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THE SPECIFICATION AND INFLUENCE OF ASSET MARKETS

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ABSTRACT

This paper is a chapter in the forthcoming Handbook of International Economics. It surveys the literature on the specification of models of asset markets and the implications of differences in specification for the macroeconomic adjustment process. Builders of portfolio balance models have generally employed "postulated" asset demand functions, rather than deriving these directly from micro foundations. The first major section of the paper lays out a postulated general specification of asset markets and summarizes the fundamental short-run results of portfolio balance models using a very basic specification of asset markets. Then, rudimentary specifications of a balance of payments equation and goods market equilibrium conditions are supplied, so that the dynamic distribution effects of the trade account under static and rational expectations with both fixed goods prices and flexible goods prices can be analyzed.

The second major section of the paper surveys and analyzes micro foundation models of asset demands using stochastic calculus. The microeconomic theory of asset demands implies some but not all of the properties of the basic specification of postulated asset demands at the macro level. Since the conclusions of macroeconomic analysis depend crucially on the form of asset demand functions, it is important to continue to explore the implications of micro foundations for macro specification.

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1. Introduction

This chapter is a discussion of two complementary approaches to the analysis of asset markets in open economies.[1/] Section 2 is devoted to portfolio balance models with postulated asset demands, asset demands broadly consistent with but not directly implied by microeconomic theory. Some implications of the microeconomic theory of portfolio selection for asset demands are spelled out in Section 3. Section 4 contains some conclusions.

2. Portfolio balance models with postulated asset demands

2.1. Overview

During the last fifteen years there has been a thorough reworking of macroeconomic theory for open economies using a portfolio balance approach.[2/] According to this approach, equilibrium in financial markets occurs when the available stocks of national moneys and other financial assets are equal to the stock demands for these assets based on current wealth, and wealth accumulation continues only until current wealth is equal to desired wealth.

In this section we review some of the important results that have been obtained using portfolio balance models. Although these models were originally developed to study movements of financial capital, variations in interest rates, and changes in stocks of international reserves under fixed exchange rates, they were quickly adapted to study movements of financial capital, variations in interest rates, and changes in the exchange rate under flexible exchange rates. Our discussion reflects the emphasis placed on the case of flexible exchange rates in more recent applications of portfolio balance models.

The builders of portfolio balance models have employed "postulated" asset demand functions. By proceeding in this way, they have not denied the desirability of deriving asset demands from explicit utility maximizing behavior. Indeed, they have attempted to establish the plausibility of their asset demands by appealing to microeconomic theory--in the case of non-monetary assets to the theory of portfolio selection and in the case of monetary assets to the theory of money demand. There is widespread agreement on the importance of exploring the implications of macroeconomic asset demand functions derived from explicit utility maximizing behavior. A new sense of urgency has been added by the argument that, in general, utility maximizing behavior leads to modifications in asset demands when the policy regime changes.[3/] While this exploration proceeds, results derived using postulated asset demands can best be regarded as suggestive hypotheses to be subjected to close scrutiny using asset demand functions with firmer microeconomic foundations.

After laying out a general specification of asset markets (subsection 2.2), we summarize the fundamental short-run results of portfolio balance models using a very basic specification of asset markets (subsection 2.3). Then, we supply rudimentary specifications of a balance of payments equation and goods market equilibrium conditions (subsection 2.4) so that we can trace out the dynamic distribution effects of the trade account under static and rational expectations with both fixed goods prices (subsection 2.5) and flexible goods prices (subsection 2.6).

2.2. The general specification of asset markets

The model contains four assets: home money, foreign money, home (currency) securities, and foreign (currency) securities.[4/] In the general specification it is assumed that residents of both countries hold all four assets. Home net wealth (W) and foreign net wealth ($E\overset{*}{W}$), both measured in units of home currency are given by

$$W = M + B + E(N + F), \quad E\overset{*}{W} = \overset{*}{M} + \overset{*}{B} + E(\overset{*}{N} + \overset{*}{F}). \tag{2.1}$$

M, B, N and F ($\overset{*}{M}$, $\overset{*}{B}$, $\overset{*}{N}$, and $\overset{*}{F}$) represent home (foreign) net private holdings of home money, home securities, foreign money, and foreign securities. E is the exchange rate defined as the home currency price of foreign currency.

Home (foreign) net wealth is allocated among the four financial assets:

$$W \equiv m(\cdot) + n(\cdot) + b(\cdot) + f(\cdot), \quad E\overset{*}{W} \equiv \overset{*}{m}(\cdot) + \overset{*}{n}(\cdot) + \overset{*}{b}(\cdot) + \overset{*}{f}(\cdot). \tag{2.2}$$

m, n, b, and f ($\overset{*}{m}$, $\overset{*}{n}$, $\overset{*}{b}$, and $\overset{*}{f}$) represent home (foreign) residents' demands for home money, foreign money, home securities, and foreign securities, all measured in units of home currency.

The home currency value of the stocks of home money ($\hat{M}$), foreign money ($E\hat{N}$), home securities ($\hat{B}$), and foreign securities ($E\hat{F}$) available for private agents to hold are assumed to be positive:

$$\hat{M} = M + \overset{*}{M} > 0,\ E\hat{N} = E(N + \overset{*}{N}) > 0,\ \hat{B} = B + \overset{*}{B} > 0,\ E\hat{F} = E(F + \overset{*}{F}) > 0. \quad (2.3)$$

The equilibrium conditions for the four asset markets are given by

$$m(0,\overset{-}{\varepsilon},\overset{-}{i},\overset{-}{\overset{*}{i}+\varepsilon},\overset{+}{PX},\overset{+}{Q},\overset{+}{W}) + \overset{*}{m}(\overset{+}{-\varepsilon},0,\overset{-}{i-\varepsilon},\overset{-}{\overset{*}{i}},\overset{+}{E\overset{*}{P}Y},\overset{+}{E\overset{*}{Q}},\overset{+}{E\overset{*}{W}}) - \hat{M} = 0, \quad (2.4a)$$

$$n(0,\overset{+}{\varepsilon},\overset{-}{i},\overset{-}{\overset{*}{i}+\varepsilon},\overset{+}{PX},\overset{+}{Q},\overset{+}{W}) + \overset{*}{n}(\overset{-}{-\varepsilon},0,\overset{-}{i-\varepsilon},\overset{-}{\overset{*}{i}},\overset{+}{E\overset{*}{P}Y},\overset{+}{E\overset{*}{Q}},\overset{+}{E\overset{*}{W}}) - E\hat{N} = 0, \quad (2.4b)$$

$$b(0,\overset{-}{\varepsilon},\overset{+}{i},\overset{-}{\overset{*}{i}+\varepsilon},\overset{-}{PX},\overset{-}{Q},\overset{+}{W}) + \overset{*}{b}(\overset{-}{-\varepsilon},0,\overset{+}{i-\varepsilon},\overset{-}{\overset{*}{i}},\overset{-}{E\overset{*}{P}Y},\overset{-}{E\overset{*}{Q}},\overset{+}{E\overset{*}{W}}) - \hat{B} = 0, \quad (2.4c)$$

$$f(0,\overset{-}{\varepsilon},\overset{-}{i},\overset{+}{\overset{*}{i}+\varepsilon},\overset{-}{PX},\overset{-}{Q},\overset{+}{W}) + \overset{*}{f}(\overset{-}{-\varepsilon},0,\overset{-}{i-\varepsilon},\overset{+}{\overset{*}{i}},\overset{-}{E\overset{*}{P}Y},\overset{-}{E\overset{*}{Q}},\overset{+}{E\overset{*}{W}}) - E\hat{F} = 0. \quad (2.4d)$$

The first four arguments in each home (foreign) asset demand function are the nominal returns associated with home money, foreign money, home securities, and foreign securities measured in home (foreign) currency.[5/] ε is the expected rate of depreciation of the home currency; it is equal to zero under static expectations and to the actual rate of depreciation ($\dot{E}/E$) under rational expectations (perfect foresight).[6/] The fifth argument in each home (foreign) asset demand function is home (foreign) nominal output measured in home currency. P ($E\overset{*}{P}$) is the home currency price of the single good produced in the home (foreign) country. X (Y) is real output of the home (foreign) good. The sixth argument in each home (foreign) asset demand function is the price of the home (foreign) consumption bundle measured in home currency. Q and $E\overset{*}{Q}$ are given by

$$Q = P^{\beta}(E\overset{*}{P})^{1-\beta}, \quad E\overset{*}{Q} = P^{\overset{*}{\beta}}(E\overset{*}{P})^{1-\overset{*}{\beta}}. \tag{2.5}$$

h ($\overset{*}{h}$) is the constant weight of the price of the home good in the price of the home (foreign) consumption bundle. The seventh argument in each home (foreign) asset demand function is home (foreign) wealth measured in home currency.

The signs of the responses of asset demands to changes in the variables on which they depend are indicated by the signs over those variables. The signs over the nominal rates of return reflect the assumption that residents of both countries regard all the assets they hold as strict gross substitutes. The signs over nominal incomes and price indices reflect the assumption that residents of both countries hold both moneys for transactions purposes. The signs over nominal wealths reflect the assumption that all assets are "normal" assets. In the special case considered below some variables do not affect some asset demands.

Equations (2.2) imply that the asset demand functions of equations (2.4) are subject to familiar restrictions:

$$m_k + n_k + b_k + f_k \equiv 0, \; k = 1,\ldots,6; \quad m_7 + n_7 + b_7 + f_7 \equiv 1; \tag{2.6a}$$

$$\overset{*}{m}_k + \overset{*}{n}_k + \overset{*}{b}_k + \overset{*}{f}_k \equiv 0, \; k = 1,\ldots,6; \quad \overset{*}{m}_7 + \overset{*}{n}_7 + \overset{*}{b}_7 + \overset{*}{f}_7 \equiv 1. \tag{2.6b}$$

The assumption that private agents do not have money illusion implies that all asset demands must be homogenous of degree one in all variables measured in home currency:

$$m \equiv m_5 PX + m_6 Q + m_7 W, \quad \overset{*}{m} \equiv \overset{*}{m}_5 E\overset{*}{P}Y + \overset{*}{m}_6 E\overset{*}{Q} + \overset{*}{m}_7 E\overset{*}{W}, \tag{2.7a}$$

$$n \equiv n_5 PX + n_6 Q + n_7 W, \quad \overset{*}{n} \equiv \overset{*}{n}_5 E\overset{*}{P}Y + \overset{*}{n}_6 E\overset{*}{Q} + \overset{*}{n}_7 E\overset{*}{W}, \tag{2.7b}$$

$$b \equiv b_5 PX + b_6 Q + b_7 W, \quad \overset{*}{b} \equiv \overset{*}{b}_5 E\overset{*}{P}Y + \overset{*}{b}_6 E\overset{*}{Q} + \overset{*}{b}_7 E\overset{*}{W}, \tag{2.7c}$$

$$f \equiv f_5 PX + f_6 Q + f_7 W, \quad \overset{*}{f} \equiv \overset{*}{f}_5 E\overset{*}{P}Y + \overset{*}{f}_6 E\overset{*}{Q} + \overset{*}{f}_7 E\overset{*}{W}. \tag{2.7d}$$

Only three of the four asset market equilibrium conditions are independent. Equations (2.3) together with equations (2.1) imply that world wealth measured in home currency is equal to the sum of the stocks of all financial assets available for private agents to hold. Equations (2.2) imply that the sum of all asset demands is identically equal to world wealth. Thus, the sum of all the excess demands given by equations (2.4) is identically equal to zero. In the algebraic analysis below attention is focused on the markets for home money, foreign money, and home securities.

2.3. The basic asset market specification

Many of the results that have been derived from portfolio balance models with postulated asset demands can be illustrated using a basic asset mark t specification. The basic specification is obtained from the general specification by imposing five simplifying assumptions. First, residents of neither country hold the other country's money: that is, there is no "currency substitution" ($\overset{*}{M} = N = \overset{*}{m} = n = \overset{*}{m}_k = n_k = 0$, $k = 1,\ldots,7$; $m_2 = b_2 = f_2 = 0$; $\overset{*}{n}_1 = \overset{*}{b}_1 = \overset{*}{f}_1 = 0$).7/ Second, in each country residents' demand for money is independent of the return on the security denominated in the other country's currency ($m_4 = \overset{*}{n}_3 = 0$). Third, in each country all changes in residents' demand for money resulting from changes in their nominal income and the price of their consumption bundle are matched by changes in their demand for the security denominated in their country's currency ($b_5 = - m_5$, $b_6 = - m_6$, $\overset{*}{f}_5 = - \overset{*}{n}_5$, and $\overset{*}{f}_6 = - \overset{*}{n}_6$). Fourth, in each country residents' demand for money is independent of nominal wealth ($m_7 = \overset{*}{n}_7 = 0$). Fifth, in each country residents' demand for money is unit elastic with respect to their nominal income ($m = m_5PX$, $\overset{*}{n} = \overset{*}{n}_5E\overset{*}{P}Y$).

The first and second assumptions imply that in each country the responsivenesses of residents' demands for the two securities to changes in the return on the security denominated in the other country's currency are equal and opposite in sign ($f_4 = - b_4$ and $\overset{*}{f}_3 = - \overset{*}{b}_3$). The first and third assumptions imply that in each country residents' demand for the security denominated in the other country's currency is independent of their nominal income and the price of their consumption bundle $f_5 = f_6 = \overset{*}{b}_5 = \overset{*}{b}_6 = 0$). The third and fourth assumptions taken together with the homogeneity assumption embodied in equations (2.7) imply that the fraction of any increase in wealth allocated by residents of each country to securities denominated in their country's currency is equal to the ratio of their total holdings of assets denominated in their country's currency to their wealth [$b_7 = (M + B)/W$ and $\overset{*}{f}_7 = (\overset{*}{N} + \overset{*}{F})/\overset{*}{W}$]. That is, in each country the sum of residents' demands for assets denominated in a given currency is homogeneous of degree one in nominal wealth. The third, fourth, and fifth assumptions taken together with the homogeneity assumption embodied in equations (2.7) imply that in each country residents' demands for money and securities denominated in their country's currency are independent of the price of their consumption bundle ($m_6 = b_6 = \overset{*}{n}_6 = \overset{*}{f}_6 = 0$).

Under the assumptions of the basic specification, equations (2.1) become equations (2.8):

$$W = M + B + EF, \qquad E\overset{*}{W} = \overset{*}{B} + E(\overset{*}{N} + \overset{*}{F}), \tag{2.8}$$

and equations (2.4) become equations (2.9):

$$m(0,\overset{0}{\varepsilon},\overset{-}{i},\overset{0}{\overset{*}{i}+\varepsilon},\overset{+}{PX},\overset{0}{Q},\overset{0}{W}) - \hat{M} = 0, \tag{2.9a}$$

$$\overset{*}{n}(-\varepsilon,\overset{0}{0},\overset{0}{i-\varepsilon},\overset{-}{\overset{*}{i}},\overset{+}{E\overset{*}{P}Y},\overset{0}{E\overset{*}{Q}},\overset{0}{E\overset{*}{W}}) - E\hat{N} = 0, \tag{2.9b}$$

$$b(0,\overset{0}{\varepsilon},\overset{+}{i},\overset{-}{\overset{*}{i}+\varepsilon},\overset{-}{PX},\overset{0}{Q},\overset{+}{W}) + \overset{*}{b}(-\varepsilon,\overset{0}{0},\overset{+}{i-\varepsilon},\overset{-}{\overset{*}{i}},\overset{0}{E\overset{*}{P}Y},\overset{0}{E\overset{*}{Q}},\overset{+}{E\overset{*}{W}}) - \hat{B} = 0, \tag{2.9c}$$

$$f(0,\overset{0}{\varepsilon},\overset{-}{i},\overset{+}{\overset{*}{i}+\varepsilon},\overset{0}{PX},\overset{0}{Q},\overset{+}{W}) + \overset{*}{f}(-\varepsilon,\overset{0}{0},\overset{-}{i-\varepsilon},\overset{+}{\overset{*}{i}},\overset{-}{E\overset{*}{P}Y},\overset{0}{E\overset{*}{Q}},\overset{+}{E\overset{*}{W}}) - E\hat{F} = 0. \tag{2.9d}$$

Appropriate modifications are made in equations (2.2), (2.3), (2.6), and (2.7).

The impact effects of asset exchanges under static expectations ($\varepsilon = 0$) are of some interest in themselves.[8/] In any case the analysis of impact effects with expected depreciation exogenous is one step in a complete analysis under rational expectations ($\varepsilon = \dot{E}/E$).

An initial asset market equilibrium is represented by the intersection of $\hat{M}_0\hat{M}_0$, $\hat{B}_0\hat{B}_0$, $\hat{N}_0\hat{N}_0$, and $\hat{F}_0\hat{F}_0$ in Figure 1. The unique home (foreign) interest rate that clears the home (foreign) money market, i_0 ($\overset{*}{i}_0$), is indicated by the horizontal $\hat{M}_0\hat{M}_0$ (vertical $\hat{N}_0\hat{N}_0$) schedule. The pairs of i and $\overset{*}{i}$ that clear the market for home (foreign) securities are represented by the upward sloping $\hat{B}_0\hat{B}_0$ ($\hat{F}_0\hat{F}_0$) schedule. An increase in the foreign interest rate lowers (raises) the demand for home (foreign) securities, so an increase in the home interest rate is required to raise (lower) the demand for home (foreign) securities if equilibrium is to be reestablished.

The assumption that residents of both countries regard the assets they hold as strict gross substitutes implies that the $\hat{B}\hat{B}$ schedule must be flatter than the $\hat{F}\hat{F}$ schedule, as shown in Figure 1. If the $\hat{B}\hat{B}$ schedule were steeper, there would be excess supply of all four assets in the region to the northwest of a_0 between the $\hat{B}_0\hat{B}_0$ and $\hat{F}_0\hat{F}_0$ schedules. However, it has been established above that the sum of the excess demands for all four assets must be zero.

Depreciation of the home currency shifts both the $\hat{B}\hat{B}$ and $\hat{F}\hat{F}$ schedules down without affecting the $\hat{M}\hat{M}$ and $\hat{N}\hat{N}$ schedules. It raises not only the home currency value of wealth in both countries but also the

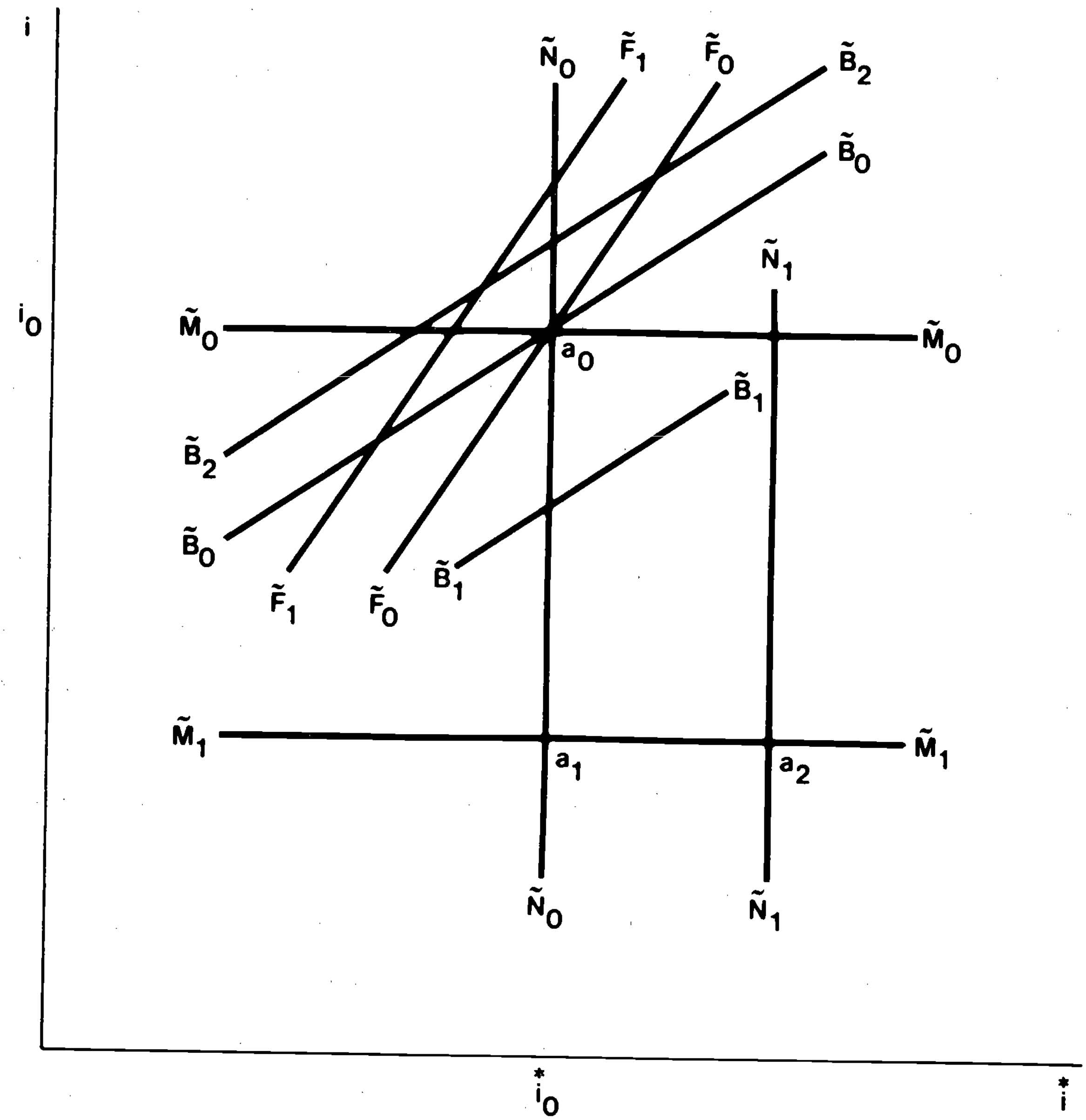

i
$\tilde{N}_0$
$\tilde{F}_1$
$\tilde{F}_0$
$\tilde{B}_2$
$\tilde{B}_0$
$\tilde{N}_1$
i_0
$\tilde{M}_0$
a_0
$\tilde{M}_0$
$\tilde{B}_1$
$\tilde{B}_2$
$\tilde{B}_0$
$\tilde{F}_1$
$\tilde{F}_0$
$\tilde{B}_1$
$\tilde{M}_1$
a_1
a_2
$\tilde{M}_1$
$\tilde{N}_0$
$\tilde{N}_1$
i^*_0
i^*

Figure 1

home currency value of the supply of foreign securities. Thus, it creates excess demand for home securities and excess supply of foreign securities.[9/] A drop in i cuts (boosts) the demand for home (foreign) securities.

