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DOES KNOWLEDGE INTENSITY MATTER?  
A DYNAMIC ANALYSIS OF RESEARCH AND DEVELOPMENT  
CAPITAL UTILIZATION AND LABOR REQUIREMENTS

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ABSTRACT

In this paper we have developed a dynamic analysis of a firm undertaking research and development (R&D) investment, physical capital accumulation and utilization, along with labor requirement decisions. Empirical work has found that there are significant costs to develop knowledge. Consequently, R&D capital is treated as a quasi-fixed factor, along with the traditional physical capital stock.

A number of empirically relevant implications arise from the analysis. It is shown that along the dynamic path as the R&D intensity of physical capital increases, knowledge per worker rises and the utilization rate of physical capital decreases. We distinguish between the intertemporal movement of the firm, and the response to unanticipated changes in demand and cost conditions. An increase in product demand causes the firm to increase both the R&D growth rate and the labor intensity of R&D capital. Contrary to a viewpoint held by many, the R&D investment does not displace labor. Finally, our model provides a framework to justify the empirically observed direct relationship between the physical capital growth and utilization rates.

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## 1. Introduction

Recent empirical findings have suggested a number of interesting and novel results on the role of research and development in the production and investment process of a firm. First, relative input prices, such as the real wage rate and the real user cost of physical capital significantly affect R&D decisions, both in the short and long runs (see Rasmussen [1973], Goldberg [1979], and Bernstein and Nadiri [1983]). Second, R&D influences the utilization of physical capital. Indeed, it was found that R&D expansion decreases the rate of physical capital utilization, implying that plant and equipment become relatively more idle (see Nadiri and Bitros [1980]). Finally, significant development costs must be incurred on the part of the firm in order to expand R&D to its desired long run level. Presence of these costs leads to intertemporal linkage of the R&D investment and imply a distributed lag structure for R&D investment expenditure. In fact this adjustment process can take anywhere from three to five years to complete (see Mansfield [1968], Griliches [1980], Nadiri [1980]).

The existing dynamic theory of investment has not been able to fully explain these important empirical findings. Investment theory ignores the empirically important interplay, both in the short run and intertemporally, between the decisions to accumulate knowledge, to utilize and invest in physical capital and to hire labor. As Griliches [1979] points out, knowledge is a stock. Thus the dual role of R&D must be explicitly recognized. In this paper the R&D investment flow is determined as part of the short run equilibrium, while as a form of capital, the stock is an input in the production process, and its level governs the intertemporal evolution of the firm.

A major difficulty in modelling the effect of R&D accumulation on phy-

sical capital utilization arises because, in general, utilization is considered to be costless. This implies that physical capital is always fully utilized, and therefore the utilization rate is independent of changes in the level of R&D capital and stock of plant and equipment. A result which, as we have observed, is not substantiated by empirical work.

In this paper the cost of physical capital utilization arises because the wage rate increases as plant and equipment are utilized at a higher rate. The rising wage schedule reflects the workers' diminishing rate of substitution between leisure and consumption. Thus the hours of operation move from the most to the least attractive. Rhythmic wage rates were first introduced by Marris [1964] and then formalized in a static framework by Lucas [1970] and Winston and McCoy [1974]. Recently Abel [1981] developed a dynamic analysis with costly labor utilization where the capital-labor ratio was fixed in the short run. In the present context, with costly physical capital utilization, knowledge and physical capital are quasi-fixed factors. The firm alters the level of these stocks through their investment decisions. Moreover, to permit general short run substitution possibilities, both labor requirements and the utilization rate are variable.

A number of empirically relevant results emerge from the present analysis. First, as the "knowledge intensity" of the firm's physical capital stock (i.e., R&D capital/physical capital) increases the physical capital stock utilization rate decreases while knowledge per worker rises. In a sense the growth of knowledge capital, given the level of demand, creates an inventory of physical capital which may be used in the future but is idle in the present. Second, there is an important distinction to be drawn bet-

ween the intertemporal unfolding of the firm, and the response to unanticipated changes in demand and cost conditions. When product demand increases the growth rates of knowledge and physical capital rise and there are increases in the utilization rate and labor requirements per unit of R&D capital. Therefore, the model illustrates that R&D investment does not displace labor, which is a result contrary to the viewpoint of many. In addition, we are able to explain the empirically observed direct relationship between the physical capital utilization and growth rates of output.

This paper is organized as follows: The structure of the model and the short run equilibrium properties are analyzed in section 2. Section 3 is devoted to the dynamic path and steady state characteristics. The effects of unanticipated changes in demand and cost conditions on the steady state are derived in section 4. The last section of the paper includes a summary of our results and some suggestions for future research.

## 2. The Model

Consider a firm whose production process can be represented as

$$(1) \quad y(t) = F(\gamma(t)K_p(t), L(t), K_r(t)),$$

where  $y(t)$  is output,  $F$  is the twice continuously differentiable production function which is homogenous of degree 1,  $\gamma(t)$  is the index of physical capital utilization,  $K_p(t)$  is the stock of physical capital,  $K_r(t)$  is the stock of R&D and  $L(t)$  are labor services. All variables are evaluated at time  $t$ . The marginal products are positive and diminishing for each of the factors.<sup>1</sup>

