NOMINAL VERSUS INDEXED DEBT:
A QUANTITATIVE HORSE RACE

Laura Alfaro
Fabio Kanczuk

Nominal versus Indexed Debt: A Quantitative Horse Race
Laura Alfaro and Fabio Kanczuk
NBER Working Paper No. 13131
May 2007
JEL No. E6,H63

ABSTRACT

The main arguments in favor and against nominal and indexed debt are the incentive to default through inflation versus hedging against unforeseen shocks. We model and calibrate these arguments to assess their quantitative importance. We use a dynamic equilibrium model with tax distortion, government outlays uncertainty, and contingent-debt service. Our framework also recognizes that contingent debt can be associated with incentive problems and lack of commitment. Thus, the benefits of unexpected inflation are tempered by higher interest rates. We obtain that costs from inflation more than offset the benefits from reducing tax distortions. We further discuss sustainability of nominal debt in developing (volatile) countries.

Laura Alfaro
Harvard Business School
Morgan Hall 263
Soldiers Field
Boston, MA 02163
and NBER
lalfaro@hbs.edu

Fabio Kanczuk
Department of Economics,
Universidade de Sao Paulo
Brazil.
kanczuk@usp.br

1 Introduction

On January 29, 1997, U.S. Treasurer Lawrence Summers auctioned $7 billion of 10 year Treasury Inflation-Protected Securities (TIPS). Before that, U.S. debt was all nominal debt. Brazil, in contrast, after introducing inflation-indexed securities in 1964, is now struggling to reduce the level of indexed-debt. In 2006, close to 20% of the Brazilian debt was nominal (not indexed). What explains these differences? What are the criteria for choosing the optimal amount of nominal debt? In what sense do these factors depend on the characteristics of the country?

The main arguments for or against nominal and indexed debt are the incentive to default through inflation versus the advantage of hedging against unforeseen shocks.[1] In this paper, we run a quantitative horse race between these different rationales. We do this by modeling the different arguments and calibrating the model in order to assess the quantitative importance of each. Despite the vast theoretical literature, few studies have analyzed the trade-offs between nominal and indexed debt by constructing calibrated economy laboratories, a methodology that yields a sharper understanding of the quantitative forces involved in economic processes.

Economists have long proposed using indexed bonds and hence have long been puzzled that most governments in the world do not index government debt.[2] The most compelling argument in favor of indexation is that it eliminates a government's incentive to inflate the economy in order to reduce the real cost of nominal liabilities. Nominal debt, however, allows the government the possibility of hedging against unexpected shocks that affect the fiscal budget and hence reduce tax distortions. Bohn (1988), for example, argues that nominal debt provides a valuable insurance against the budgetary effects of economic fluctuations. Nominal debt may be a desirable form of funding because of the covariance of inflation with government spending: high government spending tends to go with high inflation. Since nominal bonds pay poorly in real terms when inflation is surprisingly high, nominal debt has some of the characteristics of government contingent debt: nominal bonds allow the government to partially default via inflation. Similarly, Calvo and Guidotti (1993) and Chari and Kehoe (1999) show that optimal monetary policy calls for the inflation tax to be employed to pay part of the unanticipated fiscal deficit in a given period. Of course, the incentive to inflate remains. As Calvo and Guidotti (1993) mention, "enlarging the nominal base to reduce inflation and tax volatility may tempt the devil. Thus, the benefits from issuing nominal bonds need to be carefully weighed against the costs resulting from time-inconsistency."

---

[1] Lucas and Stokey (1983), Bohn (1988, 1990), Calvo and Guidotti (1993), Barro (1979, 1997), among others, have studied aspects of debt management decisions. See Fischer (1983) for an overview of the main arguments in favor and against indexed debt.

Summing up, defaulting on nominal liabilities can generate a welfare loss that must be quantitatively weighed against the gains from tax smoothing. This is the task we undertake in this paper. A good starting point to approach this problem is the model developed by Grossman and Van Huyck (1988). These authors treat sovereign debt as a contingent claim, an assumption which follows from the observation of two of the main stylized facts behind sovereign borrowing and defaulting. These are (i) defaults are usually partial and associated with identifiable bad states of nature; (ii) sovereign states often are able to borrow soon after default. Hence, in their model, sovereign defaults occur as equilibrium outcomes of debt-servicing obligations that are implicitly contingent on the realized state of the world. In their view, lenders sharply differentiate excusable defaults, which are justifiable when associated with implicitly understood contingencies, from debt repudiation, which would be inexcusable.[3]

This paper quantitatively applies this model to the problem of determining the optimal amount of contingent debt, which we take to mean nominal debt. For this purpose, we construct and calibrate a dynamic equilibrium model that focuses on the role of contingent debt to intertemporally reduce tax distortions. In the model, nominal debt allows accommodating negative shocks to the government budget. However, the model also recognizes that contingent debt is associated with incentive problems. In fact, the basic intuition behind the model is that when the method of financing is too convenient, the government is likely to abuse it – an intuition that has already been pointed out by such memorable economists as Adam Smith and David Ricardo.[4]

In order to capture this trade-off, we propose an information structure similar to the one used by Cole, Dow and English (1995) and Alfaro and Kanczuk (2005a). We assume there are two types of sovereigns. "Bad" sovereigns are extremely impatient and always default, independently of the state of the economy; "good" sovereigns default optimally (excusably) in order to smooth tax distortions. The equilibrium interest rate is determined by lenders, who signal-extract the type of government. After a default occurs through inflation, lenders become more uncertain about the type of the sovereign, and tend to charge higher interest rates. Consequently, the benefits of defaulting are tempered by higher interest rates in the future which worsen the government's financing problem.

---

[2] Proponents of indexed bonds include strong voices such as Jevons, Marshall, Keynes, Fisher, Musgrave, Friedman, and Tobin.

[3] On more theoretical grounds, an appealing property of a "contingent service" model of sovereign debt, like ours, is that it can support positive amounts of debt even if the sovereign can save after defaulting. The same is not true for "contingent debt" models. See Grossman and Han (1999).

[4] "Were the expense of war to be defrayed always by revenue raised within the year…wars would in general be more speedily concluded and less wantonly undertaken" (Adam Smith, 1791). "War-taxes, then are more economical, for when they are paid, an effort is made to save to the amount of the whole expenditure of the war, leaving the national capital undiminished" (David Ricardo, 1820).

We calibrate our artificial economy to match both the U.S. as an example of a mature economy and Brazil as an example of an emerging market. We chose the Brazilian case for various reasons. The country introduced indexation relatively early (1964) and, as mentioned before, is attempting to reduce the levels of indexed debt. Due to the high inflation experience, public debt in Brazil was virtually all linked to short-term interest rates (overnight).[5] To match the Brazilian economy stylized facts, we calibrate the model with government outlays ten times more volatile than in the United States.

We then undertake our main exercise, which is to find the optimal contingency structure, i.e. the optimal fractions of indexed and non-indexed debt for a given amount of total debt. Our artificial economy experiments suggest that the optimal amount of nominal debt is zero both for U.S. and Brazil. That is, for any amount of nominal debt, the inflation costs more than offset the benefits from tax smoothing. Our quantitative results, consequently, are at odds with the empirical evidence since, as mentioned, nominal debt tends to be the rule rather than the exception in most countries.[6]

We also study a situation in which the government *must* use some amount of non-indexed debt. In this case, the optimal structure is to have no indexed debt at all or, if possible, to have negative amounts of indexed debt (i.e., indexed assets). This perhaps surprising result has a simple explanation. Given certain government expenditure volatility, the required inflation rate in order to smooth taxes in bad periods is smaller the greater the nominal debt. Therefore, the cost associated with inflation, which is quantitatively the most important one, is smaller the greater the non-indexed debt proportion.

Finally, for each economy, we obtain a boundary on the maximum amount of sustainable nominal debt for a given degree of agency costs. Whereas for the U.S. economy this boundary is well beyond the observed values, for our Brazilian artificial economy the debt sustainability problem seems to be quantitative important.[7] That is, given the existing volatility of the government outlays, the attempt of issuing contingent debt in Brazil, as sometimes proposed, may not be feasible. We then suggest that the Brazilian high inflation history can be explained by a time-inconsistency problem, and discuss the viability of using indexed debt as a commitment device.

---

[5] In Brazil, indexing to short-term interest rates was preferred over indexing to consumer price indexes because the latter only captures inflation with one-month delay.

[6] Our calibration exercise, however, shows the welfare gains for the U.S. of changing the structure of the debt from nominal to indexed debt to be low. Other explanations for the observed levels of nominal debt in the U.S. include potential negative costs associated with a gradual transition from nominal to indexed debt, and (not-modeled) costs associated with the use of indexed debt, such as the availability of reliable price indices, and history dependence.

[7] This result can be related to the "debt intolerance" finding of Reinhart, Rogoff and Savastano (2003) which links debt sustainability levels with past histories of default and inflation. In our setup, debt intolerance levels are also associated to the volatility of the economy.

In terms of the framework, our work is similar to Chari and Kehoe's (1993) analysis of debt sustainability, where taxes on labor are the only other way to finance government outlays in addition to defaulting on the debt. Our result about the superiority of indexed debt due to the quantitative importance of incentive problems corroborate the arguments outlined in Barro (1997). This result is also related to the debate around the flexibility of monetary policy addressed in two distinct branches of the literature.

A first branch (Calvo and Guidotti (1993) and Chari and Kehoe (1999)) characterizes the "Ramsey" optimal fiscal and monetary policy in flexible-price environments. This work focuses on economies with nominal non-state-contingent government liabilities and assumes that the government effectively has access to a commitment technology, thus abstracting from time inconsistency problems. The key result that emerges is that it is optimal for the government to make the inflation rate highly volatile, as price changes play the role of shocks absorber of unexpected innovations in the fiscal deficit.

