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CAPITAL CONTROLS, THE DUAL EXCHANGE RATE, AND DEVALUATION

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ABSTRACT

This paper re-examines the effect of devaluation under capital-account restrictions, adding to traditional formulations the seemingly minor (but realistic) assumption that central-bank reserves earn interest. The extra assumption has important implications. In an intertemporal model, devaluation is no longer neutral in the long run as it is in the literature on the monetary approach to the balance of payments. Further, the economy may possess multiple stationary states, some of them unstable.

The analysis confirms, however, that even large devaluations must improve the balance of payments if the economy is initially at a stable stationary position. A by-product of the analysis is a pricing formula for the financial exchange rate in a dual exchange rate system. That formula is consistent with recent consumption-based models of asset pricing.

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Introduction

A number of controversies in international monetary economics have centered on the analysis of devaluation. Most recently, accounts of the monetary approach to the balance of payments have used the devaluation example to illustrate some basic insights of that view (see Mundell [1971], Dornbusch [1973], and Frenkel and Mussa [1984]). A typical exposition assumes that there is no private international capital mobility, so that the aggregate real money stock is predetermined. Devaluation, by instantaneously reducing real balances, leads to a flow excess demand for money, a temporary balance-of-payments surplus, but no change in the economy's unique long-run *real* equilibrium. The result is intuitively appealing, as it conforms to the principle that changes in nominal variables should have no permanent real effects.

This paper re-evaluates the foregoing predictions about devaluation in a simple optimizing model of an economy with capital controls. Unlike the models used to illustrate the monetary approach, the model used here recognizes that central-bank foreign reserves generally earn interest. This modification leads to two important conclusions. First, devaluation must lead to an increase in the economy's long-run consumption and real balances, and thus is nonneutral. Second, the economy may have multiple steady states, some of them unstable, even when all goods are normal.

In an optimizing model similar to the one employed here, Calvo [1981] has obtained the monetary-approach conclusion that devaluation is asymptotically neutral when central-bank reserves earn no interest and there is no private capital mobility. As I show below, however, devaluation is not in general neutral when capital is mobile *unless* central-bank reserves earn interest at the market rate. The assumption that reserves earn interest would therefore seem to be a natural one to make in an investigation of the effects of devaluation under capital controls. While the central bank's reserve transactions do enable the public to engage in net external saving and dissaving under capital immobility, however, the interest rate perceived by the public can differ from the one available to the bank. This distortion lies behind the economy's peculiarities, and I model the domestic real interest rate explicitly by introducing a dual foreign exchange rate for financial transactions. A characterization of the dual exchange rate and its dynamic behavior is a by-product of the analysis.[1]

The paper is organized as follows.

Section I studies the benchmark case of perfect capital mobility, showing that under certain idealized conditions (including interest-bearing official reserves), devaluation is fully neutral, even in the short run. This result was demonstrated previously in Obstfeld [1981] under assumptions about individual preferences somewhat different from those made below.

Section II modifies the previous section's model by adding an official prohibition on private capital movements. Individuals are still allowed to hold a fixed pool of "financial" foreign exchange, and the price of this asset in terms of consumption defines a floating dual exchange rate. It is shown that the equilibrium dual exchange rate is given by a standard consumption-based asset pricing formula.

Section III studies the dynamics of the resulting model. Because the return to national saving, the world interest rate, differs from the domestic interest rate motivating private saving decisions, the usual saddle-path stability condition characterizing steady states of optimizing models need not hold. Further, there may exist several long-run equilibria, some saddle-path stable and some entirely unstable.

Section IV analyzes the effect of a permanent, unanticipated devaluation on an economy initially at a stable long-run equilibrium. Devaluation always leads to a balance-of-payments surplus; but because the counterpart of this surplus is an increasing stock of official interest-bearing reserves, the economy's new long-run position is characterized by a level of national income higher than that prevailing initially. It follows that when capital is immobile, devaluation generally has real effects both in the short and long runs.

Concluding remarks are contained in Section V. An appendix deals with some technical matters.

I. Capital Mobility and the Neutrality of Devaluation

This section develops a simple optimizing model with capital mobiliity in which an unanticipated devaluation is fully neutral. As is discussed below, some idealized features of the economy described guarantee this neutrality. But the model is a useful benchmark for the discussion to follow, which shows how the imposition of capital controls renders devaluation nonneutral in the short and long runs even in a frictionless setting.

Residents of the economy consume a single perishable consumption good that may be freely imported or exported. Domestic output of the good is constant at level y. The only two assets held by residents are a domestic money not held by foreigners and an internationally traded consol-type bond paying r units of the consumption good per unit time in perpetuity. The economy is small in the world's markets for goods and bonds: both the foreign-currency price of the consumption good, P^*, and the world real bond price, q^*, are exogenous, and assumed fixed. In the model of this section, there are no official restrictions on international capital movements. An implication is that the real rate of interest, $\rho^* \equiv r/q^*$, is given by the world capital market and constant.

A representative immortal consumer inhabits the economy. The consumer's instantaneous utility is a function of consumption, c, and real balance holdings, m, where the latter variable is defined as nominal money holdings, M, divided by the domestic-currency price of consumption, P. Maximization of the intertemporal welfare criterion

$$V = \int_0^\infty u(c_t, m_t)\exp(-\delta t)dt \tag{1}$$

is the consumer's goal.[2] The parameter δ is a fixed subjective rate of time preference, assumed in this section to equal the world real interest rate ρ^*.[3] The strictly concave instantaneous utility function satisfies $u_c u_{mm} - u_m u_{cm} < 0$, $u_m u_{cc} - u_c u_{cm} < 0$, so that both goods are normal.

To simplify the subsequent discussion of the central bank's balance sheet, I consider the case in which the exchange rate (the domestic-currency price of foreign exchange) is fixed except for discrete, unanticipated devaluations. As will be evident, the analysis would easily generalize to an economy in which the exchange rate, though pegged at each instant, is expected to move continuously over time (see Calvo [1981] or Obstfeld [1981]). Under a fixed exchange rate E, goods-market arbitrage guarantees that P equals EP^*, and because P^* is fixed, the expected domestic inflation rate is zero. The opportunity cost of holding money is then the real interest rate ρ^*, so the consumer's lifetime budget constraint is

$$\int_0^\infty (c_t+\rho^* m_t)\exp(-\rho^* t)dt \leq m_0+q^* b_0+(y/\rho^*)+\int_0^\infty \tau_t \exp(-\rho^* t)dt, \qquad (2)$$

where b_0 is the (net) number of bonds issued to the domestic private sector by foreigners up to (and including) time 0 and τ_t represents expected real time-t transfer payments from the government.[4]

The central bank holds as foreign reserves consols issued by foreigners. Since the domestic public does not hold foreign currency -- which yields neither utility nor interest -- the central bank can hold the exchange rate constant at E by exchanging foreign bonds for home money whenever there is excess demand or supply in the home money market. Earnings on central-bank foreign reserves are turned over to the government and other central-bank assets earn no interest. If government consumption is zero, the consolidated public-sector budget constraint is

$$\tau_t = rf_t \qquad (3)$$

where f_t devotes the central bank's stock of foreign reserves.