First consider an expansionary open market operation in the home country ($d\hat{M} = - d\hat{B} > 0$). With the exchange rate fixed the $\hat{M}\hat{M}$ and $\hat{B}\hat{B}$ schedules shift down to $\hat{M}_1\hat{M}_1$ and $\hat{B}_1\hat{B}_1$. The shift in the $\hat{B}\hat{B}$ schedule is smaller; a reduction in the home interest rate not only reduces home residents' demand for home securities by more than it increases their demand for money because it simultaneously increases their demand for foreign securities but also reduces foreign residents' demand for home securities. The new equilibrium is at a_1 where i is lower and $\overset{*}{i}$ is unchanged. Depreciation of the home currency shifts the $\hat{B}\hat{B}$ and $\hat{F}\hat{F}$ schedules from $\hat{B}_1\hat{B}_1$ and $\hat{F}_0\hat{F}_0$ until they pass through a_1.

Now consider three types of intervention operations. Intervention of Type I is an exchange of home money for foreign money ($d\hat{M} = - d\hat{N} > 0$). This operation shifts the $\hat{M}\hat{M}$ and $\hat{N}\hat{N}$ schedules to $\hat{M}_1\hat{M}_1$ and $\hat{N}_1\hat{N}_1$. The new equilibrium is at a_2 where i is lower and $\overset{*}{i}$ is higher. Depreciation of the home currency shifts the $\hat{B}\hat{B}$ and $\hat{F}\hat{F}$ schedules down until they pass through a_2.

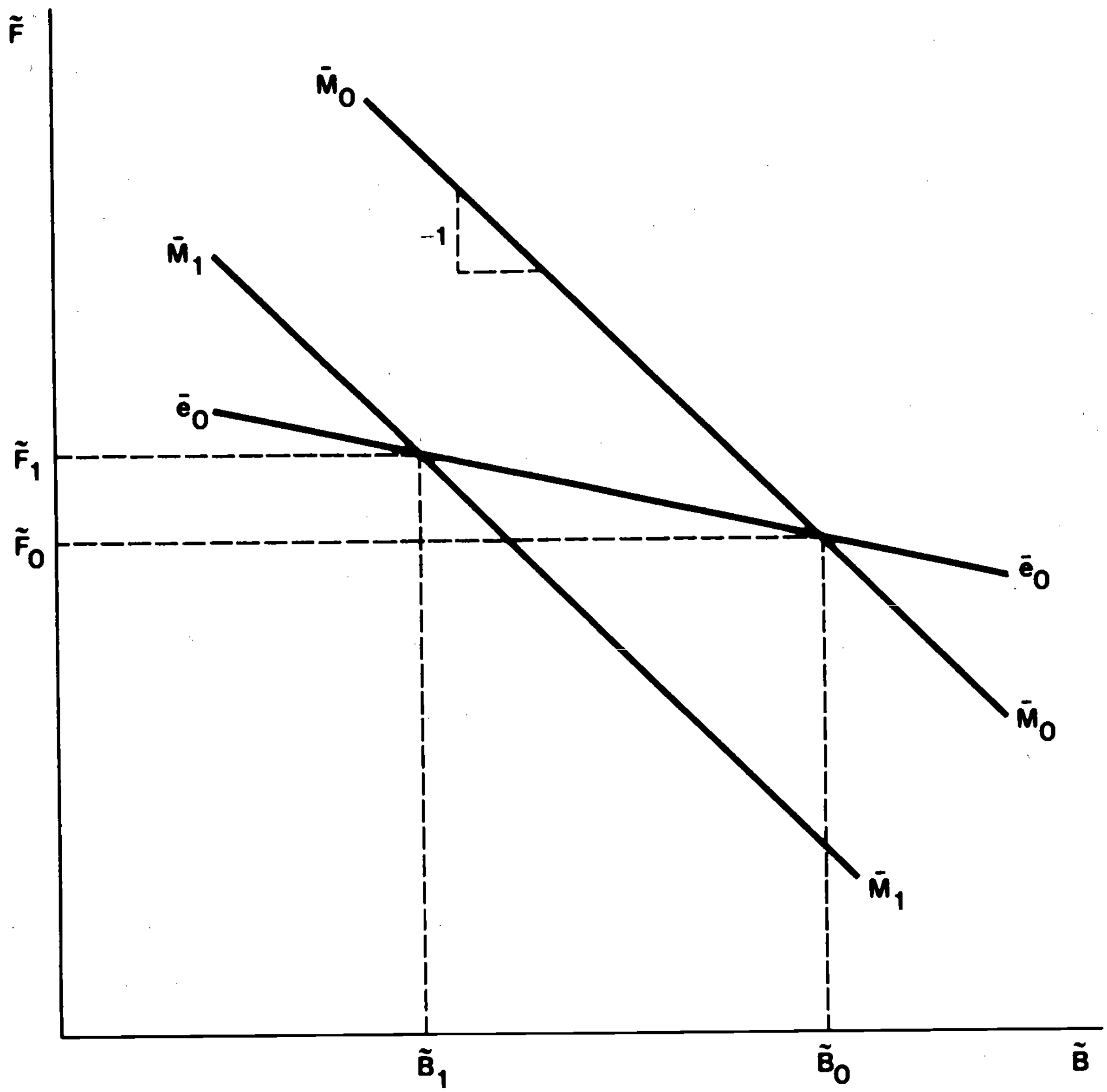
Intervention of Type II is an exchange of home money for foreign securities ($d\hat{M} = - d\hat{F} > 0$). This operation shifts $\hat{M}\hat{M}$ and $\hat{F}\hat{F}$ to $\hat{M}_1\hat{M}_1$ and $\hat{F}_1\hat{F}_1$. The new equilibrium is at a_1. The increase in the home money supply and the decline in the home interest rate are the same as they were in the case of an open market operation. Depreciation of the home currency shifts $\hat{B}\hat{B}$ and $\hat{F}\hat{F}$ down until they pass through a_1. The depreciation of the home currency is greater than it was in the case of an open market operation, since it must shift $\hat{B}\hat{B}$ from $\hat{B}_0\hat{B}_0$ to a_1 instead of from $\hat{B}_1\hat{B}_1$ to a_1.

Intervention of Type III is an exchange of home currency securities for foreign currency securities ($d\hat{B} = - d\hat{F} > 0$). Since this type of intervention leaves both money supplies unchanged, it has been called sterilized intervention. It shifts $\hat{B}\hat{B}$ and $\hat{F}\hat{F}$ to $\hat{B}_2\hat{B}_2$ and $\hat{F}_1\hat{F}_1$. The new equilibrium is at a_0 where i and $\overset{*}{i}$ are unchanged. Depreciation of the home currency shifts $\hat{B}\hat{B}$ and $\hat{F}\hat{F}$ down until they pass through a_0.

An exogenous increase in P operates exactly like an open market sale by the home authorities since it raises the excess demand for home money and lowers the excess demand for home securities by amounts that are equal in absolute value. Thus it causes i to rise and the home currency to appreciate. By analogy an exogenous increase in $\overset{*}{P}$ causes $\overset{*}{i}$ to rise and the home currency to depreciate.

We assume that there is "local asset preference:" home residents allocate a larger fraction of any increase in wealth to home securities than foreign residents ($b_7 > \overset{*}{b}_7$). With local asset preference, a transfer of wealth from home residents to foreign residents ($dw < 0$) has effects which are identical to those of a sterilized intervention operation. It lowers the excess demand for home securities and raises the excess demand for foreign securities by amounts that are equal in absolute value.

The impact effects of the home authorities' policy instruments on two possible target variables are highlighted in Figure 2. The home



$\tilde{F}$
$\bar{M}_0$
-1
$\bar{M}_1$
$\bar{e}_0$
$\tilde{F}_1$
$\tilde{F}_0$
$\bar{e}_0$
$\bar{M}_0$
$\bar{M}_1$
$\tilde{B}_1$
$\tilde{B}_0$
$\tilde{B}$

Figure 2

authorities have two independent policy instruments, open market operations and intervention operations of Type II, which they can use to achieve desired values for two target variables, the home money supply and the exchange rate, given a constant foreign money supply. Movements out the horizontal axis represent contractionary open market operations, increases in the stock of home securities ($\hat{B}$) matched by decreases in the home money supply. Movements up the vertical axis represent contractionary intervention operations of Type II, increases in the stock of foreign securities ($\hat{F}$) matched by decreases in the home money supply. The $\overline{M}_0\overline{M}_0$ schedule shows the pairs of $\hat{B}$ and $\hat{F}$ that are compatible with a constant value of the home money supply. If currency units are defined so that the exchange rate varies in the neighborhood of unity, then the $\overline{MM}$ schedule has a slope of minus one. Under the basic specification, both interest rates are constant along the $\overline{MM}$ schedule because there is a one to one correspondence between the money supply and the interest rate in each country. The $\overline{e}_0\overline{e}_0$ schedule represents the pairs of $\hat{B}$ and $\hat{F}$ that are compatible with a constant exchange rate. The $\overline{ee}$ schedule must be flatter than the $\overline{MM}$ schedule since an intervention operation of Type II has a greater effect on the exchange rate than an open market operation of equal size as shown above for the basic specification. It follows that movements down along the $\overline{ee}$ schedule lead to decreases in the home money supply and increases in the home interest rate. The home authorities can expand the home money supply from the level corresponding to $\overline{M}_0\overline{M}_0$ to the level corresponding to $\overline{M}_1\overline{M}_1$ without changing the exchange rate by conducting the open market purchase corresponding to $\hat{B}_0\hat{B}_1$ and the intervention operation of Type II corresponding to $\hat{F}_0\hat{F}_1$.

As groundwork for the dynamic analysis below it is useful to provide an algebraic derivation of the results just arrived at graphically. We first select a state variable for the system and then express the asset market equilibrium conditions in terms of deviations of the variables from their stationary equilibrium values.

It will become clear below that it is convenient to define home (foreign) residents' wealth valued at the long-run equilibrium exchange rate, $\bar{E}$, as w ($\bar{E}\overset{*}{w}$):

$$w = M + B + \bar{E}F, \quad \bar{E}\overset{*}{w} = \overset{*}{B} + \bar{E}(\overset{*}{N} + \overset{*}{F}), \tag{2.10}$$

and to choose w as the state variable of the system. The time derivative of w equals home residents' asset accumulation in the neighborhood of long-run equilibrium:

$$\dot{w} = \dot{M} + \dot{B} + \bar{E}\dot{F}. \tag{2.11}$$

The equilibrium conditions for the markets for home money, foreign money, and home securities in deviation form are given by

$$\hat{m}_i di + \hat{m}_p dp - d\hat{M} = 0, \tag{2.12a}$$

$$\hat{n}^*_i d\overset{*}{i} + \hat{n}^*_p d\overset{*}{p} - d\hat{N} = 0, \tag{2.12b}$$

$$\hat{b}_i di + \hat{b}^*_i d\overset{*}{i} + \hat{b}_e de + \hat{b}_\varepsilon \varepsilon + \hat{b}_p dp + \hat{b}_w dw - d\hat{B} = 0, \tag{2.12c}$$

$$\hat{m}_i = m_3,\ m_p = m_5 X,\ \hat{n}^*_i = \overset{*}{n}_4,\ \hat{n}^*_p = \overset{*}{n}_5 Y,\ \hat{b}_i = b_3 + \overset{*}{b}_3,\ \hat{b}^*_i = b_4 + \overset{*}{b}_4,$$

$$\hat{b}_e = b_7 F + \overset{*}{b}_7(\overset{*}{N} + \overset{*}{F}),\ \hat{b}_\varepsilon = b_4 - \overset{*}{b}_3,\ \hat{b}_w = b_7 - \overset{*}{b}_7,\ \hat{b}_p = b_5 X.$$

$\hat{m}_k$, $\hat{n}_k$, and $\hat{b}_k$, represent the partial derivatives of the excess demands for home money, foreign money, and home securities with respect to the variables that appear as subscripts under the basic specification. A variable with a d in front of it represents the deviation of that variable from its stationary equilibrium value. e, p, and $\overset{*}{p}$ are the natural logarithms of E, P, and $\overset{*}{P}$ so that de = dE/E, dp = dP/P and $d\overset{*}{p} = d\overset{*}{P}/\overset{*}{P}$. In the neighborhood of stationary equilibrium $d\varepsilon = \varepsilon = 0$ under static expectations and $d\varepsilon = \varepsilon = \dot{e}$ under rational expectations. In deriving equations (2.12) we have set $E = P = \overset{*}{P} = 1$ and have made use of the following relationships:

$$dW = dw + Fde, \quad dE\overset{*}{W} = d\overset{*}{w} + (\overset{*}{N} + \overset{*}{F})de, \tag{2.13a}$$

$$d\overset{*}{w} = d\overset{*}{B} + d\overset{*}{N} + d\overset{*}{F} = -\,dM - dB - dF = -dw. \tag{2.13b}$$

To derive equation (2.13b), sum the appropriately modified versions of equations (2.3) in deviation form to obtain

$$d\hat{M} + d\hat{N} + d\hat{B} + d\hat{F} = d\overset{*}{B} + d\overset{*}{N} + d\overset{*}{F} + dM + dB + dF. \tag{2.14}$$

Equation (2.14) implies the equality of the middle two terms in (2.13b) because world central bank intervention is governed by $d\hat{M} + d\hat{N} + d\hat{B} + d\hat{F} = 0$.

The effects of changes in the exogenous variables are given by

$$di = -(\hat{m}_p/\hat{m}_i)dp + (1/\hat{m}_i)d\hat{M}, \tag{2.15a}$$

$$d\overset{*}{i} = -(\hat{n}_p^*/\hat{n}_i^*)d\overset{*}{p} + (1/\hat{n}_i^*)d\hat{N}, \tag{2.15b}$$

$$\hat{b}_e de = [(\hat{b}_i\hat{m}_p - \hat{b}_p\hat{m}_i)/\hat{m}_i]dp + (\hat{b}_i^*\hat{n}_p^*/\hat{n}_i^*)d\overset{*}{p} \tag{2.15c}$$

$$- (\hat{b}_i/\hat{m}_i)d\hat{M} - (\hat{b}_i^*/\hat{n}_i^*)d\hat{N} - \hat{b}_\varepsilon\varepsilon - \hat{b}_w dw + d\hat{B}.$$

It is convenient not to divide through by the positive coefficient $\hat{b}_e$ since equation (2.15c) will be viewed in a different way in what follows.

2.4. The specification of the goods markets and the balance of payments

Even when the objective is to study the behavior of interest rates and the exchange rate at a point in time, it is not possible to conduct the analysis using just the asset market equilibrium conditions except under very restrictive assumptions. It was shown in the last section that if goods prices are fixed and expectations are static, then the conditions for asset market equilibrium are sufficient to determine interest rates and the exchange rate at a point in time. However, if goods prices are fixed and expectations are rational, then a balance of payments condition must be employed together with the asset market equilibrium conditions to jointly determine interest rates, the exchange rate, the percentage rate of change in the exchange rate, and the rate of transfer of wealth between home and foreign residents. Moreover, if goods prices are flexible, then under both static and rational expectations a complete model must include goods market equilibrium conditions. Of course, when the objective is to study the behavior of interest rates and the exchange rate over time, a balance of payments condition must be employed no matter whether expectations are static or rational.[10/]

In this subsection we specify equilibrium conditions for the home and foreign goods markets and a balance of payments equation. Since the focus of this chapter is asset markets, we have deliberately kept the specification of the goods market equilibrium conditions and the balance of payments equation as simple as possible.[11/]

Home (foreign) expenditure is allocated between home and foreign goods:

$$P(X - G) - s[\overset{+}{P(X - G)}, \overset{-}{W}] \equiv x(\cdot) + y(\cdot), \qquad (2.16a)$$

$$E\overset{*}{P}(Y - \overset{*}{G}) - \overset{*}{s}[\overset{+}{E\overset{*}{P}(Y - \overset{*}{G})}, \overset{-}{E\overset{*}{W}}] \equiv \overset{*}{x}(\cdot) + \overset{*}{y}(\cdot). \qquad (2.16b)$$

$X - G$ ($Y - \overset{*}{G}$) is home (foreign) real disposable income measured in the home (foreign) good.[12/] G ($\overset{*}{G}$) is home (foreign) real, balanced-budget government spending measured in home (foreign) goods. s, x, and y ($\overset{*}{s}$, $\overset{*}{x}$, and $\overset{*}{y}$) are home (foreign) saving, expenditure on the home good, and expenditure on the foreign good, all measured in home currency. Home (foreign) saving measured in home currency depends positively on home (foreign) nominal disposable income and negatively on home (foreign) nominal wealth, both measured in home currency.[13/]

The goods market equilibrium conditions and the balance of payments equation are given by

$$x[\overset{+}{P(X - G)}, \overset{+}{W}, \overset{-}{P}, \overset{+}{E\overset{*}{P}}] + \overset{*}{x}[\overset{+}{E\overset{*}{P}(Y - \overset{*}{G})}, \overset{+}{E\overset{*}{W}}, \overset{-}{P}, \overset{+}{E\overset{*}{P}}] - P(X - G) = 0, \qquad (2.17a)$$

$$y[\overset{+}{P(X - G)}, \overset{+}{W}, \overset{+}{P}, \overset{-}{E\overset{*}{P}}] + \overset{*}{y}[\overset{+}{E\overset{*}{P}(Y - \overset{*}{G})}, \overset{+}{E\overset{*}{W}}, \overset{+}{P}, \overset{-}{E\overset{*}{P}}] - E\overset{*}{P}(Y - \overset{*}{G}) = 0, \qquad (2.17b)$$

$$\overset{*}{x}[\overset{+}{E\overset{*}{P}(Y - \overset{*}{G})}, \overset{+}{E\overset{*}{W}}, \overset{-}{P}, \overset{+}{E\overset{*}{P}}] - y[\overset{+}{P(X - G)}, \overset{+}{W}, \overset{+}{P}, \overset{-}{E\overset{*}{P}}] - \dot{w} - (E - \overline{E})\dot{F} = 0. \qquad (2.17c)$$

Home (foreign) nominal spending on both home and foreign goods measured in home currency depends positively on home (foreign) nominal disposable income and nominal wealth, both measured in home currency. Increases in P ($E\overset{*}{P}$) shift both home and foreign nominal spending from home (foreign) goods to foreign (home) goods. Therefore, increases in P ($E\overset{*}{P}$) reduce (increase) the home trade surplus.[14/] $\dot{w} + (E - \overline{E})\dot{F}$ is home residents' asset accumulation. $\dot{w}$ is defined by equation (2.11).

Equations (2.16) imply that the expenditure functions of equations (2.17) are subject to familiar restrictions:

$$x_1 + y_1 \equiv 1 - s_1; \quad x_2 + y_2 \equiv - s_2; \quad x_k + y_k \equiv 0, \quad k = 3, 4; \qquad (2.18a)$$

$$\overset{*}{x}_1 + \overset{*}{y}_1 \equiv 1 - \overset{*}{s}_1: \quad \overset{*}{x}_2 + \overset{*}{y}_2 \equiv - \overset{*}{s}_2; \quad \overset{*}{x}_k + \overset{*}{y}_k \equiv 0, \quad k = 3, 4. \qquad (2.18b)$$

The assumption that private agents do not have money illusion implies that all expenditure functions and savings functions are homogenous of degree one in all nominal variables:

$$x \equiv x_1 P(X - G) + x_2 W + x_3 P + x_4 E\overset{*}{P}, \quad \overset{*}{x} \equiv \overset{*}{x}_1 E\overset{*}{P}(Y - \overset{*}{G}) + \overset{*}{x}_2 E\overset{*}{W} + \overset{*}{x}_3 P + \overset{*}{x}_4 E\overset{*}{P}, \qquad (2.19a)$$

$$y \equiv y_1 P(X - G) + y_2 W + y_3 P + y_4 E\overset{*}{P}, \quad \overset{*}{y} \equiv \overset{*}{y}_1 E\overset{*}{P}(Y - \overset{*}{G}) + \overset{*}{y}_2 E\overset{*}{W} + \overset{*}{y}_3 P + \overset{*}{y}_4 E\overset{*}{P}, \qquad (2.19b)$$

$$s \equiv s_1 P(X - G) + s_2 W, \qquad \overset{*}{s} \equiv \overset{*}{s}_1 E\overset{*}{P}(Y - \overset{*}{G}) + \overset{*}{s}_2 E\overset{*}{W}. \qquad (2.19c)$$

Below we consider two special cases of the goods market equilibrium conditions and the balance of payments equation. In both special cases real outputs in both countries are assumed to always be at their "full employment" or "natural" levels. In the first special case it is assumed that balanced budget fiscal policy is used in each country to fix the price of output in that country ($dp = d\overset{*}{p} = 0$). Furthermore, in each country the demand for the good produced in the other country is assumed to be independent of the level of nominal spending

($\overset{*}{x}_1 = \overset{*}{x}_2 = y_1 = y_2 = 0$). Therefore, the trade account of the home country is independent of nominal disposable incomes and nominal wealths. In deviation form the balance of payments equation is

$$\dot{w} = \eta de,$$

$$\eta = \overset{*}{x}_4 - y_4 = \overset{*}{x}_4 + x_4 > 0. \tag{2.20}$$

η is the effect of a depreciation of the home currency on the home trade surplus. In the neighborhood of stationary equilibrium $d\dot{w} = \dot{w}$ and $d(E - \overline{E})\dot{F} = 0$.

In the second special case it is assumed that output prices are flexible and that G and $\overset{*}{G}$ are equal to zero. In deviation form the goods market equilibrium conditions and the balance of payments equation are

$$\hat{x}_p dp + \hat{x}^*_p d\overset{*}{p} + \hat{x}_e de + \hat{x}_w dw = 0, \tag{2.21a}$$

$$\hat{y}_p dp + \hat{y}^*_p d\overset{*}{p} + \hat{y}_e de + \hat{y}_w dw = 0, \tag{2.21b}$$

$$t_p dp + t^*_p d\overset{*}{p} + t_e de + t_w dw - \dot{w} = 0, \tag{2.21c}$$

$$\hat{x}_p = -\hat{x}_p^* - x_2W - \overset{*}{x}_2\overset{*}{W} < 0, \qquad \hat{x}_e = \hat{x}_p^* + x_2F + \overset{*}{x}_2(\overset{*}{N} + \overset{*}{F}) > 0,$$

$$\hat{y}_p = y_1X + \mu > 0, \qquad \hat{y}_e = -\hat{y}_p - y_2(B + M) - \overset{*}{y}_2\overset{*}{B} < 0,$$

$$t_p = -\hat{y}_p < 0, \qquad t_e = \hat{x}_p^* + \overset{*}{x}_2(\overset{*}{N} + \overset{*}{F}) - y_2F \gtrless 0,$$

$$\hat{x}_p^* = \overset{*}{x}_1Y + \eta > 0, \qquad \hat{x}_w = x_2 - \overset{*}{x}_2 > 0,$$

$$\hat{y}_p^* = -\hat{y}_p - y_2W - \overset{*}{y}_2\overset{*}{W} < 0, \qquad \hat{y}_w = y_2 - \overset{*}{y}_2 < 0,$$

$$t_p^* = \hat{x}_p^* > 0, \qquad t_w = -\overset{*}{x}_2 - y_2 < 0.$$

$\hat{x}_k$, $\hat{y}_k$, and t_k represent the derivatives of the excess demand for home goods, the excess demand for foreign goods, and the home trade surplus with respect to the variables which appear as subscripts.15/

$\mu = y_3 + \overset{*}{y}_3 > 0$ ($\eta = x_4 + \overset{*}{x}_4 > 0$) is the effect of an increase in the price of the home (foreign) good on excess demand for the foreign (home) good given that home (foreign) nominal income is held constant.

