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ABSTRACT

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INFLATION IMPLICATIONS OF RISING GOVERNMENT DEBT*

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Abstract. The intertemporal budget constraint of the government implies a relationship between a ratio of current liabilities to the primary deficit with future values of inflation, interest rates, GDP and narrow money growth and changes in the primary deficit. This relationship defines a natural measure of fiscal balance and can be used as an accounting identity to examine the channels through which governments achieve fiscal sustainability. We evaluate the ability of this framework to account for the fiscal behaviour of six industrialised nations since 1960. We show how fiscal imbalances are mainly removed through adjustments in the primary deficit (80-100%), with less substantial roles being played by inflation (0-10%) and GDP growth (0-20%). Focusing on the relation between fiscal imbalances and inflation suggests extremely modest interactions. This post WWII evidence suggests that the widely anticipated future increases in fiscal deficits, need not necessarily have a substantial impact on inflation.

JEL classification: E31, E62

Keywords: Fiscal deficit, fiscal sustainability, government debt, inflation, intertemporal budget constraint.

1. Introduction

Figures 1 and 2 show recent fiscal trends for six large industrialised nations. Levels of government debt increased markedly during the 1970s, then stabilised and improved during the 1980s and 1990s, but have recently shown signs of further deterioration. With OECD countries experiencing an ageing population, it is widely expected that fiscal positions will worsen yet further in coming decades (see *inter alia* Roseveare, Leibfritz, Fore and Wurzel, 1998, for projections). These considerations raise three important issues: (a) Is current fiscal policy sustainable? (b) How have OECD governments achieved fiscal sustainability in past decades? (c) What are the implications for inflation of these projected rising fiscal deficits? This paper seeks to provide insights to each of these three questions.

Key to our analysis is the intertemporal budget constraint of the government. To answer the first question (on sustainability), we use the methodology of Giannitsarou and Scott (2006) and derive a log linear approximation to the intertemporal budget constraint. Using this framework, we show how debt sustainability requires an equilibrium relationship between the market value of government debt, the stock of narrow money and the levels of government revenue and expenditure. We show how to estimate

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this relationship and derive a measure of sustainability for six OECD countries and use our estimates to characterise the dynamics of fiscal adjustment for our six countries. In order to answer the second question (how do governments achieve fiscal adjustment) we show analytically how deviations from the equilibrium relationship between debt, money and the primary deficit have to be met through future changes in either primary deficits, monetary liabilities, real interest rates, inflation or GDP growth. Using the VAR methodology proposed by Campbell and Shiller (1988), we assess the relative contribution of each channel to financing fiscal activity, for the period 1960-2005. Our third and final focus is to use this framework to assess statistically whether the substantial expected increase in fiscal deficits threatens the current low levels of inflation.

Our findings can be summarised as follows. For the period under consideration and for US, Japan, Germany, UK, Italy and Canada, fiscal imbalances are mostly removed through adjustments in the primary deficit (80-100%), with less important adjustments through inflation (0-10%) and GDP growth (0-20%). In particular, the relation between fiscal imbalances and inflation suggests extremely modest statistical interactions between the two; suggesting that widely anticipated increases in fiscal deficits, due to demographic factors, are not necessarily predictors of higher future inflation. It is important to stress that this conclusion is based around an analysis which is in essence a purely accounting exercise. There exists a large and varied theoretical literature examining the *economic* linkages between fiscal deficits and inflation (see *inter alia* Sargent and Wallace, 1981, McCallum, 1984, Leeper, 1991, Sims, 1994 and Woodford, 1995) but, by contrast, our analysis pursues a different path. Using the accounting framework of the intertemporal budget constraint to empirically document the relative *statistical* role of different factors in achieving fiscal balance.

The plan of the paper is as follows. In Section 2 we examine the properties of our data set and argue that our sample period is appropriate for reviewing how fiscal adjustment is achieved in the face of large variations in government debt in response to sharp increases in social transfers. Using the period-by-period budget constraint we examine evidence for fiscal sustainability and the drivers of the debt/GDP ratio. A simple accounting exercise shows that nominal GDP growth, and especially inflation, played an important role in achieving debt stability. In Section 3 we extend our analysis and introduce our log linearised version of the intertemporal budget constraint and derive our key measure of fiscal imbalances, namely a relationship between levels of government liabilities and the primary deficit. We show how variations in this relationship are related to expected changes in future deficits, money creation and changes in inflation, real interest rates and real GDP growth. In Section 4 we show how to estimate this long run relationship between liabilities and the deficit and we then construct estimates of fiscal imbalances across the countries in our sample and draw conclusions about the dynamics of fiscal adjustment. Section 5 builds on this long run relationship and the restrictions imposed by the intertemporal budget constraint to perform a variance decomposition for how fiscal sustainability is achieved. We extend our framework to assess the separate importance of government expenditure and tax revenue in achieving fiscal balance and examine the predictive role of fiscal imbalances in predicting future inflation. A final section concludes.

2. Fiscal Sustainability: A backward looking approach

Our focus is on assessing the sustainability of fiscal policy in six industrial nations - US, Japan, Germany, UK, Italy and Canada - over the period 1960-2005. Our interest is not just to assess fiscal sustainability, but also to identify the means through which this sustainability is achieved. We choose these nations because we are interested in the implications of demographic induced deficits that are expected to affect these countries in the coming decades. We choose this time period because, in contrast to the majority of the literature, we are interested in how governments achieve fiscal balance in the face of rising non-war related expenditures.

With demographic change expected to lead to increasing social transfers, this sample is a natural environment with which to consider the likely impact of rising debt. For example, over this period, Canada increased social transfers by 4.7% of GDP, Germany by 6.6%, Italy 7.8%, Japan 9.2%, UK 5.9% and US 6.7%. which accounted for 104%, 57%, 51%, 49%, 88% and 146% respectively of the observed increase in total government expenditure during this period. In other words our period is one where increases in government expenditure were heavily linked to rises in social transfers. A further reason for focusing on this post WWII period is that it is the most recent and most relevant: our results in Giannitsarou and Scott (2006), using historical UK and US data, suggest that the means through which fiscal adjustment is achieved have changed significantly across centuries and in particular war induced increases in debt are financed through different channels than general increases in debt. We leave undiscussed the political economy features that might explain why military expenditure is financed through different means to social transfers but the implication is that focusing on modern times of peace is critical if we are to gauge the impact of demographic induced future fiscal deficits.

For this period to be informative regarding the means whereby governments achieve fiscal sustainability it is important that debt and deficits show significant variation. Another reason why so many previous researchers have focused on war related expenditure is it leads to dramatic swings in public debt and deficits and so is a natural period with which to examine fiscal sustainability. Table 1 documents some stylised facts for key fiscal variables over our post war period.[1] Debt relative to GDP shows large volatility, with a standard deviation ranging from 0.078 to 0.381. With the exception of the US, all countries see an increase in debt/GDP over the sample period and in two of our six cases, debt/GDP rises above 100% GDP. This is far from being a period of tranquil public finances or modest variation in debt, suggesting once more its relevance as a case study for coming decades.

Performing various unit root tests (see Tables 2-5) to the market value of government debt, government expenditure (excluding interest payments) and revenue, all expressed as a ratio to GDP, provides further evidence of the variability of government finances. The strong consensus that emerges from these results is that all variables appear to be non-stationary and contain unit roots. The fact that government expenditure/GDP and tax revenue/GDP are non-stationary over this period is testimony to the rise of social transfers commented on in previous paragraphs. The suggestion that debt/GDP is also non-stationary further implies that achieving fiscal sustainability in this post war period has been

[1] A detailed description of our data and its sources is provided in appendix A.

a challenge. The finding that the debt/GDP ratio is non-stationary also conflicts with the findings of Giannitsarou and Scott (2006) that use longer runs of historical data. It thus seems that the sample in this paper is one of unusual fiscal problems.

The finding that debt/GDP is non-stationary may appear to be inconsistent with fiscal sustainability, but as stressed by Bohn (2006), this is not necessarily the case. Critical for sustainability, according to this analysis, is the existence of a feedback from the level of debt to the current primary deficit. In other words, when estimating a regression of

$$PrimarySurplus_t = A(L)X_t + \alpha Debt_t + \epsilon_t,$$

where X_t denotes a set of relevant explanatory variables, it is necessary that α is positive and larger than the interest rate paid on government debt. The final column in Table 1 reports estimates of α (and associated p values) for each of our countries where X_t consisted of lagged values of the primary surplus. Only in the cases of Japan and Germany is the debt term not significant at the 5% level or less (although including additional variables such as GDP, interest rates, etc. remedied this problem for Germany), suggesting that most countries show evidence of fiscal sustainability over our period. Interestingly, Japan and Germany are the only countries where the maximum value for debt/GDP is observed in the final observation in the sample period. We include all six countries in our estimation below, but these results suggest some care should be taken in interpreting the Japanese and German findings.

To better understand the movements in the debt/GDP ratio and its relationship with other fiscal variables it is useful to consider the period-by-period budget constraint. Let $1+\varkappa_t$ denote the growth in nominal GDP and $1+i_{t-1}$ the one year holding return on nominal government bonds. Let B_t, G_t and T_t be the ratios of nominal debt, government spending and tax revenues to nominal GDP.[2] Then we have

$$\Delta B_t = (G_t - T_t) + \frac{i_{t-1}}{1+\varkappa_t} B_{t-1} - \frac{\varkappa_t}{1+\varkappa_t} B_{t-1}. \tag{1}$$

In other words, the debt to GDP ratio increases through the ratio of the primary deficit to GDP and interest payments on debt (using the growth adjusted real interest rate), and reduces through a nominal growth dividend $\varkappa_t B_{t-1}/(1+\varkappa_t)$. To investigate the role of variations in bond prices as a way of ensuring that the intertemporal budget constraint of the government holds (the focus of Marcet and Scott, 2005a), we evaluate (1) by considering i_t as the one year holding return on government bonds. In other words, we include both coupon payments and capital gains, so that our budget constraint is specified in terms of the market value of government debt, rather than the stock of outstanding debt.

Following Bohn (2005), to gain some understanding of what drives changes in the debt/GDP ratio, we can evaluate (1) using sample averages for our period. The results are shown in Table 6. As noted in Table 1, Japan and Germany show the most dramatic increases in debt, with only the US and UK showing broad debt stability. These substantial differences in debt dynamics are despite the fact that, with the exception of Italy, the total deficit/GDP ratio is reasonably similar across countries. The main

[2] For easy reference, the notation we use throughout the paper is summarised in Appendix B.

reason behind Germany and Japan's different debt dynamics is their low value of the growth dividend; this is particularly noticeable for the inflation component (the nominal growth dividend consists of a real growth and an inflation component). Reviewing Table 6, it is seems that inflation has been a significant influence in achieving fiscal sustainability. At first glance, this would seem to generate the serious concern that rising deficits induced by demographic change may lead to rising inflation.

