a policy that allows no equilibrium. Negative nominal rates are no more or less possible than letting debt explode, or running a commodity standard with high rates or low money stock. In retrospect, it doesn’t make sense to demand a Ramsey approach in setting up the problem – the Taylor rule must not prescribe negative nominal rates, because that

would violate first order conditions – and then patch it up with policy prescriptions that *do* violate first order conditions. Why not just commit to negative nominal rates in the first place?

It’s not hard to understand why the issue has become so confused. Benhabib, Schmitt-Grohé and Uribe, Woodford, and Obstfeld and Rogoff before them, did not follow my alternative suggestions – to specify policy paths that clearly and decisively forbid equilibrium. Instead, they thought about a very reasonable-sounding response to inflation or deflation, and then subtly (and doubtless unintentionally) snuck in an extra step that rules out equilibrium. It’s very easy to confuse “stopping an inflation” with “ruling out this equilibrium path.” The easy-to-miss little extra step matters, not the seductively sensible policy that gets us there.

4.3.1 Weird Taylor rules

Woodford starts “Policies to prevent an inflationary panic” by suggesting (p.136) a stronger Taylor rule, that the graph in Figure 1 becomes *vertical* at some finite inflation Π_U above Π^* , i.e. that the Fed will set an infinite interest rate target. Similarly, Alstadheim and Henderson (2006) remove the Π_L equilibrium by introducing discontinuous policy rules, or V-shaped rules that only touch the 45° line at the Π^* point. Bassetto (2004), mentioned above, likewise suggested that the Taylor rule ignore the $i \geq 0$ bound and promise negative nominal rates in a deflation.

These proposals blow up the economy directly. At one level, however, these proposals are not as extreme as they sound. After all, the Taylor principle in new-Keynesian models amounts to people believing unpleasant things about alternative equilibria. The more unpleasant the beliefs, the more effective at ruling out equilibria, so hyperinflating away the entire monetary system, introducing an arbitrage opportunity ($i < 0$), and so forth, certainly removes these equilibria, and perhaps more effectively than an inflation or fiscal imbalance that slowly gain steam.

On the other hand, the real-world economy would presumably substitute to other moneys or electronic barter of mutual fund shares before the value of money reached zero, making this hyperinflationary path possible, if admittedly unpleasant. And, while they are possible commitments one might ask a future Fed to make, none of these are vaguely plausible descriptions of current beliefs about Fed behavior, or current Fed statements.

4.3.2 Residual money demand; letting the economy blow up

Schmitt-Grohé and Uribe (2000) and Benhabib, Schmitt-Grohé and Uribe (2001) offer a similar way to rule out hyperinflations, without assuming the Fed directly blows up the economy with infinite interest rates, by adding a little money. This idea is also reviewed by Woodford (2003 p. 137), and has long roots in the literature on hyperinflations with fixed money supply and interest-elastic demand (Sims (1994)).

Schmitt-Grohé and Uribe’s idea is easiest to express with real balances in the utility function. With money, the Fisher equation contains monetary distortions:

$$1 + i_t = \Pi_{t+1} \frac{u_c(Y, M_t/P_t)}{\beta u_c(Y, M_{t+1}/P_{t+1})} = \Pi_{t+1}(1 + r_t), \quad (24)$$

where r_t denotes the real interest rate. (This is a perfect foresight model, so the expectation is missing.) Suppose the Taylor rule is

$$1 + i_t = \frac{1}{\beta} \Phi(\Pi_t).$$

Substituting i_t from the Taylor rule into (24), and rearranging the money vs. bonds first order condition as $M_t/P_t = L(Y, i_t)$, inflation dynamics follow

$$\Pi_{t+1} = \Phi(\Pi_t) \frac{u_c[Y, L(Y, \Phi(\Pi_{t+1}))]}{u_c[Y, L(Y, \Phi(\Pi_t))]} \quad (25)$$

instead of (18).

The idea, then, is that *this* difference equation may rise to require $\Pi_{t+1} = \infty$ above some bound Π_U , even if the Taylor rule for nominal interest rates $1 + i_t = \Phi(\Pi_t)/\beta$ remains bounded for all Π_t . Woodford and Schmitt-Grohé and Uribe give examples of specifications of $u(C, M/P)$ for which this can happen.

Is this the answer? First and most importantly, if we do not regard the a belief that the Fed will directly blow up the economy (i_t rises to ∞) as a reasonable characterization of expectations, why would people believe that the Fed will to take the economy to a state in which the economy blows up all on its own? Infinite inflation and finite interest rates mean infinitely negative real rates; a huge monetary distortion. Surely the Fed would notice that real interest rates are approaching negative infinity!

Second, it is delicate. In general, this approach relies on particular behavior of the utility function or the cash-credit goods specification at very low real balances. Are monetary frictions really important enough to rule out inflation above a certain limit, sending real rates to negative infinity, or to rule out deflation below another limit? We have seen some astounding hyperinflations; real rates did not seem all that affected.

Sims (1994) pursues a similar idea. Perhaps there is a lower limit on nominal money demand, so that real money demand explodes in a deflation. Perhaps not; perhaps the government can print any number it wants on bills, or will run periodic currency reforms; perhaps real money demand is finite for any price level.

In sum, these proposals require two things: First, they require expectations that the government will follow the Taylor rule to explosive hyperinflations and deflations, beyond anything ever observed, and despite the presence of equilibrium-preserving stabilization policies such as the switch to commodity standard, money growth, or non-Ricardian regime. Second, they require belief in a deep-seated monetary non-neutrality sufficient to send real rates to negative infinity or real money demand to infinity, though even the beginning of such events has never been observed. At a minimum, this sounds like a weak foundation for the founding question of macroeconomics.

5 Determinacy and identification in the three-equation model

One may well object at the whole idea of studying identification and determinacy in such a stripped-down model, with no monetary friction, no means by which the central bank can affect real rates, and a single disturbance. Typical verbal (old-Keynesian) explanations of Taylor rules and inflation, and typical Federal Open Market Committee statements, involve at least the Phillips curve and Fed control of real rates of interest: nominal rates rise, gaps appear, these gaps drive down inflation. You can't do that in a frictionless model. Empirical Taylor rule estimates are much more sophisticated than $i_t = \phi\pi_t + \varepsilon_t$ regressions. It turns out that the simple model does in fact capture the relevant issues, but one can only show that by examining "real" new-Keynesian models and regressions in detail and seeing that the same points and same logic emerge.

The excellent exposition in King (2000) makes the non-identification and determinacy theorems clear. The basic model is

$$y_t = E_t y_{t+1} - \sigma(r_t - r) + x_{dt} \quad (26)$$

$$i_t = r_t + E_t \pi_{t+1} \quad (27)$$

$$\pi_t = \beta E_t \pi_{t+1} + \gamma(y_t - \bar{y}_t) + x_{\pi t} \quad (28)$$

where y denotes output, r denotes the real interest rate, i denotes the nominal interest rate, π denotes inflation, \bar{y}_t is potential output, and the x are serially correlated structural disturbances. I use x not ε and the word “disturbance” rather than “shock” to remind us of that fact.

