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REGULATING MISINFORMATION

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ABSTRACT

The government has responded to misleading advertising by banning it, engaging in counter-advertising and taxing the product. In this paper, we consider the social welfare effects of those different responses to misinformation. While misinformation lowers consumer surplus, its effect on social welfare is ambiguous. Misleading advertising leads to overconsumption but that may be offsetting the under-consumption associated with monopoly prices. If all advertising is misinformation then a tax or quantity restriction on advertising maximizes social welfare. Other policy interventions are inferior and cannot improve on a pure advertising tax. If it is impossible to tax misleading information without also taxing utility increasing advertising, then combining taxes or bans on advertising with other policies can increase welfare.

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1 Introduction

How should government policy respond to misleading advertising? The classic economic papers on advertising assume either that advertising provides useful information about consumer products (Nelson, 1970) or that advertising shapes preferences (Dixit and Norman, 1978, Becker and Murphy, 1993), but sometimes advertising is misleading. In the nineteenth century, a variety of false claims were made about the health benefits of patent medicines that were just disguised alcohol. In the 1940s and 1950s, cigarette companies tried to convince consumers that their products were healthy (Cutler and Glaeser, 2006). Is the appropriate policy response to ban false claims or to tax the product or to produce government advertisements with an alternative viewpoint?

One laissez-faire view is that there is little cause for government intervention because these public relations efforts are ineffective. While there are many reasons to be suspicious about government intervention, it is implausible that firms would spend significantly on misinformation if that spending did nothing. A second view is that despite the flaws of private decision-making, government decision-making is worse (Glaeser, 2006). Without disputing that view, we present a simple model to examine the potential benefits of different policy responses to misinformation.

We assume that Cournot oligopolists sell a good with unobserved health costs or benefits. Following Dixit and Norman (1978) and Becker and Murphy (1993), these firms can invest in advertising which increases the taste for the good. Follow Mullainathan, Schwartzstein and Shleifer (2006), firms can also invest in advertising can manipulate beliefs and create misinformation. We focus on misinformation about the health consequences of the product.

If consumer receive none of the firms' profits, then misinformation always reduces consumer surplus, when surplus is defined to reflect true health costs .If all profits accrue to consumers, then misinformation is only harmful if it increases consumption beyond the level that would occur in a competitive market with perfect information. Since consumers typically underconsume the products of an oligopoly, misinformation helps correct this underconsumption. In cases with considerable health costs, like cigarettes, this effect is more likely to be a intellectual curiosity than an important insight. In other cases, like patented drugs where the gap between prices and marginal costs is high, misinformation that overstates the health benefits of the drug may really increase welfare by offsetting the under-consumption due to high prices. This result is similar to the idea that public misinformation overstating the private costs of risky behavior (like unsafe sex) may be optimal if that behavior has externalities.

Firms invest in misinformation and we focus on the case where advertising is product, not supplier, specific. Cigarette firms can convince smokers that cigarettes aren't harmful but not that their brand is not harmful. The earliest Federal Trade Commission interventions into cigarette advertising specifically banned brand specific health claims. This assumption means that firms don't internalize the benefits that their advertising has for other firms, and leads to the prediction that advertising will decrease with the number of firms. Monopolists reduce consumer surplus both because they set high prices and because they strongly invest in misinformation. The equilibrium level of misinformation by a monopolist will always exceed the welfare maximizing level of misinformation.

We consider the effects of three different forms of government intervention: taxes or bans on advertising, counter-advertising and taxes on profits or goods. If advertising is just misinformation, then taxes or bans on advertising yield second best options that weakly dominate all other government interventions. Counter-advertising where the government tries to refute private firms is sub-optimal because it creates a costly advertising response by the private firms. Price caps and taxes on consumption can be welfare enhancing, but they yield less social surplus than directly taxing or limiting advertising. A change in the tax code that stops firms from deducting advertising expenses is equivalent to a tax on advertising and yields similar results

If advertising both misleads and increases utility, as in Becker and Murphy (1993) then bans or taxes on advertising are less effective. In our model, good and bad forms of advertising complement each other and you cannot reduce misinformation without reducing other forms of advertising. This effect would be exacerbated if the government could not differentiate misinformation from more benevolent forms of advertising. In that case, government counter-advertising may increase welfare even if there is an optimal tax on misinformation. Since optimal taxes on advertising are low because such taxes also reduce preference-increasing advertising, counter-advertising may still have a positive effect. Taxes on consumption are not welfare increasing if there is an optimal advertising tax.

If there are multiple market segments, then firms will target segments of consumers that are more elastic in their consumption decisions. This fact suggests that it may be more remunerative to direct misleading advertising towards young people. This may mean that banning advertising towards the young raises social welfare even if the young are no more likely to be confused than the more mature.

We are not suggesting that there are markets where government action against misinformation is currently warranted. Indeed, one of our results is that misinformation may not be so bad. However, this paper does show that if all advertising is misinformation, then bans on advertising raise welfare more than government attempts to advertise an alternative view. Conversely, when firms engage in both misinformation and welfare enhancing advertising, then it is welfare enhancing to have both bans on advertising and counter-advertising.

2 Misinformation and Policy

We now review two cases where the government has responded to misleading advertising: patent medicine and cigarettes. In the case of patent medicines, the primary response was a ban on misinformation. In the case of cigarettes, the Federal Trade Commission first tried to ban misinformation, then the government supported counter-advertising and finally turned to taxation.

At the end of the nineteenth century, sixty million dollars of patent medicine was being sold annually. "In many instances, however, the medicines were ineffectual. Some of the syrups contained as much as 80 per cent alcohol; many of the tonics used cocaine and morphine. Some of the medicines destroyed health, and make drunkards and dope addicts out of their users" (Weinberg and Weinberg, 1961, p. 176). The advertisements can be stunning in their audacity. Weinberg and Weinberg (1961) cite an ad for Dr. Bye run in the socialist journal Appeal to Reason that claimed "cancer cured with soothing balmy oils." Adams (1905, contained in Weinberg and Weinberg, 1961) describes "Peruna" which was "at present the most prominent proprietary nostrum in the country." Despite the fact that Peruna's active ingredient appears only to have been alcohol, it was advertised as preventive against yellow fewer and "no matter what you've got, you will be not only enabled, but compelled, after reading Dr. Hartman's Peruna book, The Ills of Life, to diagnose your illness as catarrh, and to realize that Peruna alone will save you."

Patent medicines were misleadingly advertised, and the ads seem to have been effective. Firms spent a lot of money on advertising. The president of the National Association of Patent Medicine Men claimed in 1900 that between one-third and one-half of patent medicine revenues were spent on advertising the products. It is hard to imagine that this expenditure would have occurred if it didn't have an effect. Many patent medicines had identical medical properties to other cheaper substitutes (i.e. whiskey) and sold for much more. The price difference between whiskey and patent medicine would be hard to understand if the advertising didn't have an effect.

The government response to the patent medicine trade was the Pure Food and Drug Act of 1906. Among other things, the act forbade the sale of misbranded food or drugs "the package or label of which shall bear any statement, design, or device regarding such article, or the ingredients or substances contained therein which shall be false or misleading in any particular way." The act also gave the Food and Drug Administration authority over the sale of food and drugs and led to the requirement of prescriptions on many pharmaceuticals. The government did not engage in counter-advertising (i.e. saying that patent drugs were bad for you) or taxation. It just banned misleading advertising.

Since the 1906 ban on false advertising, there are no cases quite as egregious as 19th century patent medicines, but cigarette advertisers certainly tried to make their products seem healthy. For example, one advertisement claimed:

> "Repeated nationwide surveys show that more doctors smoke Camels than any other cigarette. A few years ago, 113,597 doctors in every branch of the medical profession were asked this question: What cigarette do you smoke, doctor? The brand named most was Camel...you see, doctors smoke for pleasure just as you and I. So what do they look for? Flavor and mildness. So smoke the cigarette that so many doctors smoke."[1]

These claims may not have been factually incorrect, but they do give the misleading impression that cigarettes were medically attractive. Of course, Camels were not the only cigarette trumpeting

[1] Text is from a television advertisement available at http://video.google.com/videoplay?docid=-1259818256007769353&q=cigarette+commercials&pl=true.

their appeal to doctors. Another slogan ran "Doctors recommend Phillip Morris." Old Golds were sold with the line "Not a Cough in a Carload."

The Cigarette industry didn't stop with catchy slogans. In the wake of the 1952 Reader's Digest article "Cancer by the Carton" that brought the medical research linking cigarettes and cancer to the wider public, tobacco firms organized "Tobacco Industry Research Committee." The scientific director of this committee, Dr. Clarence Little, then appeared on Edward R. Murrow's "See It Now," and in response to Murrow's question "have any cancer-causing agents been identified in cigarettes," Little responded "none whatsoever." After giving even-handed treatment to Little and his opponents, Murrow declared that "we have no credentials for reaching conclusions on this subject." Murrow continued smoking the cigarettes that would lead to his death at age 57 from lung cancer.[2]

The earliest public response to misleading advertising of cigarettes followed the route of the FDA. The Federal Trade Commission first complained about cigarette companies misleadingly suggesting health benefits from their brands and in 1950 received a court injunction to stop an Old Gold advertisement that claimed it was "lowest in nicotine and tars." In 1954, the FTC insisted that "no advertising should be used which refers to either the presence or absence of any physical effect of smoking." Early lawsuits, such as Cooper v. R. J. Reynolds in 1957, tried unsuccessfully to sue firms for misleadingly advertising a cancer causing product.

The Surgeon General's Report of 1964 was a major example of what we will refer to as counter-advertising: an attempt by the government to push an alternative viewpoint. The Surgeon General's report led to health warnings on cigarette packages. Continuing the counter-advertising trend, the Federal Communications Commission ruled that fairness required television stations to broadcast anti-cigarette advertising that would counter their cigarette advertising. This policy led to free air time for the public health opponents of smoking. In 1970, cigarette ads on television ended completely, although anti-cigarette advertising continued.

Litigation eventually managed to impose large judgments on cigarette companies and misleading advertising was a prominent justification for the judgments. While the early settlements, such as the 1996 Liggett Group settlement involved a lump-sum transfer, later settlements more closely resembled taxes on future sales. The Master Settlement between State Attorneys General and the tobacco industry required payments of more than $200 billion over 25 years, but those payments were indexed to operating revenue, which makes them as much a tax as a classic settlement. While conventional tobacco taxes and bans were not explicitly justified as a response to misleading advertising, some of the enthusiasm for those policies might be associated with antipathy for prior advertising policies.

The first public policy responses to misleadingly advertising cigarettes were advertising bans and counter advertising, but something like cigarette taxes eventually followed. In the next sections of the paper, we will consider the welfare effects of those different responses to misleading advertising.

[2] Text is available at http://tobaccodocuments.org/ness/31311.html

3 Misperception and welfare

We now turn to a simple model of misperception and welfare. The are J identical firms, who pay constant marginal costs (which we take to be 0 for simplicity), and compete Cournot-style in selling a product. There are N people who each receive benefits of $a \cdot i$ if they consume the product where a is a constant and i is uniformly distributed on the unit interval. There is also a true health cost of consuming the product which we denote c. The value of c is not known and we assume that all individuals believe that the health cost to them of consuming the product is $\hat{c}$, which is possibly erroneous. We let $\varepsilon \equiv c - \hat{c} \geq 0$ denote the error. We first ask about the welfare consequences of an exogenous error and then endogenize the level of error.

