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EXCHANGE-RATE DYNAMICS

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ABSTRACT

This paper discusses the dynamic behavior of exchange rates, focusing both on the exchange rate's response to exogenous shocks and the relation between exchange-rate movements and movements in important endogenous variables such as prices, interest rates, output, and the current account. Aspects of exchange-rate dynamics are studied in a variety of models, some of which are based on postulated supply and demand functions for assets and goods, and some of which are based on explicit individual utility-maximizing problems. Section 1 surveys the terrain. Section 2 explores the simplest model in which the relation among the exchange rate, price levels, and the terms of trade can be addressed--a flexible-price small-country model in which wealth effects are absent and domestic and foreign goods are imperfect substitutes. Section 3 introduces market frictions so that the role of endogenous output fluctuations can be studied. Both sticky-price models and alternative market-friction models are discussed. Section 4 studies the link between the accumulation of foreign assets and domestic capital and the exchange rate. Section 5 examines deterministic and stochastic models in which individual behavior is derived from an explicit intertemporal optimization problem. Finally, section 6 offers concluding remarks.

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1. Introduction

This chapter discusses the dynamic behavior of exchange rates. It focuses on both the exchange rate's response to exogenous disturbances and the relation between exchange-rate movements and movements in such endogenous variables as nominal and relative prices, interest rates, output, and the current account. These questions are addressed in a variety of models, some of which are based on postulated supply and demand functions for assets and goods, and some of which are based on an explicit utility-maximizing problem. Similar models are studied elsewhere in this volume (especially in chapter 14 by Frenkel and Mussa, in chapter 15 by Branson and Henderson, and in chapter 20 by Marston), but the approach taken here is different. We do not attempt to present a single, unifying model that encompasses as special cases those discussed in the literature. Instead, we try to emphasize the common or unique features of the alternative models.

An ideal treatment of exchange-rate dynamics would begin by summarizing the relevant characteristics of the empirical record. All key features of the stochastic processes that appear to govern exchange rates and other statistically related economic variables would be catalogued. Then, a set of models that are compatible with at least some of the observed relationships would be presented. The discussion would point to features of the models that are consistent with the data and to features that are not; and it would highlight implications that might allow economists to distinguish among alternative models through future empirical research.

We have not attempted to attain this ideal, in large part because it would be premature to do so on the basis of our limited data on exchange-rate behavior. Only a few central banks have allowed more than intermittent floating, and the time series covering even extended periods of floating are relatively short.

Thus, while high-frequency characteristics of exchange-rate changes have been studied with some success in recent years, studies of the lower-frequency characteristics of exchange-rate changes, corresponding to periodicities more common to macroeconomic phenomena, have proven less conclusive (Meese and Rogoff, 1983; Shafer and Loopesko, 1983). Another shortcoming of our data is the absence of quantifiable information about inherently unobservable market expectations. As the chapter illustrates, alternative expectational scenarios can give rise to very different empirical correlations between exchange-rate movements and changes in other observable variables.

In the face of limited data, economists have naturally concentrated their research on models consistent with what appear to be the stylized facts of the interwar and post-1973 experiences with floating. Earlier studies, which drew on the hyperinflationary episodes of the interwar period, emphasized the key role of monetary factors in exchange-rate determination (Frenkel, 1976). The more moderate inflation and repeated real shocks of the post-1973 period highlight different empirical regularities, however. Among these are the strong correlations between exchange-rate movements and movements in terms of trade, the high variability of exchange rates compared to that of international price-level ratios, and the on-again, off-again relationship between the exchange rate and the current account (Genberg, 1978; Frenkel and Mussa, 1980; Flood, 1981; Shafer and Loopesko, 1983). All the models discussed in this chapter grew out of attempts to reconcile exchange-rate theory with at least some of these stylized facts.

Along with the empirical regularities, the rational-expectations "revolution" in macroeconomics has had an important impact on exchange-rate theory. The models reviewed below reflect that intellectual development in a number of ways. Following Black (1973), these models endow agents with rational expectations about the future. Some extend the recent closed-economy business-cycle literature by exploring channels through which money can exert a persistent influence on output in open economies. Finally, Lucas' (1976) celebrated critique of policy evaluation finds expression in the attempts described below to base dynamic exchange-rate theory on the explicit intertemporal optimization problems of individual agents.

A recurring theme of the chapter is the distinction we make between the intrinsic and extrinsic sources of an economy's dynamics. An intrinsic source of dynamics causes movement even when all exogenous variables that affect the economy are expected to remain constant forever. An example of intrinsic dynamic behavior is the adjustment of the capital stock to its steady-state level in a growth model. In contrast, extrinsic dynamics are associated exclusively with current or anticipated future changes in exogenous variables. A system with extrinsic dynamics only is stationary in the absence of such external shocks. Our distinction between intrinsic and extrinsic dynamics corresponds closely to Samuelson's (1947) distinction between "causal" and "historical" dynamic systems.

The chapter is organized as follows. Section 2 explores the simplest model in which the relation among the exchange rate, price levels, and the terms of trade can be addressed. This flexible-price, small-country model allows domestic and foreign consumption goods to be imperfect substitutes, but it includes no monetary non-neutralities and no intrinsic dynamics. Even so, the model predicts that current or anticipated future real shocks will induce simultaneous movements of the nominal exchange rate and the terms of trade. Further, exchange rates may be more volatile than price levels when real shocks are dominant. While the exchange rate certainly displays asset-price characteristics, it also plays a role in accommodating required shifts in relative goods prices. The exchange rate's behavior is thus affected both by forces emphasized in the monetary approach to the exchange rate (Frenkel and Mussa, chapter 14) and by forces emphasized in the older elasticities approach.

Section 3 introduces market frictions so that the role of endogenous output fluctuations can be studied. Section 3.1 alters the previous section's model by assuming that the money price of domestic goods is a predetermined or non-jumping variable that must adjust gradually in the face of goods-market disequilibrium. The assumption of domestic price stickiness reinforces both the correlation between exchange-rate and terms-of-trade changes and the high short-run variability of the exchange rate compared to that of international price-level ratios. Moreover, the price-adjustment process through which goods-market imbalance is gradually eliminated adds an intrinsic component to the economy's dynamics. These intrinsic dynamics are reflected in the persistent effects of disturbances on output, prices, and interest rates. Section 3.2 investigates alternative market frictions (and alternative sources of persistence) based on more detailed descriptions of the institutional or informational environment. Some of the models are stochastic, and their solution involves rules for inducing a probability distribution function on the exchange rate from the probability distributions of various exogenous variables. These solutions are different from those of the previous deterministic models, whose equilibria can be conveniently represented as solutions to systems of differential equations.

Section 4 returns to a setting of frictionless markets to study the links between asset accumulation and the exchange rate. The adjustment of foreign assets and domestic capital to their steady-state levels provide new sources of intrinsic dynamic behavior. Within this framework, it is shown that the relationship between the exchange rate and current account, even along paths converging to a fixed long-run equilibrium, is very loose. The models studied here reveal channels through which money can influence real variables even in the absence of market frictions.

Section 5 examines deterministic and stochastic models in which individual behavior is derived from an explicit intertemporal optimization problem. These models serve at least three related purposes. First, they are suggestive of assumptions under which the aggregate behavioral relations postulated in previous sections' models are consistent with individual maximizing behavior. Second, they provide a natural setting in which some welfare consequences of macroeconomic policies can be assessed. Third, because they are built up on the basis of preferences that are invariant with respect to policy change, they provide vehicles for policy analysis that are less vulnerable than models discussed earlier to Lucas' (1976) critique. Money is introduced into these optimizing models in rather *ad hoc* ways, however, so their immunity to Lucas' criticisms is less than total. Nonetheless, the approach discussed in this section leads to a deeper perspective on the possible causes of the observed empirical regularities.

Section 6 contains concluding remarks.

2. Expectations and the Exchange Rate in a Simple Flexible-Price Model

This section studies exchange-rate determination in a rational-expectations model with flexible prices. The model abstracts from the possible intrinsic sources of dynamics to be introduced in sections 3 and 4 below, and thus highlights the extrinsic component of exchange-rate dynamics. As was noted in section 1, intrinsic dynamics lead to changes in a model's endogenous variables that need not be associated with current or expected future changes in the levels of exogenous variables that impinge on the economy. Extrinsic dynamics, in contrast, arise exclusively in response to such exogenous events.

The model set out, which comes from Mussa (1977, 1982), displays some important channels through which current and anticipated future disturbances, both monetary and real, affect exchange rates. In addition, it illustrates the implications of rational expectations for exchange-rate dynamics. (The environment assumed in this section is non-stochastic, so "rational expectations" is equivalent to "perfect foresight" here.) The model also provides a useful benchmark for the analysis in later sections, particularly section 3's discussion of exchange-rate dynamics under short-run price inflexibility.

2.1 The Model

Consider a small open economy specialized in the production of a good that is an imperfect substitute in consumption for an imported good.[1] Wealth may be held in the form of domestic fiat money (which is not held by foreigners) or in the form of interest-bearing bonds. Bonds denominated in either domestic or foreign currency are available, but these are perfect substitutes in portfolios. Thus, any difference between the nominal returns they offer is offset exactly by an expected change in the exchange rate.[2] The resulting (uncovered) interest-parity condition is written as

(2.1) $$r_t = r_t^* + \dot{e}_t,$$

where r is the nominal interest rate on domestic-currency bonds, r* is the rate on foreign-currency bonds, and e is the natural logarithm of the exchange rate, defined as the price of foreign money in terms of domestic money. (A rise in e is a depreciaton of the currency.) Unless otherwise noted, lower-case letters denote natural logarithms of the corresponding upper-case variables, except when representing rates of interest. A dot denotes a variable's (right-hand) time derivative.

Assume that agents have perfect foresight concerning all disturbances other than initial, unanticipated shocks that dislodge the economy from its previously expected trajectory. Perfect foresight is an assumption of convenience, and we could easily transplant the model explored here to an explicitly stochastic setting without changing its main implications (Mussa, 1982). The perfect-foresight assumption permits us to identify the expected rate of change of the exchange rate with the actual rate of change.

Let m denote the nominal money supply (an exogenous variable under a floating exchange rate), p the home-currency price of domestic output, p* the foreign-currency price of imports, γ the share of the home good in domestic consumption, and y domestic output. Equilibrium in the money market requires that

$$(2.2) \qquad m_t - \gamma p_t - (1 - \gamma)(e_t + p_t^*)$$

$$= \psi \left[p_t + y_t - \gamma p_t - (1 - \gamma)(e_t + p_t^*)\right] - \lambda r_t , \quad \psi \leq 1.$$

The left-hand side of (2.2) represents real money balances expressed in terms of the appropriate consumer-price index. Note that (2.2) can be rewritten as

$$(2.3) \qquad m_t - \alpha p_t - (1 - \alpha)(e_t + p_t^*) = \psi y_t - \lambda r_t ,$$

where $\alpha \equiv \gamma + \psi(1 - \gamma)$.

