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AGGREGATE DEMAND AND SUPPLY

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# Aggregate Demand and Supply

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## Abstract

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## 1 Introduction

This paper is part of a broader project that provides a microfoundation to the central ideas of the General Theory (1936). This foundation is distinct from new-Keynesian economics and has different implications for fiscal and monetary policy and for the way that one should interpret low frequency movements in the unemployment rate. The current paper focuses on Keynes’ use of the terms ‘aggregate demand’ and ‘aggregate supply’. I will argue that the meaning of these concepts has become distorted and I will demonstrate, by recovering their original usage, that the central ideas of Keynesian economics can be given a consistent microfoundation. My main purpose in the

sections that follow is to reintroduce a diagram that appeared for decades as the central expository device in undergraduate textbooks and to provide an interpretation of this diagram within the framework of a search theoretic model of the labor market.

The current dominant interpretation of the General Theory is that of new-Keynesian economics which builds on Patinkin's (1956) idea that the unemployment equilibrium of the General Theory can be interpreted as a Walrasian general equilibrium in which agents trade at 'false prices'. My main concern with this approach is that it distorts a central message of the General Theory; that in an unregulated capitalist economy, inefficiently high unemployment may exist as a feature of a steady state equilibrium.

New-Keynesian models could theoretically display very high unemployment for very long periods of time, however, when they are calibrated to sensible numbers for the rate of price adjustment they lead to the prediction that the unemployment rate does not deviate for long from its natural rate. The project, of which this paper is a part, provides an alternative micro-foundation to Keynesian economics that does not rest on sticky prices. I call the alternative models 'old Keynesian' to differentiate them from those of the new Keynesians. In old Keynesian models there is no natural rate of unemployment. Instead, the steady state of the economic system rests on self-fulfilling beliefs of economic agents - a feature for which Keynes used the term 'animal spirits'. Unlike my previous work on this topic, (1999), animal spirits determine the steady state itself and not just the path by which that state is attained. As a consequence, the implications for steady state welfare are large.

In this, and in companion papers, I build a series of economic models each of which displays an equilibrium with an unemployment rate that may be higher or lower than the social planning optimum. Some of these models are one or two period examples, some are embedded in a dynamic stochastic general equilibrium environment. They all share a single common feature. The labor market is modeled as a search equilibrium in which households and firms take the wage as given. Since the search technology has two inputs but a single price - the price taking assumption leads to a model with more unknowns than equations. Rather than use a bargaining assumption or some other game theoretic concept to close the model, I assume instead that supply adjusts to meet demand and that demand, in turn, is determined by the self-fulfilling beliefs of agents.

In the equilibria of old-Keynesian models all prices adjust to a point

where no individual has an incentive to change his behavior; nevertheless, there remain unexploited gains to trade. The prevalence of the Walrasian equilibrium concept has blunted us to the remarkable feature of a Walrasian equilibrium; that the first and second welfare theorems hold as a feature of the Walrasian decentralization. Economists as a group have become so used to this formalization of Adam Smith's doctrine of the invisible hand that we have forgotten that the optimality of equilibrium is a remarkable feature of a certain class of social organizations and not an inherent law of nature.

## 2 The Meaning of Aggregate Demand and Supply

The concepts of aggregate demand and supply are widely used by contemporary economists. They are typically explained in the context of a one commodity model in which real gdp is unambiguously measured in units of commodities per unit of time. In the General Theory there is no assumption that the world can be described by a single commodity model; to the contrary, Chapter 4 of the General Theory is devoted to a discussion of the choice of units that is designed to provide a framework that can accommodate the rich complexity of a modern industrial economy in which there are many produced commodities and many capital goods.

In his discussion of units, Keynes describes his intent to base all measurement on two homogenous units; money (I will call this a dollar) and a unit of ordinary labor. The theory of index numbers as we understand it today was not available in the early 1930's and Keynes' use of money and ordinary labor to describe aggregate relationships was clever and new. The aggregate demand and supply curves of the General Theory are relationships between these two units and they are not the same relationships as those that hold between a price index and a quantity index in modern textbook expositions of aggregate demand and supply.

Keynes chose 'an hour's unit of ordinary labor' to represent the level of economic activity because it is a relatively homogeneous unit. To get around the fact that different workers have different skills he proposed to measure labor of different efficiencies by relative wages. Thus

...the quantity of employment can be sufficiently defined for our purpose by taking an hour's employment of ordinary labour as

our unit and weighting an hour's employment of special labor in proportion to its remuneration; i.e. an hour of special labour remunerated at double ordinary rates will count as two units... [General Theory, Page 41]

In the General Theory, aggregate demand is driven primarily by the 'animal spirits' of investors. Animal spirits are a key component of autonomous investment expenditure that, in turn, is the prime cause of fluctuations in aggregate demand. To capture this idea one requires a dynamic model since investment involves plans that span at least two periods. However, autonomous expenditure is also determined by government spending and by recognizing this dependence I will be able, in this paper, to explain how a search model of the labor market can be embedded into general equilibrium in a relatively simple environment; a one period model that abstracts from investment in which output and employment are driven by fiscal policy.