While seemingly ad-hoc, the point of the entire literature is that this structure has exquisite micro-foundations, which are summarized in King (2000), and Woodford (2003). The first two equations derive from consumer first order conditions for consumption today vs. consumption tomorrow. The last equation is the “new-Keynesian Phillips curve,” derived from the first order conditions of forward-looking optimizing firms that set prices subject to adjustment costs. There is an active debate on the right specification of (28), including additional inflation dynamics and the difference between output and marginal cost, but these differences do not affect my conclusions.

For both determinacy and identification questions, we can simplify the analysis by studying alternative equilibria as *deviations* from a given equilibrium, following King (2000). Use y_t^* etc. to denote equilibrium values. y_t^* is a stochastic process, i.e. a moving average representation $y_t^*(\{x_{dt}, x_{\pi t}, x_{it}, \dots\})$ or its equivalent. There are many such equilibria. For example, given any stochastic process for $\{y_t^*\}$ you can construct the corresponding $\{\pi_t^*\}$, $\{r_t^*\}$, $\{i_t^*\}$ from (28), (26), and (27) in order.

Use tildes to denote deviations of an alternative equilibrium y_t from the $*$ equilibrium, $\tilde{y}_t \equiv y_t - y_t^*$. Subtracting, deviations must follow the same model as (26)-(28), but without constants or disturbances

$$\tilde{i}_t = \tilde{r}_t + E_t \tilde{\pi}_{t+1} \quad (29)$$

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \sigma \tilde{r}_t \quad (30)$$

$$\tilde{\pi}_t = \beta E_t \tilde{\pi}_{t+1} + \gamma \tilde{y}_t. \quad (31)$$

5.1 Determinacy

Now, if the Fed sets $i_t = i_t^*$, i.e. $\tilde{i}_t = 0$, then $\tilde{\pi}_t = 0$, $\tilde{y}_t = 0$ are *an* equilibrium. But setting $i_t = i_t^*$ does not determine that this is the *only* equilibrium. To see this point, write (29)-(31) with $\tilde{i}_t = 0$ in a standard form as

$$\begin{bmatrix} E_t \tilde{y}_{t+1} \\ E_t \tilde{\pi}_{t+1} \end{bmatrix} = \frac{1}{\beta} \begin{bmatrix} \beta + \sigma\gamma & -\sigma \\ -\gamma & 1 \end{bmatrix} \begin{bmatrix} \tilde{y}_t \\ \tilde{\pi}_t \end{bmatrix}. \quad (32)$$

Since the model only restricts the dynamics of expected future output and inflation, we have multiple equilibria. Any

$$\begin{bmatrix} \tilde{y}_{t+1} \\ \tilde{\pi}_{t+1} \end{bmatrix} = \frac{1}{\beta} \begin{bmatrix} \beta + \sigma\gamma & -\sigma \\ -\gamma & 1 \end{bmatrix} \begin{bmatrix} \tilde{y}_t \\ \tilde{\pi}_t \end{bmatrix} + \begin{bmatrix} \delta_{y,t+1} \\ \delta_{\pi,t+1} \end{bmatrix} \quad (33)$$

with $E_t \delta_{y,t+1} = 0$, $E_t \delta_{\pi,t+1} = 0$ is valid, not just $\delta_{y,t+1} = \delta_{\pi,t+1} = 0$ and hence $\tilde{y}_t = \tilde{\pi}_t = 0$ for all t .

Perhaps however the dynamics of (32) are explosive, so at least $\tilde{y} = \tilde{\pi} = 0$ is the only locally-bounded equilibrium; the only one in which $E_t(\tilde{y}_{t+j})$ and $E_t(\tilde{\pi}_{t+j})$ stay near zero. Alas, this hope is dashed as well: One of the eigenvalues of the transition matrix in (32), derived below, is less than one. We have just verified in this model the usual doctrine that an interest rate peg does not determine inflation.

To determine output and the inflation rate, then, new-Keynesian modelers add to the specification $i_t = i_t^*$ of what interest rates will be in *this* equilibrium, a specification of what interest rates would be like in *other* equilibria, in order to rule them out. King (2000) specifies Taylor-type rules in the form

$$i_t = i_t^* + \phi_\pi (\pi_t - \pi_t^*) + \phi_y (y_t - y_t^*) \quad (34)$$

or, more simply,

$$\tilde{i}_t = \phi_\pi \tilde{\pi}_t + \phi_y \tilde{y}_t.$$

(Both King and the web Appendix allow responses to expected future inflation and output as well. The simpler version in (34) is enough for all my points however.)

For example, with $\phi_y = 0$ the deviations from the * equilibrium now follow

$$\begin{bmatrix} E_t \tilde{y}_{t+1} \\ E_t \tilde{\pi}_{t+1} \end{bmatrix} = \frac{1}{\beta} \begin{bmatrix} \beta + \sigma\gamma & -\sigma(1 - \beta\phi_\pi) \\ -\gamma & 1 \end{bmatrix} \begin{bmatrix} \tilde{y}_t \\ \tilde{\pi}_t \end{bmatrix}. \quad (35)$$

The eigenvalues of this transition matrix are

$$\lambda = \frac{1}{2\beta} \left((1 + \beta + \sigma\gamma) \pm \sqrt{(1 + \beta + \sigma\gamma)^2 - 4\beta(1 + \sigma\gamma\phi_\pi)} \right). \quad (36)$$

If we impose $\sigma\gamma > 0$, then both eigenvalues are greater than one in absolute value if

$$\phi_\pi > 1$$

or if

$$\phi_\pi < - \left(1 + 2 \frac{1 + \beta}{\sigma\gamma} \right). \quad (37)$$

Thus, if the policy rule is sufficiently “active,” any equilibrium other than $\tilde{i} = \tilde{y} = \tilde{\pi} = 0$ is explosive. Ruling out such explosions, we now have the unique locally-bounded equilibrium. (The web Appendix treats determinacy conditions with output responses and responses to expected future inflation and output.)

As in the simple model, the point of policy is to induce *explosive* dynamics, eigenvalues greater than one, not to “stabilize” so that the economy always reverts after shocks. As pointed out by King (2000), p. 78, the region of negative ϕ_π described by (37), which generates oscillating explosions, works as well as the conventional $\phi_\pi > 1$ to determine inflation.

The analysis so far has exactly mirrored my analysis of the simple model of Section 2. So, in fact, that model does capture the determinacy issues, despite its absence of any frictions. Conversely, determinacy in the new-Keynesian model does not fundamentally rely on frictions, or the Fed’s ability to control real rates, “demand,” etc.

As in the simple model, no economic consideration rules out the explosive solutions. One might complain that I have not shown the full, nonlinear model in this case, as I did for the frictionless model. This is a valid complaint, especially since output may also explode in the linearized non-local equilibria. I do not pursue this question here, as I find no claim in any new-Keynesian writing

that this route can rule out the non-local equilibria. Its determinacy literature is all carried out in simpler frameworks, as I have done. And there is no reason, really, to suspect that this route will work either. Sensible economic models work in hyperinflation or deflation. If they don't, it usually reveals something wrong with the model rather than the impossibility of inflation. In particular, while linearized Phillips-curve models can give large output effects of high inflations, we know that some of their simple abstractions, such as fixed intervals between price changes, are only useful approximations for low inflation. The Calvo fairy seems to visit more often in Argentina.