The central-bank balance sheet links official foreign reserve holdings to the (high-powered) domestic money stock, m. Assume for simplicity (and without loss of generality) that no devaluations or changes in q* have occurred in the past. As long as no devaluations do occur, the real money stock is given by

$$m_t = q^* f_t + (D_0/P) \qquad (4)$$

for $t \geq 0$, where D_0 (assumed constant) is the nominal supply of domestic credit.[5]

The assumption that central-bank foreign assets earn interest plays a key role in the analysis, and will be discussed further below. It should be noted now, however, that the assumption is certainly more realistic than the alternative hypothesis that the central bank willingly forgoes interest income.[6]

To solve the individual's problem, form the Lagrangian expression

$$L = \int_0^\infty u(c_t,\ m_t)\exp(-\delta t)dt + \lambda_0\Big[m_0+q^*b_0+(y/\rho^*)+\int_0^\infty \tau_t\exp(-\rho^* t)dt-$$
$$\int_0^\infty (c_t+\rho^* m_t)\exp(-\rho^* t)dt\Big]. \qquad (5)$$

Differentiation of (5) leads to the necessary conditions

$$u_t(c_t,m_t) = \lambda_0, \qquad (6)$$

$$u_m(c_t,m_t) = \lambda_0\rho^* \qquad (7)$$

for all t. Note that (6) and (7) imply that equilibrium c_t and m_t must remain <u>constant over time</u>. To derive the unique shadow value of wealth $\tilde{\lambda}_0$ associated with an optimal individual program, use (6) and (7) to express desired consumption and real balances as explicit functions of λ_0 and ρ^*: $c_t = c(\lambda_0,\rho^*)$, $m_t = m(\lambda_0,\rho^*)$. Then $\tilde{\lambda}_0$ allows the lifetime budget constraint (2) to hold with equaliity, and thus is the solution to

$$[c(\lambda_0,\rho^*)/\rho^*]+m(\lambda_0,\rho^*) = m_0+q^*b_0+(y/\rho^*)+\int_0^\infty \tau_t\exp(-\rho^* t)dt. \qquad (8)$$

The model cannot be closed until the process generating expectatons of future government transfers is specified. For this purpose the economy's equilibrium is assumed to be a perfect foresight equilibrium, in the sense that the equilibrium expected future path of transfer payments $\{\hat{\tau}_t\}_{t=0}^{\infty}$ induces a path of official reserves $\{\hat{f}_t\}_{t=0}^{\infty} = \{[\hat{m}_t - (D_0/P)]/q^*\}_{t=0}^{\infty}$ such that condition (3), $\hat{\tau}_t = r\hat{f}_t$, holds for all t. As noted above, the money market is brought into equilibrium at each instant through official foreign exchange intervention -- in effect, exchanges of foreign bonds between the domestic public and central bank. Real balances m may therefore jump at a point in time. It is important to remember, though, that the <u>sum</u> b + f of net claims on foreigners held by the economy as a whole is a predetermined or non-jumping variable that can change only over time through current account deficits and surpluses. Instantaneous domestic portfolio adjustment can alter the money supply, but it cannot result in a transfer of real resources between countries.

Substitution of (3) and (4) into (8) with $m_t = m(\tilde{\lambda}_0, \rho^*)$ for $t > 0$ yields the equation

$$c(\tilde{\lambda}_0, \rho^*)/\rho^* = q^*(b_0+f_0)+(y/\rho^*). \tag{9}$$

When no devaluation is expected, the <u>equilibrium</u> shadow price of wealth $\hat{\lambda}_0$ is the solution to (9), and can be expressed as a function of predetermined or exogenous variables only:

$$\hat{\lambda}_0 = \hat{\lambda}_0(b_0+f_0, y, \rho^*). \tag{10}$$

($\hat{\lambda}_0$ is unique because, as is easily verified, $c_{\lambda_0} < 0$ under the normality assumptions.) Since domestic money is a nontraded asset, the economy's discounted planned consumption cannot in equilibrium exceed the capitalized value of its real resources. The equilibrium $\hat{\lambda}_0$ may be interpreted as the unique shadow price of wealth consistent with both individual optimality and this aggregate intertemporal constraint.

The effects of an unanticipated devaluation at time 0 (an increase in the exchange rate to E' from E) may now be analyzed. The devaluation occasions an incipient excess demand for domestic real balances, but this is eliminated immediately as residents sell the central bank foreign bonds to rebuild their money holdings. This process of portfolio readjustment removes foreign claims from private ownership and places then under government ownership, but leaves the sum $b_0 + f_0$ unaltered. Equation (10) therefore shows that $\hat{\lambda}_0$ -- and thus consumption and real balances -- is unchanged. Agents can instantaneously restore their real balances yet maintain their previous consumption level because the foreign interest payments they sacrifice to rebuild their money holdings are [by (3)] returned to them in the form of higher transfers. Essentially for this reason, devaluation is fully neutral.

For future reference, it is useful to derive the effect on the central-bank balance-sheet relation (4) of the unanticipated devaluation at time 0. Let $\varepsilon \equiv (E' - E)/E'$ denote the percentage devaluation. After devaluation, but before private portfolio readjustment has occurred, the <u>real</u> money supply is given, not by (4), but by

$$m_0 = q^*f_0 + (D_0/P) - \varepsilon\left[q^*f_0 + (D_0/P)\right], \tag{11}$$

provided the central bank does not monetize the nominal capital gains on its foreign reserves.[7] The devaluation thus causes an incipient reduction in real money, but as we have seen, an immediate real private capital inflow equal to $\varepsilon[q^*f_0 + (D_0/P)]$ prevents such a reduction from occurring in equilibrium. Because $\Delta m_t = q^* \Delta f_t$ if no further devaluations occur, the real central-bank balance sheet relation becomes

$$m_t = q^*f_t + (D_0/P) - \varepsilon\left[q^*f_0 + (D_0/P)\right] \tag{12}$$

in future periods. Real money is then equal to real foreign reserves plus real domestic credit <u>minus</u> an adjustment reflecting the fact that official reserves purchased up to time 0 were purchased at a money price EP^*q^* lower than the current price $E'P^*q^*$.

It was observed earlier that certain idealized features of the present economy are necessary in order that devaluation be neutral in the short run. If central bank reserves do not earn interest, the capital inflow following devaluation causes a fall in national income and in consumption (Frenkel and Rodriguez [1975]); devaluation essentially acts as a tax in this case. If bonds are claims to nominal payments in different currencies, devaluation can alter domestic wealth by changing the real value of the net external debt (Boyer [1977] and Lapan and Enders [1978]). If the economy is inhabited by overlapping generations with finite horizons, devaluation has real effects if it leads to redistribution of income across generations. Abstraction from these conventional sources of nonneutrality is necessary in order to isolate conceptually the impact of capital controls.