The one period model I will describe is simpler than a fully articulated dynamic general equilibrium model of the variety that is often used to articulate real business cycle theory. It is simpler since I abstract from dynamics and assume that all output is produced from labor and existing fixed factors. Adding dynamics would abstract from the main point of the paper; to articulate a mechanism that can provide a microfounded model of the labor market in which there may be a continuum of stationary equilibrium unemployment rates. I will tackle the problem of adding dynamics elsewhere. Dynamic Keynesian models will add additional richness and a distinct set of issues that separate them from the Cass-Ramsey model that provides the basis for RBC models.

### 3 Households

In this section and in subsequent sections I will construct a one period model that contains the essential ideas of aggregate demand and supply. I begin by describing the problem of the households. Since I am going to concentrate on the theory of aggregate supply, I will assume the existence of identical households, each of which solves the problem

$$\max_{\{C,H\}} J = j(\mathbf{C}), \tag{1}$$

subject to the constraints

$$\mathbf{p} \cdot \mathbf{C} \leq (1 - \tau) (Lw + \mathbf{r} \cdot \bar{\mathbf{K}}) + T, \quad (2)$$

$$H \leq 1, \quad (3)$$

$$L = \tilde{q}H, \quad (4)$$

$$U = H - L. \quad (5)$$

Each household has a measure 1 of members.  $\mathbf{C}$  is a vector of  $n$  commodities,  $\mathbf{p}$  is a vector of  $n$  money prices,  $w$  is the money wage,  $\mathbf{r}$  is a vector of money rental rates and  $\bar{\mathbf{K}}$  is a vector of  $m$  factor endowments. I use the symbol  $r_j$  to refer to the  $j$ 'th rental rate. The factors may be thought of as different types of land although in dynamic models they would have the interpretation of different types of capital goods. I will maintain the convention throughout the paper that boldface letters are vectors and  $\mathbf{x} \cdot \mathbf{y}$  is a vector product.

The household decides on the measure  $H$  of members that will search for jobs, and on the amounts of its income to allocate to each of the  $n$  commodities.  $T$  is the lump-sum household transfer (measured in dollars) and  $\tau$  the income-tax rate. A household that allocates  $H$  members to search will receive a measure  $\tilde{q}H$  of jobs where the employment rate  $\tilde{q}$  is taken parametrically by households. I will assume that utility takes the form

$$j(\mathbf{C}) = \sum_{i=1}^n g_i \log(C_i), \quad (6)$$

where the commodity weights sum to 1,

$$\sum_{i=1}^n g_i = 1. \quad (7)$$

Later, I will also assume that each good is produced by a Cobb-Douglas production function and I will refer to the resulting model as a logarithmic-Cobb-Douglas, or LCD, economy. Although the analysis could be generalized to allow utility to be homothetic, and technologies to be CES, this extension would considerably complicate the algebra. My intent is to find a compromise model that allows for multiple commodities but is still tractable and for this purpose, the log utility model is familiar and suitable.

The solution to the household's decision problem has the form

$$H = 1, \quad (8)$$

$$C_i = g_i Z^D, \quad (9)$$

where  $g_i$  is the budget share allocated to the  $i$ 'th good. For more general homothetic preferences these shares would be functions of the price vector  $\mathbf{p}$ .

Household income,  $Z$  is defined as

$$Z \equiv Lw + \mathbf{r} \cdot \bar{\mathbf{K}}, \quad (10)$$

and is measured in dollars. The term  $Z^D$  in Equation (9) represents disposable income and is defined by the equation

$$Z^D = (1 - \tau) Z + T. \quad (11)$$

Since all income is derived from the production of commodities it follows from the aggregate budget constraints of households, firms and government that  $Z$  is also equal to the value of the produced commodities in the economy,

$$Z \equiv \sum_{i=1}^n p_i Y_i, \quad (12)$$

and since the government budget must be balanced in equilibrium, it also follows that

$$Z = Z^D. \quad (13)$$

The equivalence of income and the value of output is a restatement of the familiar Keynesian accounting identity, immortalized in the textbook concept of the 'circular flow of income'.