We assume that aggregate (domestic plus foreign) demand for domestic output, d, is negatively related to the contemporaneous relative price of the domestic good in terms of the foreign good, $p - e - p^*$. If this relative price were constant over time, there would be a unique intertermporal price of consumption at time t_0 in terms of consumption at any other time t_1; and this relative price would depend only on the path of $r - \dot{p}$. If the contemporaneous relative price of domestic and foreign goods changes over time, however, there are two intertemporal relative prices between any two dates t_0 and t_1, one in terms of domestic goods and one in terms of foreign goods. For purposes of the present benchmark model, we assume that aggregate demand depends on an average intertemporal price expressed in terms of the home consumption bundle. We also assume that only the "instantaneous" average intertemporal relative price, the domestic real interest rate $r - \gamma\dot{p} - (1-\gamma)(\dot{e} + \dot{p}^*)$, affects aggregate demand.[3]

Under the foregoing assumptions, aggregate demand is given by

$$(2.4) \qquad d_t = \phi(e_t + p_t^* - p_t) - \sigma[r_t - \gamma\dot{p}_t - (1 - \gamma)(\dot{e}_t + \dot{p}_t^*)] + g_t ,$$

where g is a demand-shift factor such as government consumption. In this flexible-price model, aggregate demand must always equal the natural or full-employment level of output $\bar{y}$,

$$(2.5) \qquad d_t = \bar{y}_t.$$

Note that the terms of trade between domestic and foreign goods are endogenously determined. In contrast, the paths of p^* and r^* are exogenously determined in world markets where the economy under study plays an insignificant part.

The model may be reduced to a system of two non-autonomous differential equations in e and p,

$$(2.6) \qquad \dot{e}_t = \frac{1-\alpha}{\lambda}(e_t + p_t^*) + \frac{\alpha}{\lambda} p_t - \frac{1}{\lambda} x_t - \dot{p}_t^* ,$$

$$(2.7)\qquad \dot{p}_t = \left(\frac{1-\alpha}{\lambda} - \frac{\phi}{\sigma\gamma}\right)(e_t + p_t^*) + \left(\frac{\alpha}{\lambda} + \frac{\phi}{\sigma\gamma}\right) p_t - \frac{1}{\sigma\gamma} z_t - \frac{1}{\lambda} x_t ,$$

where x and z are linear combinations of exogenous variables:

$$(2.8)\qquad x_t \equiv m_t - \psi\bar{y}_t + \lambda(r_t^* - \dot{p}_t^*) ,$$

$$(2.9)\qquad z_t \equiv g_t - \bar{y}_t - \sigma(r_t^* - \dot{p}_t^*).$$

Let $\omega = \phi/\sigma\gamma$. A general solution to the differential-equation system is

$$(2.10)\qquad e_t = k_1 \exp(t/\lambda) - \frac{k_2\alpha \exp(\omega t)}{1-\alpha-\lambda\omega} + \frac{1}{\lambda}\int_t^\infty \exp\left[(t-s)/\lambda\right] x_s\, ds$$

$$+ \frac{\alpha\omega}{1-\lambda\omega}\int_t^\infty \left\{\exp\left[(t-s)/\lambda\right] - \exp[\omega(t-s)]\right\}(z_s/\phi)\, ds - p_t^*,$$

$$(2.11)\qquad p_t = k_1 \exp(t/\lambda) + k_2 \exp(\omega t) + \frac{1}{\lambda}\int_t^\infty \exp\left[(t-s)/\lambda\right] x_s\, ds$$

$$+ \frac{\alpha\,\omega}{1-\lambda\omega}\int_t^\infty \exp\left[(t-s)/\lambda\right](z_s/\phi)\, ds + \frac{\omega(1-\alpha-\lambda\omega)}{1-\lambda\omega}\int_t^\infty \exp\left[\omega(t-s)\right](z_s/\phi)\, ds ,$$

where k_1 and k_2 are arbitrary constants (see, e.g., Hirsch and Smale, 1974).

The arbitrary constants k_1 and k_2 reflect a fundamental indeterminacy in models assuming rational expectations or perfect foresight. The indeterminacy is a consequence of the self-fulfilling nature of those expectations. Returning to (2.1) and (2.3), we note that, given p, any level of the exchange rate is consistent with money-market equilibrium provided the perfectly-foreseen depreciation rate $\dot{e}$ satisfies (2.6). Similarly, (2.4) implies that any price p clears the goods market, given e and r, provided the rate of price increase $\dot{p}$, and hence the real interest rate, is appropriate. Because a higher e requires a higher $\dot{e}$, *ceteris paribus*, to clear the money market, and because a higher p calls for a higher $\dot{p}$, *ceteris paribus*, to clear the goods market, the characteristic roots

λ^{-1} and ω of the differential-equation system given by (2.6) and (2.7) are positive. Because the constants k_1 and k_2 multiply time exponentials in those positive roots, any particular solution for e and p in which k_1 or k_2 differs from zero will entail explosive price behavior unrelated to market "fundamentals" -- what might be called a "speculative bubble," since prices explode only because they are expected to do so (Sargent and Wallace, 1973; Flood and Garber, 1980).

It has become standard in the exchange-rate literature to resolve this indeterminacy by identifying as "the" equilibrium of the economy the (hopefully) unique market-clearing price vector which excludes such speculative bubbles.[4] In the present context, this amounts to taking as the equilibrium exchange rate and domestic-goods price the particular solution to (2.6) and (2.7) incorporating the initial conditions $k_1 = k_2 = 0$. This choice of initial conditions is said to place the economy on its <u>saddle path</u>. The saddle-path assumption is an appealing one because it stipulates that prices depend only on current and expected future demand and supply conditions in markets; further, as we shall see, the assumption yields intuitively reasonable results.[5] The saddle-path solutions for the flexible-price equilibrium exchange rate and domestic-goods price, given by (2.6) and (2.7) with $k_1 = k_2 = 0$, are denoted by $\tilde{e}$ and $\tilde{p}$.

Denote the price of exports in terms of imports by $q = p - e - p^*$. We will refer to q as the terms of trade. (It is sometimes referred to as the real exchange rate, though we will reserve that term for its other common usage as the price of non-traded in terms of traded goods.) Along the saddle path

$$(2.12) \qquad \tilde{q}_t = \omega\int_t^{\infty}\exp\,[\omega(t-s)]\{[g_s-\bar{y}_s-\sigma(r_s^*-\dot{p}_s^*)]/\phi\}ds.$$

Several features of these solutions are worthy of note. As (2.12) shows, the equilibrium terms of trade are independent of domestic and foreign monetary factors. q depends exclusively on current and anticipated future shocks to real variables: aggregate demand, aggregate supply, and the world real interest rate, $r^* - \dot{p}^*$. In other words, purely monetary disturbances induce proportional movements in exchange rates and international price-level ratios, as purchasing-power parity theory would predict. Because the domestic real interest rate can be written as $r^* - \gamma\dot{q} - \dot{p}^*$, it is also unaffected by monetary developments. Thus, the flexible-price model developed here implies a complete dichotomy between the real and monetary sectors.

Real as well as monetary disturbances can affect the exchange rate, however. In fact, any real disturbance requiring a movement in the equilibrium terms of trade must be accommodated in part through an exchange rate change and in part through a change in home goods prices, with the overall price level moving so as to maintain money-market equilibrium (Stockman, 1980; Obstfeld, 1980; Mussa, 1982; Helpman and Razin, 1982; Lucas, 1982; Sachs, 1983a). For example, (2.12) shows that a permanent shift in the path of aggregate demand from $\{g_s\}_{s=t}^{\infty}$ to $\{g_s + \Delta g\}_{s=t}^{\infty}$ causes the terms of trade to rise by $\Delta\tilde{q}_t = \Delta g/\phi$. To bring this rise about, the exchange rate $\tilde{e}_t$ falls (the domestic currency appreciates) by $\Delta\tilde{e}_t = -\alpha(\Delta g/\phi)$, while the output price $\tilde{p}_t$ rises by $\Delta\tilde{p}_t = (1-\alpha)(\Delta g/\phi)$. The corresponding change in the consumer price index is $\gamma\Delta\tilde{p}_t + (1-\gamma)\Delta\tilde{e}_t = -\psi(1-\gamma)(\Delta g/\phi)$, while the change in the value of domestic output (in terms of the consumption bundle) is $(1-\gamma)(\Delta\tilde{p}_t - \Delta\tilde{e}_t) = (1-\gamma)(\Delta g/\phi)$. The increased demand for money (due to the increase in real income) exactly matches the fall in the consumer price index, so money-market equilibrium is maintained [see (2.2)]. The exchange rate's role in equilibrating the domestic goods market is reminiscent of the traditional elasticities approach to the exchange rate.

Note that if $\gamma = 1$ or $\psi = 0$, the consumer price index is unaffected even though the exchange rate falls. Also note that, depending on the value of α, the change in the exchange rate may be larger or smaller than the change in the nominal price of domestic goods. Consequently, the model can be consistent with the empirical observation that exchange rates are often more volatile than the nominal prices of goods (Flood, 1981).

A corollary of the foregoing observations is that a negative correlation between movements in exchange rates and changes in the terms of trade does not imply any stickiness in domestic-goods prices. The demand shock Δg analyzed above forces the exchange rate to appreciate in real as well as nominal terms, but it does not induce goods-market disequilibrium. Nominal price rigidities can cause exchange rates and terms of trade to move in opposite directions even in response to monetary shocks, but the pattern may also emerge as the equilibrium response of the economy to real disturbances requiring adjustment in relative goods prices.

The saddle-path solutions for $\tilde{e}$ and $\tilde{p}$ imply that a change in p^*, given the paths of r^* and $\dot{p}^*$, is exactly offset by an equal and opposite change in e, so that p and q are unaffected. A flexible exchange rate therefore insulates the domestic economy against this disturbance. Similarly, the domestic economy is completely insulated against a change in the foreign inflation rate $\dot{p}^*$ if the foreign nominal interest rate fully reflects this change.[6]

The domestic economy is not insulated against foreign real disturbances, however. A change in the foreign real interest rate $r^* - \dot{p}^*$ affects both p and q as well as e. A permanent, unanticipated increase in r^* alone, for example, causes the terms of trade to change by the amount $-\sigma/\phi$. This fall in the terms of trade is accommodated partly through a depreciation of the domestic currency ($d\tilde{e}/dr^* = \lambda + (\alpha\sigma/\phi)$) and partly through a change in the nominal price of the domestic good ($d\tilde{p}/dr^* = \lambda - [(1 - \alpha)\ \sigma/\phi]$).

2.2 Anticipated Future Disturbances

Although the model just described has no intrinsic dynamics, prices will move over in time in response to anticipated changes in exogenous variables. Only when the exogenous variables are expected to be constant forever will e and p be constant as well. To illustrate these extrinsic dynamics we consider the economy's response to a permanent increase in the money supply that is announced to the public in advance of its occurrence. The exercise (similar to those performed by Sargent and Wallace, 1973, and Brock, 1975, for closed economies) yields some important insights into exchange-rate and price behavior under rational expectations.

To fix ideas, it is assumed that, prior to the announcement at time t=0 that the money stock will increase by an amount Δm at time $t = T > 0$, the money stock was expected to remain constant at level $\bar{m}$ forever. If we add the assumption that all other exogenous variables remain fixed throughout, then the economy is in a stationary state, prior to t=0, with $\tilde{e} = \bar{x} - \alpha\bar{z}/\phi - \bar{p}^*$ and $\tilde{p} = \bar{x} + (1-\alpha)\bar{z}/\phi$. Here, $\bar{x}$ and $\bar{z}$ are given by (2.8) and (2.9) evaluated at the constant levels of the exogenous variables (with $\dot{p}^* = 0$) and $\bar{p}^*$ is the fixed foreign-goods price. The announcement of the future policy action causes the exchange rate and domestic-goods price to jump upward immediately. The path of the economy from t=0 onward is described by

$$(2.13) \quad \tilde{e}_t = \begin{cases} \bar{x} + \exp\left[(t-T)/\lambda\right] \Delta m - \alpha\bar{z}/\phi - \bar{p}^* & (0 \leq t < T) \\ \bar{x} + \Delta m - \alpha\bar{z}/\phi - \bar{p}^* & (t \geq T), \end{cases}$$

$$(2.14) \quad \tilde{p}_t = \begin{cases} \bar{x} + \exp\left[(t-T)/\lambda\right] \Delta m + (1-\alpha)\bar{z}/\phi & (0 \leq t < T) \\ \bar{x} + \Delta m + (1-\alpha)\bar{z}/\phi & (t \geq T). \end{cases}$$

Both e and p jump by $\exp(-T/\lambda)\Delta m$ at t=0 and then rise smoothly until t=T, when their new stationary values are attained. The terms of trade are at no time affected by this purely monetary disturbance.