Table 6 is a useful accounting exercise, but in focusing on *ex post* realisations only provides a backward looking analysis. Equation (1) holds for all fiscal policies, regardless of whether they are sustainable or not, and as a consequence cannot be used to assess the sustainability of fiscal policy. Moreover, it does not provide information on how governments expect to achieve fiscal sustainability or how fiscal policy responds to shocks to key economic variables. In order to gain insights into these issues we next turn to a forward looking perspective by introducing the intertemporal budget constraint.

3. Fiscal Sustainability: A Forward Looking Approach

In this section we use the intertermporal budget constraint to derive an alternative accounting framework for studying fiscal sustainability. This alternative approach produces a relationship between the level of government liabilities relative to the primary deficit and projections of future values of key fiscal variables. We will argue that this relationship is a natural way to characterise imbalances in fiscal policy. Furthermore, using this expression we can examine how *adjustments* to the government's fiscal position are made in order to restore fiscal sustainability. In other words, while the backward looking accounting equation (1) helps quantify how the *average* value of fiscal variables are related this forward looking approach helps quantify how key variables *change* in order to ensure fiscal sustainability.

In deriving the government's intertemporal budget constraint we use the same log-linearisation approach as in Giannitsarou and Scott (2006). Log-linearising the intertemporal budget constraint has been used in a wide variety of applications; e.g. Campbell and Shiller (1987) and (1988) apply this approach to equity prices and dividends, Campbell and Shiller (1991) use it to analyse the yield curve, Lettau and Ludvigson (2001) examine the consumption-wealth ratio and its ability to predict capital gains, and Bergin and Sheffrin (2000) and Gourinchas and Rey (2005) apply the framework to the balance of payments. Although the overall methodology is similar to these other studies we share with Gourinchas and Rey (2005) a significant problem. Log linearisation requires approximating around stationary variables but in our case the trending nature of government expenditures and revenues prevents a straightforward approach. The non-stationarity of fiscal variables creates a number of significant difficulties that need to be overcome if we are to utilise a long linearisation approach.

Since we focus on the implications of fiscal deficits for inflation, we start with a version of the government's budget constraint (1) augmented with monetary liabilities i.e.

$$G_t - T_t = B_t - \frac{\Upsilon_{t-1}}{\Pi_t Q_t} B_{t-1} + H_t - \frac{1}{\Pi_t Q_t} H_{t-1}, \tag{2}$$

where H_t denotes the ratio of monetary liabilities to GDP, Π_t the inflation rate $(1+\pi_t)$, and Q_t is the growth in real GDP $(1+\gamma_t)$.

In seeking to derive a log linear version of the government's budget constraint Giannitsarou and Scott (2006) stress two difficulties: (a) G_t, T_t and B_t show evidence of non-stationarity and (b) it is not possible to take logs and linearise the primary deficit $D_t = G_t - T_t$ due to its possible negative values; therefore the log linearisation needs to be done separately for G_t and T_t. To overcome these difficulties and arrive at a useful version of the intertemporal budget constraint, we need to make the following assumptions :

Assumption 1: There exists a variable W_t such that $G_t/W_t, T_t/W_t, B_t/W_t$ and H_t/W_t are stationary.

Assumption 2: The real and nominal interest rate, the growth rate of GDP, inflation and the growth rate of W_t are all stationary, with steady states i, r, γ, π and ω respectively.

Assumption 3: The No-Ponzi condition holds i.e.

$$\lim_{N\to\infty}\left(\frac{1}{\mu_b}\right)^N (B_{t+N-1} + H_{t+N-1}) = 0,$$

where μ_b denotes the growth adjusted real interest rate less growth in W_t. This condition holds if

$$(1+\gamma)(1+\omega) < 1+r.$$

Assumptions 1 and 2 are required to pursue our log linear approach. Assumption 3 is a version of the standard transversality condition that ensures fiscal sustainability holds and enables us to express current fiscal policy in terms of finite valued infinite horizon present value expressions. Given the importance these assumptions it is worthwhile discussing each of them more extensively.

Assumption 1 invokes the existence of a variable W_t, that transforms the ratios of government spending, revenues and debt to GDP to stationary variables. As explained in Campbell and Shiller (1988), in order to be able to consider a first order approximation to the budget constraint, the variables that are approximated need to be stationary. The standard practice is to assume that deflating variables by GDP is sufficient to induce stationarity. However, Tables 2-5 suggest that apart from debt/GDP, expenditure, revenue and money over GDP also seem to be non-stationary. We therefore need to invoke the assumption of a common trend W_t across all four variables which is capable of inducing stationarity. An obvious issue is the identity of W_t: what variable would induce stationarity in the public finances? One possible interpretation is that W_t is a purely statistical term representing a common trend amongst these variables. Alternatively, W_t could have an economic interpretation, for instance, the value of the stock of public assets. However, because our focus lies in deriving a log-linearised version of the budget constraint, we need only assume that such a variable exists; we do not need to empirically observe this variable. This is because ultimately terms in W_t cancel out after the linearisation leaving us with just log-linear expressions for G_t, T_t, B_t.and H_t. Although W_t does not figure in our later empirical exercise, Assumption 1 does have testable implications: if these variables share a common trend W_t, we should find cointegrating relations between the variables in the pairs $\{G_t, T_t\}$, and $\{G_t,B_t\}$ and $\{B_t, H_t\}$. Indeed, evidence for such cointegrating relations is given in Table 7, which offers general support for Assumption 1.

Assumption 2 requires that certain key variables have stationary growth rates. We performed various unit root tests that provide evidence in support of Assumption 2. These tests are omitted here, but are available from the authors upon request.

Assumption 3 is more controversial and is critical in order for us to derive the intertemporal budget constraint. The form of this assumption is standard across many applications, but in our case we require that the average real interest rate be greater than the sum of real GDP growth and the growth in W_t. Clearly, testing this assumption is impossible since W_t is unobserved. Nevertheless, if we assume that W_t has positive mean growth, then a weak test of Assumption 2 is whether $r > \gamma$ or not. For our sample period this condition holds only for Canada, Germany and the US. The majority of the literature simply invokes this assumption and rarely tests whether it is true (see Ball, Mankiw and Elmendorf (1998) for an extended discussion on the validity of this assumption and the options it provides fiscal authorities if the condition fails to hold). Presumably the fact that so many studies do not test the assumption is because implicitly it is assumed that even if over the sample period r and γ do not satisfy the restriction future values will ensure it holds and so the intertemporal budget constraint is well defined. For instance, Giannitsarou and Scott (2006) show that over the sample period 1700-2005 this restriction holds for the UK, so although it is violated between 1960-2005, looking forward over an infinite horizon the assumption is justified. An alternative approach is that suggested by Bohn (2006), who shows how it is possible to derive an intertemporal budget constraint, even when $(1+\gamma)(1+\omega) > 1+r$, if one treats this as a merely technical obstacle. The trick is to use an alternative "interest" rate, $r^* > r$, which satisfies $(1+r^*) > (1+\gamma)(1+\omega)$ and then amend the budget constraint for the difference between r^* and r and then linearise this transformed budget constraint. Appendix C shows how to apply this method to our model and uses a particular example for r^* which ensures the validity of our main expressions even when $r < \gamma$.

Fundamental to our analysis is a relationship between the level of liabilities and government expenditures and revenues. Define

$$l_t \equiv \alpha_b b_{t-1} + \alpha_h h_{t-1} + \alpha_g g_t + \alpha_\tau \tau_t \tag{3}$$

where

$$\begin{aligned}
\alpha_b &= 1 - \frac{1}{\mu_b} && (4)\\
\alpha_h &= \phi \frac{\mu_h}{\mu_b}\left(1 - \frac{1}{\mu_h}\right) && (5)\\
\alpha_g &= -(\alpha_b + \alpha_h)\lambda_g && (6)\\
\alpha_\tau &= -(\alpha_b + \alpha_h)\lambda_\tau && (7)
\end{aligned}$$

then Giannitsarou and Scott (2006) show that the log-linear version of the government's present value constraint is

$$l_t \simeq \left[\left(1-\frac{1}{\mu_b}\right)+\phi\frac{\mu_h}{\mu_b}\left(1-\frac{1}{\mu_h}\right)\right]E_t\sum_{j=1}^{\infty}\left(\frac{1}{\mu_b}\right)^j\Delta d_{t+j}+\frac{\phi}{\mu_b}(1-\frac{\mu_h}{\mu_b})E_t\sum_{j=0}^{\infty}\left(\frac{1}{\mu_b}\right)^j\Delta h_{t+j}$$
$$+\left(1-\frac{1}{\mu_b}\right)E_t\sum_{j=0}^{T-1}\left(\frac{1}{\mu_b}\right)^j\left\{-r_{t+j-1}+\phi\frac{\mu_h}{\mu_b}\pi_{t+j}+\left(1+\phi\frac{\mu_h}{\mu_b}\right)\gamma_{t+j}\right\}. \tag{8}$$

Note that by Assumption 3, it must be that $\mu_b > 1$ and $\mu_h < 1$.

The variable

$$d_t \equiv \lambda_g g_t + \lambda_\tau \tau_t,$$

where λ_g and λ_τ are of opposite signs, is essentially a transformed version of the primary deficit. The critical variable for our analysis is l_t, which is a measure of the imbalances in fiscal policy. If $\lambda_g > 0$ and $\lambda_\tau < 0$, then l_t defines a relationship between government liabilities and the primary deficit; if $\lambda_g < 0$ and $\lambda_\tau > 0$, then instead the relationship is between government liabilities and the primary surplus. Since $\mu_b > 1$ and $\mu_h < 1$, the coefficient on debt is positive and the coefficient on money is negative, regardless of the signs of λ_g and λ_τ.

Equation (8) pins down a long run equilibrium relationship between the market value of government debt, monetary liabilities and a version of the current primary deficit, d_t. We interpret this expression as follows. For given values of mean interest rates, real GDP growth and money holdings, there has to be a steady state relationship between debt and the primary deficit, if debt is to be sustainable and for the intertemporal budget constraint to hold. Under our Assumption 2 and given our empirical evidence on unit roots, the right hand side of (8) is stationary, so that $E_t l_{t+j} = \kappa$, as $j \to \infty$, suggesting $l_t - \kappa$ as a natural measure of required fiscal adjustment. When $l_t = 0$, then this equilibrium relationship holds, but for $l_t > 0$ debt is too high relative to current fiscal deficits. Adjustment is achieved through variations in the right hand side of (8).

Further insights into (8) and l_t can be gained from considering the results of Trehan and Walsh (1988) and (1991), and Bohn (2006). The former show that the intertemporal budget constraint requires that the primary deficit and debt satisfy a cointegrating relationship. Given Assumption 2 and the fact that $E_t l_{t+j} = \kappa$, the same is true for our approximation to the intertemporal budget constraint.[3] Bohn (2006) focuses on an alternative insight, namely that debt sustainability requires a feedback rule from debt to deficits. Given Assumption 2, we know that l_t must be stationary, which can be achieved through a feedback rule from debt to the deficit, although in this case the feedback is from a weighted average of marketable debt and monetary liabilities.