If the product is sold at price P, then demand equals $Q(P) = \frac{N}{a}(a - \hat{c} - P)$. Cournot behavior means that $q_j = q(\varepsilon) = \frac{N(a-\hat{c})}{a(J+1)}$, so that equilibrium sales, price and profits are given by

$$Q(\varepsilon) = \frac{JN(a - \hat{c})}{a(J+1)} \tag{1}$$

$$P(\varepsilon) = \frac{a - \hat{c}}{J+1} \tag{2}$$

$$\Pi(\varepsilon) = JN\frac{(a - \hat{c})^2}{a(J+1)^2} \tag{3}$$

Total sales, price, and profits are all increasing in the error. When there are more firms, the positive effect of the error on total production becomes larger ($Q_{\varepsilon J} > 0$), because more firms increase their output in response to a higher demand (although each individual firm responds less: $q_{\varepsilon J} < 0$). At the same time, the positive effect of ε on price and profits becomes smaller as the market turns more competitive. The decreasing price effect is a direct consequence of $Q_{\varepsilon J} > 0$, since the direct effect of ε on price $P(Q) = a - \hat{c} - \frac{a}{N}Q$ is independent of the number of firms. The decreasing effect of the error on profits is a consequence of the well-known competitive externality that Cournot firms impose on each other. This externality is stronger when there are more firms, and therefore the error ε increases profits by less in this case. Under perfect competition, a small increase in the error has no first-order effect on either price or profits.

3.1 The welfare effect of misinformation

What is the impact of misinformation on welfare and consumer surplus? The primary distinction between this paper and Dixit and Norman (1978) is that we focus on consumer surplus based on the true health costs of the product, not on consumer surplus based on beliefs and preferences at the time of purchase. If we consider utility based on beliefs at the time of purchase, then almost all advertising, misleading or not, is beneficial. We refer welfare based on the true health costs of the product as "ex-post consumer surplus," $CS_{xpost}(\varepsilon)$. We also consider ex-post welfare which equal profits plus consumer surplus: $W_{xpost}(\varepsilon) \equiv \Pi(\varepsilon) + CS_{xpost}(\varepsilon)$.[3] Proposition 1 follows (all proofs are in Appendix A).

Proposition 1 *Ex-post consumer surplus is decreasing in the error ε. Ex-post welfare is increasing in the error iff*

$$\varepsilon < \frac{a-c}{J} \equiv \varepsilon^*(J). \tag{4}$$

Consumer surplus declines with misperception, ε, because misperception increases the equilibrium price in (2). This increase in price represents a transfer to firms, and condition (4) tells us that social welfare rises with misinformation as long as the error is small. Social welfare is maximized when output is equal to the competitive output where price equals zero and everyone knows the true health costs of the good. In that case, output is $Q_c(\varepsilon = 0) = \frac{N}{a}(a-c)$. Condition (4) is then necessary and sufficient for $Q(\varepsilon) < Q_c(0)$: the equilibrium output with imperfect competition and misinformation, given by (1), is less than the first-best quantity. Equivalently, condition (4) is necessary and sufficient for $P(\varepsilon) > \varepsilon$: for the equilibrium price to be larger than the error. In this model, with demand given by $Q(P) = \frac{N}{a}(a - c + \varepsilon - P)$, reducing price or raising the error has the same impact on the equilibrium quantity $Q(\varepsilon)$. As price rises, the error needs to rise to achieve the first best quantity $Q_c(0)$ which means that the optimal error is larger in less competitive markets.

Since Cournot behavior implies that too little of the good is being consumed relative to the social optimum, misinformation that increases consumption offsets this underconsumption. Since imperfect competition leads to high prices and too little consumption, unless we are in a competitive equilibrium, some misinformation will always increase welfare. Naturally, this does not mean that the level of misinformation in the case of either patent medicines or cigarettes was optimal. This positive effect of misinformation won't impact consumer surplus but it will increase profits and it is because we include profits in social welfare that this quantity rises with misinformation, as illustrated in Figure 1. As the inverse demand curve shifts out, area A is the reduction in consumer surplus of existing consumers, and area B is the losses to new consumers who buy based on the wrong beliefs. These losses are offset by an increase in firm profits, and do not reduce welfare. The area that drives welfare is C, which is the (true) utility that new consumers get from consuming the product. Even though the net surplus of these new consumers is negative because their expenditure is $B+C$, area C nevertheless represents a social gain: it is deadweight loss turned into profits. This gain will exist and welfare will increase with misinformation as long as there is any deadweight loss under the true preferences, until the error is so large that output reaches the intercept $Q_0 = \frac{N}{a}(a-c)$, which is the competitive output level under the true preferences. The level of $\varepsilon^*(J)$ defined under (4) is the first-best level of misinformation, since it guarantees that the competitive level of output is produced.

[3] We ignore all issues concerning non-comparability across individuals (or firms).

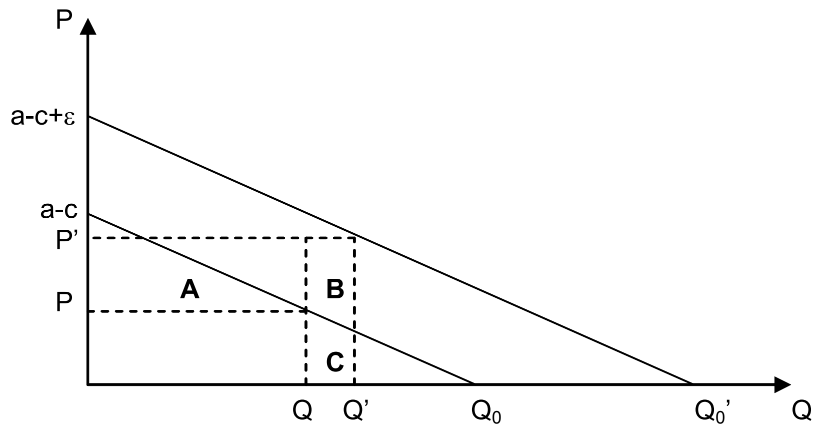

Figure 1: Welfare effect of ε.

When all consumers have the same health costs of using a product, the presence of misinformation is not enough to indicate a problem. The key question is whether too much of the good is being sold relative to the first best. In the case of cigarettes, we may be confident that health costs are so high that very few people should smoke, but in other cases, misinformation may beneficially counter other market imperfections that cause under consumption. The welfare effects of misinformation become much more complicated if there are heterogeneous health consequences of consumption in the population. In that case, misinformation can create added welfare losses by inducing the "wrong" people to consume.

Misperception might also have consequences in other areas of consumers' lives. For example, underestimating (or underestimating) the health consequences of a disease might lead to too much (or too little) of other forms of risky behavior beyond consuming the product.

The result in Proposition 1 follows Dixit and Norman (1978). They also considered the possible welfare gains of increasing demand holding preferences fixed (which they labeled advertising), but they emphasized the possibility that the resulting increase in output might increase consumer surplus. The result above is complementary to theirs, since welfare increases even though the price of the product increases and hence consumer surplus is always reduced.

3.2 Other Social Welfare Functions

Other definitions of social welfare will qualify the result in Proposition 1. If we care more about consumer surplus than profits so $W(\varepsilon, \phi) = CS(\varepsilon) + \phi\Pi(\varepsilon)$, where $\phi < 1$, then $\frac{\partial W(\varepsilon,\phi)}{\partial \varepsilon} > 0$ as long as

$$\phi > \frac{(a - \hat{c}) + (J + 1)\varepsilon}{2(a - \hat{c})} = \frac{P(\varepsilon) + \varepsilon}{2P(\varepsilon)}$$

The assumption that $\phi < 1$, of course, implies that consumers don't own the firms and that the government lacks the ability to tax away firms' profits. As long as $\phi > .5$, some misinformation will continue to be welfare enhancing.

While the "ex post" measure of consumer surplus has some merit since it reflects the true health-cost c, one can also argue for looking at "ex ante" consumer surplus CS_{xante}, based on the consumer's erroneous beliefs.[4] As Fisher and McGowan (1979) argue, the hedonic interpretation of utility can be seen as suggesting that it is the consumer's own judgement of his well-being that really matters.

If the good provides an intertemporal flow of consumption, and tomorrow the consumer is going to realize his mistake today, then perhaps we might care about a weighted sum of ex ante and ex post utility:

$$CS_\gamma(\varepsilon) \equiv (1-\gamma)CS_{xpost}(\varepsilon) + \gamma CS_{xante}(\varepsilon),$$

where $\gamma < 1$ is a constant. Define the corresponding welfare measure as $W_\gamma(\varepsilon) = \Pi(\varepsilon) + CS_\gamma(\varepsilon)$. Note that ex ante consumer surplus includes the error ε in the utility of the average consumer, so that

$$CS_{xante}(\varepsilon) = CS_{xpost}(\varepsilon) + \varepsilon Q(\varepsilon). \tag{5}$$

Putting any weight on ex-ante surplus brings us closer to Dixit and Norman (1978) and strengthens the appeal of misinformation:.

Proposition 2 *CS_γ is increasing in the error as long as*

$$\gamma > \frac{\varepsilon + \frac{a-\hat{c}}{J+1}}{a-\hat{c}+\varepsilon}.$$

$W_\gamma(\varepsilon)$ is increasing in the error as long as

$$\gamma > \frac{\varepsilon - \frac{a-\hat{c}}{J+1}}{a-\hat{c}+\varepsilon}$$

The first part of the proposition implies that compared to true preferences, a small error will improve consumer surplus as long as the weight on ex-ante consumer surplus is at least $\gamma > \frac{1}{J+1}$. When the market is perfectly competitive, any positive weight on ex-ante surplus implies that small amounts of misinformation increase consumer surplus. In this perfectly competitive market, misinformation only affects quantity, not price. The impact that misinformation has on surplus through changes in quantity consumed will be second order. The direct impact of misinformation

[4] Ex ante consumer surplus CS_{xante} is computed as $(E[ai|ai - \hat{c} > P] - P - \hat{c})Q$, where $E[ai|ai - \hat{c} > P]$ is the average utility from the product of those who chose to consume it.

on the hedonic flow of utility included in ex ante surplus is first order. In competitive markets, there is a threshold weight γ at which a small error raises this weighted consumer surplus. The weight required for ε to improve weighted social welfare is always smaller than the threshold to improve weighted consumer surplus because welfare includes profits.

4 Endogenous misinformation and welfare

4.1 Endogenous misinformation

The discussion above has taken the error as exogenous. In the remainder of the paper we assume that it is produced by the firms. We do not address the psychology of persuasion which is the topic of Mullainathan, Schwartzstein and Shleifer (2006). Instead, we assume that for a cost, firms can mislead consumers. Specifically, we assume that if each of the J identical firms spends Z_j dollars on misinforming the consumers, the error will be $\varepsilon = \varepsilon(\sum Z_j)$, where $\varepsilon(\cdot)$ is an increasing function that is sufficiently concave for second order conditions to hold.

Our assumption that advertising is a public good among the firms makes the model more relevant in some markets than in others. As we have discussed, in the 1950s the Federal Trade Commission specially banned cigarette companies from making firm-specific health claims. More generally, some public good component is arguably present in any advertisement. Even the most idiosyncratic drug advertisement informs some consumers about the risks of a particular condition. Any cigarette add showing vibrant outdoorsy people smoking suggests a connection between tobacco and health. We therefore think that our results have at least some relevance in the welfare evaluation of any misleading advertising.[5]

In the model, firms choose their spending Z_j simultaneously.[6] We focus on the symmetric equilibrium with $Z_j = Z$ for all j. The following proposition characterizes the equilibrium level of advertising and how it is affected by the parameters of the model.