II. Capital Controls and the Financial Exchange Rate

The assumption of unrestricted external asset trade is now abandoned. Under the regime of capital controls postulated here, domestic residents can neither buy nor sell bonds abroad, and any foreign exchange earnings must be converted immediately into domestic money at the central bank. A domestic bond market continues to operate, and, as before, home bonds are perfect substitutes for claims on foreigners making up a pool $\bar{b}$ of "investment currency" that the home public is allowed to hold.

The central bank now buys and sells foreign exchange for commercial purposes only, pegging the commercial exchange rate (initially at E). This suffices to peg the home price level at EP^*. Because the central bank does not intervene in the investment currency market, the stock of foreign bonds $\bar{b}$ held by the public is fixed. The real price of such bonds, q, need no longer equal the world price, q^*, and can be thought of as a dual, financial exchange rate. Correspondingly, the domestic real interest rate $\rho = (r+\dot{q})/q$ can differ from the world rate ρ^*. Both q and ρ will be determined endogenously in the model developed in this section.

Capital-account restrictions have several major effects on the functioning of the economy. First, while individuals can still borrow or lend, the private sector cannot in the aggregate alter its net claims on foreigners. Second, while individuals can instantaneously alter their money holdings, the domestic real money supply is, in the aggregate, a predetermined variable. Third, the economy as a whole (i.e., the private sector plus authorities) can alter its net foreign asset position over time through balance-of-payments disequilibria (which under capital immobility correspond to current-account disequilibria); but in this process, the central bank acts as an intermediary for the domestic public, acquiring foreign bonds when the public desires growing real balances and decumulating reserves in the opposite case. Fourth, the divergence (noted above) between the domestic equilibrium real interest rate ρ and the rate ρ^* available to the central bank introduces a new distortion into the economy.[8] As we shall see, this distortion may result in an economy with multiple stationary positions, some of them unstable.

The representative individual's problem is now to maximize V [given by

(1)] subject to the constraint

$$\int_0^\infty (c_t+\rho_t m_t)\exp(-\int_0^t \rho_s ds)dt \leq m_0+q_0b_0+\int_0^\infty (y+\tau_t)\exp(-\int_0^t \rho_s ds)dt, \tag{13}$$

where $\{\rho_t\}_{t=0}^\infty=\{(r+\dot{q}_t)/q_t\}_{t=0}^\infty$ is the expected real interest rate path. Differentiation of a Lagrangian expression analogous to (5) yields the necessary conditions

$$u_c(c_t,m_t) = \lambda_0\exp(\delta t-\int_0^t \rho_s\, ds), \tag{14}$$

$$u_m(c_t,m_t) = \lambda_0\rho_t \exp(\delta t - \int_0^t \rho_s ds), \tag{15}$$

for all t. It is clear from (14) and (15) that consumption and real balances need no longer be constant over time.

Because the financial exchange rate is endogenous, it is difficult to derive an explicit, closed-form characterization of the economy's equilibrium as in the previous section. A diagrammatic approach is therefore adopted here. Differentiation of (14) yields the relationship

$$\dot{u}_c(c_t,m_t) = u_c(c_t,m_t)\,(\delta - \rho_t), \tag{16}$$

according to which utility "capital gains" on wealth must match any excess of the subjective discount rate over the real rate of return. By (15), (16) can be written

$$u_{cc}(c_t,m_t)\dot{c}_t + u_{cm}(c_t,m_t)\dot{m}_t = u_c(c_t,m_t)\,\delta - u_m(c_t,m_t). \tag{17}$$

If $a_t \equiv m_t + q_tb_t$ denotes real marketable assets, the identity linking asset accumulation to saving is[9]

$$\dot{a}_t = y + rb_t + \tau_t + \dot{q}_t b_t - c_t. \tag{18}$$

Substituting the government budget constraint (3) and the central-bank balance-sheet relation (4) into (18), one obtains:

$$\dot{m}_t + q_t \dot{b}_t = y + rb_t + \rho^* \left[m_t - (D_0/P)\right] - c_t \tag{19}$$

In perfect-foresight equilibrium, it is also true that $b_t = \bar{b}$ for all t, so that $\dot{b}_t = 0$ for all t. In equilibrium, then, (19) implies that the evolution of real balances is governed by the equation

$$\dot{m}_t = y + r\bar{b} + \rho^*[m_t - (D_0/P)] - c_t. \tag{20}$$

Equations (17) and (20) together give the equilibrium motion of consumption:

$$\dot{c}_t = \frac{1}{u_{cc}(c_t, m_t)} \Big(u_c(c_t, m_t)\delta - u_m(c_t, m_t) - u_{cm}(c_t, m_t)\{y + r\bar{b} + \rho^*[m_t - (D_0/P)] - c_t\}\Big). \tag{21}$$

By (16), (20), and (21), a stationary state $(\bar{q}, \bar{m}, \bar{c})$ of the economy is defined by the relations

$$\bar{q} = r/\delta, \tag{22}$$

$$\bar{c} = y + r\bar{b} + \rho^*[\bar{m} - (D_0/P)], \tag{23}$$

$$u_m(\bar{c}, \bar{m})/u_c(\bar{c}, \bar{m}) = \delta \tag{24}$$

(recall the definition of ρ as $(r+\dot{q})/q$). It is assumed henceforth that at least one stationary state exists.

Equations (20) and (21) together define a complete dynamic system in real balances and consumption: the dual exchange rate q enters neither equation. It follows that co-movements in m and c may be analyzed diagrammatically without reference to movements in q. The paths of the two former variables do affect that of q, however, for equation (16) may be integrated to yield

$$q_0 = \int_0^\infty \left[\frac{u_c(c_t,m_t)}{u_c(c_0,m_0)}\right] r \exp(-\delta t)\, dt$$
$$= \int_0^\infty r \exp\left(-\int_0^t \rho_s\, ds\right) dt. \tag{25}$$

According to (25), the real financial exchange rate has two equivalent interpretations. First, as in consumption-based models of asset pricing (e.g., Grossman and Shiller [1981]), q is an integral of future physical yields, weighted by marginal rates of substitution between present and future consumption and discounted at the subjective time preference rate. Second, q may be thought of as an integral of future physical yields discounted by market real interest rates.