If the left hand side of (8) measures the degree of fiscal adjustment required, then the right hand side of (8) tells us how this fiscal adjustment ($l_t - \kappa$) is achieved through either: (a) future improvements in the primary deficit $-\Delta d_{t+j}$, (b) issuing more monetary liabilities, Δh_{t+j} or (c) variations in the growth

[3]Strictly speaking (8) states that if the components of l_t are of order of integration N, then the right hand side of (8) is order $N-1$.

adjusted real interest rate $(r_{t+j} - \pi_{t+j} - \gamma_{t+j})$. The coefficients on each of the components of the growth adjusted real interest rate differ, as the nominal dividend effect $(\pi_{t+j} - \gamma_{t+j})$ operates on both bonds and money, while r_{t+j} affects only bonds. As a result, equation (8) tells us that, *if the intertemporal budget constraint holds, any deviations in the long run relationship between debt and deficits must help predict movements in either future primary deficits, money creation, nominal interest rates, inflation or GDP growth.* We stress once more that our intertemporal budget constraint is an accounting framework and the predictive role of l_t is a purely statistical rather than necessarily causal phenomenon.

4. Estimating Fiscal Imbalances

Crucial to implementing our approach to the intertemporal budget constraint is construction of an estimate for l_t and so estimates of $\alpha_b, \alpha_h, \alpha_g$ and α_τ.A common approach to estimating (3) and its analogues in the literature is to use Stock and Watson's (1993) Dynamic OLS estimator. In our case this would involve estimating the regression:

$$g_t = \beta_1 \tau_t + \beta_2 b_{t-1} + \beta_3 h_{t-1} + \sum_{i=-k}^{k} (c_{i\tau} \Delta\tau_{t-i} + c_{ib}\Delta b_{t-i} + c_{ih}\Delta h_{t-i}). \tag{9}$$

Using (3), (4)-(7) and the fact that $\lambda_g + \lambda_\tau = 1$, we get the implied restrictions

$$\beta_2 + \beta_3 = \frac{1}{1-\lambda_\tau}, \tag{10}$$

$$\beta_1 = -\frac{\lambda_\tau}{1-\lambda_\tau}. \tag{11}$$

Table 8 shows the results from estimating this equation and the implied estimated coefficients for the parameters of interest, α_b, α_h, α_g and α_τ. These estimated coefficients are calculated as follows. First, we check that in each case, we cannot reject restriction (10), at either the 5% or 10% level. Next, equation (9) contains three key estimated parameters, namely β_1, β_2 and β_3. From these reduced form estimates we wish to estimate the structural parameters ϕ, μ_b, μ_h, λ_τ, using relations (4)-(7). However, due to (10), we have in effect only two independently estimated parameters. Therefore identifying our structural parameters requires using additional information. In particular, we can estimate ϕ as the sample average of narrow money to government debt ratio (H/B) and by definition we have $\mu_b/\mu_h = (1+\pi)(1+r)$. Using these additional restrictions we can then just identify the key structural parameters of interest and construct our estimate of l_t :

$$\hat{l}_t = \left(1 - \frac{1}{\mu_b}\right) b_{t-1} + \phi \frac{\mu_h}{\mu_b}\left(1 - \frac{1}{\mu_h}\right) h_{t-1} - \left[\left(1 - \frac{1}{\mu_b}\right) + \phi\frac{\mu_h}{\mu_b}\left(1 - \frac{1}{\mu_h}\right)\right] d_t.$$

The variable $\hat{l}_t$ is a measure of the deviation of a weighted average of government liabilities from a transformed measure of the primary deficit d_t. The larger l_t is, the larger debt is relative to the steady state level of deficits and the greater the required degree of fiscal adjustment. Note that given the estimates of Table 8, the weights on government expenditure and tax revenue (λ_g and λ_τ) are of opposite sign and approximately equal in absolute value, so that d_t is only mildly different from the primary deficit (and in the case of Japan and the UK there is very little difference). The coefficients λ_g and λ_τ, reflecting the relative importance of the primary deficit relative to liabilities, clearly show significant variations across countries. These variations reflect the fact that the size of the primary deficit needed to ensure sustainability depends on the level of interest rates, GDP growth and debt and small variations in these numbers can produce large variations in λ_g and λ_τ.

While this approach is standard in the literature our use of Assumption 1 creates additional problems which make the above analysis problematic. Assumption 1 implies that G_t, T_t, B_t and H_t all share a common stochastic trend, W_t. As a result there exist three (linearly independent) cointegrating vectors. Evidence for this was shown in Table 7, which we used earlier as support for Assumption 1. In other words, in our data we have the following cointegrating relationships:[4]

$$\begin{aligned} g_t &= \varphi_1 t_t + u_{1t} \\ b_t &= \varphi_2 h_t + u_{2t} \\ g_t &= \varphi_3 b_t + u_{3t} \end{aligned}$$

where u_{it}, $i = 1, 2, 3$ are stationary error terms, since the variables cointegrate. Given that each of these pairs is stationary, it also must be the case that any linear combination of these cointegrating relations must also be stationary. In other words, Assumption 1 implies that for *any* α_1, α_2 and α_3 the following holds :

$$\alpha_1(g_t - \varphi_1 t_t) + \alpha_2(b_t - \varphi_2 h_t) + \alpha_3(g_t - \varphi_3 b_t) = w_t$$

where w_t is a stationary error term which is a linear combination of the $u'_{it}s$. Rearranging this expression yields

$$g_t = \frac{\alpha_1\varphi_1}{\alpha_1 + \alpha_3} t_t - \frac{\alpha_2 - \alpha_3\varphi_3}{\alpha_1 + \alpha_3} b_t + \frac{\alpha_2\varphi_2}{\alpha_1 + \alpha_3} h_t + w_t$$

Assumption 1 implies that this equation must hold for *arbitrary* α_1, α_2 and α_3 while the intertemporal budget constraint implies, through (3), (4)-(7), that only one specific cointegrating vector is useful in predicting future fiscal behaviour e.g the right hand side of (8). Assumption 1 means that when we estimate (3) using DOLS we are in effect estimating some weighted average of the three cointegrating vectors, where the weights are arbitrary and not uniquely determined. While (3) and (4)-(7) imply some cross parameter restrictions, they are not enough to uniquely pin down α_1, α_2 and α_3 and overcome this indeterminacy. In other words, it is not obvious that the estimates in Table 8 truly uncover the structural parameters that we are interested in.

[4] Here we follow Gourinchas and Rey (2005) and allow the coefficients in these cointegrating relationships to differ from 1. Measurement important when dealing with fiscal data; in particular because of off-balance sheet items, official debt data does not accumulate purely as a consequence of the total deficit.

In order to validate the estimates of Table 8 as reliable measures of fiscal imbalance, we use two independent arguments. The first and most powerful is that the intertemporal budget constraint implies more than just a linear cointegrating vector of the form (3) exists. In particular the intertemporal budget constraint implies that the "true" l_t, i.e. expression (3), equals the right hand side of (8). Assumption 1 entails that there exist many cointegrating vectors between b_t, h_t, g_t and t_t but the intertemporal budget constraint implies that only one particular cointegrating vector should be useful in predicting the present value of future fiscal variables. This in turn implies that innovations in l_t should be equal to the innovations in the present value of these future fiscal variables. We show in a later section how to exploit this to test for the validity of our estimated (3) as a measure of fiscal imbalances. For the cases where this test is not rejected we can be confident that DOLS does not estimate an arbitrary cointegrating relationship but the specific one implied by the intertemporal budget constraint.

The second means of verifying our DOLS estimates of (3) as a valid measure of fiscal imbalances is to provide alternative estimates of the key structural parameters ϕ, λ_g, λ_τ, μ_b, μ_h and use these to construct an estimate of $\{l_t\}$ which can then be compared with those obtained from DOLS. Sample estimates of real interest rates, real GDP growth, government expenditure, tax revenues and money holdings as shares of GDP were constructed and an estimate of ω (the growth of the common trend W_t) formed by averaging the trend growth rates of b_t, h_t, g_t and t_t. Estimating l_t in this way is problematic for several statistical reasons and is not our preferred approach. The first problem is that g_t and t_t are non-stationary and so do not have a well defined unconditional mean. The second is that when we use DOLS we can exploit the order of integration of the variables and arrive at superconsistent estimates. However, this approach does offer, however flawed, a further means of assessing our measure of fiscal imbalances.

Table 9 shows the results for our key parameters from this alternative estimation method and also offers some information on how this measure of l_t compares with that from using DOLS. For Canada, UK and US the results are astoundingly similar, both in terms of the structural parameters and the correlation between the estimates of l_t and Δl_t. In all three cases we can reject strongly the null of non-stationarity for this alternative estimate of l_t. For Italy the coefficient estimates are broadly similar except that those on debt and money are around twice as large but the two estimates broadly track one another and show similar behaviour in their rate of change. The only countries where there are important differences are for Japan and especially Germany, note that these are the two countries for which our earlier analysis questioned the degree to which fiscal policy had been sustainable over our sample period. For Japan the main difference is that the coefficients on government expenditure and taxation are much smaller than from DOLS and so this estimate of l_t places much greater weight on the debt and monetary liabilities term. This leads to a weaker correlation between the two estimates although the fluctuations in l_t are broadly similar. The two estimates for Germany are however very different, with a near perfect *negative* correlation arising from the fact that α_g and α_t have opposite signs to that from DOLS. Note also that there is only weak evidence that this alternative measure of l_t is stationary, which our assumptions require. These independent validations of our DOLS estimates of

l_t suggest that, with the exception of Germany, they are valid measures of the degree of fiscal imbalance as suggested by the intertemporal budget constraint.

We now move to consider in more detail what these estimates imply about the dynamics of fiscal adjustment. In order to make comparisons across countries it is necessary to standardise our measures of l_t, due to the fact that the estimated coefficients in Table 8 differ so much across countries. As previously described, $\hat{l}_t$ is the deviation in the relationship between liabilities and the primary deficit. Consider the case where fiscal equilibrium, i.e. $l_t = 0$, is achieved purely through adjustments in tax rates. Given our definition of l_t this would require an increase in taxes of

$$\hat{l}_t^* = \frac{1}{-\lambda_\tau\left[\left(1-\frac{1}{\mu_b}\right)+\phi\frac{\mu_h}{\mu_b}\left(1-\frac{1}{\mu_h}\right)\right]}[\hat{l}_t - m_l].$$

where m_l is the sample mean for l_t and $\hat{l}_t^*$ is a normalised measure of fiscal imbalances which can be used to make comparisons across countries: it is the required increase in the tax revenue/GDP needed to restore fiscal equilibrium and can be thought of as an alternative measure of fiscal sustainability to those proposed by Blanchard et al (1989) and Polito and Wickens (2006).