$\hat{x}_p$, $\hat{x}_p^*$, $\hat{y}_p$, $\hat{y}_p^*$, t_p and t_p^* have the normal signs. The signs of $\hat{x}_e$, $\hat{y}_e$, t_e, and t_w reflect the assumptions that increases in wealth lead to increases in spending on both goods in both countries and that there are no negative net foreign asset positions. We assume that there is "local good preference;" home residents allocate a larger fraction of increases in spending resulting from increases in wealth to home goods than foreign residents ($x_2 > \overset{*}{x}_2$), and foreign residents allocate a larger fraction of increases in spending resulting from increases in wealth to foreign goods than home residents ($y_2 < \overset{*}{y}_2$). With local good preference, a transfer of wealth to home residents ($dw > 0$) increases demand for the home good and decreases demand for the foreign good ($\hat{x}_w > 0$, $\hat{y}_w < 0$).

It is convenient for what follows to obtain expressions for dp, $d\overset{*}{p}$, and $\dot{w}$ as functions of de and dw:

$$dp = C_1 de + C_2 dw, \tag{2.22a}$$

$$d\overset{*}{p} = - C_3 de - C_4 dw, \tag{2.22b}$$

$$\dot{w} = C_5 de - C_6 dw, \tag{2.22c}$$

$$C_1 = \{[x_2 F + \overset{*}{x}_2(\overset{*}{N} + \overset{*}{F})][y_2 W + \overset{*}{y}_2\overset{*}{W} + \hat{y}_p] + [y_2 F + \overset{*}{y}_2(\overset{*}{N} + \overset{*}{F})]\hat{x}^*_p\}/\Delta > 0,$$

$$C_2 = [(\overset{*}{x}_2 - x_2)\overset{*}{s}_2\overset{*}{W} - (s_2 - \overset{*}{s}_2)\hat{x}^*_p]/\Delta > 0,$$

$$C_3 = \{[y_2(B + M) + \overset{*}{y}_2\overset{*}{B}][x_2 W + \overset{*}{x}_2\overset{*}{W} + \hat{x}^*_p] + [x_2(B + M) + \overset{*}{x}_2\overset{*}{B}]\hat{y}_p\}/\Delta > 0,$$

$$C_4 = [(y_2 - \overset{*}{y}_2)s_2 W + (s_2 - \overset{*}{s}_2)\hat{y}_p]/\Delta > 0,$$

$$C_5 = s_2 W\overset{*}{s}_2\overset{*}{W}(\hat{x}^*_p + \overset{*}{x}_2\overset{*}{W})(b_7 - \overset{*}{b}_7)/\Delta > 0,$$

$$C_6 = s_2\overset{*}{s}_2(W + \overset{*}{W})(\hat{x}^*_p + \overset{*}{x}_2\overset{*}{W})/\Delta > 0,$$

$$\Delta = (x_2 W + \overset{*}{x}_2\overset{*}{W})(y_2 W + \overset{*}{y}_2\overset{*}{W}) + (y_2 W + \overset{*}{y}_2\overset{*}{W})\hat{x}^*_p + (x_2 W + \overset{*}{x}_2\overset{*}{W})\hat{y}_p > 0.$$

Assumptions imposed above imply that C_1, C_3, C_5 and C_6 are positive. We assume that responsivenesses of saving to wealth are the same in the two countries ($s_2 = \overset{*}{s}_2$) so that transfers of wealth between countries affect the distribution but not the level of world saving. Under this assumption C_2 and C_4 are positive. Since our assumptions imply that all the coefficients in equations (2.22) are positive, the signs preceding the coefficients indicate the signs of the effects of changes in the variables.[16/]

The relationships summarized by equations (2.22) are in accord with intuition. First, consider a transfer of wealth from foreign residents to home residents ($dw > 0$). Given that the responsivenesses of savings to wealth are the same in the two countries ($s_2 = \overset{*}{s}_2$) and that there is marginal local good preference ($x_2 - \overset{*}{x}_2 = \overset{*}{y}_2 - y_2 > 0$), the transfer raises demand for home goods and lowers demand for foreign goods by amounts that are equal in absolute value. An increase in the price of each good has an effect on excess demand for that good that is greater in absolute value than its effect on the excess demand for the other good ($|\hat{x}_p| > |\hat{y}_p|$, $|\hat{y}^*_p| > |\hat{x}^*_p|$).[17/] For example, an increase in P reduces excess demand for home goods both by increasing savings and by reducing the home trade surplus; however, it increases excess demand for the foreign good only by increasing the foreign trade surplus which, of course, is the negative of the home trade surplus. Therefore, the transfer causes P to rise and $\overset{*}{P}$ to fall. The direct effect of the transfer on the home trade surplus is to reduce it since foreigners spend less on home goods and home residents spend more on foreign goods. The indirect effect resulting from the induced price changes reinforces the direct effect.[18/]

Now consider a depreciation of the home currency ($de > 0$). Given that all security positions are positive ($B, F, \overset{*}{B}, \overset{*}{F} > 0$), the depreciation increases demand for home goods and decreases demand for foreign goods. A given percentage increase in the price of home goods has an effect on excess demand for home goods that is greater in absolute value than the effect of the same percentage depreciation ($|\hat{x}_p| > |\hat{x}_e|$) and an effect on the excess demand for foreign goods that is smaller in absolute value than the effect of the same percentage depreciation ($|\hat{y}_p| < |\hat{y}_e|$). For example, if equation (2.17a) is divided by P, then E and P always enter as the ratio of E to P except in the terms for home and foreign real wealth measured in home goods. Equiproportionate changes in E and P lower real wealth. Similarly, a given percentage increase in the price of foreign goods has an effect on excess demand for foreign goods that is greater in absolute value than the effect of the same percentage depreciation ($|\hat{y}^*_p| > |\hat{y}_e|$) and an effect on excess demand for home goods that is smaller in alsolute value than the effect of the same percentage depreciation ($|\hat{x}^*_p| < |\hat{x}_e|$). Therefore the depreciation causes p to rise and $\overset{*}{p}$ to fall.

A depreciation of the home currency increases the home trade surplus if there is local asset preference [$b_7 = (M + B)/W > \overset{*}{B}/\overset{*}{W} = \overset{*}{b}_7$], as shown in equation (2.22c). Some intuition about this result can be gained by considering two special cases. Assume temporarily that there is no local asset preference [$(M + B)/W = \overset{*}{B}/\overset{*}{W}$]. Following a depreciation let P rise by enough to keep world wealth measured in home goods $[(W + E\overset{*}{W})/P]$ constant and let $\overset{*}{P}$ fall by enough to keep world wealth measured in foreign goods $[(W + E\overset{*}{W})/E\overset{*}{P}]$ constant. Then the relative price of the foreign good $(E\overset{*}{P}/P)$ remains constant, and there is no change in excess demand for either good or in the home trade surplus. Now assume again that there is local asset preference and assume temporarily that there is no local good preference $(x_2 = \overset{*}{x}_2, y_2 = \overset{*}{y}_2)$. In this case the changes in P and $\overset{*}{P}$ considered above lower home real wealth measured in both goods and raise foreign real wealth measured in both goods. However, the goods markets remain in equilibrium since the redistribution of real wealth does not affect the excess demands for goods. Thus, there is no need for a change in the relative price of the foreign good and no need for further changes in P and $\overset{*}{P}$. The home trade surplus increases since foreign spending on home goods increases and home spending on foreign goods falls. According to equation (2.22c), the result that a depreciation increases the trade surplus when there is local asset preference is very general: it is independent of good preference and the signs of security positions.

2.5. A distribution effect of a trade surplus with goods prices fixed

If asset demands embody local asset preference, a transfer of wealth to home residents through a trade account surplus raises the demand for home securities and lowers the demand for foreign securities. In this subsection we spell out the implications of this distribution effect of a trade surplus with goods prices fixed. In this special case, our model is similar to what some have called a "partial equilibrium" model of exchange rate behavior.[19/]

The first building block of the fixed price model is the balance of payments equation from the first special case of the goods market equilibrium conditions and the balance of payments equation. This equation is reproduced here for convenience:

$$\dot{w} = \eta de, \tag{2.20}$$

$$\eta = x_4 - y_4 = \overset{*}{x}_4 + x_4 > 0.$$

The $\dot{w}$ schedule in the left-hand panel of Figure 3 represents this relationship. It slopes upward because a depreciation of the home currency increases the home trade surplus and, therefore, increases home asset accumulation ($\dot{w}$). The long-run equilibrium exchange rate ($\bar{e}_0$) is the only value of e for which the trade surplus and, therefore, asset accumulation equal zero as indicated by the horizontal $\dot{w} = 0$ schedule in the right-hand panel of Figure 3. The horizontal arrows show the direction of motion of w. When the home currency price of foreign currency is too high ($e > \bar{e}_0$), the home country runs a trade surplus and accumulates assets ($\dot{w} > 0$).

The second building block of the fixed price model is the equation for the expected rate of change of the exchange rate (ε) implied by asset market equilibrium in the basic specification of asset markets. Solving equation (2.15c) for ε with $dp = d\overset{*}{p} = 0$ yields

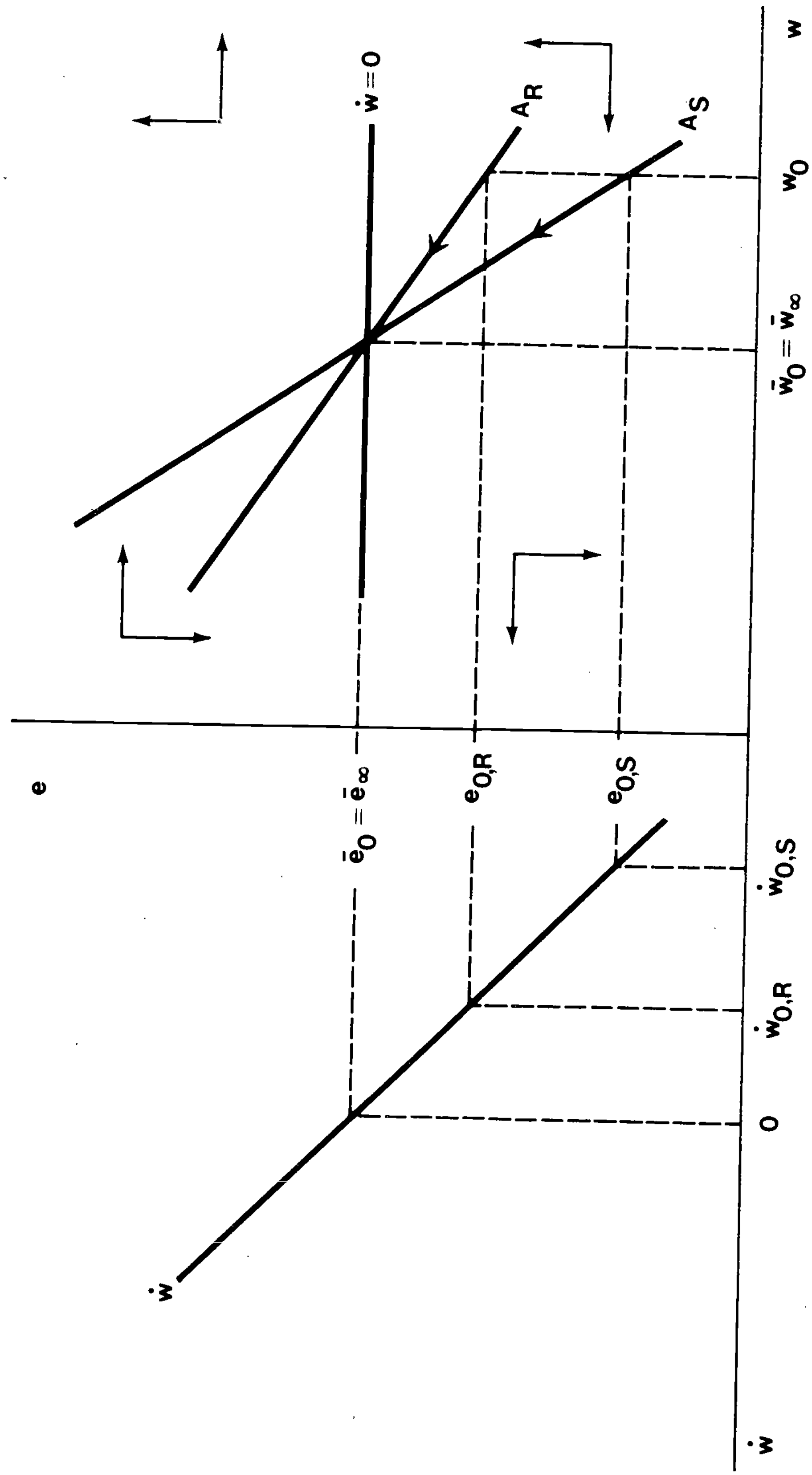


Figure 3

$$\varepsilon = \varepsilon_e de + \varepsilon_w dw - \varepsilon_{OMO} d\hat{M}, \tag{2.24}$$

$$\varepsilon_e = - \hat{b}_e / \hat{b}_\varepsilon > 0,$$

$$\varepsilon_w = - \hat{b}_w / \hat{b}_\varepsilon > 0,$$

$$\varepsilon_{OMO} = (\hat{b}_i + \hat{m}_i)/\hat{m}_i \hat{b}_\varepsilon > 0.$$

The coefficient $- \varepsilon_{OMO}$ gives the effect of an expansionary open market operation ($d\hat{M} = - d\hat{B} > 0$) on ε.

The A_S schedule in the right-hand panel of Figure 3 is the asset market equilibrium schedule under static expectations. It represents the pairs of e and w that are compatible with asset market equilibrium given that ε is equal to zero. This schedule is downward sloping under the assumption of local asset preference ($b_7 - \overset{*}{b}_7 > 0$). A transfer of wealth to home residents raises the demand for home securities, so the home currency must appreciate to reequilibrate the asset markets if ε is to remain unchanged. The more pronounced is local asset preference, that is, the greater $b_7 - \overset{*}{b}_7$, the steeper is A_S.

The A_R schedule is the asset market equilibrium schedule under rational expectations given that exchange rate expectations are compatible with the stability of long-run equilibrium. Under rational expectations the A_S schedule is simply the schedule along which $\varepsilon = \dot{e} = 0$. The vertical arrows show the direction of motion of e.

Above A_S there is excess demand for home assets with $\dot{e} = 0$, so the home currency must be expected to depreciate ($\dot{e} > 0$) to equilibrate the asset markets. Long-run equilibrium is a saddle point under rational expectations as indicated by the arrows.[20/] Following a disturbance the world economy will reach long-run equilibrium if and only if it moves along the unique saddle path represented by A_R.

The effects of an unanticipated transfer of wealth from foreign residents to home residents ($w_0 > \bar{w}_0 = \bar{w}_\infty$) are shown in Figure 3. This disturbance does not shift any of the schedules. Under static expectations the home currency appreciates ($e_{0,S} < \bar{e}_0$), and the home country begins to run a trade deficit ($\dot{w}_{0,S} < 0$). As w falls, the home currency depreciates. The economy moves along A_S back to long-run equilibrium. Under rational expectations the home currency appreciates but not as much as under static expectations ($e_{0,S} < e_{0,R} < \bar{e}_0$), and the home country begins to run a trade account deficit but one which is smaller than under static expectations ($\dot{w}_{0,S} < \dot{w}_{0,R} < 0$). Once again, as w falls, the home currency depreciates. The economy moves along A_R back to long-run equilibrium. When agents take account of the future path of the exchange rate, the initial movement in this variable is damped.

The effects of an unanticipated contractionary open market operation are shown in Figure 4. This operation shifts the A_S and A_R schedules down to A_S' and A_R'. Under both static and rational expectations the home currency appreciates, and the home country begins to run a trade account deficit. In the new long-run equilibrium, home wealth is lower

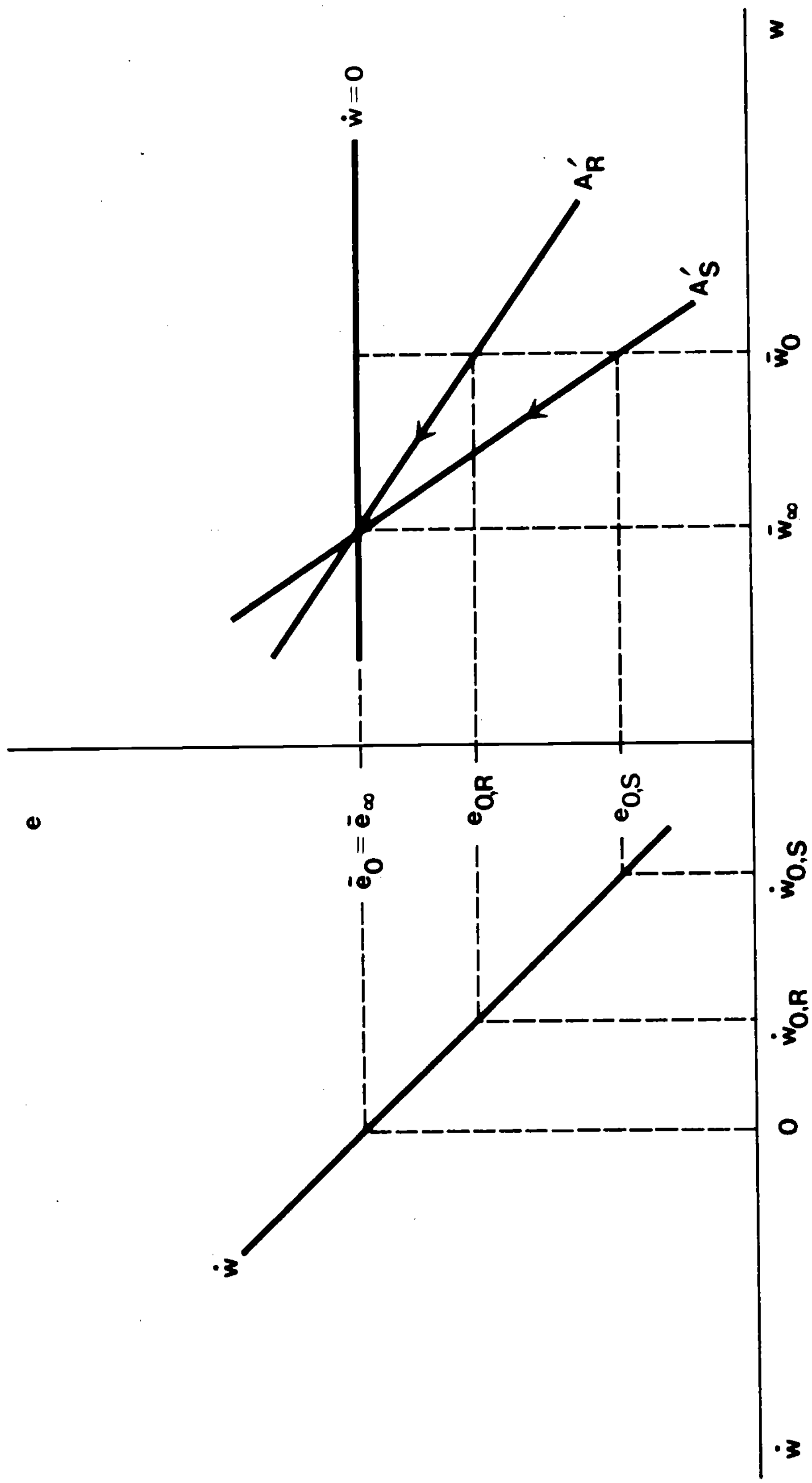

$\dot{w} = 0$
A'_R
A'_S
w
$\bar{w}_0$
$\bar{w}_\infty$
e
$\bar{e}_0 = \bar{e}_\infty$
$e_{0,R}$
$e_{0,S}$
$\dot{w}_{0,S}$
$\dot{w}_{0,R}$
0
$\dot{w}$
$\dot{w}$

Figure 4

$(\bar{w}_\infty < \bar{w}_0)$, but the exchange rate has the same value as in the initial long-run equilibrium $(\bar{e}_\infty = \bar{e}_0)$. What happens is that the home interest rate rises by enough to clear the home money market. This increase is more than enough to reequilibrate the market for home securities, so home wealth must decline to reequilibrate that market.

The effects of an unancticipated shift in spending in either country from foreign to home goods is shown in Figure 5. This disturbance shifts the $\dot{w}$ schedule, the $\dot{w} = 0$ schedule, and the A_R schedule down to $\dot{w}'$, $(\dot{w} = 0)'$, and A_R'. Under static expectations there is no effect on the exchange rate initially $e_{0,S} = \bar{e}_0$, but under rational expectations the home currency appreciates at once $(e_{0,R} < \bar{e}_0)$. Under both static and rational expectations the home country begins to run a trade account surplus. In the new long-run equilibrium, e is lower $(\bar{e}_\infty < \bar{e}_0)$ and home wealth is higher $(\bar{w}_\infty > \bar{w}_0)$. e must fall in order to reequilibrate the current account. With a lower e, w must be higher in order to reequilibrate the market for home currency securities.

2.6. Distribution effects of the trade surplus with goods prices flexible

If goods prices are flexible, not only local asset preference but also local good preference is sufficient to insure that a trade account surplus has a distribution effect. In this subsection we spell out the implications of the distribution effects of a trade surplus with goods prices flexible.

