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NEWS SHOCKS AND OPTIMAL MONETARY POLICY

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ABSTRACT

This paper studies monetary policy in a model where output fluctuations are caused by shocks to public beliefs on the economy's fundamentals. I ask whether monetary policy can offset the effect of these shocks and whether this offsetting is socially desirable. I consider an environment with dispersed information and two aggregate shocks: a productivity shock and a "news shock" which affects aggregate beliefs. Neither the central bank nor individual agents can distinguish the two shocks when they hit the economy. The main results are: (1) despite the lack of superior information an appropriate monetary policy rule can change the economy's response to the two shocks; (2) monetary policy can achieve full aggregate stabilization, that is, it can induce a path for aggregate output that is identical to that which would arise under full information; (3) however, full aggregate stabilization is typically not optimal. The fact that monetary policy can tackle the two shocks separately is due to two crucial ingredients. First, agents are forward looking. Second, current fundamental shocks will become public information in the future and the central bank will be able to respond to them at that time. By announcing its response to future information, the central bank can influence the expected real interest rate faced by agents with different beliefs and, thus, induce an optimal use of the information dispersed in the economy.

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1 Introduction

Suppose a central bank observes an unexpected expansion in economic activity. This could be due to a shift in fundamentals, say an aggregate productivity shock, or to a shift in public beliefs with no actual change in the economy's fundamentals. If the central bank could tell apart the two shocks the optimal response would be simple: accommodate the first type of shock and offset the second. In reality, however, central banks can rarely tell apart these shocks when they hit the economy. What can the central bank do in this case? What is the optimal monetary policy response? In this paper, I address these questions in the context of a model with dispersed information, which allows for a micro-founded treatment of fundamental shocks and shocks to public beliefs.

The US experience in the second half of the 90s has fueled a rich debate on these issues. The run up in asset prices has been taken by many as a sign of optimistic expectations on the presence of widespread technological innovations. In this context, the advice given by different economists has been strongly influenced by the assumptions made about the ability of the central bank to identify the economy's *actual* fundamentals. Some, e.g. Cecchetti et al. (2000) and Dupor (2002), have attributed to the central bank some form of superior information and have advocated early intervention to contain an expansion driven by incorrect beliefs. Others, e.g. Bernanke and Gertler (2001), have emphasized the uncertainty associated with the central bank's decisions and have advocated sticking to a simple inflation-targeting rule. In this paper, I explore the idea that, even if the central bank does not have superior information, a policy rule can be designed to take into account, and partially offset, "aggregate mistakes" by the private sector regarding the economy's fundamentals.

I consider an economy where aggregate productivity is subject to unobservable random shocks. Agents have access to a noisy *public signal* of aggregate productivity, which summarizes public news about technological advances, aggregate statistics, and information reflected in stock market prices and other financial variables. The noise component in this signal, or "news shock," introduces shocks to public beliefs which are uncorrelated with actual productivity shocks. In addition to the public signal, agents have access to private information regarding the realized productivity in the sector where they work. Due to cross-sectional heterogeneity, this information is not sufficient to identify the value of the aggregate shock. Therefore, agents combine public and private sources of information to forecast the aggregate behavior of the economy. The central bank has only access to public information.

In this environment, I obtain three main results. First, using a policy rule which responds to past aggregate shocks, the monetary authority *can* affect the relative response of the economy to productivity shocks and news shocks. Second, monetary policy can achieve full aggregate stabilization, that is, it can induce a path for aggregate output that is identical to that which would arise under full information. Third, full aggregate stabilization is typically suboptimal and optimal monetary policy partially accommodates the non-fundamental output fluctuations due to news.

The fact that monetary policy can tackle the two shocks separately is due to two crucial ingredients. First, agents are forward looking. Second, productivity shocks are unobservable when they are realized, but become public knowledge in later periods. At that point, the central bank can respond to them. By choosing an appropriate policy rule the monetary authority can then alter the way in which agents respond to private and public information. For example, the monetary authority can announce that it will increase the nominal price level following an actual increase in aggregate productivity. Under this policy, agents observing an increase in productivity in their own sector expect higher inflation compared to agents who only observe a positive public signal. Therefore, they face a lower expected real interest rate and choose to consume more. I will show that then equilibrium output becomes more responsive to private information and less to public information. This moderates the economy's response to news shocks. This result points to an idea which applies more generally in models with dispersed information. If future policy is set contingent on variables that are imperfectly observed today, this can change the agents' reaction to different sources of information, and thus affect the equilibrium allocation.

In the model presented, the power of policy rules to shape the economy's response to different shocks is surprisingly strong. Namely, by adopting the appropriate rule the central bank can support an equilibrium where aggregate output responds one for one to fundamentals and does not respond at all to the noise in public news. However, such a policy is typically suboptimal, since it has undesirable consequences in terms of the cross-sectional allocation. In particular, full stabilization generates an inefficient compression in the distribution of relative prices. This causes welfare losses that need to be balanced against the welfare gains associated to smaller aggregate volatility.

In this paper, I am able to derive equilibrium quantities and welfare in closed form. This is possible thanks to an assumption about the random selection of consumption baskets. In particular, I maintain the convenience of a continuum of goods in each basket, while, at the same time, I allow for baskets that differ from consumer to consumer. This technical solution may be usefully adapted to other models of information diffusion with random matching. Its main advantage is that it allows to construct models where, each period, an agent interacts with a large number of agents, and, at the same time, does not fully learn about aggregate behavior.

A number of recent papers, starting with Woodford (2002) and Sims (2003), have revived the study of monetary models with imperfect common knowledge, in the tradition of Phelps (1969) and Lucas (1972).[1] In particular, this paper is closely related to Hellwig (2005) and Adam (2006), who study monetary policy in economies where money supply is imperfectly observed by the public. In both papers consumers' decisions are essentially static, as a cash-in-advance constraint is present and always binding. Therefore, the forward-looking element, which is crucial in this paper, is absent in their models. In the earlier literature, King (1982) was the first to recognize the power of policy rules in models with imperfect information. He noticed that "*prospective* feedback actions" responding to "disturbances that are currently imperfectly known by agents" can affect real outcomes.[2] However, the mechanism in King (1982) is based on the fact that different policy rules change the informational content of prices. As I will show below, that channel is absent in this paper. Here, policy rules matter because they affect agents' incentives to respond to private and public signals.

The paper is also related to the literature on optimal monetary policy with uncertain fundamentals, including Aoki (2003), Svensson and Woodford (2003, 2005), and Reis (2003). A distinctive feature of the environment in this paper is that private agents have access to superior information about fundamentals in their local market but not in the aggregate. In this case, the monetary policy rule is designed to induce the most efficient use of the information scattered across the economy. The presence of dispersed information generates a tension between aggregate efficiency and cross-sectional efficiency in the design of an optimal policy rule.

There is a growing literature on the effect of expectations and news on the business cycle. In particular, Christiano, Motto and Rostagno (2006) and Lorenzoni (2006) show that shocks

[1] See also Moscarini (2004), Milani (2005), Nimark (2005), Bacchetta and Van Wincoop (2005), Luo (2006), Maćkowiak and Wiederholt (2006). Mankiw and Reis (2002) and Reis (2006) explore the complementary idea of lags in informational adjustment as a source of nominal rigidity.

[2] King (1982), p. 248.

to expectations about productivity can generate realistic aggregate demand disturbances in business cycle models with nominal rigidities.[3] In Christiano, Motto and Rostagno (2006) the monetary authority has full information regarding aggregate shocks and can adjust the nominal interest rate in such a way so as to essentially offset the effect of the news shock and replicate the behavior of the corresponding flexible price economy. Moreover, this offsetting is optimal in their model.[4] This leads to the question: are expectations-driven cycles merely a symptom of a suboptimal monetary regime, or is there some amount of expectations-driven volatility that survives under optimal monetary policy? This paper addresses this question in the setup of Lorenzoni (2006) and shows that optimal monetary policy does not eliminate news-driven business cycles. One may think that this result comes immediately from the assumption that the monetary authority has limited information. That is, it would seem that the central bank cannot intervene to bring output towards its "natural" level, given that this natural level is unknown. The analysis in this paper shows that the argument is subtler. The monetary authority *could* eliminate the aggregate effect of news shocks by announcing an appropriate monetary rule. However, this rule is not optimal due to its undesirable cross-sectional consequences.

A recent series of papers, starting with Morris and Shin (2002), looks at environments with imperfect common knowledge, asking whether, from the point of view of social welfare, agents put too much or too little weight on public signals relative to private signals.[5] The model in this paper has a similar flavor, since, under the wrong monetary policy rule, total output may indeed respond too much (or too little) to public news. An interesting policy question that has been addressed in this literature is what are the welfare consequences of public releases of information. Morris and Shin (2002) obtain the paradoxical result that, in a simple coordination game, making a public signal available to the players may be detrimental for welfare. This result has started a lively debate on the merits of transparency about macroeconomic policy.[6] Using my model, I can address the related question whether better information about macroeconomic fundamentals is welfare improving. Here, a number of issues open up, regarding aggregate and cross-sectional efficiency. First, it is true that an increase in the precision of the

[3] See Beaudry and Portier (2006) and Jaimovich and Rebelo (2006) for flexible price models of cycles driven by news about future productivity.

[4] See Appendix B in Christiano, Motto and Rostagno (2006).

[5] See Angeletos and Pavan (2005), Hellwig (2005), Hellwig and Veldkamp (2006), Angeletos, Lorenzoni and Pavan (2007), Amador and Weill (2007).

[6] See Amato, Morris and Shin (2002), Svensson (2005), Hellwig (2005), Morris and Shin (2005).

public signal may have negative welfare consequences, because it increases the response of the economy to news and, thus, may increase output gap volatility.[7] In Section 5, I show that, for a realistic choice of parameters, there is a hump-shaped relation between the precision of the public signal and output gap volatility. Second, a more precise public signal not only has effects on the output gap, but also on the cross-sectional allocation. As agents have more precise information on average productivity, they can set relative prices that are more responsive to their idiosyncratic shocks. Therefore, a more precise public signal is welfare improving because it allows a more efficient allocation of consumption and labor effort across sectors. What is the total welfare effect of increasing the precision of the public signal? In the examples considered, using realistic parameter values, the cross-sectional effect dominates and a more precise public signal increases total welfare. Therefore, the model implications are, overall, pro-transparency.

The model is introduced in Section 2. In Section 3, I characterize rational expectations equilibria under different monetary policy rules, show how the policy rule affects the real allocation, and show that full aggregate stabilization is feasible. In Section 4, I derive the welfare implications of different monetary policy rules and I show that full aggregate stabilization is typically suboptimal. In Section 5, I consider the model implications for transparency. Section 6 concludes.

2 The Model

2.1 Setup

I consider a dynamic model of monopolistic competition à la Dixit-Stiglitz with heterogeneous productivity shocks and imperfect information regarding aggregate shocks. Prices are set at the beginning of each period, but are, otherwise, flexible.

There is a continuum of infinitely lived households uniformly distributed on the unit interval $[0,1]$. Each household i is made of two agents: a consumer and a producer who is specialized in the production of good i. Preferences are represented by the utility function:

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\left(\ln C_{it}-\frac{1}{1+\eta}N_{it}^{1+\eta}\right)\right],$$

where C_{it} is a consumption index, described below, and N_{it} is the labor effort of producer i. Each period t, consumer i consumes a random sample of the goods produced in the economy,

[7] Here the output gap is measured with respect to the equilibrium under full information.

given by the set $J_{it} \subset [0,1]$, which will be described in detail below. The consumption index C_{it} is given by the CES aggregate:

$$C_{it} = \left(\int_{J_{it}} C_{ijt}^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

with $\sigma > 1$, where C_{ijt} is consumption of good j by consumer i in period t.

The production function for good i is

$$Y_{it} = A_{it} N_{it}.$$

Productivity is household-specific and labor is immobile across households. The productivity parameters A_{it} are the fundamental source of uncertainty in the model. Let a_{it} denote the log of individual productivity, $a_{it} = \ln(A_{it})$. Individual productivity at date t has an aggregate component, a_t, and an idiosyncratic component, ϵ_{it},

$$a_{it} = a_t + \epsilon_{it},$$

with $\int_0^1 \epsilon_{it} di = 0$. Aggregate productivity a_t follows the random walk

$$a_t = a_{t-1} + \theta_t.$$

At the beginning of period t, all households observe the value of aggregate productivity in the previous period, a_{t-1}. Next, the shocks θ_t and ϵ_{it} are realized. Agents in household i cannot observe θ_t and ϵ_{it} separately, they only observe the sum of the two, i.e., the individual productivity innovation

$$\theta_{it} = \theta_t + \epsilon_{it}.$$

Moreover, all agents observe a noisy public signal of the aggregate innovation

$$s_t = \theta_t + e_t.$$

The random variables $\theta_t, e_t, \epsilon_{it}$ are independent, serially uncorrelated, and normally distributed with zero mean and variances $\left(\sigma_\theta^2, \sigma_e^2, \sigma_\epsilon^2\right)$.