Estimates of $\hat{l}_t^*$ are shown in Figure 3, for our sample period. These suggest that the discrepancy between debt and deficit in the US is currently on a par with the Reagan years, although it has shown some signs of improvement over the last year. After a protracted correction during the 1980s and 1990s, the estimates suggest that Canadian public finances are now in rough balance as is German policy. Not surprisingly Japan's fiscal situation has shown a dramatic recent deterioration, as has the fiscal position of UK. Finally, after many consecutive years of fiscal improvements, our estimates suggest that from a long term perspective, Italian public finances have deteriorated significantly in recent times. Table 10 reports some summary statistics regarding fiscal adjustment for our countries during this period. The degree of imbalance varies between around plus and minus 5% for all economies, except for the US where the range is narrower (plus or minus 3%). In all cases fiscal adjustment is highly persistent although not a unit root process; as discussed above, it is critical for our analysis that l_t is stationary. Fiscal adjustment is a protracted process, with a half life of between 2 and 4 years. Given that our sample period contains 46 years of data for most countries we have several completed cycles of fiscal adjustment. However, the case of Japan shows weakest evidence of adjustment in fiscal policy, perhaps reflecting our earlier comments that the continual rise in Japanese government debt may reflect an uncompleted fiscal adjustment.

5. Financing the Budget

The previous section constructed estimates of our measure of fiscal imbalance and statistically characterised its variability. Equation (8) states that these fluctuations in l_t are associated with fluctuations in future deficits, money creation, real interest rates, inflation and real GDP growth in a manner that ensures the government's intertemporal budget constraint holds. If the left hand side of 8) shows a measure of fiscal imbalance then the right hand side shows which variables account for variations in fiscal imbalances. We now turn our attention to using this equation to perform a variance decomposition on l_t. Define $z_t = (l_t,\ \Delta d_t,\ \Delta h_t,\ r_{t-1},\ \pi_t,\ \gamma_t)$ and assume z_t follows a VAR(p) process, i.e.

$$z_t = A_1 z_{t-1} + A_2 z_{t-2} + ... + A_p z_{t-p} + \varepsilon_t,$$

where $A_k,\ k = 1, ..., p$ are 6×6 matrices. Denoting $\mathbf{z}_t = \left(z_t',\ z_{t-1}',\ ...,\ z_{t-p+1}'\right)$ and $\boldsymbol{\varepsilon}_t = (\varepsilon_t',\ 0, ...,\ 0)$ we can rewrite this VAR(p) as a VAR(1) so that

$$\mathbf{z}_t = \mathbf{A}\mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_t,$$

where

$$\mathbf{A} = \begin{pmatrix} A_1 & A_2 & \cdots & A_{p-1} & A_p \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{pmatrix}.$$

Noting that conditional expectations satisfy

$$E_t \mathbf{z}_{t+j} = \mathbf{A}^j \mathbf{z}_t$$

and defining appropriate indicator vectors $\mathbf{e}$ such that

$$\mathbf{e}_l' \mathbf{z}_t = l_t,\ \mathbf{e}_{\Delta d}' \mathbf{z}_t = \Delta d_t,\ \mathbf{e}_r' \mathbf{z}_t = r_{t-1},\ \mathbf{e}_{\Delta h}' \mathbf{z}_t = \Delta h_t,\ \ \mathbf{e}_\pi' \mathbf{z}_t = \pi_t,\ \mathbf{e}_\gamma' \mathbf{z}_t = \gamma_t,$$

we can then, following Campbell and Shiller (1988) rewrite (8) as

$$\begin{aligned} \mathbf{e}_l' \mathbf{z}_t \;=\;& \left[\left(1 - \frac{1}{\mu_b}\right) + \phi \frac{\mu_h}{\mu_b}\left(1 - \frac{1}{\mu_h}\right)\right] \sum_{j=1}^{\infty} (1/\mu_b)^j\, \mathbf{e}_{\Delta d}' \mathbf{A}^j \mathbf{z}_t + \frac{\phi}{\mu_b}\left(1 - \frac{\mu_h}{\mu_b}\right) \sum_{j=0}^{\infty} (1/\mu_b)^j\, \mathbf{e}_{\Delta h}' \mathbf{A}^j \mathbf{z}_t \\ & - \left(1 - \frac{1}{\mu_b}\right) \sum_{j=0}^{\infty} (1/\mu_b)^j\, \mathbf{e}_r' \mathbf{A}^j \mathbf{z}_t + \left(1 - \frac{1}{\mu_b}\right) \phi \frac{\mu_h}{\mu_b} \sum_{j=0}^{\infty} (1/\mu_b)^j\, \mathbf{e}_\pi' \mathbf{A}^j \mathbf{z}_t \\ & + \left(1 - \frac{1}{\mu_b}\right)\left(1 + \phi\frac{\mu_h}{\mu_b}\right) \sum_{j=0}^{\infty} (1/\mu_b)^j\, \mathbf{e}_\gamma' \mathbf{A}^j \mathbf{z}_t, \end{aligned}$$

or equivalently

$$\begin{aligned} \mathbf{e}_l' \mathbf{z}_t \;=\;& \left\{ \frac{1}{\mu_b}\left[\left(1 - \frac{1}{\mu_b}\right) + \phi\frac{\mu_h}{\mu_b}\left(1 - \frac{1}{\mu_h}\right)\right] \mathbf{e}_{\Delta d}' \mathbf{A}\left(I - \frac{1}{\mu_b}\mathbf{A}\right)^{-1} + \frac{\phi}{\mu_b}\left(1 - \frac{\mu_h}{\mu_b}\right) \mathbf{e}_{\Delta h}' \left(I - \frac{1}{\mu_b}\mathbf{A}\right)^{-1} \right. \\ & \left. - \left(1 - \frac{1}{\mu_b}\right)\left[\mathbf{e}_r' - \phi\frac{\mu_h}{\mu_b}\mathbf{e}_\pi' - \left(1 + \phi\frac{\mu_h}{\mu_b}\right)\mathbf{e}_\gamma'\right]\left(I - \frac{1}{\mu_b}\mathbf{A}\right)^{-1} \right\} \mathbf{z}_t. \end{aligned} \tag{12}$$

This is simply a restatement of our key equation (8), but where we have replaced the expectation terms with conditional forecasts obtained from our VAR representation for $\mathbf{z}_t$. If our log linearisation to the budget constraint is appropriate we have

$$\begin{aligned}\mathbf{e}_l^{'}\left(I-\frac{1}{\mu_b}\mathbf{A}\right) &= \frac{1}{\mu_b}\left[\left(1-\frac{1}{\mu_b}\right)+\phi\frac{\mu_h}{\mu_b}\left(1-\frac{1}{\mu_h}\right)\right]\mathbf{e}_{\Delta d}^{'}\mathbf{A}+\frac{\phi}{\mu_b}\left(1-\frac{\mu_h}{\mu_b}\right)\mathbf{e}_{\Delta h}^{'} \\ &\quad -\left(1-\frac{1}{\mu_b}\right)\left[\mathbf{e}_r^{'}-\phi\frac{\mu_h}{\mu_b}\mathbf{e}_\pi^{'}-\left(1+\phi\frac{\mu_h}{\mu_b}\right)\mathbf{e}_\gamma^{'}\right]. \qquad (13)\end{aligned}$$

This expression shows the restriction our present value formula imposes on innovations to l_t relative to innovations in deficits, money and growth adjusted real interest rates. It can therefore be used as a joint test for the adequacy of both our approximation to the intertemporal budget constraint and the forecasting system used for our variables. It is this test we referred to earlier as a means of validating our DOLS estimates of l_t. As shown in Giannitsarou and Scott (2006), an alternative and easier to implement formulation of this restriction is:

$$0=E_{t-1}\left\{\frac{1}{\mu_b}(\Delta b_t-\phi\Delta h_t)+l_t-\left[r_{t-1}-\gamma_t-\phi\frac{\mu_h}{\mu_b}(\pi_t+\gamma_t)\right]\right\}.$$

This last expression can be easily tested by regressing the expression in the expectation on variables in the t-1 information set and testing for their significance. If our log linear approximation to the intertemporal budget constraint is to hold, then these lagged variables should be insignificant.

To proceed with our variance decomposition we rewrite our system as

$$l_t=F_{\Delta d,t}+F_{\Delta h,t}+F_{r,t}+F_{\pi,t}+F_{\gamma,t},$$

where

$$\begin{aligned}F_{\Delta d,t} &= \frac{1}{\mu_b}\left[\left(1-\frac{1}{\mu_b}\right)+\phi\frac{\mu_h}{\mu_b}\left(1-\frac{1}{\mu_h}\right)\right]\mathbf{e}_{\Delta d}^{'}\mathbf{A}\left(I-\frac{1}{\mu_b}\mathbf{A}\right)^{-1}\mathbf{z}_t, \\ F_{\Delta h,t} &= \frac{\phi}{\mu_b}\left(1-\frac{\mu_h}{\mu_b}\right)\mathbf{e}_{\Delta h}^{'}\left(I-\frac{1}{\mu_b}\mathbf{A}\right)^{-1}\mathbf{z}_t, \\ F_{r,t} &= -\left(1-\frac{1}{\mu_b}\right)\mathbf{e}_r^{'}\left(I-\frac{1}{\mu_b}\mathbf{A}\right)^{-1}\mathbf{z}_t, \\ F_{\pi,t} &= \left(1-\frac{1}{\mu_b}\right)\phi\frac{\mu_h}{\mu_b}\mathbf{e}_\pi^{'}\left(I-\frac{1}{\mu_b}\mathbf{A}\right)^{-1}\mathbf{z}_t, \\ F_{\gamma,t} &= \left(1-\frac{1}{\mu_b}\right)\left(1+\phi\frac{\mu_h}{\mu_b}\right)\mathbf{e}_\gamma^{'}\left(I-\frac{1}{\mu_b}\mathbf{A}\right)^{-1}\mathbf{z}_t.\end{aligned}$$

where $F_{j,t}$ denotes the projected contribution of component j towards maintaining the intertemporal budget constraint. Figure 4 shows the value of l_t and our estimated series for $F_{\Delta d,t}+F_{\Delta h,t}+F_{r,t}+F_{\pi,t}+F_{\gamma,t}$ in the case of the US and shows that our forecasting model does an excellent job capturing the restrictions implied by the intertemporal budget constraint. This is confirmed by the χ^2 test of the orthogonality condition (13), which holds at the 5% level or less for the majority of cases, once more confirming the validity of our DOLS estimates..

The benefits of applying this VAR forecasting framework to our intertemporal budget constraint is that it can be used to perform a variance decomposition on l_t. In other words, it is possible to measure the relative importance of each of our variables (changes in primary deficit, issuance of monetary liabilities, changes in return on bonds or inflation or GDP growth) in achieving fiscal balance. As shown by Cochrane (1992), the relative importance of variable j can be measured by the estimated coefficient from a regression of $F_{j,t}$ on l_t. Table 11 shows results from our sample period and demonstrates clearly the importance of deficit fluctuations as the main source of adjustment in l_t. Compared to the importance of the deficit, the role of the other variables is minor, including that of inflation. The minor role for variations in rates of return on government debt in achieving fiscal balance is consistent with the findings of Marcet and Scott (2005b) regarding the lack of fiscal insurance offered by existing debt instruments.