It is important to note that neither the exchange rate nor the price of domestic output jumps when the money stock jumps at time t=T. Between times 0 and

T, the exchange rate depreciates at an accelerating pace, inducing a rising nominal interest rate that maintains money-market equilibrium as the price level rises and real balances shrink. At time T, when the money supply is increased, the depreciation rate drops to zero and the home interest rate drops back to the constant world level r^*. The interest rate's fall creates an increase in real money demand just equal to the increase in the real money supply at the current price level, removing any need for a discrete jump in e or p at that moment.

The economic explanation of this result is fundamental. The jump in prices at time t=0 discounts to the present the expected future monetary expansion and thereby eliminates the possibility of an anticipated discrete jump in the exchange rate (and the price of domestic goods) along the economy's subsequent path. To see how the market discounts expected future events in this manner, suppose that a sharp jump in the exchange rate could occur at time T. Because such a jump would imply an instantaneously infinite real rate of capital loss on domestic-currency assets, investors would have an incentive to move into foreign exchange an instant before time T, causing e (and p) to jump earlier than expected. The contradiction is removed if prices jump only at time t=0, when the news of the future disturbance first arrives.[7]

Another implication of the rational-expectations assumption deserves emphasis. In a world where prices move in anticipation of future events, one may not observe any clear correlation between exchange-rate movements and contemporaneous movements in the exogenous determinants of the exchange rate. The example given above shows that prices rise in advance of an expected money-supply increase; if the increase is permanent, prices and money will have risen proportionally by the time the money stock rises. There is an additional problem in a world with uncertainty: the exchange rate may react sharply to heightened probabilities of future policies which do not actually materialize, even though it was reasonable *ex ante* to expect that they might. Partly for this reason, attempts to explain historical exchange-rate movements in terms of observable "fundamentals" have often been unconvincing.

It is instructive to examine the features of the model that produce the dynamics described above. The two key parameters that generate current responses to anticipated future disturbances are the interest-elasticity of the demand for money (λ) and the interest-elasticity of aggregate demand for domestic output (σ). It is easy to see that if both λ and σ are zero, then neither e nor p responds to anticipated future changes in any exogenous variable until those changes actually occur.[8] The absence of any interest-rate sensitivity severs the link between the present and the future in this section's model.

If the interest-elasticity of money demand is negative ($\lambda > 0$) but $\sigma = 0$, the model can be written as a single differential equation in e, together with an equation for p:

$$(2.15) \quad \dot{e}_t = \frac{(e_t + p_t^*)}{\lambda} - \frac{1}{\lambda}(x_t - \alpha z_t/\phi) - \dot{p}_t^* ,$$

$$(2.16) \quad \tilde{p}_t = \tilde{e}_t + p_t^* + z_t/\phi .$$

Here, z is given by (2.9) with $\sigma = 0$. Now anticipated future changes in m, $\bar{y}$, g, or $r^* - \dot{p}^*$ affect the current exchange rate and home-output price, since the saddle-path solution to (2.15) is

$$(2.17) \quad \tilde{e}_t = \frac{1}{\lambda}\int_t^{\infty}(x_s - \alpha z_s/\phi)\exp\left[(t-s)/\lambda\right]ds - p_t^*.$$

However, it is obvious from (2.9) and (2.16) that the terms of trade $\tilde{q} = (g-\bar{y})/\phi$ are unaffected by these anticipated future changes (given the current values of $\bar{y}$ and g). Thus, while e cannot make anticipated discrete jumps, p and q can. Anticipated discrete terms-of-trade jumps imply instantaneously infinite expected domestic real interest rates. But when $\sigma = 0$, there is no incentive for the market to smooth such jumps through intertemporal substitution in consumption.

Although current real variables are unaffected by changes in anticipations of future variables in this case, current nominal variables are affected by changes in anticipations of both future nominal and future real variables. For example, the exchange-rate effect of an increase in anticipated future domestic output is ambiguous and works through two channels. First, by increasing the future demand for money and reducing the future price level and exchange rate, it increases the current demand for money and reduces the current price level and exchange rate. This effect is captured by the term x_s in (2.17). Second, a future increase in output reduces future q, partly through a rise in e. The expected fall in q is larger the smaller is ϕ, and the fraction of the fall in q that occurs as an increase in e is larger the larger is α. This second factor tends to raise the current price level and exchange rate by reducing current money demand. Its effect is captured by the term z_s in (2.17).

If the interest elasticity of aggregate demand is negative ($\sigma>0$) but $\lambda=0$, then the model becomes:

$$(2.18) \qquad \dot{q}_t = \omega q_t + [\bar{y}_t - g_t + \sigma(r_t^* - \dot{p}_t^*)]/\sigma\gamma,$$

$$(2.19) \qquad \tilde{p}_t = m_t + (1-\alpha)\tilde{q}_t - \psi\bar{y}_t.$$

[Equation (2.19) is an open-economy analogue of the monetary equilibrium condition postulated in the classical quantity theory of money.] The saddle-path solution to (2.18) is equation (2.12); the exchange rate is given by

$$(2.20) \qquad \tilde{e}_t = m_t - \alpha\tilde{q}_t - \psi\bar{y}_t - p_t^*.$$

With intertemporal substitution in the goods market ($\sigma>0$), the current terms of trade are naturally a function of future values of real variables. Discrete anticipated jumps in q are not possible. But while the exchange rate is a function of current and future values of the real variables in the model, it is not a function of future money supplies. When $\lambda = 0$, expected sharp jumps in e (and p) clearly can occur.

The influence of future real variables on e is greater for greater values of α, while the effect of future variables on p approaches zero as γ (and hence α) approaches one. Values of γ close to one are thus consistent with the idea that changes in expected future real variables have a large effect on the current exchange rate but a small effect on the current nominal price of domestic goods. (In a sense, $\gamma = 1$ means that both the exchange rate and terms of trade behave as "asset prices" -- they depend on a whole time path of future real variables -- while the nominal price of domestic goods does not.)

2.3 Expected Regime Change and Exchange Rate-Dynamics

The discussion has so far neglected the possibility of drastic institutional or structural changes in the economy. This section is concerned with the influence of expected regime change on exchange-rate behavior. The problems involved are illustrated by the example of an anticipated future return to a fixed exchange rate. Future exchange-rate pegging implies an expected transition from a regime in which the money supply is exogenous and the exchange rate is endogenous to one in which the money supply is endogenously determined.

Models involving regime change have their roots in papers by Salant and Henderson (1978) and Salant (1983) describing the breakdown of government price-fixing schemes in natural resource markets. Salant and Henderson showed that under rational expectations, the timing of speculative attacks on government resource stockpiles can be uniquely determined by the familiar requirement (cf. section 2.2) that the resource price not make an anticipated discrete jump as speculators acquire the government's reserves. Krugman (1979) extended their analysis to the foreign exchange market, demonstrating that the date at which a fixed exchange rate collapses in a sudden balance-of-payments crisis is also well defined in terms of official policies and private preferences. Flood and Garber (1983) use a stochastic model to study the problem that concerns us in this section, the influence of expected future fixing on the behavior of a currently floating exchange rate.[9]

A return to fixed exchange rates is analyzed in two steps. First, we ask what the value of the exchange rate must be just after the regime change occurs. Second, we determine the extent to which the current exchange rate must discount the expected future event if there is to be no sharp jump in prices at the moment the event takes place.

Suppose that at time $t = 0$ the monetary authority announces its intention of fixing the exchange rate permanently at a time $t = T$ in the future. The level at which the exchange rate is to be pegged is denoted by $\bar{e}$. To focus on the effect of the announcement itself, we assume that the entire future path of g and the path of the money supply between times 0 and T are unaffected by the announcement.[10]

The analysis proceeds by deriving the equilibrium that will prevail under a fixed exchange rate regime and then "working backward" to time $t = 0$. When the exchange rate is fixed at $\bar{e}$, goods-market equilibrium can be written

$$(2.21)\qquad \bar{y}_t = \phi(\bar{e} + p_t^* - p_t) - \sigma[r_t^* - \gamma\dot{p}_t - (1-\gamma)\dot{p}_t^*] + g_t ,$$

where we have made use of the fact that $r = r^*$ when no change in the exchange rate is expected. The saddle-path solution to differential equation (2.21) is

$$(2.22)\qquad p_t^f = \bar{e} + p_t^* + \omega\int_t^{\infty}\exp[\omega(t-s)](z_s/\phi)ds.$$

An "f" superscript denotes a variable's equilibrium value under a fixed-rate regime.

The domestic money supply becomes an endogenous, jumping variable under fixed rates and capital mobility. Equations (2.3) and (2.22) imply that equilibrium nominal balances are given by

$$(2.23) \qquad m_t^f = \alpha p_t^f + (1 - \alpha)\,(\bar{e} + p_t^*) + \psi\bar{y}_t - \lambda r_t^*$$

$$= \bar{e} + p_t^* + \alpha\omega\int_t^{\infty} \exp\left[\omega(t-s)\right]\,(z_s/\phi)ds + \psi\bar{y}_t - \lambda r_t^*.$$

Equation (2.22) relates the equilibrium output price to the exchange rate, to the world price level, and to current and expected values of variables that disturb the terms of trade. Monetary factors play no role: changes in money demand or supply are accommodated or offset through the capital account. According to (2.23), devaluation is neutral in the present setting. A rise in $\bar{e}$ leads to equiproportionate increases in the home-goods price and the nominal money stock, but has no real effects.

Even though the exchange rate is to be pegged at time T, the behavior of prices during the interval between times 0 and T is governed by equations (2.6) and (2.7). Because these equations do not apply after T, it is convenient to write a general solution to the implied differential-equation system as

$$(2.24) \qquad e_t = k_1'\exp\,(t/\lambda) - \frac{k_2'\alpha\exp(\omega t)}{1-\alpha-\lambda\omega} + \frac{1}{\lambda}\int_t^T \exp\left[(t-s)/\lambda\right]x_s ds$$

$$+ \frac{\alpha\omega}{1-\lambda\omega}\int_t^T \left\{\exp\left[(t-s)/\lambda\right] - \exp\left[\omega(t-s)\right]\right\}(z_s/\phi)ds - p_t^*,$$

$$(2.25) \qquad p_t = k_1'\exp(t/\lambda) + k_2'\exp(\omega t) + \frac{1}{\lambda}\int_t^T \exp\left[(t-s)/\lambda\right]x_s ds$$

$$+ \frac{\alpha\omega}{1-\lambda\omega}\int_t^T \exp\left[(t-s)/\lambda\right](z_s/\phi)ds + \frac{\omega(1-\alpha-\lambda\omega)}{1-\lambda\omega}\int_t^T \exp\left[\omega(t-s)\right](z_s/\phi)ds\ ,$$

$(0 \leq t < T)$, where k_1' and k_2' are arbitrary constants. (It is assumed that $\lambda \neq 0$, $\sigma \neq 0$.) To trace out the economy's path over the time interval $(0,T)$, we must determine appropriate values for k_1' and k_2' .