In contrast, a second strand of the literature focuses on characterizing optimal monetary policy in environments with nominal rigidities, also assuming that the government has a commitment technology (see, for example, Erceg, Henderson and Levin (2002); Khan, King and Wolman (2000); and Rotenberg and Woodford (1999)). The key result of this branch of the literature is that optimal monetary policy implies in an almost constant inflation rate for all dates and states. In some papers, this result is driven by the assumption that the government has access to (endogenous) lump-sum taxes to finance its budget. Thus, there is no need of using unanticipated inflation as a lump-sum tax. In other papers (especially Schmitt-Grohe and Uribe (2004)) this result comes from the fact that the price adjustment costs are quantitatively more important than the benefits from tax smoothing.

Our results thus indicate that even if prices were totally flexible, it would still be better to have constant inflation. This is so because the costs associated with the incentive problems surpass the benefits from tax smoothing. In other words, according to our results, when governments do not have access to a commitment technology, the introduction of incentive problems associated with the use of contingent debt is enough to overturn the results of the first branch of literature: the optimal policy is one which displays little variation in inflation. In this sense, our work is also related to Díaz-Giménez, Giovannetti, Marimon and Teles (2006) analysis of the time-inconsistency problem on monetary policy, in which nominal debt implies in lower welfare.[8]

---

[8] Both their and our paper study the government's incentive to deplete the real value of nominal debt through inflation. Among other differences, in their model government expenditure is constant and the debt level changes, whereas, in our work, the government expenditure is stochastic and the debt level stays constant, as in contingent service models, capturing the hedging role of nominal debt.

The rest of the paper is organized as follows. The model is developed in section 2. Section 3 presents the computational implementation and equilibria. Section 4 defines the data and calibration. The results are discussed in section 5. Section 6 concludes.

2 Model

Our economy is populated by a government (sovereign) who borrows funds and taxes labor income from a *continuum* of identical infinitely lived consumers (private sector), in order to finance an exogenous path of public expenditures. The sovereign type evolves over time. He can be one of two types, "good" or "bad," which describe conditions that reflect his impatience. Whereas "bad" sovereigns are extremely impatient and choose to default (through inflation) at any time independently of the state of the economy, "good" sovereigns may or may not choose optimally to default on their commitments.

In each period t, the private sector supplies labor and capital to produce a single output good, according to a Cobb-Douglas production function. The output can be used for private consumption, private investment, or government consumption. Let c_t, h_t, x_t, g_t, k_t and y_t denote the per capita levels of consumption, labor, investment, government spending, capital and output, and let C_t, H_t, X_t, G_t, K_t and Y_t denote their aggregate per capita counterparts. Feasibility requires,

$$C_t + X_t + G_t = K_t^{\alpha} H_t^{1-\alpha} = Y_t \tag{1}$$

The preferences of each consumer are given by

$$U = E \sum_{t=0}^{\infty} \beta^t u\left(c_t, \frac{m_t}{p_t}, h_t\right) \tag{2}$$

with,

$$u\left(c, \frac{m}{p}, h\right) = c + \omega_1 \frac{(m/p)^{1-\omega_2}}{1-\omega_2} - \omega_3 \frac{h^{1+1/\omega_4}}{(1+1/\omega_4)} \tag{3}$$

where ω_1, ω_2, ω_3, $\omega_4 > 0$ and $\beta \in (0, 1)$. The parameter ω_4 is usually referred to as the Frisch's intertemporal labor elasticity. In this traditional money-in-the-utility-function formulation, m_t/p_t denotes real money balances. As discussed below, the assumption that consumption enters linearly in the period utility function has the advantage of simplifying the determination of the equilibrium interest rates without affecting the tax-smoothing motive of the model.[9]

The per capita level of government consumption in each period, denoted by G_t, is exogenously specified. We assume G_t can take a finite number of values and evolve over time according to a

---

[9] Perhaps more importantly, the literature points out that the labor tax distortions are quantitatively important (Prescott, 2004), whereas risk aversion and consumption smoothing seem to be less so (Mehra and Prescott, 2003 and Hall, 1988). Thus, our results should be robust to utility functions with typical consumption curvatures.

Markov transition matrix with elements $\phi\,(G_i,\,G_j)$. That is, the probability that $G_{t+1} = G_j$ given that G_t $= G_i$ is given by the matrix ϕ element of row i and column j. Government consumption is financed through a proportional tax on labor income and with debt. Let τ_t denote the tax on labor income in period t. Government debt represents claims to consumption units in the next period. These claims can be either nominal (contingent) or indexed (non-contingent).

The sovereign debt literature tends to consider only one form of debt which is subject to default. In contrast, in this paper we assume that the sovereign can only default on nominal debt through inflation. A more general setting would consider that the sovereign can default on both types of debt but that an "explicit" default is more costly than a default through inflation. This would be consistent with the stylized fact that countries have resorted to surprise inflation a lot more often than to outright default, and would not change our substantial results.[10]

Denote the amount of nominal debt and indexed debt purchased by consumers respectively by b_t and d_t. Let $(1 + i_t)$ and $(1 + \rho)$ denote the (inverse of the) prices of these claims and v_t denote the marginal product of capital. Notice that, due to the linearity of the utility function with respect to consumption, the indexed bond yields a constant (and riskless) interest rate given by $(1 + \rho) = 1/\beta$.

The private sector's budget constraint is given by,

$$c_t + x_t + \frac{m_t}{p_t} + \frac{b_t}{p_t} + d_t = w_t(1-\tau_t)h_t + v_t k_t + \left(\frac{p_{t-1}}{p_t}\right)\left(\frac{m_{t-1}}{p_{t-1}}\right) + \left(\frac{p_{t-1}}{p_t}\right)\left(1+i_{t-1}\right)\left(\frac{b_{t-1}}{p_{t-1}}\right) + \left(1+\rho\right)d_{t-1} \quad (4)$$

As in Grossman and Van Huyck (1988), we consider a case in which the real levels of capital, indexed and real nominal debt are fixed and equal to $k_t = K$, $d_t = D$, and $(b_t / p_t) = B$ respectively. This assumption implies that debt and capital cannot be used to smooth taxes. This assumption, which greatly reduces the calculation burden, is a necessary step to making equilibrium computable.[11] Importantly, as Grossman and Han (1999) point out, this assumption is much less restrictive than it seems. These authors show that, when government can save after defaulting, contingent debt (or capital) does not allow for any additional tax smoothing. In contrast, contingent service may engender more tax smoothing than the one already attained through savings. In other words, varying b/p, d and k do not provide any additional smoothing from those already build in the model.

Because the capital level is constant, investment level must be equal to $x = \delta k$. Hence, the private sector's choices are to decide how much to work, how much money to hold, and how much to charge for government bonds.

The sovereign's preferences are given by

$$U = E \sum_{t=0}^{\infty} \beta_{sov}^t u(C_t, M_t / p_t, H_t) \tag{5}$$

where $u(.,.,.)$ is as in the consumers' preferences, but C_t, M_t and H_t denote the aggregate per capita levels of consumption, money and labor, respectively. The sovereign budget constraint is,

$$\frac{M_t}{p_t} + B + D = G_t - \tau_t w_t H_t + \left(\frac{p_{t-1}}{p_t}\right)\left[\frac{M_{t-1}}{p_{t-1}} + (1+i_t)B\right] + (1+\rho)D \tag{6}$$

where we have already suppressed the time subscripts for debt levels. We find convenient to change notation, writing the same budget constraint by,

$$\frac{M_t}{p_t} + B + D = G_t - \tau_t w_t H_t + \left(\frac{1}{1+\theta_t}\right)\left(\frac{1}{1+\pi}\right)\left[\frac{M_{t-1}}{p_{t-1}} + (1+i_t)B\right] + (1+\rho)D \tag{7}$$

where the inflation p_t / p_{t-1} was rewritten as the product of two terms: $(1 + \pi)$ denotes the (constant) "regular" inflation and $(1 + \theta_t)$ denotes the "surprise" inflation.

Now we can think of $(1 + i_t)/(1 + \pi)$ as the "contractual" real interest rate on the contingent debt, and of θ_t as the default rate. By changing θ_t the sovereign can make the effective interest rate of the nominal debt contingent on the state of the economy.

We assume θ_t can take only two values, $\theta_t \in \{0, \chi\}$, which correspond respectively to the cases of not defaulting and defaulting. This assumption, which was made due to computational reasons, implies that the default rate cannot be different in all states, and can in principle reduce the ability of the sovereign to smooth debt services. However, also due to computational reasons, this assumption is much less restrictive than it seems. As we discuss later, our calibration considers that government expenditures can only take two values. Consequently, only two levels for θ are enough to span most of the service variability.

There are two types of sovereigns who differ in the parameter β_{sov}. The "good" sovereign has the same discount factor of the private agents, $\beta_{sov} = \beta$. In contrast, the "bad" sovereign fully discounts the future: $\beta_{sov} = 0$. A direct consequence of this assumption is that the "bad" sovereign always

---

level. Hence in their case, like in ours, i. debt is not used in production and ii. capital is not used to smooth consumption. These assumptions imply that in effect ours is an endowment economy.

defaults.[12] As in Cole, Dow and English (1995) and Alfaro and Kanczuk (2005a), the government type evolves according to a Markov process (of common knowledge) with the transition probabilities given by

$$\Psi = \begin{bmatrix} 1-\psi & \psi \\ \psi & 1-\psi \end{bmatrix} \tag{8}$$

That is, a "good" type at t remains a good type at $(t + 1)$ with probability $1 - \psi$, and transitions to a bad type with probability ψ. Similarly, a "bad" type at t remains a bad type at $(t + 1)$ with probability $(1 - \psi)$ and transitions to a "good" type with probability ψ.[13] For this process to display persistence, we assume $\psi < 1/2$. We name ψ the *agency cost parameter*. Higher values of ψ imply that the current government type carries less information about the future sovereign type. Thus, when making decision, consumers are more uncertain about the sovereign's type. The limiting case $\psi = 0$ corresponds to a situation with perfect information where, effectively, there is only one type of sovereign.