Because the F_{jt} components are possibly correlated amongst themselves our variance decomposition is not an orthogonal one and so our estimates of the relative contribution of each variable are not bounded between 0 and 1. This leads to the possibility that if our inflation term is strongly correlated with fluctuations in the primary surplus then our high estimate of the role of fluctuations in the primary deficit and our low estimate for inflation may reflect the non-orthogonality of our decomposition. However, the strongest correlations in our data are between $F_{\pi,t}$, $F_{r,t}$ and $F_{\gamma,t}$ rather than with $F_{\Delta d,t}$. For instance, for US, Italy, Canada, UK, Japan and Germany the correlations between $F_{\pi,t}$ and $F_{\Delta d,t}$ are -0.26, 0.78, -0.15, -0.24, 0.42 and -0.61 compared to -0.78, 0.46, 0.25, -0.90, 0.97, 0.67 for the correlation between $F_{\pi,t}$ and $F_{\gamma,t}$. Only if the correlation between $F_{\pi,t}$ and $F_{\Delta d,t}$ is large and negative can an appeal to non-orthogonality be used to explain why our estimates of the importance of inflation may be an underestimate.

The importance of the deficit in the variance decomposition is surprising given our previous findings on the importance of the nominal growth dividend. However, as stressed earlier the growth dividend focuses on how the average level of the debt/GDP ratio is linked to *average* levels of the primary deficit, interest rates, GDP growth and inflation. By contrast our use of the intertemporal budget constraint focuses on how *changes* in these variables account for *variations* in the deficit to liabilities ratio. Examination of the raw data also reveals the importance of shifts in the primary deficit. For instance, in Italy the primary deficit started at 0%, but deteriorated to more than 8% before debt started to improve as the primary deficit moved to -6%. Similar movements in debt and primary deficits occur in the other countries in our sample.

This analysis suggests that fluctuations in the primary deficit are the main means whereby fiscal balance is acheived. This raises the question whether fluctuations in the primary deficit are driven more by government expenditure or by changes in tax revenue. We therefore extend our VAR using the decomposition

$$l_t = F_{\Delta g,t} + F_{\Delta\tau,t} + F_{\Delta h,t} + F_{r,t} + F_{\pi,t} + F_{\gamma,t},$$

where

$$\begin{aligned} F_{\Delta g,t} &= \frac{1}{\mu_b}\left[\left(1-\frac{1}{\mu_b}\right)+\phi\frac{\mu_h}{\mu_b}\left(1-\frac{1}{\mu_h}\right)\right]\lambda_g \mathbf{e}_{\Delta g}^{'}\mathbf{A}\left(I-\frac{1}{\mu_b}\mathbf{A}\right)^{-1}\mathbf{z}_t, \\ F_{\Delta\tau,t} &= \frac{1}{\mu_b}\left[\left(1-\frac{1}{\mu_b}\right)+\phi\frac{\mu_h}{\mu_b}\left(1-\frac{1}{\mu_h}\right)\right]\lambda_\tau \mathbf{e}_{\Delta\tau}^{'}\mathbf{A}\left(I-\frac{1}{\mu_b}\mathbf{A}\right)^{-1}\mathbf{z}_t. \end{aligned}$$

The results of this extended VAR are also shown in Table 11. As in Bohn (1991) both expenditure and revenue variations play a substantial role in fiscal fluctuations with a slightly more important role for expenditure fluctuations, although this effect is far more pronounced in the UK.

These variance decomposition results suggest that inflation movements play only a very minor role in accounting for shifts in the fiscal position of governments. However, this does not necessarily

mean that fiscal movements are insignificant in predicting future inflation. We now turn to another implication of equation (8), i.e. that l_t should be useful in predicting future inflation. In particular, we look at the ability of l_t to predict future inflation at horizons from 1 to 20 years. We do this by first specifying an optimal forecasting equation for inflation. We use lag selection criteria in a model where inflation depends on lagged values of inflation, nominal interest rates and GDP growth, where we consider lags of up to 8 periods for each variable. Having arrived at an optimal model, we then add l_{t-j} to gauge the additional explanatory power from our measure of fiscal imbalance. The results are shown in Table 12, where we quote the p-value for l_{t-j}, $j = 1, .., 20$.

The results are consistent with Table 1: fiscal measures have a very marginal impact on predicting inflation. The vast majority of lags are insignificant although in a few cases there is evidence of predictive ability at horizons of around 3-4 years. However, the marginal statistical contribution of l_t is fairly small. The pitfalls of such Granger causality tests are well known and their inability to successfully identify causal economic mechanisms well documented. We should therefore take care in interpreting these findings and not necessarily interpret them as implying that fiscal policy does not influence inflation. For instance, the fiscal theory of the price level would argue that expectations about future deficits, that is $E_t \sum_{j=1}^{\infty}\left(\frac{1}{\mu_b}\right)^j \Delta d_{t+j}$ would influence the current level of prices and so affect *contemporaneous* inflation. This is entirely consistent with our finding that l_t has only a minor role in predicting *future* changes in inflation i.e $E_t \sum_{j=1}^{\infty}\left(\frac{1}{\mu_b}\right)^j \Delta\pi_{t+j}$. However, while we need to be careful in the economic conclusions we draw from these findings we can draw the statistical conclusion that rising government debt is not a good predictor of rising future inflation. That is, the increased indebtedness originating from demographic change is not necessarily a statistical harbinger of rising future inflation.

6. Conclusion

This paper sought to apply a log linearised version of the intertemporal budget constraint to consider government's fiscal positions. It tried to answer three key questions (a) is current fiscal policy sustainable? (b) how have OECD governments financed their fiscal deficits in recent decades and (c) what are the implications for inflation of the expected rising deficits? The contribution of the paper is purely empirical, using an accounting identity to quantify the statistical impact of certain key variables.

In answer to the first question, for each country we estimated a measure of current fiscal imbalance, defined as the ratio between current liabilities and the primary deficit. For all countries, the current measure for this imbalance was within the historical range of variation suggesting that current policies are sustainable, with the possible exception of Japan. Using our version of the intertemporal budget constraint, we analysed how in previous years governments had achieved fiscal balance. We found an overwhelming role for changes in the primary surplus with only a minor role for inflation, growth and interest rate effects. Further we also found that fiscal imbalances had only a very weak forecasting role for future inflation at nearly all horizons, with some mild evidence that fiscal imbalances could help predict inflation three to four years ahead.

Obviously our results should be interpreted with care as they are based on a certain historical period and an assumption that governments cannot take a deficit gamble if $r < \gamma$; inevitably any attempt at an econometric approach to evaluating the intertemporal budget constraint is vulnerable to time dependence and non-stationarity. Our accounting framework also prevents us from attributing any causal role to the statistical relationships we discover. However, the statistical findings are striking: variations in fiscal imbalances and movements to fiscal sustainability are achieved mainly through variations in the primary deficit. Moreover, rising government debt amongst these countries is not a reliable predictor of higher future inflation.

A. Data

Notes on data sources for the UK and US can be found in Giannitsarou and Scott (2006). For the remaining countries details are as below. The following abbreviations are used:

- GFD: Global Financial Data
- IFS: International Financial Statistics (IMF)
- OECD-EO: OECD Economic Outlook Database
- OECD-CGD: OECD Central Government Debt Statistics
- HSoC: Historical Statistics of Canada (Statistics Canada)
- DI: DataInsight

GDP, Prices and Inflation

Country	*variable*	*sample period*	*source*	*ID/Specification*
CAN	real GDP		GFD	GDPCCANM
	nom. GDP			GDPCANM
JAP	nom. GDP	1955-2005	IFS	15899B.CZF
	Deflator			15899BIRZF
JAP, ITA, GER	nom. GDP	1960-2005	OECD-EO	
	Deflator			

The implicit GDP deflator is used as the price index.
(Gross) inflation is then obtained as the annual rate of change of the index.

Base Money

Country	*sample*	*source*	*ID/Specification*
CAN	1926-1954	HSoC	J69+J71
	1955-2005	DI	MBASENS@CN
GER		DI	M1@EURNS@GY
ITA	1960-1990	Fratianni (2005), p49ff	col BP
	1991-2005	Banca d'Italia	
JAP		DI	MBASENS@JP

For Canada, Italy, and Japan, base money is used.
For Germany, it is the national definition of M1 (currency in circulation plus overnight deposits).

Government receipts and expenditure

Country		*sample*	*source*	*ID/Spec.*
CAN	Receipts, expenditure	1926-1965	HSoC	F109, F116
	Receipts, expenditure	1966-2005	DI	REVG@CN, EXG@CN
	Interest			EXGCDINS@CN
GER, ITA, JAP	Receipts, expenditure		OECD-EO	
	Interest			

All government expenditure data is net of interest service. Revenues are net of interest receipts for Germany, Italy and Japan, but not for Canada. The primary deficit is expressed as net expenditure minus (net) receipts.

Government debt and market values. Market values are approximated for central government marketable debt, which is from OECD-CGD. For the periods before these data are available, the last available share of marketable in total debt was used to obtain marketable debt. The price of government debt is approximated as

$$p_t = \frac{1 + NC}{1 + NI},$$

where N is the average term to maturity of outstanding government securities, C is the average coupon rate, and I is the average market yield. Data on average terms to maturity and average yields is from OECD-CGD. If no average term to maturity was available, average maturities were used. For earlier periods, the last average maturity available was taken. If average yields were unavailable, yields are constant maturity benchmark yields (from GFD). For a given year, the benchmark yield closest to the average term to maturity of that year was applied. Average coupon data is approximated as the ratio of gross interest service to gross government debt, more precisely,

$$C_t = \frac{Interest_{t+1}}{DEBT_t}.$$

B. Variable Definitions and Notation

variable	definition	steady state
G_t	government spending over GDP	
T_t	tax revenues over GDP	
B_t	debt over GDP	
H_t	seignorage over GDP	
W_t	aggregate wealth	
P_t	price index	
Y_t	real GDP	
$R_t = 1 + r_t$	gross real interest rate	$R = 1 + r$
$\Upsilon_t = 1 + i_t$	gross nominal interest rate	$\Upsilon = 1 + i$
Ω_t	$= \frac{W_t}{W_{t-1}}$	$\Omega = 1 + \omega$
$\Pi_t = 1 + \pi_t$	$= \frac{P_t}{P_{t-1}}$	$\Pi = 1 + \pi$
$Q_t = 1 + \gamma_t$	$= \frac{Y_t}{Y_{t-1}}$	$Q = 1 + \gamma$
$\bar{G}_t$	$= G_t / W_t$	$\bar{G}$
$\bar{T}_t$	$= T_t / W_t$	$\bar{T}$
$\bar{B}_t$	$= B_t / W_t$	$\bar{B}$
$\bar{H}_t$	$= H_t / W_y$	$\bar{H}$
w_t	$= \ln W_t$	
g_t	$= \ln G_t$	
τ_t	$= \ln T_t$	
b_t	$= \ln B_t$	
h_t	$= \ln H_t$	
d_t	$= \lambda_g g_t + \lambda_\tau \tau_t$	
μ_b	$= \frac{\Upsilon}{\Pi Q \Omega}$	
μ_h	$= \frac{1}{\Pi Q \Omega}$	
λ_g	$= \frac{\bar{G}}{\bar{G} - \bar{T}}$	
λ_τ	$= -\frac{\bar{T}}{\bar{G} - \bar{T}}$	
ϕ	$= \frac{\bar{H}}{\bar{B}}$	
m	$= (1 - \mu_b)\, \bar{B} + (1 - \mu_h)\, \bar{H}$	
m_l	$=$ samle mean of l_t	
κ	$=$ summary of constants that we can ignore	