The physical capital utilization rate can be thought of as an index of plant and equipment (P&E) or machine usage at each time period. Resources must be brought to bear to alter the rate at which physical capital is used. These costs are manifested in the wage bill. Clearly labor prefers certain time periods to work, as the major portion of factories are operated in the daytime and during the week. Thus in order to attract workers to overtime, night and weekend shifts, a premium wage must be paid. This premium reflects, in essence, the diminishing rate of substitution between leisure and consumption for workers. The implication is that the wage rate consists of two elements, the fixed scale or basic rate,  $s$ , and the premium rate  $\omega(\gamma)$ . The premium rate is an increasing strictly convex function of the physical capital utilization rate. As the utilization rate increases each worker receives a higher wage, and as the unused physical capital stock diminishes the payment per worker increases at an increasing rate.<sup>2</sup>

In this model the distinguishing characteristic between knowledge and physical capital is that once the stock of R&D exists there are zero costs associated with its utilization. Hence although there are costs to develop knowledge capital, as there are costs to purchase and install physical capital, only the latter involve utilization costs.<sup>3</sup>

We are now in a position to describe the flow of funds for the firm,  $V$ , which is the revenue after the wage bill and investment costs have been deducted,<sup>4</sup>

$$(2) \quad V = pK_r f(\gamma k, \ell) - s\omega(\gamma)L - C(I_p/K_p)I_p - E(I_r/K_r)I_r,$$

where  $f$  has been derived from the fact that the production function exhibits constant returns to scale,  $k=K_p/K_r$  is the physical capital intensity, (of R&D capital) or  $1/k$  is the knowledge intensity of physical capital,  $l=L/K_r$  is the labor intensity (of R&D capital) or  $1/l$  is the knowledge intensity of labor (i.e. knowledge per worker), and  $p$  is the fixed product price. The costs of purchasing and installing additional physical capital is  $C$ , with  $C' > 0$ ,  $C'' > 0$  for positive investment in physical capital, i.e.  $I_p > 0$ , and  $C(0) = 0$ . There are also costs associated with developing additional R&D. These costs are captured by  $E$ , with  $E' > 0$ ,  $E'' > 0$  for  $I_r > 0$ , and  $E(0) = 0$ .<sup>5</sup>

Knowledge and physical capital accumulate according to,

$$(3) \quad \dot{K}_p = I_p - \delta K_p$$

$$(4) \quad \dot{K}_r = I_r - \mu K_r$$

where  $0 < \delta < 1$  and  $0 < \mu < 1$  are respectively the rates of P&E depreciation and R&D obsolescence.

In this model the stocks of R&D and physical capital are quasi-fixed factors of production while the labor requirements and the physical capital utilization are variable in the short run. This means that in order for the firm to maximize the present value of the flow of funds, it selects these latter two variables and the investment flows associated with the capital stocks.

The Hamiltonian of the problem is

$$(5) \quad H = p K_r f(\gamma k, l) - s \omega(\gamma) L - C(I_p / K_p) I_p \\ - E(I_r / K_r) I_r + q_1 (I_p - \delta K_p) + q_2 (I_r - \mu K_r),$$

where  $q_1$ , and  $q_2$  are respectively the shadow prices associated with P&E and R&D investment. Indeed these prices can be considered investment demand prices. The first order and canonical conditions are,

$$(6.1) \quad \frac{\partial H}{\partial L} = pf_2 - s\omega(\gamma) = 0$$

$$(6.2) \quad \frac{\partial H}{\partial \gamma} = pf_1 K_p - s\omega' L = 0$$

$$(6.3) \quad \frac{\partial H}{\partial I_p} = -C' I_p / K_p - C(I_p / K_p) + q_1 = 0$$

$$(6.4) \quad \frac{\partial H}{\partial I_r} = -E' I_r / K_r - E(I_r / K_r) + q_2 = 0$$

$$(6.5) \quad \dot{k} = k[I_p / K_p - \delta - I_r / K_r + \mu]$$

$$(6.6) \quad \dot{q}_1 = (r + \delta)q_1 - pf_1 \gamma - C'(I_p / K_p)^2$$

$$(6.7) \quad \dot{q}_2 = (r + \mu)q_2 - pf(\gamma k, \ell) + pf_1 \gamma k + pf_2 \ell - E'(I_r / K_r)^2,$$

where  $r$  is the constant discount rate which represents the opportunity costs of funds.<sup>6</sup>

In order to analyze equation set (6), it is convenient to deal with the set in two segments. First, consider equations (6.1) - (6.4) from which we can solve for  $L$ ,  $\gamma$ ,  $I_p$ , and  $I_r$  given the capital stocks and the investment demand prices. Upon solving, we can then substitute into equations (6.5) - (6.7) to determine the intertemporal paths of the stocks and shadow prices.

The solution to (6.1) - (6.4) is defined as the short run or temporary equilibrium. Equations (6.1) and (6.2) simultaneously characterize the short run equilibrium for the utilization rate and labor requirements. By combining these equations we find that



$$(7) \quad \frac{s\omega'\gamma z}{f_1} = \frac{s\omega(\gamma)}{f_2} = m$$

where  $z = L/\gamma K_p$  is worker per machine hour and  $m$  is the short run marginal production costs. In maximizing the present value of the flow of funds, at each time period the firm minimizes the cost of production given the level of output. In addition, from either (6.1) or (6.2), to determine output the firm equates the short run marginal cost of production to the product price.

Before continuing, let us consider the role of R&D capital in the determination of equilibrium. Suppose that the firm does not engage in R&D investment so that  $I_r=0$ . This could arise, for example, if development costs are too high for any  $I_r>0$ . This means that equations (6.4) and (6.7) drop out. Moreover (6.5) reverts to equation (3) and  $K_r(t)=K_r(0)>0$  for all time. The stock of knowledge is given and constant, and it ceases to be a factor of production. Hence, from equations (6.1) and (6.2) (with constant returns to scale and constant real wages), we can solve for the number of workers per machine hour and the machine utilization rate. In fact, these variables are constant over time. In other words given constant returns to scale, and constant real wages, the firm never has an incentive to alter its ratio of worker per machine hour and the utilization rate. The implication is that, in this context, knowledge capital accumulation provides the impetus for changes (apart from unanticipated exogenous shocks, as for example in the real wage) in the proportion of labor and physical capital used in the production process.