III. Dynamics

To analyze the dynamics of real balances and consumption, it is convenient initially to linearize the system consisting of (20) and (21) in the neighborhood of a stationary state. There is no guarantee, however, that the steady state (if it exists) is unique. The easiest way to see this is to graph equation (23), which gives long-run consumption as a function of long-run central-bank reserves, together with (24), the Engel curve associated with the price δ. Because (23) -- also the locus along which $\dot{m}=0$ -- defines an upward-sloping line, the Engel curve may intersect it several times even when both goods are normal, as in figure 1. (Figure 1 also shows that there could easily be no stationary state.) Of course when central bank reserves earn no interest, as in Calvo (1981) and the

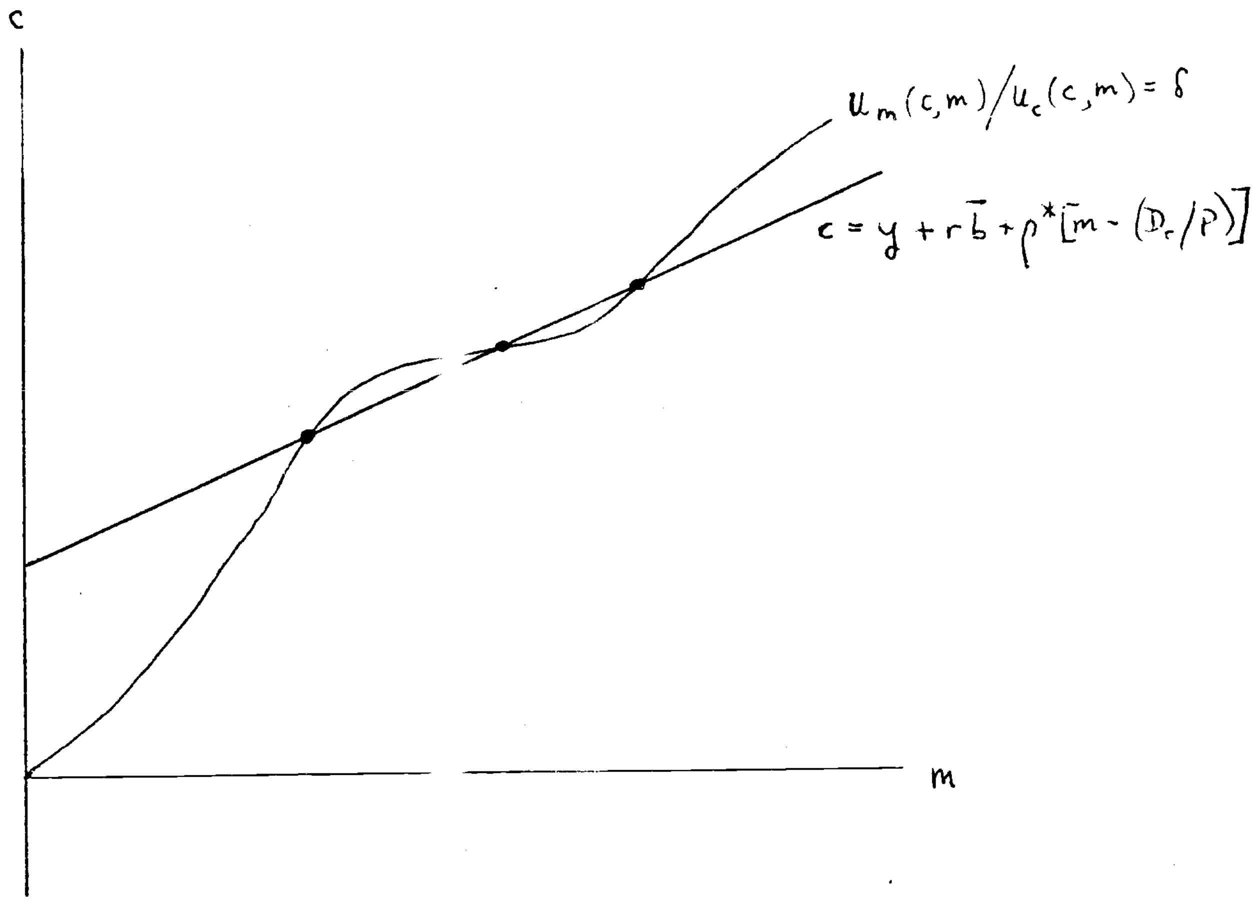

Figure 1

earlier monetary-approach literature, the $\dot{m}=0$ locus is horizontal at $\bar{c} = y+r\bar{b} > 0$, and normality of both goods ensures the existence of a unique long-run equilibrium.

While one could rule out multiple stationary states under certain assumptions (e.g., homothetic preferences), there is no compelling reason for doing so. The following section will analyze devaluation from both the local and global perspectives.

Let $\bar{m}$ and $\bar{c}$ be stationary levels of real balances and consumption. The linear approximation to (20) and (21) around those levels is

$$\begin{bmatrix} \dot{m}_t \\ \dot{c}_t \end{bmatrix} = \begin{bmatrix} \rho^* & -1 \\ \dfrac{\bar{u}_{cm}(\delta-\rho^*) - \bar{u}_{mm}}{\bar{u}_{cc}} & \delta \end{bmatrix} \begin{bmatrix} m_t - \bar{m} \\ c_t - \bar{c} \end{bmatrix}, \tag{26}$$

where functions beneath overbars are evaluated at $(\bar{m},\bar{c})$. The condition for saddle-path stability of (26) is that the system have real characteristic roots of opposite sign. As the determinant of the matrix in (26) is the product of those characteristic roots, the stability condition may be written

$$\frac{\bar{u}_{mm} - \delta\bar{u}_{cm}}{\delta\bar{u}_{cc} - \bar{u}_{cm}} > \rho^*. \tag{27}$$

Figure 2 displays the phase portrait of (26) under the assumption that (27) holds. The stability assumption implies that the slope of the $\dot{c}=0$ locus is positive, exceeding that of the $\dot{m}=0$ locus, as shown. It is assumed that, given an initial level of real balances, the equilibrium consumption level is the unique level placing the economy on the convergent

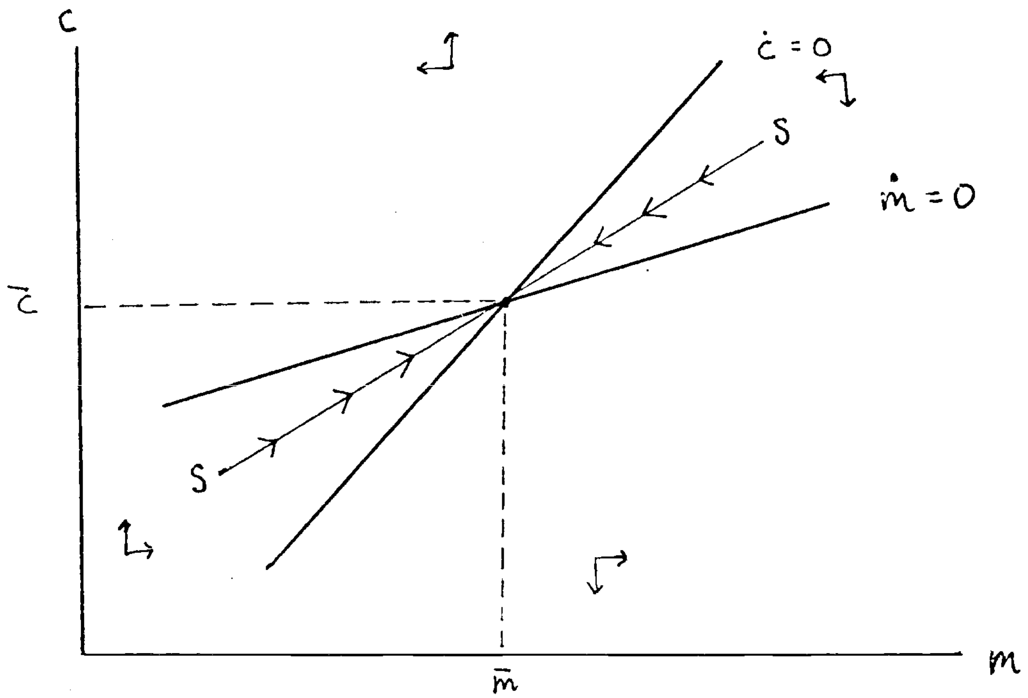

Figure 2

trajectory SS.