In one respect, this analysis is quite different from the simple model of Section 2. Determinacy is a property of the entire system, and depends on other parameters of the model, not just ϕ_π . Here, for $\sigma\gamma < 0$, there is a region with $\phi_\pi > 1$ in which both eigenvalues are *not* greater than 1, so we have indeterminacy despite an “active” Taylor rule. There is another region in which both eigenvalues *are* greater than one despite $0 < \phi_\pi < 1$, so we have local determinacy despite a “passive” Taylor rule. $\sigma\gamma < 0$ is not a plausible parameter configuration but as models become more complex, determinacy involves more parameters, and can often involve plausible values of those parameters. King (2000) and the web Appendix give some examples in this model, generalizing to Taylor rules that react to current and future output and inflation.

Fundamentally, the point of Fed policy is to perturb the dynamics so the whole *system* is unstable. The regions of determinacy are not as simple as $\phi_\pi > 1$, and testing for determinacy is really not as simple as testing the parameters of the Fed reaction function. Alas, noone has tried a test for determinacy in a more complex model.

King's expression of the Taylor rule (34) is particularly useful because it clearly separates “in-equilibrium” or “natural rate” i_t^* and “alternative-equilibrium” or determinacy $\phi_\pi(\pi_t - \pi_t^*)$ issues so neatly. One can read its instructions as: First, the Fed should set the interest rate to the “natural rate” i_t^* that appropriately reflects other shocks in the economy. Then, the Fed should react to inflation away from the desired equilibrium in order to induce local determinacy of that equilibrium. The two issues are completely separate.

For example, many theoretical treatments find that interest rates which move more than one for one with inflation are desirable, for reasons other than determinacy, or one may accept empirical evidence that they do so. But both of these observations are essentially observations that *equilibrium* interest rates i_t^* should, or do, vary more than one for one with *equilibrium* inflation, π_t^* . King's expression emphasizes that these observations tell us nothing, really, about determinacy issues, whether *deviations* from equilibrium should or do follow the same patterns. In particular, one might object on these grounds to a Non-Ricardian regime, which requires $\phi < 1$ to avoid explosions. But King's expression (34) shows us that a more than one-for-one relation between i_t^* and π_t^* is perfectly consistent with a less than one-for-one relationship $\phi < 1$ between deviations $(i - i^*)$ and $(\pi - \pi^*)$.

5.2 Identification

King's expression of the Taylor rule (34) makes the central identification point very clear. In the * equilibrium, we will always see $\pi_t - \pi_t^* = 0$ and $y_t - y_t^* = 0$. Thus, a regression estimate of (34) cannot possibly estimate ϕ_π , ϕ_y . There is no movement in the necessary right hand variables. More generally, ϕ_π and ϕ_y appear nowhere in the equilibrium dynamics characterized simply by $\tilde{\pi} = \tilde{y} = \tilde{i} = 0$, so they are not identified. Taylor determinacy depends entirely on what the Fed would do *away* from the * equilibrium, which we can never see from data *in* that equilibrium.

King recognizes the issue, in footnote 41:

“The specification of this rule leads to a subtle shift in the interpretation of the policy parameters $[\phi_\pi, \phi_y]$; these involve specifying how the monetary authority will respond to deviations of inflation from target. But if these parameters are chosen so that there is a unique equilibrium, then no deviations of inflation will ever occur.”

He does not address its implications for empirical work.

This issue is not particular to the details of the three equation model. In the general solution method for these sorts of models, we set to zero movements of the linear combinations of variables that correspond to unstable eigenvalues. As a result, we cannot measure those unstable eigenvalues. The web Appendix makes this point with equations.

So, what assumptions do people make to escape this deep problem? The prototype theoretical Taylor rule (34), repeated here,

$$i_t = i_t^* + \phi_\pi (\pi_t - \pi_t^*) + \phi_y (y_t - y_t^*), \quad (38)$$

describes how the central bank would react to potential *deviations* from the equilibrium π^*, y^* , in order to make y_t^* and π_t^* the unique locally-bounded equilibrium. To identify ϕ_π, ϕ_y , then, we have to make two assumptions:

Assumption 1: The Fed’s reaction ϕ_π, ϕ_y to a *deviation* of inflation π_t and output y_t from the desired-equilibrium value π_t^* and y_t^* is the same as the relation *between equilibrium* interest rates i_t^* and equilibrium inflation π_t^* and y_t^* ; we must assume that the ϕ in (38) are the same as the ϕ in a relation such as

$$i_t^* = \phi_\pi \pi_t^* + \phi_y y_t^* + \dots + x_{it} \quad (39)$$

where x_{it} denotes a residual combination of shocks with sufficient orthogonality properties to allow some estimation. (Of course, leads and lags and other variables may appear in both (38) and (39).)

Making this assumption is (for once) relatively uncontroversial, since there are no obvious observations one could make to refute it. Still, it’s worth making the assumption explicit and at least worth reading Fed statements to see if they support it.

The key question is whether we are *able* to make Assumption 1. Doing so requires restrictions on the model and equilibrium. The equilibrium quantities i_t^*, y_t^*, π_t^* , are functions of shocks, $i_t^*(\{x_{dt}, x_{\pi t}, x_{it}, \dots\})$, “moving averages.” To be able to make assumption 1, we need a second assumption:

Assumption 2: The model and Fed’s choice of equilibrium (different i^* imply different y^*, π^* for a given model) must be such that equilibrium quantities can be expressed in the “autoregressive” representation (39), with ϕ in the zone of determinacy, and with the error x_{it} orthogonal to something we can use as instruments.

Many models and many equilibria of a given model do not have this property. As an example, consider the failure of Section 2. The equilibrium there is, in response-to-shock form,

$$\begin{aligned} \pi_t^* &= kx_t \\ i_t^* &= r + \rho kx_t \end{aligned}$$

where k is a constant. We can express the interest rate equation as a relationship among endogenous variables,

$$i_t^* = r + \rho \pi_t^*.$$

However, $\|\rho\| < 1$, so we cannot use this relationship among endogenous variables as a Taylor rule for interest rate policy. This example violates the qualification “with ϕ in the zone of determinacy.” If we try to express this equilibrium as a rule with larger ϕ_π ,

$$i_t^* = r + \phi_\pi \pi_t^* + (\rho - \phi_\pi) k x_t$$

we obtain an “error term” – the x_{it} in (39) – that is hopelessly correlated with the right hand variables π_t^* , and all instruments, exactly the point of Section 2. This relationship violates the qualification on the error term in Assumption 2.

Even “ ϕ in the zone of determinacy” is really too loose. For example, suppose the correlations between variables in an equilibrium require $\phi_\pi = 10$. This equilibrium can be supported by $\phi_\pi = 10$, but it also can be supported by a more sensible $\phi_\pi = 1.5$. We *can* assume they are the same, identifying $\phi_\pi = 10$, but maybe we would not want to make the basic assumption in this case.