There are two alternative interpretations for the notion of different types of sovereigns. A more literary reading is that sovereigns' preferences and willingness to honor debts do actually diverge. Or, equivalently, even if the same leader stays in power, there is turnover among key advisors, who have different preferences. As experience shows, lenders face considerable uncertainty about the government's characteristics and preferences. An example of such episode was Brazil's 2002 election (see Alfaro and Kanczuk (2005b)).

For this first interpretation it is worth noting that, in the model, decision makers care about their consumption independently of whether they are in power or not. That is, they do not discount future consumption by the likelihood of their type changing, nor do they think their discount rate is time varying. Additionally, note that it is not possible for a good sovereign to signal her type, since she will always be imitated by bad type sovereigns. This is so because bad sovereigns only care about current utility and an increase in debt services (resultant from a separating equilibrium) would reduce current consumption. Importantly, this is true regardless of the assumption that debt levels are constant.

The second interpretation for the existence of different types of sovereigns is more indirect. It comes from the fact that the outcomes from having a bad type of sovereign are equivalent to the

---

[12] This assumption captures the flavor of Grossman and Van Huyck's (1988) "excusable defaults." A bad type always defaults (even in good times), which is not excusable. The good type might default, but only in relatively bad times ("excusable default").

[13] The symmetry of the transition matrix implies that the (invariant) frequency of good and bad government is equal for any agency cost ψ. Note that this simplifying assumption is immaterial for our results as we study the economy welfare conditional on having good governments in power.

outcomes from having a sudden change in market sentiment and a seizure in capital markets. On such occasions (Mexico 1994 and Brazil 1999 are natural examples), we observe hikes in the interest rate premia presumably because lenders believe that there is a high probability the government will default on debt. Thus, in the model, a bad type government can be interpreted as abrupt loss of appetite for risky investments (nominal bonds).[14]

In each period, the timing is as follows (see Figure 1). At the beginning of period t, the country inherits amounts of contingent and non-contingent debt equal to b_{t-1} and d_{t-1}, which bear an interest rate of i_{t-1} and ρ respectively. Then nature reveals the sovereign type and the public expenditure level. After observing the public expenditure level, the sovereign decides whether to default or not, θ_t, and, consequently, how much to tax labor, τ_t. Based on these decisions, consumers decide how much to work, h_t. They also update their information about the sovereign's type and decide how much to charge for the next period contingent debt, i_t, which in turn determines real money balances hold, m_t/p_t.

As mentioned before, the assumption regarding the utility functional form greatly simplifies the solution. The Euler equation for labor (labor supply) jointly with the usual firms' maximization problem (labor demand)[15] imply

$$H_t = [\frac{(1-\tau_t)(1-\alpha)K^\alpha}{\omega_3}]^{1/(1/\omega_4+\alpha)} \tag{9}$$

The Euler equation for real money (money demand) is given by

$$\frac{M_t}{p_t} = [\frac{(1+i_t)}{\omega_1 i_t}]^{1/\omega_2} \tag{10}$$

Finally, the Euler equation for consumption implies that the private sector behaves as risk-neutral lenders, with an opportunity cost given by ρ.

As previously mentioned, we assume that the private sector cannot directly observe the government's type. Therefore, the lending rate i_t depends on the perceived likelihood of default. We find it convenient to express lenders' information about the likelihood of default by defining two probabilities, P_t and Q_t. Let $P_t \in [0, 1]$ be the probability that the sovereign in period t, at the time of

---

[14] Here it is again useful to notice that since we study the economy welfare conditional on having good governments in power. Thus, the beliefs of default could even be unjustifiable, or due to externalities or contagions.

[15] As usual, we assume there is a continuum of firms (or a single price taker firm), and the demand for labor equates wages with the marginal product of labor: $w_t = (1 - \alpha)(K/H_t)^\alpha$. By the same token, the marginal product of capital is given by $v_t = \alpha(H_t/K)^{1-\alpha}$.

choosing whether or not to default, is of the "good" type.[16] Let $Q_t \in [0, 1]$ be the probability that a sovereign will default at time t *given* that the sovereign is of the "good" type. The perceived probability of default at t is given by $1 - P_t(1 - Q_t)$.

For lenders to be indifferent between the risk less asset and the contingent debt, it must be that $(1+\pi)(1+\rho) = P_t(1-Q_t)(1+i_t) + [1 - P_t(1-Q_t)](1+i_t)/(1+\theta)$, which implies that the nominal interest rate is given by

$$1 + i_t = \frac{(1+\chi)(1+\pi)(1+\rho)}{1 + \chi P_t(1-Q_t)} \tag{11}$$

To understand how the model works, consider a good sovereign that chooses to default. If a sovereign defaults, choosing $\theta_t = \chi$, expression (7) indicates that the country will enjoy a lower tax distortion today, τ_t. This decision might affect the future interest rate lenders charge and, thus, future taxes. Indeed, when lenders extract the information from the default in order to set the next period's interest rate, they will most likely consider the possibility that this period sovereign was of the "bad" type. This in turn, given (8), implies a higher probability that the sovereign country also will be of the "bad" type next period. Consequently, the private sector chooses to charge a higher interest rate (expression 11). As a consequence of defaulting, there are lower tax distortions today in exchange for higher tax distortions in the future. Following the usual assumptions regarding preferences and technology, welfare is higher for smoother tax profiles. Thus, default is a more likely outcome when the state of the economy is such that, *for a constant θ,* the government expenditure today is higher than the expected government expenditure in the future.

3 Computational Implementation and Equilibrium

The model described is a stochastic dynamic game. We restrict our attention to the Markov perfect equilibria, which we discuss next. We start by defining the *state of the economy* at period t as the ordered set (G_{t-1}, G_t, P_t), and the *excusable default set*, Δ, as

$\Delta = \{(G_{-1} \ G, \ P)$ *such that lenders believe that the good type will default*$\}$

where, for any period, G_{-1} denotes the government spending in the previous period, and G and P denote, respectively, this period government spending and lenders' assessment of the probability that the sovereign is of the good type. The excusable default set, part of the lender's strategy, corresponds to all states of the economy in which lenders believe that the sovereign will default. In Grossman and

---

[16] Note the slight abuse of notation, we use p to denote prices and P to denote the probability that a sovereign being of the "good" type will default at time t.

Van Huyck's (1988) language, Δ corresponds to the states of the economy in which defaults are excusable.

Given Δ, we can write the lenders' future probabilities assessments as a function of the state and of the sovereign's action as

$$P_{t+1} = 1 - \psi, \quad \text{if } \theta_t = 0, \tag{12a}$$

$$P_{t+1} = \psi, \quad \text{if } \theta_t = \chi \text{ and } (G_{t-1}, G_t, P_t) \notin D, \tag{12b}$$

$$P_{t+1} = P_t(1 - \psi) + (1 - P_t)\psi, \text{ otherwise} \tag{12c}$$

which corresponds to simple Bayesian updating, and

$$Q_{t+1} = \sum_{(G_t, G_{t+1}, P_{t+1}) \in \Delta} \phi(G_{t+1}, G_t) \tag{13}$$

which comes straight from the definition of the excusable default set.

Notice that the lender's strategy is completely determined by the set Δ and the expressions (11), (12), and (13). As a consequence, given Δ, we can write the sovereign's problem as

$$V(G_{t-1}, G_t, P_t) = Max_{\theta_t}\{u(C_t, M_t / p_t, H_t) + \beta_{sov} EV(G_t, G_{t+1}, P_{t+1})\} \tag{14}$$

such that (3), (7), (9), (10), (11), (12), and (13) hold.

Now we are ready for the definition of equilibrium.

A *Markov perfect equilibrium* is an excusable default set Δ, a value function *V*, and a policy function θ such that, given Δ, θ *is* a solution for the problem (14), and

$$\theta (G_{-1}, G, P) = \chi, \text{ for all } (g_{-1}, g, p) \in \Delta, \text{ and}$$

$$\theta (G_{-1}, G, P) = 0, \text{ otherwise.}$$

Although in a slightly different format much in line with one of "recursive competitive equilibrium," this definition is not different from the usual Markov perfect equilibrium definition. Given the private sectors' strategy (the set Δ and the expressions for *P, Q, H, M/p* and *i*) and the state, the sovereign maximizes utility. Given the sovereign strategy (θ) and the state, the private sector optimally chooses how much to work, and it is indifferent between buying contingent debt and earning the riskless rate. Hence, its strategy is also optimal.

4   Data and Calibration

We calibrate our model to match two economies: the United States as an example of a mature economy and Brazil as an example of an emerging market. In order to make them as comparable as possible, we use the same preference and technology parameters for both countries, calibrating them

with the U.S. data. The two economies though have different government expenditure parameters. We used data from 1957 to 2005, and calibrated our model so that each period corresponds to one year.