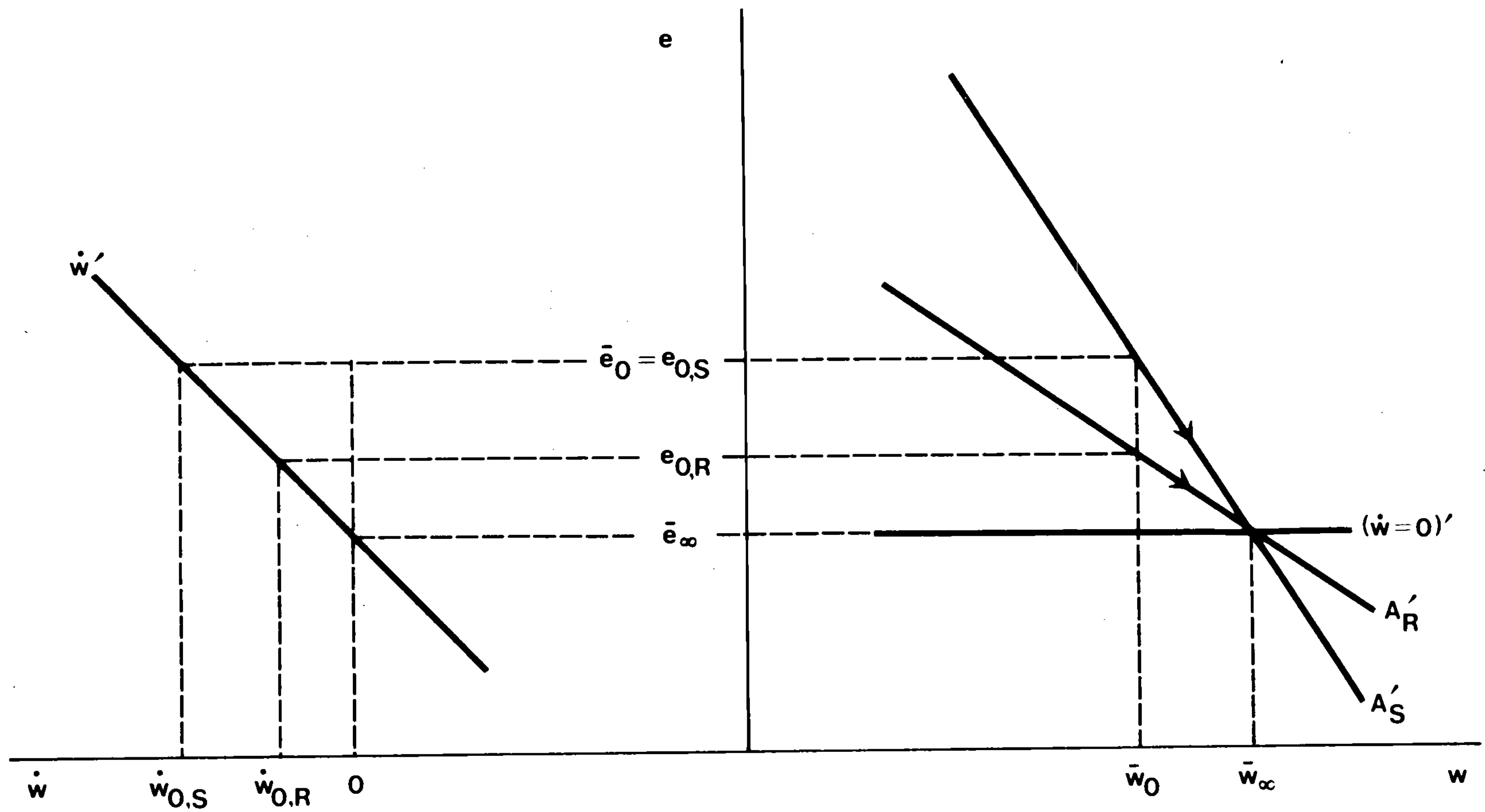

e
$\dot{w}'$
$\bar{e}_0 = e_{0,S}$
$e_{0,R}$
$\bar{e}_\infty$
$(\dot{w}=0)'$
A_R'
A_S'
$\dot{w}$
$\dot{w}_{0,S}$
$\dot{w}_{0,R}$
0
$\bar{w}_0$
$\bar{w}_\infty$
w

Figure 5

The first building block of the flexible price model is the reduced form asset accumulation equation (2.22c), which is reproduced here for convenience:

$$\dot{w} = C_5 de - C_6 dw, \qquad (2.22c)$$

$$C_5 = s_2 W \overset{*}{s}_2 \overset{*}{W}(\hat{x}_p^* + \overset{*}{x}_2 \overset{*}{W})(b_7 - \overset{*}{b}_7)/\Delta > 0,$$

$$C_6 = s_2 \overset{*}{s}_2 (W + \overset{*}{W})(\hat{x}_p^* + \overset{*}{x}_2 \overset{*}{W})/\Delta > 0.$$

The $\dot{w} = 0$ schedule in Figure 6 repesents the pairs of e and w for which $\dot{w}$ equals zero. If there is local asset preference, it slopes upward. An increase in w lowers home asset accumulation, so the home currency must depreciate in order to increase it. If there is no local asset preference, the $\dot{w} = 0$ schedule is vertical since changes in the exchange rate do not affect asset accumulation. The horizonal arrows show the motion of w. Above the $\dot{w} = 0$ schedule the home country runs a trade surplus and accumulates assets.

The second building block of the flexible price model is the reduced form equation for the expected rate of change of the exchange rate (ε) derived by solving equation (2.15c) for ε and eliminating dp and $d\overset{*}{p}$ using equations (2.22a) and (2.22b):

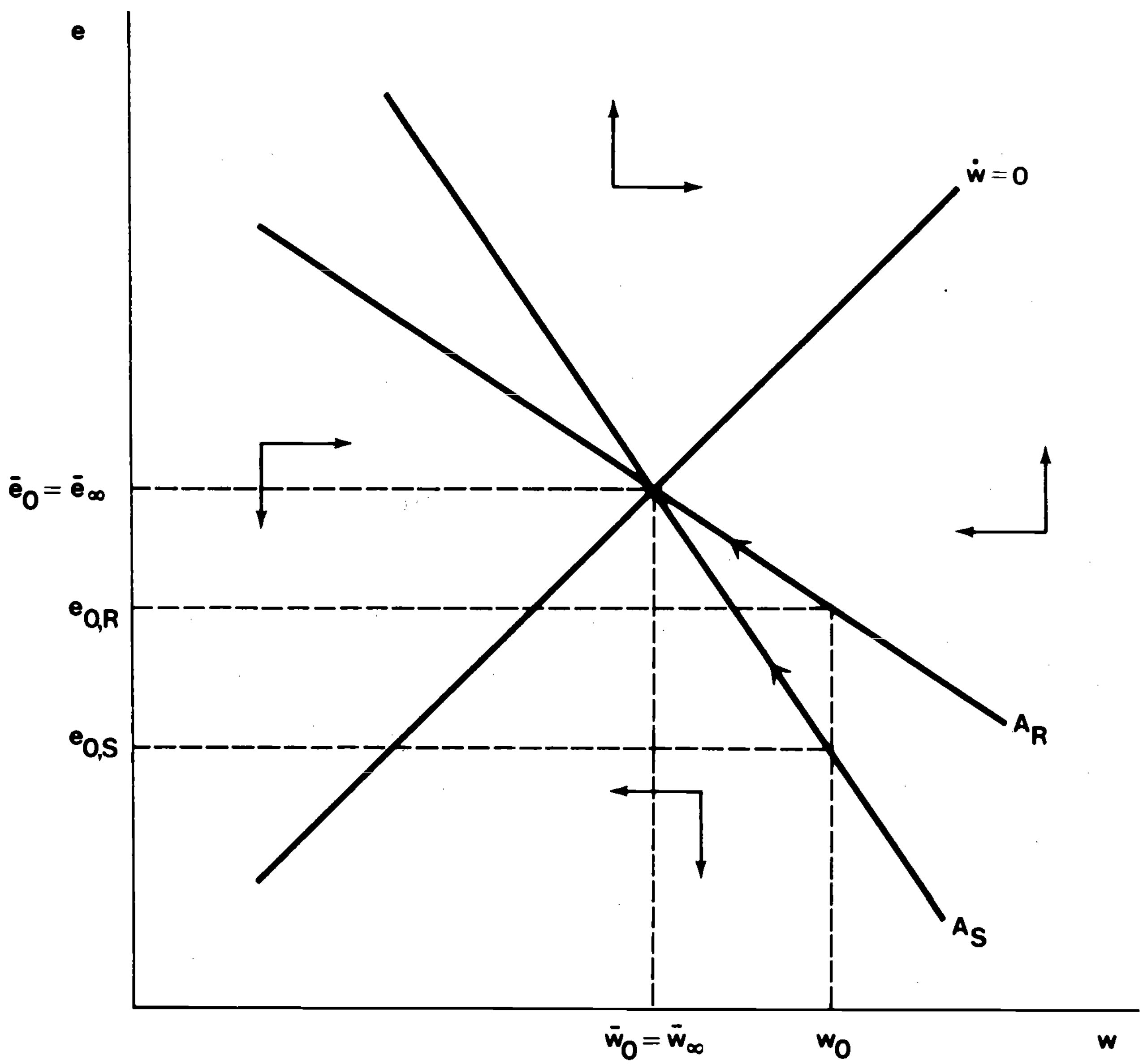


Figure 6

$$\varepsilon = \varepsilon_e' de + \varepsilon_w' dw - \varepsilon_{OMO}' d\hat{M} \qquad (2.25)$$

$$\varepsilon_e' = -(1/\hat{b}_\varepsilon)\{\hat{b}_e - [(\hat{b}_i \hat{m}_p - \hat{b}_p \hat{m}_i)/\hat{m}_i]C_1 + (\hat{b}_i^* \hat{n}_p^* / \hat{n}_i^*)C_3\} > 0,$$

$$\varepsilon_w' = -(1/\hat{b}_\varepsilon)\{\hat{b}_w - [(\hat{b}_i \hat{m}_p - \hat{b}_p \hat{m}_i)/\hat{m}_i]C_2 + (\hat{b}_i^* \hat{n}_p^* / \hat{n}_i^*)C_4\} > 0,$$

$$\varepsilon_{OMO}' = (\hat{b}_i + \hat{m}_i)/\hat{m}_i \hat{b}_\varepsilon > 0.$$

The coefficient $-\varepsilon_{OMO}'$ gives the effect of an expansionary open market operation ($d\hat{M} = -d\hat{B} > 0$) on ε.

The A_S schedule in Figure 6 is the asset market equilibrium schedule under static expectations. It represents the pairs of e and w that are compatible with asset market equilibrium given that ε is equal to zero. If there is either local asset preference or local good preference, the A_S schedule slopes downward. First consider the effect of an increase in w. If there is local asset preference, this increase in w raises the demand for home securities directly. If there is local good preference, the increase in w raises P and lowers $\overset{*}{P}$ as shown in subsection 2.4. The net effect of these induced price changes is to increase the demand for home securities as shown in subsection 2.3. Thus, either local asset preference or local good preference is a sufficient condition for an increase in w to raise the demand for home securities. Now consider the effect of an appreciation of the home currency, that is, a decrease in e. This decrease lowers the demand for home securities directly. It also lowers P and raises $\overset{*}{P}$ as shown in subsection 2.4. The net effect of these induced price changes is to lower the demand for home securities. Thus, a decrease in e unambiguously lowers the demand for home securities. If there is neither local asset preference nor local good preference, the A_S schedule is horizontal.

The A_R schedule is the asset market equilibrium schedule under rational expectations given that exchange rate expectations are compatible with the stability of long-run equilibrium. Under rational expectations, the A_S schedule is just the $\varepsilon = \dot{e} = 0$ schedule. The vertical arrows show the direction of motion of e. Above A_S there is excess demand for home assets with $\dot{e} = 0$, so the home currency must be expected to depreciate. Long-run equilibrium is a saddle point under rational expectations as indicated by the arrows.[21/] A_R is the unique saddle path along which the economy must move followng a disturbance in order to reach long-run equilibrium.

The qualitative effects of a transfer of wealth to home residents on e and w when there is local asset preference are the same whether or not there is local good preference. These effects are shown in Figure 6. This disturbance does not affect either of the schedules. Under both static and rational expectations the home currency appreciates ($e_{0,S} < e_{0,R} < \bar{e}_0$), and the home country begins to run a trade deficit. The economy moves along either A_S or A_R back to long-run equilibrium.

The qualitative effects of a contractionary open market operation on e and w when there is local asset preference are also the same whether or not there is local good preference. These effects are shown in Figure 7. The A_S and A_R schedules are shifted down to A_S' and A_R'. Under both static and rational expectations the home currency appreciates, and the

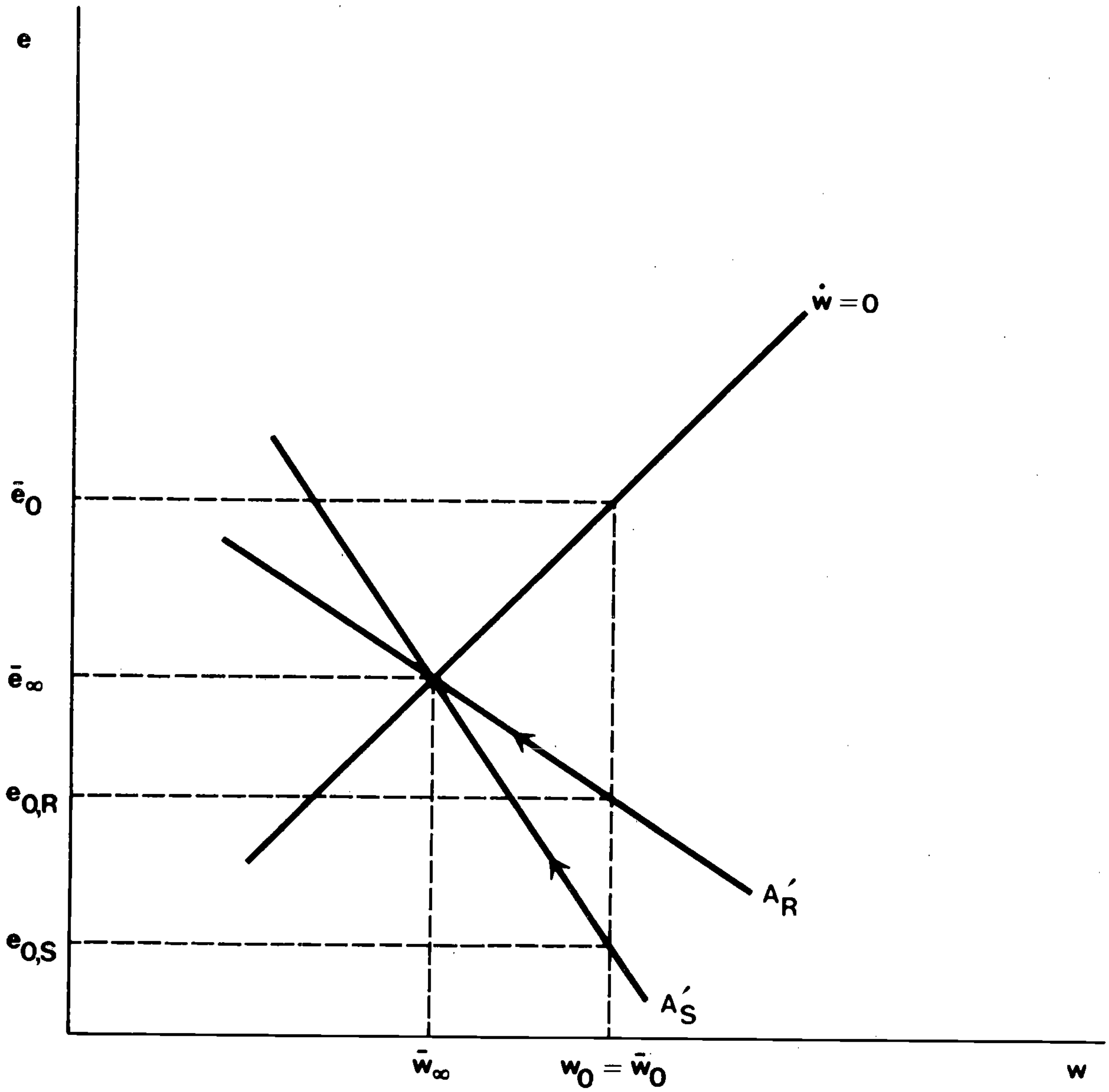


Figure 7

home country begins to run a trade deficit. In contrast to the results obtained with goods prices fixed both home wealth and the exchange rate are lower in the long-run equilibrium. Since asset accumulation now depends on w as well as e and since w has declined, e need not return all the way to its initial value in order to raise asset accumulation to zero.

While the qualitative behavior of the nominal exchange rate is the same with local asset preference whether or not there is local good preference, the behavior of the terms of trade or real exchange rate ($E\overset{*}{P}/P$) differs in the two cases. We illustrate this result with the case of a transfer of wealth. At the outset note that for this disturbance the real exchange rate is unaffected in the new long-run equilibrium. Then focus attention on the impact effects and the adjustment paths. First suppose that there is local good preference. It follows from equations (2.22a) and (2.22b) that an increase in w raises P and lowers $\overset{*}{P}$. It also follows from these equations that an appreciation of the nominal exchange rate causes an appreciation of the real exchange rate:

$$d(e + \overset{*}{p} - p)/de = (1/\Delta)(x_2\overset{*}{y}_2 - \overset{*}{x}_2y_2)(b_7 - \overset{*}{b}_7)W\overset{*}{W} > 0. \qquad (2.26)$$

As a result the impact effect of the transfer of wealth must be an appreciation of the real exchange rate, so the real exchange rate must depreciate along the adjustment path. Now suppose there is no local good preference. Increases in w and changes in the nominal exchange rate have no effect on the real exchange rate, so the real exchange rate remains unaffected by the transfer of wealth. This is a case of nominal exchange rate dynamics without real exchange rate dynamics.

The effects of a wealth transfer and an open market operation on e and w when there is local good preference but no local asset preference are shown in Figures 8 and 9. The only qualitative difference in the effects on e and w in this case is that the long-run equilibrium value of w is unchanged by an open market operation because asset accumulation is independent of the exchange rate.

An interesting special case arises when there is local good preference but no local asset preference and when open market operations are employed to peg nominal interest rates in both countries. In this special case movements in P and $\overset{*}{P}$ do not affect asset market equilibrium. The effects of a wealth transfer are shown in Figure 10 in which the A_S and A_R schedules are horizontal. A transfer of wealth has no effect on the nominal exchange rate. However, it follows from equations (2.22a) and (2.22b) that it does affect the real exchange rate. The impact effect of the transfer is to raise P and lower $\overset{*}{P}$. These variables return to their original values as home residents decumulate wealth. This is a case of real exchange rate dynamics without nominal exchange rate dynamics.

The effects of a wealth transfer and an open market operation when there is neither local asset preference nor local good preference are shown in Figures 10 and 11. The A_S and A_R schedules are horizontal even though interest rates are not pegged. There are neither nominal nor real exchange rate dynamics. A transfer of wealth has no effect on the exchange rate, but the home country begins to run a trade account deficit. An open market operation causes the home currency to appreciate immediately to its new long-run equilibrium value and has no effect on the trade surplus of the home country.