These values are determined by the requirements that the exchange rate and domestic-goods price not jump discretely at time T. In other words, the system's initial conditions must result in a path $\{\tilde{e}_t', \tilde{p}_t'\}_{t=0}^{T}$ for prices such that $\tilde{e}_T' = \bar{e}$ and $\tilde{p}_T' = p_T^f$, where the latter price is given by equation (2.22). Setting t = T in equations (2.24) and (2.25), we find that this continuity condition implies

$$(2.26) \quad k_1' = \{\bar{e} + p_T^* + \frac{\alpha\omega}{1 - \lambda\omega} \int_T^\infty \exp[\omega(T-s)]\ (z_s/\phi)ds\}\exp(-T/\lambda) \ ,$$

$$(2.27) \quad k_2' = \frac{\omega(1-\alpha - \lambda\omega)}{1 - \lambda\omega} \int_T^\infty \exp(-\omega s)\ (z_s/\phi)ds \ .$$

Substitution of (2.26) and (2.27) into (2.24) and (2.25) yields the equilibrium prices prevailing between dates 0 and T.

It was assumed above that the only changes accompanying the announcement of future pegging are changes in the monetary policy pursued after the exchange rate is fixed. That assumption yields a compact and revealing representation of the effect of future pegging. Let $\{\tilde{e}_t, \tilde{p}_t\}_{t=0}^{\infty}$ denote the price path that would have prevailed <u>in the absence</u> of any move to fix the exchange rate; $\tilde{e}$ and $\tilde{p}$ are given by equations (2.10) and (2.11) with $k_1 = k_2 = 0$. The initial conditions (2.26) and (2.27) imply that the paths of the exchange rate and the domestic-goods price between time 0 and T can be written in the form

$$(2.28) \quad \tilde{e}_t' - \tilde{e}_t = (\bar{e} - \tilde{e}_T)\exp[(t-T)/\lambda] \ ,$$

$$(2.29) \quad \tilde{p}_t' - \tilde{p}_t = (\bar{e} - \tilde{e}_T)\exp[(t-T)/\lambda] \ .$$

The foregoing expressions make clear that the change in the exchange rate's path (relative to its unperturbed level) depends on the relation between the new peg $\bar{e}$ and the exchange rate that would have prevailed at time T in the absence of the regime change. If $\bar{e} < \tilde{e}_T$, for example, the announcement of future pegging leads to an immediate appreciation of the currency relative to its previously anticipated path; $\tilde{e}'$ remains below $\tilde{e}$ for the balance of the floating-rate period, and the divergence between the two grows exponentially at rate $1/\lambda$. As was assumed in the solution procedure, $\tilde{e}'$ reaches $\bar{e}$ at time T, so that pegging can take place with no discrete movement of the exchange rate. Figure 2.1 illustrates the path just described.

The evolution of the domestic goods price relative to its predisturbance path is identical to that of the exchange rate. This is not surprising: because the regime change is a change only in the process determining nominal magnitudes, it has no effect on the path of the terms of trade. It is noteworthy that as $T \to \infty$, the effect of future pegging on the economy's path becomes progressively smaller. Further, if the exchange rate is to be pegged at time T at the value that would have materialized in the absence of a regime change, the announcement does not alter the economy's path between times 0 and T in any way.

As figure 2.1 suggests, pegging generally entails a change in the currency's depreciation rate, and hence a change in the domestic nominal interest rate. Because the real-balance deflator cannot jump at time T, a discrete change in the demand for real balances is implied. How is this change in demand accommodated when no jump in the price level is possible? To peg the exchange rate at time T, the central bank intervenes in the foreign exchange market: it sells domestic money and buys foreign reserves if pegging results in a rise in money demand, but buys money and loses reserves in the opposite case. Central-bank intervention thus facilitates the private-sector portfolio shift that may be necessary to maintain continuous money-market equilibrium. If the nominal interest rate falls at the instant of pegging, there is a momentary capital inflow, and if it rises, there is a capital outflow.

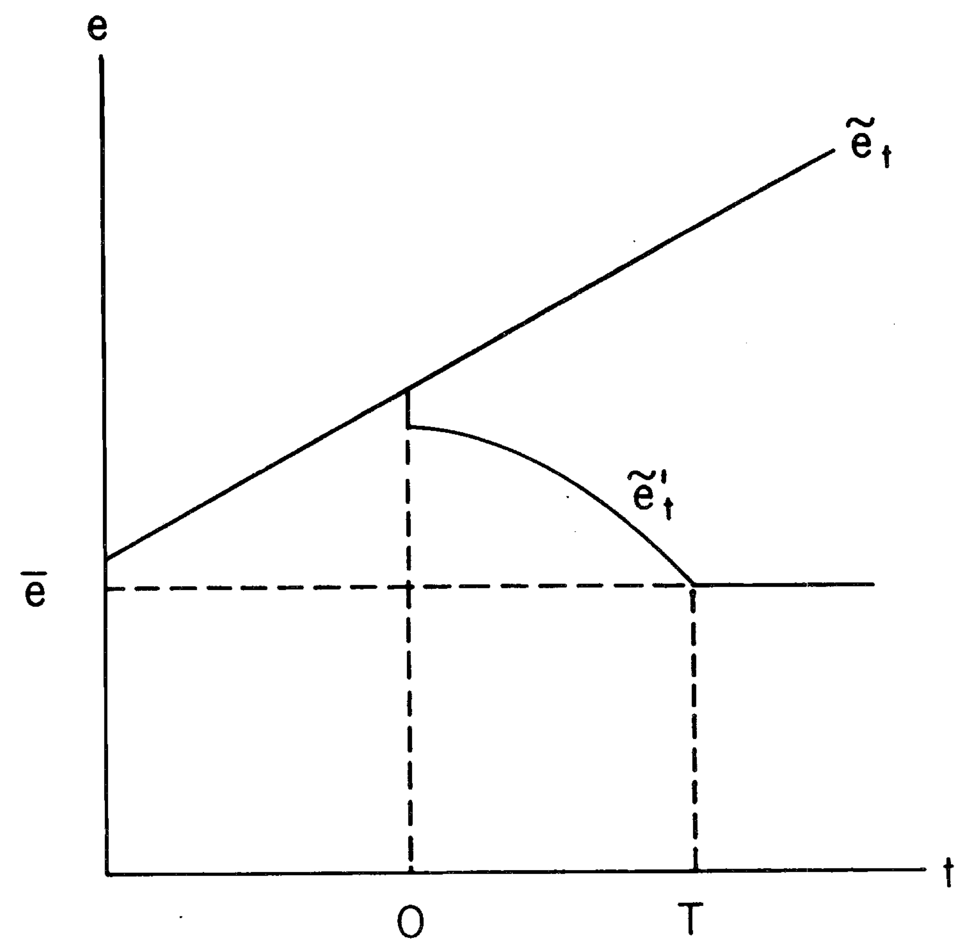


Figure 2.1

3. Market Frictions and Output Fluctuations

The assumptions of perfect price flexibility and full information are now relaxed so that we may study how market frictions influence the exchange rate's dynamic response to official policy shifts and other exogenous disturbances. We first explore a stylized "sticky-price" model in which the nominal price of domestic ouput is constrained to be a slowly-adjusting variable. Then we consider more detailed models in which pre-negotiated contracts, imperfect information, or both lie behind the tendency of goods prices to appear sluggish when compared to the exchange rate.

Models of exchange rate dynamics with sticky prices are direct descendants of the open-economy IS-LM models developed by Fleming (1962) and Mundell (1968). This type of model is studied in section 3.1, below. The Mundell-Fleming approach begins with a Keynesian economy characterized by rigid domestic prices and demand-determined output; that economy is "opened" by introducing international trade and capital movements. Shocks to the goods and asset markets lead to once-and-for-all adjustments of the exchange rate, rather than to a dynamic process of macroeconomic adjustment. These equilibrating exchange-rate movements are in fact terms-of-trade changes which are maintained indefinitely even when the initial shock is monetary.

The static Mundell-Fleming model of exchange-rate determination proved inadequate as an analytical tool in the inflationary environment of the 1970's. The dynamic Mundell-Fleming models, developed primarily by Dornbusch (1976b) and Mussa (1977, 1982), extended the earlier framework in two important respects. First, while retaining the assumption that the nominal price of domestic output is fixed (i.e., predetermined) at any moment in time, the dynamic models allow that price to adjust <u>over</u> time in response to deviations between aggregate demand and the full-employment level of output. A monetary expansion, for example, induces not only a temporary rise in output and fall in the terms of trade, but also an inflationary process in which the initial expansionary impact is dissipated and purchasing-power parity is restored. Second, the dynamic models endow market participants with rational expectations of exchange rate and price movements.

The foregoing discussion highlights two distinct sources of dynamics: the equilibrium adjustment of prices to current and anticipated future movements in exogenous variables, and the adjustment of prices and quantities as goods-market disequilibrium is eliminated over time. The first source of dynamics is extrinsic, and was contained in the flexible-price model presented in section 2. The second source of dynamics is intrinsic to the sticky-price model.

The sticky-price assumption produces models mimicking the observed tendency of international price-level ratios to exhibit considerably less volatility than the corresponding exchange rates. Moreover, the intrinsic dynamics of such a system imply that monetary and other disturbances result in temporary yet persistent deviations of output, goods prices, and asset prices from the values they would assume in a frictionless equilibrium. But while the sticky-price model is useful as a descriptive tool, it does not analyze the institutional or informational features of the economy that might result in an apparently sluggish price level.

Because the precise source of market frictions is crucial for policy analysis, attention has recently been given to exchange-rate models in which contracts and informational asymmetries give rise to monetary non-neutrality. This type of model is the subject of section 3.2. The policy implications of contracting models are of course very different from those of models based exclusively on imperfect information; however, all the models discussed in section 3.2 predict that monetary shocks (at least when imperfectly perceived) will have persistent, but not permanent, effects on output. As is illustrated below, the intrinsic dynamics of these models can arise from such sources as inventory adjustment, multi-period contracts, and external asset accumulation. Because the last source of dynamics is properly the province of section 4, it is touched on only briefly in this section.

3.1 Sticky Domestic Prices and Overshooting

The sticky-price model retains the continuous asset-market equilibrium that was a feature of section 2, but stipulates that the domestic output price is a predetermined or non-jumping variable that can adjust only over time. Both key features of the sticky-price model--instantaneous asset-market clearing and perfect short-run output-price rigidity--are surely extreme characterizations of actual market adjustment. Nonetheless, these polar extremes yield an analytically tractable model that highlights neatly the dynamic implications of different adjustment speeds between markets. (Niehans, 1977, and Frenkel and Rodriguez, 1982, study models in which some asset markets adjust slowly.) The most celebrated implication of this type of model is Dornbusch's (1976b) finding that when the price of home goods is sticky, the exchange rate may "overshoot" its eventual level in the short run in reponse to a permanent change in the money supply.

To introduce price stickiness into the exchange rate model of section 2, we replace the goods-market equilibrium condition (2.5) with the assumption that domestic output y is identically equal to aggregate demand, d. Demand-determined output might be the result of pre-negotiated nominal wage contracts which require workers to supply all the labor demanded by firms at the contract wage. However, the labor market is not modeled explicitly (see section 3.2, below). It is assumed that the price of domestic goods moves upward over time in response to both the excess of output y over its natural level $\bar{y}$ and a measure of "equilibrium" inflationary expectations. The expectational component of the price-adjustment rule is crucial. A rule omitting this component is analogous to a pre-Phelps-Friedman Phillips curve, and, as Mussa (1982) observes, yields a model in which constant monetary growth is inconsistent with an inflationary steady state unless output remains perpetually above its natural level.