The no-gap equilibrium is a particularly good example in the three-equation context, since minimizing output gaps is a natural objective for monetary policy. To see this result most simply, suppose all the shocks follow $AR(1)$ processes $x_{jt} = \rho_j x_{jt-1} + \varepsilon_{jt}$. Then, substituting $y_t^* = \bar{y}_t$ in (26)-(28), the no-gap equilibrium is

$$\pi_t^* = \frac{1}{1 - \beta \rho_\pi} x_{\pi t} \tag{40}$$

$$i_t^* = r - \frac{1 - \rho_{\bar{y}}}{\sigma} \bar{y}_t + \frac{\rho_\pi}{1 - \beta \rho_\pi} x_{\pi t} + \frac{1}{\sigma} x_{dt}. \tag{41}$$

I.e., if the Fed sets interest rates to this i_t^* , equilibrium output can always equal potential output. However, there is no way for the Fed to implement this policy and attain this equilibrium with a rule that does not depend explicitly on shocks, and thus with an error term that is uncorrelated with available instruments. We could try to substitute endogenous variables for shocks in (41) as far as

$$i_t^* = r - \frac{1 - \rho_{\bar{y}}}{\sigma} y_t^* + \rho_\pi \pi_t^* + \frac{1}{\sigma} x_{dt},$$

and implement i_t^* as a Taylor rule with $\phi_\pi = \rho_\pi$ and $\phi_y = -(1 - \rho_{\bar{y}}) / \sigma$. However, x_{dt} remains in the rule, and since it is not spanned by y_t^* and π_t^* , there is no way to remove it. x_{dt} thus must become part of the monetary policy disturbance. With no reason to rule out correlation between the disturbances x_{dt} and $x_{\pi t}$, \bar{y}_t , nor any reason to limit serial correlation of x_{dt} , we do not have any instruments.

But even this much is false progress. The coefficient $\rho_\pi < 1$ so these attempted values of ϕ_y and ϕ_π lie outside the zone of determinacy. To try implement i_t^* as a Taylor rule with coefficients in the zone of determinacy, we have to strengthen the inflation response in an almost silly way,

$$i_t^* = r - \left(\frac{1 - \rho_{\bar{y}}}{\sigma} \right) y_t^* + \phi_\pi \pi_t^* + \left\{ (\rho_\pi - \phi_\pi) \pi_t^* + \frac{1}{\sigma} x_{dt} \right\}. \tag{42}$$

The term in brackets is the new monetary policy disturbance. The right hand variable is now hopelessly correlated with the error term. (The web Appendix shows the same result directly and more generally: assuming a Taylor rule without shocks, you can't produce the no-gap equilibrium with finite coefficients.) Here, the attempt to equate the correlation between i_t^* and π_t^* in the no-gap equilibrium with the Fed's response to *alternative* equilibria must fail.

Stochastic Intercept

The term in brackets in (42) is often called a “stochastic intercept.” In order to attain the no-gap equilibrium in this model, the central bank must follow a policy in which the interest rate reacts directly to some of the structural shocks of the economy, as well as reacting to output and inflation; the Taylor rule must have an intercept that varies over time as well as output and inflation responses. Since it helps to produce equilibria with less output and inflation variability, or other desirable properties, the stochastic intercept is a crucial part of New-Keynesian policy advice. Woodford (2003) for example argues for “Wicksellian” policy in which the interest rate target varies following the “natural” rate of interest, determined by real disturbances to the economy, and *then* varies interest rates with inflation and output so as to produce local uniqueness. King’s (2000) expression (34) offers the clearest separation between “natural rate” and “determinacy” roles.

Given this fact, it is a substantial restriction to omit the intercept from empirical work, and from policy discussion surrounding empirical work. For example, Clarida, Galí and Gertler (2000) and Woodford (2003, Ch. 4) calculate the variance of output and inflation using rules with no intercepts, and discuss the merits of larger ϕ for reducing such variance. Yet all the time equilibria with zero variance of output or inflation are sitting under our noses, as in the no-gap equilibrium, if only we will allow the policy rule to depend on disturbances directly.

The stochastic intercept of theory is often left out of empirical work because it becomes part of the monetary policy disturbance in that context. It is inextricably correlated with the other structural shocks of the model, and hence with the endogenous variables which depend on other shocks of the model. Things were bad enough with genuine monetary policy disturbances – an x_{it} unrelated to other shocks of the model – because the new-Keynesian model predicts that right hand variables should jump when there are shocks to the disturbance, as highlighted in Section 2. The stochastic intercept makes things even worse, because theory then predicts strong correlation between the composite monetary policy disturbance and other shocks, and other endogenous variables which depend on those shocks.

When one assumes away the stochastic intercept – or, equivalently, assumes that the monetary policy disturbance is uncorrelated with other variables – that assumption is really a restriction on the set of equilibrium paths the economy is following, and it is an assumption on Fed policy that it does not pick, by interest rate policy i_t^* , any of those equilibria. Many equilibria are left out, including the one with no gaps.

This discussion reinforces two general principles: First, don’t take error term properties lightly. As Sims (1980) emphasizes, linear models are composed of identical-looking equations, distinguished only by exclusion restrictions and error-orthogonality properties. The IS curve

$$y_t = E_t y_{t+1} - \sigma (i_t - r - E_t \pi_{t+1}) + x_{dt},$$

after all, can be rearranged to read

$$i_t = r + E_t \pi_{t+1} + \frac{1}{\sigma} (E_t y_{t+1} - y_t) + \frac{1}{\sigma} x_{dt}.$$

If we regress interest rates on output and inflation, how do we know that we are recovering the Fed’s policy response, and not the parameters of the consumer’s IS function? Only the orthogonality of the shocks (x_{dt}) with instruments distinguishes the two equations.

Second, orthogonality is a property of the *model* and a property of the *right hand variables* not really a property of the errors. You really have to write down a full model to understand why the

endogenous right hand variables or instruments would not respond to the shocks in the monetary policy disturbance.

With this background of possibilities and implicit assumptions, I can review the explicit assumptions in classic estimates.

5.3 Clarida, Galí and Gertler

Clarida Galí and Gertler (2000) specify an empirical policy rule in partial adjustment form (in my notation),

$$i_t = (1 - \rho_1 - \rho_2) \{r + (\phi_\pi - 1) [E_t(\pi_{t+1}) - \pi] + \phi_y E_t[\Delta y_{t+1} - \Delta \bar{y}_{t+1}]\} + \rho_1 i_{t-1} + \rho_2 i_{t-2} \quad (43)$$

where

$$\begin{aligned} \pi &= \text{inflation target, estimated} \\ \Delta y_{t+1} - \Delta \bar{y}_{t+1} &= \text{growth in output gap} \\ r &= \text{“long run equilibrium real rate”, estimated} \end{aligned}$$

(See their (4) p. 153 and Table II p. 157.) What are the important identification assumptions?