When a firm begins to engage in R&D investment, it must decide on the

rate at which the stock of knowledge is to be accumulated. This stock affects the physical capital growth rate, the level of output and input proportions. Clearly, in this model, it is precisely the endogenous accumulation of knowledge which causes the firm to evolve.

Let us now revert to the discussion of the temporary equilibrium. Because physical capital utilization costs emerge through the wage bill, it is of interest to determine the region on the premium wage rate schedule on which the firm operates. To this end, it is convenient to rewrite equations (6.1) and (6.2) as

$$(8) \quad \theta_{\gamma} = e_{\gamma}/e_{\ell}$$

where  $\theta_{\gamma} = \omega'_{\gamma}/\omega$  is the utilization rate elasticity of the wage, and  $e_{\gamma} = f_1 \gamma k/f$ ,  $e_{\ell} = f_2 \ell/f$ , with  $e_{\gamma}$  the physical capital services elasticity of output and  $e_{\ell}$  is the labor services elasticity of output. Noting that the labor elasticity with respect to the wage rate is unity, equation (8) illustrates that the ratio of cost elasticities must equal the ratio of revenue elasticities. Therefore, if the output elasticity with respect to labor is greater than (is less than) the capital services elasticity, then the firm operates on the inelastic (elastic) portion of the wage schedule. Intuitively, when cost responds relatively more to labor services than to capital services (through the utilization rate), in order for the firm to be maximizing the flow of funds, revenues must also be responding more to changes in labor services. This result generalizes that found in Oi [1981] and Abel [1981], where the firm operated only on the elastic segment of the wage schedule. These authors restricted the form of the production func-

tion such that  $e_\ell < e_\gamma$ , and therefore  $\theta_\gamma$  must exceed unity.<sup>7</sup>

From equations (6.1) and (6.2) we can observe that labor demand and the utilization rate depend on the capital stocks, and the real scale wage rate. Moreover, because these first order conditions are homogenous of degree zero in  $L$ ,  $K_p$ , and  $K_r$ , we can actually solve for the labor intensity ( $\ell$ ) and the utilization rate ( $\gamma$ ) in terms of the physical capital intensity ( $k$ ) and the real scale wage rate ( $s/p$ ). If the physical capital intensity increases we find that

$$(9) \quad \frac{\partial \ell}{\partial k} = [p(f_{11} + f_{11}\gamma k)(pf_{21}k^{-s\omega'}) - pf_{21}\gamma(pf_{11}k^2 - s\omega''\ell)]/H_1$$

where  $H_1$  is the relevant Hessian determinant. In order to determine the sign of the right side of (9), consider the variable profit function per unit of R&D capital,

$$\pi = pf(\gamma k, \ell) - s\omega(\gamma)\ell.$$

We assume that  $\pi$  is strictly concave in  $\gamma$ ,  $k$  and  $\ell$ .<sup>8</sup> This implies that the matrix of second order derivatives

$$(10) \quad \begin{bmatrix} \pi_{\ell\ell} & \pi_{\ell\gamma} & \pi_{\ell k} \\ \pi_{\gamma\ell} & \pi_{\gamma\gamma} & \pi_{\gamma k} \\ \pi_{k\ell} & \pi_{k\gamma} & \pi_{kk} \end{bmatrix} = \begin{bmatrix} pf_{22} & pf_{21}k^{-s\omega'} & pf_{21}\gamma \\ pf_{12}k^{-s\omega'} & pf_{11}k^2 - s\omega''\ell & pf_{11}\gamma k + pf_1 \\ pf_{12}\gamma & pf_{11}\gamma k + pf_1 & pf_{11}\gamma^2 \end{bmatrix}$$

is negative definite. Thus the determinant of the matrix of the right side of (10), which is defined as  $H_2$ , is negative, the second principal minor, which is  $H_1$ , is positive and the diagonal terms in the matrix are negative.

Now with  $H_2 < 0$ , one of two sufficient conditions for  $H_1 > 0$  is that the numerator of the right side of (9) be positive when  $f_{12} > 0$  and negative when  $f_{12} < 0$ . Hence  $\partial \ell / \partial k \gtrless 0$  as  $f_{12} \gtrless 0$ . If increases in the physical capital intensity increase the marginal product of the labor intensity then the two intensities are directly related.

Next for the machine utilization rate

$$(11) \quad \frac{\partial \gamma}{\partial k} = [pf_{21}\gamma(pf_{12}k - s\omega') - p^2f_{22}(f_1 + f_{11}\gamma k)]/H_1.$$