To obtain the expectations term in the pricing rule, we define a price $\bar{\bar{p}}$ by

$$(3.1) \qquad \bar{y}_t = \phi(e_t + p_t^* - \bar{\bar{p}}_t) - \sigma[r_t - \gamma\dot{p}_t - (1-\gamma)(\dot{e}_t + \dot{p}_t^*)] + g_t \ .$$

Note that $\bar{\bar{p}}$ equates aggregate demand to the natural output level for current values of the other variables. We then assume the price-adjustment scheme postulated by Mussa (1977, 1982):

$$(3.2) \qquad \dot{p}_t = \theta(y_t - \bar{y}_t) + \dot{\bar{\bar{p}}}_t = \theta(d_t - \bar{y}_t) + \dot{\bar{\bar{p}}}_t \ .$$

According to (3.2), producers adjust prices to reduce excess demand and to ensure that prices "keep up" with changes in their current equilibrium level.[11]

The model is described by (2.1) - (2.4), (3.1), and (3.2). We solve the model by steps (as in Obstfeld and Rogoff, 1984), first finding the equilibrium terms of trade and then using that solution to find the equilibrium exchange rate and home-goods price. To this end, it is convenient to formulate the model in terms of deviations from the flexible-price equilibrium studied in section 2; thus, we define $\hat{q} \equiv q - \tilde{q}$, $\hat{e} \equiv e - \tilde{e}$, and $\hat{p} \equiv p - \tilde{p}$.

By (2.18), the flexible-price terms of trade $\tilde{q}$ obey the equation

$$(3.3) \qquad \dot{\tilde{q}}_t = \omega\tilde{q}_t + [\bar{y}_t - g_t + \sigma(r_t^* - \dot{p}_t^*)]/\sigma\gamma \ .$$

Differentiating (3.1) and (3.3) with respect to time and using (2.1), (3.2), and (3.3), we obtain an autonomous second-order differential equation in $\hat{q}$:

$$(3.4) \qquad \ddot{\hat{q}}_t = \omega(1-\theta\sigma\gamma)\dot{\hat{q}}_t + \omega\phi\theta\hat{q}_t \ .$$

A general solution to (3.4) is $\hat{q}_t = k_1 \exp(\omega t) + k_2 \exp(-\theta\phi t)$, where k_1 and k_2 are constants. Saddle-path equilibrium again requires that the coefficient k_1 of the explosive exponential be set at zero. But (3.4) possesses a negative root, $-\theta\phi$, associated with the predetermined nominal price of domestic goods. (The negative sign of this root reflects the stabilizing effect of excess demand on prices.) Because the initial equilibrium terms of trade q_0 will not generally coincide with the flexible-price value $\tilde{q}_0$ in this sticky-price model, the additional initial condition $k_2 = q_0 - \tilde{q}_0$ is required to obtain the saddle-path solution for q,

$$(3.5) \qquad q_t = (q_0 - \tilde{q}_0)\exp(-\theta\phi t) + \tilde{q}_t \, .$$

As (3.5) shows, the price-adjustment scheme (3.2) drives the real exchange rate toward its flexible-price value at an exponential rate.

Using the fact that $\tilde{e}$ and $\tilde{p}$ must satisfy (2.1) - (2.4), we derive the equation

$$(3.6) \qquad \dot{\hat{e}}_t = (\tfrac{1}{\lambda})\hat{e}_t - (\tfrac{\psi\phi - \alpha}{\lambda})\hat{q}_t + (\tfrac{\psi\sigma\gamma}{\lambda})\dot{\hat{q}}_t \, .$$

Differentiation of (3.5) yields

$$(3.7) \qquad \dot{\hat{q}}_t = -\theta\phi\hat{q}_t \, ,$$

which, when combined with (3.6), implies that

$$(3.8) \qquad \dot{\hat{e}}_t = (\tfrac{1}{\lambda})\hat{e}_t + [\tfrac{\alpha - \psi\phi(1 + \theta\sigma\gamma)}{\lambda}]\hat{q}_t \, .$$

Equations (3.7) and (3.8) constitute an autonomous system in $\hat{e}$ and $\hat{q}$. The saddlepath solution for $\hat{e}$ leads to the expression

$$(3.9) \qquad e_t = \frac{-(q_0 - \tilde{q}_0)[\alpha - \psi\phi(1 + \theta\sigma\gamma)]}{(1 + \lambda\theta\phi)}\exp(-\theta\phi t) + \tilde{e}_t$$

$$= \frac{-(p_0 - \tilde{p}_0)[\alpha - \psi\phi(1 + \theta\sigma\gamma)]}{[(1 - \alpha) + \psi\phi(1 + \theta\sigma\gamma) + \lambda\theta\phi]}\exp(-\theta\phi t) + \tilde{e}_t \quad .$$

From (3.5) and (3.9), the path of p is given by

$$(3.10) \qquad p_t = (p_0 - \tilde{p}_0)\exp(-\theta\phi t) + \tilde{p}_t \quad .$$

Expressions (3.9) and (3.10) show that the exchange rate and domestic-goods price will differ from their flexible-price equilibrium values whenever the predetermined initial output price p_0 differs from the value that would prevail in the hypothetical Walrasian equilibrium of the flexible-price model. The adjustment rule (3.2) drives the discrepancy $p - \tilde{p}$ to zero at rate $\theta\phi$. According to (3.9), e converges to $\tilde{e}$ at that rate.

As in the flexible price model of section 2, current and anticipated changes in exogenous variables contribute to the system's dynamics. They do so by altering $\tilde{e}$ and $\tilde{p}$ over time, thus changing the long-run equilibrium toward which the economy converges. The process of convergence, however, is a second, intrinsic component of dynamics. Differentiation of (3.9) and (3.10) yields

$$\dot{e}_t = -\theta\phi(e_t - \tilde{e}_t) + \dot{\tilde{e}}_t \ , \tag{3.11}$$

$$\dot{p}_t = -\theta\phi(p_t - \tilde{p}_t) + \dot{\tilde{p}}_t \ . \tag{3.12}$$

Equations (3.11) and (3.12) show that the motion of the system is in fact the sum of two sources of motion: The (extrinsic) movement in the system's flexible-price equilibrium and the (intrinsic) adjustment of prices to their current flexible-price values in response to goods-market disequilibrium.

We now consider an unanticipated, permanent shock to the money supply occurring at time t=0, i.e., a shift in the anticipated path of the money supply from $\{m_t\}_{t=0}^{\infty}$ to $\{m_t + \Delta m\}_{t=0}^{\infty}$. To highlight the effects of this shock, we suppose that the economy is in full equilibrium before it occurs, with $p_0 = \tilde{p}_0$. As the analysis of the flexible-price model showed, both $\tilde{e}_0$ and $\tilde{p}_0$ jump immediately by the amount Δm; indeed, the paths of $\tilde{e}$ and $\tilde{p}$ jump uniformly by that amount. Because the price of domestic output is predetermined, however, it remains temporarily at p_0. As (3.12) shows, the divergence between p_0 and $\tilde{p}_0$ raises the rate of domestic price inflation.

An interesting aspect of the exchange rate's response to a monetary shock is the possibility that it may "overshoot" its new flexible-price or full equilibrium level.[12] From (3.9), the initial depreciation Δe_0 exceeds or falls short of $\Delta\tilde{e}_0$ as

$$\alpha - \psi\phi(1 + \theta\sigma\gamma) \gtrless 0 \ . \tag{3.13}$$

Intuitively, overshooting arises as follows. Because domestic prices are predetermined, the initial depreciation of the currency is a real depreciation that shifts demand from foreign toward domestic goods. Aggregate demand is stimulated further through a fall in the real interest rate, so output unambiguously rises. The concomitant increase in the demand for money reduces the initial excess supply occasioned by the monetary expansion, as does the rise in the overall price level implied by the currency's depreciation. But if an excess supply of money remains after these adjustments, the nominal domestic interest rate must fall to preserve asset-market equilibrium. Since a permanent increase in the level of the money supply does not affect the depreciation rate in the flexible-price model, (2.1) and (3.11) imply that the home interest rate can fall only if the currency depreciates so far on impact that it is expected to appreciate thereafter toward its flexible-price value. In contrast, if the increase in output raises money demand sufficiently to produce excess demand at the initial nominal interest rate, then overshooting will not occur. The nominal interest rate must rise to clear the money market in this case, and the exchange rate will be expected to depreciate thereafter toward its long-run level $\tilde{e}$. This is the "undershooting" case, in which the impact depreciation of the currency falls short of the depreciation that would take place in a flexible-price economy.

The adjustment of the system after the shock at time $t = 0$ is determined endogenously through the workings of the price-adjustment rule (3.2). p rises gradually toward $\tilde{p}$ and e may fall or rise toward $\tilde{e}$ depending on whether over- or undershooting has occurred. The real rate of interest rises over time as the terms of trade return to the initial level and output falls. Monetary policy is neutral in the long run but sluggish price adjustment gives it the power to alter output and relative prices in the short run. In the sticky-price setting, therefore, deviations from purchasing power parity may result both from real disturbances and, temporarily, from monetary shocks.

We now consider an anticipated permanent increase in the future money stock announced at time $t = 0$. It is assumed, as in section 2.2, that the money supply, previously expected to be constant at $\bar{m}$, is now expected to increase by the amount Δm at $t = T$. All else remains fixed. If the economy is initially at long-run equilibrium with $p_0 = \tilde{p}_0$, (3.9) and (3.10) imply that its subsequent path is

$$(3.14)\qquad e_t = \frac{[\alpha - \psi\phi(1 + \theta\sigma\gamma)]\exp(-T/\lambda)\Delta m}{[(1 - \alpha) + \psi\phi(1 + \theta\sigma\gamma) + \lambda\theta\phi]}\exp(-\theta\phi t)$$

$$+ \begin{cases} \bar{x} + \exp[(t-T)/\lambda]\Delta m - \alpha\bar{z}/\phi - \bar{p}^* & (0 \leq t < T) \\ \bar{x} + \Delta m - \alpha\bar{z}/\phi - \bar{p}^* & (t \geq T)\ , \end{cases}$$

$$(3.15)\qquad p_t = -\exp(-T/\lambda)\Delta m \cdot \exp(-\theta\phi t)$$

$$+ \begin{cases} \bar{x} + \exp[(t-T)/\lambda]\Delta m + (1 - \alpha)\bar{z}/\phi & (0 \leq t < T) \\ \bar{x} + \Delta m + (1 - \alpha)\bar{z}/\phi & (t \geq T)\ . \end{cases}$$

The announcement causes both the exchange rate and nominal interest rate to jump upward, with the impact depreciation smaller than in the case of an unanticipated occurrence of the same shock. The domestic-goods price is sticky and cannot jump in response to the announcement, but it begins to rise gradually in response to a rise in output and inflationary expectations. As in the flexible-price model, neither the exchange rate nor the price of domestic goods jumps at time T when the anticipated increase in money occurs. Rather, the nominal interest rate falls at T as the rate of currency depreciation falls. In the overshooting case, the exchange rate will abruptly begin to appreciate at T, but without a discrete change in its level.

Note that goods-market disequilibrium remains even after the increase in money has taken place. Because the price level is sluggish, nominal prices cannot adjust fully to the anticipated disturbance by time T, as they did in the flexible-price case. Therefore, the real effects of anticipated money persist until the flexible-price equilibrium is asymptotically attained.

A very useful diagrammatic rendition of the model is possible under the assumption that x, z, and p^* are expected to remain constant at $\bar{x}$, $\bar{z}$, and $\bar{p}^*$ forever. Because $\tilde{e} = \bar{x} - \alpha\bar{z}/\phi - \bar{p}^*$ and $\tilde{p} = \bar{x} + (1-\alpha)\bar{z}/\phi$ are then constant, the diagram highlights the intrinsic dynamics of the system. As we shall see, however, it is also possible to use the diagram to analyze the effects of anticipated shocks. (An alternative diagram, in which real balances and the terms of trade appear on the axes, is sometimes used. See, e.g., Buiter and Miller, 1982.)