Inequality (27) has a straightforward interpretation. The left-hand side of the inequality is just the slope of an Engel curve such as that pictured in figure 1. The inequality thus states that no stable trajectory will exist unless an increase in real balances induces an increase in consumption exceeding the world interest rate ρ^*. The logic of this condition is evident. Suppose there is a current-account (i.e., balance-of-payments) surplus. An increase Δm in real balances is associated with an increase $q^* \Delta f$ in central-bank foreign reserves and hence an increase $\rho^* \Delta m$ in national income. Because real balances are a normal good, the rise in income induces a further rise in desired real balances, and unless absorption is rising faster than national income (i.e., unless $\Delta c > \rho^* \Delta m$), the current account surplus will widen over time rather than shrink. Figure 3 depicts a stationary position around which (27) does not hold.[10]

The foregoing discussion implies an easy saddle-path stability criterion for the multiple stationary states shown in figure 1. Because the $\dot{m}=0$ locus has slope ρ^*, any point at which the Engel curve cuts it from below is a saddle point. Similarly, any point at which the Engel curve cuts the $\dot{m}=0$ locus from above is an unstable stationary state. Figure 4 shows an example of a dynamic system with several stationary states. Note that the Engel curve coincides with the global $\dot{c}=0$ locus only when the instantaneous utility function is separable in consumption and real balances ($u_{cm}=0$).

There are obviously several potential pathologies. For example, if the $\dot{c} = 0$ locus first intersects the $\dot{m} = 0$ locus from above, there is no convergent equilibrium to the left of the first (i.e., lowest real-balance) steady state. Similarly, if the $\dot{c} = 0$ locus intersects the $\dot{m} = 0$ locus