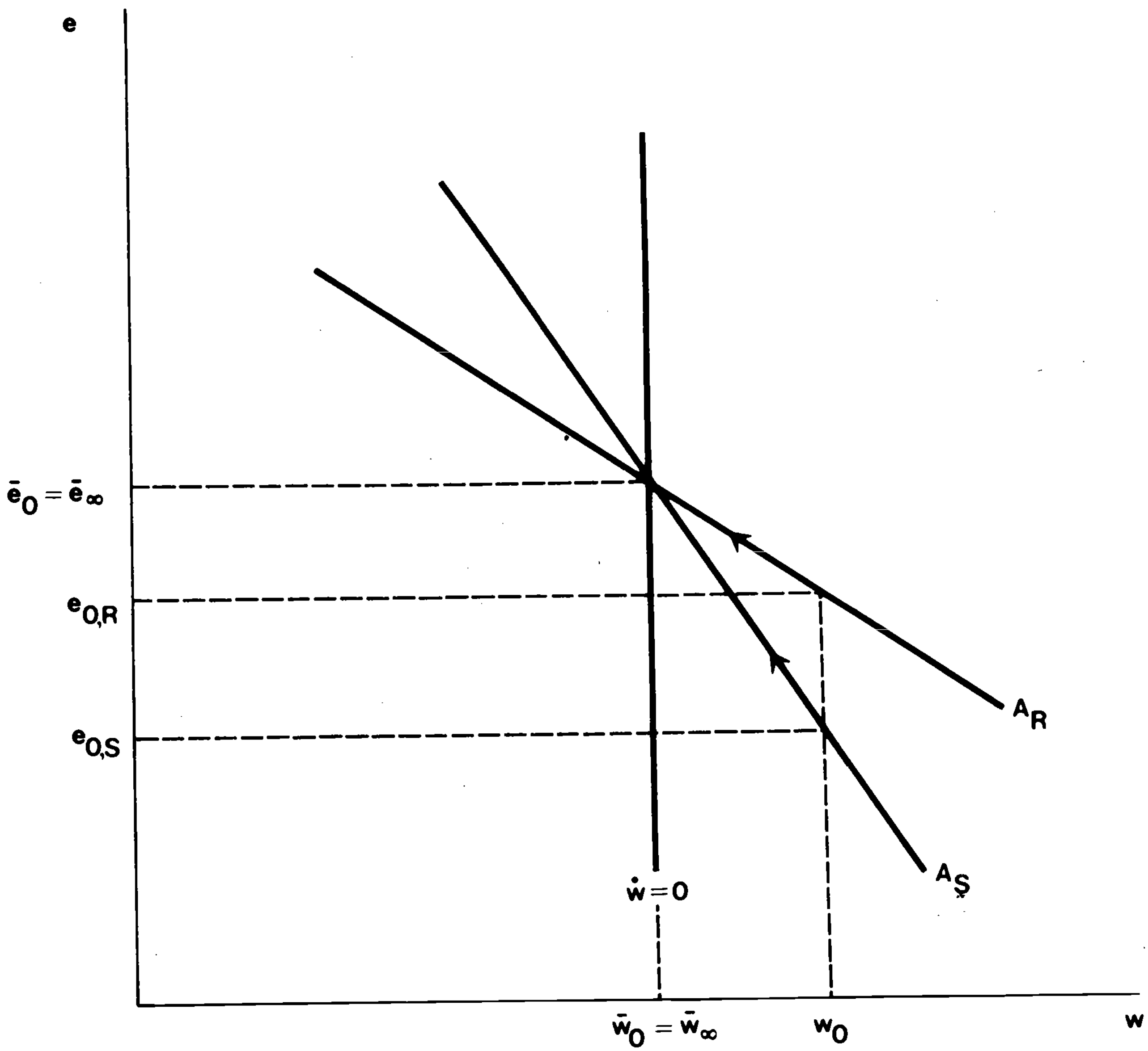


Figure 8

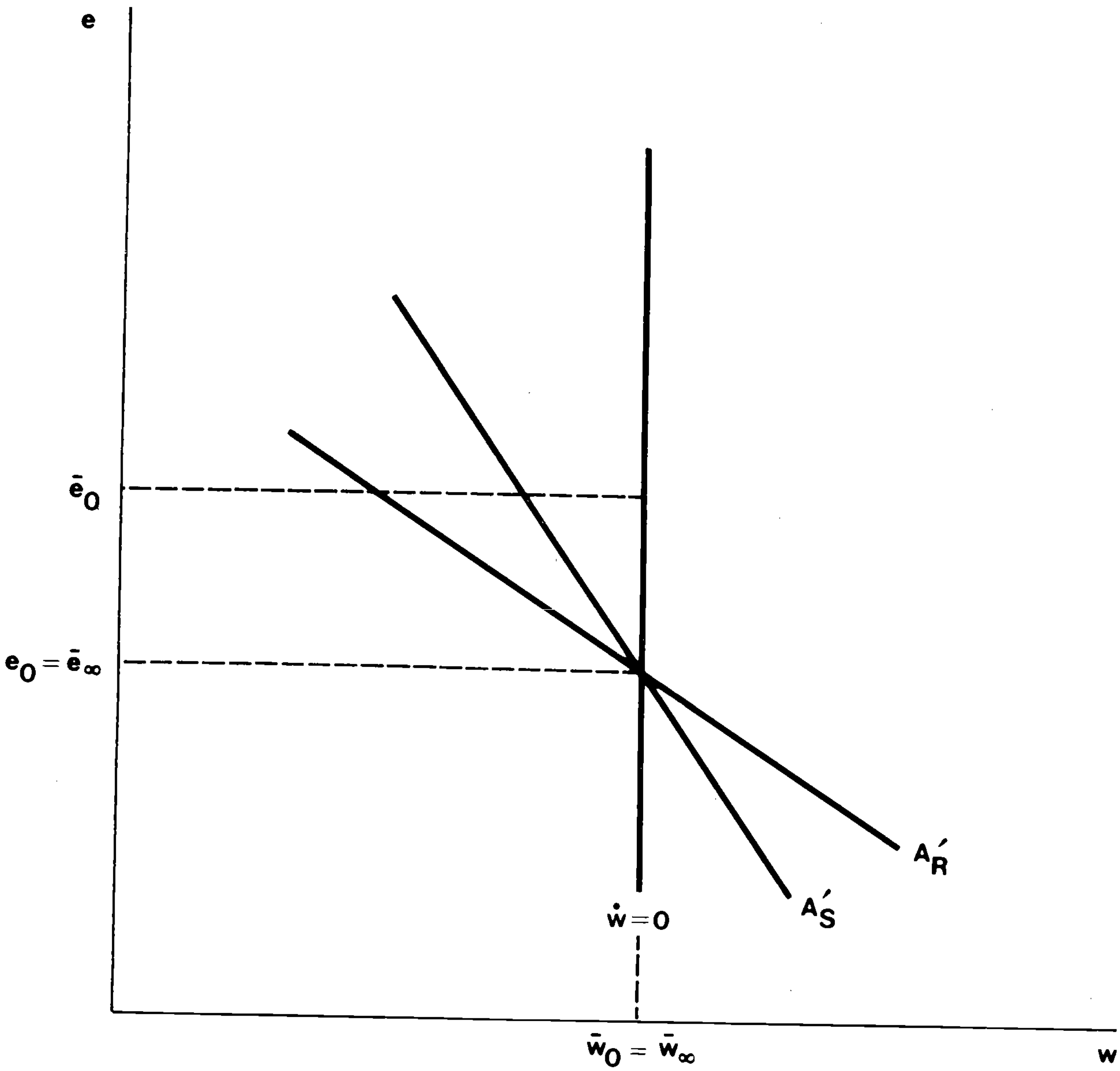


Figure 9

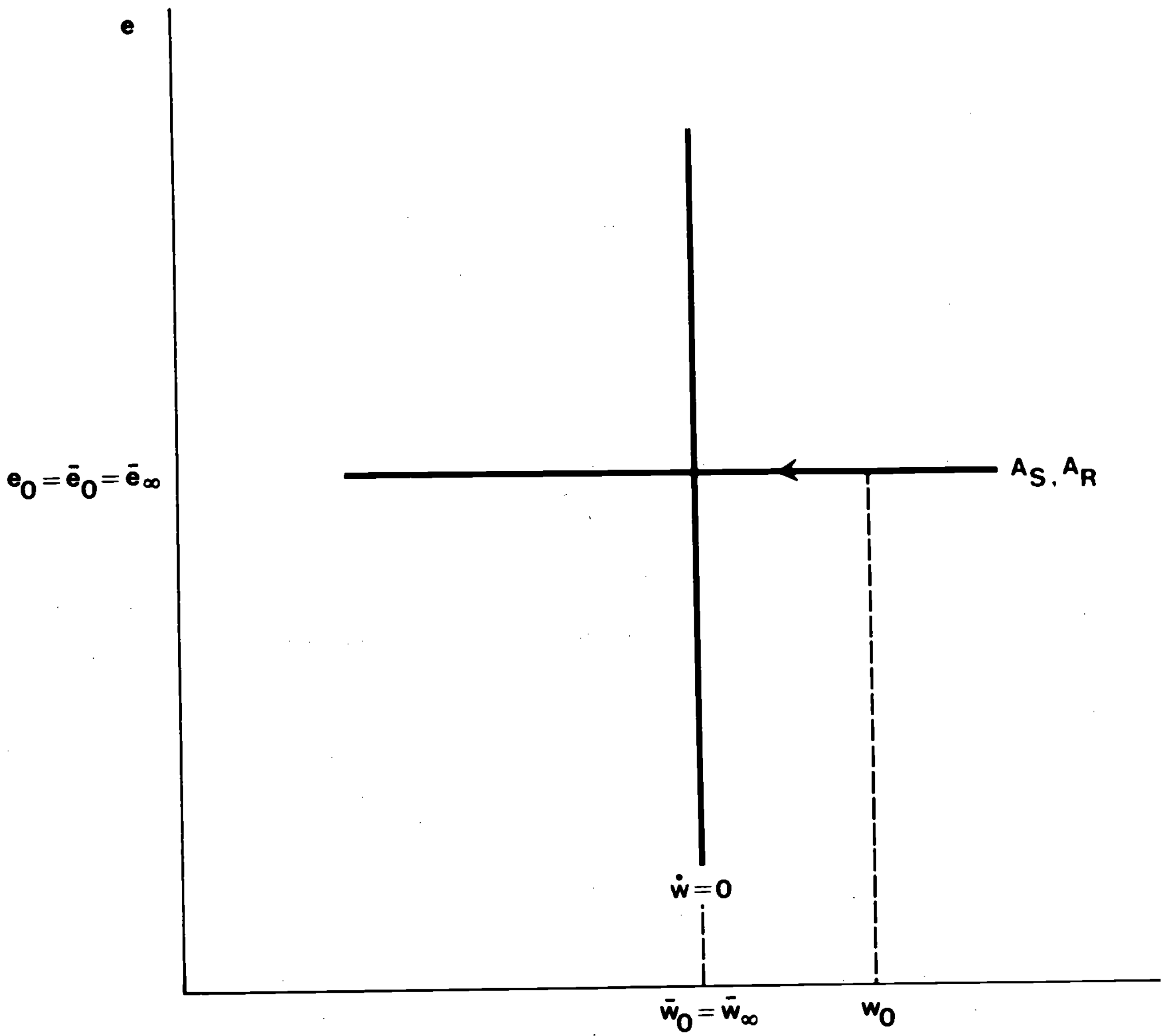


Figure 1(

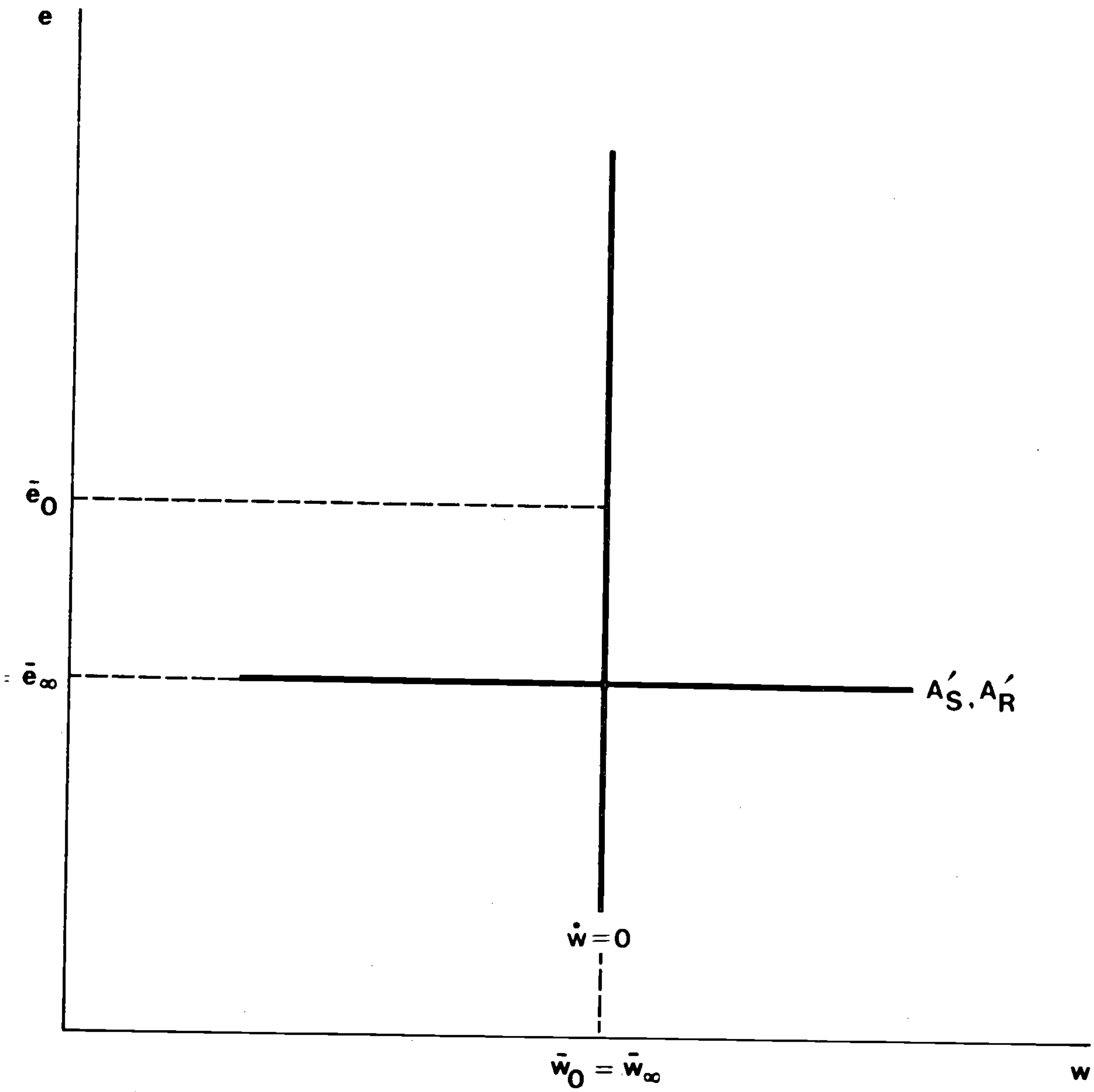


Figure 11

2.7. Negative net foreign asset positions and stability

A negative net foreign asset position is a net debt of residents of one country denominated in the currency of the other country (F or $\overset{*}{B} < 0$). Several writers have suggested that negative net foreign asset positions alone can be a source of dynamic instability.[22/] According to an alternative view presented in this subsection, negative net foreign asset positions are not an independent source of instability. Instability can arise only under nonrational expectations or because of destabilizing speculation.[23/]

The exposition is simplified by retaining the assumption of local asset preference $[(M + B)/W > \overset{*}{B}/\overset{*}{W}$ and, therefore, $(\overset{*}{N} + \overset{*}{F})/\overset{*}{W} > F/W]$. This assumption taken together with our other assumptions implies that only net foreign asset positions can be negative; net domestic asset positions are always positive ($M + B$, $\overset{*}{N} + \overset{*}{F} > 0$). W, $\overset{*}{W}$, $\hat{B}$, $\hat{F}$, $\hat{M} = M$, and $\hat{N} = N$ are all positive. If $\overset{*}{B}$ and F are positive, B and $\overset{*}{F}$ may be negative. However, if there is local asset preference, $M + B$ and $\overset{*}{N} + \overset{*}{F}$ must still be positive. If $\overset{*}{B}$ and F are negative, B and $\overset{*}{F}$ and, therefore, $M + B$ and $\overset{*}{N} + \overset{*}{F}$ are positive.

Suppose goods prices are fixed. The $\dot{w}$ and $\dot{w} = 0$ schedules are the same as those shown in Figure 3 whether or not there are negative net foreign asset positions because net foreign asset positions do not enter the balance of payments equation.

In contrast, the slope of the asset market equilibrium schedule under static expectations (A_S) may be different when there are negative net foreign asset positions. A_S is downward sloping as in Figure 3 in the "normal" case. In this case, a depreciation of the home currency raises the value of ε required to clear the asset markets ($\varepsilon_e > 0$). As shown by (2.24), since $b_7 > \overset{*}{b}_7$, an increase in w raises the value of ε required to clear the asset markets ($\varepsilon_w > 0$). Thus, an increase in w must be matched by an appreciation of the home currency if the asset markets are to remain in equilibrium. However, the A_S schedule is upward sloping as in Figure 12 in the "perverse" case. In this case, a depreciation of the home currency lowers the value of ε required to clear the asset markets ($\varepsilon_e < 0$). An increase in w must be matched by a depreciation of the home currency if the asset markets are to remain in equilibrium.

The restriction required for the normal case ($\varepsilon_e > 0$) is always satisfied if there are no negative net foreign asset positions ($F, \overset{*}{B} > 0$). When F and $\overset{*}{B}$ are positive, a depreciation of the home currency unambiguously raises the demand for home securities ($\delta_e > 0$) because it raises home and foreign wealth. The restriction required for the normal case may be violated if either F or $\overset{*}{B}$ is negative and is definitely violated if both are negative. If F is negative, a depreciation lowers home residents' wealth and, therefore, reduces their demand for home securities. If $\overset{*}{B}$ is negative, a depreciation raises foreign residents' wealth. However, their demand for home securities falls as their wealth rises.

We investigate the stability of long-run equilibrium by analyzing an unanticipated transfer of wealth from foreign residents to home residents. Long-run equilibrium is stable under static expectations in the normal case. The analysis of a wealth transfer in subsection 2.5,

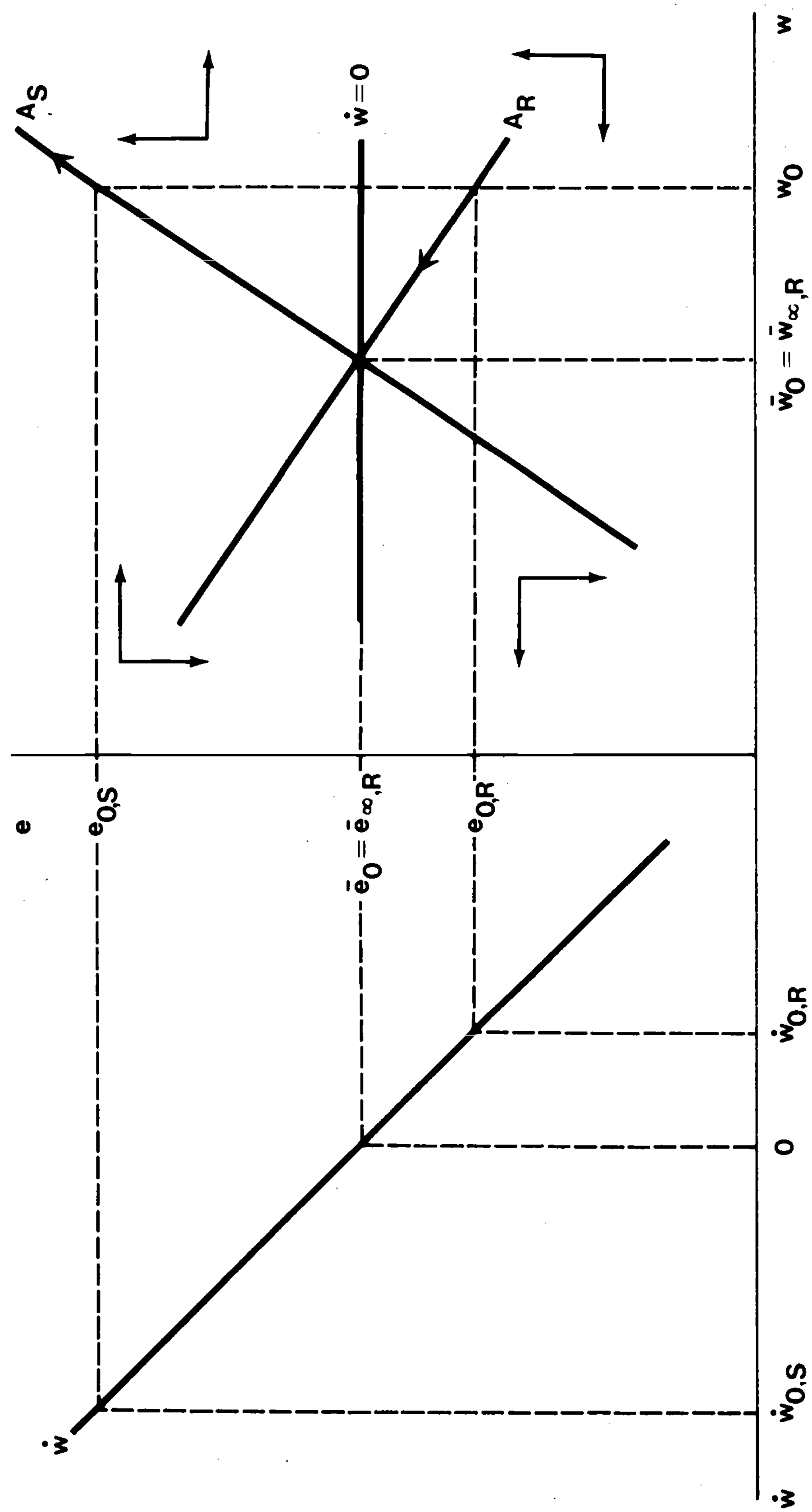

w
A_S
$\dot{w}=0$
A_R
w_O
$\bar{w}_O = \bar{w}_{\infty,R}$
e
$e_{O,S}$
$\bar{e}_O = \bar{e}_{\infty,R}$
$e_{O,R}$
$\dot{w}_{O,R}$
0
$\dot{w}_{O,S}$
$\dot{w}$
$\dot{w}$

Figure 12

which is summarized in Figure 3, applies without modification. In the normal case negative net foreign asset positions do not alter the qualitative effects of wealth transfers and exchange rate changes on ε. Long-run equilibrium is definitely unstable under static expectations in the perverse case as shown in Figure 12. The transfer of wealth from foreign residents to home residents ($w_0 > \bar{w}_0$) raises demand for home securities. In the perverse case, asset market equilibrium is restored by a depreciation ($e_{0,S} > \bar{e}_0$) rather than an appreciation of the home currency, and the home country begins to run a trade surplus ($\dot{w}_{0,S} > 0$). As w rises, the home currency depreciates further. The economy moves along A_S away from long-run equilibrium.[24/]

If speculation is stabilizing, long-run equilibrium is stable under rational expectations in both the normal and perverse cases.[25/] As the arrows in Figures 3 and 12 indicate, long-run equilibria are saddle points under rational expectations. What is remarkable is that this result holds not only in the normal case but also in the perverse case. It is usual to find that if long-run equilibrium is stable under static expectations, it is a saddle point under rational expectations. However, here long-run equilibrium is a saddle point under rational expectations even if it is unstable under static expectations.

Under rational expectations the exchange rate jumps to clear the asset markets, just as it did under static expectations. Following a transfer, the world economy reaches long-run equilibrium if and only if it moves along the unique saddle path represented by A_R in Figure 3 in the normal case and by A_R in Figure 12 in the perverse case. If the bidding of market participants causes the exchange rate to jump to $e_{0,R}$, the exchange rate on the A_R schedule corresponding to w_0, it will be said that speculation is stabilizing. When speculation is stabilizing, the home currency appreciates, and long-run equilibrium is stable no matter what the sign of ε_e. If the exchange rate remains unchanged at $\bar{e}_0$ or jumps to any value other than $e_{0,R}$, it will be said that speculation is destabilizing. When speculation is destabilizing, long-run equilibrium is unstable, as indicated by the arrows in Figures 3 and 12. Under rational expectations, instability can arise only because of destabilizing speculation and not because of perverse valuation effects associated with negative net foreign asset positions.26/

Now suppose goods prices are flexible. The $\dot{w} = 0$ schedule is upward sloping as in Figure 6 whether or not there are negative net foreign asset positions. An increase in w reduces the home trade surplus. If the trade surplus is to be restored to its previous level, the home currency must depreciate under our assumption of local asset preference.

With goods prices flexible, just as with goods prices fixed, the slope of the asset market equilibrium schedule under static expectations (A_S) may be different when there are negative net foreign asset positions. A_S is downward sloping as in Figure 6 in the normal case. In this case, a depreciation of the home currency raises the value of ε required to clear the asset markets ($\varepsilon_e' > 0$). As shown by (2.25), the net impact of the direct and indirect effects of an increase in w is to raise the value of ε required to clear the asset markets ($\varepsilon_w' > 0$). Thus, an increase in w must be matched by an appreciation of the home currency if the asset markets are to remain in equilibrium. However, the A_S schedule is upward sloping as in Figure 13 in the perverse case. In

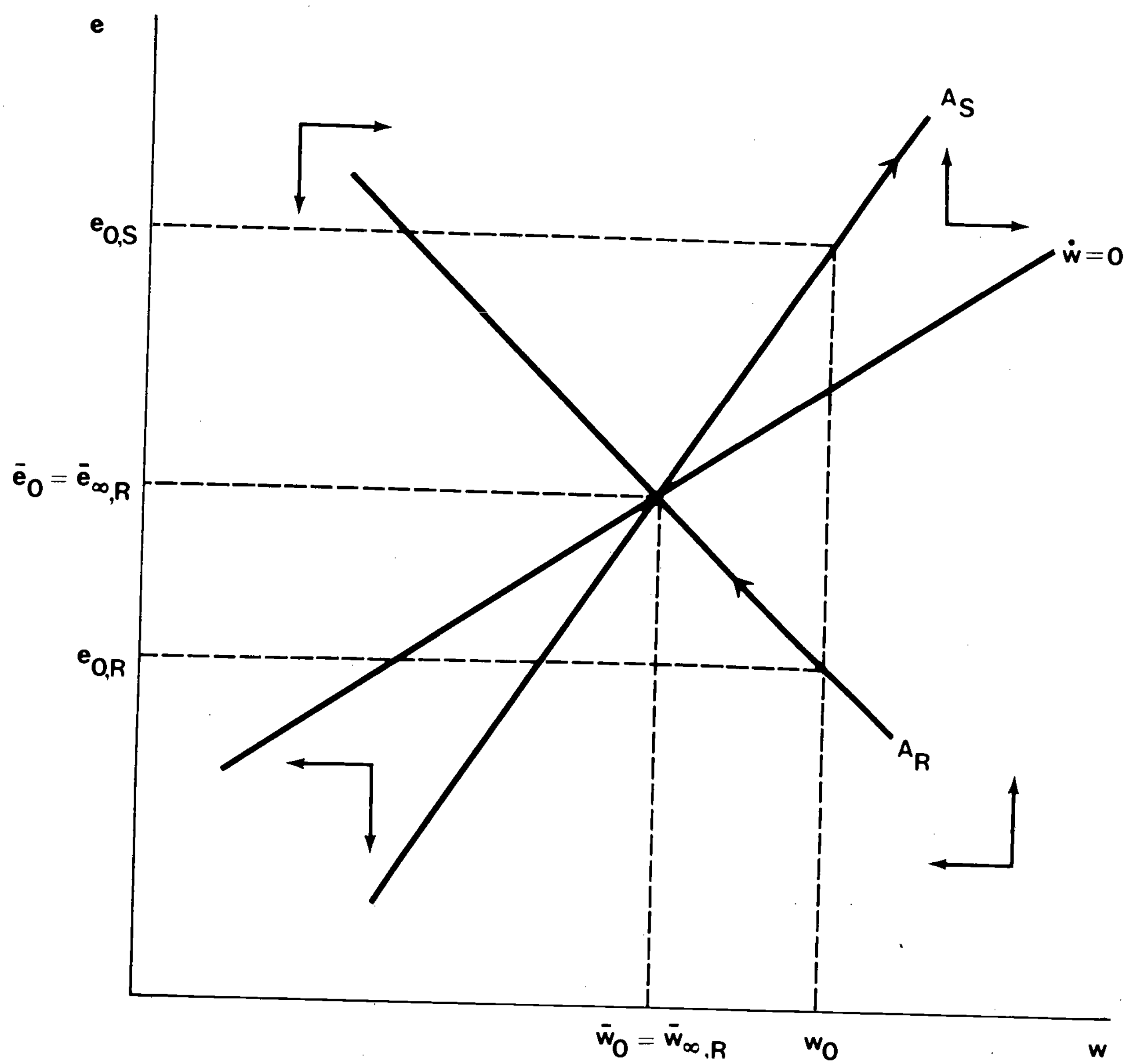


Figure 13

this case, a depreciation of the home currency lowers the value of ε required to clear the asset markets ($\varepsilon'_e < 0$). An increase in w must be matched by a depreciation of the home currency if the asset markets are to remain in equilibrium.