C. Derivation of the Log-Linear Budget Constraint[5]

The budget constraint for the government, after having adjusted with GDP and prices, can be written as

$$G_t - T_t = B_t - \frac{\Upsilon_{t-1}}{\Pi_t Q_t} B_{t-1} + H_t - \frac{1}{\Pi_t Q_t} H_{t-1}.$$

Dividing through with W_t, we get

$$\frac{G_t}{W_t} - \frac{T_t}{W_t} = \frac{B_t}{W_t} - \frac{\Upsilon_{t-1}}{\Pi_t Q_t \Omega_t} \frac{B_{t-1}}{W_{t-1}} + \frac{H_t}{W_t} - \frac{1}{\Pi_t Q_t \Omega_t} \frac{H_{t-1}}{\Omega_{t-1}},$$

i.e.

$$\bar{G}_t - \bar{T}_t = \bar{B}_t - \frac{\Upsilon_{t-1}}{\Pi_t Q_t \Omega_t} \bar{B}_{t-1} + \bar{H}_t - \frac{1}{\Pi_t Q_t \Omega_t} \bar{H}_{t-1}.$$

In this last expression, all variables are by assumption stationary. Thus we can log-linearise the expression. To do this, we rewrite it as

$$\Omega_t \bar{G}_t - \Omega_t \bar{T}_t = \Omega_t \bar{B}_t - \frac{\Upsilon_{t-1}}{\Pi_t Q_t} \bar{B}_{t-1} + \Omega_t \bar{H}_t - \frac{1}{\Pi_t Q_t} \bar{H}_{t-1},$$

We then use the approximations

$$\begin{aligned} \exp(z) &\approx z + 1, \\ r_{t-1} &\approx i_{t-1} + \pi_t \end{aligned}$$

and the steady state relationship

$$\begin{aligned} \bar{G} - \bar{T} &= (1-\mu_b)\,\bar{B} + (1-\mu_h)\,\bar{H} \equiv m \quad \text{or} \\ \left(\bar{G} - \bar{T}\right) - \left(\bar{B} + \bar{H}\right) &= -\left(\mu_b \bar{B} + \mu_h \bar{H}\right), \end{aligned}$$

to obtain

$$b_{t-1} = \frac{1}{\mu_b} b_t - \frac{1}{\mu_b} \frac{m}{\bar{B}} d_t + \frac{\phi}{\mu_b} s_t - r_{t-1} + \phi \frac{\mu_h}{\mu_b} \pi_t + \left(1 + \phi \frac{\mu_h}{\mu_b}\right) \gamma_t.$$

Substituting forward we get

$$b_{t-1} = \left(\frac{1}{\mu_b}\right)^T b_{t+T-1} + \sum_{j=0}^{T-1} \left(\frac{1}{\mu_b}\right)^j \left\{ -\frac{1}{\mu_b} \frac{m}{\bar{B}} d_{t+j} + \frac{\phi}{\mu_b} s_{t+j} - r_{t+j-1} + \phi \frac{\mu_h}{\mu_b} \pi_{t+j} + \left(1 + \phi \frac{\mu_h}{\mu_b}\right) \gamma_{t+j} \right\}$$

Let

$$l_{t-1} = \left(1 - \frac{1}{\mu_b}\right) b_{t-1} + \phi \frac{\mu_h}{\mu_b} \left(1 - \frac{1}{\mu_h}\right) h_{t-1} - \left[\left(1 - \frac{1}{\mu_b}\right) + \phi \frac{\mu_h}{\mu_b} \left(1 - \frac{1}{\mu_h}\right)\right] d_t$$

[5] See Table ?? for the notation of the variables and parameters we use.

so that with some manipulations and under Assumption 3, we get that in the limit, i.e. as $T \to \infty$,

$$l_{t-1} = -\frac{1}{\mu_b}\left[(1-\mu_b) + (1-\mu_h)\,\phi\right]\sum_{j=1}^{\infty}\left(\frac{1}{\mu_b}\right)^j \Delta d_{t+j}$$
$$+\frac{\phi}{\mu_b}\left(1-\frac{\mu_h}{\mu_b}\right)\sum_{j=0}^{\infty}\left(\frac{1}{\mu_b}\right)^j \Delta h_{t+j}$$
$$+\left(1-\frac{1}{\mu_b}\right)\sum_{j=0}^{\infty}\left(\frac{1}{\mu_b}\right)^j\left\{-r_{t+j-1} + \phi\frac{\mu_h}{\mu_b}\pi_{t+j} + \left(1+\phi\frac{\mu_h}{\mu_b}\right)\gamma_{t+j}\right\}$$

More details about the steps of the derivations can be found in Giannitsarou and Scott (2006).

C.1. Violation of Assumption 3. We next consider the case where Assumption 3 is violated. As above, the budget constraint

$$\bar{G}_t - \bar{T}_t = \bar{B}_t - \frac{\Upsilon_{t-1}}{\Pi_t Q_t \Omega_t}\bar{B}_{t-1} + \bar{H}_t - \frac{1}{\Pi_t Q_t \Omega_t}\bar{H}_{t-1}$$

Assume the existence of a nominal interest rate Υ_t^*, with an associated real interest rate r_t^* for which $(1+r^*) > (1+\gamma)(1+\omega)$. We can then rewrite our budget constraint as

$$\bar{G}_t - \bar{T}_t = \bar{B}_t - \frac{\Upsilon_{t-1}^*}{\Pi_t Q_t \Omega_t}\bar{B}_{t-1} + \frac{\Upsilon_{t-1} - \Upsilon_{t-1}^*}{\Pi_t Q_t \Omega_t}\bar{B}_{t-1} + \bar{H}_t - \frac{1}{\Pi_t Q_t \Omega_t}\bar{H}_{t-1}$$

Bohn's (2006) suggestion is to define

$$\bar{G}_t^* = \bar{G}_t - \frac{\Upsilon_{t-1} - \Upsilon_{t-1}^*}{\Pi_t Q_t \Omega_t}\bar{B}_{t-1}$$

and then use the budget constraint

$$\bar{G}_t^* - \bar{T}_t = \bar{B}_t - \frac{\Upsilon_{t-1}}{\Pi_t Q_t \Omega_t}\bar{B}_{t-1} + \bar{H}_t - \frac{1}{\Pi_t Q_t \Omega_t}\bar{H}_{t-1}$$

However for our purposes, this would require constructing a synthetic government expenditure series whose interpretation would be more difficult. Instead we use the following approach and make the assumption that

$$\Upsilon_{t-1} - \Upsilon_{t-1}^* = \varsigma \bar{H}_{t-1}/\bar{B}_{t-1}$$

Given the sign of H and B and in order for $r^* > r$, we need $\varsigma < 0$. Under this assumption we have

$$\bar{G}_t - \bar{T}_t = \bar{B}_t - \frac{\Upsilon_{t-1}^*}{\Pi_t Q_t \Omega_t}\bar{B}_{t-1} + \frac{\varsigma \bar{H}_{t-1}/\bar{B}_{t-1}}{\Pi_t Q_t \Omega_t}\bar{B}_{t-1} + \bar{H}_t - \frac{1}{\Pi_t Q_t \Omega_t}\bar{H}_{t-1}$$

or

$$\bar{G}_t - \bar{T}_t = \bar{B}_t - \frac{\Upsilon_{t-1}^*}{\Pi_t Q_t \Omega_t}\bar{B}_{t-1} + \bar{H}_t - \frac{1-\varsigma}{\Pi_t Q_t \Omega_t}\bar{H}_{t-1} \tag{14}$$

This last equation is exactly the same as (2), except that the coefficient on $\bar{H}_{t-1}$ is different. All the above steps for deriving a log-linear present value constraint can now be replicated by substituting μ_b and μ_h with

$$\begin{aligned} \mu_b^* &= \frac{1+r^*}{(1+\gamma)(1+\omega)} \text{ and} \\ \mu_h^* &= \frac{1-\varsigma}{(1+\gamma)(1+\omega)} \end{aligned}$$

respectively.

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Country	Mean	Min	Max	Initial	Final	St. Dev	Feed Coef	P-Value
Canada	0.378	0.292	0.718	0.292	0.464	0.193	-0.024	0.03
Germany	0.246	0.049	0.700	0.052	0.700	0.213	-0.006	0.45
Italy	0.702	0.304	1.203	0.374	1.076	0.328	-0.048	0.00
Japan	0.333	0.041	1.296	0.069	1.296	0.226	-0.003	0.67
UK	0.523	0.312	0.928	0.928	0.429	0.381	-0.034	0.01
US	0.300	0.185	0.457	0.353	0.337	0.078	-0.057	0.01

Table 1: **Debt Variability and Sustainability**. First 5 columns report the average, minimum, maximum, initial and final period values of the market value of government debt to GDP ratio. The next column reports the standard deviation of the debt/GDP ratio. The penultimate column shows estimates of α in a regression of a country's primary surplus on four lags of its own value and the lagged value of the debt/GDP ratio with the final column reporting the p value (to two decimal places) of the feedback coefficient.

Debt/GDP				
Country	**Test Statistic**	**statistic**	**5% CV**	**verdict**
CAN	ADF(8) C	-3.450	-2.860	stationary
	ADF(8) C T	-3.910	-3.410	trend stationary
	KPSS(4)	0.718	0.463	unit root
	KPSS(4) T	0.159	0.146	unit root
GER	ADF(1)	-2.280	-1.940	stationary
	ADF(1) C T	-2.440	-3.410	unit root
	KPSS(4)	0.994	0.460	unit root
	KPSS(4) T	0.150	0.146	unit root
ITA	ADF(1)	-2.110	-1.940	stationary
	KPSS(4)	0.959	0.463	unit root
	KPSS(4) T	0.151	0.146	unit root
JAP	ADF(1)	-3.310	-1.940	stationary
	ADF(3) C T	-2.270	-3.410	unit root
	KPSS(4)	0.969	0.463	unit root
	KPSS(4) T	0.163	0.146	unit root
UK	ADF(2) C	-2.510	-2.860	unit root
	KPSS(4)	0.581	0.463	unit root
	KPSS(4) T	0.204	0.146	unit root
US	ADF(1) C	-1.790	-2.860	unit root
	ADF(1) C T	-2.440	-3.410	unit root
	KPSS(4)	0.381	0.463	stationary
	KPSS(4) T	0.146	0.146	unit root
C means constant included. T means trend included.				