Proposition 3 *The equilibrium advertising level Z solves*

$$\frac{2N}{a(J+1)^2}(a - c + \varepsilon(JZ))\varepsilon'(JZ) - 1 = 0. \tag{6}$$

Total advertising expenditure increases with market-size ($\frac{\partial(JZ)}{\partial N} > 0$), decreases with the true health-cost ($\frac{\partial(JZ)}{\partial c} < 0$), and decreases with the number of firms ($\frac{\partial(JZ)}{\partial J} < 0$).

This proposition describes the determinants of misinformation. Larger markets will inspire more misinformation because the benefits of misinformation are proportional to market size but the costs are not. If the costs of misinformation rose with market size, then this result could disappear. As

[5] Appendix C discusses an extension in which misleading advertising is a private good.

[6] Given symmetry, the assumption of simultaneous moves only affects the distribution of profits among the firms. It has no impact on ε or any of the other quantities of interest.

real health costs rise, the incentive to misinform declines, because the impact the error will have on demand and price will be smaller. As competition rises, misinformation falls. This is a classic example of the free rider problem. All firms benefit by confusing consumers about the costs of the product, but if there are too many firms, they will fail to invest in this industry-level public good. This result would disappear if misinformative advertising was firm-specific, which is why the highly competitive field of patent medicine had abundant amounts of misinformative advertising.

4.2 Welfare effects

If we are concerned with ex post consumer surplus, then Proposition 3 should lead consumer advocates to fear monopoly because of high prices and misinformation. Monopolists have stronger incentives to mislead consumer which further reduces ex post consumer surplus. As the market approaches perfect competition, then Proposition 3 implies that the equilibrium level of misinformation goes to zero.[7]

We now include advertising costs in profits so that $\Pi(Z) = \Pi(\varepsilon(JZ)) - JZ$, where $\Pi(\varepsilon(JZ))$ is total profits defined under (3). Ex post welfare is then given by $W_{xpost}(Z) = CS_{xpost}(\varepsilon(JZ)) + \Pi(Z)$. Let $\varepsilon^{**}(J)$ denote the welfare-maximizing level of misinformation, given that it is supplied at a cost born by competing firms, which we define as the second-best level of misinformation. If firms invest in misinformation to maximize their profits, then the level of equilibrium misinformation can still be either too high or too low:

Proposition 4 *The second-best level of misinformative advertising Z^{**} satisfies*

$$\frac{JN}{a(J+1)^2}(a - c - J\varepsilon(JZ^{**}))\varepsilon'(JZ^{**}) = 1 \tag{7}$$

and we denote the corresponding error $\varepsilon^{**}(J)$.

The value of $\varepsilon^{*}(J)$ is always greater than $\varepsilon^{**}(J)$ because $\varepsilon^{**}(J)$ takes the costs of advertising into account and optimal misinformation must therefore be lower. Even though misinformation will increase welfare in the monopoly case, the equilibrium level of misinformation is always too high. The monopolist doesn't internalize the negative impact that misinformation has on social welfare and therefore produces too much misinformation.

5 Regulating misleading advertising

In this section, we study some of the possible regulatory responses to misleading advertising and their welfare consequences. The interventions we consider are (i) a tax or a ban on advertising; (ii)

[7] Moreover, Proposition 2 implies that if ex ante consumer surplus enters into our CS measure with any positive weight, starting from a competitive equilibrium, a small exogenous increase in the error is beneficial. In this case, perfect competition provides too little error.

product market regulations, including sales and profit taxes and price control (iii) rival advertising or other changes in the firms' technology of misinformation. A direct tax (or, equivalently, a partial ban) on misinformation can implement the second-best. We then show that indirect instruments, such as product market regulations or rival advertising, are generally inferior. One exception is a tax on profits gross of advertising costs, which in this model also yields the second-best.

5.1 Taxing or banning advertising

The second-best level of misinformation ε^{**} can be implemented using either a tax or a quantity restriction (partial ban) on misinformation Z. The level of misleading advertising Z^{**} for which $\varepsilon^{**} = \varepsilon(JZ^{**})$ is determined implicitly by (7). To achieve this level of misleading advertising, the regulator may set a quantity limit on Z equal to $\bar{Z} = Z^{**}$. Alternatively, a tax rate τ_Z (with a lump-sum rebate) may be set such that

$$\tau_Z = \frac{2N}{a(J+1)^2}(a - c + \varepsilon(JZ^{**}))\varepsilon'(JZ^{**}) - 1,$$

in which case, from (6), the equilibrium level of misinformation will be exactly Z^{**}.

The simple equivalence of taxes and bans breaks down if we allow for entry (which we study in Appendix C), or the possibility of targeting regulations to specific market segments (which we discuss in Section 7 below). Appendix B shows the difficulties which arise if regulation affects different firms differently, perhaps because some of them find a way to circumvent the ban.

5.2 Rival advertising and changes in the technology of persuasion

Assume now that the government can take some action Z_g affecting the technology of persuasion, so that the error becomes $\varepsilon(\sum Z_j, Z_g)$. We will refer to Z_g as "rival advertising" aimed at educating the consumers. Assume $\varepsilon_1 > 0$, $\varepsilon_{11} < 0$ as before, and let $\varepsilon_2 < 0$. Define $x \doteq \frac{-\varepsilon_2\varepsilon_1}{a-\hat{c}}$ and $x' \doteq \frac{\varepsilon_2\varepsilon_{11}}{\varepsilon_1}$ (from second order conditions and the assumptions on ε we know that $0 < x < x'$).

We first discuss the effects of rival advertising on misinformation and then turn to the welfare effects of these policies.

5.2.1 Rival advertising and misinformation

Proposition 5 *An increase in rival advertising Z_g (i) reduces both misleading advertising and the error if $\varepsilon_{12} < x$; (ii) increases misleading advertising and reduces the error if $x < \varepsilon_{12} < x'$; (iii) increases both if $\varepsilon_{12} > x'$.*

When government advertising reduces the marginal effect of firm advertising, because $\varepsilon_{12} \leq 0$, an increase in government advertising always reduces misinformation. However, when $\varepsilon_{12} > 0$, the effect on deception is no longer unambiguous. If ε_{12} is large enough so that $\varepsilon_{12} > x'$, firms will react

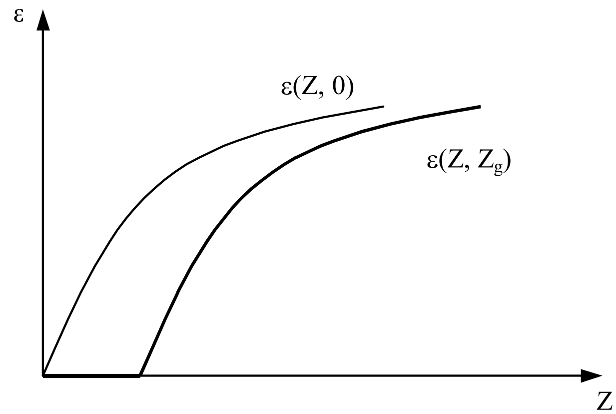

Figure 2: Rival advertising with $\varepsilon_{12} > 0$.

to government advertising by increasing their own advertising to such an extent that the amount of consumer error (ε) actually rises. A different interpretation of this result is that *increasing* the effectiveness of firms' misinformation technology can be useful if that increase ends up reducing the amount of firms' investment in misinformation. When $\frac{\varepsilon_{11}}{\varepsilon_1} + \frac{\varepsilon_1}{a-\hat{c}}$ is large in absolute value, then firm advertising responds less to public advertising and in that case, the range of ε_{12} for which public advertising will both increase firm advertising and reduce consumer error is larger.

We illustrate Proposition 5 in the case of three different misinformation technologies.

Example 1 *Assume $\varepsilon(\sum Z_j, Z_g) = \varepsilon(\sum Z_j - Z_g)$, so that government advertising simply reduces the "stock" of advertising responsible for misinformation. One can check that in this case $\varepsilon_{12} = x'$, therefore Proposition 5 implies that ε remains constant, and government advertising has no effect on the equilibrium level of misinformation.*

Example 2 *The reduction in the stock of misleading advertising that the government can achieve is inversely proportional to the stock itself: $\Delta Z = \frac{\mu_1 Z_g}{Z}$ and the relationship between the stock of advertising and misinformation is linear: $\varepsilon(Z, Z_g) = \mu_2(Z - \Delta Z) = \mu_2(Z - \mu_1 \frac{Z_g}{Z})$ (where μ_1, μ_2 are constants). In this case, $\varepsilon_{12} > x'$ for $\varepsilon > 0$, therefore, from Proposition 5 such an intervention will always result in an increase in misinformation. Examples 1 and 2 are illustrated in Figure 2. In both cases, as Z_g increases holding Z constant, ε shifts out horizontally.*

Example 3 *Consumers believe that a product is either "unhealthy", with health cost c, or "healthy", with health cost $c - \varepsilon_0$, with probabilities $(1 - r)$ and r, respectively. Firms can influence these (subjective) probabilities by investing in advertising (for example, by increasing the number of ads claiming or suggesting that the product is "healthy"), so that $r = r(Z)$, $r' > 0$, $r'' < 0$. The government can influence consumers' perceptions of what "healthy" and "unhealthy" mean, i.e., it*

can affect the consumers' estimate of the difference in health costs between the two products.[8] *Let* $\varepsilon_0 = \varepsilon_0(Z_g)$ *with* $\varepsilon_0 < 0$*, so that the government can take actions to show that a claim of healthiness implies a lower difference in health costs that consumers would have thought.*
Because in this formulation the expected health cost is $c - r\varepsilon_0$*, we may write* $\varepsilon(Z, Z_g) = r(Z)\varepsilon_0(Z_g)$*. One can check that the assumptions of Proposition 5 are satisfied with* $\varepsilon_{12} < 0$*, therefore such an intervention always reduces misinformation.*

Example 4 *Misinformation only affects the beliefs of a fraction* u *of the population (the "uniformed"), while fraction* $(1-u)$ *always holds correct beliefs.*[9] *As long as the* uN *uninformed individuals have the same distribution of taste parameters* i *as the population, the model is equivalent to one where misinformation affects everyone, but the error that the firms can create is* $u\varepsilon(JZ)$*.*
To see this, note that with uN *uninformed consumers, the demand function becomes*

$$\begin{aligned} Q(P) &= \frac{uN}{a}(a - \hat{c} - P) + \frac{(1-u)N}{a}(a - c - P) \\ &= \frac{N}{a}(a - c + u\varepsilon - P), \end{aligned}$$

which is equivalent to a model with $\hat{c} = c - u\varepsilon$*.*
If the government has the ability to reduce the fraction of uninformed individuals, so that $u = u(Z_g)$*, then we are back in the previous example with* $\varepsilon(Z, Z_g) = \varepsilon(Z)u(Z_g)$*, and increasing* Z_g *always reduces misinformation.*[10]

How does rival advertising compare to a direct tax or quantity control on misinformation in terms of reducing the error ε? One way of making the comparison is to focus on small changes in policies, and ask whether a change in Z_g, τ_Z, or a combination of these is more *effective* (i.e. yields a larger decrease in the error). The result is given in the following proposition (where $x'_- \equiv x' - \varepsilon_1$ and $x'_+ \equiv x' + \varepsilon_1$).