To calibrate the government expenditure, we use data on the ratio of government expenditure to GDP. This choice reflects an attempt to capture the tax smoothing motive, that is, the variation of government expenditures relative to that of tax revenues. Notice that in our model there are no technology shocks. These shocks have a direct impact on GDP and tax revenues. Thus, we "internalize" tax revenue shocks into the government expenditure shocks, by studying the ratio G_t/Y_t. We assume G can take one of two possible levels, G_H and G_L, and assume that the transition probability matrix that defines the Markov process is symmetric. To calibrate the necessary parameters, we first detrend the G_t/Y_t series. We then calculate their mean, standard deviation, and autoregressive coefficients. For both the United States and Brazil, the mean is about $G/Y = 20\%$, and the autoregressive coefficient is about 0.97. In contrast, the standard deviation is very different in the two countries: government expenditure in Brazil ($\sigma_{G/Y-BRAZIL} = 0.40$) is about ten times more volatile than in the United States ($\sigma_{G/Y-U.S.} = 0.04$). To match these facts, we set

$$\phi = \begin{bmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{bmatrix}$$

for both countries. For the U.S., we set $G_H/Y = 0.208$ and $G_L/Y = 0.192$. And for Brazil, we set $G_H/Y = 0.28$ and $G_L/Y = 0.12$.

Following the Real Business Cycle literature, we can calibrate most of the technology and preferences parameters. The (the inverse of the) price of the noncontingent debt, which corresponds to the riskless real interest rate, is given by $\rho = 0.05$. This implies a discount parameter of $\beta = 0.95$. The depreciation level is $\delta = 0.05$ and $\alpha = 0.33$.

We follow Lucas (2000) and set the interest elasticity of money demand equal to 0.5, what implies $\omega_2 = 2$. For the 1957 to 2004 period, the average stock of real money balances was 17% of GDP, and the average nominal interest rate was 8.2%. These together with the Euler equation for money imply $\omega_1 = .00071$. We also use the Frisch elasticity proposed by Domeij and Floden (2003), $\omega_4 = 0.5$, and that the average hours of work is $H = 0.3$.

We make many experiments for different values of the contingent debt level, B, but we always set the total debt level constant. We impose this assumption of a constant total debt in order to make more explicit the role of the noncontingent debt, D, and make welfare comparisons. For both the United States and Brazil the total debt is $(B + D)/Y = 60\%$.

The total amount of debt allows us to determine the amount of tax revenues required to satisfy the government's budget constraint for a given level of government consumption, g. From this follows

that the mean value of the tax rate on labor is $\tau = 35.1\%$. With this value and the Euler equation for labor, we can calibrate $\omega_3 = 8.71$.

Because the transition probabilities of government type are unobservable, we experiment with many different agency costs parameters (ψ). Recall that higher ψ are associated with less information about the type of next government. In this sense, we expect developed economies to display smaller values of ψ than the developing economies. For the U.S. economy, in particular, one can arguably consider there are no agency costs, and that a good benchmark is $\psi = 0$.

We assume the parameter χ is set in order to minimize the standard deviation of the tax rate, what is actually a crucial aspect of our approach. By doing so, we are modeling nominal bonds as "pure" contingent debt, in the sense that their role is exclusively to smooth taxes. This is without loss of generality, since any default rate could be emulated by combining adequate amounts of nominal and indexed debt, i.e., by making a linear combination of χ and zero.

Putting it differently, our welfare maximization has two steps. In the first step we set χ to minimize tax variance and, in the second step, we combine nominal and indexed debt in different amounts. This approach has the advantage of untangling the two effects we are studying, by associating them with different type of bonds: the nominal bonds role is to smooth taxes, and the indexed bonds role is to avoid incentive problems. In order to maximize welfare, the two types of bonds are then combined in the proportion desirable to achieve the right amount of each effect. This optimal proportion is the meaning of optimal debt structure in our environment.

Table 1 contains the full list of parameter calibrations.

5 Results

5.1 Equilibrium Selection and Characterization

In this model, as in any repeated game, one lacks a good way to choose among the many possible equilibria. These different equilibria display different default patterns, corresponding to different contingent claims and, in this sense, are not really comparable. It turns out, however, that for most economies we studied (i.e., for most combinations of b and ψ), only three very simple equilibria emerged. We restrict our focus to these in order to make welfare comparisons and discussions.[17]

The first of these equilibria, which we name *always*-default equilibrium, corresponds to the case in which the sovereign defaults for all possible states. In fact, it is worth noticing that our model

---

[17] To be precise, some economies displayed a fourth equilibrium, extremely similar to one of the three already considered.

is set up such that this equilibrium exists regardless of the parameters. To see that, consider that the excusable default set Δ contains all possible states. Then, from expression (13), one sees that $Q = 1$, and from expression (11), that $1 + i_t = (1 + \chi)(1 + \pi)(1 + \rho)$, for all states. These expressions indicate that punishment is independent of what a sovereign does or, better, that investors are not drawing any information from a sovereign's actions. Consequently, the sovereign has no incentive not to default and chooses to default in any state. This strategy validates the equilibrium.

The second equilibrium considered is the case in which the good government does not default, regardless of the state of the economy. We name this one the *never*-default-equilibrium. Note that due to the existence of governments of the bad type, even in this equilibrium there could be a premium to compensate for the possibility of default, which depends on the agency cost parameter.

In the third equilibrium we focus, government debt corresponds to a claim that is contingent on the current state of government expenditure level. This claim determines that there should be default if $G_t = G_H$, and that there should not be a default if $G_t = G_L$. This is so regardless of the last period's government expenditure and of any beliefs about the type of the sovereign. We call this the *sometimes*-default-equilibrium.

In order to characterize these equilibria, we calculate some average prices and allocations conditional on the states of the economy for which we observe G_H and G_L. To do so, we first obtain the invariant distribution of the occurrence of the possible 12 states, considering that the government is of the good type. To better contrast the equilibria, we chose to focus on the U.S. economy with nominal debt equal to 30% of GDP and agency costs equal to zero;[18] the results for other parameters are quantitative similar and identical from a qualitative perspective. Table 2 reports the results.

Note first that the surprise inflation rate θ is immaterial in the never-default equilibrium, since it never occurs. For the always-default equilibrium, the surprise inflation is always the same, and thus is not really a surprise. In contrast, for the sometimes-default equilibrium, there is only surprise inflation when $G_t = G_H$, creating the possibility of tax smoothing.

As expected, nominal interest rates respond to the surprise inflation rate. And the interest rates in turn determine the real money balances (equation (10)). For the never-default equilibrium, nominal interest rates are always equal to its riskless value of 8.2%. In the always default equilibrium they are constant but at 115.5%. For the sometimes-default equilibrium they differ depending on the state of the economy. Notice, though, that even when government expenditure level is low, the interest rate is higher than 8.2%, since consumers take into account the probability of having surprise inflation in the next period.

---

[18] Actually, we set $\psi = 10^{-9}$.

In line with the surprise inflation rate, labor taxes τ are (approximately) constant in the sometimes-default equilibrium, due to tax smoothing. In contrast, labor taxes fluctuate a lot in the never-default and always-default equilibria. Notice too that taxes are lower in the always-default equilibrium. As we discuss later, this happens because in the always default equilibrium seignorage becomes an important source of revenue, easing the need for labor taxes.

Labor taxes in turn determine consumption levels *C*, given government expenditure levels *G*. That is, it is true that higher consumption levels are associated with lower government expenditures levels. But, controlling for government expenditures, consumption is higher for lower labor taxes. This is so because higher labor taxes imply in lower wages, less hours of work, and lower output.

5.2 Optimal Debt Management

We now undertake our main exercise, which is to find the optimal contingency structure, i.e., the amounts of indexed and nominal debt that lead to maximum welfare. In order to do so, as discussed above, we concentrate exclusively on the sometimes-default-equilibrium, as this is the equilibrium in which nominal debt acts as a contingent claim. We first focus on the U.S. economy and study the case of no agency costs. Once more, other values of the agency cost do not modify the qualitative results. We postpone the discussion of the Brazilian case for section 5.3 and 5.4. Finally, it is worth reminding the reader that the results reported correspond to weighed averages (by the occurrence of each state) over the different states of the economy conditional on the government being of the good type.

Before analyzing welfare, it is useful first to get some intuition from the equilibrium prices and allocations. Figure 2 depicts the surprise inflation rate (χ) and the mean of the nominal interest rate (*i*) for different amounts of nominal debt (*B/Y*). Both curves are smooth and decreasing for positive values of nominal debt, but display a spike at *B/Y* = 0.

The strong discontinuity at *B/Y* = 0, which turns out to be important for our discussion, happens because at this level of debt there is a change in the equilibrium. Since *B/Y* = 0 refers to a situation in which all debt is non-contingent, the results reported for this point refer to the never-default-equilibrium, which displays very different characteristics from the sometimes-default-equilibrium. In effect, notice that the surprise inflation rate for *B/Y* = 0, which was reported to be zero, is really immaterial, since surprise inflation does not happen in equilibrium. At this point, the nominal interest rate is constant (riskless), and equal to 8.2%

For positive *B/Y*, notice that higher stock of nominal debt are associated with lower surprise inflation rates. This is so because greater stock of nominal debt implies that the "inflation tax base" is

bigger, that is, the same default rate implies in higher government revenues for greater nominal debt stocks. Thus, with greater nominal debt stock outstanding, the sovereign can smooth taxes with a lower inflation rate. As expected, the nominal interest rate displays a similar behavior. Consumers anticipate part of the surprise inflation rate, regardless of the state of the economy, by increasing the nominal interest rate. And, for large B/Y, as the surprise inflation rate converges to zero, nominal interest rates converge to their riskless value of 8.2%.

Noticeably, when analyzing equilibrium for large values of B/Y, we are not constraining the stock of nominal debt to be at most the stock of total outstanding debt. This is so because there is in principle no natural bound for the nominal debt B/Y, even if total debt, $(B + D)/Y$, is constant. For example, one could have a negative value for D/Y, indicating that the sovereign is a debt creditor in indexed bonds.