## 4 Firms

This section deals with the production of commodities in a decentralized economy. The main aspect that is novel is my description of the hiring process in which I will assume that a firm must use part of its labor force in the activity of recruiting.

I assume that there are  $n \geq m$  commodities. Output of the  $i$ 'th commodity is denoted  $Y_i$ , and is produced by a constant returns-to-scale neoclassical production function

$$Y_i = \Psi_i(\mathbf{K}_i, X_i), \quad (14)$$

where  $\mathbf{K}_i$  is a vector of  $m$  capital goods used in the  $i$ 'th industry and  $X_i$  is labor used in production in industry  $i$ . The  $j$ 'th element of  $\mathbf{K}_i$ , denoted  $K_{i,j}$ ,

is the measure of the  $j$ 'th capital good used as an input to the  $i$ 'th industry and  $\mathbf{K}_i$  is defined as,

$$\mathbf{K}_i \equiv (K_{i,1}, K_{i,2} \dots, K_{i,m}). \quad (15)$$

The function  $\Psi_i$  is assumed to be Cobb-Douglas,

$$\Psi_i(\mathbf{K}_i, X_i) \equiv A_i K_{i,1}^{a_{i,1}} K_{i,2}^{a_{i,2}} \dots K_{i,m}^{a_{i,m}} X_i^{b_i}, \quad (16)$$

where the constant returns-to-scale assumption implies that the weights  $a_{i,j}$  and  $b_i$  sum to 1,

$$\sum_{j=1}^m a_{i,j} + b_i = 1. \quad (17)$$

Since the assumption of constant returns-to-scale implies that the number of firms in each industry is indeterminate I will refer interchangeably to  $Y_i$  as the output of a firm or of an industry.

Each firm recruits workers in a search market by allocating a measure  $V_i$  of workers to recruiting. The total measure of workers  $L_i$ , employed in industry  $i$ , is

$$L_i = X_i + V_i. \quad (18)$$

Each firm takes parametrically the measure of workers that can be hired, denoted  $q$ , and employment at firm  $i$  is related to  $V_i$  by the equation,

$$L_i = qV_i. \quad (19)$$

I have assumed that labor, rather than output, is used to post vacancies in contrast to most search models. This innovation is not important and is made for expositional simplicity and to allow me, in related work, to write down models that can easily be compared with more familiar real business cycle economies. The timing of the employment decision deserves some discussion since by implication I am allowing the firm to use workers to recruit themselves.

If a firm begins the period with no workers, and if workers are an essential input to recruiting, it might be argued that the firm can never successfully hire a worker. Since I will be thinking of the time period of the model as a quarter or a year, this assumption should be seen as a convenient way of representing the equilibrium of a dynamic process. The firm puts forward a plan that consists of a feasible  $n$ -tuple  $\{V_i, Y_i, L_i, \mathbf{K}_i, X_i\}$ . Given the

exogenous hiring elasticity,  $q$ , a plan to use  $V_i$  workers in recruiting results in  $qV_i$  workers employed of whom  $X_i$  are used to produce commodities.

We are now equipped to describe the profit-maximization of a firm in industry  $i$ . The firm solves the problem

$$\max_{\{\mathbf{K}_i, V_i, X_i, L_i\}} p_i Y_i - w_i L_i - \mathbf{r} \cdot \mathbf{K}_i \quad (20)$$

$$Y_i \leq A_i K_{i,1}^{a_{i,1}} K_{i,2}^{a_{i,2}} \dots K_{i,m}^{a_{i,m}} X_i^{b_i}, \quad (21)$$

$$L_i = X_i + V_i, \quad (22)$$

$$L_i = qV_i. \quad (23)$$

Using equations (22) and (23) we can write labor used in production,  $X_i$ , as a multiple,  $Q$ , of employment at the firm,  $L_i$

$$X_i = L_i Q, \quad (24)$$

where  $Q$  is defined as

$$Q = \left(1 - \frac{1}{q}\right). \quad (25)$$

$Q$  acts like a productivity shock to the firm but it is in fact an externality that represents ‘labor market tightness’. I will return to the role of  $Q$  below. Using this definition we may write the firm’s profit maximization problem in reduced form,

$$\max_{\{\mathbf{K}_i, V_i, X_i, L_i\}} p_i Y_i - w_i L_i - \mathbf{r} \cdot \mathbf{K}_i, \quad (26)$$

$$Y_i \leq A_i L_i^{b_i} Q^{b_i} K_{i,1}^{a_{i,1}} K_{i,2}^{a_{i,2}} \dots K_{i,m}^{a_{i,m}}. \quad (27)$$

The solution to this problem is characterized by the first-order conditions

$$a_{i,j} p_i Y_i = K_{i,j} r_j, \quad j = 1, \dots, m, \quad (28)$$

$$b_i p_i Y_i = w L_i. \quad (29)$$

Using these conditions to write  $L_i$  and  $K_{i,j}$  as functions of  $w$ ,  $\mathbf{r}$ , and  $p_i$  and substituting them the production function leads to an expression for  $p_i$  in terms of factor prices,

$$p_i = p_i \left( \frac{w}{Q}, \mathbf{r} \right). \quad (30)$$

The function  $p_i : R^{m+1} \rightarrow R_+$  is known as the factor price frontier and is homogenous of degree 1 in the vector of  $m$  money rental rates  $\mathbf{r}$  and in the productivity adjusted money wage,  $w$ .