The second sufficient condition for  $H_1 > 0$  when  $H_2 < 0$  is that the numerator of the right side of (11) is positive when  $f_1 + f_{11}\gamma k > 0$  and negative when  $f_1 + f_{11}\gamma k < 0$ . By defining  $e_{\gamma\gamma} = f_{11}\gamma k / f_1$ , then  $f_1 + f_{11}\gamma k = pf_1(e_{\gamma\gamma} + 1)$ , and thus we find  $\partial \gamma / \partial k \gtrless 0$  as  $e_{\gamma\gamma} + 1 \gtrless 0$ . The intuition can be easily explained. An increase in the physical capital intensity effects the value of the marginal product of the utilization rate (which is  $pf_1k$ ) by the amount  $p(f_{11}\gamma k + f_1) = pf_1(e_{\gamma\gamma} + 1)$ . Moreover, the increase in this intensity does not affect the marginal input cost of the utilization rate (which is  $s\omega'\ell$ ). Therefore, if  $e_{\gamma\gamma} + 1 > 0$  then the value of the marginal product increases above the marginal input cost. In order to restore equilibrium, the utilization rate must increase. The higher rate simultaneously lowers the value of the marginal product and raises the marginal input cost, until the equality between the two is once more established. Conversely, if  $e_{\gamma\gamma} + 1 < 0$  then the value of the marginal product of the utilization rate declines with the increase in physical capital intensity. This means that the rate must fall to bring about an equilibrium.<sup>9</sup>

As the R&D intensity of physical capital increases (i.e.  $1/k$  increases), knowledge per worker rises and physical capital becomes relatively more idle, when  $f_{12} > 0$  and  $e_{\gamma\gamma} + 1 > 0$ . In this case, the R&D intensity of physical capital displaces labor per unit of R&D, and also physical capital through the utilization rate. In addition, from the production function in R&D intensive form, when  $k$  declines, leading to decreases in  $\ell$  and  $\gamma$ , output per unit of knowledge capital falls. Thus as the R&D intensity of physical capital increases, and since  $p$  is exogenous to the firm, the R&D intensity of sales ( $py$ ) increases.

In the situation with  $f_{12} > 0$  and  $e_{\gamma\gamma} + 1 < 0$ , the expansion of the stock of knowledge per unit of physical capital leads to a higher utilization rate. The firm produces output with a relatively higher R&D intensity of physical capital, and increased use of physical capital, while the labor intensity declines.<sup>10</sup>

Increases in the real scale wage rate ( $s/p$ ) make it more expensive to produce output whether through a higher physical capital utilization rate or by hiring labor. The reason is that the costs of altering utilization, given the stock of physical capital is manifested in the premium wage rate. Thus we find

$$(12) \quad \frac{\partial \ell}{\partial (s/p)} = [\omega(pf_{11}k^2 - s\omega''\ell) - \omega'\ell(pf_{21}k - s\omega')]p/H_1 < 0.$$

$$(13) \quad \frac{\partial \gamma}{\partial (s/p)} = [pf_{22}\omega'\ell - \omega(pf_{12}k - s\omega')]p/H_1 < 0.$$

The right sides of (12) and (13) are negative because the variables profit function is strictly concave, and therefore it is also strictly quasi-

concave. This latter condition is used to establish that the numerators in the right side of (12) and (13) are negative.

We can summarize our results from equations (9) and (11)-(13) by defining  $\gamma = \Gamma(k, s/p)$  with  $\Gamma_1 \geq 0$  as  $e_{\gamma\gamma} + 1 \geq 0$ ,  $\Gamma_2 < 0$  and  $\lambda = G(k, s/p)$  with  $G_1 > 0$ ,  $G_2 < 0$ . The R&D intensity of physical capital affects the utilization rate in accordance with the value of the elasticity of the marginal product of physical capital services, while increases in this intensity (i.e.  $1/k$ ) decreases the labor intensity of R&D. Finally a rising real scale wage rate decreases the labor intensity and the utilization rate.

Turning to the firm's investment behavior, we can observe from (6.3) and (6.4) that both types of investment rates are positively related to their respective shadow prices. Thus

$$(14) \quad I_p/K_p = J_p(q_1), \quad J'_p = 1/[C'' I_p/K_p + 2C'] > 0$$

$$(15) \quad I_r/K_r = J_r(q_2), \quad J'_r = 1/[E'' I_r/K_r + 2E'] > 0.$$

An increase in the marginal value of investment to the firm, other things constant, increases the growth rates for both types of capital. These results are similar to those found in Lucas [1967], Gould [1968], Treadway [1970], and Hayashi [1982]. The investment decisions highlight the inter-temporal link, as the capital growth rates depend on the investment demand prices, which are equal to the present value of the rentals accruing to units of the stocks installed at the current time, but brought into service over the remaining time horizon.

### 3. Dynamics and the Steady State

Given the short run solution, we are in a position to analyze the intertemporal path of the firm and the steady state properties.

Substituting  $\Gamma(k, s/p)$ ,  $G(k, s/p)$ ,  $J_p(q_1)$  and  $J_r(q_2)$  into (6.5) yields

$$(16.1) \quad \dot{k} = k[J_p(q_1) - \delta - J_r(q_2) + \mu] .$$

From equation (16.1) we can determine how the physical capital intensity evolves. The growth rate of the physical capital intensity ( $\dot{k}/k$ ) is a function of the demand prices of both types of investment, the depreciation rate of physical capital and the rate of obsolescence of the stock of knowledge. It is independent of the physical capital intensity itself. This result is due to the separable nature of the adjustment costs associated with the quasi-fixed factors, and because the depreciation and obsolescence rates are exogenous. We find that  $\partial \dot{k} / \partial q_1 = k J'_p > 0$  and  $\partial \dot{k} / \partial q_2 = -k J'_r < 0$  (for  $k > 0$ ). Therefore, with  $\dot{k} = 0$  there is a locus in  $(q_2, q_1)$  space (see Figure 1) which is positively sloped with  $dq_1/dq_2$  (at  $\dot{k} = 0$ )  $= J'_r / J'_p > 0$ .