Proposition 6 *A tax is more effective at reducing the error than rival advertising iff* $x'_- < \varepsilon_{12}$*. Combining taxation and rival advertising is more effective than a tax alone iff* $\varepsilon_{12} < x'_+$*.*

Proposition 6 is illustrated in Figure 3, where the instruments are ranked according to their effectiveness at reducing the error in the various ranges of ε_{12}. When $\varepsilon_{12} < x$, government advertising

[8] *This is a simple way of modelling "truth-content" regulations which effectively define what certain words or phrases commonly used in advertisements have to mean. More generally, this assumption also captures the notion that firms usually provide information about the products they produce, while government campaigns might provide more generic information about the desirability of general classes of products, technologies, inputs/ingredients etc.*

[9] *For example,* Z *might be an advertisment that has the potential of being misunderstood as claiming something which is untrue (e.g. that a drug is less risky than it in fact is). The assumption is that a fraction* u *of the population misunderstands the ad.*

[10] *For example, the government could send* Z_g *messages about true health-cost, which are received randomly in the whole population. Then, measure of informed agents is* $(1 - u_0 + u_0\frac{Z_g}{N})N$*, and the measure of the uninformed is* $Nu_0(1 - \frac{Z_g}{N})$*.*

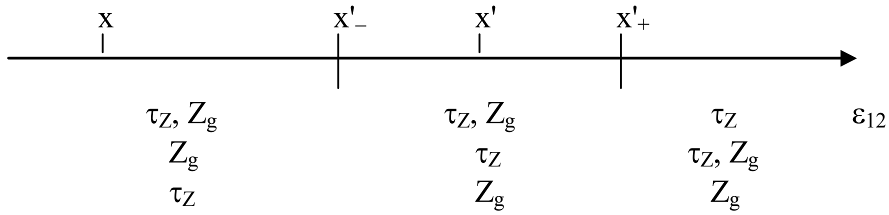

Figure 3:

has a multiplicator effect on consumer error by inducing firms to reduce their private advertising, as seen in Proposition 5. In that case, rival advertising will always be more effective than a tax (combining public advertising and a tax is even more effective). When $x'_{-} > x$ (the case depicted in the Figure), this ranking will also hold for a range above x, so that government advertising is more effective than a tax despite the induced increase in private advertising. This requires that the direct effect of public advertising on consumer error to be sufficiently large.[11]

Once $\varepsilon_{12} > x'_{-}$, taxation will become more effective than rival advertising. However, combining rival advertising with a tax will be even more effective as long as $\varepsilon_{12} < x'_{+} = \varepsilon_2 \frac{\varepsilon_{11}}{\varepsilon_1} + \varepsilon_1$. Intuitively, if the impact of public advertising on consumer error is large enough, combining public advertising with a tax to limit firms' response will lead to a larger effect on consumer error than a tax alone. This holds even in a range above x', where public advertising alone would *increase* the error. Finally, when $\varepsilon_{12} > x'_{+}$, the induced increase in private advertising will always outweigh the direct benefit of rival advertising, and using only a tax dominates the combined policy.

5.2.2 Rival advertising and welfare

Using the results in Section 3, the welfare analysis of rival government advertising can be undertaken based on Proposition 5.

Corollary 1 *Ex post consumer surplus is increased by rival advertising Z_g if and only if $\varepsilon_{12} < x'$.*

This follows from the fact that private advertising only enters consumer surplus through consumer error. Even though $\varepsilon_{12} \in [x, x']$ implies an increase in private advertising in response to government advertising, the cost of which are born by the firms, these costs do not affect consumer utility, and ex post consumer surplus increases if and only if consumer error is reduced. We now consider total welfare, which includes consumer surplus, profits, and the cost of firm advertising. To make the best case for government advertising, we ignore its costs.

[11]Formally, $x'_{-} > x$ iff $\varepsilon_2 \cdot SOC^* > \varepsilon_1$.

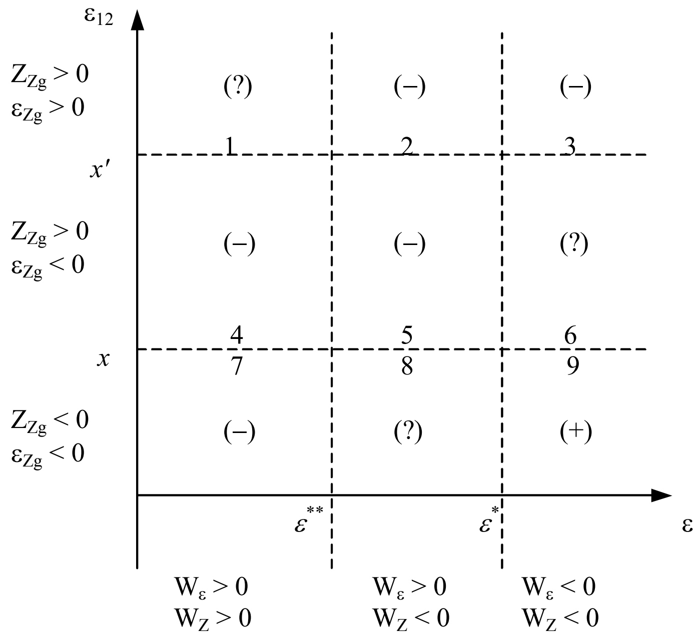

Figure 4: Welfare effect of Z_g (positive: +, negative: -, ambiguous: ?).

Corollary 2 *The welfare effects of rival advertising Z_g are positive when ($\varepsilon_{12} < x$ and $\varepsilon > \varepsilon^*$), ambiguous when ($\varepsilon_{12} < x$ and $\varepsilon^* > \varepsilon > \varepsilon^{**}$), ($x' > \varepsilon_{12} > x$ and $\varepsilon > \varepsilon^*$) or ($\varepsilon_{12} > x'$ and $\varepsilon < \varepsilon^{**}$), and negative otherwise.*

This corollary is illustrated in Figure 4, showing the welfare effects of public advertising are a function of the equilibrium error and the impact of public advertising on the effectiveness of private advertising, ε_{12}. The figure shows the optimal direction of the policy, if the government can decide to increase or decrease its advertising. Even if we assume that the equilibrium error is excessive, reducing misinformation through rival advertising unambiguously increases welfare only if the intervention does not increase the effectiveness of firm advertising too much ($\varepsilon_{12} < x$). In all other cases, the welfare effect of public advertising is at best ambiguous, and often strictly negative.

These difficulties with rival advertising suggest that, even if free, such a policy is inferior to the direct tax or quantity control on misinformation studied above. This observation is formalized in the following proposition.

Proposition 7 *Once an optimal tax or quantity limit on misleading advertising is in place, rival advertising by the government cannot improve welfare.*

5.3 Product market regulations

5.3.1 Taxing production

Suppose that the government levies a tax τ on the product (with revenues rebated lump sum), so that each firm's objective function in the Cournot game becomes

$$(a - \hat{c} - \tau - \frac{aQ}{N})q_j. \tag{8}$$

As the following Proposition shows, such a tax always reduces misinformation. Nevertheless, its effect on consumer surplus may well be negative.

Proposition 8 *An increase in the product tax always reduces the error ε. However, it increases prices, and its effect on consumer surplus is negative whenever*

$$\frac{\varepsilon'[a - \tau - c - (J+2)\varepsilon]}{-[(\varepsilon')^2 + (a - \hat{c} - \tau)\varepsilon'']} < J(a - c - \tau) - \varepsilon. \tag{9}$$

A tax on production makes misinformation less profitable, and the resulting decrease in misleading advertising raises consumer surplus. This effect is shown on the left hand side of (9). At the same time, the tax leads to an increase in prices as firms pass the tax burden on to consumers (for given ε). The resulting decrease in consumer surplus is the right hand side of (9). This negative effect is larger the more firms there are, because each reduces its production slightly without internalizing the full effect of the resulting price increase. Whenever (9) holds, consumer surplus is reduced.

5.3.2 Price caps

Because misinformation increases the price of the product, capping prices may be among the regulatory responses considered. The effects of introducing a price cap $\bar{P}$ below the equilibrium price $P(\varepsilon(JZ))$ are described by the following proposition.

Proposition 9 *A smaller price cap $\bar{P}$ reduces misinformation. However, in general, the second-best level of misinformation cannot be implemented with a price cap.*

The price cap reduces the profitability of misinformation and therefore reduces the level of misleading advertising. At the same time, the cap also has a direct positive effect on demand, and the resulting increase in sales moves the equilibrium closer to the competitive output. This implies a decrease in the level of the error that is desirable in the second-best. By changing the price cap the regulator is changing both the equilibrium and the target level of ε, and in general the second-best cannot be implemented in equilibrium.

5.3.3 Taxing profits without allowing advertising cost deductions

Consider taxing profits instead of production. Taxing profits net of advertising costs has no effect on firm behavior, merely redistributes wealth from the firms to the consumers (under a rebate). If however, tax policy does not allow advertising costs to be deducted from firms' tax base then the firms' objective in the advertising game becomes

$$\pi(Z_j, \tau_\pi) = (1 - \tau_\pi) \frac{N(a - \hat{c})^2}{a(J + 1)^2} - Z_j, \tag{10}$$

where τ_π is the profit tax.

Assuming that tax revenues are rebated to consumers lump sum, the tax has no direct effect on welfare. Thus, the welfare effects work entirely through the firms' choice of advertising level Z, and we have the following result.

Proposition 10 *A tax on profits always reduces Z, and can implement the second-best level of misinformation ε^{**}.*

While the second part of the Proposition may seem surprising, it is a simple consequence of the fact that a tax $(1 - \tau_\pi)$ on profits gross of advertising costs is equivalent to a tax $1 + \tau_Z = \frac{1}{1-\tau_\pi}$ on misleading advertising (see (10)), and we saw in Section 5.1 that the latter policy can always implement the second-best.

6 Advertising that increases utility

6.1 Firms' problem and welfare

We now turn to the more realistic assumption that advertising has both positive and negative effects. Like Becker and Murphy (1993), we allow advertising to directly increase utility. We also continue assuming that advertising misleads consumers. We now assume that people's utility from the product is $a_0 + a \cdot i$ with $i \sim U[0, 1]$. Demand is $Q(P) = \frac{N}{a}(\hat{a} - \hat{c} - P)$, where $\hat{a} \equiv a_0 + a$. Firms now can invest in both misleading advertising Z_j and advertising that increases consumers' utility. We let Y_j denote expenditure on utility-increasing advertising, and we assume that $a_0 = a_0(\sum Y_j)$, where a_0 is increasing and concave. For symmetry reasons, we assume that both forms of advertising have the same public good aspect to them.

Utility-increasing advertising tends to raise the desirability of having at least a small exogenous error. Proposition 1 implies that a larger level of utility-increasing advertising (a larger a_0) will increase the positive effect of misperception on welfare. The next proposition implies that this effect is reinforced when advertising levels are chosen optimally, because a larger error in turn implies a higher equilibrium level of utility-increasing advertising.

Proposition 11 *The equilibrium levels of utility-increasing advertising Y and misleading advertising Z are complements for the firms ($\frac{\partial Y}{\partial Z} > 0$). The comparative statics of Proposition 3 continue to hold: a rising N and a declining c increase both Z and Y, and increasing the number of firms J reduces both total misleading advertising JZ and total truthful advertising JY.*

We also find that.

Proposition 12 *An increase in misleading advertising Z raises welfare if and only if*

$$\varepsilon(JZ) < (\hat{a} - c)\frac{J-2}{J^2+2} + \frac{\varepsilon''}{a_0''}\frac{(\hat{a} - c)(J2 + J^2 - 2) + \varepsilon(J-2)}{(J^2+2)}. \tag{11}$$

On the right-hand side of (11), the first term is the threshold below which misleading advertising is suboptimal in a market equilibrium with no useful advertising. The second term on the right-hand side is positive, so that with useful advertising, the equilibrium level of misinformation will more often be suboptimal. This effect reflects the complementarity between misleading and utility-increasing advertising described in Proposition 11. As a lower level of misleading advertising also implies a lower level of utility increasing advertising, improving welfare requires tolerating a higher level of misinformation. As the curvature of ε relative to a_0 ($\frac{\varepsilon''}{a_0''}$) rises, a given change in misleading advertising yields a larger change in utility enhancing advertising, so that the complementarity becomes stronger, and a higher level of misinformation becomes desirable, as (11) shows.

