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R&D, PRODUCTION STRUCTURE, AND PRODUCTIVITY GROWTH IN THE U.S., JAPANESE AND GERMAN MANUFACTURING SECTORS

Pierre A. Mohnen

M. Ishaq Nadiri

Ingmar R. Prucha

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ABSTRACT

The paper analyzes the production structure and the demand for inputs in three major industrialized countries, the U.S., Japan and Germany. A dynamic factor demand model with two variable inputs (labor and energy) and two quasi-fixed inputs (capital and R&D) is derived directly from an intertemporal cost-minimization problem formulated in discrete time. Adjustment costs are explicitly specified. The model is estimated for the manufacturing sector of the three countries using annual data from 1965 to 1977. Particular attention is given to the role of R&D. For all countries the rate of return on R&D is found to be higher than that on capital. Their respective magnitudes are similar across countries. We find considerable differences in factor demand schedules; we also find that for all countries the speed of adjustment for capital is higher than that of R&D. Adjustment costs are of importance in the demand equations for capital and R&D, but play a minor role in the decomposition of total factor productivity growth.

M. Ishaq Nadiri
Department of Economics
15-19 West 4th Street
New York, NY 10012

Pierre A. Mohnen
Department of Economics
15-19 West 4th Street
New York, NY 10012

Ingmar R. Prucha
Department of Economics
University of Maryland
College Park, MD 20742

Introduction*

There is a considerable literature on the contribution of research and development expenditure (R&D) to the growth rates of output and productivity in various U.S. industries.[1] The questions of what are the determinants of R&D expenditure and what are the rates of return on R&D investment in different sectors of the U.S. economy are also extensively examined.[2] The issue of lags between R&D expenditure, innovative activities, and growth of output have been active areas of research.[3] Similarly, the role of energy as a factor of production in the evolution of the production structure of the U.S. economy and various industries has been debated for a considerable period of time. Of particular interest has been the effect of increase in oil prices on the rate of growth of output, productivity growth and direction of technical change.[4]

These discussions have been largely focused on the U.S. economy and its various industries. Very few econometric studies are available that have explored the role of R&D in other industrialized economies. Further, most of the available studies are based on static equilibrium models and therefore do not adequately explore the intertemporal nature of some of the issues. To take account of these two issues, we shall develop a dynamic factor demand model that takes explicit account of adjustment costs inherent in the investment process and estimate the model using data for the manufacturing sectors of the U.S., Japan and Germany. In the context of our dynamic framework we shall explore the role played by labor and energy as well as that of the quasi-fixed factors, the stocks of plant and equipment and R&D in the evolution of the structure of the production process of the manufacturing sectors of these three countries. These countries were chosen because they are the major economies among the OECD countries and also provide a reasonable regional representation. The manufacturing sectors were selected because of their importance in the industrial structure of these economies and the availability of reasonable sets of data.

In our model a system of dynamic input demand equations is derived from an intertemporal model with internal costs of adjustment. Because of the adjustment costs, the stocks of plant and equipment and R&D will not adjust instantaneously to their optimal levels, and hence those factors will be quasi-fixed in the short run. Labor and energy, the other factors, are assumed to be variable. Because of the adjustment costs, an intertemporal optimization process must be formulated. Furthermore particular care is necessary in the calculation of short- and long-run input elasticities, factor productivity and rates of return on quasi-fixed factors. These measures are likely to differ from the conventional measures reported in the literature.

Our estimating equations are derived directly from an intertemporal cost-minimization problem formulated in discrete time. The technology is represented by a restricted cost function. The technology is assumed to be homogeneous, technical change is considered to be disembodied, and the adjustment costs are specified to be separable.

Using our theoretical framework, we would like to explore several issues: Our first objective is to examine the evolution of the production process of the manufacturing sectors of the U.S., Japan and Germany. Of particular interest would be the short- and long-run responses of employment, energy, investment in plant and equipment and in research and development to changes in the relative prices, output and technical change. Our second aim is to note the magnitude of the adjustment costs related to investment in plant and equipment and R&D. The third objective is to formulate a concept of average net rates of return on the quasi-fixed inputs in the context of an intertemporal model and calculate these rates for capital and R&D for the different manufacturing industries. Finally, we will decompose total factor productivity in the context of our dynamic model into several components and examine the contribution of different inputs, technical change and adjustment costs to the growth of output in the manufacturing sectors of the U.S., Japan and Germany in the period 1965 to 1977.

The paper is organized as follows: In section 1 we formulate the analytical framework and econometric specification of our model. In section 2 we present our estimation results and their interpretation. Section 3 is devoted to the examination of the differential responses of the inputs in different countries to changes in input prices, output, and technical change in the short, intermediate and long run. The average net rates of return in the individual quasi-fixed inputs are formulated and calculated in section 4. In section 5 we examine the decomposition of total factor productivity. The contribution of various factors to the growth rate of output in the manufacturing sectors of these economies will also be discussed in this section. The paper is concluded with a summary and some suggestions for future research. The types and sources of the data used in estimating our model are described in the appendix.

1. Theoretical Model and Econometric Specification

1.1 Theoretical Model

The model underlying this paper is close to that of Denny, Fuss and Waverman (1981) and Morrison and Berndt (1981). Denny et al. and Morrison and Berndt specify their model in continuous time and then employ a discrete approximation of the continuous factor demand equations in their empirical investigation. Instead, we shall specify the entire model in discrete time. It turns out that the two approaches lead to different specifications.[5] The importance of this difference depends, as is explained in more detail below, on the magnitude of certain a priori unknown parameters of the model.

Consider a firm in period t that employs m variable inputs $v_{it}(i=1,\ldots,m)$ and n quasi-fixed inputs $x_{jt}(j=1,\ldots,n)$ in producing the single output good Q_t. More specifically, the firm's production process is described by the following generalized production function:

(1) $$Q_t = F(v_t, x_{t-1}, \Delta x_t, T_t)$$

where $v_t = \{v_{it}\}$ is the vector of variable inputs, $x_t = \{x_{jt}\}$ is the vector of end-of-period stocks of quasi-fixed factors, and T_t is a technology index. The vector $\Delta x_t = \{x_{jt} - x_{j,t-1}\}$ represents internal adjustment costs in terms of forgone output due to changes in the quasi-fixed factors, i.e. $F_{\Delta x_j} < 0$. The production function satisfies standard assumptions with respect to the traditional factors v_t and x_{t-1} and is assumed to be concave in all inputs. This implies that the marginal products of the traditional factors of production are decreasing and that the marginal adjustment costs are increasing.

The firm is assumed to face perfectly competitive markets with respect to its factor inputs. We denote the acquisition price for the variable and quasi-fixed factors as $\tilde{w}_{it}$ $(i = 1,\ldots,m)$ and $\tilde{q}_{jt}$ $(j = 1,\ldots,n)$, respectively. It proves convenient to normalize all prices in terms of the price of the first variable factor, and we denote those normalized prices as $w_{it} = \tilde{w}_{it}/\tilde{w}_{1t}$ and $q_{jt} = \tilde{q}_{jt}/\tilde{w}_{1t}$.

The production technology (1) can be described alternatively in terms of the normalized restricted cost function. Let $\{\hat{v}_{it}\}$ denote the cost-minimizing variable factor inputs needed to produce output Q_t conditional on x_{t-1} and Δx_t; then the normalized restricted cost function is defined as

$$(2) \qquad G(w_t, x_{t-1}, \Delta x_t, Q_t, T_t) = \sum_i w_{it}\hat{v}_{it}$$

with $w_t = \{w_{it}\}_{i=2}^{m}$. This function has the following properties (see Lau (1976)): $G_{x_j} < 0$, $G_{\Delta x_j} > 0$, $G_Q > 0$, $G_{w_i} > 0$. Furthermore, $G(\cdot)$ is convex in x_{t-1} and Δx_t is concave in w_t.