The  $\dot{k} = 0$  locus shows us that in order to maintain the equality between the growth rates for both types of capital, each investment demand price must rise, thereby generating increases in the growth rates. Moreover, if the demand price of P&E ( $q_1$ ) is above that defined by the  $\dot{k} = 0$  curve, for any value of  $q_2$ , then the P&E growth rate outruns the rate for R&D, causing  $\dot{k} > 0$ . The converse occurs for values of  $q_2$  below the  $\dot{k} = 0$  locus.

Turning to the evolution of the demand price of physical capital investment, substituting the temporary equilibrium into (6.6) yields,

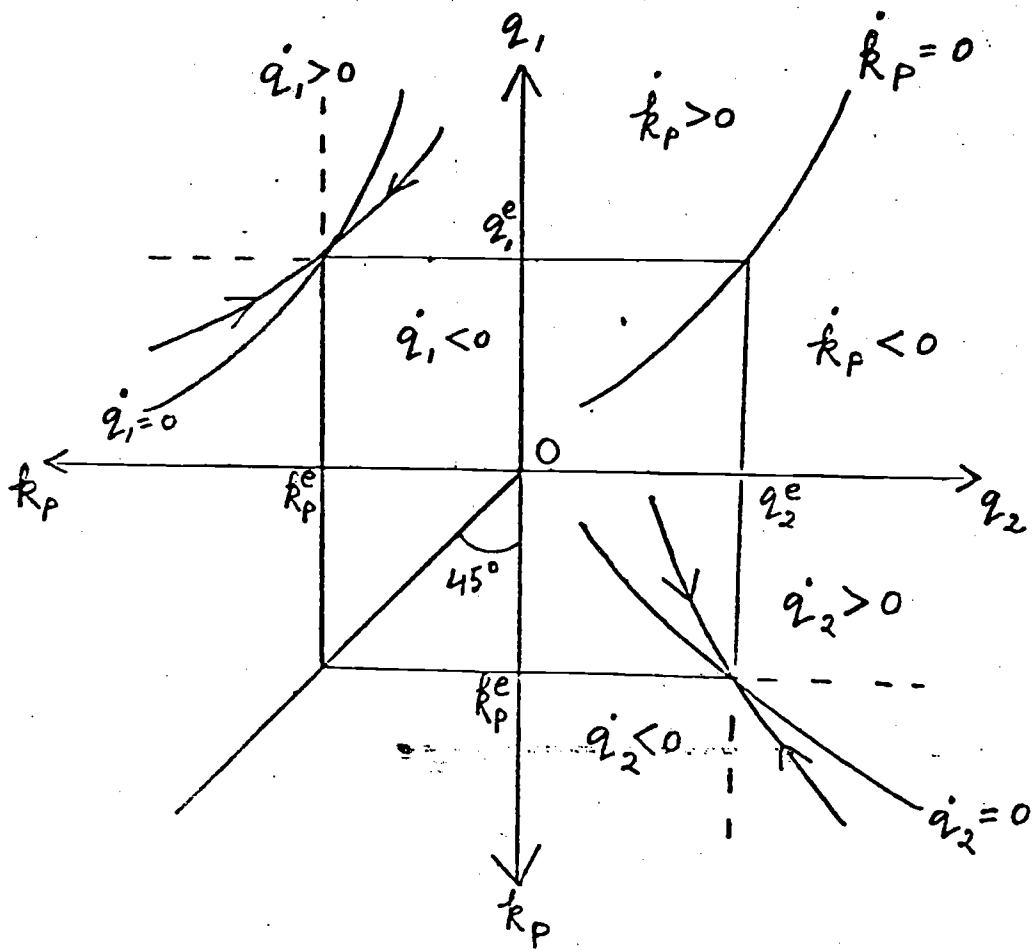


Figure 1. The Steady State and Dynamic Path.



$$(16.2) \quad r = [pf_1(\Gamma(k, s/p)k, G(k, s/p))\Gamma(k, s/p) - \delta q_1 \\ + C'(J_p(q_1))(J_p(q_1))^2 + \dot{q}_1]q_1.$$

This equation points out that the firm equates the rate of return on physical capital (the right side of (16.2)) to the opportunity cost of funds ( $r$ ). The rate of return consists of three elements, the value of the marginal product net of depreciation, the reduction in physical investment costs brought about by the stock expansion and any capital gains that arise. From (16.2) we can discern how the time path of the demand price changes in response to changes in the shadow price itself and the physical capital intensity. First, an increase in the demand price, given the physical capital intensity, leads to a decrease in the rate of return on physical capital. Hence, given the opportunity cost of funds, a capital gain must accrue to the firm. In fact  $\partial \dot{q}_1 / \partial q_1 = r + \delta - (I_p / K_p) > 0$ , which is positive if the present value of the flow of funds is to be positive in the neighborhood of the steady state.<sup>11</sup> Next an increase in the physical capital intensity decreases the value of the marginal product of physical capital. This implies that the rate of return drops below the opportunity cost of funds. In order to restore the equalities a capital gain must arise. Thus  $\partial \dot{q}_1 / \partial k = -H_2 / H_1 > 0$ . Recall that  $H_2 < 0$  and  $H_1 > 0$  from the strict concavity of the variable profit function in R&D intensive form. Combining these results yields a  $\dot{q}_1 = 0$  locus in  $(k, q_1)$  space in Figure 1, which is negatively sloped since  $dq_1 / dk$  (at  $\dot{q}_1 = 0$ ) =  $H_2 / H_1 (r + \delta - (I_p / K_p)) < 0$ . Points above the  $\dot{q}_1 = 0$  locus define  $\dot{q}_1 > 0$ , while points below illustrate  $\dot{q}_1 < 0$ .