Equations (3.6) and (3.7) may be transformed to yield

$$(3.16)\qquad \dot{e}_t = \frac{[(1-\alpha) + \psi\phi(1 + \theta\sigma\gamma)]}{\lambda}(e_t - \tilde{e}) + \frac{[\alpha - \psi\phi(1 + \theta\sigma\gamma)]}{\lambda}(p_t - \tilde{p}) ,$$

$$(3.17)\qquad \dot{p}_t = \frac{[(1-\alpha) + \psi\phi(1 + \theta\sigma\gamma) + \lambda\theta\phi]}{\lambda}(e_t - \tilde{e}) + \frac{[\alpha - \psi\phi(1 + \theta\sigma\gamma) - \lambda\theta\phi]}{\lambda}(p_t - \tilde{p}).$$

The phase diagram for this system in the case

$$(3.18)\qquad 0 < \alpha - \psi\phi(1 + \theta\sigma\gamma) < \lambda\theta\phi$$

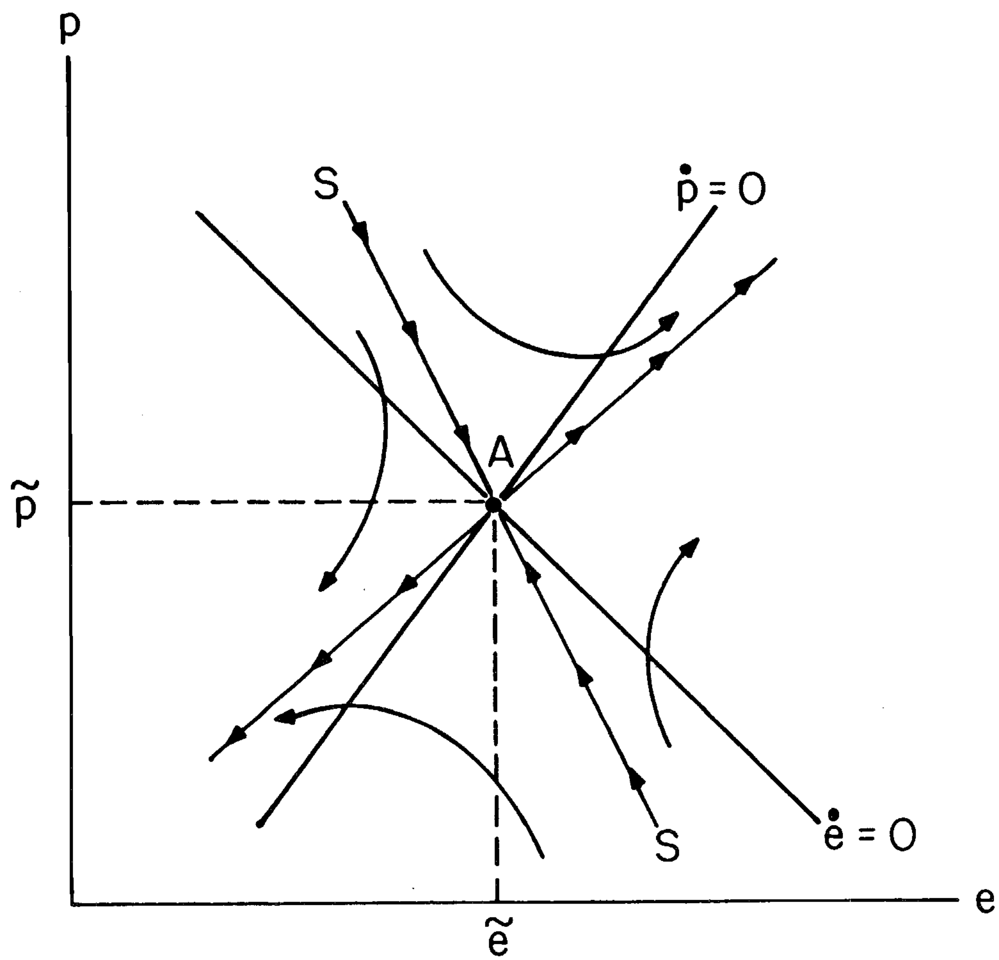
is shown in figure 3.1.[13] The long-run equilibrium A is a saddle point. A unique path SS converges to that stationary position, and all other paths diverge explosively.

To interpret the diagram, consider the general solution to (3.16) and (3.17):

$$(3.19)\qquad e_t - \tilde{e} = k_1\exp(t/\lambda) - \frac{k_2[\alpha - \psi\phi(1 + \theta\sigma\gamma)]}{[(1 - \alpha) + \psi\phi(1 + \theta\sigma\gamma) + \lambda\theta\phi]}\exp(-\theta\phi t),$$

$$(3.20)\qquad p_t - \tilde{p} = k_1\exp(t/\lambda) + k_2\exp(-\theta\phi t) .$$

The trajectory emanating from any point in the phase plane can be obtained by appropriate choice of the arbitrary initial conditions k_1 and k_2. Because the exponential $\exp(t/\lambda)$ induces explosive behavior, initial conditions with $k_1 \neq 0$ necessarily place the economy on one of the divergent paths. The saddle-path condition $k_1 = 0$, in contrast, places the economy on the convergent trajectory SS. Its position on that path is determined by the time that has elapsed since the



$$0<\alpha-\psi\phi(1+\theta\sigma\gamma)<\lambda\theta\phi$$

Figure 3.1

last unanticipated shock and by the additional initial condition $k_2 = p_0 - \tilde{p}$. By setting $k_1 = 0$ in (3.19) and (3.20), we see that the equation for the saddle path SS is

$$(3.21) \qquad p_t - \tilde{p} = \frac{-[(1-\alpha) + \psi\phi(1 + \theta\sigma\gamma) + \lambda\theta\phi]}{[\alpha - \psi\phi(1 + \theta\sigma\gamma)]}(e_t - \tilde{e}) .$$

SS slopes downward or upward according to condition (3.13).

Figure 3.2 illustrates the effects of a permanent, unanticipated fiscal expansion when (3.18) holds. Initially, the system is at the long-run equilibrium A. The shift in policy moves the system's long-run equilibrium to the new point A′ which, as (3.21) shows, is to the left of the original saddle path SS. Because the output price p cannot jump, the instantaneous post-disturbance equilibrium is at the point B′on the new convergent path S′S′. Thus, the currency appreciates, and the terms of trade and output rise. Because the currency is expected to appreciate further, the nominal interest rate falls, and because the terms of trade are expected to rise, the real interest rate falls. Over time, expectations are fulfilled: as the economy moves along S′S′toward its new stationary position, the domestic currency appreciates, the price of the home good rises, and output and interest rates return to their pre-disturbance levels. Note that while (3.18) implies exchange-rate overshooting in reponse to unanticipated, permanent monetary shocks, the present example shows that it does not imply overshooting in response to all shocks.

Although the diagrammatic analysis is predicated on the assumption that the exogenous variables are expected to remain constant, the apparatus may be rigorously used to study the effects of anticipated future disturbances (see Wilson, 1979; Rogoff, 1979; Gray and Turnovsky, 1979; and Boyer and Hodrick, 1982a). Consider again an anticipated, permanent increase Δm in the money supply. The long-run equilibrium after the increase in money is point A′in figure 3.3. The path of the economy from the moment of the announement on is described by