The firm is assumed to hold static expectations on relative factor prices, output, the technology and the discount rate, r_t. In each period the firm is assumed to derive, for given initial stocks x_{t-1} and subject to the production function constraint (1), an optimal input path such that the present value of the expected cost stream is minimized. Making use of the restricted cost function (2) and choosing a "first order certainty equivalence" representation we can state the firm's optimization problem in period t as

$$(3) \qquad \min_{\{x_{t+\tau}\}_{\tau=0}^{\infty}} \sum_{\tau=0}^{\infty} \{G(w_t, x_{t+\tau-1}, \Delta x_{t+\tau}, Q_t, T_t) + \sum_j q_{jt}(x_{j,t+\tau} - (1-\delta)x_{j,t+\tau-1})\}(1+r_t)^{-\tau}$$

where δ_j is the depreciation rate of the j-th quasi-fixed factor. The optimization problem (3) is a standard dynamic programming problem. The following conditions are necessary for a minimum (j=1,...,n)

$$(4) \qquad \frac{\partial G(t+\tau)}{\partial \Delta x_j} + q_{jt} + \frac{1}{1+r_t}\left[\frac{\partial G(t+\tau+1)}{\partial x_j} - \frac{\partial G(t+\tau+1)}{\partial \Delta x_j} - (1-\delta_j)q_{jt}\right] = 0, \qquad \tau = 0,\ldots,\infty;$$

where $G(t+\tau) = G(w_t, x_{t+\tau-1}, \Delta x_{t+\tau}, Q_t, T_t)$.[6]

The firm's demand for its variable factors can be derived from the normalized restricted cost function via Shephard's lemma as

$$(5) \qquad \hat{v}_{it} = \frac{\partial G(t)}{\partial w_{it}} \qquad i = 2,\ldots,m.$$

The demand for the first variable factor can be computed as $\hat{v}_{1t} = G(t) - \sum_{i=2}^{m} w_{it}\hat{v}_{it}$.

The first order conditions describing the firm's demand for quasi-fixed factors can be given the following economic interpretation: The firm plans to invest in a quasi-fixed input until, at the margin, the cost of adjustment plus the purchase price of one unit of investment in period t+τ plus the discounted depreciation losses equals the discounted sum of the production and adjustment cost savings plus the purchase price of the investment unit in period t+τ+1.

1.2 Econometric Specification

The functional form of the normalized restricted cost function used in our empirical analysis (dropping all t-subscripts) is as follows:

$$
\begin{aligned}
\text{(6a)} \quad G(\cdot) = Q\Big[\alpha_0 &+ \alpha_{0T}T + \sum_{i=2}^{m} \alpha_i w_i + \frac{1}{2}\sum_{i=2}^{m}\sum_{\ell=2}^{m} \alpha_{i\ell} w_i w_\ell \\
&+ \sum_{i=2}^{m} \alpha_{iT} w_i T + \frac{1}{2}\alpha_{TT}T^2\Big] \\
&+ \sum_{j=1}^{n} \gamma_j z_j + \sum_{j=1}^{n} \dot{\gamma}_j \Delta x_j + \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} \gamma_{jk}\frac{z_j \cdot z_k}{Q} \\
&+ \frac{1}{2}\sum_{i=2}^{m}\sum_{j=1}^{n} \beta_{ij} w_i z_j + \sum_{j=1}^{n} \gamma_{jT} z_j T \\
&+ \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} \ddot{\gamma}_{jk}\frac{\Delta x_j \cdot \Delta x_k}{Q} + \frac{1}{2}\sum_{i=2}^{m}\sum_{j=1}^{n} \dot{\beta}_{ij} w_i \Delta x_j \\
&+ \sum_{j=1}^{n} \dot{\gamma}_{jT} \Delta x_j T + \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} \dot{\gamma}_{jk}\frac{\Delta x_j z_k}{Q}
\end{aligned}
$$

where $z_{jt} = x_{j,t-1}$ and $\alpha_{i\ell}=\alpha_{\ell i}$, $\gamma_{jk}=\gamma_{kj}$, $\ddot{\gamma}_{jk}=\ddot{\gamma}_{kj}$. This functional form of a normalized restricted cost function was first introduced by Denny, Fuss and Waverman (1981) and Morrison and Berndt (1981). It can be viewed as a second order approximation to a general normalized restricted cost function corresponding to a constant returns to scale technology.[7] We impose the usual parameter restriction:

$$\text{(6b)} \qquad \dot{\gamma}_j = \dot{\beta}_{ij} = \dot{\gamma}_{jt} = \dot{\gamma}_{jk} = 0, \qquad j,k = 1,\ldots,n, \quad i = 2,\ldots,m,$$

$$\text{(6c)} \qquad \gamma_{jk} = \ddot{\gamma}_{jk} = 0 \qquad j \neq k,\ j,k = 1,\ldots,n.$$

Restriction (6b) implies that the marginal adjustment costs at $\Delta x=0$ are zero. Restriction (6c) is rather strong; it implies a zero long-run cross-price elasticity between quasi-fixed factors. However, the restriction is imposed to make our problem in case of more than one quasi-fixed factor analytically and empirically tractable.

Making use of Shephard's lemma (5) we obtain the following demand equations for the variable factors:

$$\text{(7a)} \qquad \frac{\hat{v}_{it}}{Q_t} = \alpha_i + \sum_{\ell=2}^{m} \alpha_{i\ell} w_{\ell t} + \alpha_{iT} T_t + \sum_{j=1}^{n} \beta_{ij} \frac{x_{j,t-1}}{Q_t} \qquad i = 2,\ldots,m$$

$$\begin{aligned} \text{(7b)} \qquad \frac{\hat{v}_{1t}}{Q_t} &= \left[G(t) - \sum_{i=2}^{m} w_{it}\hat{v}_{it}\right]/Q_t \\ &= \alpha_o + \alpha_{oT}T_t - \frac{1}{2}\sum_{i=2}^{m}\sum_{\ell=2}^{m} \alpha_{i\ell} w_{it} w_{\ell t} + \frac{1}{2}\alpha_{TT}T_t^2 \\ &\quad + \sum_{j=1}^{n} \gamma_j \frac{x_{j,t-1}}{Q_t} + \sum_{j=1}^{n} \gamma_{jT} \frac{x_{j,t-1}}{Q_t} T_t \\ &\quad + \frac{1}{2}\sum_{j=1}^{n} \gamma_{jj}\left[\frac{x_{j,t-1}}{Q_t}\right]^2 + \frac{1}{2}\sum_{j=1}^{n} \ddot{\gamma}_{jj}\left[\frac{\Delta x_{jt}}{Q_t}\right]^2 . \end{aligned}$$

The necessary first order conditions corresponding to (4) describing the optimal input plan of the quasi-fixed factors are given by $(j = 1,\ldots,n)$

$$\begin{aligned} \text{(8)} \qquad & -\ddot{\gamma}_{jj}x_{j,t+\tau-1} + \left[\gamma_{jj} + (2 + r_t)\ddot{\gamma}_{jj}\right]x_{j,t+\tau} - (1 + r_t)\ddot{\gamma}_{jj}x_{j,t+\tau-1} \\ &= -\left[\gamma_j + \sum_{i=2}^{m} \beta_{ij} w_{it} + \gamma_{jT}T_t + q_{jt}(r_t + \delta_j)\right]Q_t, \qquad \tau = 0,\ldots,\infty. \end{aligned}$$

Note that the unstable root of each of the above sets of second order difference equations is (analogously to the continuous model) ruled out by a transversality condition. It is then not difficult to see that solving (8) yields the following set of accelerator equations for the demand of the quasi-fixed factors:

$$\text{(9a)} \qquad \frac{\hat{x}_{jt}}{Q_t} - \frac{x_{j,t-1}}{Q_t} = m_{jj,t}\left[\frac{x^*_{jt}}{Q_t} - \frac{x_{j,t-1}}{Q_t}\right] \qquad j = 1,\ldots,n,$$

where

$$\text{(9b)} \qquad x^*_{jt} = -\frac{1}{\ddot{\gamma}_{jj}}\left[\gamma_j + \sum_{i=2}^{m} \beta_{ij} w_{jt} + \gamma_j T_t + q_{jt}(r_t + \delta_j)\right] Q_t$$

is the long run equilibrium stock and

$$\text{(9c)} \qquad m_{jj,t} = -\frac{1}{2}\{r_t + \frac{\gamma_{jj}}{\ddot{\gamma}_{jj}} - \left[(r_t + \frac{\gamma_{jj}}{\ddot{\gamma}_{jj}})^2 + 4\frac{\gamma_{jj}}{\ddot{\gamma}_{jj}}\right]^{\frac{1}{2}}\}.$$

Our entire system of estimable demand equations thus consists of the m variable factor demand equations (7) and the n quasi-fixed factor demand equations (9). For the empirical estimation we replace $\hat{v}_{it}$ and $\hat{x}_{jt}$ by the actually observed values v_{it} and x_{jt} and add a stochastic disturbance term to equations (7a,b) and (9a).

The discrete approximations of the accelerator equations obtained from a continuous model are also of the form (9) but with $m_{jj,t}$ replaced by

$$\text{(9c')} \qquad m^+_{jj,t} = -\frac{1}{2}\{r_t - \left[r_t^2 + 4\frac{\gamma_{jj}}{\ddot{\gamma}_{jj}}\right]^{\frac{1}{2}}\}.$$

Note that as the adjustment costs go to zero, i.e. $\ddot{\gamma}_{jj} \to 0$, we have $m_{jj,t} \to 1$ but $m^+_{jj,t} \to \infty$. Hence within the context of our discrete model we can test and allow for the possibility that some of the factors that are a priori expected to be quasi-fixed are actually variable, i.e. $\hat{x}_{jt} = x^*_{jt}$. The discrete demand equations derived from a continuous model become meaningless as adjustment costs go to zero.