The restriction required for the normal case ($\varepsilon'_e > 0$) is always satisfied if there are no negative net foreign asset positions ($F, \overset{*}{B} > 0$). A depreciation of the home currency raises the demand for home securities directly by raising home and foreign wealth ($\delta_e > 0$) and indirectly by raising the price of home goods thereby raising the home interest rate and by lowering the price of foreign goods thereby lowering the foreign interest rate. The restriction required for the normal case may be violated if either F or $\overset{*}{B}$ is negative. It has been shown above that the direct effect of a depreciation on the demand for home securities may be perverse ($\delta_e < 0$) if F or $\overset{*}{B}$ is negative and is definitely perverse if both are negative. The indirect effects may also be perverse: the price of home goods may fall if F is negative; the price of foreign goods may rise if $\overset{*}{B}$ is negative. With goods prices fixed, a necessary and sufficient condition for the slope of A_S to be perverse is that the direct effect of a depreciation on the demand for home securities be perverse ($\delta_e < 0$). However, with goods prices flexible, $\delta_e < 0$ is neither a necessary nor a sufficient condition for the slope of A_S to be perverse. For example, suppose residents of both countries have negative net foreign asset positions ($F, \overset{*}{B} < 0$) and that there is no local good preference ($x_2 = \overset{*}{x}_2$ and $y_2 = \overset{*}{y}_2$). Under those circumstances the direct effect of a depreciation is perverse ($\delta_e < 0$), but the indirect effects are normal, so the overall effect is indeterminate.

The results of stability analysis with goods prices flexible are similar to those with goods prices fixed. Long-run equilibrium is stable under static expectations in the normal case. The analysis of a wealth transfer in subsection 2.6, which is summarized in Figure 6, applies without modification since in the normal case negative net foreign asset positions do not alter the qualitative effects of wealth transfers and exchange rate changes on ε. Long-run equilibrium is definitely unstable under static expectations in the perverse case as shown in Figure 13 because the A_S schedule is steeper than the $\dot{w} = 0$ schedule.[27/] The transfer of wealth to home residents $(w_0 > \bar{w}_0)$ leads to a depreciation of the home currency $(e_{0,S} > \bar{e}_0)$ and a trade surplus $(\dot{w}_{0,S} > 0)$, and the economy moves along A_S away from equilibrium.[28/] If speculation is

stabilizing, long-run equilibrium is stable under rational expectations in both the normal and perverse cases.[29/] As the arrows in Figures 6 and 13 indicate, long-run equilibria are saddle points under rational expectations. The unique saddle path is represented by A_R in Figure 6 in the normal case and by A_R in Figure 13 in the perverse case. When speculation is stabilizing, the exchange rate jumps to $e_{0,R}$. The home currency appreciates, and long-run equilibrium is stable no matter what the sign of ε_e'. When speculation is destabilizing, the exchange rate remains unchanged or jumps to some value other than $e_{0,R}$, and long-run equilibrium is unstable. No matter whether goods prices are fixed or flexible, under rational expectations instability can arise only because of destabilizing speculation and not because of perverse valuation effects associated with negative net foreign asset positions.[30/]

3. The Microeconomic foundations of asset demands in open economies

3.1. Overview

Demand equations for assets denominated in different currencies are based on the solution to a maximization problem faced by an individual investor. One specification of the problem is very common. The investor consumes a bundle of goods each of which is produced in a different country and priced in the currency of the country in which it is produced. In each currency denomination there is a security with a fixed nominal value and a certain nominal return. The investor has initial holdings of some or all of the securities and an uncertain stream of future labor income. Percentage changes in goods prices and exchange rates are assumed to follow "geometric Brownian motion." This assumption implies that successive percentage changes in these variables are independently distributed no matter how short the time interval and that the levels of the variables are log normally distributed. The investor maximizes the expected value of discounted lifetime utility.

In early analyses of portfolio selection in a closed economy, the specification of the investor's maximization problem was simplified by several assumptions.[31/] First, it was assumed that the portfolio allocation decision in each period was separable from the saving decision. Under this assumption the optimal portfolio rule could be obtained by maximizing the expected utility of return in each period. Second, no distinction was made between nominal and real returns because the price level was assumed to be fixed. Third, it was assumed either that uncertain asset returns were normally distributed or that utility was quadratic in portfolio return. Fourth, it was assumed that there was an asset with a known return, the "safe" asset. These assumptions yielded the classic portfolio separation results.

More recent analyses of portfolio selection and saving in the closed economy have employed the tools of stochastic calculus.[32/] An implicit solution for a general version of the investor's lifetime utility maximization problem has been obtained by applying the "Fundamental Theorem of Stochastic Dynamic Programming" and Ito's Lemma on stochastic differentials. In this general case the portfolio allocation problem is not separable from the saving decision and the classic portfolio separation results do not hold.

Recognizing the implications of some special assumptions in the more recent continuous time framework provides some perspective on earlier contributions. The assumption that the investor's instantaneous utility function exhibits constant relative risk aversion implies that the portfolio allocation decision is separable from the saving decision.[33/] It is comforting to know that the separability of these decisions, which was simply assumed in earlier contributions, is implied by a class of utility functions. The assumption that the percentage changes in asset prices follow geometric Brownian motion so that the prices themselves are log normally distributed implies the classic portfolio separation results.[34/] The very similar assumption that percentage returns are normally distributed yields these separation results in the earlier analyses.

The investor in the open economy must take account of both exchange rate and price index uncertainty.[35/] Although a foreign security has a certain nominal return denominated in foreign currency, its nominal return in home currency is uncertain. Uncertainty about real returns arises not only because future values of exchanges rate are unknown but also because future values of the price index used to deflate nominal wealth are unknown. Exchange rate and price index changes are related in general, and the covariance between nominal returns inclusive of exchange rate changes and price index changes plays an important role in portfolio choice in an open economy. Changing the stochastic specification of the price index can have significant effects on this important covariance.

We begin by laying out a basic model with two assets, a home security and a foreign security, in subsection 3.2. The implications of a popular specification of the price index are spelled out in subsection 3.3. In subsection 3.4 we show the effect of imposing relative purchasing power parity on this popular specification, and in section 3.5 we trace out some consequences of violating the law of one price. Subsection 3.6 contains a three asset model that is generalized in subsection 3.7. Finally, in subsection 3.8 we illustrate the integration of money into the open economy portfolio allocation problem.

3.2. Asset demands in a two asset model with the exchange rate and the home price index stochastic

Analysis of demands for assets denominated in different currencies with the tools of stochastic calculus has usually proceeded under two simplifying assumptions. First, it has been assumed that percentage changes in prices follow geometric Brownian motion. Second, it has been assumed that the instantaneous utility function exhibits constant relative risk aversion [$U(\hat{C}) = (1/\gamma)\hat{C}^{\gamma}$, where $\hat{C}$ is real consumption, and $\gamma < 1$]. Under these assumptions, the solution for optimal wealth allocation is the same as the one implied by maximization of an objective function that is linear in expected return and variance of return. Thus, the consumer can be viewed as deciding on the allocation of his wealth by maximizing the objective function

$$V = E(d\hat{W}/\hat{W}) - (1/2)R[var(d\hat{W}/\hat{W})]. \tag{3.1}$$

$\hat{W}$ is real wealth, and R is the coefficient of relative risk aversion $[-\hat{C}U''(\hat{C})/U'(\hat{C}) = 1 - \gamma]$.

In the two asset model, a home resident allocates a fraction λ of his nominal wealth W to foreign (currency) securities F and the remaining fraction $1 - \lambda$ to home (currency) securities B:

$$\lambda W = EF, \tag{3.2a}$$

$$(1 - \lambda)W = B. \tag{3.2b}$$

The exchange rate E is the home currency price of foreign currency. Home and foreign securities are short bonds and have certain nominal returns represented by i and $\overset{*}{i}$ respectively:[36/]

$$dB/B = idt, \tag{3.3a}$$

$$dF/F = \overset{*}{i}dt. \tag{3.3b}$$

Real wealth $\hat{W}$ is nominal wealth deflated by the relevant price index Q:

$$\hat{W} = W/Q = (B + EF)/Q \tag{3.4}$$

Below we will discuss the alternative assumptions about the stochastic properties of Q that have been made by different authors. For what follows it is useful to note that equations (3.2) and equation (3.4) imply that

$$1/\hat{W} = Q/W = \lambda Q/EF = (1 - \lambda)Q/B. \tag{3.5}$$

We begin by postulating stochastic processes for E and Q:37/

$$dE/E = \varepsilon dt + \sigma_e dz_e, \tag{3.6}$$

$$dQ/Q = \pi_q dt + \sigma_q dz_q. \tag{3.7}$$

ε and π_q are the means and σ_e^2 and σ_q^2 are the variances of the stochastic processes. z_e and z_q are standard normal random variables, so dz_e and dz_q are Wiener processes or Brownian motion often referred to in the literature as "Gaussian white noise." The covariance between the stochastic processes is denoted by ρ_{qe}. The investor's objective function depends on the mean and variance of the stochastic process followed by the percentage change in real wealth $d\hat{W}/\hat{W}$. In order to find $d\hat{W}/\hat{W}$, we make use of Ito's Lemma. Let $H = J(K_1,\ldots, K_n,t)$ be a twice continuously differentiable function defined on $R^n X[0,\infty)$. Suppose the K_i follow geometric Brownian motion:

$$dK_i/K_i = \pi_i dt + \sigma_i dz_i, \quad i = 1,\ldots,n. \tag{3.8}$$

According to Ito's Lemma the stochastic differential of H is given by

$$dH \equiv \sum_i(\partial J/\partial K_i)dK_i + (\partial J/\partial t)dt + (1/2)\sum_i\sum_j(\partial^2 J/\partial K_i\partial K_j)dK_i dK_j, \tag{3.9}$$

and the product $dK_i dK_j$ is defined by

$$dz_i dz_j = r_{ij}dt, \quad i,j = 1,\ldots,n, \tag{3.10a}$$

$$dz_i dt = 0, \quad i = 1,\ldots,n, \tag{3.10b}$$

where r_{ij} is the instantaneous correlation coefficient between the Wiener processes dz_i and dz_j.38/

The stochastic differential of real wealth $d\hat{W}$ is derived from the expression for real wealth $(B + EF)/Q$ in equation (3.4). $d\hat{W}$ is equal to the conventional first differential of this expression plus one half times the conventional second differential:

$$\begin{aligned} d\hat{W} = {} & (1/Q)dB + (E/Q)dF + (F/Q)dE - (W/Q^2)dQ \\ & + (1/2)[-(1/Q^2)dQdB + (1/Q)dEdF - (E/Q^2)dQdF \\ & + (1/Q)dFdE - (F/Q^2)dQdE \\ & - (1/Q^2)dBdQ - (E/Q^2)dFdQ - (F/Q^2)dEdQ + (2W/Q^3)dQ^2]. \end{aligned} \tag{3.11}$$

Note that $\hat{W}$ is not explicitly dependent on time, so there is no dt in the stochastic differential.

Multiplying equation (3.11) by $1/\hat{W}$, taking account of the relationships in (3.5), and combining terms yields an expression for $d\hat{W}/\hat{W}$:

$$d\hat{W}/\hat{W} = (1 - \lambda)dB/B + \lambda dF/F + \lambda dE/E - dQ/Q$$

$$+ (1/2)[- 2(1 - \lambda)(dQ/Q)(dB/B) - 2\lambda(dQ/Q)(dF/F)$$

$$- 2\lambda(dE/E)(dF/F) - 2\lambda(dQ/Q)(dE/E) + 2(dQ/Q)^2]. \tag{3.12}$$

Application of Ito's Lemma to the products of the stochastic processes yields

$$(dQ/Q)(dB/B) = 0, \qquad (dQ/Q)(dF/F) = 0, \quad (dE/E)(dF/F) = 0, \tag{3.13a}$$

$$(dQ/Q)(dE/E) = \rho_{qe}dt, \quad (dQ/Q)^2 = \rho_{qq}dt, \tag{3.13b}$$

ρ_{ij} is defined as $\sigma_i\sigma_j r_{ij}$ is the covariance.

The following example shows how the terms in (3.13) follow from Ito's Lemma. The product of (3.6) and (3.7) is

$$(dQ/Q)(dE/E) = \pi_q\varepsilon dt^2 + \pi_q\sigma_e dtdz_e + \varepsilon\sigma_q dtdz_q + \sigma_q\sigma_e dz_q dz_e. \tag{3.14}$$

The first term on the right hand side is second order of magnitude, approximately zero. The product $dtdz_i$ is zero because dz_i is white noise. Therefore, the second and third terms disappear. Since the variance of a continuous time process is proportional to time, the standard deviation term dz_i is of the order of magnitude of the square root of dt.[39/] Therefore, the last term becomes

$$\sigma_p\sigma_e dz_p dz_e = \sigma_p\sigma_e r_{pe} dt = \sigma_{pe} dt.$$

Thus (3.14) reduces to the expression for (dQ/Q)(dE/E) in (3.13b).

The final expression for $d\hat{W}/\hat{W}$ is obtained by substituting equations (3.3), (3.6), (3.7), and (3.13) into equation (3.12):

$$d\hat{W}/\hat{W} = [(1 - \lambda)i + \lambda\overset{*}{i} + \lambda\varepsilon - \pi_q - \lambda\rho_{qe} + \sigma_q^2]dt$$

$$+ \lambda\sigma_e dz_e - \sigma_q dz_q. \tag{3.15}$$

The expected value of $d\hat{W}/\hat{W}$ is given by the coefficient of dt since the expected value of the dz_i terms is zero:

$$E(d\hat{W}/\hat{W}) = (1 - \lambda)i + \lambda\overset{*}{i} + \lambda\varepsilon - \pi_q - \lambda\rho_{qe} + \sigma_q^2. \tag{3.16}$$

Using Ito's Lemma to evaluate $(d\hat{W}/\hat{W})^2$ yields

$$(d\hat{W}/\hat{W})^2 = (\lambda^2\sigma_e^2 - 2\lambda\rho_{qe} + \sigma_q^2)dt. \tag{3.17}$$

The variance of $d\hat{W}/\hat{W}$ is the coefficient of dt in (3.17):

$$\mathrm{var}(d\hat{W}/\hat{W}) = \lambda^2\sigma_e^2 - 2\lambda\rho_{qe} + \sigma_q^2. \tag{3.18}$$

The home consumer maximizes his objective function

$$V = E(d\hat{W}/\hat{W}) - (1/2)R[\mathrm{var}(d\hat{W}/\hat{W})] \tag{3.1}$$

with respect to his choice variable λ, the share of foreign securities in his portfolio. The optimal portfolio rule is

$$\lambda = (1/R\sigma_e^2)[\overset{*}{i} + \varepsilon - i + (R - 1)\rho_{qe}]. \tag{3.19}$$

The home investor's demands for foreign and home securities are given by equations (3.2) which are repeated here for convenience:

$$EF = \lambda W, \tag{3.2a}$$

$$B = (1 - \lambda)W, \tag{3.2b}$$

where λ is given by equation (3.18). The partial derivatives of these demands for securities with respect to the expected return differential in favor of foreign securities are

$$\partial EF/\partial(\overset{*}{i} + \varepsilon - i) = W/R\sigma_e^2 = - \partial B/\partial(\overset{*}{i} + \varepsilon - i). \tag{3.20}$$

As risk aversion or the variance of the exchange rate increases, demands for securities become less sensitive to changes in the expected return differential

In a model with two securities, the securities must be gross substitutes:

$$\partial EF/\partial(\overset{*}{i} + \varepsilon) = W/R\sigma_e^2 = - \partial B/\partial(\overset{*}{i} + \varepsilon), \tag{3.21a}$$

$$\partial EF/\partial i = - W/R\sigma_e^2 = - \partial B/\partial i. \tag{3.21b}$$

However, we show below that in a model with three securities, the securities need not be gross substitutes.

3.3. Implications of a popular specification of the home price index

Additional results can be obtained by assuming a particular specification of the home price index Q:40/

$$Q = P^{1-\beta}(E\overset{*}{P})^{\beta}. \tag{3.22}$$

P is the home currency price of home goods. $\overset{*}{P}$ is the foreign currency price of foreign goods. The "law of one price" holds, so the domestic currency price of foreign goods is $E\overset{*}{P}$. β is the share of expenditure devoted to foreign goods.

The exchange rate follows the stochastic process (3.6), and the prices of both goods follow geometric Brownian motion:41/

$$dP/P = \pi_p dt + \sigma_p dz_p, \tag{3.23a}$$

$$d\overset{*}{P}/\overset{*}{P} = \pi_p^* dt + \sigma_p^* dz_p^*. \tag{3.23b}$$

π_p and π_p^* are the means and σ_p^2 and σ_p^{*2} are the variances of the price processes. dz_p and dz_p^* are Brownian motion. The covariance between the two price processes is ρ_{pp^*}. The covariances of the exchange rate process with the two price processes are ρ_{pe} and ρ_{pe}^*. We note here in passing that equations (3.6) and (3.22) imply that purchasing power parity ($P = E\overset{*}{P}$) does not hold in general. We return to this point in subsection 3.4.

In subsection 3.2 we specified a stochastic process for the domestic price index Q. In this subsection the stochastic process for Q is implied by the specification of the price index given by equation (3.21) and the stochastic processes for E, P, and $\overset{*}{P}$ given by equations (3.6), (3.22), and (3.23) respectively. In subsection 3.2 we showed that the only parameter of the stochastic process for Q that enters the optimal portfolio rule is the covariance of this process with the stochastic process for the exchange rate ρ_{qe}. The expression for ρ_{qe} implied by equations (3.21), (3.6), (3.22), and (3.23) is obtained by applying Ito's Lemma twice. First, it is used to find dQ/Q. Then, it is employed to evaluate the product (dQ/Q)(dE/E):[42/]

$$(dQ/Q)(dE/E) = [(1 - \beta)\rho_{pe} + \beta\sigma_e^2 + \beta\rho_{pe}^*]dt. \tag{3.24}$$

That is,

$$\rho_{qe} = (1 - \beta)\rho_{pe} + \beta\sigma_e^2 + \beta\rho_{pe}^*. \tag{3.25}$$

The specification of the home price index given by equation (3.21) implies that the optimal portfolio rule depends on the share of expenditure devoted to the foreign good. Substituting (3.25) into (3.19) yields:

$$\lambda = (1/R\sigma_e^2)\{i^* + \varepsilon - i + (R - 1)[(1 - \beta)\rho_{pe} + \beta\sigma_e^2 + \beta\rho_{pe}^*]\}. \quad (3.26)$$

In section 2 it was shown that the properties of portfolio balance models with postulated asset demands depend critically on whether there is local asset preference. It seems clear that there is local asset preference in most countries. A widely accepted explanation for local asset preference is that foreigners allocate a larger share of their portfolios to foreign assets than home residents because they devote a larger share of their expenditure to foreign goods. Therefore, it is interesting to ask whether the portfolio rule of equation (3.26) implies a positive association between the share of wealth devoted to foreign securities λ and the share of expenditure devoted to foreign goods β.

It turns out that λ does not necessarily rise when β increases. The derivative of λ with respect to β can be written as

$$\partial\lambda/\partial\beta = (\partial\lambda/\partial\rho_{qe})(\partial\rho_{qe}/\partial\beta), \quad (3.27)$$

where

$$\partial\lambda/\partial\rho_{qe} = (R - 1)/R\sigma_e^2, \quad (3.28)$$

$$\partial\rho_{qe}/\partial\beta = -\rho_{pe} + \sigma_e^2 + \rho_{pe}^*. \quad (3.29)$$

An increase in the correlation between the price index and the exchange rate ρ_{qe} raises λ if and only if the coefficient of relative risk aversion R is greater than one. If E is the only stochastic variable so that $\rho_{pe} = \rho^*_{pe} = 0$, then an increase in β definitely raises ρ_{qe}. In this case, $R > 1$ is a necessary and sufficient condition for an increase in β to raise λ.[43/] If E, P and $\overset{*}{P}$ are all stochastic variables, the analysis is somewhat more complicated. In this case, $R > 1$ implies that an increase in β raises λ if and only if an increase in β raises ρ_{qe}. Presumably $\rho_{pe} > 0$ and $\rho^*_{pe} < 0$, so $\partial\rho_{qe}/\partial\beta > 0$ if and only if the exchange rate variance σ^2_e is larger than the sum of the absolute values of the covariances of the exchange rate with the two prices.[44/] The result that $\partial\lambda/\partial\rho_{qe} > 0$ if and only if $R > 1$ arises because real wealth is the ratio of two stochastic variables, nominal wealth and the price index. Applying Ito's Lemma to this ratio yields an expression for the mean of the percentage change in wealth in equation (3.16) which includes $- \lambda\rho_{qe}$. Therefore $- \rho_{qe}$ is included in the numerator of the portfolio rule.[45/]

The portfolio rule (3.19) can be rewritten in two intuitively appealing forms whenever the exchange rate and the price index follow geometric Brownian motion. This rule can be rewritten in a third intuitively appealing form in the special case of the popular specification of the price index in (3.22).