Table 2: **Unit Root Tests, Gov Debt/GDP**. ADF denotes Augmented Dickey Fuller Test and KPSS the Kwiatowski, Phillips, Schmidt and Shin test. C denotes a constant also included in the test and T a trend. Number in parantheses indicates number of lags with which test was augmented. The final column shows the inference from evaluating null of test statistic at 95 per cent confidence intervals.

Money				
Country	**test**	**statistic**	**5% CV**	**verdict**
CAN	ADF(1) C	-1.440	-2.860	unit root
	KPSS(4)	0.971	0.463	unit root
	KPSS(4) T	0.133	0.146	trend stationary
GER	ADF(1) C	1.390	-2.860	unit root
	ADF(1) C T	-1.110	-3.410	unit root
	KPSS(4)	0.839	0.460	unit root
	KPSS(4) T	0.253	0.146	unit root
ITA	ADF(0)	1.410	-1.940	unit root
	ADF(4) C T	-3.610	-3.410	trend stationary
	KPSS(4)	0.880	0.463	unit root
	KPSS(4) T	0.143	0.146	trend stationary
JAP	ADF(1)	-1.730	-1.940	unit root
	KPSS(4)	0.590	0.463	unit root
	KPSS(4) T	0.182	0.146	unit root
UK	ADF(5) C	-0.430	-1.940	unit root
	KPSS(4)	0.940	0.463	unit root
	KPSS(4) T	0.203	0.146	unit root
US	ADF(1) C	-2.080	-2.860	unit root
	ADF(0) C T	-0.450	-3.410	unit root
	KPSS(4)	0.666	0.463	unit root
	KPSS(4) T	0.252	0.146	unit root
C means constant included. T means trend included.				

Table 3: **Unit Root Tests, H/GDP**. The table reads as in table 2.

Expenditure/GDP				
Country	**test**	**statistic**	**5% CV**	**verdict**
CAN	ADF(1), C	-2.110	-2.860	unit root
	KPSS(4)	0.748	0.463	unit root
	KPSS(4) T	0.217	0.146	unit root
GER	ADF(0)	-1.820	-1.940	unit root
	KPSS(4)	0.600	0.460	unit root
	KPSS(4) T	0.198	0.146	unit root
ITA	ADF(2) C	-2.670	-2.86	unit root
	ADF(2) C T	-1.360	-3.410	unit root
	KPSS(4)	0.920	0.463	unit root
	KPSS(4) T	0.236	0.146	unit root
JAP	ADF(3)	-1.310	1.940	unit root
	ADF(3) C T	-2.455	-3.410	unit root
	KPSS(4)	0.941	0.463	unit root
	KPSS(4)	0.137	0.146	trend stationary
UK	ADF(9) C	-1.604	-2.860	unit root
	ADF(10) C T	-3.745	-3.410	trend stationary
	KPSS(4)	0.398	0.463	stationary
	KPSS(4) T	0.204	0.146	unit root
US	ADF(7) C	-1.628	-2.860	unit root
	ADF(6) C T	-1.993	-3.410	trend stationary
	KPSS(4)	0.193	0.463	stationary
	KPSS(4) T	0.194	0.146	unit root
C means constant included. T means trend included.				

Table 4: **Unit Root Tests, G/GDP**. The table reads as in table 2.

Revenue/GDP				
Country	**test**	**statistic**	**5% CV**	**verdict**
CAN	ADF(0)	-1.359	-1.940	unit root
	KPSS(4)	0.915	0.463	unit root
	KPSS(4) T	0.155	0.146	unit root
GER	ADF(1) C	-2.490	-2.860	unit root
	KPSS(4)	0.713	0.460	unit root
	KPSS(4) T	0.210	0.146	unit root
ITA	ADF(0)	-2.430	-1.940	stationary
	KPSS(4)	0.969	0.463	unit root
	KPSS(4) T	0.121	0.146	unit root
JAP	ADF(0)	-2.440	1.940	stationary
	KPSS(4)	0.871	0.463	unit root
	KPSS(4)	0.215	0.146	unit root
UK	ADF(1) C	-2.950	-2.860	stationary
	KPSS(4)	0.186	0.463	stationary
	KPSS(4) T	0.184	0.146	unit root
US	ADF(1) C	-3.730	-2.860	stationary
	KPSS(4)	0.141	0.463	stationary
	KPSS(4) T	0.045	0.146	trend stationary
C means constant included. T means trend included.				

Table 5: **Unit Root Tests, T/GDP**. The table reads as in table 2.

Country	Total Def.	Prim. Def.	Int. Paym.	Nom. Gr. Div.	Real Gr.Term	Infl. Term	$\%\Delta\frac{Debt}{GDP}$
US	0.021	0.000	0.021	0.019	0.010	0.009	0.003
UK	0.017	-0.021	0.039	0.041	0.014	0.027	-0.014
GER	0.016	-0.001	0.021	0.009	0.005	0.004	0.064
JAP	0.018	-0.008	0.024	0.011	0.008	0.003	0.080
ITA	0.064	0.021	0.043	0.059	0.015	0.044	0.025
CAN	0.029	-0.005	0.035	0.023	0.011	0.012	0.013

Table 6: **Debt Dynamics**. The first column shows the average total deficit/GDP for each country over the sample period. The next colums show average primary deficit/GDP and average interest payments/GDP. The fourth column shows the average nominal growth dividend $B_{t-1}\gamma_t/(1+\gamma_t))$ and the next two columns decompose this into a real growth term and an inflation term. The final column shows the average percentage change in the Debt/GDP ratio over the period.

	G_t, T_t					G_t, B_t					B_t, H_t				
Lags	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4
Canada	1%	10%	5%*	5%	10%	1%	10%*	10%	5%	5%	5%	10%*			10%
Germany	1%	10%	10%	10%*	5%	1%*	10%	10%	5%	5%	5%		10%*	10%	10%
Italy		10%	5%*	5%	5%	1%	1%*	1%	1%	1%	1%	1%*	1%	1%	5%
Japan	5%			10%	10%*	1%	1%*	1%	1%	1%	5%	5%	5%*	10%	10%
UK	10%	10%	5%*	5%	5%	5%	5%*	1%	1%	5%	1%	5%	10%*	5%	5%
US	5%	5%*	5%	1%	1%	1%	5%	5%*	5%	5%	1%	5%	1%	5%*	5%

Table 7: **Cointegration tests**. The Table shows the p-value of test for level of rejection of the null of no hypothesis between variables listed in top row using Johansen (1991). A * indicates the lag augmentation selected by the AIC criteria.

	US	**Canada**	**Germany**	**Japan**	**UK**	**Italy**
ϕ	0.302	0.174	1.060	0.612	0.103	0.267
γ	1.078	1.083	1.065	1.057	1.090	1.075
μ_b	1.027	1.033	1.008	1.021	1.008	1.041
μ_h	0.953	0.954	0.947	0.965	0.925	0.969
λ_g	-6.968	-15.650	7.690	-2564.103	-56.526	-3.073
λ_τ	7.968	16.650	-6.690	2565.103	57.526	4.073
α_b	0.026	0.032	0.008	0.020	0.008	0.040
α_h	-0.014	-0.008	-0.056	-0.021	-0.008	-0.008
α_g	0.088	0.376	0.376	-1.458	0.047	0.097
α_τ	-0.100	-0.399	-0.319	1.458	-0.048	-0.129
$H_0 : \beta_2 + \beta_3 = \frac{1}{1-\lambda_\tau}$	0.033	0.046	0.071	0.029	0.061	0.039

Table 8: **Estimates of Equilibrium Relationship Debt and Deficits**. The first row reports the sample average of H/B and the second row reports the sample average of nominal GDP growth. The parameters μ_b and μ_τ are estimated, as are λ_g and λ_τ (subject to the restriction that $\lambda_g + \lambda_\tau = 1$). The next four rows show the implied estimated coefficients for defining $l_t = \alpha_b b_t + \alpha_h h_t + \alpha_g g_t + \alpha_\tau \tau_t$. The last row shows p-values for the hypothesis that $\beta_2 + \beta_3 = 1/(1 - \lambda_\tau)$.

	US	Japan	Germany	UK	Italy	Canada
μ_b	1.026	1.015	1.006	1.008	1.010	1.014
	1.027	1.021	1.008	1.008	1.041	1.033
μ_h	0.947	0.980	0.973	0.933	0.944	0.978
	0.953	0.965	0.947	0.925	0.969	0.954
λ_g	-9.371	34.703	-19.672	-52.589	20.196	-36.460
	-6.968	-2564.100	7.690	-56.526	-3.073	-15.650
λ_t	10.371	-33.703	20.672	53.589	-19.196	37.460
	7.968	2565.100	-6.690	57.526	4.073	16.650
α_b	0.026	0.015	0.006	0.008	0.010	0.014
	0.026	0.020	0.008	0.008	0.004	0.032
α_h	-0.016	-0.012	-0.028	-0.007	-0.015	-0.004
	-0.014	-0.021	-0.056	-0.008	-0.008	-0.008
α_g	0.095	-0.093	-0.441	0.040	0.105	0.383
	0.088	-1.458	0.376	0.047	0.097	0.376
α_t	-0.105	0.089	0.464	-0.041	-0.100	-0.394
	-0.100	1.458	-0.319	-0.048	-0.129	-0.399
Corr(l_t)	0.999	0.874	-0.978	0.999	0.749	0.966
Corr(Δl_t)	0.998	0.893	-0.617	0.998	0.824	0.997
ADF	0.010	0.030	0.080	0.010	0.030	0.010

Table 9: **Alternative Estimates of Fiscal Imbalances**. The first row for each country shows estimates of structural parameters and coefficients to form l_t using sample averages and the second row shows estimates from DOLS. The row labelled Corr(l_t) shows the correlation coefficient between the two estimates of l_t and the row labelled Corr(Δl_t) shows the correlation between the first differences of the two estimates. The final row shows the p-value from an Augmented Dickey Fuller test for stationarity.