6.2 Regulating misinformation in the presence of utility-increasing advertising

We now turn to the impacts of different policies when firms provide both misleading and useful advertising. Under a tax, the reduction in misinformation is larger than in the benchmark case of no useful advertising because of the complementarity between the two types of advertisements.

Proposition 13 *A tax (or partial ban) on misleading advertising reduces the equilibrium levels of both Z and Y. Moreover, the presence of Y magnifies the effect of the policy on Z: the reduction in misleading advertising is larger than what it would be holding a_0 constant.*

When utility increasing advertising is also present, regulating Z alone cannot achieve the second-best level of misinformation ε^{**}. The most a tax (or a ban) can achieve is some third best, $\varepsilon^{***}(J)$, which is the level of misinformation providing highest welfare taking into account the equilibrium relationship between misleading advertising and utility-increasing advertising. This third-best level of misinformation is given implicitly by the right-hand side of (11). To achieve it, the tax rate τ_Z should be set so that

$$\tau_Z = \frac{\varepsilon'(JZ^{***})}{a_0'(JY^{***})} - 1,$$

where Y^{***} denotes the equilibrium level of utility-increasing advertising corresponding to $Z = Z^{***}$.

The difficulties with a ban on misinformation would be even more severe if the government cannot differentiate misinformation from useful advertising. This case corresponds more to cigarettes than to patent medicine. The Camel adds which shows doctors smoking Camels and Marlboro advertisements showing healthy cowboys smoking are probably both utility enhancing and misleading. The complementarity of misleading and utility-increasing advertising implies that a tax (or quantity restriction) on misleading advertising reduces both the level of misinformative advertising and the level of useful advertising.

We now turn to rival advertising. As before, when government advertising aimed at reducing the error raises the effectiveness of firm advertising, firms' optimal response may lead to an increase in the error. On the other hand, when rival advertising is effective at reducing the error, it will also induce a reduction in useful advertising, which will yield a further decrease in misleading advertising. Thus, in the presence of utility-increasing advertising, the effect of rival advertising on misinformative advertising will more often be negative.

Proposition 14 *Using the notation $x \doteq \frac{-\varepsilon_2 \varepsilon_1}{\hat{a}-\hat{c}}$ and $x' \doteq \frac{\varepsilon_2 \varepsilon_{11}}{\varepsilon_1}$ corresponding to Proposition 5, rival advertising reduces misleading advertising Z iff $\varepsilon_{12} < x \frac{(\hat{a}-\hat{c})a_0''}{(\hat{a}-\hat{c})a_0''+(a_0')^2}$. It reduces the error as well as utility-increasing advertising Y if and only if $\varepsilon_{12} < x'$.*

Because $\frac{(\hat{a}-\hat{c})a_0''}{(\hat{a}-\hat{c})a_0''+(a_0')^2} > 1$, the threshold for public advertising to have a negative effect on misleading advertising is higher than before. Because private misinformative advertising and public advertising only affect the equilibrium level of utility-increasing advertising through the amount of consumer error, the condition for rival advertising to reduce the error is the same as in the benchmark model. A smaller error also reduces utility-increasing advertising.

With utility-increasing advertising, the result of Proposition 7 no longer holds: even if an optimal tax or quantity limit on misleading advertising is in place, rival advertising by the government can sometimes improve welfare.

Proposition 15 *Rival advertising improves upon the optimal direct regulation of misleading advertising whenever*

$$\frac{\varepsilon_{12}}{a_0''} \frac{(\hat{a}-c)(J2+J^2-2)+\varepsilon^{***}(J-2)}{\hat{a}-\hat{c}} + \frac{JN}{a(J+1)^2}(\hat{a}-c-J\varepsilon^{***})\varepsilon_2 > 0. \tag{12}$$

The second term in (12) is the direct effect of rival government advertising on welfare through the consumer error. As in Proposition 7, this is always negative. The first term represents the direct effect of public advertising on useful advertising.[12] It is positive as long as $\varepsilon_{12} < 0$, in which case public advertising has a direct positive effect on useful advertising. If this first term is large

[12]This direct effect is not accounted for by the change in Z, and it is therefore not optimized out when Z is optimally regulated.

enough, the effect of rival advertising on welfare will be positive even if an optimal tax or quantity limit on private advertising is already in place.

Can a tax on production also improve upon an optimal direct tax or ban on misleading advertising? As the following Proposition shows, the answer is negative.

Proposition 16 *Once an optimal tax or quantity limit on misleading advertising is in place, a tax on production cannot improve welfare.*

In Proposition 15, rival advertising was able to improve upon direct regulation because the former policy has an asymmetric impact on misleading and utility-increasing advertising. Therefore, rival advertising is able to implement changes in advertising levels which a single instrument cannot replicate. With product taxes, things are different. Such an instrument affects the optimal levels of misleading and useful advertising symmetrically, i.e. it does not change their relationship in equilibrium. Therefore, any change in advertising levels brought about by a product tax can be implemented with a direct tax on misinformation, and an optimal advertising tax cannot be improved upon.

It is easy to see that this intuition carries over to the case of profit taxes (without allowing for the cost deduction of either type of advertising). Because such a tax also affects misinformation and useful advertising symmetrically, it cannot improve upon an optimal advertising tax, as the following proposition shows.

Proposition 17 *Once an optimal tax or quantity limit on misleading advertising is in place, a tax on profits cannot improve welfare.*

7 Market Targeting

Some of the most contentious discussions of misleading information concerns advertising to children who are presumably more prone to believe misinformation. To address this, we assume there are two market segments and return to the case where all advertising is misinformation. The first one is the "high-valuation" segment, where the utility from the product is given by $a \cdot i_H$ with $i_H \sim U[\psi a, 1]$, where $\psi < 1$ is a constant. In the second, "low-valuation" segment, utility is $a \cdot i_H$, with $i_H \sim U[0, \psi a]$. For simplicity, assume that the relative size of the two segments reflect their valuations, so that there are ψN and $(1-\psi)N$ individuals in each segment respectively.

If each firm can choose how much to invest in advertising in each of the two market segments (denoted by Z_L and Z_H respectively), the following proposition shows that they will never invest in both segments.

Proposition 18 *Consider the level of advertising Z^* which satisfies $\frac{2N}{a(J+1)^2}(a-c+\varepsilon(JZ^*))\varepsilon'(JZ^*) = 1$. If*

$$J(\psi a - c) + (\psi - 1)a > \varepsilon(JZ^*), \tag{13}$$

then in equilibrium, only the low-valuation segment gets positive advertising, with $Z_L^* = Z^*$ *and* $Z_H^* = 0$. *If the reverse holds, then only the high-valuation segment gets positive advertising, with* $Z_H^* = Z^*$ *and* $Z_L^* = 0$.

Firms earn higher profits if they can target their ads to consumers that will respond to them more. The proposition implies that regulatory policies will only be effective at reducing misinformation if they encompass the market segment that firms are targeting. For example, if (13) holds, a ban which only affects the high-valuation segment will be ineffective. If a ban has differential impact in the two segments, only its impact in the targeted segment matters.

The possibility of targeting regulation may affect regulatory costs (hence welfare). If targeted bans or taxes are feasible, the enforcement costs of such policies may be lower than attempting to regulate misinformation in the entire market. Similarly, counter-advertising targeted at the relevant market segment may be cheaper than also providing information to non-marginal consumers. The proposition suggests that banning advertising to the young may be efficient even if the young think as clearly as adults. If firms target the young because they are new consumers who are particularly likely to respond to advertising, then it may make sense to particularly ban advertising against this group.

8 Conclusion

In this paper, we have examined the impact of misinformation on social welfare and the impacts of different governmental responses to misinformation. Our first result was that misinformation may not be socially inefficient. If a monopoly has high prices and then misleads people into consuming more, and if monopoly profits are distributed across the population, then misinformation can be welfare enhancing. Consumer error leads to more consumption which offsets the underconsumption due to monopoly prices. Misinformation is more likely to be welfare reducing when prices are closer to marginal costs than in a more monopolistic setting. When misleading advertising was endogenized, we found that monopolies will always produce too much misinformation.

When advertising only acts to misinform, then the second best outcome can be created by a tax on advertising or an equivalent quantity control. Counter-advertising by the government is inefficient both because it may have its own costs and because it can increase firm advertising. Taxes on sales and price caps also fail to replicate the second best outcome. These results suggest that quantity restrictions on false advertising in the spirit of the Pure Food and Drug Act of 1906 may have been an efficient response to the problem of misleading advertising of patent medicine.

When advertising both misinforms and increases utility, then the results are more nuanced. A simple tax on advertising cannot yield the second best outcome because the tax reduces both good and bad forms of advertising even if the ban only applies to misinformation because the two types of advertising are complements. If an optimal tax is put in place, then it may still be optimal for the government to engage in counter advertising detailing the health costs of the product. Taxes on the product do not increase welfare given an optimal tax or ban on advertising. This result suggest that the government policy towards cigarettes that both limited some forms of firm advertising and engaged in counter-advertising may have been efficient. However, responding to misleading advertising with a tax on cigarettes seems less likely to be efficient relative to using other policy levers.

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A Appendix: Proofs

A.1 Misperception and misleading advertising

Proof of Proposition 1

Ex-post consumer surplus may be computed as the average utility of those individuals who chose to consume the product, $\frac{J(a-c)-\varepsilon(J+2)}{2(J+1)}$,[13] times the number of consumers, $Q(\varepsilon) = J\frac{N(a-\hat{c})}{a(J+1)}$.

[13] To find this, write the average utility as $E[ai|ai-\hat{c}-P>0]-c-P$. Using $P=\frac{a-\hat{c}}{J+1}$, this is $\frac{1}{2}(\frac{a-\hat{c}}{J+1}+a+\hat{c})-c-\frac{a-\hat{c}}{J+1}$. Rearranging gives the expression in the text.

This gives

$$CS_{xpost}(\varepsilon) = JN\frac{(a-c)^2J - (a-c)\varepsilon - \varepsilon^2(J+2)}{2a(J+1)^2}. \tag{14}$$

Ex-post welfare is $W_{xpost}(\varepsilon) \equiv \Pi(\varepsilon) + CS_{xpost}(\varepsilon)$. Adding (3) and (14) and taking derivatives one can check that (4) is necessary and sufficient for $\frac{dW_{xpost}}{d\varepsilon} > 0$.

Proof of Proposition 2

For the first part, write $CS_\gamma(\varepsilon) = CS_{xpost}(\varepsilon) + \gamma\varepsilon Q(\varepsilon)$ and use the definitions in (1) and (14) to find the derivative as

$$-JN\frac{a-c+\varepsilon(J+2)}{a(J+1)^2} + \gamma\frac{JN(a-c+2\varepsilon}{a(J+1)}.$$

Rearrange to get the condition. Use (3) and the expression above to derive the second condition.