The optimal portfolio rule (3.19) can be viewed as a weighted average of the minimum variance portfolio rule and the logarithmic or "international investor's" portfolio rule.[46/] The minimum variance portfolio rule (λ_M) is obtained by minimizing (3.17) with respect to λ:

$$\lambda_M = \rho_{qe}/\sigma_e^2. \tag{3.30}$$

If the investor's utility function is logarithmic $[U(\hat{C}) = \ln\hat{C}]$, then $R = 1$. The logarithmic portfolio rule (λ_L) is obtained by setting $R = 1$ in equation (3.26):

$$\lambda_L = (1/\sigma_e^2)(\overset{*}{i} + \varepsilon - i). \tag{3.31}$$

This rule has often been referred to as the international investor's portfolio rule because it is independent of expenditure shares. The optimal portfolio rule can be written as a weighted average of λ_M and λ_L:

$$\lambda = [(R - 1)/R](\rho_{qe}/\sigma_e^2) + (1/R)[(1/\sigma_e^2)(\overset{*}{i} + \varepsilon - i)]. \tag{3.32}$$

As the coefficient of relative risk aversion R approaches infinity the optimal rule approaches the minimum variance rule. As R approaches one the optimal rule approaches the logarithmic rule. The covariance term ρ_{qe} enters only through the minimum variance portfolio, and the return differential enters only through the logarithmic portfolio.

The optimal portfolio can also be written as the sum of the minimum variance portfolio and a zero net worth "speculative" portfolio.[47/] Writing the shares of the optimal portfolio in terms of deviations from the shares of minimum variance portfolio yields

$$\lambda = \rho_{qe}/\sigma_e^2 + (1/R\sigma_e^2)(\overset{*}{i} + \varepsilon - i - \rho_{qe}), \tag{3.33a}$$

$$1 - \lambda = (1 - \rho_{qe}/\sigma_e^2) - (1/R\sigma_e^2)(\overset{*}{i} + \varepsilon - i - \rho_{qe}). \tag{3.33b}$$

The shares of the minimum variance portfolio, λ_M and $1 - \lambda_M$, sum to unity. Therefore, the shares of the speculative portfolio, λ_S and $- \lambda_S$ where

$$\lambda_S = (1/R\sigma_e^2)(\overset{*}{i} + \varepsilon - i - \rho_{qe}), \tag{3.34}$$

must sum to zero.

Finally, in the special case of the popular specification of the price index in (3.22), the optimal portfolio can be written as the sum of an "expenditure share" portfolio and two zero net worth portfolios. In this case ρ_{qe} is given by (3.25). Therefore, the minimum variance portfolio can be written as the sum of the expenditure share portfolio and a zero net worth "hedge" portfolio:

$$\lambda_M = \beta + (1/\sigma_e^2)[(1 - \beta)\rho_{pe} + \beta\rho^*_{pe}], \tag{3.35a}$$

$$1 - \lambda_M = 1 - \beta - (1/\sigma_e^2)[(1 - \beta)\rho_{pe} + \beta\rho^*_{pe}]. \tag{3.35b}$$

The expenditure shares sum to unity. Therefore, the shares of the hedge portfolio, λ_H and $- \lambda_H$ where

$$\lambda_H = (1/\sigma_e^2)[(1 - \beta)\rho_{pe} + \beta\rho^*_{pe}], \tag{3.36}$$

must sum to zero. Note that if E is the only stochastic variable so that $\rho_{pe} = \rho^*_{pe} = 0$, then the minimum variance shares are simply the expenditure shares. If E, P, and $\overset{*}{P}$ are stochastic, the minimum variance shares deviate from the expenditure shares when exchange rate changes are associated with changes in goods prices. Substituting equations (3.35) into equations (3.33) confirms that the optimal portfolio can be written as the sum of the expenditure share portfolio, the hedge portfolio, and the speculative portfolio. Of course, the hedge portfolio and the speculative portfolio could be added together so that the optimal portfolio could be expressed as the sum of the expenditure share portfolio and a single zero net worth portfolio.

3.4. Implications of relative purchasing power parity

When separate stochastic processes are specified for E, P, and $\overset{*}{P}$ as in subsection 3.3, the relative price of foreign goods ($E\overset{*}{P}/P$) is free to vary. Here we explore the implications of assuming that relative purchasing power parity holds, that is, that the relative price of the foreign good is constant ($E\overset{*}{P}/P = k$, so $E = kP/\overset{*}{P}$). Given relative PPP the stochastic differential of E is

$$dE/E = (dP/P) - (d\overset{*}{P}/\overset{*}{P}) + (d\overset{*}{P}/\overset{*}{P})^2 - (dP/P)(d\overset{*}{P}/\overset{*}{P}). \tag{3.37}$$

Substituting in expressions for dP/P, $d\overset{*}{P}/\overset{*}{P}$, $(d\overset{*}{P}/\overset{*}{P})^2$, and $(dP/P)(d\overset{*}{P}/\overset{*}{P})$ obtained using (3.22), (3.23) and Ito's Lemma yields

$$dE/E = (\pi_p - \pi^*_p + \sigma^{*2}_p - \rho_{pp^*})dt + \sigma_p dz_p - \sigma^*_p dz^*_p. \tag{3.38}$$

Thus, the expected percentage change in the exchange rate is

$$\varepsilon = \pi_p - \pi_{p^*} + \sigma_{p^*}^2 - \rho_{pp^*}. \tag{3.39}$$

Evaluating $(dE/E)^2$, $(dP/P)(dE/E)$, and $(dP^*/P^*)(dE/E)$ yields

$$(dE/E)^2 = (\sigma_p^2 + \sigma_{p^*}^2 - 2\rho_{pp^*})dt, \tag{3.40a}$$

$$(dP/P)(dE/E) = (\sigma_p^2 - \rho_{pp^*})dt, \tag{3.40b}$$

$$(dP^*/P^*)(dE/E) = (-\ \sigma_{p^*}^2 + \rho_{pp^*})dt. \tag{3.40c}$$

Thus, the variance of the exchange rate and the covariances of the two prices with the exchange rate are

$$\sigma_e^2 = \sigma_p^2 + \sigma_{p^*}^2 - 2\rho_{pp^*}, \tag{3.41a}$$

$$\rho_{pe} = \sigma_p^2 - \rho_{pp^*}, \tag{3.41b}$$

$$\rho_{pe}^* = -\ \sigma_{p^*}^2 + \rho_{pp^*}. \tag{3.41c}$$

It has been argued that when relative PPP holds, the investor does not face exchange risk.[48/] It is true that ε, σ_e^2, ρ_{pe}, and ρ_{pe}^* can be eliminated from the optimal portfolio rule (3.26) with the use of the relationships in (3.39) and (3.41):

$$\lambda = [1/R(\sigma_p^2 + \sigma_p^{*2} - 2\rho_{pp^*})][i^* + \pi_p - \pi_p^* + \sigma_p^{*2} - \rho_{pp^*}$$

$$- i + (R - 1)(\sigma_p^2 - \rho_{pp^*})]. \qquad (3.42)$$

However, whether this observation confirms the view that the investor does not face exchange risk is a question of semantics. Other expressions for the optimal portfolio besides (3.42) are consistent with relative PPP. For example, using the relationships in (3.41) to solve for ρ_{pe}^* in terms of σ_e^2, σ_p^2, and ρ_{pe}; substituting the result into (3.26); and collecting terms yields

$$\lambda = (1/R\sigma_e^2)[i^* + \varepsilon - i + (R - 1)\rho_{pe}] \qquad (3.43)$$

in which π_p^*, σ_p^{*2}, ρ_{pe}^*, and ρ_{pp^*} do not appear. All that relative PPP implies is that E, P, and P^* are tied together so that specification of stochastic processes for any two of the three variables implies a stochastic process for the third.

3.5. Price index - exchange rate covariance and the law of one price

The optimal portfolio rule depends on the covariance between the price index and the exchange rate ρ_{qe} unless the coefficient of relative risk aversion equals one. In their survey of the literature on international portfolio diversification, Adler and Dumes (1983) report that for many countries the covariance between the consumer price index and the exchange rate is low in monthly data. This finding suggests that it is worth asking what might cause the covariance to be low.

In exploring for possible causes of a low price index - exchange rate covariance it is useful to adopt a general specification of the price index, one in which neither relative PPP nor the law of one price is imposed. Suppose that the home currency price of foreign goods P^f is equal to the product of the exchange rate E, the foreign currency price of foreign goods, and a variable representing the (proportional) deviation from the law of one price V:

$$P^f = E\overset{*}{P}V. \tag{3.44}$$

Then, the price index is given by

$$Q = P^{1-\beta}(E\overset{*}{P}V)^{\beta}. \tag{3.45}$$

The exchange rate, the price of home goods, and the price of foreign goods follow the stochastic processes (3.6), (3.22), and (3.23) respectively, and the deviation from the law of one price follows geometric Brownian motion:

$$dV/V = \pi_v dt + \sigma_v dz_v. \tag{3.46}$$

The expression for ρ_{qe} implied by equations (3.45), (3.6), (3.22), (3.23), and (3.46) is obtained by applying Ito's Lemma twice, first to find dQ/Q and then to evaluate (dQ/Q)(dE/E):49/

$$(dQ/Q)(dE/E) = [(1 - \beta)\rho_{pe} + \beta\sigma_e^2 + \beta\rho^*_{pe} + \beta\rho_{ve}]dt. \tag{3.47}$$

That is,

$$\rho_{qe} = (1 - \beta)\rho_{pe} + \beta\sigma_e^2 + \beta\rho^*_{pe} + \beta\rho_{ve}. \tag{3.48}$$

Given that $\rho_{ij} = \sigma_i\sigma_j r_{ij}$, ρ_{qe} is equal to zero if and only if

$$0 = (1 - \beta)\sigma_p r_{pe} + \beta\sigma_e + \beta\sigma^*_p r^*_{pe} + \beta\sigma_v r_{ve}. \tag{3.49}$$

If goods prices are nonstochastic or if the correlations of E with P and $\overset{*}{P}$ are zero, zero covariance between E and Q implies that the correlation between the exchange rate and the deviation from the law of one price must satisfy:

$$r_{ve} = - \sigma_e/\sigma_v. \tag{3.50}$$

If $\sigma_e = \sigma_v$, a perfect negative correlation between E and V makes the covariance between Q and E equal zero. If goods prices are nonstochastic or $r_{pe} = r^*_{pe} = 0$, $\rho_{qe} = 0$ implies systematic deviations from the law of one price. In more general circumstances, condition (3.49) might be satisfied even if there were no deviations from the law of one price.

3.6. Asset demands in a three asset model with exchange rates and the price index stochastic

In the two asset model of subsection 3.2 assets are gross substitutes as they are in all two asset models. However, in models with three or more assets, the possibility arises that some assets may be complements. Whether assets are substitutes or complements depends on the association between the returns on the assets. If the interest rate on the first asset rises and the returns on the second and third assets are highly correlated, the demand for second asset may rise while the demand for the third asset falls, or vice versa.

In this subsection we spell out the conditions under which assets are complements in a three asset generalization of the two asset model of subsection 3.2. The objective function V is given by equation (3.1). The investor allocates a proportion λ_1 of his nominal wealth W to the first foreign security F_1, a proportion λ_2 to the second foreign security F_2, and the remainder to the home security B:

$$\lambda_1 W = E_1 F_1, \tag{3.51a}$$

$$\lambda_2 W = E_2 F_2, \tag{3.51b}$$

$$(1 - \lambda_1 - \lambda_2) W = B. \tag{3.51c}$$

E_i, i = 1, 2, is the home currency price of foreign currency i. The first foreign security, the second foreign security, and the home security have certain nominal returns represented by $\overset{*}{i}_1$, $\overset{*}{i}_2$, and i respectively:

$$dF_1/F_1 = \overset{*}{i}_1 dt, \tag{3.52a}$$

$$dF_2/F_2 = \overset{*}{i}_2 dt, \tag{3.52b}$$

$$dB/B = idt. \tag{3.52c}$$

Real wealth $\hat{W}$ is nominal wealth deflated by the price index:

$$\hat{W} = W/Q = (B + E_1F_1 + E_2F_2)/Q. \tag{3.53}$$

Equations (3.51 and (3.53) imply

$$1/\hat{W} = Q/W = \lambda_1 Q/E_1F_1 = \lambda_2 Q/E_2F_2 = (1 - \lambda_1 - \lambda_2)Q/B. \tag{3.54}$$

The two exchange rates and the price index follow geometric Brownian motion:

$$dE_1/E_1 = \varepsilon_1 dt + \sigma_1 dz_1, \tag{3.55a}$$

$$dE_2/E_2 = \varepsilon_2 dt + \sigma_2 dz_2, \tag{3.55b}$$

$$dQ/Q = \pi_q dt + \sigma_q dz_q. \tag{3.55c}$$

The second order terms utilized below are

$$(dQ/Q)^2 = \sigma_q^2 dt, \tag{3.56a}$$

$$(dQ/Q)(dE_1/E_1) = \rho_{q1} dt, \tag{3.56b}$$

$$(dQ/Q)(dE_2/E_2) = \rho_{q2} dt, \tag{3.56c}$$

$$(dE_1/E_1)(dE_2/E_2) = \rho_{12} dt. \tag{3.56d}$$

Calculating the stochastic differential of $\hat{W}$ using Ito's Lemma, dividing through by $\hat{W}$, and substituting in the expressions in (3.56) yields

$$\begin{aligned} d\hat{W}/\hat{W} = {} & [(1 - \lambda_1 - \lambda_2)i + \lambda_1 \overset{*}{i}_1 + \lambda_2 \overset{*}{i}_2 + \lambda_1 \varepsilon_1 + \lambda_2 \varepsilon_2 - \pi_q \\ & + \sigma_q^2 - \lambda_1 \rho_{q1} - \lambda_2 \rho_{q2}]dt \\ & - \sigma_q dz_q + \lambda_1 \sigma_1 dz_1 + \lambda_2 \sigma_2 dz_2. \end{aligned} \tag{3.57}$$

The expected value and variance of $d\hat{W}/\hat{W}$ are given by

$$\begin{aligned} E(d\hat{W}/\hat{W}) = {} & (1 - \lambda_1 - \lambda_2)i + \lambda_1 \overset{*}{i}_1 + \lambda_2 \overset{*}{i}_2 + \lambda_1 \varepsilon_1 + \lambda_2 \varepsilon_2 - \pi_q \\ & + \sigma_q^2 - \lambda_1 \rho_{q1} - \lambda_2 \rho_{q2}, \end{aligned} \tag{3.58a}$$

$$\mathrm{var}\ (d\hat{W}/\hat{W}) = \sigma_q^2 + \lambda_1^2 \sigma_1^2 + \lambda_2^2 \sigma_2^2 - 2\lambda_1 \rho_{q1} - 2\lambda_2 \rho_{q2} + 2\lambda_1 \lambda_2 \rho_{12}. \tag{3.58b}$$

Substituting these expressions into the objective function (3.1) and setting the partial derivatives with respect to λ_1 and λ_2 equal to zero yields two equations in λ_1 and λ_2:

$$R(\sigma_1^2\lambda_1 + \rho_{12}\lambda_2) = \overset{*}{i}_1 + \varepsilon_1 - i + (R - 1)\rho_{q1}, \tag{3.59a}$$

$$R(\rho_{12}\lambda_1 + \sigma_2^2\lambda_2) = \overset{*}{i}_2 + \varepsilon_2 - i + (R - 1)\rho_{q2}. \tag{3.59b}$$

These equations can be rewritten in matrix form:

$$R\underline{\Omega\lambda} = \underline{\delta} + (R - 1)\underline{\rho}. \tag{3.60}$$

where

$$\underline{\Omega} = \begin{bmatrix} \sigma_1^2 & \rho_{12} \\ \rho_{12} & \sigma_2^2 \end{bmatrix}, \quad \underline{\lambda} = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}, \quad \underline{\delta} = \begin{bmatrix} \overset{*}{i}_1 + \varepsilon_1 - i \\ \overset{*}{i}_2 + \varepsilon_2 - i \end{bmatrix}, \quad \underline{\rho} = \begin{bmatrix} \rho_{q1} \\ \rho_{q2} \end{bmatrix}.$$

$\underline{\Omega}$ is the variance-covariance matrix for exchange rate changes. $\underline{\lambda}$ is the vector of the portfolio shares devoted to the first and second foreign securities. $\underline{\delta}$ is the vector of differentials between the expected nominal returns on the first and second foreign securities and the home security. $\underline{\rho}$ is the vector of covariances of the price index with the two exchange rates.

The portfolio rule can be obtained by inverting $R\underline{\Omega}$:

$$\underline{\lambda} = [(R - 1)/R]\underline{\Omega}^{-1}\underline{\rho} + (1/R)\underline{\Omega}^{-1}\underline{\delta}. \tag{3.61}$$

This rule is analogous to the rule obtained in the two asset model. The share of wealth devoted to a single foreign security is replaced by a vector of shares. The inverse of the variance of exchange rate changes is replaced by the inverse of the varince-covariance matrix of changes in exchange rates. The single expected nominal return differential is replaced by the vector of expected nominal return differentials. The covariance of the price index with one exchange rate is replaced by a vector of covariances of the price index with exchange rates.

In equation (3.60) the optimal portfolio shares are expressed as a weighted average of the shares of the minimum variance portfolio $\underline{\lambda}_M = \underline{\Omega}^{-1}\underline{\rho}$ and the logarithmic portfolio $\underline{\lambda}_L = \underline{\Omega}^{-1}\underline{\delta}$. The structure of the logarithmic and minimum variance portfolios in the three asset model is analogous to the structure of those portfolios in the two asset example: return differentials enter only the logarithmic portfolio and covariances of the price index with exchange rates enter only the minimum variance portfolio.

The portfolio rule can also be written as the sum of the minimum variance portfolio $\underline{\lambda}_M = \underline{\Omega}^{-1}\underline{\rho}$ and a zero net worth speculative portfolio $\underline{\lambda}_S = \underline{\Omega}^{-1}(\underline{\delta} - \underline{\rho})$:

$$\underline{\lambda} = \underline{\Omega}^{-1}\underline{\rho} + (1/R)\underline{\Omega}^{-1}(\underline{\delta} - \underline{\rho}). \tag{3.62}$$

$\underline{\delta} - \underline{\rho}$ is a vector of expected real return differentials.

In a three asset model it is possible for assets to be complements. The partial derivatives of the three security demand functions (3.51) with respect to $\overset{*}{i}$, are

$$\partial \lambda_1 W/\partial \overset{*}{i}_1 = W/R\sigma_1^2(1 - r_{12}^2), \tag{3.63a}$$

$$\partial \lambda_2 W/\partial \overset{*}{i}_1 = - [W(\sigma_1/\sigma_2)r_{12}]/R\sigma_1^2(1 - r_{12}^2), \tag{3.63b}$$

$$\partial(1 - \lambda_1 - \lambda_2)W/\partial \overset{*}{i}_1 = - W[1 - (\sigma_1/\sigma_2)r_{12}]/R\sigma_1^2(1 - r_{12}^2). \tag{3.63c}$$

An increase in the nominal return on the first foreign security raises the demand for that security, as always. The two other assets are gross substitutes for the first foreign security if both cross partials are negative, that is, if the correlation between the nominal returns on the two foreign securities, which is just the correlation between the two exchange rates, is positive but less than σ_2/σ_1. If the two exchange rates are positively correlated and have variances that are roughly equal, the three securities are definitely gross substitutes. Negative correlation between the two exchange rates implies that the two foreign securities are complements. Positive correlation and a large enough value of σ_1/σ_2 imply that the first foreign security and the home security are complements. Thus, in a three asset model, making the assumption that the assets are gross substitutes is equivalent to imposing restrictions on the correlations between the nominal returns on assets.

3.7 Asset demands in a general model with exchange rates and the price index stochastic

The three asset model can be easily generalized to the case in which there are N foreign securities and a home security. By analogy the optimal portfolio rule can be written in two ways:

$$\underline{\lambda} = [(R - 1)/R]\underline{\Omega}^{-1}\underline{\rho} + (1/R)\underline{\Omega}^{-1}\underline{\delta}, \tag{3.64a}$$

$$\underline{\lambda} = \underline{\Omega}^{-1}\underline{\rho} + (1/R)\underline{\Omega}^{-1}(\underline{\delta} - \underline{e}). \tag{3.64b}$$

$\underline{\lambda}_M = \underline{\Omega}^{-1}\underline{\rho}$ is the minimum variance portfolio, $\underline{\lambda}_L = \underline{\Omega}^{-1}\underline{\delta}$ is the logarithmic portfolio, and $\underline{\lambda}_S = \underline{\Omega}^{-1}(\underline{\delta} - \underline{e})$ is the zero net worth speculative portfolio. $\underline{\lambda}$ is the N dimensional column vector of the shares of the N foreign securities in the optimal portfolio. $\underline{\Omega}$ is the NxN variance-covariance matrix for the changes in the N exchange rates, which are defined as foreign currency prices of the home currency. $\underline{\rho}$ is the N dimensional vector of covariances of the price index with the N exchange rates. $\underline{\delta}$ is the N dimensional vector of return differentials, $\overset{*}{i}_n + \varepsilon_n - i$, $n = 1,\ldots,N$. $\underline{\lambda}_M, \underline{\lambda}_L$, and $\underline{\lambda}_S$ are all N dimensional vectors. For the rules $\underline{\lambda}$, $\underline{\lambda}_M$, and $\underline{\lambda}_L$, the home security share is one minus the sum of the N foreign security shares; for the rule $\underline{\lambda}_S$, the home security share is the negative of the sum of the N foreign security shares.

The basic structure of the general model is the same as that of the two and three asset models. The logarithmic portfolio is not sensitive to the choice of assumption regarding price index dynamics. However, the minimum variance portfolio is sensitive to this choice. Different assumptions about price index dynamics made by various analysts are reflected in different $\underline{\rho}$ vectors.

3.8. Integrating money into the microeconomic theory of asset demands

The portfolio rules of subsections 3.2 to 3.7 are rules for allocating nominal wealth among interest bearing securities. If the mean and variance of the change in real wealth are the only arguments in the objective function, non interest bearing money is not held in portfolios because it is dominated by securities denominated in the same currency that pay a certain nominal return. Money has been integrated into the microeconomic theory of asset demands by assuming that real money balances enter the investor's objective function. Some analysts justify the procedure by arguing that real balances as well as goods are inputs into a "production function" for consumption, so utility can be expressed as a function of real balances and goods.[50/] Others argue that an investor with higher real balances has more leisure because he need make fewer trips to the bank.[51/] There is a lively debate about whether it is useful to assume that real balances enter the investor's objective function. We make no attempt to summarize that debate here.[52/] Rather we report some of the results that have been derived under the assumption that real balances enter the investor's objective function.