A similar set of results can be derived for the demand price of R&D investment, since

$$(16.3) \quad r = [pf(\Gamma(k, s/p)k, G(k, s/p)) - pf_1(\Gamma(k, s/p)k, \\ G(k, s/p))\Gamma(k, s/p)k - pf_2(\Gamma(k, s/p)k, \\ G(k, s/p))G(k, s/p) - \mu q_2 \\ + E(J_r(q_2))(J_r(q_2))^2]/q_2.$$

Clearly  $\partial \dot{q}_2 / \partial q_2 = r + \mu - (I_r / K_r) > 0$  and  $\partial \dot{q}_2 / \partial k = kH_2H_1 < 0$ . Thus a  $\dot{q}_2 = 0$  locus is defined which is positively sloped in  $(k, q_2)$  space in Figure 1 and  $dq_2/dk$  (at  $\dot{q}_2 = 0$ ) =  $-kH_2/H_1 (r + \mu - (I_r / K_r)) > 0$ . Moreover, below  $\dot{q}_2 = 0$ ,  $\dot{q}_2 < 0$ , and above,  $\dot{q}_2 > 0$ .

Using the four quadrant technique developed by Abel [1981] we can characterize the steady state solution ( $\dot{k} = \dot{q}_1 = \dot{q}_2 = 0$ ) for the firm from Figure 1 at  $(k^e, q_1^e, q_2^e)$ . There exists a unique steady state which is a saddle point. The steady state magnitudes are denoted by the formation of the rectangle and the dynamic paths are monotonic and illustrated in  $(k, q_1)$  and  $(k, q_2)$  spaces in Figure 1.<sup>12</sup>

From Figure 1 we can characterize the nature of the path that the firm follows to the steady state. The paths of  $k$ ,  $q_1$ , and  $q_2$  are illustrated in the northwest and southwest quadrants. Suppose that  $k^0 < k^e$  (i.e. the initial physical capital intensity is less than the steady state magnitude) then  $q_1^0 > q_1^e$  and  $q_2^0 < q_2^e$ . The intuition is quite clear. In order for the firm to be able to increase its physical capital intensity up to the steady state level, it must be investing in physical capital at a rate above and knowledge capital below that needed to maintain long run equilibrium.

It is also possible to discern the intertemporal paths of the labor intensity. For  $k^0 < k^e$ , since the physical capital and labor intensity are

directly related (as  $G_1 > 0$ ) then  $\ell^o < \ell^e$ . This means that with the R&D intensity of physical capital above its steady state level, knowledge per worker is also too high. A further implication, which combines the previous results on the dynamic path, is that in order to reduce the knowledge per worker from the excessive level, the firm must accumulate physical capital at a rate above and knowledge capital at a rate below that found in the steady state. Thus the growth rate of R&D capital is directly related to, while the growth rate of physical capital is inversely related to the labor intensity.

Along the dynamic path the relationship between the R&D intensity of physical capital and the utilization rate depends on the elasticity of the marginal product of physical capital services. If increases in the physical capital intensity increase the value of the marginal product of the utilization rate (i.e.  $e_{\gamma\gamma} + 1 > 0$ ), then the elasticity of the marginal product of physical capital services is inelastic. In this situation, as the physical capital intensity increases towards the long run level, physical capital becomes relatively less idle. Thus  $\gamma^o < \gamma^e$ , which follows from  $\gamma = \Gamma(k, s/p)$  with  $\Gamma_1 > 0$  as  $e_{\gamma\gamma} + 1 > 0$ . In other words the R&D intensity of physical capital increases, the degree to which physical capital is utilized decreases.

Notice that when the utilization rate is below its long run magnitude, the physical capital growth rate is above the steady state rate. Clearly, the firm is utilizing physical capital at too low a rate because the stock is growing too rapidly. Moreover, we see that the stock is expanding at an excessive rate, because the firm desires to increase the physical capi-

tal intensity (or lower the R&D intensity of physical capital). As a consequence, in this situation (with  $e_{\gamma\gamma} + 1 > 0$ ), there is a negative correlation between the physical capital growth rate and the utilization rate, while there is a direct relationship between the latter rate and knowledge accumulation.<sup>13</sup>

#### 4. Comparative Steady States

In this section we consider the effects of unanticipated changes in the exogenous variables, such as the product price and the cost of capital, which characterize the firm's environment.

First, suppose that there is an increase in the opportunity cost of funds. This increase implies (from (16.2) and (16.3)) that the rates of return on capital must correspondingly increase. Hence at the original steady state with  $k=k^e$ , in order to maintain  $\dot{q}_1 = \dot{q}_2 = 0$ , the investment demand prices must fall. However, when both  $q_1$  and  $q_2$  decrease, the physical capital intensity of R&D responds in an ambiguous manner, because the growth rates of both physical and R&D capitals are falling. These effects can be discerned by setting equation set (16) equal to zero and differentiating with respect to the discount rate,

$$(17.1) \quad \frac{\partial(I_p/K_p)}{\partial r} = \frac{\partial(I_r/K_r)}{\partial r} = - \frac{kJ'_p J'_r H_2 (q_2 + q_1 k)}{H_1 H_3} < 0$$

where  $H_3 = knH_2(J'_p + kJ'_r)/H_1 < 0$  and  $n = r + \delta - I_p/K_p = r + \mu - I_r/K_r > 0$ . Clearly, from (17.1) the increase in the discount rate leads to the identical effects on the capital growth rates, and consequently physical capital intensity is

not unambiguously altered. Notice that since the utilization rate and labor intensity depend on the physical capital intensity, ambiguities in the latter translate into ambiguities for the former variables. In fact from

$$(17.2) \quad \frac{\partial k}{\partial r} = nk \left[ \frac{J'_p q_1}{p} - \frac{J'_r q_2}{r} \right] / H_3$$

we can observe that the movement of the physical capital intensity depends on the relative responsiveness of physical and R&D investment to their respective demand prices. If, for example, physical capital responds relatively more to its demand price, then, as the opportunity cost of funds increase, physical capital intensity declines. The higher discount rate exacts its toll on the stock of physical capital relative to R&D capital.