Interestingly, as noted above, the greater B/Y the larger will be the inflation tax base. And a larger tax base implies that the same degree of labor tax smoothing could be obtained by smaller changes in inflation. In the limit, as B/Y goes to infinity, the default rate goes to zero and perfect smoothing could still be achievable. This raises the possibility of an implication which is opposite to a result in Persson, Persson and Svensson (1988), where the government, in order to reduce the incentive to generate surprise inflation, should buy nominal assets (become a net creditor in nominal bonds) and issue indexed debt.

Figure 3 depicts average consumption and real money balances for different amounts of nominal debt. These results are expressed as fractions of the allocations of a lump-sum tax economy. That is, we normalize both consumption and real money balances of an economy which has access to lump-sum taxes to be equal to 100%. Noticeably, such an economy can completely avoid tax distortions, and at the same time keep inflation and interest rates low.

Both consumption and real money balances spike at $B/Y = 0$, but one curve is the mirror image of the other. Notice too that whereas money balances are very sensitive to B/Y—ranging from 100% to 51%—consumption levels are not so—varying from 83.8% to 84.5%—. Explaining the pattern of real money balances is fairly easy, as it is a direct consequence of the money demand equation (10) and figure 2, i.e., higher nominal interest rates imply in lower real money balances. Understanding the behavior of consumption is more complicated, as it is a function of the labor tax distortions, which in turn are affected through different channels. On the one hand, tax distortions are a function of the tax variability (or variance), which is higher for an economy with no nominal bonds. On the other hand, tax distortions are a function of the average (or mean) level of taxes, which also changes with the debt structure. In particular, as nominal debt increases, inflation rate decreases, implying lower seignorage revenues. As a consequence, for higher nominal debt there must be more taxes on labor, and thus more

16

distortions. Accordingly, our calibrated results show that the higher the nominal debt, the lower is the consumption level. More interestingly, they indicate that the tax variance smoothing effect seems to be of second order, since consumption for $B/Y = 0$ is comparable to the consumption level for very high B/Y values. More importantly, however, our results suggest that the effects the debt structure have over the consumption level seem to be small compared to the effects over the real money balances.

Finally, figure 4 reports welfare in consumption terms, as in Lucas (1987). Analogously to before, we assume that the case in which in the economy has access to lump-sum taxes corresponds to a welfare equal to 100%. This figure again displays a spike at $B/Y = 0$, and for $B/Y > 0$ shows that welfare increases as the amount of nominal debt increases. This suggest that the welfare result can be directly associated with the level of real money balances, or, equivalently, that anticipated inflation costs seems to be the dominant factor. In other words, the tax smoothing role of nominal debt is of secondary importance from the quantitative point of view.

Let us turn now to the policy implications of the results. For low nominal debt values it is clear that the economy is better off having exclusively indexed debt. As we increase the amount of nominal debt up to 325% of GDP, welfare gradually improves, and becomes close to the case with only indexed debt. Notice that since we assumed a fixed amount of total debt equal to 60% of GDP, this case corresponds to one in which the government is the creditor of indexed bonds amounting to 265% of GDP. Even in this extreme case, the welfare level never surpasses that achieved when there is only indexed debt.

For nominal debt levels higher than 325% of GDP, the economy does not display the sometimes-default equilibrium. As we will discuss later, higher amounts of debt are not sustainable, even if they were relevant from an empirical point of view. Hence, as a conclusion, our quantitative results suggest that the best debt composition is simply issuing only indexed (non-contingent) debt. Or, in other words, the loss from anticipated inflation is higher than the gains from tax smoothing for all sustainable equilibrium.

As a pretend experiment, consider that the United States decides to increase the proportion of indexed debt outstanding, keeping total debt constant. Suppose that although this policy is gradual, consumers do not anticipate it, such that our model is still an adequate tool to analyze welfare.[19] In the United States, the amount of indexed debt in 2004 is close to 1.5% of GDP. Thus, the potential welfare increase of a change in the current composition of debt to one in which there is only indexed debt is 0.5% of the consumption level. However, if the transition process from nominal to indexed debt is gradual, the economy has to go through periods of low amounts of nominal debt before reaching the

---

[19] If consumers did anticipate changes in government debt composition, their maximization problem would be potentially very different from the one we analyze. See Díaz-Giménez et al. (2006) for such an analysis.

state in which all debt is indexed, as seen in Figure 4. This process implies considerable welfare losses in amounts close to 4% of the consumption level.[20]

Thus, our model suggests very different recommendations, depending on whether the debt structures changes are swift or gradual. An abrupt switch from nominal to indexed debt could be beneficial. However, in a more realistic scenario of a gradual transition, this process could imply important welfare losses, and it would be safer to increase even more the fraction of nominal debt. This could be a rationale for the observed levels of nominal debt in the U.S. An additional rationale is simply the costs associated with the use of indexed debt, such as those related to the availability of reliable price indices (See Fischer (1983) for further discussion).

5.3 Debt sustainability

We now investigate the model implications for debt sustainability, motivating it as an emerging market concern. The distinctive characteristic of most emerging markets economies is that volatility is much bigger than in the United States. In the case of Brazil, as noted before, the volatility of government expenditure (as a fraction of GDP) is about ten times bigger than in the United States. Also as noted before, the Brazilian government stated its intention to increase the proportion of non indexed debt, but seems to have been unsuccessful in doing so. Our interpretation of this is that the Brazilian government is facing sustainability problem, which makes issuing nominal debt a difficult task.

To analyze sustainability, it is necessary to assess the benefits and costs of a government default. In most sovereign debt analyses (e.g. Chari and Kehoe (1993)) the cost of defaulting includes permanent exclusion from the capital markets and, therefore, forfeiture of tax smoothing. The benefit of defaulting, according to these papers, is increased present consumption, which is directly linked to the size of the debt: the higher the debt level, the higher the consumption increase in the event of default. Consequently, there is a maximum amount of debt that is sustainable. Beyond some threshold of debt level, the benefits of defaulting surpass the costs of defaulting and debt ceases to be sustainable.

In a framework such as ours, in which default is a repeated phenomenon and governments can continue to smooth taxes after default, the analysis of sustainability is more complicated. The benefit from defaulting (as in the other type of models) is associated with a higher consumption level today. However, since the country does face not permanent exclusion from the international capital markets, the costs from defaulting are not associated with the impossibility of smoothing taxes. Rather, they are

---

[20] Note that this result depends on the assumption that χ is set to minimize tax variances.

associated with two other factors. One is lower future consumption levels due to higher future interest rates. Because investors signal-extract the type of government, a default leads to higher future interest rates, which, in turn, imply higher debt services and lower consumption. The other relevant cost of defaulting is associated with the traditional inflation cost, an analysis which goes back to Bailey's calculation of the area under the inverse demand function for real money (see Lucas (2000)). A rise in interest rates post default begets a drop in real money balances, implying in lower welfare levels.

We can now deduce the effects of agency costs and debt level on sustainability. As agency costs increase, the amount of information about the future type of government decreases and thus the punishment for a defaulting government decreases. Consequently, as agency costs increase, sustainability decreases. Regarding debt level, a default implies a bigger increase in consumption when debt is greater, hence, the benefit of defaulting increases with the amount of debt. The costs of default also increase with debt, but less so. The first factor we discussed, the increase in debt service, is directly proportional to debt. But the second factor, which is the reduction of money balances due to the increase in the inflation rate, is not proportional to the debt level. In fact, we have assumed that the default rate was set to minimize tax variances and, as we discussed, this implies a negative relationship between debt levels and surprise inflation rates. Consequently, the costs of defaulting are less elastic to the debt level than the benefits from defaulting. Hence, there is a threshold of the debt level after which debt ceases to be sustainable.[21]

It is also noteworthy that the minimum level of sustainable nominal debt is zero, i.e., negative amounts of nominal debt are not sustainable. This is so because, with negative amounts of nominal debt, the surprise inflation rate that minimizes labor tax variances is negative (that is, a surprise deflation on nominal assets). However, a surprise deflation would imply in a reduction of the future interest rates, reducing the punishment associated with a default, and making debt unsustainable.

Figures 5 and 6 depict the region of sustainability for the nominal debt for the U.S. and Brazilian economies, respectively. The horizontal axis represents the agency cost (ψ), the vertical axis the nominal debt level (as a fraction of GDP). As just discussed, both higher debt level and higher agency costs decrease sustainability. The region below and to the left of the frontiers corresponds to situations in which debt is sustainable. In the region on the top and to the right of the frontier, debt is not sustainable. Notice that the two pictures are similar, but their scales are very different. For zero agency costs, for example, the maximum amount of sustainable debt is 325% in the U.S. and 110% for

---

[21] Notice that even if we had assumed that the surprise inflation rate were constant, the same result would hold. Namely, the costs from defaulting would not be less than proportional to debt, and there would be a maximum level of sustainable debt.

Brazil. Whereas any debt level ceases to be sustainable for agency costs higher than 1.5% in Brazil, some debt is still sustainable for agency costs lower than 25% in the U.S.

This result indicates that the volatility of the economy, in particular the volatility of the government budget, is also an important factor behind debt sustainability. More volatile government budget imply in higher benefits from defaulting, without a correspondent increase in costs. Thus, in our model, debt is less sustainable in more volatile (developing) economies. Interestingly, this result stands in contrast with the papers that assume that default causes exclusion from markets, such as Eaton and Gersovitz (1981, Proposition 4). As it turns out, in these papers the costs from defaulting are in general more sensitive to volatility than the benefits, what makes debt more sustainable in more volatile countries. As Chari and Kehoe (1993, section 5) put it, "For a large class of parametric examples we have found that the future losses outweighed the current gains. Thus we found that increased variability typically makes it easier to support good outcomes."