The investor's augmented objective function V^A is assumed to be the sum of V in equation 3.1 and a function of real balances $Z(M/Q)$, where $Z' > 0$ and $Z'' < 0$:

$$V^A = E(d\hat{W}/W) - (1/2)R[\mathrm{var}(d\hat{W}/\hat{W})] + Z(M/Q). \qquad (3.65)$$

The investor allocates a proportion λ of his nominal wealth to foreign securities, a proportion μ to home money M, and the remainder to home securities:

$$\lambda W = EF, \tag{3.66a}$$

$$\mu W = M, \tag{3.66b}$$

$$(1 - \lambda - \mu)W = B. \tag{3.66c}$$

The expected value and variance of $d\hat{W}/\hat{W}$ become

$$E(d\hat{W}/\hat{W}) = (1 - \lambda - \mu)i + \lambda \overset{*}{i} + \lambda\varepsilon - \pi_q - \lambda\rho_{qe} + \sigma_q^2 , \tag{3.67a}$$

$$\text{var}(d\hat{W}/\hat{W}) = \lambda^2\sigma_e^2 - 2\lambda\rho_{qe} + \sigma_q^2. \tag{3.67b}$$

Substituting the expressions (3.67) into V^A, noting that $M/Q = \mu W/Q$, and setting the partial derivatives with respect to λ and μ equal to zero yields

$$\overset{*}{i} + \varepsilon - i - \rho_{qe} - R(\lambda\sigma_e^2 - \rho_{qe}) = 0, \tag{3.68a}$$

$$i - [Z'(\mu W/Q)](W/Q) = 0. \tag{3.68b}$$

Solving (3.68a) for the share of foreign securities λ yields exactly the same expression as the one in equation (3.19), which is derived from the two asset model with no money. Equation (3.68b) implies a value for the share of money μ given values for W/Q and i. Below we assume some specific forms for the function Z(M/Q) and solve explicitly for μ. The share of home securities is determined as a residual.

The optimal portfolio has some interesting properties. The investor can be viewed as making his portfolio allocation decision in two steps. First, he divides his portfolio between foreign securities and total home assets, money and securities, according to equation (3.68a). Then, he expands his money holdings until equation (3.68b) is satisfied. The rest of the portfolio goes into home securities. A change in wealth or the home interest rate alters the holdings of all assets, but a change in the foreign interest rate affects only holdings of home and foreign securities. Changes in the transactions demand for money are changes in Z'. Since these changes do not affect equation (3.68a), the resulting adjustments in money holdings are matched one for one by adjustments in home security holdings. All of these properties are reflected in the basic specification of asset markets of section 2, except that in the basic specification money demand does not depend on wealth.

Assuming specific forms for Z(M/Q) makes it possible to solve explicitly for μ or the demand for real balances. First, suppose $Z(M/Q) = \alpha \ln(M/Q)$.[53/] Then $Z' = \alpha/(M/Q)$, and according to (3.68b) the demand for real balances is

$$M/Q = (\alpha/i)(W/Q). \tag{3.69}$$

Now, suppose $Z(M/Q) = (M/Q)^{\phi}/\phi$. Then $Z' = (M/Q)^{\phi-1}$ and the demand for real balances is

$$M/Q = (W/iQ)^{\frac{1}{1-\phi}}. \tag{3.70}$$

If the underlying utility function displays constant relative risk aversion, $U(\hat{C}) = \hat{C}^{\gamma}/\gamma$, the solution of the lifetime consumption problem implies that optimal real consumption $\hat{C}$ is a constant fraction of real wealth $\hat{W}$ at every point in time.[54/] In this case the demand for money can be written as

$$M/Q = (\hat{C}/i)^{\frac{1}{1-\phi}},$$

with real consumption as the "activity" variable.

Under an alternative set of assumptions money demand depends on real income $\hat{Y}$. Suppose that a measure of real transactions is given by $k\hat{Y}$ and that the augmented objective function V^A is equal to the sum of V in equation (3.1) and a function that is linear homogeneous in real balances $\hat{M}$ and real transactions $Z = \hat{M}^{\psi}(k\hat{Y})^{1-\psi}$ where $0 < \psi < 1$. In this case $Z_{\hat{M}} = \psi\hat{M}^{\psi-1}(k\hat{Y})^{1-\psi}\hat{W}$ and money demand is given by

$$\hat{M} = k\hat{Y}(\psi\hat{W}/i)^{\frac{1}{1-\psi}}.$$

The arguments of this money demand function are the same as those of the money demand function in the basic specification of asset markets of section 2 except that the arguments of this money demand function include real wealth.

4. Conclusions

The microeconomic theory of asset demands discussed in section 3 implies some but not all of the properties of the basic specification of asset markets in section 2. Under the assumptions of section 3 the demand for the sum of assets denominated in each currency is homogeneous of degree one in nominal wealth, and the demand for money in each country depends on the return on the security denominated in that country's currency but not on the return on securities denominated in other currencies. However, under these same assumptions the demand for money depends on real wealth. Since the conclusions of macroeconomic analysis often depend crucially on the form of asset demand functions, it is important to continue to explore the implications of the microeconomic theory of section 3 and other microeconomic approaches.

The consumer of section three arrives at his asset demands by maximizing his utility given interest rates and the parameters of the distributions of prices and exchange rates. Of course, the distributions of prices and exchange rates are not invariant to changes in the distributions of policy variables and stochastic components of tastes and technology. It has been recognized that a very important item on the research agenda is imbedding consumers' asset demands based on utility maximization in a general equilibrium model in which the distributions of prices and exchange rate are determined endogenously.[55]

Appendix 1

In this appendix it is shown that the determinant of the differential equation system made up of equation (2.22c) and equation (2.25) with $\varepsilon = \dot{e}$ is always negative. Thus, with flexible prices stationary equilibrium is always a saddle point. In the perverse case of subsection 2.7 this result implies that the A_S schedule is steeper than the $\dot{w} = 0$ schedule.

Let G represent the matrix of coefficients of the differential equation system made up of equation (2.22c) and equation (2.25) with $\varepsilon = \dot{e}$. Then,

$$\det G = -\,(\varepsilon_e' C_6 + \varepsilon_w' C_5), \tag{A1.1}$$

where C_5 and C_6 are defined below equations (2.22) and ε_e' and ε_w' are defined below equation (2.25). A lengthy and tedious derivation yields

$$\begin{aligned}\det G = H_1\{&(M + B + \overset{*}{B})(\overset{*}{N} + \overset{*}{F} + F)\Delta \\ &- H_2(\overset{*}{N} + \overset{*}{F} + F)[\hat{x}_p^*(s_2W + \overset{*}{s}_2\overset{*}{W}) + \overset{*}{s}_2\overset{*}{W}(x_2W + \overset{*}{x}_2\overset{*}{W})] \\ &- H_3(M + B + \overset{*}{B})[\hat{y}_p(s_2W + \overset{*}{s}_2\overset{*}{W}) + s_2W(y_2W + \overset{*}{y}_2\overset{*}{W})] < 0,\end{aligned} \tag{A1.2}$$

$$H_1 = (s_2\overset{*}{s}_2/\hat{b}_\varepsilon\Delta^2)(\hat{x}^*_p + \overset{*}{x}_2\overset{*}{\hat{W}}) < 0,$$

$$H_2 = -(\hat{b}_i\hat{m}_p - \hat{b}_p\hat{m}_i)/\hat{m}_i > 0,$$

$$H_3 = \hat{b}^*_i\hat{n}^*_p/\hat{n}^*_i > 0.$$

$\Delta > 0$ is defined below equations (2.22). In the derivation use is made of the relationship in footnote 8.

With flexible prices the difference between the slope of the A_S schedule and the slope of the $\dot{w} = 0$ schedule is given by

$$(de/dw)_{A_S} - (de/dw)_{\dot{w}=0} = -(1/\varepsilon'_e C_5)(\varepsilon'_e C_6 + \varepsilon'_w C_5), \qquad \text{(A1.3)}$$

$$= (1/\varepsilon'_e C_5)(\det G).$$

If C_5, C_6, and $\varepsilon'_w > 0$ but $\varepsilon'_e < 0$, both $(de/dw)_{A_S}$ and $(de/dw)_{\dot{w}=0}$ are positive. A_S is steeper because det G is always negative. If C_6, ε'_e, and $\varepsilon'_w > 0$ but $C_5 < 0$, both $(de/dw)_{A_S}$ and $(de/dw)_{\dot{w}=0}$ are negative. A_S is flatter because det G is always negative.

Appendix 2

In this appendix we derive an expression for the covariance of the stochastic processes for the domestic price index and the exchange rate ρ_{qe} when the exchange rate E, the home currency price of home goods P, the foreign currency price of foreign goods $\overset{*}{P}$, and the deviation from the law of one price all follow geometric Brownian motion. The home price index is given by

$$Q = P^{1-\beta}(E\overset{*}{P}V)^{\beta}. \tag{A2.1}$$

The stochastic processes for E, P, $\overset{*}{P}$, and V are reproduced here for convenience:

$$dE/E = \varepsilon dt + \sigma_e dz_e, \tag{A2.2a}$$

$$dP/P = \pi_p dt + \sigma_p dz_p, \tag{A2.2b}$$

$$d\overset{*}{P}/\overset{*}{P} = \pi_p^* dt + \sigma_p^* \, dz_p^*, \tag{A2.2c}$$

$$dV/V = \pi_v dt + \sigma_v dz_v. \tag{A2.2d}$$

Calculating the stochastic differential dQ, multiplying it by 1/Q, and collecting terms yields

$$dQ/Q = (1 - \beta)dP/P + \beta dE/E + \beta d\overset{*}{P}/\overset{*}{P} + \beta dV/V$$

$$+(1/2)\{- \beta(1 - \beta)[(dP/P)^2 + (dE/E)^2 + (d\overset{*}{P}/\overset{*}{P})^2 + (dV/V)^2]$$

$$+ 2\beta(1 - \beta)[(dP/P)(dE/E) + (dP/P)(d\overset{*}{P}/\overset{*}{P}) + (dP/P)(dV/V)]$$

$$+ 2\beta^2[(d\overset{*}{P}/\overset{*}{P})(dE/E) + (d\overset{*}{P}/\overset{*}{P})(dV/V) + (dE/E)(dV/V)]\}. \quad (A2.3)$$

Substituting the processes (A2.2) into (A2.3) and using Ito's Lemma to evaluate the products of processes transforms (A2.3) into

$$dQ/Q = [(1 - \beta)\pi_p + \beta\varepsilon + \beta\pi_p + \beta\pi_v - \beta(1 - \beta)(\sigma_p^2 + \sigma_e^2 + \sigma_{p}^{*2} + \sigma_v^2)$$

$$+ 2\beta(1 - \beta)(\rho_{ep} + \rho_{pp}^{*} + \rho_{pv}) + 2\beta^2(\rho_{pe}^{*} + \rho_{pv}^{*} + \rho_{ev})]dt$$

$$+ (1 - \beta)\sigma_p dz_p + \beta\sigma_e dz_e + \beta\sigma_p^* dz_p^* + \beta\sigma_v dz_v. \quad (A2.4)$$

Using Ito's Lemma to evaluate the product (dQ/Q)(dE/E) yields

$$(dQ/Q)(dE/E) = [(1 - \beta)\rho_{pe} + \beta\sigma_e^2 + \beta\rho_{pe}^* + \beta\rho_{ve}]dt. \quad (A2.5)$$

That is,

$$\rho_{qe} = (1 - \beta)\rho_{pe} + \beta\sigma_e^2 + \beta\rho_{pe}^* + \beta\rho_{ve}. \quad (A2.6)$$

Footnotes

* The authors appreciate helpful comments made by Michael Adler, Shoichi Katayama, and participants at the conference held at Princeton University in May 1982 to discuss preliminary drafts of chapters 13 through 23, especially Peter Kenen, Jorge de Macedo, and the two discussants of this chapter, Jacob Frenkel and Tim Padmore. This paper represents the views of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or other members of its staff.

1/ Almost all the contributions to the literature on asset markets in open economies in the references of this chapter were published in 1975 or later. Many important contributions were published before 1975. Bryant (1975) provides an excellent assessment of empirical research on financial capital flows up to the mid 1970's. He includes in his references most of the important theoretical and empirical analyses dealing specifically with asset markets in open economies that were available when he wrote.

2/ The portfolio balance approach to macroeconomic modeling was developed by Metzler (1951) and Tobin (1969).

3/ This argument is often referred to as the Lucas (1976) critique.

4/ Throughout the rest of this paper home currency securities and foreign currency securities are referred to as home securities and foreign securities respectively. In order to simplify the analysis, we assume that there are no home and foreign capital stocks and, therefore, no equity claims on those capital stocks. Models with capital stocks and equity claims are discussed by Bruce and Purvis in chapter 16 and by Obstfeld and Stockman in chapter 17.

5/ We assume that the sum of the partial derivatives of each asset demand function with respect to its first four arguments is zero so that, for example $\sum_{k=0}^{4} m_k = 0$. Under this assumption expressing any asset demand as a function of nominal returns is equivalent to expressing it as a function of real returns since the expected rate of change of the price of a country's consumption bundle in terms of its currency can be subtracted from all nominal returns without changing the value of the asset demand.

6/ Precisely, the assumption that expectations are static implies that ε is exogenous. However, it is usual to specify paths for the exogenous variables other than ε that lead to a unique value for the steady state actual rate of depreciation, and it is natural to set ε equal to that value. Throughout this section it is assumed that the asset stocks available for the public to hold do not change continuously over time ($\dot{\hat{M}} = \dot{\hat{N}} = \dot{\hat{B}} = \dot{\hat{F}} = 0$). Steady states are stationary states in which the actual rate of depreciation is equal to zero, so it makes sense to set ε equal to zero under static expectations. Kouri (1976) develops a model in which residents of the home country face a fixed foreign currency price of the single world good and allocate their wealth between home and foreign money. He sets the exogenous expected rate of depreciation equal to the exogenous positive rate of growth of home money under static expectations.

7/ Models that allow for currency substitution have been constructed by Girton and Roper (1981), Kareken and Wallace (1981), Lapan and Enders (1980), and Nickelsburg (1983) among others. In most contributions currency substitution is defined as substitution among national moneys defined as currency and coin plus deposits that bear non market related or zero rates of interest. According to Girton and Roper, currency substitution warrants special study because moneys are the only financial assets that have their stated returns, if any, fixed in terms of themselves. Girton and Roper employ postulated money demand functions. Kareken and Wallace, Lapan and Enders, and Nickelsburg assume that moneys are the only stores of value in models with overlapping generations composed of individuals who maximize explicit utility functions.

8/ If goods prices, outputs, and initial asset holdings are taken as exogenous, the model of equations (2.9) is representative of short-run portfolio balance models of international financial markets. Other models of this type are employed by Black (1973), Dooley and Isard (1982), Frankel (1983), Freedman (1977), Girton and Henderson (1977, 1976a, 1976b), Henderson (1979), Herring and Marston (1977a, 1977b), Hewson and Sakakibara (1975), Kouri and Porter (1974), and Marston (1980). When short-run portfolio balance models are used to analyze a regime of flexible exchange rates, it is usually assumed that exchange rate expectations are static or regressive.

9/ Under the basic specification the derivative of the excess demand for foreign securities with respect to the exchange rate,

$$f_7F + \overset{*}{f}_5\overset{*}{P}Y + \overset{*}{f}_7\ (\overset{*}{N} + \overset{*}{F}) - \hat{F},$$

is negative if $0 < f_7$, $\overset{*}{f}_7 < 1$ since $\overset{*}{f}_5\overset{*}{P}Y = -\ \overset{*}{n}_5\overset{*}{P}Y = -\ \overset{*}{N}$.

10/ A balance of payments condition or one or more goods market equilibrium condition or both are added to the asset market equlibrium conditions in the portfolio balance models of Allen and Kenen (1980), Boyer (1978, 1977, 1975), Branson (1977, 1974), Bryant (1980), Calvo and Rodriguez (1977), Dornbusch (1975), Enders (1977), Flood (1979), Frenkel and Rodriguez (1975), Henderson (1980, 1979), Henderson and Rogoff (1982), Kenen (1981, 1976), Kouri (1983a, 1983b), Kouri and de Macedo (1978), Masson (1981, 1980), McKinnon and Oates (1966), Melitz (1982), Myhrman (1975), Obstfeld (1982, 1980), Tobin and de Macedo (1981), and Wallace (1970).

11/ The implications of several alternative specifications of goods markets are considered by Bruce and Purvis in chapter 16.

12/ For simplicity we adopt the system of taxes and transfers under which interest payments do not enter the analysis suggested by Allen and Kenen (1980). Each government taxes away all the interest received by the residents of its country and transfers to the government of the other country an amount equal to the interest received by the residents of its country from the other country. Under this system, the current account surplus and trade account surplus of a country are equal, and if the budget of the government of a country is balanced, the disposable income of its residents is equal to output minus government spending.

13/ For simplicity we assume that saving in each country does not depend on the real returns on home and foreign securities. If this assumption were relaxed in the case of flexible prices and rational expectations, it would be necessary to analyze a system of four differential equations rather than a system of two differential equations.

14/ Our assumption about the effects of increases in P and EP^* on home and foreign spending on home and foreign goods is sufficient but not necessary to insure that increases in P reduce the home trade surplus and that increases in $E\overset{*}{P}$ increase it. Of course, without our assumption it is possible that increases in P increase the home trade surplus and that increases in $E\overset{*}{P}$ reduce it.

15/ In the derivation of equations (2.21) we make use of the appropriately modified versions of equations (2.1) and equations (2.19).

16/ In the relatively lengthy and tedious derivation of equations (2.22) we make use of the appropriately modified versions of equations (2.1), equations (2.18), and equations (2.19). We approximate the goods market equilibrium conditions and the balance of payments equation around long-run equilibrium where home and foreign saving are zero. In the neighborhood of long run equilibrium equations (2.18) and (2.19) imply a key relationship:

$$\hat{y}_p \equiv \hat{x}_p^* + \overset{*}{x}_2\overset{*}{W} - y_2W.$$

17/ This assertion can be confirmed by substituting relationships implied by equations (2.18) and 2.19) into the definitions of $\hat{x}_p$, $\hat{y}_p$, $\hat{x}_p^*$, and $\hat{y}_p^*$.

18/ If s_2 is not equal to $\overset{*}{s}_2$, the indirect effect from the induced price changes does not necessarily reinforce the direct effect. However, the overall effect of the transfer on the trade surplus is always to reduce it; that is C_6 is always positive.

19/ Kouri (1983a) uses this terminology.

20/ The determinant of the differential equation system made up of equation (2.20) and equation (2.24) with $\varepsilon = \dot{e}$ is $- \varepsilon_w\eta < 0$.

21/ The determinant of the differential equation system made up of equation (2.22c) and equation (2.25) with $\varepsilon = \dot{e}$ is $-(\varepsilon_e' C_6 + \varepsilon_w' C_5) < 0$.

22/ The instability problem associated with negative net foreign asset positions is a central issue in several recent papers: Boyer (1977); Branson, Halttunen, and Masson (1979); and Martin and Masson (1979). It is also discussed by Tobin and de Macedo (1981). Tobin (1980) summarizes a main conclusion reached in these papers. The problem is considered further by Henderson and Rogoff (1982), Kouri (1983a), Masson (1981), and Melitz (1982). The conclusions presented here are similar to those of Henderson and Rogoff and Kouri but somewhat different from those of Masson. Melitz argues that the transactions demand for money is an important stabilizing influence.

23/ The negative net foreign asset case is not the only portfolio constellation that has led analysts to question the stability of open economy portfolio balance models. Enders (1977) and Masson (1980) discuss the possibility that instability might arise when positive net foreign asset positions are "too large." See footnotes 24, 26, 28, and 30.

24/ If ε_e is positive but there is foreign asset preference ($b_7 < \overset{*}{b}_7$), long-run equilibrium is definitely unstable under static expectations. With goods prices fixed, $b_7 < \overset{*}{b}_7$ is a necessary and sufficient condition for ε_w to be negative. In this case, as in the perverse case of the text, the A_S schedule is upward sloping. It can be shown that if the assumption that in each country the demand for the good produced in the other country is independent of nominal spending is dropped, long-run equilibrium is stable for some, but not all, parameter values. Thus, with goods prices fixed, the Enders (1977) problem of instability caused by large net foreign asset positions can arise even if it is assumed that the home trade surplus depends on nominal spending in both countries.

25/ See footnote 20.

26/ If $\varepsilon_e > 0$ but $b_7 < \overset{*}{b}_7$ so that $\varepsilon_w < 0$, long-run equilibrium is definitely not a saddle point. The determinant of the differential equation system made up of equation (2.20) and equation (2.24) with $\varepsilon = \dot{e}$ is $-\varepsilon_w \eta > 0$. It can be shown that either the two roots of the characteristic equation are real and positive or they are complex conjugates with positive real parts. It can also be shown that if the assumption that in each country the demand for the good produced in the other country is independent of nominal spending is dropped, long-run equilibrium is a saddle point under rational expectations if and only if it is stable under static expectations. Thus, with goods prices fixed, the Enders (1977) problem can arise under rational expectations even if it is assumed that the home trade surplus depends on nominal spending in both countries.

27/ This assertion is proved in Appendix 1.

28/ If ε_e is positive but there is foreign asset preference ($b_7 < \overset{*}{b}_7$), long-run equilibrium is definitely stable under static expectations. With goods prices flexible and $b_7 < \overset{*}{b}_7$, ε_w' may be negative and C_5 is definitely negative. Thus, the A_S schedule may be upward sloping, and the $\dot{w} = 0$ schedule is definitely downward sloping. It is shown in

Appendix 1 that if the A_S schedule is downward sloping, it is flatter than the $\dot{w} = 0$ schedule. Although the effect of a wealth transfer on the exchange rate depends on the slope of the A_S schedule, long-run equilibrium is stable whatever the slope of this schedule. Thus, with good prices flexible, the Enders (1977) problem cannot arise.

29/ It is shown in Appendix 1 that the determinant of the differential equation system made up of equation (2.22c) and equation (2.25) with $\varepsilon = \dot{e}$ is always negative.

30/ If ε_e is positive but there is foreign asset preference ($b_7 < b_7^*$), long-run equilibrium is definitely a saddle point under rational expectations. See footnote 21.

31/ Markowitz (1959) and Tobin (1965) laid the foundations of portfolio selection theory.

32/ Merton (1971) pioneered this approach.

33/ Merton (1969) shows that the portfolio allocation decision is independent of the saving decision in a continuous time model. He assumes both that the investor's utility function exhibits constant relative risk aversion and that percentage changes in asset prices follow geometric Brownian motion. Samuelson (1969) derives the same result in a discrete time model. He assumes constant relative risk aversion but puts no restrictions on the distribution of returns.

34/ Merton (1971) proves this result.

35/ Solnik (1974) was the first to analyze portfolio selection in an open economy using stochastic calculus. He assumes that residents of each country consume only the good produced in that country and that goods prices are non stochastic.