First, *there is no error term*, no monetary policy disturbance *at all*. The central problem of my simple example is that any monetary policy disturbance is correlated with right hand variables, since the latter must jump endogenously when there is a monetary policy disturbance. Clarida, Galí, and Gertler simply assume this problem away. They also assume away the stochastic intercept, the component of the monetary policy disturbance that reflects adaptation to other shocks in the economy.

A regression error term appears when Clarida, Galí, and Gertler replace expected inflation and output with their ex-post realized values, writing

$$i_t = (1 - \rho_1 - \rho_2) \{r + (\phi_\pi - 1) [\pi_{t+1} - \pi] + \phi_y \Delta y_{t+1}\} + \rho_1 i_{t-1} + \rho_2 i_{t-2} + \varepsilon_{t+1}. \quad (44)$$

In this way, Clarida, Galí, and Gertler avoid the 100% R^2 prediction which normally results from assuming away a regression disturbance. The remaining error ε_{t+1} is a pure forecast error, so it is serially uncorrelated. This fact allows Clarida, Galí and Gertler to use variables observed at time t as instruments to remove correlation between the forecast error ε_{t+1} and the ex-post values of the right hand variables π_{t+1} and Δy_{t+1} . Validity for this purpose does not mean such instruments would be valid if we were to recognize a genuine monetary policy disturbance.

Clarida, Galí and Gertler (1998) consider a slightly more general specification that does include a monetary policy disturbance.¹ In this case, they specify (their equation 2.5, my notation)

$$i_t = \rho i_{t-1} + (1 - \rho) [\alpha + \phi_\pi E(\pi_{t,t+n}|\Omega_t) + \phi_y E(y_t - \bar{y}_t|\Omega_t)] + v_t \quad (45)$$

where \bar{y} denotes potential output, separately measured, and Ω_t is the central bank’s information set at time t , which they assume does not include current output y_t . v_t is now the monetary policy disturbance, defined as “an exogenous random shock to the interest rate.” They add, “Importantly, we assume that v_t is i.i.d.” They estimate (45) by instrumental variables, using lagged output, inflation, interest rates, and commodity prices as instruments.

¹Curiously, Clarida Gali and Gertler (2000) mention the disturbance v_t below their equation (3), p. 153, but it does not appear in the equations or the following discussion. I presume the mention of v_t is a typo in the 2000 paper.

Obviously, the assumption of an i.i.d. disturbance is key, and restrictive. Many commentators accuse the Fed of deviating from the Taylor rule for years at a time in the mid 2000s, for example. This assumption means that any other shocks in the monetary policy disturbance – stochastic intercepts, variation in the “natural rate,” – are also i.i.d. There is no reason preference shifts (IS curve) or marginal cost shocks (Phillips curve shifts) should be i.i.d. But *some* other shock must not be i.i.d., so that there is persistent variation in the right hand variables. Therefore, the monetary policy disturbance must not include a “stochastic intercept” that responds to the non-i.i.d. shocks.

5.4 Giannoni, Rotemberg, Woodford

Rotemberg and Woodford (1997, 1998, 1999), followed by Giannoni and Woodford (2005) (see also the summary in Woodford 2003, Ch. 5) follow a different identification strategy, which allows them to estimate the parameters of the Taylor rule by OLS rather than IV regressions. Giannoni and Woodford (2005 p. 36-37) write the form of the Taylor rule in these papers:

We assume that the recent U.S. monetary policy can be described by the following feedback rule for the Federal funds rate

$$i_t = \bar{i} + \sum_{k=1}^n \phi_{ik}(i_{t-k} - \bar{i}) + \sum_{k=0}^{n_w} \phi_{wk} \hat{w}_{t-k} + \sum_{k=0}^{n_w} \phi_{\pi k} (\pi_{t-k} - \bar{\pi}) + \sum_{k=0}^{n_y} \phi_{yk} \hat{Y}_{t-k} + \varepsilon_t \quad (46)$$

where i_t is the Federal funds rate in period t ; π_t denotes the rate of inflation between periods $t - 1$ and t ; \hat{w}_t is the deviation of the log real wage from trend at date t , \hat{Y}_t is the deviation of log real GDP from trend, \bar{i} and $\bar{\pi}$ are long-run average values of the respective variables. The disturbances ε_t represent monetary policy “shocks” and are assumed to be serially uncorrelated. ...To identify the monetary policy shocks and estimate the coefficients in [(46)], we assume ... that a monetary policy shock at date t has no effect on inflation, output or the real wage in that period. It follows that [(46)] can be estimated by OLS...(p.36-37)

Since they lay out so many of the assumptions that identify this policy rule with such clarity, we can easily examine their plausibility. First, they assume that the monetary policy disturbance ε_t is i.i.d. – uncorrelated with lags of itself and past values of the right hand variables. This is again a strong assumption, given that ε_t is not a forecast error, but instead represents the natural rate or stochastic intercept, responding to the “natural rate” or structural disturbances in the economy, preferences of the Fed, and other endogenous variables that the Fed may respond to.

Second, they assume that the disturbance ε_t is also not correlated with *contemporaneous* values of \hat{w}_t , π_t and \hat{Y}_t . This is an especially surprising result of a new-Keynesian model, because \hat{w}_t , π_t and \hat{Y}_t are endogenous variables. From the very simplest model in this paper, endogenous variables have jumped in the new-Keynesian equilibrium when there is a monetary policy (or any other) disturbance. The whole *point* of new-Keynesian, forward-looking solutions is that endogenous variables jump, so that disturbances do not lead to accelerating inflation. How can \hat{w}_t , π_t and \hat{Y}_t *not* jump when there is a shock ε_t ? To achieve this result, Giannoni, Rotemberg and Woodford assume as part of their economic model that \hat{w}_t , π_t and \hat{Y}_t must be predetermined by at least one quarter, so they cannot move when ε_t moves. (In the model as described in their Technical Appendix, output \hat{Y} is actually fixed two quarters in advance, and the marginal utility of consumption μ_t is also

fixed one quarter in advance.) On the one hand, it is wonderful that Giannoni, Rotemberg and Woodford explain the properties of the *model* which generate the needed correlation properties of the *instruments*. But needless to say, these are strong assumptions. Are wages, prices, output and marginal utility really fixed one to two quarters in advance in our economy, and therefore unable to react within the quarter to monetary policy disturbances? They certainly aren't forecastable one to two quarters in advance!

Most of all, if $\hat{w}_t, \pi_t, \hat{Y}_t$ do not jump when there is a monetary policy disturbance, *something else must jump*, to head off the explosive equilibria. What jumps in this model are expectations of future values of these variables, among others $\hat{w}_{t+1} = E_t \hat{w}_{t+1}, \pi_{t+1} = E_t \pi_{t+1}$, and $\hat{Y}_{t+2} = E_t \hat{Y}_{t+2}$ as well as the state variable $E_t \mu_{t+1}$, the marginal utility of consumption. All of these variables are determined at date t . Now, we see another implicit assumption in the policy function (46) – none of these future variables are present in the policy rule. Thus, Giannoni, Rotemberg, and Woodford achieve identification by a classic exclusion restriction. In contrast to the literature that argues for the empirical necessity and theoretical desirability of Taylor rules that react to expected future output and inflation, and to other variables that the central bank can observe, those reactions are assumed to be absent here.