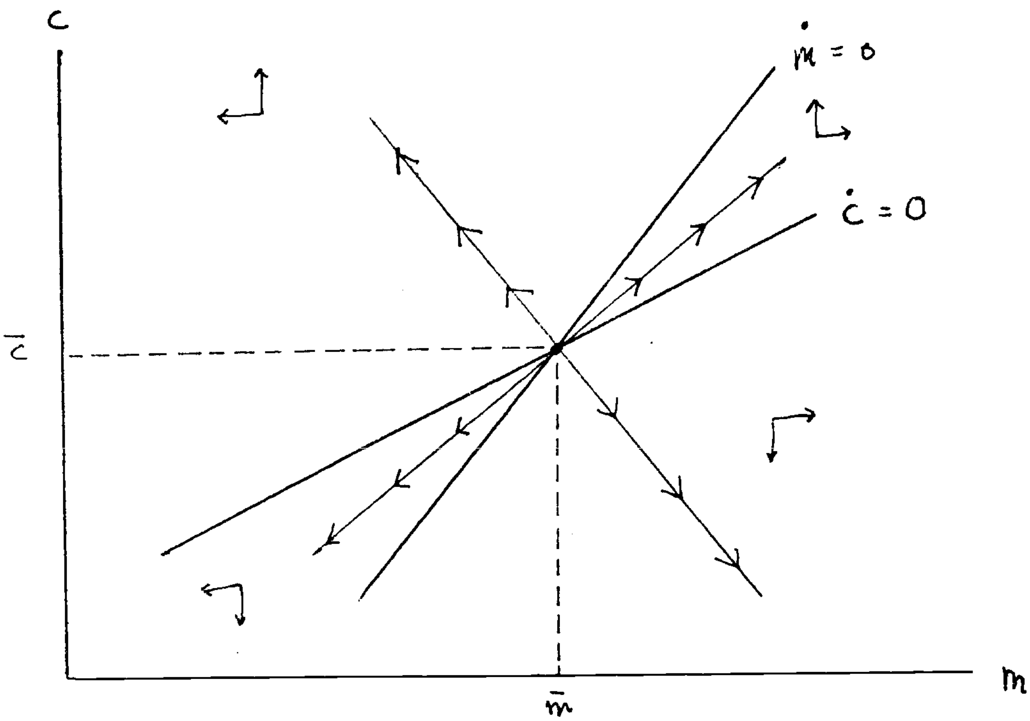

Figure 3

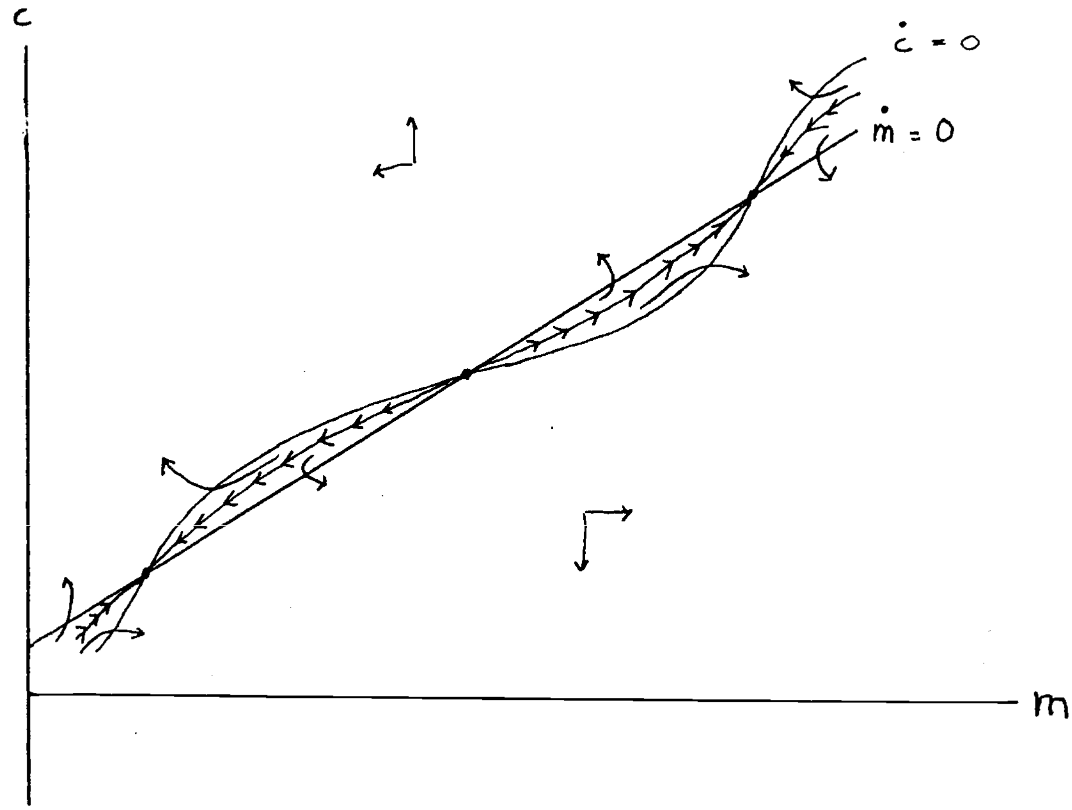

Figure 4

from above at its last intersection, there is no convergent equilibrium to the right of the last (i.e., highest real-balance) steady state. To rule out these cases, I assume that for every real-balance level, there exists a value of consumption placing the economy on a stable trajectory.[11] The usual sufficient conditions imply that these stable paths are equilibria for the economy.

Using (25), it is easy to describe the behavior of the dual exchange rate q along a convergent path. When the balance of payments is in deficit, say, consumption is high relative to its ultimate level, and the marginal utility of consumption is low. Thus, q exceeds its long-run level $\bar{q} = r/\delta$, and, by (25), falls toward it as the economy converges to external equilibrium. This heuristic argument indicates that a deficit is accompanied by a falling (or appreciating) financial exchange rate, and a surplus by a rising (or depreciating) rate. An appendix presents a rigorous proof of the proposition.[12]

IV. Nonneutral Devaluation

The preceding apparatus may be used to demonstrate that when central-bank reserves earn interest and there are no private capital movements, devaluation alters the economy's long-run equilibrium. This section first analyzes the effect of a small, unanticipated devaluation when the economy is initially at a saddle-path stable stationary equilibrium. The result of the experiment is always a balance-of-payments surplus that leads to a rise in long-run consumption and real balances. A global analysis of devaluation yields essentially the same outcome. Even when there are multiple equilibria, a large devaluation can never "leap-frog" an unstable stationary state and cause a balance-of-payments deficit.

Consider first the local analysis of a small, unanticipated devaluation on the assumption that the economy is initially at a stable long-run equilibrium $(\bar{m},\bar{c})$ such as that shown in figure 2. A change in the exchange rate (to E' from E) generally shifts both the $\dot{m}=0$ and the $\dot{c}=0$ loci, for this policy action alters the balance-sheet relation linking the real money stock and the stock of foreign reserves. By (4) and (12), the post-devaluation equations of motion for the economy are

$$\dot{m}_t = y + r\bar{b} + \rho^* \left[m_t - (D_0/P) + \varepsilon\bar{m}\right] - c_t \ , \tag{28}$$

$$\dot{c}_t = \frac{1}{u_{cc}(c_t,m_t)} \Big(u_c(c_t,m_t)\,\delta - u_m(c_t,m_t) - u_{cm}(c_t,m_t) \left\{y+r\bar{b}+\rho^*\left[m_t - (D_0/P) + \varepsilon\bar{m}\right] - c_t\right\}\Big), \tag{29}$$

where again, $\varepsilon \equiv (E'-E)/E'$ and $\bar{m}$ is the pre-devaluation real-balance level.

Near $(\bar{m},\bar{c})$, the vertical shift of the $\dot{m}=0$ locus is

$$\left.\frac{dc}{d\varepsilon}\right|_{\dot{m}=0} = \rho^*\bar{m} > 0 \ ,$$

while that of the $\dot{c}=0$ locus is

$$\left.\frac{dc}{d\varepsilon}\right|_{\dot{c}=0} = \frac{\bar{m}\ \bar{u}_{cm}\ \rho^*}{\bar{u}_{cc}\ \delta} \gtrless 0 \ .$$

If the $\dot{c}=0$ schedule shifts downward, long-run consumption and real balances necessarily rise. But long-run consumption and real balances rise also if the $\dot{c}=0$ schedule shifts upward, for

$$\left.\frac{dc}{d\varepsilon}\right|_{\dot{m}=0} - \left.\frac{dc}{d\varepsilon}\right|_{\dot{c}=0} = \bar{m}\rho^* \left(\frac{\bar{u}_{cc}\ \delta - \bar{u}_{cm}}{\bar{u}_{cc}\ \delta}\right) > 0.$$

Figure 5 shows the impact and long-run effects of a small unanticipated devaluation for the case in which the $\dot{c}=0$ locus shifts upward. Because the impact fall in real balances equals the leftward shift of the $\dot{m}=0$ locus,

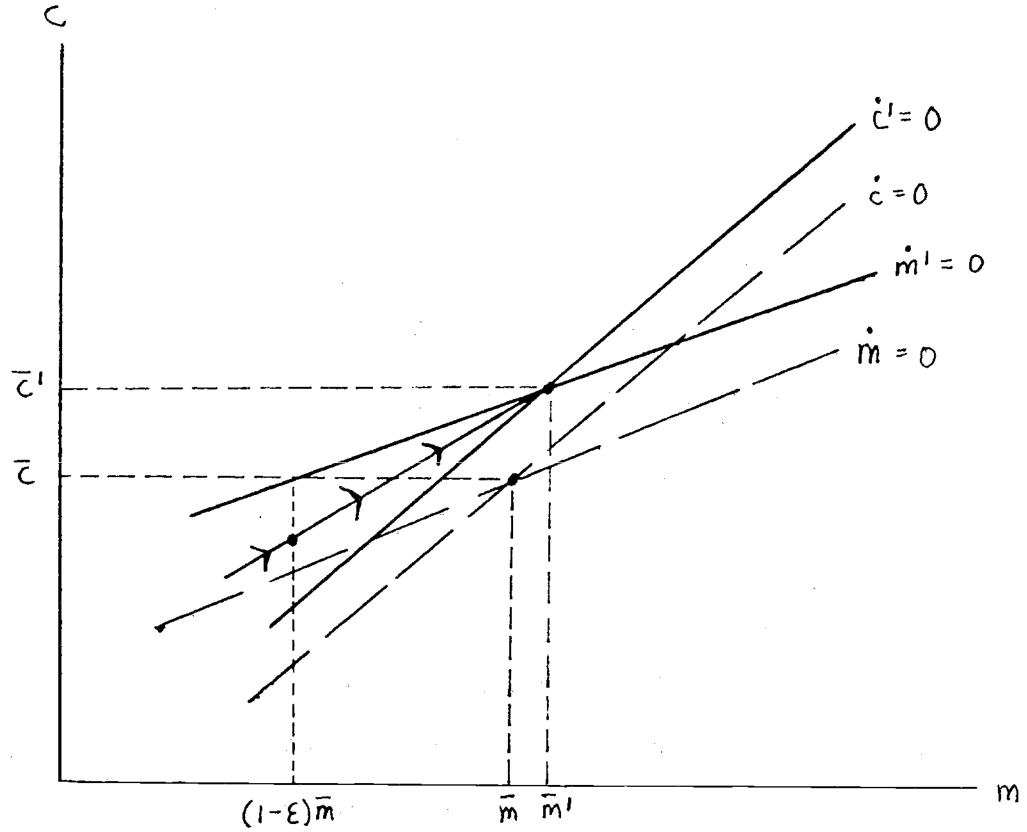
c
$\dot{c}' = 0$
$\dot{c} = 0$
$\dot{m}' = 0$
$\dot{m} = 0$
$\bar{c}'$
$\bar{c}$
$(1-\varepsilon)\bar{m}$
$\bar{m}$
$\bar{m}'$
m