Summarizing, there are two aggregate shocks: the productivity shock θ_t and the shock e_t, representing noise in the public signal. The latter will be called "news shock." Both shocks are unobservable during period t, but are fully revealed at the beginning of $t+1$, when a_t is observed. The public history of the economy at the beginning of date t is denoted by

$$h_t \equiv \langle \theta_{t-1}, e_{t-1}, \theta_{t-2}, e_{t-2}, ... \rangle .$$

Let me turn now to the set J_{it}, the consumption basket of consumer i. The set J_{it} is randomly selected each period choosing goods with correlated productivity shocks. In this way, even though each consumer consumes a large number of goods (a continuum), the law of large numbers does not apply, and consumption baskets differ across consumers. This limits consumers' ability to infer the underlying aggregate state of the economy from price observations. In the appendix, I give a full description of the matching process between consumers and producers. Here, I summarize the properties of the consumption baskets that arise from that process. Each consumer receives a "sampling shock" v_{it} (unobserved by the consumer himself) and the goods in J_{it} are selected so that the distribution of the shocks ϵ_{jt} for $j \in J_{it}$ is normal with mean v_{it} and variance $\overline{\sigma}_{\epsilon}^2$. The sampling shocks v_{it} are normally distributed across consumers, with zero mean and variance σ_v^2. They are independent of all other shocks and satisfy $\int_0^1 v_{it} di = 0$. To ensure consistency of the matching process the variances $\sigma_v^2, \overline{\sigma}_{\epsilon}^2$ and σ_{ϵ}^2 have to satisfy $\sigma_v^2 + \overline{\sigma}_{\epsilon}^2 = \sigma_{\epsilon}^2$. Let me introduce here the parameter γ, given by

$$\gamma = \frac{\sigma_v^2}{\sigma_{\epsilon}^2},$$

which reflects the degree of heterogeneity in consumption baskets. The limit case $\gamma = 0$ corresponds to the case where all consumers consume the same representative sample of goods. The limit case $\gamma = 1$ arises when a consumer's basket is made of goods with identical productivity shocks. Let $\overline{\theta}_{it}$ denote the average productivity innovation in the basket of consumer i,

$$\overline{\theta}_{it} = \theta_t + v_{it}.$$

2.2 Trading, financial markets and monetary policy

The central bank acts as an account keeper for the agents in the economy. Each household holds a checking account, denominated in dollars, directly with the central bank. The account is debited whenever the consumer makes a purchase and credited whenever the producer makes a sale. Households start with a balance $M > 0$ at date 0. Their balance must be non-negative at the beginning of each period, but it is allowed to go negative within the period. At the beginning of period t the central bank pays the (gross) interest rate R_{t-1} on nominal balances carried over from last period. The presence of interest-paying money balances allows me to study a monetary environment where money and nominal bonds are perfect substitutes. This modeling choice is made for simplicity, as it allows a closed-form derivation of equilibrium

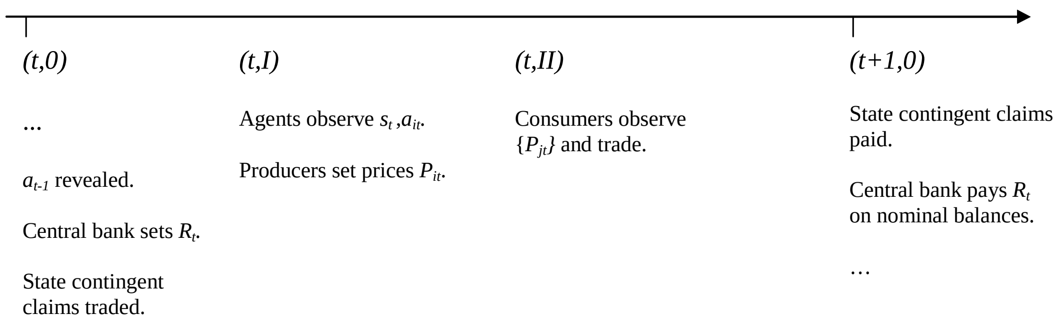

Figure 1: Timeline.

quantities and welfare.[8] The only other policy instrument available is a proportional subsidy τ on sales, which is financed by a lump-sum tax T_t. As usual in the literature, the subsidy will be used to eliminate the distortions due to monopolistic competition.

To describe the trading environment, it is convenient to divide each period t in three stages, $(t, 0)$, (t, I), and (t, II). In stage $(t, 0)$, the central bank sets the interest rate R_t for the current period, households observe a_{t-1} and trade state-contingent claims on a centralized financial market. These claims will be paid at the beginning of $(t + 1, 0)$. In stage (t, I), all aggregate and individual shocks are realized, producer i sets the dollar price of good i, P_{it}, and stands ready to deliver any quantity of good i at that price. Finally, in stage (t, II), consumer i observes the prices of the goods in his consumption basket, $\{P_{jt}\}_{j\in J_{it}}$, chooses his consumption vector, $\{C_{ijt}\}_{j\in J_{it}}$, and buys C_{ijt} from each producer $j \in J_{it}$. In this stage, consumer i and producer i are spatially separated, so the consumer does not observes the production of good i when choosing his consumption vector. Figure 1 summarizes the events taking place during period t.

Agents do not have access to the financial market in (t, I) and (t, II). Therefore, given the presence of idiosyncratic uncertainty, households will generally end up with different end-of-period nominal balances. However, households can fully smooth these shocks by trading

[8] For an alternative approach, see Obstfeld and Rogoff (2000), who start from a model with money in the utility function (and zero interest on cash balances) and look at the “cashless” limit to derive welfare in closed form.

contingent claims in stage $(t, 0)$ which are paid at the beginning of $(t+1, 0)$. This implies that the nominal balances M_{it} will be constant and equal to M in equilibrium.[9] In this way, I can eliminate the distribution of wealth from the state variables of the problem, which greatly simplifies the analysis.[10]

Let $Z_{it}(\omega_{it})$ denote the state-contingent claims purchased by household i in $(t, 0)$, where $\omega_{it} \equiv \{\epsilon_{it}, v_{it}, \theta_t\}$. The price of these claims is denoted by $q_t(\omega_{it})$. Let M_{it-1} denote the nominal balances of household i at the beginning of period t. The household budget constraint in period t is, then,

$$M_{it+1} = R_{t-1} M_{it} + (1+\tau) P_{it} Y_{it} - \int_{J_{it}} P_{jt} C_{ijt} dj - \int Z_{it}(\tilde{\omega}_{it}) q_t(\tilde{\omega}_{it}) d\tilde{\omega}_t - T_t + Z_{it}(\omega_{it}).$$

Nominal bonds are in zero net supply, and I omit them for simplicity. It is easy to show that in all the equilibria studied below their price is $1/R_t$.

The behavior of the monetary authority is described by a policy rule. For the definition of the policy rule it is useful to define an aggregate price index. For ease of notation, it will be convenient to use the geometric mean[11]

$$P_t \equiv \exp\left(\int_0^1 \ln P_{it} di\right). \tag{2}$$

In period $(t, 0)$, the central bank sets R_t based on the public history h_t and the past realizations of the price index. A monetary policy rule is described by the map $\mathcal{R}$, which gives $R_t = \mathcal{R}(h_t, P_{t-1}, P_{t-2}, ...)$. Allowing the monetary policy to condition R_t on the public signal s_t would not alter any of the results.

Finally, the supply of nominal balances is kept constant at M. Therefore, the government budget balance condition is

$$T_t = (R_t - 1) M + \tau \int_0^1 P_{it} Y_{it} di.$$

2.3 Equilibrium

Household behavior is captured by three functions, $\mathcal{Z}$, $\mathcal{P}$ and $\mathcal{C}$. The first gives the optimal holdings of state-contingent claims as a function of the initial balances M_{it} and of the public

[9] Note that the nominal balances M_{it} are computed after the claims from period $t-1$ have been settled.

[10] The use of this type of assumption to simplify the study of monetary models goes back to Lucas (1990).

[11] An alternative price index is $\hat{P}_t \equiv \left(\int_0^1 P_{it}^{1-\sigma} di\right)^{\frac{1}{1-\sigma}}$. All results stated for P_t hold for $\hat{P}_t$, modulo a multiplicative constant.

history h_t, $Z_{it+1}(\omega_{it}) = \mathcal{Z}(\omega_{it}; M_{it}, h_t)$. The second gives the optimal price for household i, as a function of the same variables plus the current realization of individual productivity and of the public signal, $P_{it} = \mathcal{P}(M_{it-1}, h_t, a_{it}, s_t)$. The third gives optimal consumption as a function of the same variables plus the observed price vector, $C_{it} = \mathcal{C}\left(M_{it}, h_t, a_{it}, s_t, \{P_{ij}\}_{j \in J_{it}}\right)$. Before defining an equilibrium, I need to introduce two more objects. The map $\Delta(h_t)$ gives the distribution of nominal balances M_{it} for each public history h_t, and $q(\omega_{it}; h_t)$ denotes the price of a ω_{it}-contingent claim in period $(t, 0)$ after public history h_t.

A *symmetric rational expectations equilibrium* under the policy rule $\mathcal{R}$ is given by an array of functions $\{\mathcal{Z}, \mathcal{P}, \mathcal{C}, \Delta, q\}$ that satisfy three conditions: optimality, market clearing and consistency. Optimality requires that the individual rules $\mathcal{Z}, \mathcal{P}$ and $\mathcal{C}$ are optimal for the individual household, taking as given the policy rule $\mathcal{R}$, the distributions Δ, and the fact that all other households follow $\mathcal{Z}, \mathcal{P}, \mathcal{C}$. Market clearing requires that the goods markets and the market for state-contingent claims clear for each h_t. Consistency requires that the dynamics of the distribution of nominal balances, described by Δ, are consistent with individual decision rules.

3 Policy Rules and Expectations

In this section, I characterize the equilibrium behavior of output and prices, and show how it is affected by the choice of a monetary policy rule.

3.1 Linear equilibria under a price targeting rule

Given the form of the agents' preferences and the normality of the shocks, it is possible to study linear rational expectations equilibria in closed form. To describe monetary policy I concentrate on a specific type of interest rate rule, representing a form of price level targeting. I will show later that this is without loss of generality. The nominal interest rate is set as follows,[12]

$$r_{t+1} = \rho + \xi\left(p_t - p_t^*\right), \tag{3}$$

where p_t is the price index defined in (2), p_t^* is the state-contingent target

$$p_t^* = \mu a_{t-1} + \phi_\theta \theta_t + \phi_s s_t, \tag{4}$$

[12] Throughout the paper, I adopt the convention that a lowercase variable denotes the natural logarithm of the corresponding uppercase variable.

and $\rho, \xi, \mu, \phi_\theta$, and ϕ_s are constant parameters. Under this rule, I look at equilibria where individual prices and consumption are given by

$$p_{it} = \mu a_{t-1} + \phi_\theta \theta_{it} + \phi_s s_t, \tag{5}$$

$$c_{it} = \psi_0 + a_{t-1} + \psi_\epsilon \theta_{it} + \psi_v \overline{\theta}_{it} + \psi_s s_t, \tag{6}$$

where $\psi_0, \psi_\epsilon, \psi_v$ and ψ_s are constant coefficients.[13] Due to the presence of complete financial markets the equilibrium distribution of nominal balances is degenerate, with balances equal to M for all households at the beginning of each period. In equilibrium, the aggregate price level is always equal to the target p_t^* and the nominal interest rate is constant and equal to ρ.

An equilibrium of the form (5)-(6) exists if the parameters of the policy rule satisfy certain restrictions. In particular, next proposition shows that the choice of the parameter μ in (4) uniquely pins down ρ, ϕ_θ, and ϕ_s in the policy rule and all the remaining coefficients of the linear equilibrium. The proofs of this and of all following results are in the appendix. Let $\boldsymbol{\phi}$ and $\boldsymbol{\psi}$ denote the vectors $\{\phi_\theta, \phi_s\}$ and $\{\psi_0, \psi_\epsilon, \psi_v, \psi_s\}$.

Proposition 1 *For every $\mu \in \mathbb{R}$ there exists a unique vector $\{\boldsymbol{\phi}, \boldsymbol{\psi}\}$ and a unique ρ such that (5)-(6) constitute a rational expectations equilibrium under the monetary policy rule (3)-(4) for any value of ξ. If $\xi > 1$ the equilibrium is locally determinate.*

Equilibrium behavior can be described as follows. At the beginning of period t, the monetary authority observes a_{t-1} and announces a menu of price targets p_t^* conditional on the current realization of the aggregate shock θ_t, which it cannot currently observe. During trading, each agent i sets his price and consumption responding to the variables in his information set. At the beginning of period $t+1$, the central bank observes the realized price level p_t and the aggregate shock θ_t, and checks whether aggregate behavior did conform to the target p_t^*. As this is always true in equilibrium, the central bank leaves the nominal interest rate unchanged at ρ.

Aggregate output is measured by the index[14]

$$y_t \equiv \int_0^1 c_{it} di.$$

[13] Recall that $\overline{\theta}_{it}$ is the average productivity innovation for the goods in the basket of consumer i. I will clarify later why this variable enters (6).

[14] An alternative quantity index is

$$\hat{y}_t = \log \frac{\int_0^1 P_{it} Y_{it} di}{P_t}.$$

All results stated for y_t also hold for $\hat{y}_t$, modulo a constant term.

The equilibrium coefficients $\boldsymbol{\psi}$ determine the response of aggregate output to aggregate shocks. Summing (6) across agents, and recalling that $s_t = \theta_t + e_t$, gives

$$y_t = \psi_0 + a_{t-1} + \psi_\theta \theta_t + \psi_s (\theta_t + e_t), \tag{7}$$

where $\psi_\theta \equiv \psi_\epsilon + \psi_v$. The first question raised in the introduction can now be stated in formal terms. How does the choice of a policy rule, μ, affects the equilibrium coefficients ψ_θ and ψ_s, which determine the response of output to fundamental and news shocks? The rest of this section addresses this question.

Before proceeding further, it is useful to clarify why the consumption of agent i, in (6), depends on $\overline{\theta}_{it}$. If all producers follow (5) then the population of log prices observed by consumer i follows a normal distribution with mean $\mu a_{t-1} + \phi_\theta \overline{\theta}_{it} + \phi_s s_t$ and variance $\phi_\theta^2 \overline{\sigma}_\epsilon^2$. Given that consumer i observes the public signal s_t, he can back out $\overline{\theta}_{it}$ from the mean of this distribution, as long as $\phi_\theta \neq 0$. Two conclusions follow: (i) knowing $\overline{\theta}_{it}$ one can derive the entire distribution of prices faced by consumer i, and (ii) $\overline{\theta}_{it}$ is a sufficient statistic for all the information regarding θ_t contained in the price vector $\{P_{jt}\}_{j \in J_{it}}$ (if $\phi_\theta \neq 0$). The second conclusion is stated formally in the following lemma.

Lemma 1 *If prices are given by (5) and $\phi_\theta \neq 0$ then the information of consumer i regarding the current shock θ_t is summarized by the three independent signals s_t, θ_{it} and $\overline{\theta}_{it}$.*

A useful consequence of this lemma is that in all linear equilibria with price targeting the informational content of prices is the same and is independent of the monetary policy rule chosen, except in the knife-edge case $\phi_\theta = 0$ in which prices are uninformative.