## 5 Search

I have described how individual households and firms respond to the aggregate variables  $w, \mathbf{p}, \mathbf{r}, q$  and  $\tilde{q}$ . This section describes the process by which searching workers find jobs. To model the job finding process I assume that there is an aggregate match technology of the form,

$$\bar{m} = \bar{H}^{1/2} \bar{V}^{1/2}, \quad (31)$$

where  $\bar{m}$  is the measure of workers that find jobs when  $\bar{H}$  unemployed workers search and  $\bar{V}$  workers are allocated to recruiting in aggregate by all firms. I have used bars over variables to distinguish aggregate from individual values. Since leisure does not yield disutility and hence  $\bar{H} = 1$ , this equation simplifies as follows,

$$\bar{m} = \bar{V}^{1/2}. \quad (32)$$

Further, since all workers are initially unemployed, employment and matches are equal,

$$\bar{L} = \bar{V}^{1/2}. \quad (33)$$

In a dynamic model employment at each firm will become a state variable that acts much like capital in the RBC model. In the one period model I have abstracted from this role by assuming that all workers begin the period unemployed.

Equation (33) describes how aggregate employment is related to the aggregate search effort of all firms. I will refer to the measure of workers that can be hired by a single employee engaged in recruiting as the hiring effectiveness of the firm. To find an expression for the hiring effectiveness of the  $i$ 'th firm I assume that jobs are allocated in proportion to the fraction of aggregate recruiters attached to firm  $i$ ; that is,

$$L_i \equiv \bar{V}^{1/2} \frac{V_i}{\bar{V}}, \quad (34)$$

where  $V_i$  is the number of recruiters at firm  $i$ . The hiring effectiveness of firm  $i$  is given by the expression

$$\frac{1}{\bar{V}^{1/2}}, \quad (35)$$

which is a decreasing function of aggregate recruiting efforts. This reflects the fact that when many firms are searching there is congestion in the market and it becomes harder for an individual firm to find a new worker.

## 6 The Social Planner

The purpose of this paper is to describe a micro-founded model of the Keynesian idea that equilibrium is determined by the intersection of aggregate demand and supply. A secondary purpose is to study the properties of a Keynesian equilibrium and to formulate the idea of Keynesian unemployment. To describe how employment may be too low I will need a benchmark of efficient employment against which it can be measured; this section provides such a bench-mark by studying the problem that would be solved by a social planner who operates production and search technologies to maximize the following welfare function,

$$\max_{\{C,V,L,H\}} j(\mathbf{C}) = \sum_{i=1}^n g_i \log(C_i), \quad (36)$$

subject to the resource constraints,

$$C_i \leq A_i K_{i,1}^{a_{i,1}} K_{i,2}^{a_{i,2}} \dots K_{i,m}^{a_{i,m}} (L_i - V_i)^{b_i}, \quad i = 1, \dots, n, \quad (37)$$

$$\sum_{i=1}^n K_{i,j} \leq \bar{K}_j, \quad j = 1, \dots, m \quad (38)$$

$$L_i = \left(\frac{H}{V}\right)^{1/2} V_i, \quad (39)$$

$$H \leq 1. \quad (40)$$

Since the optimal value of  $H$ , denoted  $H^*$ , will equal 1, Equations (39) and (40) can be rearranged to give the following expression for aggregate employment as a function of aggregate labor devoted to recruiting,

$$L \equiv \sum_{i=1}^n L_i = V^{1/2}. \quad (41)$$

Combining this expression with Equation (39) leads to the following relationship between labor used in recruiting at firm  $i$ , employment at firm  $i$ , and aggregate employment,

$$V_i = L_i L. \quad (42)$$

Equation (42) restates the implication of (34) in a different form. The social planner recognizes that it takes more effort on the part of the recruiting

department of firm  $i$  to hire a new worker when aggregate employment is high and he will take account of this relationship between hiring activities of different industries when allocating labor to alternative activities.