Figure 5

consumption falls in the short run. Corresponding to the public's desire to rebuild its real balances is a balance-of-payments surplus. As real balances rise over time, however, central-bank reserves rise, and because these earn interest at rate ρ^*, national income grows as well. At the new long-run equilibrium $(\bar{m}', \bar{c}')$, consumption and real money demand are therefore higher than before the devaluation.[13]

Section III's results imply that the dual exchange rate q is also affected by the devaluation. The sudden reduction in real wealth reduces private demand for foreign bonds; equilibrium requires a fall in their price and a depreciating financial exchange rate. Over time, expectations are fulfilled as q rises toward r/δ. In the short run, therefore, an unanticipated devaluation causes the real interest rate to rise.

The preceding results were derived using "local" arguments. Must the conclusions be modified when large devaluations are considered? It turns out that no modification is needed: a large devaluation can never lead to a long-run decline in consumption and real balances.

Figure 6, which combines the consumer's Engel curve with the $\dot{m}=0$ locus, is used to make this point. If initial real balances at the stable equilibrium A are $\bar{m}$, devaluation reduces them to $(1-\varepsilon)\bar{m}$, and also shifts the $\dot{m}=0$ schedule leftward by $\varepsilon\bar{m}$. The Engel curve's position does <u>not</u> change.

The economy will now travel to one of the two stationary positions closest to C, labelled B and D in figure 6. But B must be the stable equilibrium, as shown: with both goods normal, the Engel curve cannot cut $\dot{m}'=0$ from above between C and B and still pass through A, the initial position. Thus, the closest stationary state to left of C is unstable, while

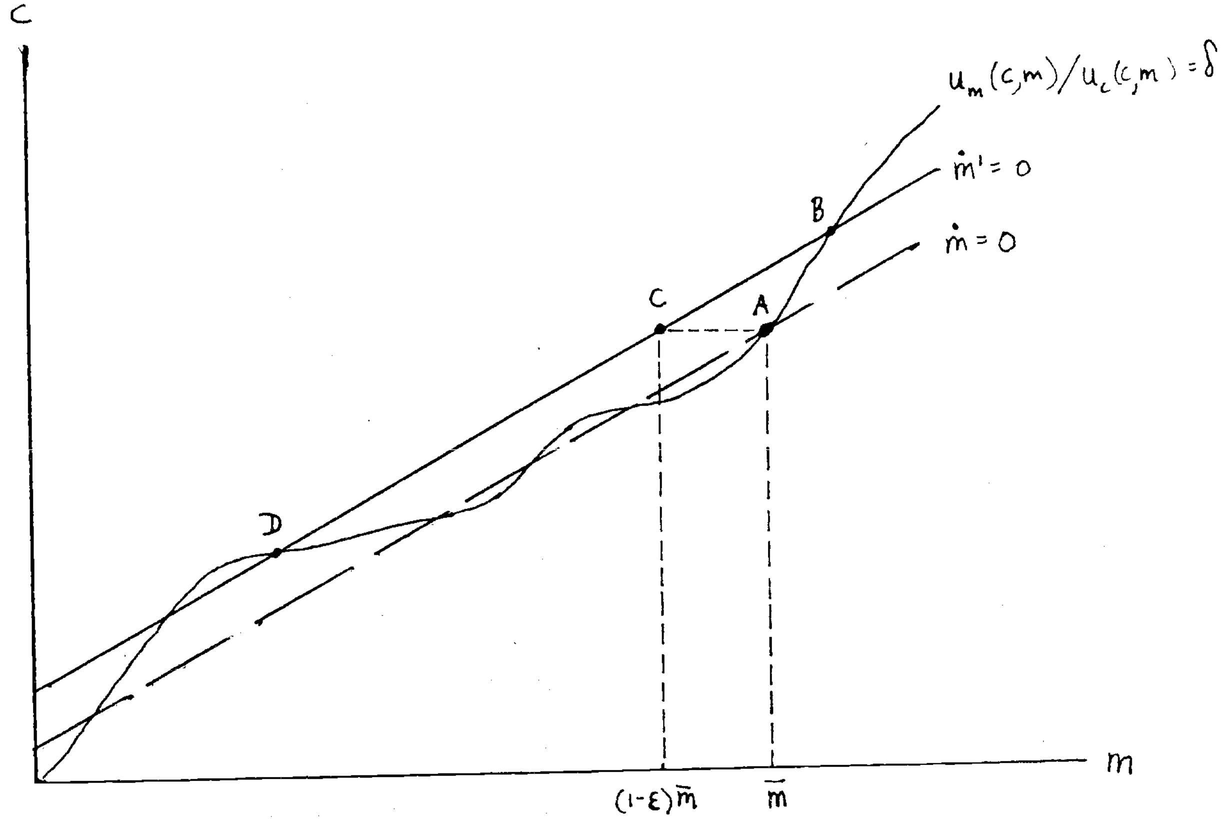

Figure 6

the closest to the right is stable. It follows again that the short-run result of devaluation is a fall in consumption and a balance-of-payments surplus. Ultimately, real balances and consumption rise above their original values as the central bank accumulates foreign reserves.[14]

V. Conclusion

This paper has re-examined the effect of devaluation when there are capital-account restrictions, adding to traditional formulations the realistic assumption that central-bank reserves earn interest. In an intertemporal model, devaluation is no longer neutral in the long run as it is in the monetary-approach literature. Further, the economy may possess multiple long-run equilibria, some of them unstable.

The analysis confirms, however, that even a large devaluation must improve the balance of payments if the economy is initially at a stable stationary position. A by-product of the analysis is a pricing formula for the financial exchange rate in a dual exchange rate system. That formula is consistent with recent consumption-based models of asset pricing.