2. Empirical Results

For empirical estimation of the model we consider two variable factors, labor (L) and energy (E), and two quasi-fixed inputs, stocks of plant and equipment (K) and R&D (R). The estimating equations consist of (7a), (7b), and (9a), with m=n=2 and where $v_1 = L$, $v_2 = E$, $x_1 = K$, and $x_2 = R$. The data used to estimate the production structure and factor demand in the manufacturing sector of the U.S., Japan and Germany cover the period 1965-1977. The sources of the data and method of constructing the variables of the model are described in the appendix. Data on output, energy, capital and R&D are in constant 1970 prices. The technology index is represented by a simple time trend. The estimation technique used is full information maximum likelihood. When necessary, a correction was made for first order autocorrelation of the disturbances. All estimation was performed with TSP.

The parameter estimates of the model are shown in Table 1. The fit of the model is quite good, and the estimated coefficients are generally statistically significant. The R^2 for the labor equation is low for Japan; the inidividual parameter estimates, as expected, vary across countries. However, the important restrictions susggested by economic theory to insure the concavity conditions of the underlying technology are met in all cases, i.e., the parameters $\ddot{\gamma}_{KK}$, γ_{KK}, $\ddot{\gamma}_{RR}$ and γ_{RR} are all positive, and γ_{vv} are all negative.

The adjustment coefficients $m_{jj,t}$ (j = K,R) as implied by the estimates in Table 1 can be calculated using the expression (9c). Their 1970 values are shown in Table 2.

Table 1

Maximum Likelihood Estimates of the Cost of Adjustment

Model for the Total Manufacturing Sectors of the U.S., Japan, and Germany*

1965 - 1977

Parameter	U.S.	JAPAN	GERMANY
α_0	1.79 (9.76)	5.01 (61.71)	3.31 (8.64)
α_{0T}	0.16 (2.06)	-3.86 (-19.54)	-1.12 (-10.69)
γ_K	-0.93 (-2.84)	-2.41 (-18.11)	-2.45 (-3.60)
γ_R	-0.32 (-4.32)	-0.14 (-34.67)	-0.19 (-7.85)
α_{VV}	-4.40 (-10.57)	-0.53 (-2.79)	-13.43 (-3.71)
γ_{KK}	0.75 (2.18)	0.90 (6.19)	1.79 (2.91)
γ_{RR}	0.22 (3.02)	0.05 (10.82)	0.16 (5.80)
$\ddot{\gamma}_{KK}$	4.22 (3.47)	1.67 (4.05)	2.54 (4.08)
$\ddot{\gamma}_{RR}$	5.42 (8.07)	0.42 (14.69)	1.42 (3.51)
γ_{KT}	-0.04 (-1.20)	0.67 (11.29)	0.28 (5.03)
γ_{RT}	-0.06 (-3.93)	0.03 (21.78)	-0.03 (-3.02)
α_{TT}	-0.19 (-4.33)	2.16 (13.39)	0.38 (6.95)

Table 1 (Continued)

Parameter	U.S.	JAPAN	GERMANY
α_V	0,71 (7,06)	1,27 (32,95)	-0,39 (-1,81)
α_{VT}	-0.06 (-2,54)	-0.25 (-10.67)	-0,03 (-0,38)
β_{VK}	-0,07 (-0,53)	0.38 (6.77)	2,24 (8.33)
β_{VR}	0,74 (6,70)	-0,30 (-55,35)	-0.40 (-1.81)
L - equation; R^2	0,77	0.34	0.86
E - equation; R^2	0,92	0.97	0.78
K - equation; R^2	0,98	0,98	0.98
R - equation; R^2	0,99	0,99	0.99

*The ratios of parameter estimates to asymptotic standard errors are given in parentheses. The R^2 values are the squared correlation coefficients between the actual variables (L, E, K, R) and their fitted values as calculated from the reduced form.

Table 2

Maximum Likelihood Estimates of the Adjustment Coefficients

m_{KK} (Capital) and m_{RR} (R&D) in the Manufacturing Sectors

of the U.S., Japan, and Germany for 1970

Adjustment Coefficients	U.S.	JAPAN	GERMANY
m_{KK}	0.32	0.49	0.53
m_{RR}	0.15	0.26	0.26

Several points are of interest with respect to these results. First, the adjustment coefficient for capital, m_{KK}, is generally greater by a factor of two in each country than the corresponding adjustment coefficient for R&D, m_{KK}. These estimates imply that about 75% of the adjustment in capital is completed after two or three years, but that it takes much longer to adjust the stock of R&D. There is some evidence confirming our results in the case of capital stock. Mayer (1960) concluded from a survey of 276 U.S. companies that there was a lag of two to three years involved in plant investment and of seven to eight months in investment in equipment. Similar results were obtained by Almon (1968), Berndt, Fuss and Waverman (1980), Bischoff (1969, 1971), Coen and Hickman (1970), and Jorgenson and Stephenson (1967) on U.S. data. However, not much evidence exists on the adjustment of the stock of R&D to its desired level. Nadiri (1980) and Nadiri and Bitros (1980) obtained an own-adjustment coefficient for R&D from .16 to .32 on U.S. firm and total manufacturing data. Second, the adjustment coefficients of capital and R&D differ among the manufacturing sectors of the countries. For the U.S., both stocks of plant and equipment and R&D adjust to their desired levels much more slowly than in the other countries. The pattern of adjustments of plant and equipment and R&D in Japan and Germany are quite similar, i.e., about 50% of the adjustment of the capital stock and 26% of the adjustment of the R&D stock takes place in the first year.

Table 3

Costs of Adjustment in Relation to Investment Expenditure and Equilibrium Rental Price for Total Manufacturing Sector of the U.S., Japan, Germany for 1970 (in percentages)

	U.S.	JAPAN	GERMANY
	A. Total Adjustment Costs as % of Investment Expenditures		
Capital	5.4	4.4	6.2
R&D	27.9	6.2	23.6
	B. Gap Between the Ratios of the Marginal Products of the Quasi-Fixed Factors and Labor as a % of the Equilibrium Rental Price		
Capital	12.38	10.50	46.50
R&D	57.94	23.50	89.02

Another interesting feature of the model is the look at the magnitude of the average adjustment cost for one investment unit as a percentage of the acquisition price of investment. These calculations are indicated in panel A of Table 3. The figures for both the average costs of adjustments for plant and equipment and R&D show that the Japanese manufacturing sector experienced lower adjustment cost than the U.S. and German manufacturing sectors. This was especially true in the case of R&D.

Because of adjustment cost we observe in the short run a wedge between the rate of technical substitution and relative input prices. This is evident by inspection from equation (4). In the long run, adjustment costs are zero and hence $-\frac{\partial G(\cdot)}{\partial x_j} = q_j (r + \delta_j)$, i.e., the rate of technical substitution will equal the (normalized) equilibrium user cost of the inputs. But, in the short run, the adjustment costs serve as a wedge. This implies that the marginal productivity of the quasi-fixed inputs is in the short run much larger than their equilibrium values to compensate for both the user costs and the adjustment costs of investment. In panel B of Table 3, we present some estimates of the percentage gap between the ratios of the marginal products of the quasi-fixed factors and labor as compared with the equilibrium rental prices normalized by the wage rate. Again, these results confirm the conclusions stated above that the Japanese manufacturing sector faced in the period under consideration a much smaller adjustment cost than the manufacturing sectors of the U.S. and Germany. This was particularly true in the case of R&D.