3.2 Monetary policy and real allocations

Let me turn now to the effects of different monetary policy rules, parametrized by μ, on the real equilibrium allocation. Let me begin with the case of full information, which arises when s_t is a noiseless signal, $\sigma_e^2 = 0$.

Proposition 2 *With full information the real allocation in all linear equilibria with price targeting is constant and independent of μ. Aggregate output satisfies*

$$y_t = \psi_0 + a_t.$$

This proposition gives a neutrality result: under full information the choice of the policy rule is immaterial for the determination of the real allocation.[15] Furthermore, this proposition gives a characterization of aggregate dynamics: under full information aggregate output moves one for one with aggregate productivity a_t. The value of the constant ψ_0 depends on the subsidy τ. In particular, it is possible to find a subsidy τ^* that replicates the first best allocation, that is, the allocation that would be chosen by a social planner maximizing the ex ante expected utility of the representative household under full information.[16] For this reason, I will use the full-information equilibrium as a benchmark in the rest of the analysis.

When agents have imperfect information, the choice of the policy rule is no longer neutral, as shown in the following proposition.

Proposition 3 *With imperfect information ($\sigma_\theta^2 > 0, \sigma_e^2 > 0, \sigma_\epsilon^2 > 0$) the real allocation in a linear equilibrium with price targeting depends on μ. The effects of changing μ on the equilibrium coefficients $\{\phi_\theta, \phi_s, \psi_\theta, \psi_s\}$ are:*

$$\begin{aligned} \frac{\partial \phi_\theta}{\partial \mu} &> 0, \qquad \frac{\partial \phi_s}{\partial \mu} > 0, \\ \frac{\partial \psi_\theta}{\partial \mu} &> 0, \qquad \frac{\partial \psi_s}{\partial \mu} < 0. \end{aligned}$$

Moreover, $\partial \left(\psi_\theta + \psi_s\right) / \partial \mu > 0$.

The main result of this proposition is that an increase in μ increases the response of aggregate output to fundamental shocks and reduces its response to news shocks. To shed light on this result, I will now discuss the way in which the choice of μ affects household behavior. Consider first the optimality condition that determines optimal consumption

$$c_{it} = \mathbb{E}_{i,(t,II)} \left[c_{it+1} - \left(r_t - \overline{p}_{it+1} + \overline{p}_{it}\right)\right],$$

where $\mathbb{E}_{i,(t,II)}[.]$ denotes the expectation of agent i at date (t, II) and $\overline{p}_{it}$ denotes the price index for the consumption basket of consumer i.[17] Apart from the fact that price indexes are consumer-specific, this is the standard consumer's Euler equation with iso-elastic preferences, linking the expected growth rate of consumption to the expected real interest rate.

[15] See McCallum (1979) for an early neutrality result in a model with pre-set prices.

[16] See Lemma 4 in the appendix.

[17] A constant term is omitted in the optimality condition. See (13) in the appendix for the formal definition of $\overline{p}_{it}$.

In a linear equilibrium (5)-(6), expected future consumption is given by $\mathbb{E}_{i,(t,II)}\left[c_{it+1}\right] = \psi_0 + \mathbb{E}_{i,(t,II)}\left[a_t\right]$ and is independent of the policy rule. The reason why the choice of μ affects consumption is that it affects the expected real interest rate

$$\mathbb{E}_{i,(t,II)}\left[r_t - \overline{p}_{it+1} + \overline{p}_{it}\right] = \rho - \mu a_{t-1} - \mu \mathbb{E}_{i,(t,II)}\left[\theta_t\right] + \overline{p}_{it}.$$

For a given $\overline{p}_{it}$, an increase in μ increases the response of consumers' demand both to a fundamental shock and to a news shock, given that both shocks increase the expectation $\mathbb{E}_{i,(t,II)}\left[\theta_t\right]$, and, thus, the expected future price level. This would seem to suggest that an increase in μ makes output more sensitive to both shocks. However, in general equilibrium, the choice of μ also affects the response of the current prices in $\overline{p}_{it}$. Therefore, to understand the full effect of monetary policy, I need to turn to the pricing conditions.

The optimality condition for p_{it} takes the form

$$p_{it} = \frac{\eta}{1+\sigma\eta}\left(\mathbb{E}_{i,(t,I)}\left[d_{it}\right] - a_{it}\right) + \frac{1}{1+\sigma\eta}\left(\mathbb{E}_{i,(t,I)}\left[\overline{p}_{it} + c_{it}\right] - a_{it}\right), \tag{8}$$

where d_{it} is the demand index

$$d_{it} = \int_{j \in \tilde{J}_{it}} \left(c_{jt} + \sigma \overline{p}_{jt}\right) dj,$$

capturing the intercept of the demand curve faced by producer i.[18] After a fundamental or a news shock, producers expect consumers' demand to increase more if μ is larger, following the reasoning above. As a consequence, they tend to set higher prices. Therefore, an increase in μ magnifies the response of prices to both shocks.[19] This is confirmed by the results for ϕ_θ and ϕ_s in Proposition 3.

Let me go back now to the response of the expected real interest rate. Summing up the discussion so far, the net effect on the expected real rate depends on the relative strength of two effects: a direct effect, by which a higher μ increases the expected future price level, and an indirect effect, by which a higher μ increases current prices. Proposition 3 shows that the first effect dominates in the case of a fundamental shock, while the second effect dominates in the case of a news shock. The reason for this difference is the following. After a positive news

[18] The demand for good i is

$$Y_{it} = D_{it} P_{it}^{-\sigma}.$$

See Lemma 3 in the appendix for the full derivation of D_{it}. Constant terms are omitted both in (8) and in the expression for d_{it}.

[19] This discussion focuses on the effect working through $\mathbb{E}_{i,(t,I)}\left[d_{it}\right]$ in (8). It is possible to show that the effect working through $\mathbb{E}_{i,(t,I)}\left[\overline{p}_{it} + c_{it}\right]$ goes in the same direction.

shock producers tend to over-estimate the demand increase, given that they update upward their beliefs on θ_t, while θ_t has not changed. If, instead, a positive productivity shock hits, producers tend to under-estimate the demand increase, given that they only have access to noisy observations of θ_t. Therefore, an increase in μ leads to higher prices in response to both shocks, but the increase is sharper (relative to the actual demand increase) after a news shock.

This discussion also helps to clarify the mechanism behind the neutrality result in Proposition 2. In the case of full information producers perfectly anticipate any aggregate change in demand, i.e., they never over-estimate or under-estimate demand conditions. This accounts for the fact that the response of prices at date t fully offsets the effects of μ on future prices, and the real rate is unchanged.

The key assumptions behind the non-neutrality result are that agents have dispersed information regarding θ_t, that they are forward-looking, and that the value of θ_t will be fully revealed in the future. The central bank can then affect individual behavior by announcing that the price target p^*_{t+1} will respond to θ_t. If the price target p^*_{t+1} was set as a function of variables that are common knowledge at date t, e.g. s_t, this would have no effects on the real allocation at date t. In this case, the change in demand would be perfectly forecasted by producers, leading to a neutrality result analogous to the one derived in the case of perfect information.

To conclude this discussion, let me remark that the parameters $\{\phi_\theta, \phi_s, \psi_\theta, \psi_s\}$ also affect the sensitivity of *individual* consumption and prices to idiosyncratic shocks. Therefore, the choice of μ has implications not only for the aggregate responses, but also for the cross-sectional distribution of consumption and relative prices. This observation will turn out to be crucial in evaluating the welfare consequences of different monetary rules.

3.3 Full aggregate stabilization

Let me now compare aggregate output dynamics under full information and under imperfect information. Recall that, with full information, aggregate output responds one for one to productivity shocks. Inspecting (7) shows that the same property holds under imperfect information if and only if $\psi_s = 0$ and $\psi_\theta = 1$. To achieve a path for aggregate output that tracks the changes in the full information benchmark, the central bank has to eliminate the effect of news shocks and ensure, at the same time, that output responds one for one to fundamental shocks. Next proposition shows that, even though the monetary authority can only set μ, it can still

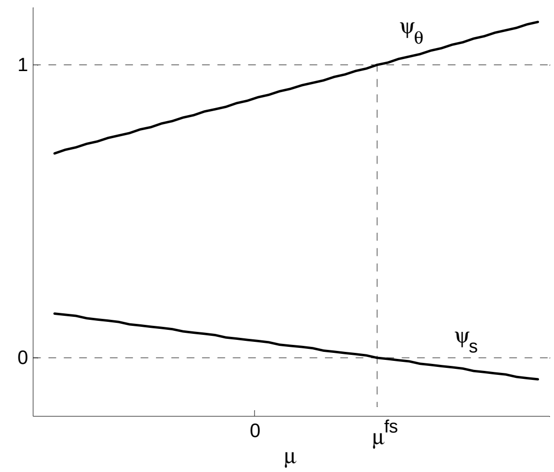

Figure 2: Monetary Policy and Full Aggregate Stabilization

achieve both objectives. This type of policy rule is called “full aggregate stabilization.”

Proposition 4 *There exists a monetary policy rule, given by μ^{fs}, that achieves full aggregate stabilization, $y_t = \psi_0 + a_t$.*

To illustrate this result Figure 2 shows the equilibrium relation between μ and the parameters ψ_θ and ψ_s. Consistent with the results in Proposition 3, an increase in μ increases ψ_θ and reduces ψ_s. The surprising result is that the value of μ that sets ψ_s to zero is the same that sets ψ_θ equal to one.

Notice that the value of the constant ψ_0 in the equilibrium described in Proposition 4 is, in general, different from the one in Proposition 2. However, the subsidy τ can be chosen so as to achieve the same value of ψ_0. Therefore, by choosing the right pair (τ, μ) the monetary authority can perfectly replicate the behavior of aggregate output arising under full information.

3.4 General policy rules

Let me consider now more general interest rate rules. In particular, consider all the policy rules under which the equilibrium behavior of prices and consumption is given by

$$p_{it} = \phi_{0,t} + \phi_{\theta,t}\theta_{it} + \phi_{s,t}s_t, \tag{9}$$

$$c_{it} = \psi_{0,t} + a_{t-1} + \psi_{\epsilon,t}\theta_{it} + \psi_{v,t}\overline{\theta}_{it} + \psi_{s,t}s_t, \tag{10}$$

where the coefficients $\boldsymbol{\phi}_t$ and $\boldsymbol{\psi}_t$ are linear functions of h_t, that is, of all the variables that are common knowledge at the beginning of period t. For simplicity, suppose the subsidy τ is constant over time. The following proposition shows that the real allocation arising in any of these equilibria can also be achieved under a price targeting rule analogous to (3)-(4).

Proposition 5 *For any linear equilibrium of the form (9)-(10) there exists a sequence $\{\rho_t, \mu_t\}$ such that the same allocation is supported in a linear equilibrium under the policy rule*

$$\begin{aligned} r_{t+1} &= \rho_{t+1} + \xi \left(p_t - p_t^*\right), \\ p_t^* &= \mu_t a_{t-1} + \phi_{\theta t}\theta_t + \phi_{st} s_t. \end{aligned}$$

4 Optimal Monetary Policy

4.1 Welfare

Let me turn now to welfare analysis and to the characterization of optimal monetary policy. I will evaluate the choice of μ by looking at its implications for expected utility in period t

$$W_t = \mathbb{E}_t \left[\int_0^1 \left(\ln C_{it} di - \frac{1}{1+\eta} N_{it}^{1+\eta} \right) di \right],$$

where $\mathbb{E}_t [.]$ denotes the expectation conditional on the public history h_t. In a linear equilibrium (5)-(6) the only variables that are relevant for consumers' utility are consumption, c_{it}, and relative prices, $p_{it} - p_t$.[20] This implies that the term μa_t in p_{t+1} has no effects on welfare in period $t+1$, although, as argued in the previous section, it matters for the real allocation in period t. This allows me to conduct welfare analysis period-by-period and evaluate the monetary rule μ by looking at its effects on expected utility in period t. Thanks to log-normality the welfare measure W_t can be derived analytically.

Lemma 2 *Given a linear equilibrium, characterized by the coefficients $\boldsymbol{\phi}$ and $\boldsymbol{\psi}$, consumers' welfare is equal to*

$$W_t = a_{t-1} + \psi_0 - \frac{1}{1+\eta} \exp\left\{ (1+\eta) \left(\psi_0 - \frac{1}{2} w \right) \right\}$$

where

$$\begin{aligned} w = & - (1+\eta) \left((\psi_s + \psi_\theta - 1)^2 \sigma_\theta^2 + \psi_s^2 \sigma_e^2 \right) + \sigma (\sigma - 1) \phi_\theta^2 \overline{\sigma}_\epsilon^2 \\ & - (1+\eta) (1 + \sigma\phi_\theta - (\psi_v + \sigma\phi_\theta) \gamma)^2 \sigma_\epsilon^2 + \\ & - \psi_\epsilon^2 \sigma_\epsilon^2 - (\psi_v + \sigma\phi_\theta)^2 \gamma \overline{\sigma}_\epsilon^2 \end{aligned} \tag{11}$$

[20] The distribution of labor supply can be derived from these quantities.

The choice of the interest rate rule affects the equilibrium allocation both in terms of levels (captured by the constant term ψ_0) and in terms of the economy's response to aggregate and idiosyncratic shocks (captured by the coefficients $\phi_\theta, \psi_\epsilon, \psi_v$, and ψ_s). However, any level effect of μ can be offset by adjusting the subsidy τ. Therefore, the optimal value of μ can be found by looking at its effects on the welfare measure w, defined in (11), as shown in next proposition.