Using Equation (42) to eliminate  $V_i$  from the production function we can rewrite (37) in terms of  $\mathbf{K}_i$  and  $L_i$ ,

$$C_i = A_i K_{i,1}^{a_{i,1}} K_{i,2}^{a_{i,2}} \dots K_{i,m}^{a_{i,m}} L_i^{b_i} (1 - L)^{b_i}. \quad (43)$$

Equation (43) makes clear that the match technology leads to a production externality across firms. When all other firms have high levels of employment it becomes harder for the individual firm to recruit workers and this shows up as an external productivity effect, this is the term  $(1 - L)$ , in firm  $i$ 's production function. The externality is internalized by the social planner but may cause difficulties that private markets cannot effectively overcome.

To find a solution to the social planning solution, we may substitute Equation (43) into the objective function (36) and exploit the logarithmic structure of the problem to write utility as a weighted sum of the logs of capital and labor used in each industry, and of the externality terms that depend on the log of  $(1 - L)$ . The first-order conditions for the problem can then be written as

$$\frac{g_i b_i}{L_i} = \frac{\sum_{i=1}^n g_i b_i}{(1 - L)}, \quad i = 1, \dots, n, \quad (44)$$

$$\frac{g_i a_{i,j}}{K_{i,j}} = \lambda_j, \quad i = 1, \dots, n, \quad j = 1, \dots, m, \quad (45)$$

$$\sum_{j=1}^m K_{i,j} = \bar{K}_j, \quad j = 1, \dots, m. \quad (46)$$

The variable  $\lambda_j$  is a Lagrange multiplier on the  $j$ 'th resource constraint. Using Equation (44) and the fact that  $L = \sum_{i=1}^n L_i$ , it follows from some simple algebra, that the social planner will allocate half of the labor force in the LCD economy to employment;

$$L^* = 1/2. \quad (47)$$

The remaining half will be unemployed. The fraction 1/2 follows from the Cobb-Douglas elasticity of the matching function. The fact that one half of the labor force remains unemployed in equilibrium follows from the fact

that resources would have to be diverted from production to recruiting in order to increase employment and increasing employment beyond 1/2 would be counterproductive and result in a loss of output in each industry. In this economy, the ‘natural rate of unemployment’ is 50%.

The social planning solution provides a clear candidate definition of full employment - it is the level of employment  $L^*$  that maximizes per-capita output. In the General Theory, Keynes argued that a laissez-faire economic system would not necessarily achieve full employment and he claimed the possibility of equilibria at less than full employment as a consequence of what he called a failure of effective demand.

The first-order conditions can also be used to derive the following expression for the labor  $L_i^*$  used in industry  $i$ ;

$$L_i^* = \frac{g_i b_i}{\sum_{i=1}^n g_i b_i} L^*. \quad (48)$$

To derive the capital allocation across firms for capital good  $j$ , the social planner solves Equations (45) and (46) to yield the optimal allocation of capital good  $j$  to industry  $i$ ;

$$K_{i,j}^* = \frac{g_i a_{i,j}}{\sum_{i=1}^n g_i a_{i,j}} \bar{K}_j. \quad (49)$$

To recap, in the LCD economy, the social planner sets employment at 1/2 and allocates factors across industries using weights that depend on a combination of factor shares and consumption weights in preferences.

## 7 Aggregate Demand and Supply

This section derives the properties of the Keynesian aggregate supply curve for the LCD economy. When I began this project I thought of aggregate supply as a relationship between employment and output. Intuition that was carried over from my own undergraduate training led me to think of this function as analogous to a movement along a production function in a one-good economy. This intuition is incorrect: A more appropriate analogy would be to compare the aggregate supply function to the first order condition for labor in a one good model.

Consider a one-good economy in which output,  $Y$ , is produced from labor  $L$  and capital  $K$  using the function

$$Y = A(\bar{L}) K^\alpha L^{1-\alpha}, \quad (50)$$

where  $A$  may be a function of aggregate employment because of the search externalities discussed above. It would be a mistake to call the function,

$$A(L) K^\alpha L^{1-\alpha}, \quad (51)$$

where  $\bar{L}$  is replaced by  $L$ , the Keynesian aggregate supply function.

If this is not what Keynes meant by aggregate supply then what did he mean? The points that I want to make are worth emphasizing because the original intent of the General Theory has been obfuscated by decades of misinterpretation. The first point is that although the aggregate demand and supply diagram plots demand and supply prices against quantities, these are not prices in the usual sense of the term. Keynes defined the *aggregate supply price* of a given volume of employment to be the

‘expectation of proceeds which will just make it worth the while of the entrepreneurs to give that employment’ (Keynes, 1936, Page 24).