Proof of Proposition 3

Using (1) and (2), in the advertising game, firm j's profit is given by

$$\pi(Z_j) = \frac{N(a-c+\varepsilon(\sum Z_{j'}))^2}{a(J+1)^2} - Z_j.$$

The equilibrium value of the average advertising level $Z \equiv \frac{1}{J}\sum Z_j$ is determined by the first order condition (6). The second-order condition is

$$\frac{2N}{a(J+1)^2}[(\varepsilon')^2 + (a-c+\varepsilon)\varepsilon''] < 0. \tag{15}$$

It will be convenient to use the notation $SOC \equiv (\varepsilon')^2 + (a-\hat{c})\varepsilon''$, and $SOC^* \equiv \frac{SOC}{(a-\hat{c})\varepsilon'}$ (the latter is SOC around the equilibrium, obtained by substituting (6) into the left-hand side of (15)). Clearly, with constant symmetric marginal costs of advertising, any vector $(Z_1, ..., Z_J)$ such that $\frac{1}{J}\sum Z_j = Z^*$ is an equilibrium. We focus on the symmetric equilibrium with $Z_j = Z^*\ \forall\ j$.

The comparative statics can be obtained directly from (6) and (15).

Proof of Proposition 4

Welfare is

$$CS(\varepsilon) + \Pi(\varepsilon) - JZ = \frac{JN}{2a(J+1)^2}(a-\hat{c})((J+2)(a-c) - J\varepsilon) - JZ.$$

Taking the derivative,

$$\frac{\partial(CS + \Pi - JZ)}{\partial Z} = J\left[\frac{JN}{a(J+1)^2}(a-c-J\varepsilon)\varepsilon' - 1\right]$$

This equation implies that the socially optimal level of adverting is

$$\frac{JN}{a(J+1)^2}(a-c-J\varepsilon(JZ^{**}))\varepsilon'(JZ^{**}) = 1.$$

and we define $\varepsilon^{**} = \varepsilon(JZ^{**})$

A.2 Regulation

Proof of Proposition 5

Comparative statics w.r.t. Z_g yield

$$\frac{\partial Z^*}{\partial Z_g} = \frac{-\varepsilon_2\varepsilon_1 - (a-\hat{c})\varepsilon_{12}}{J((\varepsilon_1)^2 + (a-\hat{c})\varepsilon_{11})}, \tag{16}$$

which is negative iff $\varepsilon_{12} < x$. Moreover,

$$\frac{d\varepsilon}{dZ_g} = J\varepsilon_1\frac{\partial Z}{\partial Z_g} + \varepsilon_2 = \frac{(a-\hat{c})(\varepsilon_2\varepsilon_{11} - \varepsilon_1\varepsilon_{12})}{(\varepsilon_1)^2 + (a-\hat{c})\varepsilon_{11}}, \tag{17}$$

which is negative iff $\varepsilon_{12} < x'$.

Proof of Proposition 6

For the first part, use (16) and (17) to write $\frac{\partial\varepsilon}{\partial\tau_Z} = \frac{\varepsilon_1}{SOC^*} < \frac{(a-\hat{c})(\varepsilon_2\varepsilon_{11}-\varepsilon_1\varepsilon_{12})}{SOC} = \frac{d\varepsilon}{dZ_g}$, and rearrange using the definitions of SOC and SOC^*. For the second part, write the total derivative of ε as

$$\begin{aligned} d\varepsilon &= J\varepsilon_1(\frac{\partial Z}{\partial Z_g}dZ_g + \frac{\partial Z^*}{\partial \tau_Z}d\tau_Z) + \varepsilon_2 dZ_g \\ &= \frac{\varepsilon_1}{SOC}\{[-\varepsilon_2\varepsilon_1 - (a-\hat{c})\varepsilon_{12}]dZ_g + (a-\hat{c})\varepsilon_1 d\tau_Z\} + \varepsilon_2 dZ_g. \end{aligned} \tag{18}$$

We want to compare (18), and $d\varepsilon$ under the tax only policy, $\frac{\varepsilon_1}{SOC^*}d\tau_Z$. Letting $dZ_g = d\tau_Z = \delta$, we find

$$\begin{aligned} \frac{\varepsilon_1}{SOC}\{[-\varepsilon_2\varepsilon_1 - (a-\hat{c})\varepsilon_{12}] + (a-\hat{c})\varepsilon_1\}\delta + \varepsilon_2\delta &< \frac{\varepsilon_1}{SOC}\delta \\ -\varepsilon_2\varepsilon_1 - (a-\hat{c})\varepsilon_{12} + (a-\hat{c})\varepsilon_1 + \frac{\varepsilon_2}{\varepsilon_1}[(\varepsilon_1)^2 + (a-\hat{c})\varepsilon_{11}] &> 0 \\ x' + \varepsilon_1 &> \varepsilon_{12}, \end{aligned}$$

where we have substituted in $SOC = (\varepsilon_1)^2 + (a-\hat{c})\varepsilon_{11}$.

Proof of corollary 2

The effects of changing Z_g on W_{xpost} are given by

$$\frac{d[W_{xpost}(\varepsilon(JZ, Z_g), Z^*) - \gamma Z_g]}{dZ_g} = \frac{\partial W}{\partial \varepsilon}\left(J\varepsilon_1 \frac{\partial Z}{\partial Z_g} + \varepsilon_2\right) - J\frac{\partial Z}{\partial Z_g}. \tag{19}$$

Rewrite expression (19) alternatively as

$$\frac{dW(\varepsilon, Z)}{dZ_g} = \frac{\partial W}{\partial \varepsilon}\frac{d\varepsilon}{dZ_g} - J\frac{\partial Z}{\partial Z_g}, \tag{20}$$

or as

$$\frac{dW(\varepsilon, Z)}{dZ_g} = \frac{dW}{dZ^*}\frac{\partial Z}{\partial Z_g} + \frac{\partial W}{\partial \varepsilon}\varepsilon_2. \tag{21}$$

Based on our previous results regarding the signs of $\frac{\partial W}{\partial \varepsilon}, \frac{dW}{dZ^*}, \frac{dZ}{dZ_g}$ and $\frac{d\varepsilon}{dZ_g}$ in Propositions 4 and 5, the welfare effects may be signed unambiguously for a large portion of the parameter space using (20) and (21).

Proof of Proposition 7

Write the derivative of welfare w.r.t. Z_g as

$$\frac{\partial(CS + \Pi - JZ)}{\partial Z_g} = \left[\frac{\partial(CS + \Pi)}{\partial \varepsilon}\varepsilon_1 - J\right]\frac{\partial Z}{\partial Z_g} + \frac{\partial(CS + \Pi)}{\partial \varepsilon}\varepsilon_2$$

Under an optimal tax or quantity limit, the term in brackets is 0. Because $\varepsilon = \varepsilon^{**}$ under such a policy, the second term is

$$\frac{\partial(CS + \Pi)}{\partial \varepsilon}\varepsilon_2 = \frac{JN}{a(J+1)^2}(a - c - J\varepsilon^{**})\varepsilon_2.$$

Because $\varepsilon^{**} < \varepsilon^* = \frac{a-c}{J}$, and $\varepsilon_2 < 0$ by assumption, this expression is negative. Thus, starting from an optimal tax, rival government advertising can only reduce welfare.

Proof of Proposition 8

Solving the Cournot game with the objectives given in (8), we find $q(\varepsilon, \tau) = \frac{N(a-\hat{c}-\tau)}{a(J+1)}$, $P(\varepsilon, \tau) = \frac{a-\hat{c}+J\tau}{J+1}$, and the equilibrium profit is $\pi(\varepsilon, \tau) = \frac{N(a-\hat{c}-\tau)^2}{a(J+1)^2}$. Thus, price increases in the production tax.

The first order condition of the advertising game is

$$\frac{2N}{a(J+1)^2}(a - \hat{c} - \tau)\varepsilon' - 1 = 0, \tag{22}$$

and write the corresponding second order condition as $SOC^*(\tau) = \frac{2N}{a(J+1)^2}\left\{(\varepsilon')^2 + (a - \hat{c} - \tau)\varepsilon''\right\} < 0$. The comparative statics yield $\frac{\partial Z}{\partial \tau} = \frac{\varepsilon'}{JSOC^*(\tau)} < 0$.

Consumer surplus is

$$CS(\varepsilon,\tau) = \frac{JN}{2a(J+1)^2}[(a-\tau-c)^2 J - 2(a-\tau-c)\varepsilon - \varepsilon^2(J+2)].$$

The derivative w.r.t. τ is proportional to

$$-[a-\tau-c+(J+2)\varepsilon]\frac{\partial\varepsilon}{\partial\tau} - J(a-c-\tau)+\varepsilon.$$

Using (22) to find $\frac{\partial\varepsilon}{\partial\tau} = \frac{\varepsilon'}{-[(\varepsilon')^2+(a-\hat{c}-\tau)\varepsilon'']}$ and rearranging yields the condition in the text.

Proof of Proposition 9

With capped prices, equilibrium sales are given by $Q^* = \frac{N}{a}(a-\hat{c}-\bar{P})$. A firm's profit becomes $\pi_j(Z_j) = \bar{P}(a-\hat{c}-\bar{P})\frac{N}{aJ}$, and the equilibrium advertising level solves the first order condition

$$\frac{\bar{P}N}{aJ}\varepsilon'(JZ^*) = 1. \tag{23}$$

With $\varepsilon'' < 0$, we get $\frac{\partial Z}{\partial \bar{P}} > 0$.

Here, $CS(\varepsilon,\bar{P}) = \left(\frac{a-\hat{c}-\bar{P}}{2} - \varepsilon\right)\frac{a-\hat{c}-\bar{P}}{a}N$, and the condition $\frac{\partial(\Pi+CS-JZ)}{\partial Z} = 0$ defining the second-best is $\frac{N}{a}(\bar{P}-\varepsilon)J\varepsilon' - J = 0$. Using the first order condition (23), this becomes $\bar{P}\frac{J-1}{J} = \varepsilon$. Clearly, this condition and (23) will in general yield a different ε.

Proof of Proposition 10

For the first part, the comparative statics give

$$\frac{\partial Z}{\partial \tau_\pi} = \frac{1}{J\cdot SOC^*(\tau_\pi)} < 0,$$

where $SOC^*(\tau_\pi) \equiv (1-\tau_\pi)(\frac{\varepsilon'}{a-\hat{c}} + \frac{\varepsilon''}{\varepsilon'})$. For the second part, note simply that a tax $(1-\tau_\pi)$ on profits gross of advertising costs is equivalent to a tax $1+\tau_Z = \frac{1}{1-\tau_\pi}$ on misleading advertising (see 10), and we saw in Section 5.1 that the latter policy can always implement the second-best.

A.3 Utility-increasing advertising

Proof of Proposition 11

Allowing firms to optimally set both Z_j and Y_j, each firm solves

$$\max_{Z_j,Y_j}\frac{N(A+a_0(\sum Y_{j'})+\varepsilon(\sum Z_{j'}))^2}{a(J+1)^2} - Z_j - Y_j.$$

First-order conditions are

$$\frac{2N(A + a_0 + \varepsilon)\varepsilon'}{a(J+1)^2} = 1$$

and

$$\frac{2N(A + a_0 + \varepsilon)a_0'}{a(J+1)^2} = 1.$$

For future reference, write the second order condition as the requirement that the Hessian $\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} \\ h_{12} & h_{22} \end{bmatrix}$ be negative semi-definite, where

$$\begin{aligned} h_{11} &= \frac{2N}{a(J+1)^2}[(\varepsilon')^2 + (\hat{a} - \hat{c})\varepsilon''] \\ h_{12} &= \frac{2N}{a(J+1)^2}\varepsilon' a_0' \\ h_{22} &= \frac{2N}{a(J+1)^2}[(a_0')^2 + (\hat{a} - \hat{c})a_0'']. \end{aligned}$$

(As before, we focus on symmetric equilibria in which $Z_j = Z$ and $Y_j = Y$ for all j.) The first order conditions imply that $\varepsilon'(JZ^*) = a_0'(JY^*)$ in equilibrium. Because $\varepsilon'' < 0$ and $a_0'' < 0$, it follows that $\frac{\partial Y^*}{\partial Z^*} > 0$.