Proposition 6 *The optimal monetary policy rule is found by choosing the μ that maximizes (11), where the relation between μ and the parameters $\{\phi_\theta, \psi_\epsilon, \psi_v, \psi_s\}$ is the one derived in Proposition 1.*

For the sake of interpretation w can be expressed as:

$$\begin{aligned} w &= -(1+\eta)\,\mathbb{E}\left[(y_t - \psi_0 - a_t)^2 \,|a_{t-1}\right] + \sigma(\sigma-1)\,Var\,(p_{jt}|j \in J_{it}) \\ &\quad -(1+\eta)\,Var\,(n_{it}) - Var\left(c_{jt} + \sigma\overline{p}_{jt}|j \in \tilde{J}_{it}\right), \end{aligned} \tag{12}$$

This expression shows that the welfare effects of monetary policy can be split in four parts. The first term in (12) captures the volatility of an aggregate "output gap" measure $y_t - \psi_0 - a_t$. This reflects deviations of aggregate output from the path that would arise under full information.[21] The remaining three terms capture cross-sectional effects. The second term reflects the effect of dispersion in relative prices on welfare. Note that this term is positive, that is, larger price dispersion is beneficial in terms of welfare. This is due to the fact that higher price dispersion reduces the price index $\overline{P}_{it}$ for each consumer —since the price index is concave in individual prices—, and increases consumers' purchasing power. The third term captures the welfare loss associated to the dispersion in labor supply across producers. The fourth term captures the welfare loss due to the dispersion in consumption across consumers, corrected for the difference in relative prices faced by different consumers.

The fact that higher price dispersion increases welfare might seem at odds with standard results in the sticky-price literature, where price dispersion is associated to a welfare loss. However, in standard sticky-price models there are no idiosyncratic productivity shocks, so any dispersion in relative prices is harmful, since producers have identical marginal costs. Here, instead, relative price dispersion is beneficial as long as it reflects productivity differences. Price dispersion that does not reflect productivity differences is still costly, since it determines greater dispersion in labor supply across producers, as reflected in the third term of (12).

[21] Recall, from the discussion on page 16, that the level of ψ_0 is generally different with full information and with imperfect information. The measure $\mathbb{E}\left[(y_t - \psi_0 - a_t)^2 \,|a_{t-1}\right]$ disregards this level effect, which, again, is taken care of by τ.

4.2 Optimal accommodation of news shocks

Having obtained a general characterization of optimal monetary policy, I can turn to the question: is full aggregate stabilization optimal? That is, should monetary policy completely eliminate the aggregate demand disturbances driven by news shocks and set $\psi_s = 0$? The next proposition shows that, typically, full stabilization is suboptimal and optimal monetary policy involves some degree of accommodation of news shocks.

Proposition 7 *If $\eta > 0$ and $\gamma \in (0, 1)$ then full aggregate stabilization is suboptimal and the optimal monetary policy μ^o satisfies*

$$\mu^o < \mu^{fs}.$$

At the optimal monetary policy news shocks have a positive effect on aggregate output

$$\psi_s^o > 0.$$

Full aggregate stabilization is optimal, $\mu^o = \mu^{fs}$, if any one of the following conditions is satisfied: (i) $\eta = 0$, (ii) $\gamma = 0$, (iii) $\gamma = 1$.

Full stabilization is only optimal in three special cases. The first, $\eta = 0$, corresponds to the case of linear disutility of labor. To interpret the other two cases, recall that the parameter γ reflects how representative is the sample of goods purchased by a given consumer. The limit case $\gamma = 0$ corresponds to the case where all consumers consume all goods. In this case, prices are fully revealing and there is full information at the consumption stage. The limit case $\gamma = 1$ arises when each consumer consumes essentially one type of good. In this case, there is no asymmetric information among price-setters, since each producer knows all the prices faced by his consumers. This shows that both asymmetric information among consumers and among producers are needed to obtain a meaningful trade-off between aggregate stabilization and cross-sectional efficiency.

To illustrate the result in Proposition 7 I will use a numerical example. The parameters for the example are in Table 1. The values for σ and η are chosen in the range of values used in the sticky-price literature. The values for the variances σ_θ^2 and σ_ϵ^2 are set conventionally at 1, while I assume that the value of σ_e^2 is 1/3, implying that the public signal is relatively more precise than the private signals. For γ I choose a value of 0.5.

Figure 3 illustrates how the choice of the monetary policy rule affects the four variance terms in (12). The figure identifies the value of μ that achieves full aggregate stabilization,

σ	7	η	2
σ_θ^2	1	σ_ϵ^2	1
σ_e^2	1/3	γ	0.5

Table 1: Parameters for the benchmark example

μ^{fs}, and the value corresponding to optimal monetary policy, μ^o. Panel (a) plots the relation between μ and aggregate output gap volatility. Not surprisingly, the minimum for this curve is reached at μ^{fs}, that is, at the monetary policy that achieves full aggregate stabilization. In accordance with Proposition 4, output gap volatility is zero at μ^{fs}. Panel (b) shows the effects of monetary policy on equilibrium price dispersion. In a neighborhood of μ^{fs}, an increase in μ has the effect of reducing the dispersion of relative prices by reducing the value of $|\phi_\theta|$, which determines the response of individual prices to individual productivity shocks.[22] This reduction in price dispersion can be interpreted as follows. A household facing a positive productivity shock, $\theta_{it} > 0$, increases its consumption more if μ is larger. As a consequence the marginal utility of consumption decreases and, at the price-setting stage, household i has a weaker incentive to lower the relative price of its good, even though its productivity has increased. Therefore, relative prices respond less to differences in individual productivities. Finally, under the parametric assumptions made, an increase in μ tends to reduce the dispersion of labor supply and consumption. Both effects are depicted in panel (c). Remember that price dispersion has a positive effect on welfare, while labor supply and consumption dispersion have negative effects. Therefore, the total effect of an increase in μ in a neighborhood of μ^{fs} depends on the relative strength of the effects depicted in panels (b) and (c).

Figure 4 plots the total welfare function w. It shows that the price-dispersion term dominates and makes the loss function decreasing at μ^{fs}. This implies that optimal monetary policy is characterized by $\mu^o < \mu^{fs}$. Proposition 7 shows that this is a generic property of the model. Since the relation between ψ_s and μ is decreasing (from Proposition 3) and ψ_s is zero when $\mu = \mu^{fs}$, it follows that optimal monetary policy entails $\psi_s > 0$.

5 Transparency

So far, I have assumed that the source of public information, the signal s_t, is exogenous and outside the control of the monetary authority. Suppose, now, that the central bank has

[22] Lemma 5 in the appendix shows that as long as $\psi_\theta \leq 1$ the value of ϕ_θ is negative. This, together with Proposition 3 implies that $|\phi_\theta|$ is decreasing in μ, for $\mu \leq \mu^{fs}$.

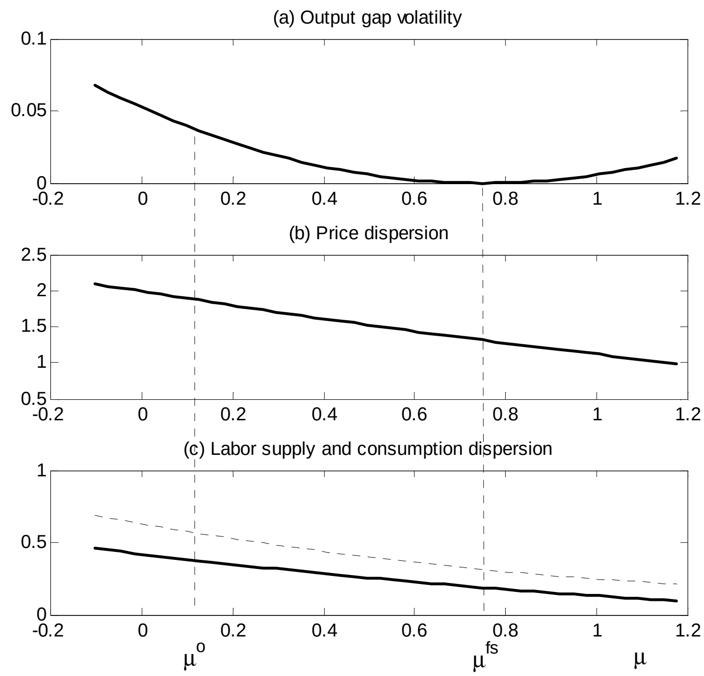

Figure 3: The Effects of Monetary Policy on Aggregate Volatility and on Cross-Sectional Variances
Panel (a): variance of $y_t - a_t$. Panel (b): variance of p_{jt} for $j \in J_{it}$. Panel (c): solid line, variance of n_{it}; dashed line, variance of $c_{jt} + \sigma\overline{p}_{jt}$ for $j \in \tilde{J}_{it}$.

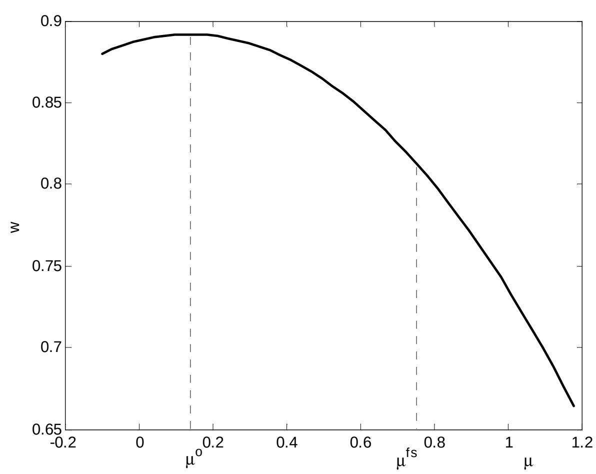

Figure 4: Monetary Policy and Total Welfare

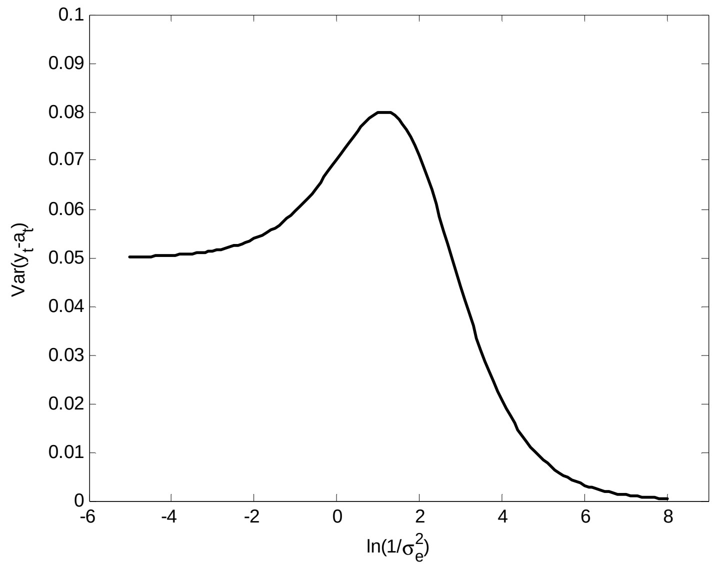

Figure 5: Precision of Public Information and Output Gap Volatility

some control on the information received by the private sector. For example, it can decide whether or not to release some aggregate statistics, which would increase the precision of public information. What are the welfare effects of this decision? To address this question I look at the effects of changing the precision of the public signal, measured by $1/\sigma_e^2$, on total welfare, assuming that the policy rule is set optimally for each level of $1/\sigma_e^2$. This exercise connects this paper to a growing literature on the welfare effects of public information.[23]

Figure 5 shows the relation between the precision of the public signal and the volatility of the output gap.[24] This relation is non-monotone, increasing for low values of $1/\sigma_e^2$ and decreasing for high values. When the signal is very imprecise agents disregard it and the coefficient ψ_s in (7) goes to zero. In this case aggregate volatility is low, since there is no volatility due to the news shock e_t. When the signal becomes more precise, agents rely more on the public signal and this, initially, increases aggregate volatility. Therefore, for low values of $1/\sigma_e^2$, more precise public information has a destabilizing effect on the economy. Eventually, when the signal precision is very large, the economy converges towards the full information equilibrium and output gap volatility goes to zero.

[23] See Morris and Shin (2002) and the references in footnotes 5 and 6.

[24] The latter is measured by $\mathbb{E}\left[(y_t - \psi_0 - a_t)^2 \, | a_{t-1}\right]$. To make the graph easier to read I use a log scale for $1/\sigma_e^2$. The parameters are those in Table 1.

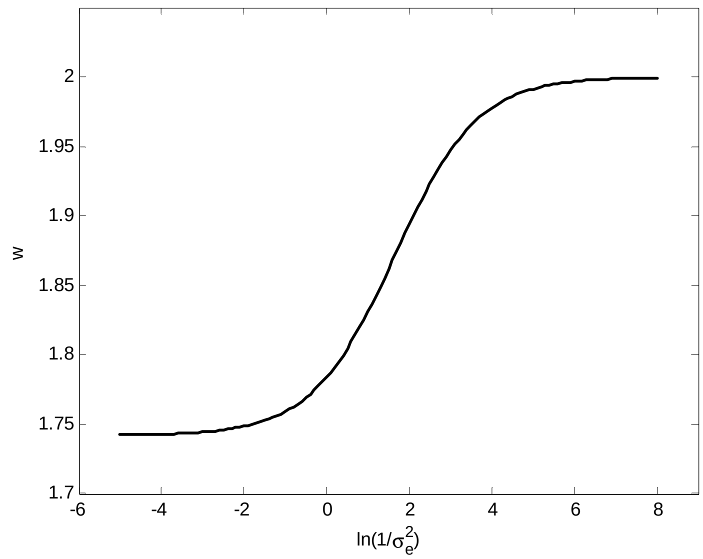

Figure 6: Precision of Public Information and Total Welfare

This figure, however, is only telling a piece of the story. Figure 6 shows the effect of changing the signal precision on the total welfare measure w. Recall from (12) that w corresponds to the sum of aggregate volatility (with a negative sign) and three terms reflecting cross-sectional distortions. Figure 6 shows that, putting together all four terms, welfare is always increasing in the signal precision.

To understand the difference between the two graphs notice that, when the public signal is very imprecise, agents have to use their own individual productivity to estimate aggregate productivity. This makes them underestimate the idiosyncratic component of their productivity and leads to a compressed distribution of relative prices. An increase in the signal precision helps producers set relative prices that reflect more closely the underlying productivity differences. The associated gain in allocative efficiency is positive and more than compensates for the welfare loss due to higher aggregate volatility, which is present at low values of $1/\sigma_e^2$.