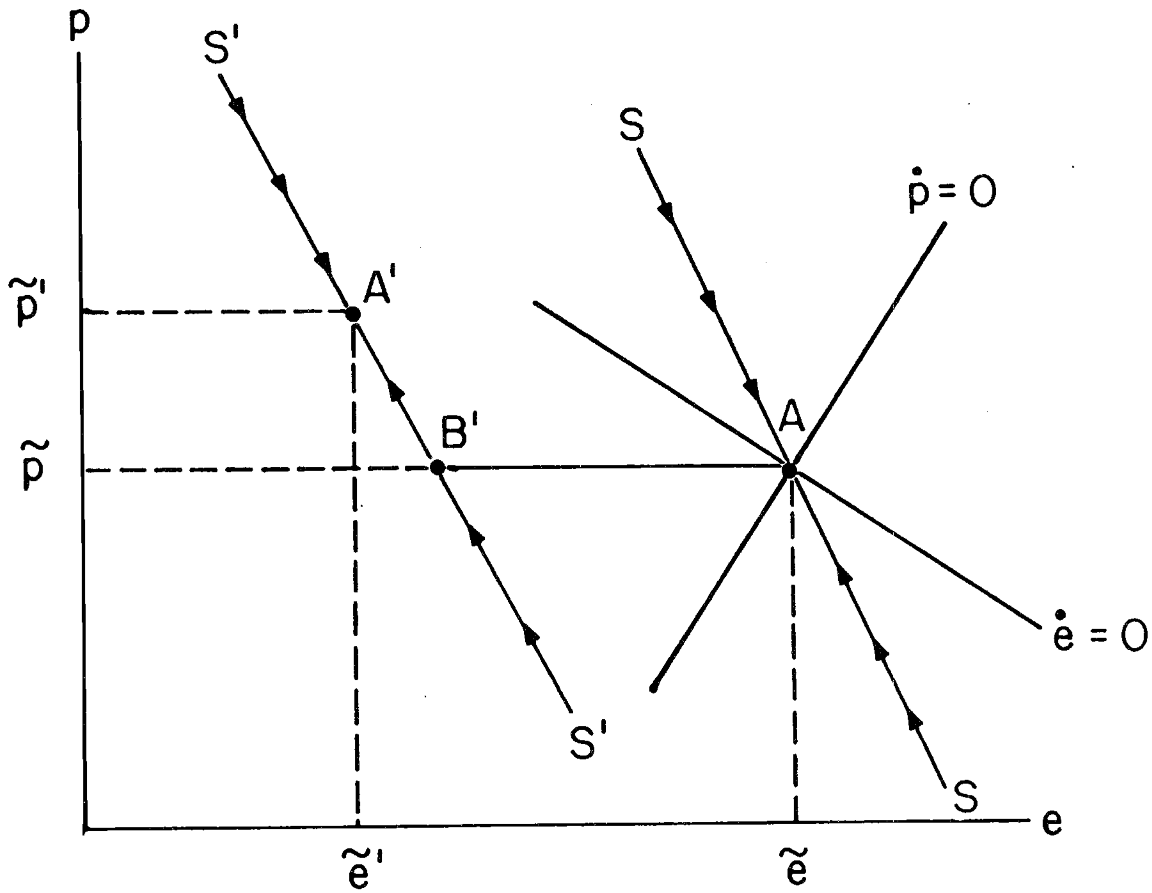


Figure 3.2

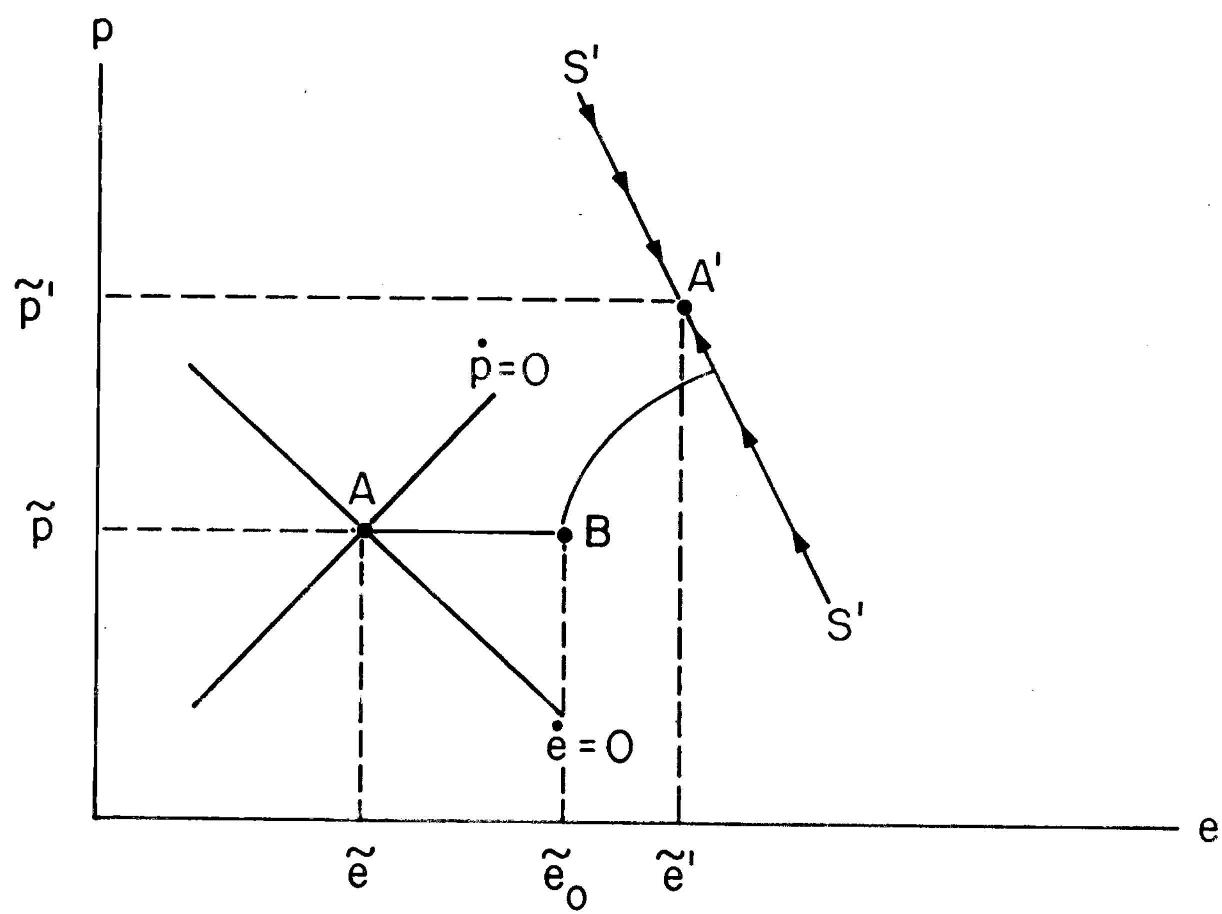


Figure 3.3

(3.14) and (3.15). Between 0 and T, that path corresponds to the particular solution of (3.16) and (3.17) obtained by setting $k_1 = -k_2 = \exp(-T/\lambda)\Delta m$ in (3.19) and (3.20). Thus, the economy's motion over that time interval is described by one of the unstable paths of the system with steady state A, though because (3.14) and (3.15) incorporate the saddle-path assumption, the economy's path is in no sense a "bubble" path. After time T, however, the coefficient of $\exp(t/\lambda)$ in (3.14) and (3.15) is 0, and the economy may therefore be regarded as traveling along the convergent path $S'S'$ toward the long-run equilibrium A'. The additional information that the exchange rate cannot jump discretely at time T leads to the adjustment path depicted in figure 3.3. When the future disturbance is announced, the exchange rate jumps immediately to $\tilde{e}_0$ (at point B), and then rises further to meet the new convergent path $S'S'$ precisely at time T. Only one initial exchange rate $\tilde{e}_0$ places the economy on a trajectory hitting $S'S'$ at time T; $\tilde{e}_0$ is the equilibrium exchange rate because it alone is consistent with the absence of infinitely large expected arbitrage gains along the economy's path.

3.2 Exchange-Rate Dynamics with Contracting and Imperfect Information

The sticky-price assumption of section 3.1 resulted in short-run monetary non-neutrality and an intrinsic component of exchange-rate dynamics. Here we briefly discuss the predictions of some stochastic exchange-rate models in which monetary non-neutrality and intrinsic dynamics arise from other sources. For comparability with the main references on these models, we switch from the continuous time framework of previous sections to discrete time.

There are three channels through which a change in the money supply affects the exchange rate in the models discussed here. First, the exchange rate rises as other nominal variables rise in response to an increase in the money supply. Second, a change in money may affect output or aggregate demand and, by changing the demand for money, affect the exchange rate. Third, a change in money may change the equilibrium terms of trade or relative price of non-traded goods. As noted in section 2, part of such a change will be accommodated by the exchange-rate.

The first two of these channels were present in the sticky-price model of section 3.1, and the first channel always dominates the second: an increase in money raises output (and money demand) only when the exchange rate rises. After its initial increase, the exchange rate falls or rises toward its long-run level depending on the sign of (3.13). Different exchange-rate dynamics are obtained by Flood and Hodrick (1983) in a model with "semi-sticky" prices. In that model, goods prices are determined simultaneously with the exchange rate and are sticky only in the sense of being set prior to the time when sales (which are stochastic) occur and output decisions are made. Unexpectedly high demand is met at the pre-set prices with an increase in output and a fall in inventories of final goods held by firms. An unperceived increase in the money supply causes asset-price movements that lead consumers and firms to the rational but mistaken inference that there has been a decrease in demand for the domestic good. As a result, domestic firms set a nominal price of goods that is consistent, given e and p^*, with a lower terms of trade. Because a fall in demand for the domestic good fails to materialize, the actual quantity demanded at the lower terms of trade is greater than firms had anticipated. Firms respond both by increasing output and reducing inventories.

Unlike in the model of section 3.1, the fall in the terms of trade here is explicitly the result of an optimal choice made by firms with incomplete information. Further, output responds only to monetary changes that are not fully perceived as such by firms. As usual, the fall in the terms of trade occurs partly through a rise in the exchange rate. Although the exchange rate initially rises in the Flood-Hodrick model as in the model of section 3.1, and may overshoot its new long-run level, the subsequent dynamics are very different. The fall in inventories eventually causes an increase in the equilibrium terms of trade as firms re-build their goods stocks; and this increase occurs partly through a fall in the exchange rate. After its initial rise at the time of the monetary shock, the exchange rate therefore falls below its new long-run equilibrium level and then rises monotonically to that level. In the Flood-Hodrick setup, the persistent real effects of monetary shocks reflect the intrinsic dynamics implied by the inventory adjustment process.

The Gray (1976) and Fischer (1977) models of nominal wage contracting are discussed in an open-economy context by Marston in chapter 20, section 5.[14] In the model presented by Marston, the exchange rate initially rises in response to an unanticipated permanent increase in money, and subsequently rises or falls to its new long-run equilibrium. The intrinsic dynamics are similar to those in the sticky-price model of section 3.1, but the speed of adjustment to the new long-run equilibrium now depends on how rapidly wages can move to eliminate labor-market imbalance. The degree of wage inertia is in general a function of contract lengths, indexation provisions, and the extent to which contract periods in different sectors of the economy overlap. Taylor (1980), for example, shows how staggered nominal wage contracts result in a persistent effect of disturbances on output. Burgstaller (1980), Sachs (1980), Obstfeld (1982), and Calvo (1983) study dynamic open-economy models assuming sluggish wage adjustment.

Models developed by Lucas (1975) and Barro (1980) ascribe the short-run nonneutrality of money entirely to incomplete information in decentralized markets: there is no role for pre-set prices. Koh (1982) extends this setup to the open-economy. (See also Saidi, 1980; Harris and Purvis, 1981; Stockman and Koh, 1982; and Kimbrough, 1983.) In contrast to the other models discussed above, Koh's model predicts that the exchange rate may either rise or fall initially in response to an unperceived permanent increase in the money supply. There are several possible post-disturbance adjustment paths that reflect an intrinsic dynamic process powered by external asset accumulation.

To illustrate these results, we consider an "island" model of the type analyzed by Lucas and Barro. Each island in the small domestic economy can exchange a "traded" good with other islands in the domestic economy or with the outside world. This international good can be exchanged for domestic money or for an asset denominated in foreign currency. In addition, there is on each island a non-traded good that cannot be exchanged either with other islands or the outside world. Demand for and supply of the traded good T and the non-traded good N on each island z are functions of the relative price of the nontraded good on island z, $p^N(z) - p^T$; the stock of foreign assets held on island z, f(z); and (as in Barro, 1980) a wealth term in unperceived nominal money, $m(z) - \underline{E}_z m$, where $m_t(z)$ is money held on island z at time t and $\underline{E}_z m_t$ is the expected per-island value of the aggregate domestic money stock, conditional on information available at time t on island z. These demand and supply functions are written as

$$y_t^{T,i}(z) = \gamma_1^i\,(p_t^N(z) - p_t^T) + \gamma_2^i\, f_t(z) + \gamma_3^i\,(m_t(z) - \underline{E}_z m_t) + \nu_t^i\,(z)\ , \tag{3.22}$$

$$y_t^{N,i}(z) = \beta_1^i\,(p_t^N(z) - p_t^T) + \beta_2^i\, f_t(z) + \beta_3^i\,(m_t(z) - \underline{E}_z m_t) + \varepsilon_t^i(z) \tag{3.23}$$

$(i = d, s)$. The parameters γ_1^d, γ_2^d, γ_3^d, β_1^s, β_2^d, and β_3^d are positive, while γ_1^s, γ_2^s, γ_3^s, β_1^d, β_2^s, and β_3^s are negative. Assume that $\nu^d(z)$ and $\varepsilon^s(z)$ are identically zero. $\nu^s(z)$ and $\varepsilon^d(z)$ are then random disturbances to the supply of traded goods and the demand for non-traded goods on island z.

Because money, like the foreign asset and the international good, can be traded across islands, there is an economy-wide money market in which aggregate money demand must equal the aggregate money supply. Let p^N, $y^{N,s}$, and $y^{T,s}$ be averages of all island-specific values of the corresponding prices and supplies. The money-market equilibrium condition is similar to (2.3), with $\lambda = 0$ for simplicity:

$$(3.24) \qquad m_t - \alpha p_t^T - (1-\alpha)p_t^N = \Omega y_t^{N,s} + (\eta - \Omega)y_t^{T,s} .$$

It is also necessary for equilibrium that the demand for and supply of the non-traded good be equal on each individual island.