Appendix

This appendix proves that in the neighborhood of a stable steady state, a balance-of-payments surplus is accompanied by a rising (or depreciating) financial exchange rate q and a deficit by a falling (or appreciating) rate. To see this, linearize the model around a long-run equilibrium $(\bar{q}, \bar{m}, \bar{c})$, defined by (22)-(24). By (14) and (15), $u_m(c_t,m_t)/u_c(c_t,m_t) = \rho_t = (r+\dot{q}_t)/q_t$, so that the dynamics of q are governed by

$$\dot{q}_t = \{[u_m(c_t,m_t)/u_c(c_t,m_t)] - (r/q_t)\}q_t \ .$$

The approximate equations of motion for the economy are therefore

$$\dot{q}_t = \delta(q_t - \bar{q}) + \frac{[r(\bar{u}_c \bar{u}_{mm} - \bar{u}_m \bar{u}_{cm})]}{\delta \bar{u}_c^2} (m_t - \bar{m})$$

$$+ \frac{[r(\bar{u}_c \bar{u}_{cm} - \bar{u}_m \bar{u}_{cc})]}{\delta \bar{u}_c^2} (c_t - \bar{c})$$

and (26).

If stability condition (27) is assumed, the foregoing system has two positive characteristic roots (one of which is δ) and one negative root. Let θ denote the negative root, and $\vec{\omega} = (\omega_1, \omega_2, \omega_3)$ a characteristic vector belonging to θ. The saddle-path equilibrium of the economy is written as

$$q_t - \bar{q} = (\omega_1/\omega_2)(m_0 - \bar{m}) \exp(\theta t),$$

$$m_t - \bar{m} = (m_0 - \bar{m}) \exp(\theta t),$$

$$c_t - \bar{c} = (\omega_3/\omega_2)(m_0 - \bar{m}) \exp(\theta t).$$

Differentiation of this solution yields

$$\dot{q}_t = \theta(\omega_1/\omega_2)(m_t - \bar{m}),$$

$$\dot{m}_t = \theta(m_t - \bar{m}),$$

$$\dot{c}_t = \theta(\omega_3/\omega_2)(m_t - \bar{m}).$$

Because $\omega_3/\omega_2 = \rho^* - \theta > 0$, consumption and real balances rise or fall together during the transition to a stable long-run equilibrium (as the diagrams in the text showed). To establish that q rises during a surplus (when $m < \bar{m}$) and falls during a deficit (when $m > \bar{m}$), it must be shown that $\omega_1/\omega_2 > 0$.

By direct calculation,

$$\omega_1/\omega_2 = \frac{r}{\bar{u}_c \quad \delta(\delta-\theta)}\left[(\theta-\rho^*)(\bar{u}_{cm} - \delta\bar{u}_{cc}) - (\bar{u}_{mm} - \delta\bar{u}_{cm})\right].$$

Thus ω_1/ω_2 is positive if and only if

$$\rho^*\delta\bar{u}_{cc} - \bar{u}_{mm} + (\delta-\rho^*)\bar{u}_{cm} > -\theta(\bar{u}_{cm} - \delta\bar{u}_{cc}) \tag{A1}$$

[the left-hand side of (A1) is positive by (27)]. Because

$$\theta = \frac{\rho^* + \delta}{2} - \left\{\frac{(\rho^*+\delta)^2}{4} - \left[\rho^*\delta + \frac{\bar{u}_{cm}(\delta-\rho^*)-\bar{u}_{mm}}{\bar{u}_{cc}}\right]\right\}^{1/2}$$

and $\bar{u}_{cm} - \delta\bar{u}_{cc} > 0$, (A1) is equivalent to

$$\frac{\rho^* + \delta}{2} + \frac{\rho^*\delta\bar{u}_{cc} - \bar{u}_{mm} + (\delta-\rho^*)\bar{u}_{cm}}{\bar{u}_{cm} - \delta\bar{u}_{cc}} > \left\{\frac{(\rho^*+\delta)^2}{4} - \left[\rho^*\delta + \frac{\bar{u}_{cm}(\delta-\rho^*)-\bar{u}_{mm}}{\bar{u}_{cc}}\right]\right\}^{1/2} \tag{A2}$$

Squaring both (positive) sides of (A2) yields an inequality which, after some manipulation, can be shown to be equivalent to

$$\bar{u}_{cc}\bar{u}_{mm} - \bar{u}_{cm}^2 > 0 \quad . \tag{A3}$$

But (A3) is true because the instantaneous utility function is strictly concave. Thus $\omega_1/\omega_2 > 0$, as was to be demonstrated.

Footnotes

*This paper was written while the author was a visitor at the Department of Economics, Tel-Aviv University. Financial support from the National Science Foundation is gratefully acknowledged.

1. Descriptive models of two-tier exchange-rate systems are developed by Flood [1978], Marion [1981], Aizenman [1983], Kiguel [1983], and Cumby [1984]. Calvo [1979] presents a related model in which nonreproducible "land" plays the role of foreign assets under capital controls. Adams and Greenwood [1983] use a two-period optimizing model to study a regime in which the authorities intervene to fix both the commerical and financial exchange rates.

2. The neutrality result established in this section is not sensitive to the particular preference setup adopted. Obstfeld [1981] gives an example of devaluation neutrality under time-inseparable tastes.

3. The assumption $\delta=\rho^*$ is made only to ensure the existence of a non-degenerate stationary equilibrium for the economy; see, for example, Obstfeld and Stockman [1984], section 5.1. It should be clear, however, that devaluation is neutral in the present model when $\delta\neq\rho^*$. The assumption $\delta=\rho^*$ is relaxed in the following sections.

4. Because there is a single representative individual, the individual's net holding of domestically-issued bonds is zero.

5. In a more general formulation, D_0 would would reflect also past devaluations (as discussed below) and past changes in q^*.

6. Of course if central-bank reserves f were negative -- implying past official borrowing to support the exchange rate -- the central bank would

not have a choice about paying interest (barring default). In this case, according to (3), the public would be taxed to meet official debt-service obligations.

7. Typically the central bank creates a fictitious non-monetary accounting liability instead.

8. There is already a distortion in the economy, even under capital mobility, because the private opportunity cost of real balances is positive and thus exceeds the zero marginal cost at which money can be provided by the central bank.

9. The following equation reflects the assumption that interest payments on foreign bonds are repatriated at the commercial exchange rate.

10. Models based on intertemporal optimization typically imply the existence of a convergent saddle path. That implication cannot be obtained here because of the interest-rate distortion. Descriptive models of external asset accumulation typically need to assume a stability condition like (27), however. For a discussion, see Obstfeld and Stockman [1984], section 4.1.

11. Tangencies are also excluded by assumption.

12. This result also emerges from the descriptive literature cited in footnote 1.

13. Clearly a decrease in the money supply would have a similar long-run impact. Monetary policy (like devaluation) is therefore nonneutral in the present model.

14. A natural question is whether a devaluation has similar effects when the economy is out of long-run equilibrium. It appears that devaluation, in this case, may improve or worsen the balance of payments on impact. However, arguments similar to that in the preceding paragraph suggest that

the long-run equilibrium attained after a devaluation is characterized by higher consumption and real balances than would have been attained in the absence of devaluation.

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