The notion that more precise information about aggregate variables has important cross-sectional implications is also highlighted in Hellwig (2005). In that paper there are no idiosyncratic productivity shocks, so the cross-sectional gain is reflected in a *reduction* in price dispersion. However, the underlying principle is the same: in both cases a more precise public signal leads to relative prices more in line with productivity differentials (which are zero in

Hellwig (2005)).

The graphs above have been derived under the assumption that monetary policy is set optimally. However, given the parameters in Table 1, similar outcomes emerge if μ is kept constant and only the signal precision changes. I have experimented with several parameter combinations, both adjusting the policy rule and leaving it unchanged, and could not obtain an example of a welfare-reducing increase in precision. Whether this is a general proposition or counter-examples can be constructed is an open question for future work.

6 Conclusions

In this paper, I have explored the effect of policy rules in a model where information about macroeconomic fundamentals is dispersed across the economy. The emphasis has been on the ability of the policy rule to shape the economy's response to different shocks. In particular, the monetary authority is able to reduce the economy's response to news shocks by manipulating agents' expectations about the real interest rate. The principle behind this result goes beyond the specific model used in this paper: by announcing that policy actions will respond to future information, the monetary authority can affect differently agents with different pieces of information. In this way, it can change the aggregate response to fundamental and news shocks. The result that full aggregate stabilization is feasible is clearly more dependent on the specific features of the model. Nonetheless, that result is useful in clarifying why eliminating the effects of news is not optimal. Not necessarily because it is not possible, but because, even when possible, it is socially costly.

The optimal policy rule used in this paper can be implemented both under commitment and under discretion. To offset an expansion driven by optimistic beliefs, the central bank announces that it will make the real interest rate higher if good fundamentals do not materialize. With flexible prices, this effect is achieved with a jump in the price level between t and $t+1$. As argued in Section 4.1, this jump has no welfare consequences from period $t+1$ on. Therefore, the central bank has no incentive to deviate from its announced policy. In economies with sluggish price adjustment, a similar effect could be obtained by a combination of a price level change and an increase in nominal interest rates. In that case, however, commitment problems are likely to arise, because both type of interventions have additional distortionary consequences ex post. The study of models where lack of commitment interferes with the central bank's ability to deal with informational shocks seems an interesting area for further research.

Finally, in the model presented, the information sets of consumers, producers, and of the central bank, are independent of the monetary rule chosen. Morris and Shin (2005) have recently argued that stabilization policy may have adverse effects, if it reduces the informational content of prices. Here this concern does not arise, as the information in the price indexes observed by consumers is independent of monetary policy (as seen in Lemma 1). A natural extension of the model in this paper would be to introduce additional sources of noise in prices, so as to make their informational content endogenous and sensitive to policy.

Appendix

Random consumption baskets

At the beginning of each period, household i is assigned two random variables, ϵ_{it} and v_{it}, independently drawn from normal distributions with mean zero and variances, respectively, σ_ϵ^2 and σ_v^2. These variables are not observed by the household. The first random variable represents the idiosyncratic productivity shock, the second is the sampling shock, that will determine the sample of firms visited by consumer i. Consumers and producers are then randomly matched so that the density of producers of type ϵ_{it} matched to consumers of type v_{jt} is given by $\phi(\epsilon_{it}, v_{jt})$, where ϕ is a bivariate joint normal density, with covariance matrix $\begin{bmatrix} \sigma_\epsilon^2 & \sqrt{\gamma}\sigma_v\sigma_\epsilon \\ . & \sigma_v^2 \end{bmatrix}$, where $\gamma \in [0,1]$. Since the variable v_{it} has no direct effect on payoffs I can normalize its variance and set $\sigma_v = \sqrt{\gamma}\sigma_\epsilon$.

Let the sets J_{it} and $\tilde{J}_{it}$ be defined as in the text. Given the matching process described above the following properties follow. The distribution $\{\epsilon_{jt} : j \in J_{it}\}$ is a normal $N\left(v_{it}, (1-\gamma)\sigma_\epsilon^2\right)$. That is, the average productivity innovation for the goods observed by consumer i is

$$\overline{\theta}_{it} = \theta_t + v_{it}.$$

The distribution $\left\{v_{jt} : j \in \tilde{J}_{it}\right\}$ is a normal, $N\left(\gamma\epsilon_{it}, \gamma(1-\gamma)\sigma_\epsilon^2\right)$.

Throughout the paper, I use the following notation:

$$\begin{aligned} \overline{\sigma}_\epsilon^2 &= (1-\gamma)\sigma_\epsilon^2, \\ \overline{\sigma}_v^2 &= \gamma(1-\gamma)\sigma_\epsilon^2, \end{aligned}$$

where $\overline{\sigma}_\epsilon^2$ is the variance of the productivity shocks across the goods in the basket of a given consumer and $\overline{\sigma}_v^2$ is the variance of the sampling shocks across the consumers purchasing from a given producer.

Proof of Proposition 1

The proof is split in four steps. First, I introduce price and demand indexes that apply in the linear equilibrium conjectured. Second, I use them to characterize the individual optimization problem. Then, I define and solve a fixed point problem that gives the desired equilibrium. Finally, I turn to local determinacy.

Price and demand indexes

Suppose that individual behavior is characterized by the linear rules (5) and (6). Individual optimization implies that, given C_{it}, the consumption of good j by consumer i is:

$$C_{ijt} = \left(\frac{P_{jt}}{\overline{P}_{it}}\right)^{-\sigma} C_{it},$$

where $\overline{P}_{it}$ is the price index

$$\overline{P}_{it} \equiv \left(\int_{j\in J_{it}} P_{jt}^{1-\sigma} dj\right)^{\frac{1}{1-\sigma}}. \tag{13}$$

Consider now producer i. Aggregating the demand for good i across all the consumers $j \in \tilde{J}_{it}$ gives

$$Y_{it} = D_{it} P_{it}^{-\sigma},$$

where D_{it} is the demand index

$$D_{it} \equiv \int_{j\in\tilde{J}_{it}} \overline{P}_{jt}^{\sigma} C_{jt} dj. \tag{14}$$

The aggregate price index and aggregate output are, as defined in the main text, $p_t = \int_0^1 p_{it} di$ and $y_t = \int_0^1 c_{it} di$. Given (5) and (6) they are equal to

$$\begin{aligned} p_t &= \mu a_{t-1} + \phi_\theta \theta_t + \phi_s s_t, \\ y_t &= \psi_0 + a_{t-1} + \psi_\theta \theta_t + \psi_s s_t, \end{aligned}$$

where $\psi_\theta = \psi_\epsilon + \psi_v$.

Lemma 3 *If individual prices and quantities are given by (5) and (6) then the price index for consumer i and the demand index for producer i are equal to*

$$\begin{aligned} \overline{P}_{it} &= V_p \exp\left\{p_t + \phi_\theta v_{it}\right\}, && (15) \\ D_{it} &= V_d \exp\left\{y_t + \sigma p_t + \left(\psi_v + \sigma \phi_\theta\right) \gamma \epsilon_{it}\right\}, && (16) \end{aligned}$$

where V_p and V_d are constant terms equal to

$$\begin{aligned} V_p &= \exp\{\frac{1-\sigma}{2} \phi_\theta^2 \overline{\sigma}_\epsilon^2\}, \\ V_d &= \exp\{\frac{1}{2} \psi_\epsilon^2 \sigma_\epsilon^2 + \frac{1}{2} \left(\psi_v + \sigma \phi_\theta\right)^2 \overline{\sigma}_v^2 + \sigma \frac{(1-\sigma)}{2} \phi_\theta^2 \overline{\sigma}_\epsilon^2\}. \end{aligned}$$

Proof. Given (5) the prices observed by consumer i are log-normally distributed, with mean $p_t + \phi_\theta v_{it}$ and variance $\phi_\theta^2 \overline{\sigma}_\epsilon^2$, therefore,

$$\int_{j \in J_{it}} P_{jt}^{1-\sigma} dj = e^{(1-\sigma)(p_t + \phi_\theta v_{it}) + \frac{(1-\sigma)^2}{2} \phi_\theta^2 \overline{\sigma}_\epsilon^2},$$

the price index is derived substituting this in (13). Using this result and expression (6), the demand index for producer i, (14), can be written as

$$D_{it} = e^{y_t + \sigma p_t} V_p^\sigma \int_{j \in \tilde{J}_{it}} e^{\psi_\epsilon \epsilon_{jt} + \psi_v v_{jt}} \left(e^{\phi_\theta v_{jt}}\right)^\sigma dj.$$

Note that the distribution $\left\{v_{jt} : j \in \tilde{J}_{it}\right\}$ is normal with mean $\gamma \epsilon_{it}$ and variance $\overline{\sigma}_v^2$. It follows that

$$\int_{j \in \tilde{J}_{it}} e^{\psi_\epsilon \epsilon_{jt} + \psi_v v_{jt} + \sigma \phi_\theta v_{jt}} dj = e^{\frac{1}{2} \psi_\epsilon^2 \sigma_\epsilon^2} e^{(\psi_v + \sigma \phi_\theta) \gamma \epsilon_{it} + \frac{1}{2} (\psi_v + \sigma \phi_\theta)^2 \overline{\sigma}_v^2}.$$

■

Individual optimization

In the conjectured linear equilibrium the nominal interest rate is constant and equal to $R = e^\rho$. Let $f\left(\epsilon_{it}, v_{it}, \theta_t\right)$ denote the joint density of the shocks ϵ_{it}, v_{it} and θ_t. Let the prices of state-contingent claims at $(t, 0)$ be

$$q\left(\omega_{it}\right) = g\left(\theta_t\right) f\left(\epsilon_{it}, v_{it}, \theta_t\right), \tag{17}$$

where $g\left(\theta_t\right) \equiv \exp\left\{-\left(1+\mu\right) \theta_t + \frac{1}{2} \left(1+\mu\right)^2 \sigma_\theta^2\right\}$ (recall that $\omega_{it} \equiv \{\epsilon_{it}, v_{it}, \theta_t\}$).

Given the conjectures made about aggregate behavior the only state variables for the optimization problem of the individual household are M_{it} and a_{t-1}. The problem can then be analyzed using the Bellman equation

$$V\left(M_{it}, a_{t-1}\right) = \max_{\substack{\{P(s_t, \theta_{it})\}, \left\{C\left(s_t, \theta_{it}, \overline{\theta}_{it}\right)\right\}, \\ \{Z_{it}(\omega_{it})\}, \{M_{it+1}(\omega_{it})\}}} \mathbb{E}_t \left[\log C_{it} - \frac{1}{1+\eta} N_{it}^{1+\eta} + \beta V\left(M_{it+1}, a_t\right)\right]$$

subject to the constraints

$$
\begin{aligned}
& M_{it+1}(\omega_{it}) = RM_{it} + (1+\tau) P_{it}Y_{it} - \overline{P}_{jt}C_{it} - \int Z_{it}(\tilde{\omega}_{it}) q(\tilde{\omega}_{it}) d\tilde{\omega}_{it} - T_t + Z_{it}(\omega_{it}), \\
& Y_{it} = D_{it}P_{it}^{-\sigma}, \\
& Y_{it} = A_{it}N_{it}, \\
& P_{it} = P(s_t, \theta_{it}), C_{it} = C\left(s_t, \theta_{it}, \overline{\theta}_{it}\right), \\
& (15), (16).
\end{aligned}
$$

where $\mathbb{E}_t[.]$ represents the expectation formed at $(t, 0)$. Given the conjecture made about the equilibrium $\mathbb{E}_t[.]$ can be replaced by $\mathbb{E}[.|a_{t-1}]$.

From this problem I can derive the optimality condition for prices and consumption

$$
\mathbb{E}_{i,(t,I)}\left[\frac{1+\tau}{\overline{P}_{it}C_{it}}Y_{it} - \frac{\sigma}{\sigma-1}\frac{1}{A_{it}}\left(\frac{Y_{it}}{A_{it}}\right)^{\eta}\frac{Y_{it}}{P_{it}}\right] = 0, \tag{18}
$$

$$
\mathbb{E}_{i,(t,II)}\left[\frac{1}{\overline{P}_{it}C_{it}} - \beta R\frac{1}{\overline{P}_{it+1}C_{it+1}}\right] = 0. \tag{19}
$$

where $\mathbb{E}_{i,(t,I)}[.]$ and $\mathbb{E}_{i,(t,II)}[.]$ denote the expectations of agent i at (t, I) and (t, II). Given the conjectured equilibrium, and given Lemma 1, they can be replaced, respectively, by $\mathbb{E}[.|s_t, \theta_{it}, a_{t-1}]$ and $\mathbb{E}\left[.|s_t, \theta_{it}, \overline{\theta}_{it}, a_{t-1}\right]$.

By Lemma 3 all the random variables in the expressions above are log-normal, including the output of firm i which is equal to $Y_{it} = P_{it}^{-\sigma}D_{it}$. Rearranging and substituting in (18) and (19) gives

$$
p_{it} = \kappa_p + \frac{\eta}{1+\sigma\eta}\left(\mathbb{E}_{i,(t,I)}[d_{it}] - a_{it}\right) + \frac{1}{1+\sigma\eta}\left(\mathbb{E}_{i,(t,I)}[\overline{p}_{it} + c_{it}] - a_{it}\right), \tag{20}
$$

$$
\overline{p}_{it} + c_{it} = \kappa_c + \mathbb{E}_{i,(t,II)}\left[\overline{p}_{it+1} + c_{it+1}\right]. \tag{21}
$$

where κ_p and κ_c are equal to

$$
\begin{aligned}
\kappa_p &= \frac{1}{1+\sigma\eta}\left(H(\psi_\epsilon, \psi_v, \psi_s, \phi_\theta) - \ln(1+\tau)\right), \\
\kappa_c &= -\rho + G(\psi_\epsilon, \psi_v, \psi_s, \phi_\theta),
\end{aligned}
$$

where H and G are known functions which depend on the model parameters.