3. Short-Run, Intermediate-Run and Long-Run Responses

To examine how factor inputs respond to changes in relative prices, output and technical change in the context of a dynamic model, it requires careful formulation of the various concepts of elasticity. We calculate various elasticities along the optimal adjustment path. Let $\hat{x}_{j,t+\tau}$ $(\tau = 0,1,\ldots)$ be the sequence of the optimal quasi-fixed factor inputs defined by (9). We then have

$$\hat{x}_{jt} = m_{jj,t}x^*_{jt} + (1 - m_{jj,t})x_{j,t-1}$$

$$\hat{x}_{j,t+1} = m_{jj,t}(2 - m_{jj,t})x^*_{jt} - (1 - m_{jj,t})^2 x_{j,t-1}$$

and $\hat{x}_{j,t+\infty} = x^*_{jt}$. We refer to the elasticities of $\hat{x}_{jt}$, $\hat{x}_{j,t+1}$ and x^*_{jt} with respect to input prices and output as, respectively, the short-run, intermediate-run and the long-run elasticities of the j-th quasi-fixed factor. We denote them as $\varepsilon^S_{x_j s}$, $\varepsilon^I_{x_j s}$ and $\varepsilon^L_{x_j s}$, where $s = \tilde{w}_{1t},\ldots,\tilde{c}_{1t},\ldots,Q_t,T$ and $\tilde{c}_{jt} = \tilde{q}_{jt}(r_t + \delta_j)$:

$$\varepsilon^S_{x_j s} = \frac{s}{\hat{x}_{jt}}\frac{\partial \hat{x}_{jt}}{\partial s} = m_{jj,t}\frac{s}{\hat{x}_{jt}}\frac{\partial x^*_{jt}}{\partial s},$$

$$\text{(10a)} \qquad \varepsilon^I_{x_j s} = \frac{s}{\hat{x}_{j,t+1}}\frac{\partial \hat{x}_{j,t+1}}{\partial s} = m_{jj,t}(2 - m_{jj,t})\frac{s}{\hat{x}_{j,t+1}}\frac{\partial x^*_{jt}}{\partial s};$$

$$\varepsilon^L_{x_j s} = \frac{s}{x^*_{jt}}\frac{\partial x^*_{jt}}{\partial s}.$$

Let $\hat{v}_{i,t+\tau}$ be the sequence of optimal variable factor inputs associated with $\hat{x}_{j,t+\tau}$ $(\tau = 0,1,\ldots)$. Write (7) more compactly as $\hat{v}_{it} = G_i(\tilde{w}_{1t},\ldots,x_{1,t-1},\ldots,Q_t,T)$; we then have $\hat{v}_{i,t+1} = G_i(\tilde{w}_{1t},\ldots,\hat{x}_{1t},\ldots,Q_t,T)$ and $\hat{v}_{i,t+\infty} = v^*_{it} = G_i(\tilde{w}_{1t},\ldots,x^*_{1t},\ldots,Q_t,T)$ for $i = 1,\ldots,n$. Analagously to the above, we define the following short run, intermediate-run and long-run elasticities of the i-th variable factor with respect to input prices, output, and technology index

$$\varepsilon^{S}_{v_i s} = \frac{s}{\hat{v}_{it}} \frac{\partial \hat{v}_{it}}{\partial s} = \frac{s}{\hat{v}_{it}} \frac{\partial G_i}{\partial s}\bigg|_{x_{t-1}} ,$$

$$(10b) \qquad \varepsilon^{I}_{v_i s} = \frac{s}{\hat{v}_{i,t+1}} \frac{\partial \hat{v}_{i,t+1}}{\partial s} = \frac{s}{\hat{v}_{i,t+1}} \left[\frac{\partial G_i}{\partial s} + \sum_j \frac{\partial G_i}{\partial \hat{x}_{jt}} \frac{\partial \hat{x}_{jt}}{\partial s}\right]_{\hat{x}_t} ,$$

$$\varepsilon^{L}_{v_i s} = \frac{s}{v^*_{it}} \frac{\partial v^*_{it}}{\partial s} = \frac{s}{v^*_{it}} \left[\frac{\partial G_i}{\partial s} + \sum_j \frac{\partial G_j}{\partial x^*_{jt}} \frac{\partial x^*_{jt}}{\partial s}\right]_{x^*_t} .$$

Because the quasi-fixed factors do not adjust immediately to their long-run equilibrium values, some of the variable factors have to overshoot in the short run their long-run equilibrium levels. That is, the short-run elasticities of some of the variable factors have to be larger than the long-run elasticities.

Using the expressions (10a) and (10b), we calculate the own- and cross-price elasticities of the inputs as well as their output elasticities in the short, intermediate and long run. We have also derived and calculated the response pattern of the inputs to technical change, which are reported below.

3.1 Price elasticities

The own-price elasticities of labor, energy, capital and R&D are reported in Table 4. The first row contains own-price elasticities of employment; the short-run values of this elasticity are small and positive in every case, but its intermediate- and long-run values are larger and have the expected negative signs. The positive sign of this elasticity is due to the fact that, in the short run, the firm cannot substitute labor for its quasi-fixed input, but must bear the costs of adjustment in terms of labor input. The long-run labor price elasticity is low in the U.S. and German manufacturing sectors but very high in

Table 4

Short-Run, Intermediate-Run, and Long-Run Own-Price Elasticities,

U.A., Japan, Germany; Manufacturing Sectors, 1970

Elasticity Estimate	U.S.			JAPAN			GERMANY		
	SR	IR	LR	SR	IR	LR	SR	IR	LR
ε_{LPL}	.01	-.03	-.12	.08	-.45	-1.02	.01	-.05	-.07
ε_{EPE}	-.22	-.23	-.29	-.04	-.08	-.21	-.39	-.39	-.42
ε_{KPK}	-.08	-.13	-.23	-.52	-.75	-.96	-.08	-.11	-.13
ε_{RPR}	-.05	-.09	-.28	-.46	-.72	-1.27	-.07	-.11	-.22

ε_{LPL}: Own-price elasticity of labor

ε_{EPE}: Own-price elasticity of energy

ε_{KPK}: Own-price elasticity of capital

ε_{RPR}: Own-price elasticity of R&D

the Japanese manufacturing sector.

The own-price elasticity of energy is negative in all cases but very small in Japanese manufacturing. It has the largest magnitude in the case of Germany, followed by the U.S. The own-price elasticity of capital is negative in every case; its magnitude is small in the short run but becomes relatively large in the intermediate run and long run. Surprisingly, the magnitudes of the own-price elasticity of capital are for the U.S. and Germany larger than those of labor. The long-run own-price elasticity of the capital stock is highest in Japan. The pattern of the own-price elasticities of the stock of R&D is, in general, similar to that of the capital stock, and the ranking of countries does not change significantly. It seems that in general the own-price elasticities of the inputs are much greater in the Japanese manufacturing sector than in the other two countries. As expected, the elasticities of the quasi-fixed inputs are higher in the long run than in the short run.

The cross-price elasticities of inputs are shown in Table 5. They reveal the following patterns: The cross-price elasticities are generally small except for the elasticities of capital with respect to the price of labor in the U.S. and Germany, and the elasticities of R&D with respect to price of labor and energy. The intermediate- and long-run cross-price elasticities of labor with respect to user cost of capital and the elasticities of energy with respect to prices of the quasi-fixed factors are also fairly large. Labor seems to be a substitute for capital, R&D and energy in all countries; the degree of substitution between labor and the quasi-fixed inputs are much stronger in Japan than in the manufacturing sectors of the U.S. and Germany over the period under consideration. Capital and energy are complements in Japan and Germany and substitutes in the U.S., but the strength of these elasticities is quite weak.

These patterns of substitution and complementarity among inputs are generally in accordance with the findings of some of the studies reported in the literature. Capital and energy are often reported to be complements in time series studies but substitutes in studies using panel data.[8] Previous studies based on cost-of-adjustment models using U.S. data (Berndt, Fuss and Waverman (1980), Morrison and Berndt (1981), and Pindyck and Rotemberg (1982)) have found capital and energy to be complements, while labor and energy were found to be substitutes. However, a more careful analysis is required to compare our findings on the input elasticities to those reported in the literature, a task that we have postponed for the present.

Table 5

Short-Run, Intermediate-Run and Long-Run Cross-Price Elasticities,

U.S., Japan, Germany: Manufacturing Sectors, 1970

Elasticity Estimate	U.S.			JAPAN			GERMANY		
	SR	IR	LR	SR	IR	LR	SR	IR	LR
ε_{LPE}	.01	.01	.03	.01	.01	.01	.02	.02	.02
ε_{LPK}	-.01	.01	.05	-.07	.41	.92	-.01	.03	.05
ε_{LPR}	-.01	.00	.04	-.02	.02	.09	-.01	.00	.01
ε_{EPL}	.22	.26	.48	.04	.13	.10	.39	.52	.61
ε_{EPK}		.01	.01		-.17	-.36		-.16	-.28
ε_{EPR}		-.04	-.20		.11	.48		.02	.09
ε_{KPL}	.07	.12	.23	.54	.77	.99	.09	.13	.17
ε_{KPE}	.00	.00	.01	-.02	-.03	-.03	-.02	-.03	-.03
ε_{RPL}	.08	.14	.42	.31	.49	.86	.05	.08	.16
ε_{RPE}	-.03	-.05	-.14	.15	.23	.41	.02	.03	.06

ε_{LPE}: elasticity of labor with respect to the price of energy

ε_{LPK}: elasticity of labor with respect to the user cost of capital

ε_{LPR}: elasticity of labor with respect to the user cost of R&D

ε_{EPL}: elasticity of energy with respect to the price of labor

ε_{EPK}: elasticity of energy with respect to the user cost of capital

ε_{EPR}: elasticity of energy with respect to the user cost of R&D

ε_{KPL}: elasticity of capital with respect to the price of labor

ε_{KPE}: elasticity of capital with respect to the price of energy

ε_{RPL}: elasticity of R&D with respect to the price of labor

ε_{RPE}: elasticity of R&D with respect to the price of energy

3.2 Output Elasticities

The output elasticities of employment, energy, capital stock, and stock of R&D are shown in Table 6. The long-run elasticities of the inputs are equal to unity, as is implied by the underlying linear homogeneous technology. The results indicate that, in all cases, employment responds very strongly in the short run to a change in output; the reason is that employment overshoots its long-run equilibrium value in the short run to compensate for the sluggish adjustments of the two quasi-fixed inputs. It slowly adjusts toward its long-run equilibrium value as capital and R&D adjust. The short- and intermediate-run output elasticities of employment are somewhat higher in Japan, but, in general, the pattern of adjustment is remarkably similar among the manufacturing sectors of these countries.