5.4 Remarks on Emerging Markets

Turning our attention back to Brazil, as we mentioned before, the government has declared its intention to increase the amount of non-indexed debt, which was only about 15% of GDP, whereas total debt is 60% (see Ministerio da Fazenda (2006)). Our analysis suggests that this policy is simply not feasible. The private sector would not be willing to buy bigger amounts of nominal debt at any interest rate, believing the government would never honor it. In other words, using our language, if the government insisted in raising the amount of nominal debt the Brazilian economy would end up in an always-default equilibrium.

To better see this point, in Figure 7 we depict the welfare levels for alternative debt structures for the Brazilian economy with agency costs equal to 1%. This figure is Brazilian counterpart of figure 4, but with an important difference. Instead of only showing the sometimes-default equilibrium outcomes, it shows the best equilibrium from the welfare point of view. That is, when the nominal debt level is so high that the sometimes-default equilibrium is not sustainable, it shows the welfare for the always-default equilibrium, which is always sustainable. When the nominal debt is positive but not so high, it shows the welfare for the sometimes-default equilibrium, since it is higher than the never-default equilibrium welfare.

Notice that when $B/Y = 0$, the welfare level attain its maximum level. This point corresponds to the spike of figure 4, and reports the never-default equilibrium, since the economy only has indexed non-contingent liabilities outstanding. Small positive values of B/Y refer to the sometimes-default equilibrium. In this region welfare increases with the nominal debt level, since lower surprise inflation

rates are needed to smooth taxes when the tax base is higher. Similarly to the U.S. analysis, one could expect that high *B/Y* could deliver the best structure solution, by combining tax smoothing with lower inflation rates. However, again as in the U.S. case, the sometimes-default equilibrium ceases to be sustainable before such outcome happens. As previously seen, for the U.S., the maximum amount of sustainable sometimes-default equilibrium is 325% of GDP, a level so high that is perhaps empirically irrelevant. In contrast, for Brazil, this threshold is at fairly low levels, which deserve further discussion. For *B/Y* equal of higher than 15% of GDP, the only sustainable equilibrium is the never-default equilibrium. Under this circumstance, the nominal debt ceases to be contingent, and the economy ends up with low welfare levels. The economy displays high inflation rates, but these are fully anticipated by the private sector - a situation that resembles some of the Brazilian past.

Indeed, we think our results for volatile economies can be motivated in different forms, and raise some interesting questions. Firstly, following the debt intolerance argument of Reinhart, Rogoff and Savastano (2003), our results suggest that countries with history of inflation and defaulting can only sustain low amounts of nominal debt. In fact, such countries may issue indexed bonds simply because they cannot avoid it.[22]

Secondly, we could argue that the history of high inflation of these countries actually corresponds to the always-default equilibrium of our model. Recall from table 2 that in the always default equilibrium the average inflation rate is higher than 100%, and seignorage revenues help reducing the labor tax. These outcomes are in fact similar to the Brazilian economy before the stabilization plan of 1994. From 1957 to 1994 the average and the median inflation rate in Brazil were, respectively, 336% and 53%, and inflation taxes were an important source of government revenues. At the same time, most of the inflation was anticipated, real money balances were small—about 3% of GDP, in line with the model—, and the welfare level was also probably very low—at least from anecdotal evidence from opinion polls that pinned down inflation as people's number one dissatisfaction—.

Worth mentioning, according to this motivation, we are explaining the Brazilian history of high inflation as a time-inconsistency problem, or "expectation trap" (Albanesi, Chari and Christiano (2002)). The only crucial difference from models a la Kydland and Prescott (1977) and Barro and Gordon (1983) being that the gains from unexpected inflation are not higher output due to price rigidities but, rather, to lower distortionary labor taxes. And this indeed seems a more plausible channel in countries with high inflation, where the correlation between inflation and output is clearly negative.

---

[22] Fischer (1983) wrote "Governments in inflationary difficulties issue indexed bonds and those that can avoid, it, do not."

Finally, there is a question about the possibility of avoiding the always-default equilibrium, by making all debt indexed. Or in other words, the possibility of using indexed debt as a commitment device. In our model this is perfectly achievable, and would imply a substantial gain in welfare --for the Brazilian economy, the never-default equilibrium welfare is 5% higher than the always default equilibrium welfare, in consumption terms--. However, we know that in some cases is not really possible to have perfectly indexed debt or truly non-contingent debt. In the case of Brazil, even the debt linked to the short-term interest rate (Selic) suffered losses when inflation accelerated and, in this sense, was not perfectly indexed. Furthermore, even if it was possible to have only indexed debt, one should also consider the possibility of having explicit defaults, not considered in the paper. In Brazil, the process of debt indexation, caused by high inflation, made outright default (through "tablitas" or price freezes) more common. That is, although all debt became indexed, it also became state-contingent.

When indexed debt is contingent, our exercise degenerates into the standard sovereign debt analysis, which considers only one form of debt which is subject of default. In Alfaro and Kanczuk (2005a) we have analyzed such an environment, and showed that default inevitably occurs in equilibrium when the economy displays high volatility and/or high agency costs. In other words, indexed debt ceases to be a commitment device when the fundamentals are very poor. Indeed, it might as well be the case that the fundamentals of an emerging country are sometimes so unfavorable that debt structure becomes irrelevant: any composition of debt implies in equilibria that display default and low levels welfare levels. Equivalently, the choice between nominal and indexed debt become only relevant when the country fundamentals are good enough to have some form of debt which is non-contingent. This is surely the case of the U.S., and it seems also to be the case of Brazil during the last years.

6 Conclusions

In this paper, we used the methodology of simulating a calibrated artificial economy to study sovereign debt default. In particular, we ran a quantitative horse race between the main arguments in favor of and against nominal debt which are the incentive to default through inflation versus hedging against unforeseen shocks. We use a dynamic equilibrium model with tax distortion, government outlays uncertainty, and contingent debt service. In the model, the benefits of defaulting through inflation are tempered by higher future interest rates. We use the model to calibrate the case of the U.S. economy. We further discuss sustainability of nominal debt in volatile (developing) countries where issues related to debt sustainability have taken center stage among policy circles.

A noteworthy result of our calibration exercises is that equilibria with mostly nominal debt (like the one observed in the U.S.) cannot be sustained in Brazil. Our results not only cast doubt about the possibility of volatile developing countries, such as Brazil, successfully reducing their indexed debt levels but also caution against the optimality of doing so from a welfare point of view. Indeed, our results suggest that state contingent contracts are not easy to sustain where incentive problems are present**.**

Our work also contributes to the literature on optimal monetary policy, which tends to have very different recommendations depending on the assumption about price flexibility. Whereas flexible price models suggest that volatile inflation is the "Ramsey" optimal manner to reduce tax distortions (Chari and Kehoe (1999)), fixed price models point out that constant inflation is best, due to price adjustment costs (Schmitt-Grohé and Uribe (2004)). Our results indicate that, even under the assumption of flexible prices, it is better to have constant inflation, once incentive problems are considered.

However, one should keep in mind that our results depend on simplifications needed to keep the computational analysis manageable. In particular, we assumed that the debt levels were constant, and we completely abstracted from analyzing different debt maturity structures. Exploring these directions further is an important topic for future research.

7 References

Albanesi, Stefania, V. V. Chari and Lawrence J. Christiano (2003). "Expectation Traps and Monetary Policy," *Review of Economic Studies* 70, 715-741

Alfaro, Laura and Fabio Kanczuk (2005a). "Sovereign Debt as a Contingent Claim: A Quantitative Approach," *Journal of International Economics* 65, 297-314.

Alfaro, Laura and Fabio Kanczuk (2005b). "Debt Maturity: Is Long Term Debt Optimal?" mimeo.

Barro, Robert J. (1979). "On the Determination of Public Debt," *Journal of Political Economy* 87, 940-971.

Barro, Robert J. (1997). "Optimal Debt Management of Indexed and Nominal Debt," *NBER Working Paper* 6197.

Barro, Robert J. and David Gordon (1983). "A Positive Theory of Monetary Policy in a Natural Rate Model?" *Journal of Political Economy* 91, 589-610.

Bohn, Henning (1988). "Why Do We Have Nominal Government Debt?" *Journal of Monetary Economics* 21, 127-140.

Bohn, Henning (1990). "Tax Smoothing with Financial Instruments," *American Economic Review* 80, 1217-1230.

Calvo, Guillermo and Pablo Guidotti (1993). "On the Flexibility of Monetary Policy: The Case of the Optimal Inflation Tax," *The Review of Economics Studies* 60, 667-687.

Chari, V.V. and Patrick Kehoe (1993). "Sustainable Plans and Mutual Default," *The Review of Economic Studies,* 60, 175-195.

Chari, V.V. and Patrick Kehoe (1999). "Optimal Fiscal and Monetary Policy," In John B. Taylor and Michael Woodford, eds., *Handbook of Macroeconomic, vol 1*.Amsterdam: Elsevier Science

Cole, Harold L., James Dow, and William B. English (1995). "Default, Settlement, and Signaling: Lending Resumption in a Reputational Model of Sovereign Debt," *International Economic Review* 36, 365-385.

Díaz-Giménez, Javier, Giorgia Giovannetti, Ramon Marimon and Pedro Teles (2006) "Nominal Debt as a Burden on Monetary Policy," *mimeo.*

Domeij, David and Martin Floden (2003). "The Labor Supply Elasticity and Borrowing Constraints: Why Estimates are Biased," *Review of Economic Dynamics*, forthcoming

Erceg, Christopher. J., Dale W. Henderson, and Andrew Levin. (2000) "Optimal monetary policy with staggered wage and price contracts," *Journal of Monetary Economics* 46, 281-313

Fischer, Stanley (1983). "Welfare Effects of Government Issue of Indexed Bonds," in Rudiger Dornbusch and Mario H. Simonsen, eds., *Inflation, Debt and Indexation*. Cambridge: MIT Press.