Let me show that the market for state-contingent claims clears and that nominal balances are constant and equal to M. Suppose $M_{it} = M$. Let the portfolio of state-contingent claims be

$$
Z_{it}(\omega_{it}) = M - RM - (1+\tau) P_{it}Y_{it} + \overline{P}_{jt}C_{it} + T_t.
$$

For each realization of the aggregate shock $\hat{\theta}_t$ goods markets clearing implies that

$$
\int\int \left(P_{it}Y_{it} - \overline{P}_{jt}C_{it}\right) f\left(\epsilon_{it}, v_{it}, \hat{\theta}_t\right) d\epsilon_{it}dv_{it} = 0.
$$

Substituting the government budget constraint, this implies that: (1) the market for state-contingent claims clears for each aggregate state θ_t,

$$
\int Z_{it}(\omega_{it}) f\left(\epsilon_{it}, v_{it}, \hat{\theta}_t\right) d\epsilon_{it}dv_{it} = -(R-1)M - \tau\int_0^1 P_{it}Y_{it} - T_t = 0,
$$

and (2) that the portfolio $\{Z_{it}(\omega_{it})\}$ has zero value at date $(t, 0)$,

$$
\int Z_{it}(\omega_{it}) q(\omega_{it}) d\omega_{it} = \int\left(\int Z_{it}(\omega_{it}) f\left(\epsilon_{it}, v_{it}, \hat{\theta}_t\right) d\epsilon_{it}dv_{it}\right) g(\theta_t) d\theta_t = 0.
$$

Substituting in the household budget constraint shows that $M_{it+1} = M$.

Let me now check that the portfolio just described is optimal. The optimality condition for a security contingent on ω_{it} can be written as

$$\frac{\partial V(M, a_t)}{\partial B_{it}} f(\omega_{it}) = \int q(\omega_{it}) \frac{\partial V(M, a_t)}{\partial B_{it}} f(\omega_{it})\, d\tilde{\omega}_t.$$

Using the envelope conditions, this gives

$$\mathbb{E}\left[\frac{1}{\overline{P}_{it+1} C_{it+1}} | a_{t-1}, \theta_t\right] f(\omega_{it}) = q(\omega_{it}) \int \mathbb{E}\left[\frac{1}{\overline{P}_{it+1} C_{it+1}} | a_{t-1}, \tilde{\theta}_t\right] dF\left(\tilde{\theta}_t\right).$$

Substituting equilibrium prices and consumptions (5)-(6), and the security prices (17), I obtain

$$e^{-(1+\mu)\theta_t} = g(\theta_t)\, \mathbb{E}\left[e^{-(1+\mu)\tilde{\theta}_t}\right],$$

which is satisfied, given the definition of g.

Fixed point

To check optimality substitute the conjectures made for individual behavior (5) and (6) in the optimality conditions (20) and (21). Notice that all the shocks are i.i.d. so the expected value of all future shocks is zero. Let β_θ, β_s and $\delta_\epsilon, \delta_\eta, \delta_s$ be coefficients such that

$$\begin{aligned}
\mathbb{E}[\theta_t | \theta_{it}, s_t] &= \beta_\theta \theta_{it} + \beta_s s_t, \\
\mathbb{E}\left[\theta_t | \theta_{it}, \overline{\theta}_{it}, s_t\right] &= \delta_\epsilon \theta_{it} + \delta_v \overline{\theta}_{it} + \delta_s s_t,
\end{aligned}$$

and define

$$\delta_\theta \equiv \delta_\epsilon + \delta_v.$$

Defining the precision of the generic random variable x as $\pi_x \equiv \left(\sigma_x^2\right)^{-1}$ the coefficients β_θ, β_s and $\delta_\epsilon, \delta_\eta, \delta_s$ are

$$\beta_\theta = \frac{\pi_\epsilon}{\pi_\theta + \pi_\epsilon + \pi_e}, \quad \beta_s = \frac{\pi_e}{\pi_\theta + \pi_\epsilon + \pi_e}, \tag{22}$$

$$\delta_\epsilon = \frac{\pi_\epsilon}{\pi_\theta + \pi_\epsilon + \pi_v + \pi_e}, \quad \delta_v = \frac{\pi_v}{\pi_\theta + \pi_\epsilon + \pi_v + \pi_e}, \quad \delta_s = \frac{\pi_e}{\pi_\theta + \pi_\epsilon + \pi_v + \pi_e}. \tag{23}$$

Using the law of iterated expectations one can replace $\mathbb{E}_{i,(t,I)}[\overline{p}_{it} + c_{it}]$ on the right hand side of (20) with $\kappa_c + \mathbb{E}_{i,(t,I)}\left[\overline{p}_{it+1} + c_{it+1}\right]$. Substitute the conjectures (5) and (6) on both sides of (20) and (21). Matching the coefficients for $\theta_{it}, \overline{\theta}_{it}$ and s_t gives the conditions

$$(1+\sigma\eta)\phi_\theta = \eta\left((\psi_\theta + \sigma\phi_\theta)\beta_\theta - 1 + (\psi_v + \sigma\phi_\theta)\gamma(1-\beta_\theta)\right) + (1+\mu)\beta_\theta - 1, \tag{24}$$

$$(1+\sigma\eta)\phi_s = \eta\left((\psi_\theta + \sigma\phi_\theta)\beta_s + \psi_s + \sigma\phi_s - (\psi_v + \sigma\phi_\theta)\gamma\beta_s\right) + (1+\mu)\beta_s, \tag{25}$$

$$\psi_\epsilon = (1+\mu)\delta_\epsilon, \tag{26}$$

$$\psi_v = (1+\mu)\delta_v - \phi_\theta, \tag{27}$$

$$\psi_s = (1+\mu)\delta_s - \phi_s, \tag{28}$$

and matching the constant terms gives

$$0 = -\ln(1+\tau) + H(\psi_\epsilon, \psi_v, \psi_s, \phi_\theta) + \ln V_d + (1+\eta)\psi_0, \tag{29}$$

$$0 = -\rho + G(\psi_\epsilon, \psi_v, \psi_s, \phi_\theta). \tag{30}$$

The value of ψ_ϵ is immediately given by (26). Conditions (24)-(28) give the following values for ϕ_θ and ϕ_s,

$$\phi_\theta = \frac{\eta\left(\left(1+\mu\right)\delta_\theta\beta_\theta - 1\right) + \left(1+\mu\right)\beta_\theta - 1 + \eta\left(1+\mu\right)\delta_v\gamma\left(1-\beta_\theta\right)}{1+\sigma\eta-\eta\left(\sigma-1\right)\left(\beta_\theta+\gamma\left(1-\beta_\theta\right)\right)}, \tag{31}$$

$$\begin{aligned}\phi_s = {} & \frac{1}{1+\eta}\{\eta\left(\left(1+\mu\right)\delta_\theta+\left(\sigma-1\right)\phi_\theta\right)\beta_s+\eta\left(1+\mu\right)\delta_s+\left(1+\mu\right)\beta_s \\ & -\eta\left(\left(1+\mu\right)\delta_v+\left(\sigma-1\right)\phi_\theta\right)\gamma\beta_s\}.\end{aligned} \tag{32}$$

Note that a solution for ϕ_θ exists since

$$\begin{aligned}1+\sigma\eta-\eta\left(\sigma-1\right)\left(\beta_\theta+\gamma\left(1-\beta_\theta\right)\right) &\geq \\ 1+\sigma\eta-\eta\left(\sigma-1\right) &> 0,\end{aligned}$$

where the first inequality follows since $\beta_\theta \leq 1$ and $\gamma \leq 1$. Substituting in (27)-(28) gives ψ_v and ψ_s. Finally, substituting $\psi_\epsilon, \psi_v, \psi_s$, and ϕ_θ in (29)-(30) gives ψ_0 and ρ.

Local determinacy

Let variables with a tilde denote deviations from the equilibrium derived above. A first order approximation of the optimality conditions gives

$$\begin{aligned}\mathbb{E}_{i,(t,I)}\left[\tilde{p}_{it}-\frac{\eta}{1+\sigma\eta}\tilde{d}_{it}+\frac{1}{1+\sigma\eta}\left(\tilde{\overline{p}}_{it}+\tilde{c}_{it}\right)\right] &= 0, \\ \mathbb{E}_{i,(t,II)}\left[\tilde{\overline{p}}_{it}+\tilde{c}_{it}+\tilde{r}_t-\tilde{\overline{p}}_{it+1}-\tilde{c}_{it+1}\right] &= 0.\end{aligned}$$

Taking expectations at time $(T,0)$ and integrating across agents, in that order, gives

$$\begin{aligned}\mathbb{E}_T\left[\tilde{p}_t-\frac{\eta}{1+\sigma\eta}\tilde{d}_t+\frac{1}{1+\sigma\eta}\left(\tilde{p}_t+\tilde{y}_t\right)\right] &= 0, \\ \mathbb{E}_T\left[\tilde{p}_t+\tilde{y}_t+\tilde{r}_t-\tilde{p}_{t+1}-\tilde{y}_{t+1}\right] &= 0,\end{aligned} \tag{33}$$

for all $t \geq T$. Since $\tilde{d}_t = \tilde{y}_t + \sigma\tilde{p}_t$ the first condition implies that $\mathbb{E}_T\left[\tilde{y}_t\right] = 0$. Moreover, notice that $\tilde{r}_t = r_t - \rho$ and $\tilde{p}_t = p_t - p_t^*$. Therefore, under the policy rule (3)

$$\tilde{r}_t = \xi\tilde{p}_{t-1} \text{ for all } t \geq T.$$

Substituting these conditions in (33) and letting $h_t = \mathbb{E}_T\left[\tilde{p}_t\right]$ gives the following difference equation for h_t:

$$h_{t+1} - h_t - \xi h_{t-1} = 0 \text{ for all } t \geq T$$

with $h_{T-1} = \tilde{p}_{T-1}$. The assumption $\xi > 1$ ensures that any $h_{T-1} \neq 0$ gives an explosive solution. This shows that any equilibrium in a neighborhood of the original equilibrium must display $p_t = p_t^*$ for all realizations of the aggregate shocks. Using this result one can show that the individual prices and consumption are the same as under the original equilibrium.

Proof of Lemma 1

In the text.

Proof of Proposition 2

Under full information $\beta_\theta = \delta_\epsilon = \delta_v = 0$ and $\beta_s = \delta_s = 1$. Substituting in (31)-(32) gives

$$\begin{aligned}\phi_\theta &= -\frac{1+\eta}{1+\sigma\eta-\eta(\sigma-1)\gamma}, \\ \phi_s &= \frac{\eta(\sigma-1)}{1+\eta}\phi_\theta(1-\gamma)+1+\mu.\end{aligned}$$

Next, (26)-(28) give

$$\begin{aligned}\psi_\epsilon &= 0, \\ \psi_v &= -\phi_\theta, \\ \psi_s &= 1+\mu-\phi_s = \\ &= -\frac{\eta(\sigma-1)}{1+\eta}\phi_\theta(1-\gamma),\end{aligned}$$

Finally, ψ_0 can be determined from (29). Notice that only ϕ_s depends on μ. The real equilibrium allocation only depends on the consumption levels c_{it} and on the relative prices $p_{it}-p_t$, and, given (5) and (6), these are independent of ϕ_s. To conclude the proof, notice that

$$\psi_\theta+\psi_s = -\phi_\theta-\frac{\eta(\sigma-1)}{1+\eta}\phi_\theta(1-\gamma)=1.$$

The next lemma compares the full information equilibrium with a first-best benchmark.

Lemma 4 *There is a τ^* such that the equilibrium under full information replicates the first-best allocation.*

Proof. Given that there is no capital the first-best allocation can be derived period by period solving the problem

$$\begin{aligned}\max_{\{C_{ijt}\}\{N_{it}\}} \quad & \int_0^1\left(\ln C_{it}-\frac{1}{1+\eta}N_{it}^{1+\eta}\right)di \\ s.t. \quad & C_{it}=\left(\int_{J_{it}} C_{ijt}^{\frac{\sigma-1}{\sigma}}dj\right)^{\frac{\sigma}{\sigma-1}} \quad \forall i, \\ & \int_{\tilde{J}_{it}} C_{jit}dj = A_{it}N_{it} \ \forall i.\end{aligned}$$

The first order conditions can be re-arranged to obtain

$$\begin{aligned}C_{it}^{\frac{1}{\sigma}-1}C_{ijt}^{-\frac{1}{\sigma}} &= \lambda_{jt} \text{ for each } i\in[0,1] \text{ and each } j\in J_i, \\ A_{it}\lambda_i &= N_{it}^{\eta} \text{ for each } i, \end{aligned} \tag{34}$$

where λ_{jt} is the Lagrange multiplier on the j-th resource constraint. Since the problem is concave these conditions are sufficient for an optimum.

Let τ^* be the τ that satisfies $1+\tau^*=\frac{\sigma}{\sigma-1}$. Consider the equilibrium allocation arising under full information, let

$$\lambda_{jt}=\frac{P_{jt}}{\overline{P}_{jt}C_{jt}}.$$

With full information the Euler equation (19) implies that $\overline{P}_{jt}C_{jt}$ is constant across agents. The demand for good j by consumer i is given by $C_{ijt} = \overline{P}_{it}^{\sigma} P_{jt}^{-\sigma} C_{it}$. These two facts imply that

$$C_{it}^{\frac{1}{\sigma}-1} C_{ijt}^{-\frac{1}{\sigma}} = \frac{P_{jt}}{\overline{P}_{it}C_{it}} = \frac{P_{jt}}{\overline{P}_{jt}C_{jt}} = \lambda_{jt}.$$

Moreover, under full information the optimal pricing condition (18) gives

$$\frac{1}{\overline{P}_{it}C_{it}} = \frac{1}{A_{it}} \frac{1}{P_{it}} N_{it}^{\eta},$$

which can be rearranged to give (34). Therefore, the equilibrium allocation satisfies the first order conditions of the problem. ■

Proof of Proposition 3

For the following derivations recall that under imperfect information all the coefficients $\beta_\theta, \beta_s, \delta_\epsilon, \delta_v, \delta_s$ are in $(0,1)$ and $\gamma \in [0,1]$.