Table 6

Short-Run, Intermediate-Run, and Long-Run Output Elasticities,

U.S., Japan, Germany; Manufacturing Sectors, 1970

Elasticity Estimate	U.S.			JAPAN			GERMANY		
	SR	IR	LR	SR	IR	LR	SR	IR	LR
ε_{LQ}	1.43	1.31	1.00	1.85	1.44	1.00	1.65	1.25	1.00
ε_{EQ}	.39	.49	1.00	.93	1.01	1.00	-.84	.34	1.00
ε_{KQ}	.33	.54	1.00	.54	.78	1.00	.57	.81	1.00
ε_{RQ}	.18	.33	1.00	.36	.57	1.00	.35	.52	1.00

ε_{LQ}: elasticity of labor with respect to output

ε_{EQ}: elasticity of energy with respect to output

ε_{KQ}: elasticity of capital with respect to output

ε_{RQ}: elasticity of R&D with respect to output

The output elasticities of energy are fairly high for Japan, about unity in the short and intermediate run. This contrasts to the situation in the U.S. and German manufacturing sectors, where the short-run output elasticity of energy is fairly small. This contrast between Japan and the other two countries is quite interesting in light of the high degree of the Japanese economy's dependence on imported energy supplies.

The output elasticities of capital and R&D are small in the short run, but they increase over time. The short- and intermediate-run elasticities of capital with respect to output are fairly high for Japan and Germany in comparison to those for the U.S. manufacturing sector. Similarly, the output elasticities of stock of R&D are high for Japan and Germany, and quite alike. On the whole the output elasticities of both the variable and the quasi-fixed factors exceed substantially their own-price elasticities noted earlier. Also, in Japan factor inputs respond to both changes in relative prices and output much more than in the U.S. and German manufacturing sectors.

3.3 Impact of Technical Change

The impact of technological change on factor inputs is shown in Table 7. The signs of these estimates indicate the direction of technical change; i.e., if the coefficient is positive, technological change is using more of the particular input and, if it is negative, it is using less of the input. In all cases the estimated technical change is labor-saving. The impact of technical change is small in the short run but increases in the long run. The results for the U.S. are similar to those reported by Berndt et al. (1980) but the magnitude of our long-run effect of technological change is somewhat higher. It is interesting to note that the magnitude of labor-saving technological change is high in Japan and Germany in comparison to that in the U.S. The long-run magnitude of labor-saving due to technical progress is more than twice in manufacturing sectors of these countries than in the U.S. manufacturing sector.

Technological advance is also energy-saving in all three countries, with the highest impact registered in Japan. The energy-saving effect of technical change in Germany is small in the short run but becomes fairly large in the

Table 7

The Short-Run, Intermediate-Run, and Long-Run Responses

of the Inputs to Technical Change;

U.S., Japan, Germany: Manufacturing Sectors, 1970

Elasticity Estimate	U.S.			JAPAN			GERMANY		
	SR	IR	LR	SR	IR	LR	SR	IR	LR
ε_{LT}	-.013	-.015	-.021	-.124	-.087	-.043	-.055	-.051	-.050
ε_{ET}	-.006	-.003	-.012	-.026	-.036	-.040	-.003	-.024	-.045
ε_{KT}	.002	.003	.006	-.046	-.066	-.085	-.010	-.013	-.017
ε_{RT}	.005	.008	.025	-.020	-.031	-.054	.005	.008	.016

ε_{LT}: elasticity of labor with respect to technical change

ε_{ET}: elasticity of energy with respect to technical change

ε_{KT}: elasticity of capital with respect to technical change

ε_{RT}: elasticity of R&D with respect to technical change

intermediate and long run. The magnitude of the energy-saving effect of technical change in the U.S. manufacturing sector seems to be fairly small.

It seems that technical change is capital-saving in both Japan and Germany but capital-using in the U.S. manufacturing sector. The magnitude of the capital saving is very high in Japan in comparison to those in Germany; the capital-using effect of technical change in the U.S. is fairly small. Technical change is R&D-using in the U.S. and German manufacturing sectors but R&D-saving in the

Japanese manufacturing sector . Again, the magnitude of this biased technical change is much larger in Japan than in the other two countries.

These results suggest that the effect of technical change on different inputs varies across countries. As one would expect, the effect of technical change is much stronger in the long run. The technological bias seems to be factor-saving in the majority of the cases, but it is R&D- and capital-using in the U.S. manufacturing sector. Finally, the magnitudes of the effect of technological progress on all factors of production, like the effect of changes in relative input prices and output, are much stronger in Japan than in the other two countries.

4. Average Rate of Return on Individual Factors

In this section we define a measure for the rate of return on the investment expenditures on an individual factor in period t. Because of adjustment costs, the firm's invesment decisions are intertemporally connected. In defining the rate of return we hence have to be specific about the firm's behavior in future periods.

The maintained hypothesis in this paper is that the firm chooses its inputs such that it minimizes, for a given output stream, the discounted value of its costs. However, for expository reasons, consider for a moment a firm whose objective is to maximize the discounted value of its net profit stream. In our empirical model we considered the case of two quasi-fixed factors, i.e. $x_{1t} = K$ and $x_{2t} = R$, and two variable factors, i.e. $v_{1t} = L$ and $v_{2t} = E$.[9] The firm's net revenues in period t can then be written as

$$(11) \qquad R(x_{t-1}, \Delta x_t, v_t) = \tilde{p}_t F(x_{1,t-1}, x_{2,t-1}, \Delta x_{1t}, \Delta x_{2t}, v_t, T_t) - \sum_{i=1}^{2} \tilde{w}_{it} v_{it} - \sum_{j=1}^{2} \tilde{q}_{jt}(x_{j,t} - (1-\delta)x_{j,t-1})$$

where $\tilde{p}_t$ is the output price. Given that the firm has static expectations on all prices, we can state the firm's objective as to choose its inputs such that it maximizes

$$(12) \qquad \sum_{\tau=0}^{\infty} R(x_{t+\tau-1}, \Delta x_{t+\tau}, v_{t+\tau})/(1 + r_t)^{\tau}$$

subject to the initial conditions x_{t-1}. Let $\{\hat{x}_{t+\tau}, \hat{v}_{t+\tau}\}_{\tau=0}^{\infty}$ denote that maximizing input sequence.

Assuming that the firm realizes the initial portion of its investment plan, the firm's net investment expenditures on (say) the first quasi-fixed factor are $\tilde{q}_{1t}\Delta x_{1t} = \tilde{q}_{1t}(\hat{x}_{1t} - x_{1,t-1})$. Clearly the expected returns on this investment (discounted by the opportunity rate r_t) are maximal only if the firm plans to follow the entire plan also with respect to the other factors. To calculate the net returns from this investment we have to compare these returns with the returns from an input sequence where that particular investment is not undertaken. To capture the entire effect of the firm's investment we assume that this alternative input sequence is conditionally optimal, i.e. optimal subject to the condition that the firm's investment in the first quasi-fixed factor in period t is not undertaken and hence zero.[10] More formally, we consider as the alternative input sequence, say $\{\tilde{x}_{t+\tau}, \tilde{v}_{t+\tau}\}_{\tau=0}^{\infty}$, the input sequence that maximizes (12) subject to the constraint $\Delta x_{1t} = 0$. We now define as our rate of return the internal rate ρ that equates the present value of the differences in the two net return streams with the initial investment expenditure, i.e.:

$$(13) \qquad \tilde{q}_{1t}\Delta\hat{x}_{1t} = \tilde{p}_t F(x_{1,t-1}, x_{2,t-1}, \Delta\hat{x}_{1t}, \Delta\hat{x}_{2t}, \hat{v}_t, T_t)$$

$$- \tilde{p}_t F(x_{1,t-1}, x_{2,t-1}, 0, \Delta\tilde{x}_{2t}, \tilde{v}_t, T_t)$$

$$- \sum_i \tilde{w}_{it}(\hat{v}_{it} - \tilde{v}_{it}) - \tilde{q}_{2t}(\hat{x}_{2t} - \tilde{x}_{2t})$$

$$+ \sum_{\tau=1}^{\infty} \{R(\hat{x}_{t+\tau-1}, \Delta\hat{x}_{t+\tau}, \hat{v}_{t+\tau}) - R(\tilde{x}_{t+\tau-1}, \Delta\tilde{x}_{t+\tau}, \tilde{v}_{t+\tau})\}(1+\rho)^{-\tau}.$$