Grossman, Herschel I. and John B. Van Huyck (1988). "Sovereign Debt as a Contingent Claim: Excusable Default, Repudiation, and Reputation," *American Economic Review* 78, 1088-1097.

Grossman, Herschel I. and Taejoon Han (1999). "Sovereign Debt and Consumption Smoothing," *Journal of Monetary Economics* 44, 149-158.

Eaton, Jonathan and Mark Gersovitz (1981). "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *The Review of Economic Studies* 48, 289-309.

Hall , Robert (1988). "Intertemporal Substitution in Consumption," *Journal of Political Economy* 96, 339-57.

Kahn, Aubhik., Robert G. King, and Alexander L. Wolman, (2002) "Optimal Monetary Policy," *NBER Working Paper* 9402.

Kydland, Finn and Edward Prescott (1977). "Rules Rather than Discretion: The Inconsistency of Optimal Plans," *Journal of Political Economy* 85, 473-491.

Lucas, Robert E. Jr. (1987). *Models of Business Cycles.* Cambridge, Mass.: Basil Blackwell.

Lucas, Robert E. Jr. (2000). "Inflation and Welfare," *Econometrica* 68, 247-274.

Lucas, Robert E. Jr. and Nancy Stokey (1983). "Optimal Fiscal and Monetary Policy in an Economy without Capital," *Journal of Monetary Economics* 12, 55-93.

Ministério da Fazenda (2006). "Divida Publica: Plano Anual de Financiamento 2006", www.tesouro.fazenda.gov.br

Persson Mats, Torsten Persson, and Lars E. O. Svensson (1988). "Time Consistency of Fiscal and Monetary Policy," *Econometrica* 55, 1419-1432.

Prescott, Edward (2004). "Why do Americans Work so Much More than Europeans?" NBER Working Paper 20316.

Prescott Edward and Rajnish Mehra (2003). "The Equity Premium in Retrospect," NBER Working Paper 9525.

Reinhart, Carmen, Kenneth Rogoff and Miguel A. Savastano (2003). "Debt Intolerance," *Brookings Papers in Economic Activity* 1, 1-74.

Ricardo, David (1951). *Funding System*, reprinted in P. Safra, ed., *The Works and Correspondence of David Ricardo*, v. IV, Cambridge, Cambridge University Press.

Rotemberg, Julio and Woodford, Michael D. (1999). "Interest Rate Rules in an Estimated Sticky Price Model," in J. B. Taylor (Ed.) *Monetary Policy Rules*, The University of Chicago Press, Chicago, pp 57-119.

Schmitt-Grohé, Stephanie and Martín Uribe (2004). "Optimal Fiscal and Monetary Policy Under Sticky Prices," *Journal of Economic Theory* 114, 198-230.

Smith, Adam (1971). *An Inquiry into the Nature and Causes of the Wealth of Nations.* W. Strahan and T. Cadell eds, London.

Table 1: Calibration

Preferences

$\beta = 0.95$ (time discount factor)

$\omega_1 = .00071$ (money utility parameter)

$\omega_2 = 2$ (inverse of the interest elasticity of money)

$\omega_3 = 8.71$ (labor disutility parameter)

$\omega_4 = 0.5$ (labor intertemporal elasticity)

Technology

$\alpha = 0.33$ (capital share)

$\delta = 0.05$ (depreciation rate)

U.S. Government

$G_H/Y = 0.208$ (high government expenditure level)

$G_L/Y = 0.192$ (low government expenditure level)

$\phi_{1,1} = 0.95$ (government expenditure persistency)

$(B + D)/Y = 60\%$ (total debt)

Brazilian Government

$G_H/Y = 0.28$ (high government expenditure level)

$G_L/Y = 0.12$ (low government expenditure level)

$\phi_{1,1} = 0.95$ (government expenditure persistency)

$(B + D)/Y = 60\%$ (total debt)

Table 2: Equilibrium Characterization

	Never		Sometimes		Always	
	G_H	G_L	G_H	G_L	G_H	G_L
θ (%)	-	-	24.1	0.0	99.2	99.2
i (%)	8.2	8.2	34.2	9.2	115.5	115.5
M/p (% GDP)	16.4	16.4	8.0	15.4	4.4	4.4
τ (%)	33.7	31.0	31.7	31.6	30.9	28.3
C (% GDP)	60.7	63.5	61.6	63.3	61.9	64.7

Figure 1: Time Line

t   t + 1

b_{t-1} Nature: Default decision θ_t b_t

d_{t-1} G_t (and thus τ_t) d_t

m_{t-1}/p_{t-1} gov't type m_t/p_t

i_{t-1} i_t
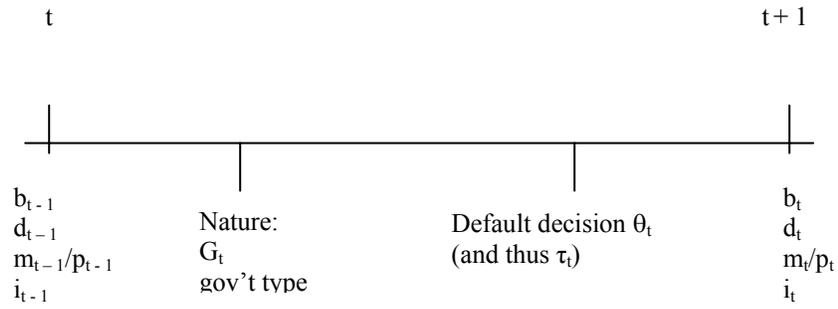
Figure 2: Equilibrium Prices (U.S. Economy, $\psi = 0$)