Next suppose that there is an autonomous change in the obsolescence rate on R&D, such that knowledge becomes obsolete at a faster rate. In this instance there is a shift towards the capital stock with the relative increase in its life. Thus the physical capital intensity increases by,

$$(18.1) \quad \frac{\partial k}{\partial \mu} = - n [n + q_2 k J'_r] / H_3 > 0.$$

The increase in physical capital intensity lowers the value of the marginal product of P&E and therefore its rate of return. In order to restore the equality between this rate of return and the opportunity cost of funds, the physical capital investment demand price must decrease. In other words, the growth rate of physical capital declines by,

$$(18.2) \quad \frac{\partial \left( \frac{I_p}{K_p} \right)}{\partial \mu} = - J'_p H_2 [n + q_2 k J'_r] / H_3 < 0.$$

Interestingly, the growth rate of R&D capital is subject to two opposing forces, as its useful life diminishes at a faster rate. The increase in the R&D depreciation rate causes its investment demand price to decline at the initial steady state ( $k=k^e$ ), to retain the equality between the opportunity cost of funds and the rate of return on R&D capital. However, as the physical capital intensity increases, the value of the marginal product of R&D capital increases, and this movement shifts the burden of adjustment away from the price and onto the stock.

The increase in the physical capital intensity implies that the labor intensity also increases in the steady state. Moreover, when  $e_{\gamma\gamma}+1 < 0$  the utilization rate decreases, while for  $e_{\gamma\gamma}+1 > 0$  the rate increases.<sup>14</sup> Notice that, for the unanticipated increase in the R&D obsolescence rate, when  $e_{\gamma\gamma}+1 > 0$  the growth of physical capital and its utilization rate is inversely related, while the converse occurs for  $e_{\gamma\gamma}+1 < 0$ .

The last unanticipated change that is considered is due to a shift in product demand which causes an increase in the output price. The increase in the product price increases the value of the marginal product for physical and R&D capital, and thereby also the rates of return. Given the opportunity cost of funds, and in order to remain in long run equilibrium, the investment demand prices must increase. Consequently the shift in demand causes the capital growth rates to increase by

$$(19.1) \quad \frac{\partial(I_p/K_p)}{\partial p} = \frac{\partial(I_r/K_r)}{\partial p} = - \frac{k J' J' H_2}{p r} \left( \frac{\partial \dot{q}_2}{\partial p} + \frac{\partial \dot{q}_1}{\partial p} \right) / H_1 H_3 > 0,$$

where  $\partial \dot{q}_1 / \partial p < 0$ ,  $\partial \dot{q}_2 / \partial p < 0$ . Nevertheless, the effect on the physical capital

intensity is ambiguous. As the product price increases, given the investment demand prices, the intensity must increase in order to bring the rate of return of physical capital into line with the opportunity cost of funds, while for the rate of return on R&D capital, the intensity must decrease.<sup>15</sup>

Although ambiguities associated with the physical capital intensity pose difficulties in determining the effects on the labor intensity and the utilization rate, there are situations when unambiguous results can be discerned. From the definition of  $\Gamma(k, s/p)$  and  $G(k, s/p)$ ,

$$(19.2) \quad \frac{\partial \gamma}{\partial p} = \Gamma_1 \frac{\partial k}{\partial p} - \Gamma_2 \frac{s}{p^2}$$

$$(19.3) \quad \frac{\partial \lambda}{\partial p} = G_1 \frac{\partial k}{\partial p} - G_2 \frac{s}{p^2}$$

We know that  $G_1 > 0$ ,  $G_2 < 0$  and  $\Gamma_2 < 0$  (by equations (9), (12) and (13)). Hence if  $\text{sgn } \Gamma_1 = \text{sgn } \partial k / \partial p$  then  $\partial \gamma / \partial p > 0$  and if  $\partial k / \partial p > 0$  then  $\partial \lambda / \partial p > 0$ . Consequently if the unexpected rise in the product price increases the physical capital intensity and if  $e_{\gamma\gamma} + 1 > 0$  (so  $\Gamma_1 > 0$ ), the labor intensity and utilization rate increase. The response to the price shock is that the physical capital growth rate, the utilization rate, the labor requirements per unit of R&D capital, and the growth rate of knowledge capital are positively correlated. Moreover, from the production function we can see that, in this case, output in R&D intensive form also increases. Hence, unlike the movement along the intertemporal path where the utilization rate, the labor intensity and the R&D capital growth rate are inversely related to the physical capital rate of growth, we now have a

situation where there are co-movements of these variables.<sup>16</sup> In this context, as opposed to the conventional wisdom, R&D capital accumulation does not displace labor. In addition, we are able to provide a rationale for empirically observed direct relationship between physical capital accumulation and its utilization rate.

##### 5. Conclusion

In this paper we have developed a dynamic analysis of a firm undertaking physical and knowledge capital accumulation, along with labor hiring and utilization decisions. Utilization costs were introduced through a rising wage rate associated with the greater flow of physical capital services per unit of the stock.