By aggregate demand he meant,

the proceeds which entrepreneurs expect to receive from the employment of  $L$  men, the relationship between  $D$  and  $L$  being written  $D = f(L)$  which can be called the *Aggregate Demand Function*. (Keynes, 1936, Page 25, ‘L’ is substituted for ‘N’ from the original).

Keynes then asked us to consider what would happen if, for a given value of employment, aggregate demand  $D$  is greater than aggregate supply  $Z$ . In that case...

there will be an incentive to entrepreneurs to increase employment beyond  $L$  and, if necessary, *to raise costs by competing with one another for the factors of production*, up to the value of  $L$  for which  $Z$  has become equal to  $D$ . (Keynes, 1936, Page 25, N replaced by L and italics added).

It is not possible to understand this definition without allowing relative prices to change since the notion of competing for factors requires an adjustment of factor prices. In a general equilibrium environment Walras law

implies that one price can be chosen as numeraire; the price chosen by Keynes was the money wage. Given a value of  $w$ , competition for factors requires adjustment of the money price  $p$  and the rental rate  $r$  to equate aggregate demand and supply.

In the one-good economy, the equation that triggers competition for workers is the first-order condition

$$\frac{(1 - \alpha)Y}{L} = \frac{w}{p}. \quad (52)$$

Aggregate demand  $Z$ , is price times quantity. Using this definition, Equation (52) can be rearranged to yield the expression,

$$Z \equiv pY = \frac{wL}{(1 - \alpha)}, \quad (53)$$

which is the Keynesian aggregate supply function for a one-good economy. By fixing the money wage Keynes was not assuming disequilibrium in factor markets; he was choosing a numeraire. Once this is recognized, the Keynesian aggregate supply curve takes on a different interpretation from that which is given in introductory textbooks. A movement along the aggregate supply curve is associated with an increase in the price level that reduces the real wage and brings it into equality with a falling marginal product of labor.

In an economy with many goods the aggregate supply price,  $Z$ , is defined by the expression

$$Z \equiv \sum_{i=1}^n p_i Y_i. \quad (54)$$

For the LCD economy the aggregate supply function has a particularly simple form since the logarithmic and Cobb-Douglas functional forms allow individual demands and supplies to be aggregated. The first order condition for the use of labor at firm  $i$  has the form

$$L_i = \frac{b_i Y_i p_i}{w}. \quad (55)$$

To aggregate labor across industries we need to know how relative prices adjust as the economy expands. To determine relative prices we must turn to preferences and here the assumption of logarithmic utility allows a simplification since the representative agent allocates fixed budget shares to each commodity

$$Y_i p_i = g_i Z^D = g_i Z, \quad (56)$$

where the equality of  $Z$  and  $Z^D$  follows from the government budget constraint. Combining Equation (55) with (56) yields the expression,

$$L_i = \frac{b_i g_i Z}{w}, \quad (57)$$

and summing Equation (57) over all  $i$  industries and choosing  $w = 1$  as the numeraire leads to the expression

$$Z = \frac{1}{\chi} L \equiv \phi(L), \quad (58)$$

where

$$\chi \equiv \sum_{i=1}^n g_i b_i. \quad (59)$$

Equation (58) is the Keynesian aggregate supply curve for the multi-good logarithmic-Cobb-Douglas economy.

To reiterate; the aggregate supply curve in a one-good economy is not a production function; it is the first-order condition for labor. In the LCD economy it is an aggregate of the first order conditions across industries with a coefficient that is a weighted sum of preference and technology parameters for the different industries.

Can this expression be generalized beyond the LCD case? The answer is yes, but the resulting expression for aggregate supply depends, in general, on factor supplies, that is,  $Z$  will be a function not only of  $L$  but also of  $\bar{K}_1, \dots, \bar{K}_m$ . The following paragraph demonstrates that, given our special assumptions about preferences and technologies, these stocks serve only to influence rental rates.

The first order condition for the  $j$ 'th factor used in firm  $i$  can be written as

$$K_{i,j} = \frac{a_{i,j} Y_i p_i}{r_j}. \quad (60)$$

Combining the first order conditions for factor  $j$  and summing over all  $i$  industries leads to the expression

$$\bar{K}_j = \sum_{i=1}^n K_{i,j} = \frac{\sum_{i=1}^n a_{i,j} Y_i p_i}{r_j}. \quad (61)$$

Exploiting the allocation of budget shares by consumers, Equation (56), one can derive the following expression,

$$r_j = \frac{\chi_j Z}{\bar{K}_j}, \quad (62)$$

where

$$\chi_j \equiv \sum_{i=1}^n a_{i,j} g_i. \quad (63)$$

Equation (62) determines the nominal rental rate for factor  $j$  as a function of the aggregate supply price  $Z$  and the factor supply  $\bar{K}_j$ .