In sum, Giannoni and Woodford identify the Taylor rule in their model, by two assumptions about Fed behavior: 1) The disturbance, including “natural rate” “stochastic intercept” reactions to other shocks, is not predictable by any variables at time $t - 1$, and 2) The Fed does not react to expected future output, or wage, price inflation, or other state variables. There is an assumption about the economy: 3) Wages, prices, and output are fixed a period in advance.

6 Old-Keynesian models

Determinacy and identification are properties of specific models, not general properties of variables and parameters. Old-Keynesian models reverse many of the determinacy and identification propositions. In these models, an inflation coefficient greater than one is the key for *stable* dynamics, to produce system eigenvalues *less* than one, and to solve the model *backward*. Since the model does not have expected future terms, such a backward solution gives determinacy. The policy rules are identified, at least up to the usual (Sims 1980) issues with simultaneous-equation macro models. I think much of the determinacy and identification confusion stems from misunderstanding the profound differences between new-Keynesian and old-Keynesian models. Alas, the old-Keynesian models lack economic foundations, so can't be a serious competitor for the basic question I started with: What, fundamentally, determines the price level or inflation rate?

Taylor (1999) gives us a nice explicit example of a “old-Keynesian” model (my terminology) which forms a good basis for explicit discussion of these points. (As everywhere else, this is just a good example, not a critique of a specific paper; thousands of papers adopt “old-Keynesian” models.) Taylor adopts a “simple model” (p. 662, in my notation)

$$y_t = -\sigma(i_t - \pi_t - r) + u_t \quad (47)$$

$$\pi_t = \pi_{t-1} + \gamma y_{t-1} + e_t \quad (48)$$

$$i_t = r + \phi_\pi \pi_t + \phi_y y_t. \quad (49)$$

We see a striking difference – *all the forward-looking terms are absent*.

Taylor states (p. 663) that “it is crucial to have the interest rate response coefficient on the inflation rate ... above a critical ‘stability threshold’ of one,” (p. 664)

The case on the left [$\phi_\pi > 1$] is the stable case...The case on the right [$\phi_\pi < 1$] is unstable... This relationship between the stability of inflation and the size of the interest rate coefficient in the policy rule is a basic prediction of monetary models used for policy evaluation research. In fact, because many models are dynamically unstable when ϕ_π is less than one... the simulations of the models usually assume that ϕ_π is greater than one.

This is exactly the *opposite* philosophy from the new-Keynesian models. In new-Keynesian models, $\phi_\pi > 1$ is the condition for a “dynamically *unstable*” model. New-Keynesian models want unstable dynamics, in order to rule out multiple equilibria and force forward-looking solutions. In Taylor’s model, $\phi_\pi > 1$ is the condition for *stable* dynamics, eigenvalues *less than* one, in which we solve for endogenous variables (including inflation) by *backward-looking* solutions. “ $\phi_\pi > 1$ ” sounds superficially similar, but in fact its operation is diametrically the opposite.

A little more formally, and to parallel the analysis of the new-Keynesian model following (26)-(28), the standard form of Taylor’s model is

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} \sigma\gamma \frac{1-\phi_\pi}{1+\sigma\phi_y} & \sigma \frac{1-\phi_\pi}{1+\sigma\phi_y} \\ \gamma & 1 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \end{bmatrix} + \begin{bmatrix} \frac{1}{1+\sigma\phi_y} & \sigma \frac{1-\phi_\pi}{1+\sigma\phi_y} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_t \\ e_t \end{bmatrix}. \quad (50)$$

(Substitute (49) into (47).) The eigenvalues of this transition matrix are

$$\lambda_1 = 1 + \sigma\gamma \frac{1 - \phi_\pi}{1 + \sigma\phi_y}; \quad \lambda_2 = 0.$$

Therefore, $\phi_\pi > 1$ (with the natural restrictions $\phi_y > -1/\sigma$, $\sigma\gamma > 0$) generates values of the first eigenvalue *less* than one. Following the usual decomposition, we can then write the unique solution of the model as a *backward-looking* average of its shocks,

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \frac{1}{1 + \sigma\phi_y + \sigma\gamma(1 - \phi_\pi)} \begin{bmatrix} \lambda_1 - 1 & \lambda_1 \\ 1 + \sigma\phi_y & 0 \end{bmatrix} \sum_{j=0}^{\infty} \lambda_1^j \begin{bmatrix} u_{t-j} \\ e_{t-j} \end{bmatrix}.$$

There is no multiple-equilibrium or indeterminacy issue in this backward-looking solution.

More intuitively, take $\phi_y = 0$, and assume $\phi_\pi > 1$. Then if inflation π_t rises in the policy rule (49), the Fed ends up raising the real rate (as defined here without forward-looking terms) $i_t - \pi_t$. In the IS curve (47) this lowers output y_t and lower output in the Phillips curve (48) lowers $E_t(\pi_{t+1})$. This model thus embodies the classic concept of “stabilization” that more inflation makes the Fed raise real interest rates which lowers demand and lowers future inflation. This is exactly the opposite of the new-Keynesian dynamics. In the New-Keynesian model, a rise in inflation π_t leads to an explosion; “stabilization” by $\phi_\pi > 1$ means we count on π_t to have jumped to the unique value that heads off such explosions.

Why do the two models disagree so much on the desired kind of dynamics? The equations of Taylor’s model have no expected future terms. Hence, there are no expectational errors. All the shocks (u_t, e_t) driving the system are exogenous economic disturbances. By contrast, the new-Keynesian model has expected future values in its “structural” equations (26)-(28), so the shocks in its standard representation such as (33) contain expectational errors. To avoid multiple equilibria, we have to change Fed behavior to induce unstable dynamics, and then solve forward to remove these expectational errors as shocks to the economy. The difference isn’t happenstance; the whole

point of the New-Keynesian enterprise is to microfound behavioral relationships, and microfounded behavior is driven by expectations of the future, not memory of the past.

Taylor regards this model as a “reduced form” in which expectations have been “solved out,” so that parameters γ, σ, r may change if ϕ changes. He claims that nonetheless “these equations summarize more complex forward-looking models” (p. 662). I do not think this is true. Taylor’s model is fundamentally different, not a simpler “reduced form,” or rough guide to give intuition formalized by a more complex new-Keynesian effort. The difference between this model and the new-Keynesian model (26)-(28) is not about “policy invariance.” We want to analyze dynamics for *given* policy parameters ϕ_π, ϕ_y . Even if γ, σ and r change with different ϕ_π, ϕ_y , they are constant for a given ϕ_π, ϕ_y . Equations (47)-(49) are not a “simpler” or “reduced form” version of (26)-(28). They are the same equations – with the same algebraic complexity – with different t subscripts. Different t subscripts dramatically change dynamics, including stability and determinacy. The operation of the models is completely different. The response to shocks in an old-Keynesian model represents the means by which structural equations are brought back to balance. The response to shocks of a new-Keynesian model represents a jump to a different equilibrium, a choice among many different possibilities, in each of which the structural equations are all in balance. King (2000) p. 72 also details a number of fundamental differences between “new” and “old” Keynesian models of this sort.