	US	Canada	Germany	Japan	UK	Italy
Max	0.031	0.065	0.050	0.045	0.055	0.049
Min	-0.025	-0.053	-0.053	-0.060	-0.049	-0.062
Std Dev	0.015	0.027	0.022	0.027	0.024	0.026
Sum AR	0.758	0.860	0.718	0.608	0.756	0.590
Unit Root	0.042	0.031	0.100	0.046	0.054	0.027
25%	0.960	3.300	0.930	1.300	0.210	3.000
50%	2.500	4.100	2.100	2.100	2.500	4.100
75%	5.000	5.200	4.100	2.900	4.960	6.900

Table 10: **Dynamics of Fiscal Adjustment**. The first row shows minimum value of l_t over sample period, while the second row shows the maximum value. The third row is the standard deviation of l_t and the fourth row shows the sum of the AR coefficients when l_t is modelled as an AR(P) process where P is chosen optimally using AIC criteria. The next row is the p-value from an ADF test that l_t is a unit root process. The last three rows show the number of periods it takes l_t to adjust by 25%, 50% and 75% respectively to a shock to its value.

	$F_{\Delta d}$	$F_{\Delta g}$	$F_{\Delta\tau}$	$F_{\Delta h}$	F_r	F_π	F_γ	**Orth. Test**
US	1.321			-0.057	0.007	0.058	0.191	0.019
		0.815	0.718	0.073	-0.219	0.067	-0.059	0.032
Canada	0.985			0.010	-0.013	0.002	-0.018	0.022
		0.534	-0.437	-0.015	-0.039	0.006	-0.002	0.044
Germany	0.930			0.053	-0.021	0.042	0.008	0.045
		0.467	0.571	0.021	-0.016	0.034	0.004	0.061
Japan	1.071			-0.066	0.042	0.066	0.059	0.017
		0.819	0.675	-0.121	0.126	0.116	0.165	0.013
UK	0.912			0.113	-0.141	0.025	-0.003	0.038
		1.001	0.221	0.182	-0.121	-0.002	-0.002	0.031
Italy	0.881			0.007	-0.054	0.068	0.037	0.062
		0.614	0.471	0.312	-0.036	0.096	0.067	0.044

Table 11: **Variance Decomposition of Fiscal Adjustments**. For each country the first row shows the basic variance decomposition and the second row shows the extended variance decomposition.

	US	**Canada**	**Germany**	**Japan**	**UK**	**Italy**
1	0.04	0.08	0.21	0.13	0.23	0.16
2	0.07	0.05	0.26	0.11	0.31	0.13
3	0.08	0.07	0.32	0.07	0.17	0.12
4	0.13	0.11	0.21	0.06	0.13	0.08
5	0.09	0.17	0.17	0.08	0.04	0.04
6	0.07	0.32	0.16	0.12	0.03	0.15
7	0.21	0.25	0.35	0.15	0.05	0.31
8	0.32	0.21	0.49	0.16	0.11	0.24
9	0.36	0.42	0.45	0.21	0.19	0.42
10	0.48	0.56	0.26	0.21	0.23	0.46
11	0.63	0.54	0.32	0.19	0.35	0.53
12	0.54	0.59	0.54	0.26	0.46	0.41
13	0.52	0.58	0.56	0.29	0.32	0.36
14	0.59	0.65	0.63	0.32	0.41	0.39
15	0.73	0.78	0.78	0.33	0.53	0.55
16	0.86	0.79	0.74	0.34	0.52	0.45
17	0.88	0.82	0.72	0.30	0.46	0.34
18	0.75	0.91	0.76	0.26	0.64	0.46
19	0.82	0.96	0.73	0.45	0.71	0.74
20	0.95	0.99	0.74	0.54	0.55	0.65

Table 12: **Predicting Inflation**. The Table shows p-values of significance of l_{t-j} (where j is listed in the first column) in a forecasting equation for inflation, containing lagged values of inflation, interest rates and GDP growth.

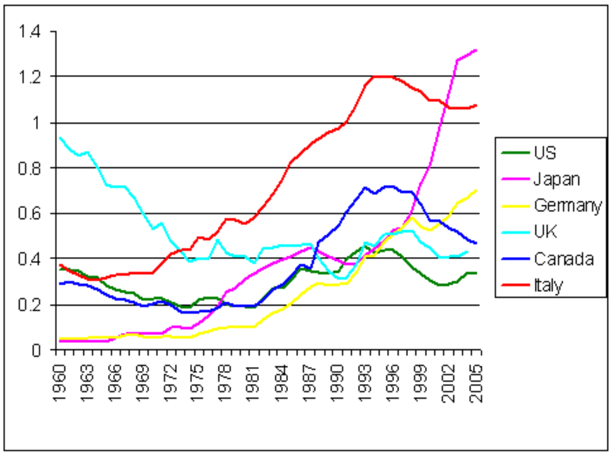

Figure 1: Debt over GDP, for six industrialised countries, 1960 - 2005.

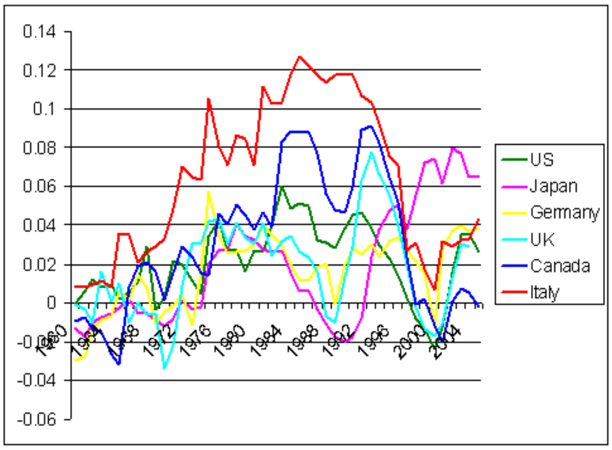

Figure 2: Total deficit over GDP, for six industrialised countries, 1960 - 2005.

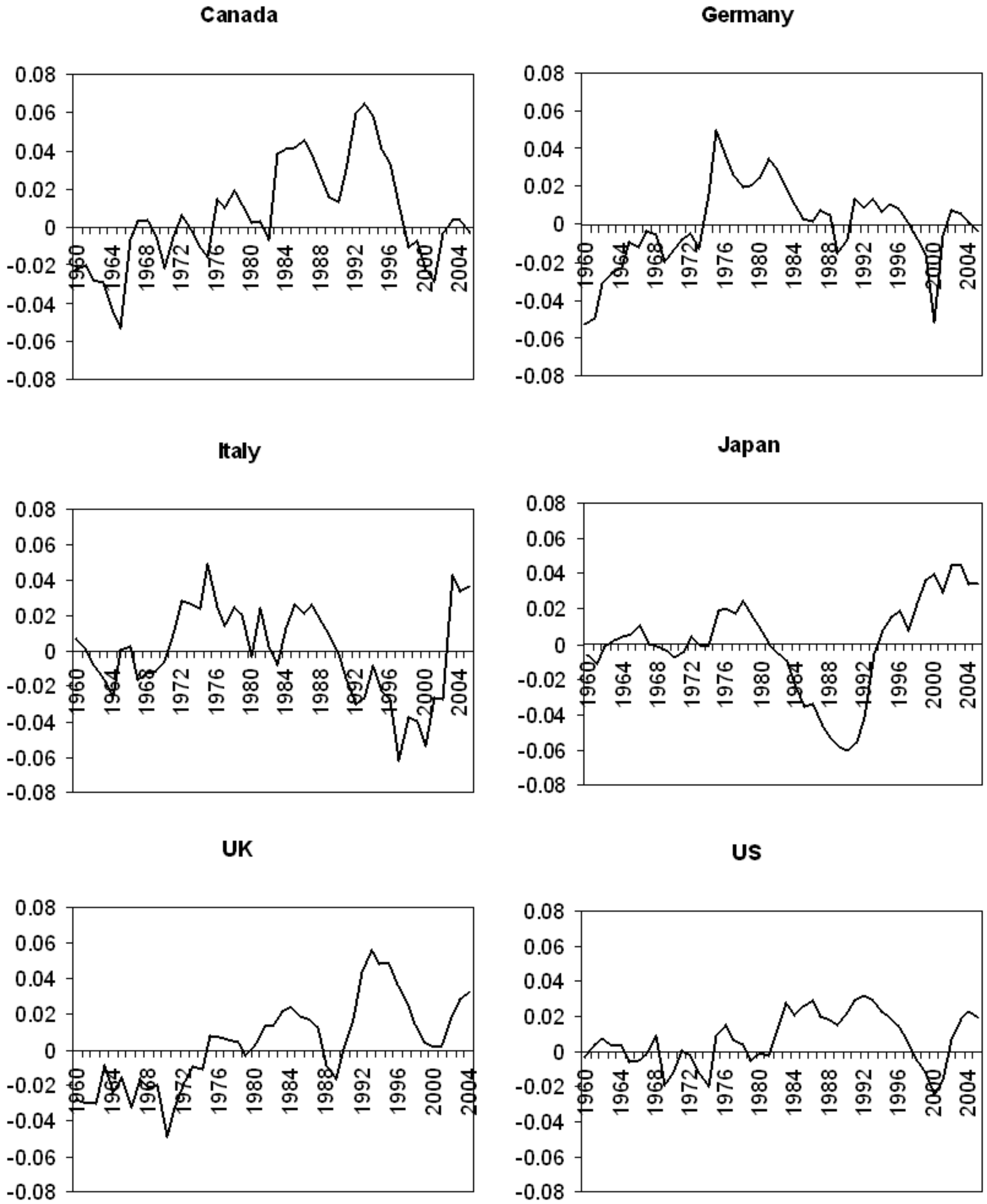

Figure 3: Fiscal Position.

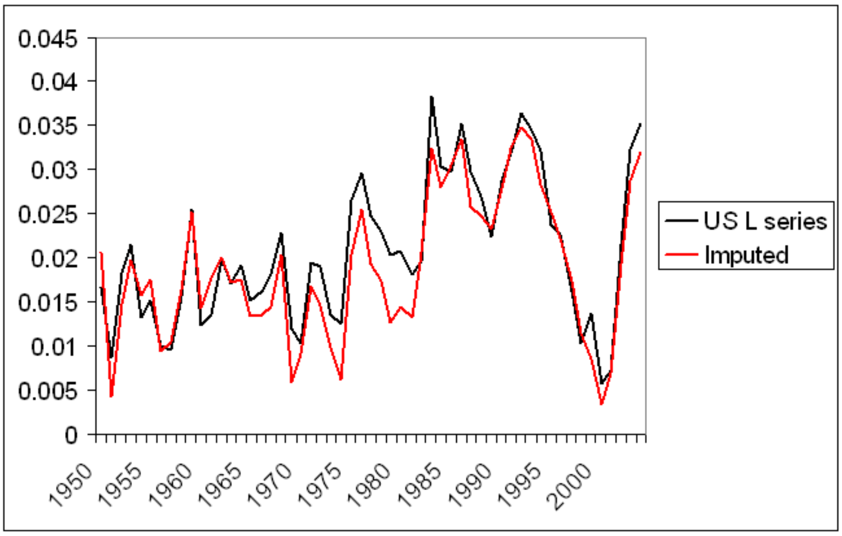

Figure 4: US Decomposition.