Proof of Proposition 12

The sum of consumer surplus and profits is $\frac{JN}{2a(J+1)^2}[(2+J)(\hat{a} - c)^2 + 2(\hat{a} - c)\varepsilon - J\varepsilon^2]$. The derivative w.r.t. Z is

$$\frac{\partial(CS + \Pi - JZ - JY)}{\partial Z} = J\left[\frac{JN}{a(J+1)^2}[(\hat{a} - c - J\varepsilon)\varepsilon' + ((\hat{a} - c)(2+J) + \varepsilon)a_0' Y'] - 1 - Y'\right].$$

From $\varepsilon'(JZ) = a_0'(JY)$, Y' is $\frac{\varepsilon''}{a_0''}$, and using both, we get

$$\frac{\partial(CS + \Pi - JZ - JY(Z))}{\partial Z} \sim [(\hat{a} - c)\frac{J-2}{J^2+2} - \varepsilon] + \frac{\varepsilon''}{a_0''}\frac{(\hat{a} - c)(J2 + J^2 - 2) + \varepsilon(J-2)}{J^2+2}.$$

Proof of Proposition 13

The comparative statics w.r.t. a change in the tax τ_Z can be written as

$$J\mathbf{H}\begin{bmatrix} \frac{\partial Z}{\partial \tau_Z} \\ \frac{\partial Y}{\partial \tau_Z} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Using Cramer's rule and the definition of $\mathbf{H}$ above, we find

$$\begin{aligned}
\frac{\partial Z}{\partial \tau_Z} &= \frac{h_{22}}{J \det \mathbf{H}} < 0 \\
\frac{\partial Y}{\partial \tau_Z} &= -\frac{h_{12}}{J \det \mathbf{H}} = \frac{2NJ}{a(J+1)^2} \frac{-a_0' \varepsilon'}{J \det \mathbf{H}} < 0
\end{aligned}$$

Comparing $\frac{\partial Z}{\partial \tau_Z} = \frac{h_{22}}{J \det \mathbf{H}}$ to $\frac{\partial Z}{\partial \tau_Z}|_{a_0 = const.} = \frac{1}{J h_{11}}$, one can verify that $\frac{\partial Z}{\partial \tau_Z} < \frac{\partial Z}{\partial \tau_Z}|_{a_0 = const.} < 0$, so that the presence of Y magnifies the effect of τ_Z on Z.

Proof of Proposition 14

The comparative statics w.r.t. Z_g can be written as

$$J \bar{\mathbf{H}} \begin{bmatrix} \frac{\partial Z}{\partial Z_g} \\ \frac{\partial Y}{\partial Z_g} \end{bmatrix} = \begin{bmatrix} -(\hat{a} - \hat{c})\varepsilon_{12} - \varepsilon_2 \varepsilon_1 \\ -\varepsilon_2 a_0' \end{bmatrix}$$

where $\bar{\mathbf{H}} = \frac{a(J+1)^2}{2N} \mathbf{H}$. Solving, we find

$$\frac{\partial Z}{\partial Z_g} = -(\hat{a} - \hat{c}) \frac{[(\hat{a} - \hat{c})a_0'' + (a_0')^2]\varepsilon_{12} + \varepsilon_2 \varepsilon_1 a_0''}{J \det \bar{\mathbf{H}}}$$

therefore $\frac{\partial Z}{\partial Z_g} < 0$ iff $\varepsilon_{12} < x \frac{(\hat{a} - \hat{c})a_0''}{(\hat{a} - \hat{c})a_0'' + (a_0')^2}$.

Similarly,

$$\frac{d\varepsilon}{dZ_g} = J\varepsilon_1 \frac{\partial Z}{\partial Z_g} + \varepsilon_2 = \frac{(\hat{a} - \hat{c})[(\hat{a} - \hat{c})a_0'' + (a_0')^2](\varepsilon_{11}\varepsilon_2 - \varepsilon_1 \varepsilon_{12})}{J \det \bar{\mathbf{H}}}$$

The middle term in the numerator is (a constant times) h_{22}, which is negative, therefore $\frac{d\varepsilon}{dZ_g} < 0$ iff $\varepsilon_{12} < x'$.

Finally,

$$\frac{\partial Y}{\partial Z_g} = a_0'(\hat{a} - \hat{c}) \frac{-\varepsilon_2 \varepsilon_{11} + \varepsilon_{12} \varepsilon_1}{J \det \bar{\mathbf{H}}},$$

therefore $\frac{\partial Y}{\partial Z_g} < 0$ iff $\varepsilon_{12} < x'$.

Proof of Proposition 15

We saw that in equilibrium, $\varepsilon' = a_0'$. This now becomes $\varepsilon'(JZ, Z_g) = a_0'(JY)$, we can therefore write $Y(Z, Z_g)$ as the equilibrium level of utility-increasing advertising as a function of Z and Z_g. Denoting $\frac{\partial Y}{\partial Z_g}$ the derivative of Y holding Z constant, write the derivative of welfare with respect to Z_g as

$$\frac{dW}{dZ_g} = \left[\frac{dW}{dZ} + \frac{\partial W}{\partial Y}\frac{dY}{dZ}\right]\frac{dZ}{dZ_g} + \frac{\partial W}{\partial Y}\frac{\partial Y}{\partial Z_g} + \frac{\partial W}{\partial Z_g}.$$

When the direct regulation of misleading advertising Z is optimally set, the term in brackets is equal to 0 (as in Proposition 12). Therefore, we get

$$\begin{aligned}\frac{dW}{dZ_g} &= \frac{\partial W}{\partial Y}\frac{\partial Y}{\partial Z_g} + \frac{\partial W}{\partial Z_g} \\ &= \frac{\varepsilon_{12}}{a_0''}\frac{(\hat{a}-c)(J2+J^2-2)+\varepsilon^{***}(J-2)}{\hat{a}-\hat{c}} + \frac{JN}{a(J+1)^2}(a-c-J\varepsilon^{***})\varepsilon_2.\end{aligned}$$

Therefore $\frac{dW}{dZ_g}$ is positive if and only if (12) holds.

Proof of Proposition 16

Under a product tax, the first-order conditions become

$$\frac{2N(A+a_0+\varepsilon-\tau)\varepsilon'}{a(J+1)^2} = 1$$

and

$$\frac{2N(A+a_0+\varepsilon-\tau)a_0'}{a(J+1)^2} = 1.$$

Therefore $\varepsilon'(JZ) = a_0'(JY)$ holds in equilibrium, which in turn implies that $\frac{\partial Y(Z)}{\partial \tau} = 0$: the product tax has no effect on the equilibrium relationship between Y and Z.

Using this observation, we may proceeding as in Proposition 15, writing

$$\frac{dW}{d\tau} = \left[\frac{dW}{dZ} + \frac{\partial W}{\partial Y}\frac{dY}{dZ}\right]\frac{dZ}{d\tau} + \frac{\partial W}{\partial \tau}.$$

When the direct regulation of misleading advertising Z is optimally set, the term in brackets is equal to 0. Therefore, we have

$$\frac{dW}{d\tau} = \frac{\partial W}{\partial \tau} = \frac{-JN}{a(J+1)^2}(a-c-J\varepsilon^{***}+J\tau),$$

which is negative.

Proof of Proposition 17

Under a product tax, the first-order conditions become

$$(1-\tau_\pi)\frac{2N(A+a_0+\varepsilon)\varepsilon'}{a(J+1)^2} = 1$$

and

$$(1-\tau_\pi)\frac{2N(A+a_0+\varepsilon)a_0'}{a(J+1)^2}=1.$$

We again have $\varepsilon'(JZ)=a_0'(JY)$, and since the profit tax τ_π has no direct impact on welfare, $\frac{dW}{d\tau_\pi}=\left[\frac{dW}{dZ}+\frac{\partial W}{\partial Y}\frac{dY}{dZ}\right]\frac{dZ}{d\tau_\pi}=0$ when a tax on Z has been optimally set.

A.4 Market Targeting

Proof or Proposition 18

Denote $\varepsilon(JZ_L)=\varepsilon_L$ and $\varepsilon(JZ_H)=\varepsilon_H$. Given the distribution of tastes, the demand function is given by the following expression

$$Q(P)=\min\left[1,\frac{a-c+\varepsilon_H-P}{(1-\psi)a}\right](1-\psi)N+\max\left[0,\frac{\psi a-c+\varepsilon_L-P}{\psi a}\right]\psi N. \tag{24}$$

Assume first that the equilibrium price satisfies $P<\psi a-c$. In this case, the whole high-valuation segment will consume, and (24) becomes

$$Q(P)=(1-\psi)N+\frac{\psi a-c+\varepsilon_L-P}{\psi a}\psi N$$

or

$$P(Q)=a-c+\varepsilon_L-\frac{a}{N}Q.$$

This inverse demand function has the same form as in the baseline case, so that the equilibrium is characterized by the same first-order condition $\frac{2N}{a(J+1)^2}(a-c+\varepsilon(JZ_L^*))\varepsilon'(JZ_L^*)=1$. Since ε_H does not affect the price, firms will set $Z_H^*=0$. Because the equilibrium price is given by $P=\frac{a-c+\varepsilon_L}{J+1}$, the condition $P<\psi a-c$ is equivalent to the condition (13) in the proposition.

Next, suppose that $P>\psi a-c+\varepsilon_L$ in equilibrium, so that no-one from the low-valuation segment consumes. Then (24) becomes

$$Q(P)=\frac{a-c+\varepsilon_H-P}{(1-\psi)a}(1-\psi)N,$$

and similar reasoning to the previous case shows that in equilibrium $Z_L^*=0$ and Z_H^* is given by $\frac{2N}{a(J+1)^2}(a-c+\varepsilon(JZ_H^*))\varepsilon'(JZ_H^*)=1$. Since $\varepsilon_L=0$, the condition $P>\psi a-c+\varepsilon_L$ is exactly the reverse of (13).

What is left to show is that advertising in both segments cannot be an equilibrium. It follows from our previous argument that only the case $\psi a-c<P<\psi a-c+\varepsilon_L$ needs to be checked. Clearly, if $P<\psi a-c+\varepsilon_H$, then the whole high-valuation segment will consume, and targeting advertising to the lower segment is optimal. Thus $P<\psi a-c$, contradicting the assumption we started with. If $P > \psi a - c + \varepsilon_H$, both segments consume, and the demand function becomes

$$Q(P) = \frac{a - c + \varepsilon_H - P}{(1-\psi)a}(1-\psi)N + \frac{\psi a - c + \varepsilon_L - P}{\psi a}\psi N,$$

yielding

$$P(Q) = (1+\psi)a - 2c + \varepsilon_L + \varepsilon_H - \frac{a}{N}Q.$$

Because the two errors affect the price symmetrically, concavity implies that $\varepsilon'(JZ_H^*) = \varepsilon'(JZ_L^*)$ in equilibrium, so that firms advertising in both segments would do so in equal amounts. However, this implies that $P < \psi a - c + \varepsilon_L$ and $P > \psi a - c + \varepsilon_H$ cannot both be true.

B Appendix: Asymmetric regulation

The discussion in the main text has relied on the fact that symmetric firms are regulated in a symmetric manner. In reality, regulation might affect firms differently for a number of reasons. Enforcement of the regulation might be imperfect, or firms differ in some attribute that does not affect productivity, but affects the cost that a tax or ban imposes on them. The next proposition shows that asymmetries in regulation can lead to surprising redistributions of profits across the firms. Moreover, as long as at least one firm escapes taxation or finds some way around the ban, regulation has no impact on total advertising (hence welfare).