## 8 Keynesian Equilibrium

What determines relative outputs in the Keynesian model and how are aggregate employment,  $L$ , and the aggregate supply price  $Z$ , determined? Aggregate demand follows from the gdp accounting identity,

$$D = \sum_{i=1}^n p_i C_i. \quad (64)$$

In an economy with government purchases and investment expenditure this equation would have two extra terms as in the textbook Keynesian accounting identity that generation of students have written as

$$Y = C + I + G. \quad (65)$$

In our notation  $C$  is replaced by  $\sum_{i=1}^n p_i C_i$ ,  $Y$  is replaced by  $D$ , and  $G$  and  $I$  are absent from the model. The Keynesian consumption function is simply the budget equation

$$\sum_{i=1}^n p_i C_i = (1 - \tau) Z + T, \quad (66)$$

and since  $D = \sum_{i=1}^n p_i C_i$  and  $\chi Z = L$ , from Equation (58), the aggregate demand function for the LCD economy is given by the expression,

$$D = (1 - \tau) \frac{L}{\chi} + T. \quad (67)$$

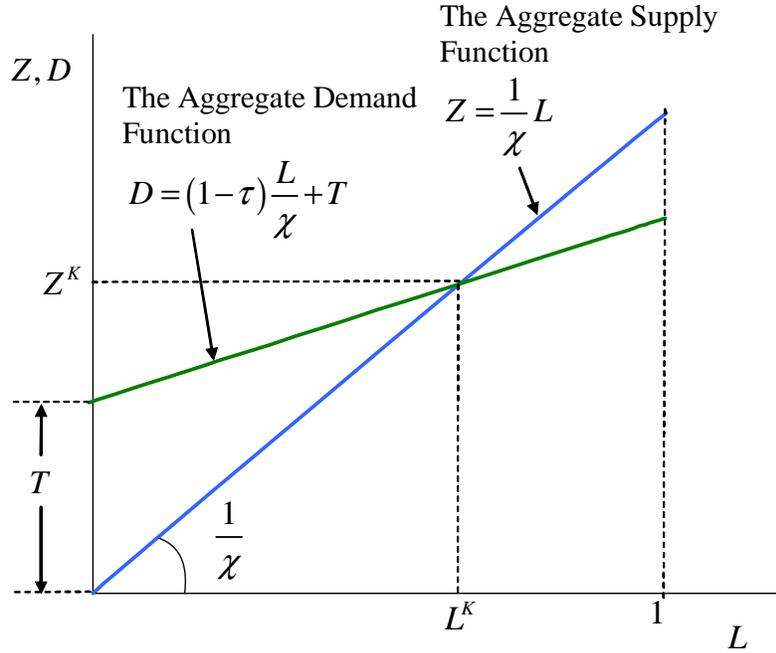


Figure 1: Aggregate Demand and Supply

In a Keynesian equilibrium, when  $D = Z$ , the value of income,  $Z^K$  is given by the equality of aggregate demand and supply; that is,

$$Z^K = \frac{T}{\tau}, \quad (68)$$

and equilibrium employment is given by the expression.

$$L^K = \chi Z^K.$$

The Keynesian aggregate demand and supply functions for the LCD economy are graphed in Figure 1.

## 9 Keynes and the Social Planner

How well do markets work and do we require government micro-management of individual industries to correct inefficient allocations of resources that are

inherent in capitalist economies? Keynes gave a two part answer to this question. He argued that the level of aggregate economic activity may be too low as a consequence of the failure of effective demand and here he was a strong proponent of government intervention. But he was not a proponent of socialist planning. In this section I will show that the model outlined in this paper provides a formalization of Keynes' arguments. If effective demand is too low, the model displays an inefficient level of employment and in this sense there is an argument for a well designed fiscal policy. But the allocation of factors across industries, for a given volume of employment, is the same allocation that would be achieved by a social planner.

To make the argument for fiscal intervention one need only compare aggregate employment in the social planning solution with aggregate employment in a Keynesian equilibrium. The social planner would choose

$$L^* = \frac{1}{2}. \tag{69}$$

The Keynesian equilibrium at

$$L^K = \frac{\chi T}{\tau}, \tag{70}$$

may result in any level of employment in the interval  $[0, 1]$ . For any value of  $L^K < L^*$  we may say that the economy is experiencing Keynesian unemployment and in this case there is a possible Pareto improvement that would make everyone better off by increasing the number of people employed.