For the intertemporal movement, it was established that increases in the knowledge intensity of physical capital led the firm to decrease its labor intensity (measured in R&D terms), while the effect on the utilization rate depended on the magnitude of the elasticity of the marginal product of physical capital services. In general when increases in the knowledge intensity of physical capital decrease the marginal product of the utilization rate, then physical capital becomes relatively more idle. Consistent with this context, the physical capital growth rate is inversely correlated with the utilization rate, the R&D capital growth rate, and the labor intensity. However, we have shown that it is important to distinguish between the evolution of the firm in the absence of unanticipated demand and cost changes, and the movement in response to these shocks. Indeed, an unexpected price rise, in general, causes the firm to



increase its labor intensity, physical capital growth and utilization, and R&D accumulation.

To understand the process of R&D accumulation and the integration with physical capital growth and utilization there are important areas of future research. In this paper we have distinguished R&D and P&E in the basic way of recognizing that physical capital is costly to use. However, the variable utilization rate did not affect the rate at which physical capital was depreciated. By allowing endogenous depreciation, it would then be possible to investigate the relationship between R&D growth and P&E durability.

Also, the only type of knowledge that we have considered is that which is not embodied in the physical capital stock. In fact, the firm can undertake R&D activities or it can buy other factors of production which embody technical advances undertaken by other firms. Modelling this aspect would entail a vintage physical capital model with the integration of R&D investment. However, since the current state of vintage capital models relies on very specific technologies and generally only investigates steady state properties, much work needs to be done in this area.

Finally, R&D can affect the demand conditions confronting the firm, and consequently interfirm rivalry becomes an important consideration. Thus an interesting extension would be to develop a dynamic model of industry equilibrium with both physical and R&D capital accumulation.

### Footnotes

1. The production function reflects that the utilization elasticity of output equals the physical capital elasticity of output. A related form is found in Nadiri and Rosen [1969], Taubman and Wilkinson [1970] and Abel [1981].
2. See Oi [1981] for a survey of the use of rhythmic wage rates in static models of production.
3. We could introduce a utilization rate for R&D capital. In the present paper the rate is fixed and normalized to unity. If the rate is variable, we could assume it to be positively related to the physical capital rate. The results would not be materially affected and the distinction between the two quasi-fixed factors would still stand, since only physical capital exhibits costly utilization. Finally, labor hiring is anonymous with labor utilization, since the latter is a variable factor of production.
4. We now drop the symbol (t) for notational simplicity.
5. Adjustment costs are by now quite standard. We adopt the separable form which depends on gross investment. See Lucas [1967], Gould [1968] and Hayashi [1982].
6. There are also the transversality conditions  $\lim_{t \rightarrow \infty} q_i > 0$   $i=1,2$ ,  $\lim_{t \rightarrow \infty} q_1 K_p = \lim_{t \rightarrow \infty} q_2 K_r = 0$  and the Legendre-Clebsch conditions, which state that the matrix of second order derivatives with respect to the control variables (L,  $\gamma$ ,  $I_p$ ,  $I_r$ ) is negative definite.
7. A form of the production function which is usually adopted, in the absence of R&D capital, is  $y = \gamma L f(k_p)$ , where  $k_p = K_p/L$  (see Oi [1981] for a survey). In this instance equation (8) becomes  $\theta \gamma = 1/e_\gamma$ . Thus the technology restricts  $e_\gamma$  to be a constant and equal to unity. Since  $e_\gamma < 1$  then  $\theta \gamma > 1$ . In the present paper  $e_\gamma$  is neither a constant nor is it restricted to be greater than  $e_\gamma$ .
8. Although the strict concavity of the variable profit function is only a sufficient condition for the stability of the short run equilibrium, it is a necessary condition for the stability of the steady state.
9. This result extends the previous work on utilization in the context of rising factor prices. The usual form of the production function restricted  $e_\gamma \gamma + 1 > 0$ , and so the stock of machines and its utilization rate were directly related. Indeed, if  $y = \gamma L f(k_p)$ , as in footnote 7, then  $e_\gamma = 1$  and  $e_\gamma \gamma = 0$ , which means that  $e_\gamma \gamma + 1 > 0$ .

10. The results are accordingly modified when  $f_{12} < 0$ . From this juncture we impose the reasonable assumption that  $f_{12} > 0$ .
11. We ignore the possibility that  $\dot{\partial}q_1/\partial q_1$  can be nonpositive where  $\dot{k} \neq 0$ .
12. The relevant boundary conditions on the production function prevent  $k^e$  and thereby  $q_1^e$  and  $q_2^e$  from being either zero or infinite.
13. When  $e_{\gamma\gamma} + 1 < 0$ , for  $k^o < k^e$ , we find  $\gamma^o > \gamma^e$ . Thus the utilization rate is positively related to the physical capital growth rate and inversely related to the physical capital intensity and the knowledge capital growth rate.
14. The results from an increase in the physical capital depreciation rate follow from the effects of an increase in the R&D obsolescence rate. Here the P&E intensity and R&D growth rate decline, while the P&E growth rate moves in an ambiguous direction.
15. This discussion centers around  $\partial k/\partial p = kn[(J'_p \dot{\partial}q_1/\partial p) - (J'_r \dot{\partial}q_2/\partial p)]/H_3$ .
16. Actually, the condition is less stringent for the utilization rate because all that is needed is that  $\text{sgn } \Gamma_1 = \text{sgn } \partial k/\partial p$ . We do not need  $\partial k/\partial p > 0$ . In addition, an unanticipated decrease in the scale wage rate generates the same qualitative effects as the product price increase.

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