For additional interpretation of the above definition, consider the special case of only one quasi-fixed factor. In this case the rate of return defined by (13) is identical to the rate obtained by specifying as the alternative policy that the firm maintains forever the quasi-fixed factor at its initial level, i.e. x_{t-1}, while choosing the variable inputs correspondingly optimal. To see this, note that in case of one quasi-fixed factor $\tilde{x}_{t+\tau+1} = \hat{x}_{t+\tau}$ and $\tilde{v}_{t+\tau+1} = \hat{v}_{t+\tau}$ for $\tau \geq 1$. Consider further the case of no adjustment costs, i.e., $\partial F/\partial \Delta x = 0$. It is not difficult to see that in this case definition (13) implies the following expression for the rate of return on optimal investment in period t:

$$(14) \qquad \rho = \frac{\tilde{p}_t\{F(v_t^*, x_t^*) - F(v_t^0, x_{t-1})\} - \sum_i^2 \tilde{w}_{it}(v_{it}^* - v_{it}^0) - \tilde{q}_t\delta(x_t^* - x_{t-1})}{q_t(x_t^* - x_{t-1})}$$

where we adopt obvious simplifications in our notation. The equilibrium values x_t^* and v_t^* are defined by $\partial F(v_t^*, x_t^*)/\partial x_t = \tilde{q}_t(r_t + \delta)/\tilde{p}_t$ and $\partial F(v_t^0, x_{t-1})/\partial v_{it} = \tilde{w}_{it}/\tilde{p}_t$ for $i = 1,2$. Similarly v_t^0 is defined by $\partial F(v_t^0, x_{t-1})/\partial v_{it} = \tilde{w}_{it}/\tilde{p}_t$ for $i = 1,2$. In case the initial stock is zero, the above formula reduces to the following conventional measure for the rate of return on the capital stock:

$$(15) \qquad \rho = \frac{\tilde{p}_t F(v_t^*, x_t^*) - \sum_i \tilde{w}_{it} v_{it}^* - \tilde{q}_t \delta x_t^*}{\tilde{q}_t x_t^*}.$$

In case of a cost-minimizing firm the input sequences $\{\hat{x}_{t+\tau},\hat{v}_{t+\tau}\}_{\tau=0}^{\infty}$ and $\{\tilde{x}_{t+\tau},\tilde{v}_{t+\tau}\}_{\tau=0}^{\infty}$ are established under the additional constraint that $F(v_{t+\tau},x_{t+\tau},\Delta x_{t+\tau},T_t) = Q_t$ for all $\tau \geq 0$. We can still use (13) as our measure for the rate of return on investment. Note however that gross revenues will be identical for both input sequences. Hence in case of a cost-minimizing firm we actually compare the difference in cost streams.

In Table 8 we present the estimated internal rates of return on net investment in plant and equipment and R&D for the period 1965-1977. These rates are net of the adjustment costs and depreciation rates of the two quasi-fixed inputs. They are calculated using equation (13). Several interesting points should be noted: First, that the average rates of return over the period for capital and R&D are fairly similar except in Germany, where the rate of return on R&D is about 50% larger than that on capital. Second, there are variations in the rates of return over time for both capital and R&D, but especially in the rate of return on capital. The timing of these variations differs from country to country. Finally, the approximate equality of the rates of return across countries is interesting, for it suggests that none of the countries is earning excessive return on its investment in plant and equipment and R&D exclusive of the depreciation rates and the costs of adjustments of the investments.

Table 8

Internal Rates of Return on the Net Investments of Capital and R&D for Total Manufacturing in the U.S., Japan and Germany

Year	U.S. Capital	U.S. R&D	JAPAN Capital	JAPAN R&D	GERMANY Capital	GERMANY R&D
1965	.09	.08	.09	.07	.14	.13
1966	.10	.10	.08	.07	.12	.14
1967	.09	.10	.08	.08	.07	.12
1968	.10	.12	.08	.08	.10	.14
1969	.10	.13	.09	.09	.15	.16
1970	.08	.12	.10	.09	.16	.18
1971	.07	.11	.13	.10	.12	.17
1972	.11	.13	.15	.10	.10	.16
1973	.14	.16	.15	.10	.13	.20
1974	.12	.13	.13	.12	.11	.18
1975	.08	.11	.10	.11	--*	.16
1976	.11	.13	.10	.11	.08	.17
1977	.13	.14	.09	.10	.08	.17
Yearly Average (1965-1977)	.10	.12	.11	.09	.11	.16

*In 1975, the German manufacturing sector disinvested in capital. The rate of return was not computed for that year.

5. Decomposition of Total Factor Productivity

The Tōrnquist approximation of the standard Divisia index for total factor productivity is given by

$$\text{(16)} \qquad \Delta TFP_t = \Delta \log Q_t - \frac{1}{2}\sum_i \left[s_{v_i}(t) + s_{v_i}(t-1)\right]\Delta \log v_{it}$$

$$- \frac{1}{2}\sum_j \left[s_{x_j}(t) + s_{x_j}(t-1)\right]\Delta \log x_{j,t-1}$$

where s_{v_i} and s_{x_j} are factor shares in total cost: $s_{v_i}(t) = \tilde{w}_{it}v_{it}/TC_t$, $s_{x_j}(t) = \tilde{c}_{jt}x_{j,t-1}/TC_t$ and $TC_t = \sum_i \tilde{w}_{it}v_{it} + \sum_j \tilde{c}_{jt}x_{j,t-1}$, with $\tilde{c}_{jt} = \tilde{q}_{jt}(r_t + \delta_j)$. It is not difficult to see that the standard ΔTFP measure can be decomposed for our cost-of-adjustment technology into the following three components:

$$\text{(17)} \qquad \Delta TFP_t = R_1(t) + R_2(t) + R_3(t)$$

where

$$R_1(t) = -\frac{1}{2}\left[\frac{\partial G(t)}{\partial T}\right]\Big/\left[\frac{\partial G(t)}{\partial Q_t}\,Q_t\right] - \frac{1}{2}\left[\frac{\partial G(t-1)}{\partial T}\right]\Big/\left[\frac{\partial G(t-1)}{\partial Q_{t-1}}\,Q_{t-1}\right],$$

$$R_2(t) = \frac{1}{2}\sum_j \phi_{jt}\Delta \log N_t^{*} + \frac{1}{2}\sum_j \phi_{j,t-1}\Delta \log N_t^{**}$$

$$- \frac{1}{2}\sum_j (\phi_{jt}x_{j,t-1} + \phi_{j,t-1}x_{j,t-2})\Delta \log x_{j,t-1},$$

$$R_3(t) = \frac{1}{2}\sum_j \psi_{jt}\Delta x_{jt}\Delta \log N_t^{*} + \frac{1}{2}\sum_j \psi_{j,t-1}\Delta x_{j,t-1}\Delta \log N_t^{**}$$

$$- \frac{1}{2}\sum_j (\psi_{jt} + \psi_{j,t-1})(\Delta x_{jt} - \Delta x_{j,t-1})$$

and

$$\phi_{jt} = \left[\frac{\partial G(t)}{\partial x_{j,t-1}} + \frac{\tilde{\omega}_{jt}}{\tilde{w}_{1t}}\right] \Big/ \left[\frac{\partial G(t)}{\partial Q_t} Q_t\right] ,$$

$$\psi_{jt} = \left[\frac{\partial G(t)}{\partial \Delta x_{jt}}\right] \Big/ \left[\frac{\partial G(t)}{\partial Q_t} Q_t\right] ,$$

$$\Delta \log N_t^* = \sum_i s_{v_i}(t) \Delta \log v_{it} + \sum_j s_{x_j}(t) \Delta \log x_{j,t-1} ,$$

$$\Delta \log N_t^{**} = \sum_i s_{v_i}(t-1) \Delta \log v_{it} + \sum_j s_{x_j}(t-1) \Delta \log x_{j,t-1} .$$

All expressions in the above decomposition can be evaluated directly from the restricted cost function. The first component, $R_1(t)$, is attributable to technical change: Note that $\partial F/\partial T = -(\partial G/\partial T)/(\partial G/\partial Q)$. The second term, $R_2(t)$, is attributable to the short-run inequality between rate of technical substitution and the equilibrium relative factor prices: Note that $(\partial F/\partial x_j)/(\partial F/\partial v_1) = - \partial G/\partial x_j$. The third component, $R_3(t)$, stems from the presence of Δx in the production function. In a long-run equilibrium situation both $R_2(t)$ and $R_3(t)$ will be zero.