The formalization of Keynesian economics based on search contains the additional implication that there also may be overemployment since  $L^K$  may be greater than  $L^*$ . Overemployment is also Pareto inefficient and welfare would, in this case, be increased by employing fewer workers in all industries. Although a value of  $L^K$  greater than  $L^*$  is associated with a higher value of nominal gdp ( $Z^K > Z^*$ ), there is too much production on average and by lowering  $L$  back towards  $L^*$  the social planner will be able to increase the quantity of consumption goods available in every industry.

In an overemployment equilibrium the additional workers spend more time recruiting their fellows than in productive activity. In the limit, as employment tends to 1, nominal gdp tends to its upper bound,  $1/\chi$ . But although gdp measured in wage units always increases as employment increases, for very high values of employment the physical quantity of output produced in each industry is very low and in the limit at  $L = 1$ ,  $Y_i$  is equal

to zero in each industry and  $p_i$  is infinite. Every employed worker is so busy recruiting additional workers that he has no time to produce commodities.

What about the allocation of factors across industries. Here the capitalist system fares much better. Equations (48) and (49), that determine factor allocations in the social planning solution, are reproduced below

$$L_i^* = \frac{g_i b_i}{\sum_{i=1}^n g_i b_i} L^*, \quad (71)$$

$$K_{i,j}^* = \frac{g_i a_{i,j}}{\sum_{i=1}^n g_i a_{i,j}} \bar{K}_j. \quad (72)$$

Given the resources  $\bar{K}_j$  for  $j = 1, \dots, m$ , Equation (72) implies that these resources will be allocated across industries in proportion to weights that depend on the preference parameters  $g_i$  and the production elasticities  $a_{i,j}$ . Equation (71) implies that the volume of resources employed,  $L^*$ , will be allocated across industries in a similar manner.

Contrast these equations with their counterparts for the competitive equilibrium. The factor demand equations (60), and the resource constraints (61) are reproduced below,

$$K_{i,j} = \frac{a_{i,j} Y_i p_i}{r_j}, \quad (73)$$

$$K_j = \sum_{i=1}^n K_{i,j} = \frac{\sum_{i=1}^n a_{i,j} Y_i p_i}{r_j}. \quad (74)$$

Consumers with logarithmic preferences will set budget shares to utility weights

$$p_i Y_i = g_i Z. \quad (75)$$

Combining this expression with Equations (60) and (74) leads to the following equation that determines the allocation of factor  $j$  to industry  $i$  in a Keynesian equilibrium,

$$K_{i,j} = \frac{g_i a_{i,j}}{\sum_{i=1}^n g_i a_{i,j}} \bar{K}_j. \quad (76)$$

This expression is identical to the social planning solution, Equation (72).

What about the allocation of labor across industries? The first order conditions for firms imply

$$b_i p_i Y_i = w L_i. \quad (77)$$

Combining this expression with Equation (75) and using the fact that  $\chi Z^K = L^K$  gives,

$$L_i = \frac{g_i b_i}{\sum_{i=1}^n g_i b_i} L^K, \quad (78)$$

where I have used the fact that  $\chi \equiv \sum_{i=1}^n g_i b_i$ . Equation (78) that determines the allocation of labor across industries is identical to the social planning solution with one exception; the efficient level of aggregate employment  $L^*$  is replaced by the Keynesian equilibrium level  $L^K$ . It is in this sense that Keynes provided a *General* theory of employment; the classical value  $L^*$  is just one possible rest point of the capitalist system, as envisaged by Keynes, and in general it is not one that he thought would be found by unassisted competitive markets.

## 10 Concluding Comments

It is difficult to read the General Theory without experiencing a disconnect between what is in the book and what one has learned about Keynesian economics as a student. The most egregious misrepresentation is the notion of aggregate demand and supply that we teach to undergraduates and that bears little or no relationship to what Keynes meant by these terms. The representative textbook author has adopted the Humpty Dumpty approach that, “...when I use a word, it means just what I choose it to mean - neither more nor less.”<sup>1</sup>

The textbook aggregate demand curve slopes down; the Keynesian aggregate demand curve slopes up. The textbook aggregate demand curve plots a price against a quantity; so does the Keynesian aggregate demand curve, at least in name, but the “aggregate demand price” and the “aggregate supply price” of the General Theory are very different animals from the price indices of modern theory. Beginning with Patinkin (1956), textbook Keynesians have tried to fit the round peg of the General Theory into the square hole of Walrasian general equilibrium theory. The fact that the fit is less than perfect has caused several generations of students to abandon the ideas of the General Theory and to follow the theoretically more coherent approach of real business cycle theory. The time has come to reconsider this decision.

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<sup>1</sup>The quote is from *Alice's Adventures in Wonderland*, by Lewis Carroll.

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