New-Keynesian models are often described with old-Keynesian intuition, or old-Keynesian intuition is used to explain quantitative results from new-Keynesian models. This is a mistake. There is a deep tension between how many economists write about inflation determination and how their new-Keynesian models actually work.

Identification in Taylor’s model does not suffer the central problem of identification in new-Keynesian models. The behavior we are assessing is not how the Fed would respond to the emergence of alternative equilibrium paths, it is completely revealed by the Fed’s behavior in equilibrium. The parameters ϕ_π, ϕ_y appear in the equilibrium dynamics (50) and hence the likelihood function. That doesn’t mean identification is easy; it means we “only” have to face the standard issues in simultaneous-equation models as reviewed by Sims (1980) and studied extensively by the VAR literature since then.

Since it easily delivers a unique equilibrium, and thus inflation determinacy, why not conclude that Taylor’s model is the right one to use? Alas, our quest is for *economic* models of price determinacy. This model fails on the crucial qualification – as Taylor’s (p. 662) discussion makes very clear. If in fact inflation has nothing to do with expected future inflation, so inflation is mechanistically *caused* by output gaps which are directly under the Fed’s control; if in fact the output gap driving inflation has nothing to do with expected future output, then, yes, the Taylor rule does lead to inflation determinacy. But despite a half-century of looking for them, *economic* models do not deliver the “if” part of these statements. If we follow this model, we are giving up on an *economic* understanding of price-level determination, in favor of (at best) a mechanistic description.

7 Extensions and responses

The web Appendix contains an extensive critical review of the literature, responses to many objections, and extensions left out of the text for reasons of space. If you want to know “What about x’s approach to determinacy or identification?” you are likely to find an answer there.

I investigate identification in the other equilibria of the simple model. For $\|\phi\| < 1$, ϕ is not identified in any equilibrium. For $\|\phi\| > 1$, however, you can identify ϕ for every equilibrium *except* the new-Keynesian local equilibrium. If explosions occur, you can measure their rate. I explore the impulse response functions of the simple model, and contrast new-Keynesian and non-Ricardian choices in terms of impulse-response functions. I generalize the simple mode to include an “IS” shock in (1). In this case, we can’t even estimate ρ .

I address the question, what happens if you run Taylor rule regressions in artificial data from new-Keynesian models? This discussion generalizes the finding in section 2, in which regressions recovered the shock autocorrelation process rather than the Taylor rule parameter, to the three-equation model. Unsurprisingly, Taylor-rule regressions do not recover Taylor-rule parameters in artificial data from typical models.

One may ask, “well, if not a change in the Taylor rule, what did Clarida Galí and Gertler measure?” The right answer is really “it doesn’t matter” – once a coefficient loses its structural interpretation, who cares how it comes out? At a minimum, the right answer is “you need a different model to interpret the coefficient.” However, the web Appendix gives an example of how other changes in behavior can show up as spurious Taylor rule changes. In the example, the Taylor rule coefficient is constant at $\phi = 1.1$, but the Fed gets better at offsetting IS shocks, i.e. following better the “natural rate.” This change in policy causes mis-measured Taylor rule coefficients to rise as they do in the data.

An obvious question is whether full likelihood approaches, involving dynamics of the entire model, might be able to identify parameters where the single-equation methods I surveyed here are faltering. Equivalently, perhaps the impulse response function to *other* shocks can identify the Taylor-rule parameters (or, more generally, system eigenvalues). I survey these issues in the web Appendix. While identification in full systems has been studied and criticized, nobody has tried to use full-system methods to test for determinacy. This literature *imposes* determinacy and explores model specification to better fit second moments. This is not a criticism; “fitting the data” rather than “testing the model” is a worthy goal. But it means I have no useful results to report or literature to review on whether this approach can overcome identification problems in order to test for determinacy.

I explore leads and lags in Taylor rules in the context of the simple frictionless model, continuous time models, and the three-equation model. It turns out that determinacy questions depend quite sensitively on the timing assumptions in the Taylor rule. The problem is particularly evident on taking the continuous-time limit. Given that changing a time index, e.g. $E_t\pi_{t+1}$ in place of π_{t-1} , can reverse stability properties, this finding is not surprising, but it does counter the impression that stability is “robust” to changes in specification.

A separate web Technical Appendix collects documentation and calculation details. I explain budget constraints from Section 3.1 in more depth. I also collect analytic solutions to the standard three-equation model.

8 Conclusions and implications

Determinacy

Practically all *verbal* explanations for the wisdom of the Taylor principle – the Fed should increase interest rates more than one-for-one with inflation – use old-Keynesian, stabilizing, logic:

This action will raise real interest rates, which will dampen demand, which will lower future inflation. New-Keynesian *models* operate in an entirely different manner: by raising interest rates in response to inflation, the Fed induces accelerating inflation or deflation, or at a minimum a large “non-local” movement, unless inflation jumps to one particular value.

Alas, there is no economic reason why the economy should pick this unique initial value, as inflation and deflation are valid economic equilibria. No supply/demand force acts to move inflation to this value. The attempts to rule out multiple equilibria basically state that the government will blow up the economy should accelerating inflation or deflation occur. This is not a reasonable characterization of anyone’s expectations. Such policies also violate the usual criterion that the government must operate in markets just like agents. I conclude that inflation is just as indeterminate, in microfounded new-Keynesian models, when the central bank follows a Taylor rule with a Ricardian fiscal regime, as it is under fixed interest rate targets.

The literature – understandably, I think – confused “stopping an inflation” with “ruling out an equilibrium path.” Alas, now that confusion is lifted, we can see the latter goal is not achieved.

Identification

The central empirical success of New Keynesian models are estimates such as Clarida, Galí and Gertler’s (2000) that say inflation was stabilized in the U.S. by a switch from an “indeterminate” to a “determinate” regime. My main point is that the crucial Taylor-rule parameter is not identified in the new-Keynesian model, so we cannot interpret regressions in this way. The new-Keynesian model has nothing to say about inflation in an indeterminate regime, so Taylor-rule regressions in the 1970s are doubly uninterpretable in the new-Keynesian context.

Clarida, Galí, and Gertler’s coefficients range from 2.15 (Table IV) to as much as 3.13 (Table V). These coefficients are not only greater than one, they are a *lot* greater than one. These coefficients imply that if the US returned to the 12% inflation of the late 1970s, (a 10 percentage point rise), the Federal Reserve would raise the funds rate by 21.5 to 31.3 percentage points. If these predictions seem implausibly large, digesting the estimates as something less than structural helps a great deal.

The identification issue stems from the heart of all new-Keynesian models. The models have multiple equilibria. The modelers specify policy rules which lead to explosive dynamics, and then pick only the locally-bounded equilibrium. But locally-bounded equilibrium variables are stationary, so cannot reveal the strength of the explosions, which only occur in the equilibria we do not observe. In the model, endogenous variables jump in response to disturbances, to head off explosions. Such jumps induce correlation between right hand variables of the policy rule and its error, so that rule will be exquisitely hard to estimate. One can only begin to get around these central problems by strong assumptions, in particular that the central bank does not respond to many variables, and to “natural rate” shocks in particular, in ways that would help it to stabilize the economy.