In Table 9 we present the decomposition of the growth rate of total factor productivity (ΔTFP) with R_1, R_2, R_3 and the unexplained residual. Technical change plays the dominant role in explaining the growth of total factor productivity in all the countries. The contribution of R_1 ranges from .90 in Germany to .80 in Japan to .70 for the U.S. The magnitude of R_2 seems to be small. As a percentage of the growth of TFP it ranges from .09 for the U.S. to about .06 for Japan. Also the magnitude of R_3 is very small, practically zero,

Table 9

Decomposition of the TFP growth for 1965 - 1977 and of the TFP slowdown for 1965-73/1973-77 in the total manufacturing of the U.S., Japan and Germany (in percentages)

	U.S.	JAPAN	GERMANY
TOTAL	1.64	5.12	3.79
R_1	1.11	4.04	3.54
R_2	0.15	0.34	0.24
R_3	-0.06	0.06	-0.02
RESIDUAL	0.44	0.68	0.04

R_1: portion of total factor productivity growth attributable to technical change

R_2: portion of total factor productivity growth attributable to inequality between short-run marginal products of quasi-fixed factors and their equilibrium service price

R_3: portion of total factor productivity growth attributable to the presence of Δx_j in the production function

in all countries. The magnitude of the unexplained residual as a percentage of TFP growth remains fairly large in the U.S., but is very small in Japan and Germany. What emerges is that technical change plays the most dominant role while the disequilibrium factors, the sum of R_2 and R_3, have a minor role in explaining growth of the traditional TFP measure. Thus, the presence of adjustment cost makes a substantial difference to the process of accumulation of capital and R&D and to the dynamic behavior of factor demand, but its implications for the traditional TFP measure are small.

The contributions of inputs, technical change and adjustment costs to growth of output in the manufacturing sectors of the three countries are shown in Table 10. In this period the growth rate of manufacturing output has been very high in Japan, about 9.5% per annum, while it was about 4% in Germany and slightly more than 3% in the U.S. The contributions of various inputs to the growth of output differs substantially across countries. The most significant source of growth of output in all the countries is technical change; it contributes over 30%, 40%, and 90%, respectively to the growth of output in the manufacturing sector of the U.S., Japan, and Germany. The next most important sources of growth of output can be attributed to capital accumulation; it accounts for about 20 to 25% of the growth of output in the American and German manufacturing, while its contribution is a dramatic 42% in the growth of the Japanese manufacturing sector. The contribution of labor is fairly high in U.S. manufacturing, but very small in Japan, about 2%, partly because of the low growth of employment over this period. The contribution of employment is negative, and fairly large, to the growth rate of output in the German manufacturing sector. The reason for this is the decline in both employment and hours worked in German manufacturing over the period under consideration.[11] If technical change and capital accumulation had not contributed so significantly as they did, the growth rate of output in German manufacturing could have been small or even negative.

Table 10

Sources of Growth of Output:

Average Annual Rates of Growth 1965-1977,

Total Manufacturing in Four OECD Countries.

Countries	Output	Inputs				Adjustment Costs		Technical Change
		Labor	Energy	Capital	R&D	Adjustment in Capital	Adjustment in R&D	
U.S.A.	3.24	0.70	0.05	0.69	0.36	-0.34	-0.07	1.11
JAPAN	9.50	0.20	0.38	3.98	0.46	0.04	-0.02	4.04
GERMANY	3.76	-0.96	0.02	0.95	0.34	-0.05	-0.01	3.54

The contribution of R&D to the growth of output seems to be highest in the U.S. manufacturing, about 11%, followed by Germany, about 9%. It contributed only half as much, about 5%, to the growth rate of manufacturing output in Japan. Given that the share of R&D is small, these rates are fairly significant. The contribution of energy to growth of output is generally very small, about 4% in the Japanese, 1.5% in the U.S., and less than 1% in the German manufacturing sector. Finally, the contribution of the adjustment costs of capital and R&D is generally small in the rapidly growing economies of Japan and Germany, and fairly large, over 10%, in the U.S. manufacturing sector. The adjustment-costs effect of capital is much larger in comparison to that of R&D, which partly reflects the larger share of capital. The inference that can be drawn is that if the adjustment costs were lower, the growth of output would have been larger, especially in the U.S. manufacturing sector.

Summary and Conclusions

A dynamic factor demand model with two variable inputs (labor and energy) and two quasi-fixed factors (capital stock and stock of R&D) was formulated and estimated using data for the period 1965 to 1977 for the manufacturing sectors of the three major industrialized countries, the U.S., Japan, and Germany. The model was derived from an intertemporal cost-minimization problem formulated in discrete time. The empirical results suggest that:

(a) The adjustment cost model explains the behavior of inputs fairly well. The speed of adjustment of stocks of plant and equipment and R&D investments are generally different from each other and vary across countries. It takes a considerably longer time for the stock of R&D to adjust to its optimum value than for the capital stock.

(b) The patterns of own- and cross-price elasticities of the inputs vary considerably among countries. The magnitudes of the own-elasticities differ across countries, and there is a substitutional relation between labor and other inputs. Capital and energy are complements in some countries and substitutes in others. The output elasticities of the inputs in the short and intermediate runs differ from each other and across countries. The labor input overshoots, in the short run, its long-run equilibrium value to compensate for the sluggish adjustments of the two quasi-fixed inputs; the output elasticities of capital stock are generally larger than those of R&D in the short and medium runs. Technical change seems to be generally labor-, capital-, and R&D-saving. There are, however, exceptions: In the U.S., technology seems to be R&D- and capital-using. The magnitude of the effect of technical change is often larger on capital than on R&D.

(c) An interesting result is that the average net rate of return exclusive of the costs of adjustment and depreciation rate are similar for both R&D and capital in the manufacturing sectors of the three countries. The rate of return on R&D is often somewhat greater than that on capital in each sector.

(d) The presence of the cost of adjustment does not contribute a great deal to the growth of total factor productivity, but makes a significant difference in the process and timing of investment and calculation of the rates of return in the short run. The adjustment lags differ greatly among the manufacturing sectors and between capital and R&D; we find an average lag of 2 years for capital in the U.S. and about one year for Japan and Germany. The average lag for R&D is about 5½ years in the U.S. and about 3 years in Japan and Germany. The results for U.S. manufacturing are similar to some evidence reported in the literature. Our results as a whole confirm the notion that it takes a much longer period for R&D to contribute to output than does capital.

(e) Factor demands in Japanese manufacturing behave quite differently than in the other countries. The contrast is fairly sharp between the U.S. and Japanese experiences. The quasi-fixed factors adjust more quickly and the inputs respond much more to changes in output, prices and technical change in the Japanese manufacturing than in the U.S. Also, the contributions of various factors of production to growth of output are quite different in the two countries.

There are several issues that require further research. One is the relaxation of the assumption of constant returns to scale. Another possibility is to relax the assumption of instantaneous adjustment of labor. Our assumption of static expectation is very restrictive. Some attempt has recently been made to incorporate non-static expectations with adjustment costs in a unified analytical framework (see Prucha and Nadiri (1982)), which could be extended to this study. The strong assumption of separability between the two quasi-fixed inputs, which rules out any substitution between them, and the interdependency of their adjustment pattern requires further work. Finally, there is a need for increasing the span of time for the study by collecting new data for the recent years and re-estimating the model.

Footnotes

*This study is part of an ongoing research project on the role of R&D in productivity growth, at NBER's New York Office. Similar studies on other European countries are currently underway involving the authors and Professor Angelo Cardani. Miguel Oviedo provided extremely useful assistance in preparation of this study.

1. For a brief survey of contributions of R&D to growth of output and the determinants of R&D expenditure see Nadiri (1980). Also see Griliches (1980,a).

2. See Mansfield (1980) and Griliches (1980,b) for a discussion of the rates of return on R&D in various U.S. sectors and industries.

3. Pakes (1981) has recently examined the lags between R&D and patents.

4. There is a vast literature on the role of energy in growth of output, productivity growth and technical change; see Jorgenson and Fraumeni (1981) and Berndt (1980) for some contrasting findings.

5. For a general dynamic factor demand model see Prucha and Nadiri (1982).

6. In case of nonstatic expectations it is generally difficult to solve first-order conditions explicitly. For a discussion of how to estimate cost-of-adjustment models under general nonstatic expectations see Prucha and Nadiri (1982).

7. The normalized restricted cost function corresponding to a linear homogeneous technology is in general of the form

$$QG(w, \frac{x_{-1}}{Q}, \frac{\Delta x}{Q}, T).$$

   For a generalization of the above functional form to the homothetic case see Prucha and Nadiri (1983).