Differentiating (31) with respect to μ gives

$$\frac{\partial \phi_\theta}{\partial \mu} = \frac{\eta \delta_\theta \beta_\theta + \beta_\theta + \eta \delta_v \gamma (1-\beta_\theta)}{1 + \sigma\eta - \eta(\sigma-1)(\beta_\theta + \gamma(1-\beta_\theta))} > 0. \tag{35}$$

To prove the inequality notice that the numerator is positive as it involves all non-negative terms and $\beta_\theta > 0$. The denominator is positive since

$$\beta_\theta + \gamma(1-\beta_\theta) \leq 1,$$

and

$$\eta(\sigma-1) \leq \eta\sigma,$$

(since $\eta \geq 0$), and these two inequalities imply

$$\sigma\eta \geq \eta(\sigma-1)(\beta_\theta + \gamma(1-\beta_\theta)).$$

Differentiating (32) gives

$$\begin{aligned} \frac{\partial \phi_s}{\partial \mu} = \quad & \frac{1}{1+\eta}\{\eta\left(\delta_\theta + (\sigma-1)\frac{\partial \phi_\theta}{\partial \mu}\right)\beta_s + \beta_s + \eta\delta_s \\ & -\eta\left(\delta_v + (\sigma-1)\frac{\partial \phi_\theta}{\partial \mu}\right)\gamma\beta_s\} > 0. \end{aligned} \tag{36}$$

The inequality follows since $\gamma \leq 1$ and $\delta_v < \delta_\theta$ imply that

$$\left(\delta_\theta + (\sigma-1)\frac{\partial \phi_\theta}{\partial \mu}\right) - \gamma\left(\delta_v + (\sigma-1)\frac{\partial \phi_\theta}{\partial \mu}\right) > 0.$$

Recall that $\psi_\theta = \psi_v + \psi_\epsilon$, then substituting (26) and (27) and differentiating gives

$$\frac{\partial \psi_\theta}{\partial \mu} = \delta_\theta - \frac{\partial \phi_\theta}{\partial \mu} > 0.$$

To prove this inequality I need to prove that

$$\delta_\theta > \frac{\eta \delta_\theta \beta_\theta + \beta_\theta + \eta \delta_v \gamma (1-\beta_\theta)}{1 + \sigma\eta - \eta(\sigma-1)(\beta_\theta + \gamma(1-\beta_\theta))},$$

where I am using (35). Dividing by δ_θ on both sides and rearranging this is equivalent to

$$1+\sigma\eta-\eta\left(\sigma-1\right)\left(\beta_\theta+\gamma\left(1-\beta_\theta\right)\right)>\frac{\beta_\theta}{\delta_\theta}+\eta\beta_\theta+\eta\delta_v\gamma\left(1-\beta_\theta\right),$$

and this follows from the following two inequalities

$$1>\frac{\beta_\theta}{\delta_\theta},$$

and

$$\begin{aligned} & \sigma\eta-\eta\left[\left(\sigma-1\right)\left(\beta_\theta+\gamma\left(1-\beta_\theta\right)\right)+\beta_\theta+\delta_v\gamma\left(1-\beta_\theta\right)\right]> \\ = \quad & \sigma\eta\left[1-\left[\beta_\theta+\left(1-\frac{\left(1-\delta_v\right)\gamma}{\sigma}\right)\left(1-\beta_\theta\right)\right]\right]>0. \end{aligned}$$

Differentiating (28) gives

$$\frac{\partial\psi_s}{\partial\mu}=\delta_s-\frac{\partial\phi_s}{\partial\mu}<0.$$

To prove the inequality notice that (36) implies

$$\frac{\partial\phi_s}{\partial\mu}\geq\frac{\beta_s+\eta\delta_s}{1+\eta}$$

and

$$\frac{\beta_s+\eta\delta_s}{1+\eta}>\delta_s.$$

Some lengthy but straightforward algebra, together with the inequalities derived above, can be used to show that

$$\delta_\theta+\delta_s>\frac{\partial\phi_\theta}{\partial\mu}+\frac{\partial\phi_s}{\partial\mu},$$

which proves the last statement.

Proof of Proposition 4

To prove the statement it is sufficient to find a μ and a vector $\{\psi_\epsilon,\psi_v,\psi_s,\phi_\theta,\phi_s\}$ that satisfy (24)-(28) and such that $\psi_\epsilon+\psi_v=1$ and $\psi_s=0$. Let me set

$$\phi_\theta^{fs}=\frac{\eta+\eta\gamma\frac{\delta_v}{\delta_\theta}+\frac{1}{1-\beta_\theta}\left(1-\frac{\beta_\theta}{\delta_\theta}\right)}{\eta\sigma+\eta\gamma\left(\sigma-\frac{\delta_\epsilon}{\delta_\theta}\right)+\frac{1}{1-\beta_\theta}\left(1-\frac{\beta_\theta}{\delta_\theta}\right)}, \tag{37}$$

and set the following values for the remaining variables

$$\mu^{fs}=\frac{1+\phi_\theta^{fs}}{\delta_\theta}-1, \tag{38}$$

$$\phi_s^{fs}=\eta\left(\left(1+\sigma\phi_\theta^{fs}\right)\beta_s-\left(\left(1+\mu^{fs}\right)\delta_v+\left(\sigma-1\right)\phi_\theta^{fs}\right)\gamma\beta_s\right)+\left(1+\mu^{fs}\right)\beta_s, \tag{39}$$

$$\begin{aligned} \psi_\epsilon^{fs} &= \left(1+\mu^{fs}\right)\delta_\epsilon, \\ \psi_v^{fs} &= \left(1+\mu^{fs}\right)\delta_v-\phi_\theta^{fs}, \\ \psi_s^{fs} &= \left(1+\mu^{fs}\right)\delta_s-\phi_s^{fs}. \end{aligned}$$

Checking that $\psi_\epsilon^{fs} + \psi_v^{fs} = 1$ is straightforward (recall that $\delta_\theta = \delta_\epsilon + \delta_v$). To check that $\psi_s^{fs} = 0$ I need to check the following equality:

$$\left(1+\mu^{fs}\right)\delta_s = \eta\left(\left(1+\sigma\phi_\theta^{fs}\right)\beta_s - \left(\left(1+\mu^{fs}\right)\delta_v + (\sigma-1)\,\phi_\theta^{fs}\right)\gamma\beta_s\right) + \left(1+\mu^{fs}\right)\beta_s.$$

Substitute μ^{fs} from (38) in this expression. Rearranging terms, this gives

$$\phi_\theta^{fs} = \frac{\eta + \eta\gamma\frac{\delta_v}{\delta_\theta} + \frac{1}{\delta_\theta}\frac{\beta_s - \delta_s}{\beta_s}}{\eta\sigma + \eta\gamma\left(\sigma - \frac{\delta_\epsilon}{\delta_\theta}\right) + \frac{1}{\delta_\theta}\frac{\beta_s-\delta_s}{\beta_s}}.$$

To check that this expression is identical to (37) it suffices to show that

$$\frac{\beta_s}{1-\beta_\theta} = \frac{\beta_s - \delta_s}{\delta_\theta - \beta_\theta}.$$

After some lengthy manipulation of the inference coefficients, one can show that both sides of this equation are equal to $\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\epsilon^2}$.

Finally, I can check that these values satisfy the equilibrium conditions (24)-(28). Conditions (26)-(28) follow immediately from the definitions of $\left\{\psi_\epsilon^{fs}, \psi_v^{fs}, \psi_s^{fs}\right\}$. Substituting $\psi_\theta^{fs} = 1$, $\psi_s^{fs} = 0$, and

$$\psi_v^{fs} = \frac{1+\phi_\theta^{fs}}{\delta_\theta}\delta_v - \phi_\theta^{fs},$$

in (24)-(25) gives me the expressions (37)-(39). This confirms that also (24) and (25) are satisfied.

Proof of Proposition 5

First, using the household's optimality condition I will derive necessary conditions for the coefficients $\{\psi_{\epsilon t}, \psi_{vt}, \psi_{st}, \phi_{\theta t}\}$. Next, I will show that under the appropriate choice of μ_t and ρ_t the same coefficients arise in equilibrium and induce the same real allocation.

The consumer Euler equation can be derived as in Proposition 1, and, using the law of iterated expectations, can be written as

$$\overline{p}_{it} + c_{it} = \kappa_{c,t} + \mathbb{E}_{i,(t,II)}\left[\mathbb{E}_{t+1}\left[\overline{p}_{it+1} + c_{it+1}\right]\right].$$

Since all idiosyncratic shocks are i.i.d.

$$\mathbb{E}_{t+1}\left[\overline{p}_{it+1} + c_{it+1}\right] = \phi_{0,t+1} + \psi_{0,t+1}.$$

Moreover, recall that $\phi_{0,t+1}$ and $\psi_{0,t+1}$ are linear functions of the variables in $h_{t+1} = \langle \theta_t, e_t, \theta_{t-1}, e_{t-1}, ...\rangle$. Therefore, it is possible to find a vector $\left\{\zeta_{t+1}, \xi_{s,t+1}, \xi_{\theta,t+1}\right\}$ such that

$$\phi_{0,t+1} + \psi_{0,t+1} = \zeta_{t+1}\cdot h_t + \xi_{s,t+1}s_t + \xi_{\theta,t+1}\theta_t.$$

Using the optimality conditions (20) and (21) one can proceed as in the proof of Proposition 1 and show that the coefficients $\left\{\phi_{s,t}, \phi_{\theta,t}, \psi_{\epsilon,t}, \psi_{v,t}, \psi_{s,t}\right\}$ must satisfy the following set of conditions

$$\begin{aligned}
(1+\sigma\eta)\,\phi_{\theta,t} &= \eta\left(\left(\psi_{\theta,t} + \sigma\phi_{\theta,t}\right)\beta_\theta - 1 + \left(\psi_{v,t} + \sigma\phi_{\theta,t}\right)\gamma\left(1-\beta_\theta\right)\right) + \xi_{\theta,t+1}\beta_\theta - 1,\\
(1+\sigma\eta)\,\phi_{s,t} &= \eta\left(\left(\psi_{\theta,t} + \sigma\phi_{\theta,t}\right)\beta_s + \psi_{s,t} + \sigma\phi_{s,t} - \left(\psi_{v,t} + \sigma\phi_{\theta,t}\right)\gamma\beta_s\right) + \xi_{\theta,t+1}\beta_s,\\
\psi_{\epsilon,t} &= \xi_{\theta,t+1}\delta_\epsilon,\\
\psi_{v,t} &= \xi_{\theta,t+1}\delta_v - \phi_{\theta,t},\\
\psi_{s,t} &= \xi_{\theta,t+1}\delta_s - \phi_{s,t}.
\end{aligned}$$

This shows that the parameter $\xi_{\theta,t+1}$ uniquely pins down the coefficients $\{\phi_{s,t}, \phi_{\theta,t}, \psi_{\epsilon,t}, \psi_{v,t}, \psi_{s,t}\}$ in period t. Set

$$\mu_t = \xi_{\theta,t+1} - 1$$

one can proceed as in the proof of Proposition 1, show that this choice of μ_t delivers the same coefficients $\phi_{\theta,t}, \psi_{0,t}, \psi_{\epsilon,t}, \psi_{v,t}, \psi_{s,t}$, and derive the value of ρ_t consistent with μ_t.

Proof of Lemma 2

Using the demand index D_{it} defined in (16), the equilibrium labor supply of consumer i is given by

$$N_{it} = \frac{D_{it} P_{it}^{-\sigma}}{A_{it}} = V_d e^{c_t - a_t} e^{(-1-\sigma\phi_\theta + (\psi_v + \sigma\phi_\theta)\gamma)\epsilon_{it}}.$$

Substituting in the consumer utility function gives the per-period utility

$$c_{it} - \frac{1}{1+\eta} V_d^{1+\eta} \left(e^{c_t - a_t - (1+\sigma\phi_\theta - (\psi_v + \sigma\phi_\theta)\gamma)\epsilon_{it}} \right)^{1+\eta}.$$

Taking expectations with respect to the idiosyncratic shocks ϵ_{it} and v_{it} gives

$$y_t - \frac{1}{1+\eta} V_d^{1+\eta} \exp\{(1+\eta)\psi_0 + (1+\eta)(\psi_s + \psi_\theta - 1)\theta_t + (1+\eta)\psi_s e_t +$$
$$+\frac{1}{2}(1+\sigma\phi_\theta - (\psi_v + \sigma\phi_\theta)\gamma)^2 (1+\eta)^2 \sigma_\epsilon^2\}.$$

Taking expectations with respect to the aggregate shocks θ_t and e_t gives

$$\begin{aligned} W_t &= \psi_0 + a_{t-1} - \frac{1}{1+\eta} V_d^{1+\eta} \exp\{(1+\eta)\psi_0 + \\ &+ \frac{(1+\eta)^2}{2} \left((\psi_s + \psi_\theta - 1)^2 \sigma_\theta^2 + \psi_s^2 \sigma_e^2 + (1+\sigma\phi_\theta - (\psi_v + \sigma\phi_\theta)\gamma)^2 \sigma_\epsilon^2 \right)\}. \end{aligned}$$

Substituting the expression for V_d derived in Lemma 3 gives the expression in the Lemma.

Proof of Proposition 6

The policy problem is to choose a subsidy τ, an interest rate rule $(\mu, \phi_\theta, \phi_s, \rho, \xi)$, and equilibrium coefficients $(\psi_0, \psi_v, \psi_\epsilon, \psi_s)$ that maximize W_t subject to the constraints (24)-(30). Notice that the subsidy τ only appears in the constraint (29) and, for any value of the remaining variables, the constraint (29) is satisfied by setting

$$\tau = \exp\{H(\psi_\epsilon, \psi_v, \psi_s, \phi_\theta) + \ln V_d(\psi_\epsilon, \psi_v, \psi_s, \phi_\theta) + (1+\eta)\psi_0\} - 1.$$

A similar reasoning applies to ρ and constraint (30), while the choice of ξ has no effects on the equilibrium allocation. Therefore, the problem can be restated eliminating τ, ρ and ξ from the choice variables and (29) and (30) from the constraints. As shown in the proof of Proposition 1 the constraints (24)-(28) define a map between μ and $(\phi_\theta, \phi_s, \psi_v, \psi_\epsilon, \psi_s)$. This can be used to define a map $w(\mu)$ between μ and the variable w defined in (11). In short the policy problem can be stated as

$$\max_{\mu,\psi_0} \psi_0 - \frac{1}{1+\eta} \exp\{(1+\eta)\left(\psi_0 - \frac{1}{2} w(\mu)\right)\}.$$

For any value of ψ_0 the objective function in this problem is a monotone increasing function of w. Therefore, the problem can be solved in two steps, first setting $\mu^o = \arg\max_\mu w(\mu)$, and then choosing ψ_0 to maximize $\psi_0 - \frac{1}{1+\eta} \exp\{(1+\eta)\left(\psi_0 - \frac{1}{2} w(\mu^o)\right)\}$.