The literature – understandably, I think – did not appreciate that “determinacy” and “desirable rate in equilibrium” are separate issues; that new-Keynesian models, unlike their old-Keynesian counterparts, achieve “determinacy” by responses to alternative equilibria, which are not measurable, not by responses to equilibrium variation in inflation, which are; and that “achieving determinacy” is a different reading of history than “raising rates to lower inflation.” Again, however, now that the distinction is clear we need not persist in mis-interpreting the regressions.

If not this, then what?

The contribution of this paper is negative, establishing that one popular theory does not, in the end, determine the price level or the inflation rate. So what theory *can* determine the price level, in an economy like ours? Commodity standards and $MV=PY$ can work in theory, but do not apply to our economy, with fiat money, interest-elastic money demand and no attempt by the central bank to target quantities.

The price level *can* be determined for economies like ours in models that adopt – or, perhaps, recognize – that governments follow a fiscal regime that is at least partially non-Ricardian. Such models solve all the determinacy and uniqueness problems in one fell swoop. And the change is not really so radical. Though the deep question of where the price level comes from changes, the vast majority of the new-Keynesian ingredients can be maintained. Whether the results are the same, and whether such models can account for the data is an open question.

“Economic” is an important qualifier. Most of the case for Taylor rules in popular and central bank writing, FOMC statements, and too often in academic contexts, emphasizes the old-Keynesian stabilizing story. This is a pleasant and intuitively pleasing story to many. However, it throws out the edifice of theoretical coherence – explicit underpinnings of optimizing agents, clearing markets etc. – that is the hallmark achievement of the new-Keynesian effort. If inflation is, in fact, stabilized in modern economies by interest rate targets interacted with *backward*-looking IS and Phillips curves, economists really have no idea *why* this is so.

9 References

- Alstadheim, Ragna, and Dale W. Henderson, 2006, "Price-Level Determinacy, Lower Bounds on the Nominal Interest Rate, and Liquidity Traps," *The B.E. Journal of Macroeconomics* 6:1 (Contributions), Article 12.
<http://www.bepress.com/bejm/contributions/vol6/iss1/art12>
- Atkeson, Andrew, Chari, V.V. and Patrick J. Kehoe, 2009, "Sophisticated Monetary Policies," Federal Reserve Bank of Minneapolis Research Department Staff Report 419.
- Bassetto, Marco, 2004, "Negative Nominal Interest Rates," *American Economic Review* 94 (2), 104-108.
- Benhabib, Jess, Stephanie Schmitt-Grohé, and Martín Uribe, 2002, "Avoiding Liquidity Traps," *Journal of Political Economy* 110, 535-563
- Canzoneri, Matthew, Robert Cumby and Behzad Diba, 2001, "Is the Price Level Determined by the Needs of Fiscal Solvency," *American Economic Review* 91, 1221 - 1238.
- Clarida, Richard, Jordi Galí, and Mark Gertler, 1998, "Monetary Policy Rules in Practice: Some International Evidence," *European Economic Review*, 42,1033-1067.
- Clarida, Richard, Jordi Galí, and Mark Gertler, 2000, "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *Quarterly Journal of Economics*, 115, 147-180.
- Cochrane, John H., 1998, "A Frictionless model of U.S. Inflation," in Ben S. Bernanke and Julio J. Rotemberg, Eds., *NBER Macroeconomics Annual 1998* Cambridge MA: MIT press, p. 323-384.
- Cochrane, John H., 2005, "Money as Stock," *Journal of Monetary Economics* 52, 501-528.
- Giannoni, Marc P., and Michael Woodford, 2005, "Optimal Inflation Targeting Rules," in Bernanke, Benjamin S. and Michael Woodford Eds., *Inflation Targeting*, Chicago: University of Chicago Press, 2005.
(pdf available at <http://www.columbia.edu/~mw2230/>; Technical Appendix at http://www2.gsb.columbia.edu/faculty/mgiannoni/prog/nberit_programs/model_nberit.pdf.)
- King, Robert G., 2000 "The New IS-LM Model: Language, Logic, and Limits," Federal Reserve Bank of Richmond *Economic Quarterly* 86, 45-103.
- Klein, Paul, 2000, "Using the Generalized Schur Form to Solve a Multivariate Linear Rational Expectations Model," *Journal of Economic Dynamics and Control* 24, 1405-1423.
- Leeper, Eric , 1991, "Equilibria Under 'Active' and 'Passive' Monetary and Fiscal Policies," *Journal of Monetary Economics* 27, 129-147.
- Obstfeld, Maurice and Kenneth Rogoff, 1983, "Speculative Hyperinflations in Maximizing Models: Can we Rule them Out?," *Journal of Political Economy* 91, 675-687.
- Obstfeld, Maurice and Kenneth Rogoff, 1986, "Ruling out Divergent Speculative Bubbles," *Journal of Monetary Economics* 17, 349-362.

- Rotemberg, Julio and Michael Woodford, 1997, "An Optimization-Based Econometric Model for the Evaluation of Monetary Policy," *NBER Macroeconomics Annual* 12, 297-346
- Rotemberg, Julio and Michael Woodford, 1998, "An Optimization-Based Econometric Model for the Evaluation of Monetary Policy," *NBER Technical Working Paper* no. 233.
- Rotemberg, Julio and Michael Woodford, 1999, "Interest-Rate Rules in an Estimated Sticky-Price Model," in Taylor, John B., Ed., *Monetary Policy Rules*, Chicago: University of Chicago Press.
- Sargent, Thomas J., and Neil Wallace, 1975, "'Rational' Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule," *Journal of Political Economy* 83, 241-254.
- Schmitt-Grohé, Stephanie and Martín Uribe, 2000, "Price Level Determinacy and Monetary Policy Under a Balanced Budget Requirement," *Journal of Monetary Economics* 45, 211-246.
- Sims, Christopher A., 1980, "Macroeconomics and Reality," *Econometrica* 48, 1-48.
- Sims, Christopher A., 1994, "A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy," *Economic Theory* 4, 381-99.
- Taylor, John B. 1999, "The Robustness and Efficiency of Monetary Policy Rules as Guidelines for Interest Rate Setting by the European Central Bank," *Journal of Monetary Economics* 43, 655-679.
- Woodford, Michael, 1994, "Monetary Policy and Price Level Determinacy in a Cash-in-Advance Economy," *Economic Theory* 4, 345-380.
- Woodford, Michael, 1995, "Price Level Determinacy Without Control of a Monetary Aggregate," *Carnegie Rochester Conference Series on Public Policy*, 43, 1-46.
- Woodford, Michael, 2001, "Monetary Policy in the Information Economy," in *Economic Policy for the Information Economy*, Kansas City: Federal Reserve Bank of Kansas City.
- Woodford, Michael, 2003, *Interest and Prices*, Princeton: Princeton University Press.