Proposition 19 *(i) Assume that different firms face different limits on advertising. Unless the overall limit imposed on firms is smaller than equilibrium total advertising with no regulation (JZ_0^*), the policy has no effect on total advertising. When the overall limit is smaller than JZ_0^*, every firm will advertise the maximum amount it is allowed to.*
(ii) Assume that different firms face different tax rates on advertising. In equilibrium, the firm with the lowest tax rate will be the only one to advertise. If the lowest tax rate is 0, the advertising of this firm will be JZ_0^*, so that regulation has no effect on total advertising.

Proof. Let Z_0^* denote equilibrium average advertising with no regulation, and assume the government imposes a ban $\bar{Z}_j$ on firm j.

(i) Since the equilibrium of the advertising game, (6), only pins down total advertising, $\sum\limits_j \bar{Z}_j \geq JZ_0^*$ implies $JZ^* = JZ_0^*$. If $\sum\limits_j \bar{Z}_j < JZ_0^*$, the first order conditions of the advertising game become

$$\frac{2N(a - c + \varepsilon(JZ))\varepsilon'(JZ)}{a(J+1)^2} - 1 \geq 0 \text{ with } Z_j \leq \bar{Z}_j \text{ complementarity},$$

implying that $Z_j^* = \bar{Z}_j$ for all j.

(ii) Let τ_Z^j be the tax on firm j, and, w.l.o.g., let $\tau_Z^1 = \min_j \tau_Z^j$. The first order conditions of the advertising game with taxation are

$$\frac{2N(a-c+\varepsilon(JZ))\varepsilon'(JZ)}{a(J+1)^2} - (1+\tau_Z^j) \leq 0 \text{ with } Z_j \geq 0 \text{ complementarity.}$$

Because $\tau_Z^1 < \tau_Z^j$, only firm 1's FOC will hold with equality, and it will be the only firm to advertise a positive amount: $Z_1 = Z^*$. If $\tau_Z^1 = 1$, the equilibrium condition is the same as with no taxation, therefore the same JZ^* obtains. ■

The first result implies that in this model, a ban will be ineffective as long as at least one firm finds a way around regulation.

The second result implies that the firm with the lowest tax rate will bear the full burden of providing the equilibrium level of advertising.[14] When at least one firm escapes taxation, regulation is purely redistributive. Compared to the symmetric equilibrium, that firm's profit decreases by $\frac{(J-1)Z^*}{J}$ and each of the other firms experiences an increase in profits of $\frac{Z^*}{J}$. In this case, regulation has no effect on misinformation or welfare.

With more general (increasing) marginal costs, taxation will reduce total advertising, but less than if the tax was uniform. Profits will still be redistributed towards the firms facing a higher tax rate. To illustrate this point, assume linear marginal costs kz. It can be shown that increasing firm 1's marginal cost k_1 slightly above k reduces Z_1 while increasing Z_j $j \neq 1$. For a small change starting from $k_1 = k$, the change in total advertising is the same as with a uniform increase in k. The only difference is the redistribution of profits from $j = 2, ..., J$ to $j = 1$. When the change is larger in magnitude, or $k_1 \neq k$ to start with, differential taxation leads to a smaller change in Z than a uniform policy.

C Appendix: Private misinformation and entry

Consider now a version of the model in which misinformation is a private good. Write $\varepsilon_j(Z_j)$, and assume that consumers have the same preferences as above. This implies that if $\varepsilon_j(Z_j) < \varepsilon_k(Z_k)$, everyone will buy from firm k and no-one buys from j. Assume $J = 2$ to simplify the discussion.

Given firm 1's choice, firm 2's profit may be written as

$$\pi_2(Z_2) = \left\{ \begin{array}{c} 0 \text{ if } Z_2 < Z_1 \text{ (in which case } Z_2 = 0 \text{ is optimal)} \\ N\frac{(a-c+\varepsilon(2Z_2))^2}{9a} - Z_2 \text{ if } Z_2 = Z_1 \\ N\frac{(a-c+\varepsilon(Z_2))^2}{4a} - Z_2 \text{ if } Z_2 > Z_1 \end{array} \right\}.$$

One may check that if firms choose advertising levels simultaneously as before, no pure-strategy

[14] This is because in a public good game with constant marginal costs, everyone will free-ride on the contributions of the lowest cost agent as long as contributions are perfect substitutes.

Nash equilibrium exists.[15] Let us therefore assume that firm 1 chooses Z_1 first, and firm 2 responds by choosing Z_2.[16]

For any Z_1, firm 2 will choose Z_2 slightly above it, unless that would yield $\pi_2 \leq 0$, in which case he prefers $Z_2 = 0$. Given this, the unique Subgame Perfect Nash Equilibrium is for 1 to choose the Z_1 yielding 0 profits,[17] defined by

$$\pi(Z_1) = N\frac{(a - c + \varepsilon(Z_1))^2}{4a} - Z_1 = 0. \tag{25}$$

Firm 2's best response is to choose $Z_2 = 0$. Thus, firm 1 uses misinformation to deter entry. In the product market, Firm 1 acts as a monopoly, creates a deadweight loss by underproducing, but earns 0 profits. The equilibrium level of Z given in (25) is higher than any level observed in the case where misinformation was a public good, including the case of public good with monopoly. This discussion verifies the following proposition:

Proposition 20 *Allowing for entry, equilibrium misinformation will be higher than the profit-maximizing level under monopoly.*

The effects of regulating advertising are described in the following propositions. We see that the case for regulating advertising is much stronger in this case than when misinformation was a public good.

Proposition 21 *A (partial) ban on advertising set sufficiently high increases profits as well as consumer surplus, hence raises welfare.*

Proof. Suppose the limit $\bar{Z}$ on advertising is close to Z_1 given in (25), so that it satisfies

$$N\frac{(a - c + \varepsilon(2\bar{Z}))^2}{9a} - \bar{Z} < 0. \tag{26}$$

Then the ban increases the profit of Firm 1 without changing the structure of the equilibrium. To see this, note that if $\pi' \geq 0$ at Z_1, then firm 2 would enter with a higher Z, contradicting the equilibrium. Thus, $\pi'(Z_1) < 0$ necessarily. Moreover, if (26) holds, Firm 2 will still have no incentive to enter. Therefore, a ban $\bar{Z} < Z_1$ raises the incumbent's profit.

The derivative of consumer surplus is $\frac{dCS}{dZ} = -JN\frac{(a-c)^2 J-(a-c)\varepsilon-\varepsilon^2(J+2)}{2a(J+1)^2} < 0$ as before, thus, banning Z increases both profits and consumer surplus, leading to an increase in welfare. ■

[15] Firms' best response is to outbid each-other until the monopoly profit is 0. However, if both firms choose this advertising level, they become a duopoly, with negative profits, so they both prefer $Z = 0$. But if one chooses $Z = 0$, the other's best response is to reduce his Z and make positive profits.

[16] In the public good case, introducing sequential moves would have no effect on the equilibrium Z^*, and therefore on any quantity of interest. The only difference would be to redistribute profits between firms by inducing every firm to free-ride on the one choosing its advertising level last.

[17] This result is analogous to that found in the theory of contestable markets (explaining why the threat of entry could force monopoly profits to 0).

Proposition 22 *(i) A small uniform tax of τ_Z leaves profits unchanged while increasing consumer surplus, so that welfare is raised.*

(ii) If the entrant (firm 2) is taxed at rate τ_Z and the incumbent (firm 1) is taxed at a lower rate, consumer surplus is raised by the same amount as in (i), but profit increases, leading to a higher welfare gain.

(iii) If the incumbent (firm 1) is taxed at rate τ_Z and the entrant (firm 2) is taxed at a lower rate, 2 will become a monopoly. This case yields the largest increase in consumer surplus, profit, and hence welfare.

Proof. (i) A uniform tax rate of τ_Z reduces Z as (25) becomes

$$N\frac{(a-c+\varepsilon(Z_1))^2}{4a} - (1+\tau_Z)Z_1 = 0. \tag{27}$$

Profit is still 0, but consumer surplus increases due to reduction in misinformation. Thus, welfare is raised.

(ii) If $\tau_{Z,1} < \tau_{Z,2} = \tau_Z$, equilibrium condition is again (27), yielding the same decrease in Z and increase in consumer surplus as a uniform tax. However, the profit of the incumbent is now higher: $N\frac{(a-c+\varepsilon(Z_1))^2}{4a} - \tau_{Z,1}Z_1 > N\frac{(a-c+\varepsilon(Z_1))^2}{4a} - \tau_Z Z_1$, so welfare is increased by a larger amount.

(iii) If $\tau_{Z,2} < \tau_{Z,1} = \tau_Z$, then firm 1 cannot afford the Z_1 in (25) required to keep firm 2 out of the market. Knowing this, he is forced to choose $Z_1 = 0$. As a response, firm 2 will be a monopolist in the advertising market, setting Z_2 at its profit maximizing level. Because Z_2 is even lower than in (ii), and profit is maximized, this yields even higher welfare. ■

When misleading advertising is a private good, the equilibrium level of misinformation is higher than in the public good case. Thus, the reason for regulatory intervention is stronger, and regulation may yield unambiguous improvements in welfare.

In this model, the threat of entry forces the incumbent firm to overinvest in misinformation. As Proposition 22, part (ii) shows, the regulator can increase welfare by creating entry barriers in the form of higher tax rates for entrants. As part (iii) shows, even higher welfare is achieved if the regulator eliminates the possibility of entry altogether, by forcing the incumbent firm out of the market through higher taxes. (This outcome is equivalent to simply prohibiting one of the firms from entering the market.)

In terms of optimal regulation in the presence of entry, the fundamental difference between the policies is that some can create entry, while others cannot. In particular, a tax will never give rise to entry, therefore the second-best under duopoly, $\varepsilon^{**}(2)$, is not attainable with this policy. It can however implement $\varepsilon^{**}(1)$, the second best under monopoly. A ban can create entry, and therefore can implement the second-best under duopoly. Whether or not the second-best under monopoly can be achieved depends on whether the corresponding level of advertising would induce entry.

Proposition 23 *A ban can achieve the second-best level of misinformation under duopoly, and may or may not achieve the second-best under monopoly. Taxation can achieve the second-best under monopoly, and cannot achieve the second-best under duopoly.*

Proof. Let $Z^{opt,J}$ denote the advertising level which gives the second-best error $\varepsilon^{**}(J)$ with J firms. Then, $Z^{opt,J}$ solves the FOC of welfare maximization:

$$\frac{a-c-J\varepsilon(JZ^{opt,J})}{a(J+1)^2}JN\varepsilon'(JZ^{opt,J}) = J.$$

A ban $\bar{Z} = Z^{opt,2}$ achieves the second-best under duopoly. It can achieve the second-best under monopoly if and only if $Z^{opt,1}$ satisfies

$$N\frac{(a-c+\varepsilon(2Z^{opt,1}))^2}{9a} - Z^{opt,1} < 0.$$

If this is not the case, firm 2 will enter with the same advertising level $Z^{opt,1}$, and the market will be a duopoly.

As explained in Proposition 22, taxation can never give rise to entry, therefore the duopoly solution cannot be achieved with this policy. Under monopoly, the optimal policy is to choose the tax τ_Z so as to solve

$$N\frac{(a-c+\varepsilon(Z^{opt,1}))^2}{4a} - (1+\tau_Z)Z^{opt,1} = 0.$$

In this case, $Z^{opt,1}$ will be the level of advertising chosen by the incumbent to prevent entry. ■