Proof of Proposition 7

Rearranging the constraints and substituting away the choice variable μ, the problem described in Proposition 6 can be restated as

$$\begin{aligned} \max_{\psi_\theta,\psi_v,\psi_\epsilon,\psi_s,\phi_\theta} \quad & -(1+\eta)\left((\psi_\theta+\psi_s-1)^2\sigma_\theta^2+\psi_s^2\sigma_e^2\right)+\sigma(\sigma-1)\phi_\theta^2\overline{\sigma}_\epsilon^2 \\ & -(1+\eta)(1+\sigma\phi_\theta-(\psi_v+\sigma\phi_\theta)\gamma)^2\sigma_\epsilon^2+ \\ & -\psi_\epsilon^2\sigma_\epsilon^2-(\psi_v+\sigma\phi_\theta)^2\gamma\overline{\sigma}_\epsilon^2, \end{aligned}$$

subject to

$$(1+\sigma\eta)\phi_\theta = \eta((\psi_\theta+\sigma\phi_\theta)\beta_\theta-1)+(\psi_\theta+\phi_\theta)\frac{\beta_\theta}{\delta_\theta}-1+\eta(\psi_v+\sigma\phi_\theta)\gamma(1-\beta_\theta), \tag{40}$$

$$0 = \eta(\psi_\theta+\sigma\phi_\theta)\beta_s+(1+\eta)\psi_s+(\psi_\theta+\phi_\theta)\frac{\beta_s-\delta_s}{\delta_\theta}-\eta(\psi_v+\sigma\phi_\theta)\gamma\beta_s \tag{41}$$

$$\psi_\epsilon = \frac{\psi_\theta+\phi_\theta}{\delta_\theta}\delta_\epsilon, \tag{42}$$

$$\psi_v = \psi_\theta-\frac{\psi_\theta+\phi_\theta}{\delta_\theta}\delta_\epsilon. \tag{43}$$

It can be easily shown that the constraints (40)-(43) define a linear map that for each ψ_θ gives the remaining choice variables, $\psi_\epsilon,\psi_v,\psi_s,\phi_\theta$. Moreover, Proposition 4 implies that the first term in the objective function is zero when $\psi_\theta=1$.

Therefore, to prove the statements in the proposition it is sufficient to study the properties of the following function

$$\begin{aligned} f(\psi_\theta) \equiv\; & -\sigma(\sigma-1)\phi_\theta^2(1-\gamma)+(1+\eta)(1+\sigma\phi_\theta-(\psi_v+\sigma\phi_\theta)\gamma)^2+ \\ & +\psi_\epsilon^2+(\psi_v+\sigma\phi_\theta)^2\gamma(1-\gamma) \end{aligned}$$

where $\psi_\epsilon,\psi_v,\psi_s,\phi_\theta$ are those that satisfy the constraints (40)-(43). To derive this expression I have used the fact that $\overline{\sigma}_\epsilon^2=(1-\gamma)\sigma_\epsilon^2$.

In particular, μ^{fs} is optimal if and only if $f'(1)=0$ and $\mu^o<\mu^{fs}$ if $f'(1)<0$.

The expression for $f'(\psi_\theta)$ is given by:

$$\begin{aligned} f'(\psi_\theta) =\; & 2\{-\sigma(\sigma-1)(1-\gamma)\phi_\theta\frac{d\phi_\theta}{d\psi_\theta}+ \\ & +(1+\eta)(1+\sigma\phi_\theta-(\psi_v+\sigma\phi_\theta)\gamma)\left(\sigma\frac{d\phi_\theta}{d\psi_\theta}-\gamma\left(\frac{1}{1+\gamma}-\frac{\gamma}{1+\gamma}\frac{d\phi_\theta}{d\psi_\theta}+\sigma\frac{d\phi_\theta}{d\psi_\theta}\right)\right)+ \\ & \psi_\epsilon\left(1+\frac{d\phi_\theta}{d\psi_\theta}\right)\frac{\gamma}{1+\gamma}+(\psi_v+\sigma\phi_\theta)\gamma(1-\gamma)\left(\frac{1}{1+\gamma}-\frac{\gamma}{1+\gamma}\frac{d\phi_\theta}{d\psi_\theta}+\sigma\frac{d\phi_\theta}{d\psi_\theta}\right)\}, \end{aligned}$$

where

$$\frac{d\phi_\theta}{d\psi_\theta}=\frac{\eta\beta_\theta+\frac{\beta_\theta}{\delta_\theta}+\eta\gamma(1-\beta_\theta)\frac{1}{\gamma+1}}{\eta\sigma(1-\beta_\theta)-\frac{\beta_\theta}{\delta_\theta}-\eta\gamma(1-\beta_\theta)\left(\sigma-\frac{\gamma}{\gamma+1}\right)+1}.$$

Setting $\psi_\theta=1$, it is possible to show that $\eta\geq 0$ implies that the corresponding value for ϕ_θ satisfies $\phi_\theta\in[-1,0)$ and $\phi_\theta=1$ only if $\eta=0$. This property, together with the properties $\sigma>1,\beta_\theta,\delta_\theta\in(0,1),\beta_\theta<\delta_\theta$ and $\gamma\in[0,1]$ can be used to show that: (i) $f'(1)>0$ if $\gamma\in(0,1)$ and $\eta>0$, (ii) $f'(1)=0$ in all remaining cases. The complete derivations are available from the author.

The next lemma shows that if $\mu\leq\mu^{fs}$ producers with higher productivity tend to set lower prices.

Lemma 5 *If $\psi_\theta \leq 1$ the response of prices to individual productivity is negative, $\phi_\theta < 0$.*

Proof. By substitution one can show that ϕ_θ can be written as:

$$\phi_\theta = -\frac{1 - \frac{\beta_\theta}{\delta_\theta}\psi_\theta + \eta\left(1 - \beta_\theta\psi_\theta\right) - \eta\gamma\left(1 - \beta_\theta\right)\psi_\theta\left(1 - \frac{\delta_\epsilon}{\delta_\theta}\right)}{1 - \frac{\beta_\theta}{\delta_\theta} + \eta\sigma\left(1 - \beta_\theta\right) - \eta\gamma\left(1 - \beta_\theta\right)\left(\sigma - \frac{\delta_\epsilon}{\delta_\theta}\right)}.$$

Notice that $\beta_\theta < \delta_\theta$, $\beta_\theta < 1$, $\delta_\epsilon < \delta_\theta, \sigma > 1$ and $\gamma \in (0, 1)$. Therefore, the denominator of the fraction on the right-hand side is positive, since

$$\eta\sigma\left(1 - \beta_\theta\right) \geq \eta\gamma\left(1 - \beta_\theta\right)\left(\sigma - \frac{\delta_\epsilon}{\delta_\theta}\right).$$

Moreover, if $\psi_\theta \leq 1$ then

$$\eta\left(1 - \psi_\theta\beta_\theta\right) \geq \eta\gamma\left(1 - \beta_\theta\right)\psi_\theta\left(1 - \frac{\delta_\epsilon}{\delta_\theta}\right)$$

and the numerator is also positive. ■

References

[1] Klaus Adam, 2006, "Optimal Monetary Policy with Imperfect Common Knowledge," *Journal of Monetary Economics* (forthcoming).

[2] Manuel Amador and Pierre-Olivier Weill, 2007, "Learning from Private and Public Observations of Others Actions," mimeo UCLA.

[3] Jeffery Amato, Stephen Morris and Hyun Song Shin, 2002, "Communication and Monetary Policy," *Oxford Review of Economic Policy* 18, pp. 495-503.

[4] George Marios Angeletos, Guido Lorenzoni and Alessandro Pavan, 2006, "Beauty Contests," mimeo MIT.

[5] George Marios Angeletos and Alessandro Pavan, 2005, "Efficient Use of Information and Welfare Analysis with Complementarities and Asymmetric Information," mimeo MIT.

[6] Kosuke Aoki, 2003, "On the Optimal Monetary Policy Response to Noisy Indicators," *Journal of Monetary Economics* 50, pp. 501-523.

[7] Philippe Bacchetta and Eric Van Wincoop, 2005, "Higher Order Expectations in Asset Pricing," mimeo, Study Center Gerzensee.

[8] Paul Beaudry and Franck Portier, 2006, "When Can Changes in Expectations Cause Business Cycle Fluctuations in Neo-Classical Settings?," *Journal of Economic Theory*, forthcoming.

[9] Ben S. Bernanke and Mark Gertler, 2001, "Should Central Banks Respond to Movements in Asset Prices?" *American Economic Review*, Vol. 91, No. 2, pp. 253-257

[10] Stephen Cecchetti, Hans Genberg, John Lipsky, and Sushil Wadhwani, 2000, *Asset Prices and Central Bank Policy.* London, International Center for Monetary and Banking Studies.

[11] Larry Christiano, Roberto Motto, and Massimo Rostagno, 2006, Monetary Policy and Stock Market Boom-Bust Cycles, mimeo, Northwestern.

[12] Jean-Pierre Danthine, John Donaldson, and Thore Johnsen 1998, "Productivity Growth, Consumer Confidence, and the Business Cycle," *European Economic Review* 42, pp. 1113-1140.

[13] Bill Dupor, 2002, "Nominal Price versus Asset Price Stabilization," mimeo.

[14] Bill Dupor, 2005, "Stabilizing Non-Fundamental Asset Price Movements under Discretion and Limited Information," *Journal of Monetary Economics* 62, pp. 727-747.

[15] Christian Hellwig, 2005, "Heterogeneous Information and the Welfare Effects of Public Information Disclosures" UCLA Working Paper.

[16] Christian Hellwig and Laura Veldkamp, 2006, "Knowing What Others Know: Coordination Motives in Information Acquisition," mimeo NYU.

[17] Nir Jaimovich and Sergio Rebelo, 2005, "Can News about the Future Drive the Business Cycle?" mimeo, Northwestern University.

[18] Robert King, "Monetary Policy and Information Content of Prices," *Journal of Political Economy*, April 1982, pp. 247-279.

[19] Guido Lorenzoni, 2006, "A Theory of Demand Shocks," NBER WP 12477.

[20] Yulei Luo, 2006, "Consumption Dynamics under Information Processing Constraints," mimeo, University of Hong Kong.

[21] Robert E. Lucas Jr.,1972, "Expectations and the Neutrality of Money," *Journal of Economic Theory* 4 (2), pp. 103-24.

[22] Robert E. Lucas Jr.,1990, "Liquidity and Interest Rates." *Journal of Economic Theory*, 1990, 50(2), pp. 237-64.

[23] Bartosz Maćkowiak and Mirko Wiederholt, 2006, "Optimal Sticky Prices under Rational Inattention," mimeo, Humboldt University.

[24] N. Gregory Mankiw and Ricardo Reis, 2002, "Sticky Information Versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve," *Quarterly Journal of Economics* 117, pp. 1295-1328.

[25] Bennett T. McCallum, 1979, "A Monetary Policy ineffectiveness Result in a Model with a Predetermined Price Level," *Economics Letters*, Vol. 3, No. 1, 1979, 1-4.

[26] Fabio Milani, 2005, "Expectations, Learning and Macroeconomic Persistence," mimeo, UC Irvine.

[27] Stephen Morris and Hyun Song Shin, 2002, "The Social Value of Public Information," *American Economic Review* 92, pp. 1521-1534.

[28] Stephen Morris and Hyun Song Shin, 2005, "Central Bank Transparency and the Signal Value of Prices," *Brookings Papers on Economic Activity* 2, pp. 1-66.

[29] Giuseppe Moscarini, 2004, "Limited Information Capacity as a Source of Inertia," *Journal of Economic Dynamics and Control*, vol. 28 (10), pp. 2003-2035.

[30] Kristoffer P. Nimark, 2005, "Calvo Pricing and Imperfect Common Knowledge: A Forward Looking Model of Rational Inflation Inertia," ECB Working Paper No. 474.

[31] Maurice Obstfeld and Kenneth Rogoff, 2000, "New Directions for Stochastic Open Economy Models," *Journal of International Economics* 50, pp. 117-53.

[32] Athanasios Orphanides, 2003, "Monetary Policy Evaluation with Noisy Information," *Journal of Monetary Economics* 50 (3), pp. 605-631.

[33] Edmund S. Phelps et al., 1969, *Microeconomic Foundations of Employment and Inflation Theory*, New York: Norton.

[34] Ricardo Reis, 2003, "Where Is the Natural Rate? Rational Policy Mistakes and Persistent Deviations of Inflation from Target," *Advances in Macroeconomics, The B. E. Journals in Macroeconomics*, vol. 3 (1).

[35] Ricardo Reis, 2006, "Inattentive Producers," *Review of Economic Studies*, vol. 73 (3).

[36] Christopher A. Sims, 2003, "Implications of Rational Inattention," *Journal of Monetary Economics* 50 (3), pp. 665-690.

[37] Lars E.O. Svensson, 2006, "Social Value of Public Information: Morris and Shin (2002) Is Actually Pro Transparency, Not Con," *American Economic Review* 96, pp. 448-451.

[38] Lars E.O. Svensson and Michael Woodford, 2003, "Indicator Variables for Optimal Policy," *Journal of Monetary Economics* 50, pp. 691-720.

[39] Lars E.O. Svensson and Michael Woodford, 2005, "Indicator Variables for Optimal Policy under Asymmetric Information," *Journal of Economic Dynamics and Control.*

[40] Michael Woodford, 2002, "Imperfect Common Knowledge and the Effects of Monetary Policy", in P. Aghion, R. Frydman, J. Stiglitz, and M. Woodford, eds., *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, Princeton: Princeton University Press.

[41] Michael Woodford, 2003, *Interest and Prices*, Princeton: Princeton University Press.