Surprise Inflation Rate (%)   Mean Nominal Interests (%)
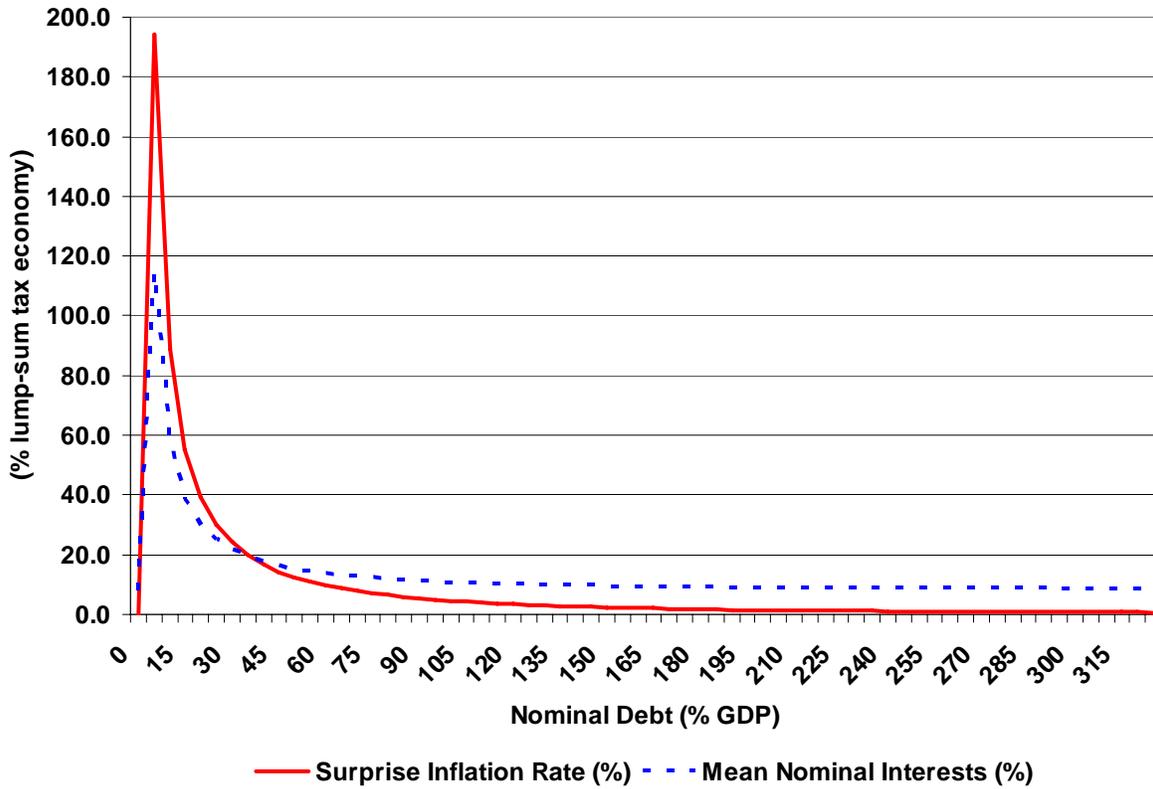
**Figure 3: Equilibrium Allocations (U.S. Economy, ψ = 0)**
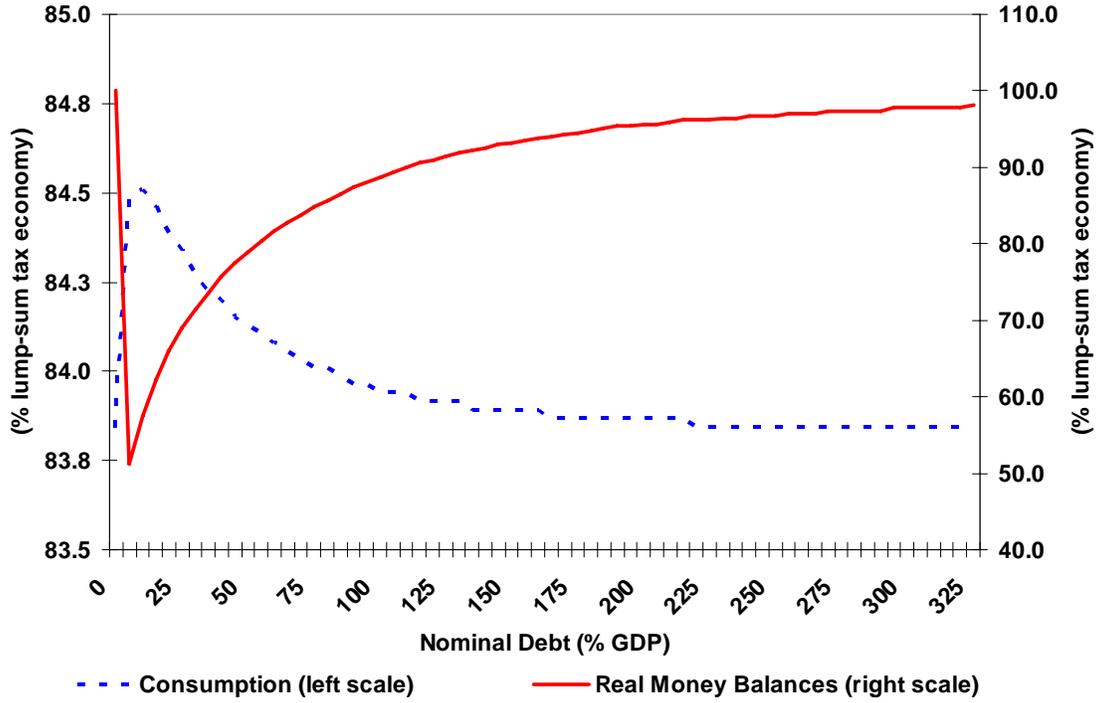
Figure 4: Welfare in Consumption Units (U.S. Economy, ψ = 0)
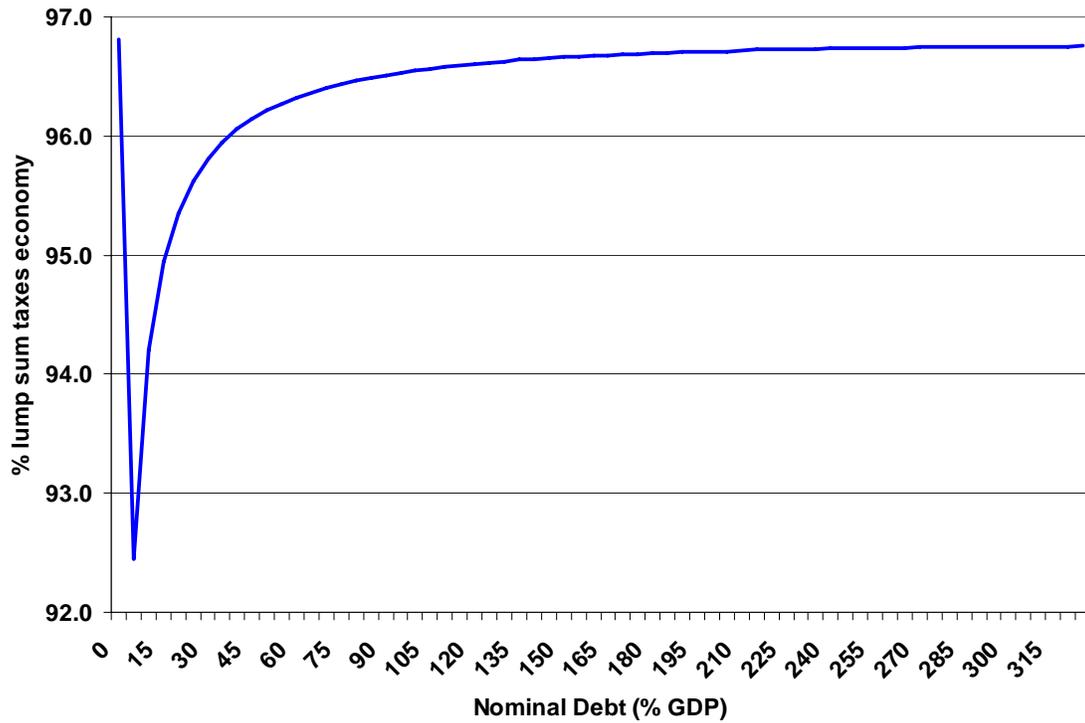
Figure 5: Sustainability (U.S. Economy)
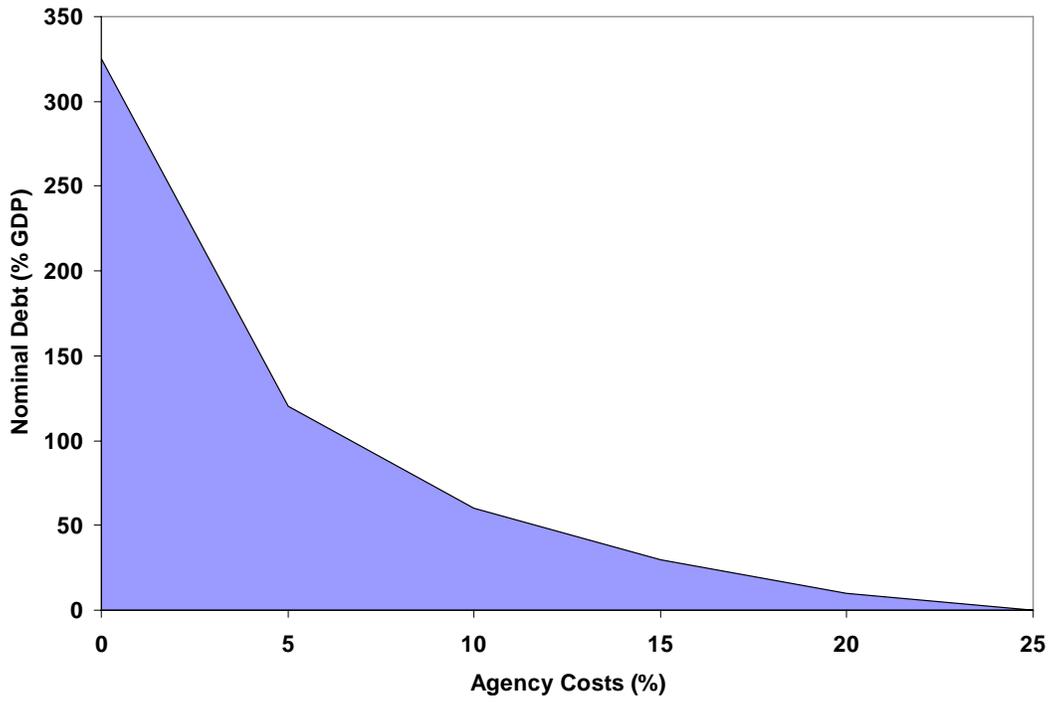

Figure 6: Sustainability (Brazilian Economy)
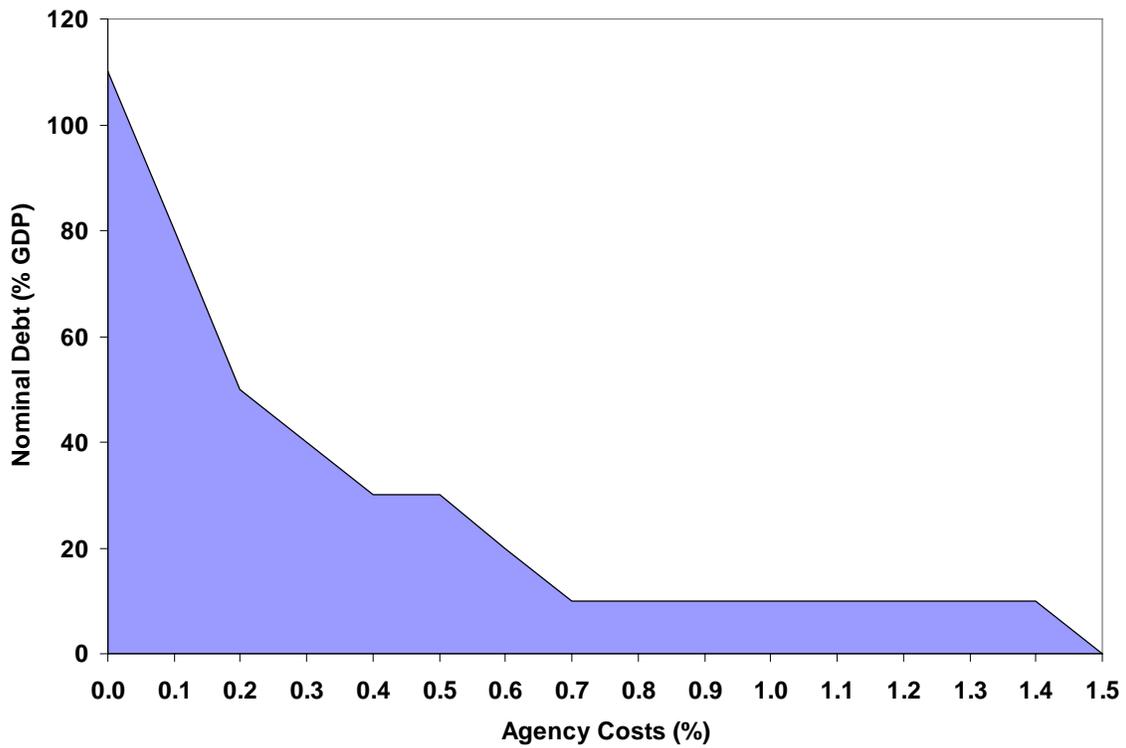
Figure 7: Welfare in Consumption Units (Brazilian Economy, $\psi = 1\%$)