8. For a review of the empirical studies on this issue see Mittelstädt (1983).

9. We note that the subsequent discussion generalizes in a trivial way to more than two quasi-fixed factors.

10. In case we want to calculate the rate of return on the last, say, 50 units, we have to make a comparison with returns attainable from an input sequence for which net investment in period 5 is $\Delta x_{1t} - 50$.

11. Man-hours worked in the German manufacturing sector declined over the period 1965 to 1977, especially after 1974. Similar figures are reported by Kendrick (1981), Table 1, p. 128.

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Appendix

Data Sources and Constructions

The data cover the period from 1965 to 1977 and pertain to the total manufacturing sectors of the U.S., Japan and Germany. The data have been assembled from various sources, indicated below:

Labor: Employment (L) is measured in man-hours per year. The employment data for the U.S. and Japan are from the OECD (1981). Those for Germany were obtained directly from the Statistical Office of the European Communities, Luxembourg. For all countries the figures on hours worked are provided in ILO (1980) and earlier publications.

Capital: The figures on net capital stock (K) are obtained from various sources. The capital stock series for the U.S. comes from the U.S. Department of Commerce (1982). For Japan, the gross capital stock series reported by the Economic Planning Agency (1977) is converted to a net capital stock series using the gross-to-net capital stock ratios contained in Denison and Chung (1973). The capital stock series for Germany is taken from the OECD (1983). All capital stocks are measured as end-of-period stocks in 1970 prices.

Energy: The source for energy consumption (E) is OECD (1980) and earlier issues. The data are spliced to obtain a consistent series. Energy is measured in millions of tons of oil equivalent.

R&D: The R&D stock (R) is constructed by the perpetual inventory method with a depreciation rate of .10. The benchmark is obtained from the first period R&D expenditure divided by the depreciation rate and the growth rate in real value-added. The nominal R&D expenditures are from the OECD (1979) and (1982). The GNP deflator is used as a deflator for R&D.

Wage Rate: Total compensations per hour worked (w^L) are obtained from U.S. Department of Labor (1980).

User Cost of Capital: The user cost of capital (c^K) is constructed as $c^K = q^K(\delta_K + r)$, where q^K = investment deflator, δ_K = depreciation rate of capital stock, and r = government bond yield. The nominal and real investment data used to compute the implicit investment deflator are from the same sources as the capital stock data. For Japan, we use the investment deflator for machinery and equipment published by the Bank of Japan (1981). The IMF (1979) has the figures on the government bond yields. The depreciation rates are obtained implicitly from the perpetual inventory formula, using the gross investment and net capital stock figures.

User Cost of R&D: The user cost of R&D (c^K) is constructed in the same way as c^K, using the R&D depreciation rate and the GNP deflator.

Energy Price: The energy price (w^E) for the United States in 1970 is constructed by dividing the nominal energy consumption, provided by Norsworthy and Harper (1981), by the real energy consumption figures, published by the OECD. The 1970 value for Japan and Germany is computed by multiplying the U.S. figure by a relative price factor derived from Mittelstädt and Hall (1981). The 1970 values are then linked to country indices, which have been provided by the OECD.

Output: Output (Q) is measured as gross value-added at 1970 prices plus the energy expenditures and the R&D serivces, both in 1970 prices. The real value-added figures are taken from the OECD (1982) for the U.S. and Japan. Those for Germany were obtained directly from the Statistical Office of the European Communities, Luxembourg. The energy expenditures in 1970 prices are obtained by multiplying the energy series by their 1970 price. The R&D services are obtained by multiplying the stock of R&D (in 1970 dollars) by the sum of the depreciation and interest rates.

For the data of different countries to be comparable, all currencies are converted to U.S. dollars, using the purchasing power parities for gross domestic income for 1970 computed by R. Summers, I.B. Kravis and A. Heston (1980). Variations in the exchange rates are therefore eliminated.

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Appendix A: Derivation of the Decomposition of Total Factor Productivity

The Törnquist approximation of the standard Divisia index is based on a translog expansion of the production function in terms of the traditional inputs. Analogously we consider the following approximation:

$$(A.1)\qquad \Delta\log Q_t = \frac{1}{2}\sum_{i=1}^{m}\left[\varepsilon_{v_i}(t) + \varepsilon_{v_i}(t-1)\right]\Delta\log v_{it}$$

$$+ \frac{1}{2}\sum_{j=1}^{n}\left[\varepsilon_{x_j}(t) + \varepsilon_{x_j}(t-1)\right]\Delta\log x_{j,t-1}$$

$$+ \frac{1}{2}\sum_{j=1}^{n}\left[\frac{1}{Q_t}\frac{\partial F(t)}{\partial \Delta x_j} + \frac{1}{Q_{t-1}}\frac{\partial F(t-1)}{\partial \Delta x_j}\right](\Delta x_{jt} - \Delta x_{j,t-1})$$

$$+ \frac{1}{2}\left[\frac{1}{Q_t}\frac{\partial F(t)}{\partial T} + \frac{1}{Q_{t-1}}\frac{\partial F(t-1)}{\partial T}\right]$$

where $\varepsilon_{v_i}(t) = \dfrac{\partial F(t)}{\partial v_{it}}\dfrac{v_{it}}{Q_t}$ and $\varepsilon_{x_j}(t) = \dfrac{\partial F(t)}{\partial x_{j,t-1}}\dfrac{x_{j,t-1}}{Q_t}$.

The reason for expanding in the above approximation ln Q in terms of Δx rather than ln Δx is that Δx can take on the value of zero. By standard calculations we get $(i = 2,\ldots,m;\ j = 1,\ldots,n)$

$$(A.2)\qquad \frac{\partial F(t)}{\partial v_{1t}} = 1\Big/\left[\frac{\partial G(t)}{\partial Q_t}\right], \qquad \frac{\partial F(t)}{\partial v_{it}} = \left[\frac{\tilde{w}_{it}}{\tilde{w}_{1t}}\right]\Big/\left[\frac{\partial G(t)}{\partial Q_t}\right],$$

$$\frac{\partial F(t)}{\partial x_{j,t-1}} = -\left[\frac{\partial G(t)}{\partial x_{j,t-1}}\right]\Big/\left[\frac{\partial G(t)}{\partial Q_t}\right], \qquad \left[\frac{\partial F(t)}{\partial \Delta x_{jt}}\right] = -\left[\frac{\partial G(t)}{\partial \Delta x_{jt}}\right]\Big/\left[\frac{\partial G(t)}{\partial Q_t}\right].$$

Substituting those into (A.1) yields

$$(A,3)\qquad \Delta\log Q_t = \frac{1}{2}\sum_i \left[a(t)s_{v_i}(t) + a(t-1)s_{v_i}(t-1)\right]\Delta\log v_{it}$$

$$+ \frac{1}{2}\sum_j \left[a(t)s_{x_j}(t) + a(t-1)s_{x_j}(t-1)\right]\Delta\log x_{j,t-1}$$

$$+ \frac{1}{2}\left[\frac{1}{Q_t}\frac{\partial F(t)}{\partial T} + \frac{1}{Q_{t-1}}\frac{\partial F(t-1)}{\partial T}\right]$$

$$- \frac{1}{2}\sum_j \left[\phi_{jt}x_{j,t-1} + \phi_{j,t-1}x_{j,t-2}\right]\Delta\log x_{j,t-1}$$

$$- \frac{1}{2}\sum_j [\psi_{jt} + \psi_{j,t-1}][\Delta x_{jt} - \Delta x_{j,t-1}]$$

where $a(t) = TC_t \Big/ \left[\tilde{w}_{1t}\frac{G(t)}{Q_t}Q_t\right]$ and ϕ_{jt} and ψ_{jt} are defined in the text. It is readily seen that

$$(A,4)\qquad 1 = \left[1 - \sum_j \frac{\partial G(t)}{\partial x_{j,t-1}}\frac{x_{j,t-1}}{G(t)} - \sum_j \frac{\partial G(t)}{\partial \Delta x_{jt}}\frac{\Delta x_{jt}}{G(t)}\right] \Big/ \left[\frac{\partial G(t)}{\partial Q_t}\frac{Q_t}{G(t)}\right]$$

(This reflects that the scale elasticity of the underlying technology is one -- see Caves, Christensen and Swanson (1981) for analagous expressions in a slightly different context). Using (A,4) we get

$$(A,5)\qquad a(t) = \left[\tilde{w}_{1t}G(t) + \sum_j \tilde{c}_{jt}x_{j,t-1}\right] \Big/ \left[\tilde{w}_{1t}\frac{\partial G(t)}{\partial Q_t}Q_t\right]$$

$$= 1 + \sum_j \phi_{jt}x_{j,t-1} + \sum_j \psi_{jt}\Delta x_{jt} .$$

It is now readily seen that substitution of (A,3) and (A,5) into (16) yields the decomposition (17).