The exchange rate is given by

$$(3.25) \qquad e_t = p_t^T - p_t^{T*},$$

where p^{T*} is the exogenous foreign price of the traded good. Let $\underline{E}m$ and f denote the averages of $\underline{E}_z m$ and f(z) over all islands, and define $\gamma_i \equiv \gamma_i^d - \gamma_i^s$ and $\beta_i \equiv \beta_i^d - \beta_i^s$. Then (3.22) - (3.25) and the goods-market equilibrium conditions imply

$$(3.26) \qquad e_t = m_t - \nu_t^s (1 - \alpha + \eta - \Omega) - \frac{\beta^s}{\beta_1} \Omega \varepsilon_t^d .$$

$$- (m_t - \underline{E}m_t) \left[\frac{\beta_3}{\beta_1} (1 - \alpha + \Omega\beta_1^s - (\eta-\Omega)\gamma_1^s) - \Omega\beta_3^s - (\eta-\Omega)\gamma_3^s\right]$$

$$- f_t\left[\frac{\beta_2}{\beta_1} (1 - \alpha + \Omega\beta_1^s - (\eta-\Omega)\gamma_1^s) - \Omega\beta_2^s - (\eta-\Omega)\gamma_2^s\right] - p_t^{T*} .$$

When consumers observe two prices -- the exchange rate (or price of traded goods) and the price of non-traded goods on their own island -- they are unable to infer the precise realized values of the three random variables m, ν^s, and ε^d. Since the coefficient on unperceived money m - $\underline{E}m$ may be of either sign and may be arbitrarily large, an increase in m accompanied by a smaller increase in $\underline{E}m$ can either increase or decrease the exchange rate initially.[15]

The subsequent path of the exchange rate depends on the adjustment of the net stock of foreign assets in the economy, f. Because external assets adjust gradually, monetary shocks will have persistent real effects. It can be shown that an increase in m coupled with a smaller increase in $\underline{E}m$ raises $y^{T,d}$ and lowers $y^{T,s}$ in each island. The resulting trade deficit implies a reduction in next period's stock of foreign assets, and this reduction affects the exchange rate in subsequent periods. The role of foreign asset accumulation in exchange-rate dynamics has received considerable attention in the literature, and this is the subject to which we now turn.[16]

4. Portfolio Balance, Wealth, and the Exchange Rate

The portfolio-balance approach introduces private wealth as an explicit determinant of the demands for both money and goods.[17] Stocks of external assets and domestic capital are predetermined variables that influence the rate at which new wealth is accumulated through current-account surpluses and investment; and changes in wealth, in turn, move the economy's short-run equilibrium over time. The introduction of wealth thus adds an intrinsic component to the economy's dynamics. In a rational-expectations context, the overall dynamics of the system will result from foreseen changes in exogenous variables as well as from the adjustment of external claims and capital to the long-run levels desired by firms and individuals. We simplify this section's discussion of the portfolio approach by assuming perfect price flexibility throughout.

Dynamic portfolio-balance models of exchange rate determination spring from two distinct sources in the closed-economy macro-dynamic literature. The first source is the work on inflation and growth exemplified by models of Tobin (1965) and Foley and Sidrauski (1970). The second source is the work of Blinder and Solow (1973) and others on the long-run effects of policies in models where the government's budget constraint is taken into account.

Fixed exchange rates were assumed in the seminal studies of Branson (1974), Dornbusch (1975), and Frenkel and Rodriguez (1975), which applied the dynamic portfolio approach to the open economy. With the monetary base endogenous, asset markets adjusted to disturbances in part through stock-shift capital flows -- instantaneous private portfolio shifts accommodated by the central bank's willingness to trade foreign bonds for money at the posted exchange rate. As wealth evolved over time through investment and external saving, a sequence of short-run portfolio equilibria was traced out, implying a path for the balance of payments.

The basic approach was easily applied to the study of floating exchange rates, as the papers of Dornbusch (1976c), Kouri (1976), and Calvo and Rodriguez (1977) demonstrated. The nominal money supply could no longer change over time to ensure continuous equilibrium as wealth changed, but the exchange rate did so, altering both the real money stock and, through its effect on expectations, the relative real yields on domestic and foreign assets.

4.1 Foreign Bonds in a Portfolio-Balance Model

The simplest possible model, due in its essentials to Kouri (1976), is used to develop the main elements of the portfolio approach. We consider a world in which a single composite consumption good is available and examine a small economy whose residents may hold wealth in the form of domestic fiat money or bonds denominated in foreign currency and paying the fixed world interest rate r^*.[18] Foreigners do not hold domestic money, and because the central bank does not intervene in the foreign-exchange market, all net intertemporal trade between the home country and the rest of the world is accomplished through the private exchange of goods for foreign bonds.

On the assumption that the foreign-currency price P^* of the single consumption good is fixed and equal to 1, the domestic price level P may be identified with the exchange rate E and the domestic price inflation rate π with the proportional rate of increase of E. Output is perishable, with its home supply exogenous and equal to Y. When domestic investment is introduced explicitly below, home output of the consumption good will become an endogenous variable and the menu of available assets will expand by one.

The focus on external asset accumulation and wealth calls for a careful description of individuals' lifetime consumption possibilities. For simplicity, we assume throughout that an economy is inhabited by a single, representative agent. The typical individual's lifetime budget constraint limits the real present value of planned expenditure to total real wealth V, where wealth includes the present value of expected future output plus transfers from the government. Let M denote nominal money holdings, F the foreign-currency (and, because $P^* = 1$, real) value of external claims, and T real transfers. If an infinite planning horizon is assumed, the lifetime budget constraint takes the form:

$$(4.1) \qquad \int_t^{\infty} [C_s + (r^* + \pi_s)(M_s/P_s)] \exp[-r^*(s-t)]\, ds$$
$$= (M_t/P_t) + F_t + \int_t^{\infty} (Y_s + T_s) \exp[-r^*(s-t)]\, ds \equiv V_t.$$

Constraint (4.1) reflects an expenditure concept that includes both spending on the consumption good and spending on the services of real balances, where the latter good is valued at the opportunity cost of holding money, $r^* + \pi$.

Consumption C is assumed to be an increasing function of both current disposable income Y^d and wealth V,[19]

$$(4.2) \qquad C_t = C(Y_t^d, V_t), \quad 1 > C_{Y^d} > 0, \quad C_V > 0.$$

Disposable income is the sum of current output, real interest payments on foreign bond holdings, lump-sum transfers from the government, and real capital gains on asset holdings. Thus, $Y^d = Y + r^*F + T - \pi(M/P)$, where $\pi(M/P)$ is the inflation tax.

Instantaneous equilibrium in asset markets requires that the demand for real money balances equal the supply. The fraction of real wealth allocated to real money holdings is assumed to be a positive declining function $L(r^*+\pi)$ of the nominal interest rate. In equilibrium, therefore,

$$(4.3) \qquad M_t/P_t = L(r^* + \pi_t)V_t, \quad L' < 0.$$

If μ denotes the (positive) growth rate of the nominal money supply, $\dot{M}/M$, and ℓ denotes desired real money balances, logarithmic differentiation of (4.3) shows that in equilibrium,

$$(4.4)\qquad \dot{\ell}_t/\ell_t = \mu_t - \pi_t.$$

The model is closed by specification of the government's flow budget constraint. Real government consumption G and transfer payments must be financed through money creation; there is no government-issued interest-bearing debt.[20] This implies a public-sector finance constraint of the form

$$(4.5)\qquad T_t + G_t = \dot{M}_t/P_t = \mu_t\ell_t.$$

Because the level of real balances ℓ is an endogenous variable, the government cannot choose the paths of μ, T, and G independently while continuously satisfying (4.5). The analysis therefore takes μ and T to be the variables controlled by the government and assumes that government consumption adjusts passively according to the equation $G_t = \mu_t\ell_t - T_t$.

We now describe how expectations and asset stocks evolve over time in equilibrium.

The law of motion for real balances ℓ is derived by combining equilibrium conditions (4.3) and (4.4) to obtain the relation

$$(4.6)\qquad L[r^* + \mu_t - (\dot{\ell}_t/\ell_t)] = \ell_t/V_t = 1/\{1 + [(F_t+\tilde{Y}_t+\tilde{T}_t)/\ell_t]\}.$$

In (4.6), $\tilde{Y}_t = \int_t^\infty Y_s \exp[-r^*(s-t)]\, ds$ and $\tilde{T}_t = \int_t^\infty T_s \exp[-r^*(s-t)] ds$.

Inversion of (4.6) yields

$$(4.7)\qquad \dot{\ell}_t = \{r^* + \mu_t - \Phi[\ell_t/(F_t+\tilde{Y}_t+\tilde{T}_t)]\}\ell_t,\quad \Phi'<0.$$

Φ is a decreasing function because a higher portfolio share will be willingly allocated to money only if the inflation rate falls; and, given the monetary growth rate μ, a fall in inflation implies more rapid growth of real balances.

The system's second differential equation describes the motion of the foreign asset stock F. Because there is no domestic investment in the present model, the difference between disposable income and consumption equals the change in holdings of real money and real foreign bonds:

$$(4.8)\qquad \dot{\ell}_t + \dot{F}_t = Y_t^d - C_t = Y_t + r^*F_t + T_t - \pi_t \ell_t - C_t \ .$$

Equations (4.4) and (4.8) together give the equilibrium current-account balance $\dot{F}$ as

$$(4.9)\qquad \dot{F}_t = Y_t + r^*F_t - C(Y_t+r^*F_t+T_t+\dot{\ell}_t-\mu_t \ell_t,\ \ell_t+F_t+\tilde{Y}_t+\tilde{T}_t) + T_t - \mu_t \ell_t.$$

The public-sector budget constraint (4.5) implies that the current account equals the difference between national income, $Y + r^*F$, and national absorption, $C + G$.

If one is willing to consider a global linearization of the system consisting of (4.7) and (4.9) it is possible to study the economy's response to various expected trajectories for the forcing variable μ, T, and Y (see, e.g., Barro, 1978; Flood, 1979; Rodriguez, 1980; Boyer and Hodrick, 1982b; and Mussa, 1984). We assume instead that these variables are constant at levels $\bar{\mu}$, $\bar{T}$, and $\bar{Y}$ except for permanent unanticipated jumps; and we therefore focus on the intrinsic component of the system's dynamics fueled by the adjustment of foreign asset stocks to long-run desired levels. (As in the discussion of section 3.1, however, the diagram we now develop to illustrate this adjustment process may be used also to study certain anticipated and transitory shocks.)

By requiring that the exogenous variables follow constant paths, we reduce (4.7) and (4.9) to an autonomous differential-equation system in ℓ and F. That system is assumed to possess a unique stationary point $(\bar{\ell},\bar{F})$ at which the growth rate of real balances $\dot{\ell}$ and the equilibrium current account $\dot{F}$ are simultaneously zero. The linear Taylor approximation to the system around $(\bar{\ell},\bar{F})$ is

$$(4.10)\quad \begin{bmatrix} \dot{\ell}_t \\ \dot{F}_t \end{bmatrix} = \begin{bmatrix} -\phi' L/(1-L) & \phi' L^2/(1-L)^2 \\ -\bar{\mu}(1-C_{Yd})-C_V+[C_{Yd}\phi' L/(1-L)] & r^*(1-C_{Yd})-C_V-[C_{Yd}\phi' L^2/(1-L)^2] \end{bmatrix} \begin{bmatrix} \ell_t-\bar{\ell} \\ F_t-\bar{F} \end{bmatrix},$$

where all functions are evaluated at long-run equilibrium. The determinant of the above matrix is assumed to be negative:

$$(4.11)\quad [\phi' L/(1-L)]\{C_V - r^*(1-C_{Yd})+[L/(1-L)][C_V + \bar{\mu}(1-C_{Yd})]\} < 0.$$

Because the determinant is the product of the system's characteristic roots, the system must possess a negative root (associated with the predetermined variable F) and a positive root (associated with the jumping variable ℓ, which varies inversely with E). Thus, the long-run equilibrium $(\bar{\ell},\bar{F})$ is a saddle point.

Stability condition (4.11) requires that equilibrium public plus private absorption increase faster than income as foreign assets increase. Section 5, below, shows that side conditions like (4.11) can sometimes be replaced by assumptions on preferences in models based on explicit intertemporal optimization: if a stationary position exists, the model's inherent logic will then imply saddle-path stability. As we shall see, however, there do exist optimizing models with no well-behaved long-run equilibrium, as well as optimizing models with multiple stationary points.

Figure 4.1 is the local phase diagram of the system described by equations (4.7) and (4.9). Differentiation of (4.7) shows that the locus of points along which $\dot{\ell} = 0$ is of slope $[F + (\bar{Y} + \bar{T})/r^*]/\ell$, which is a positive number if the stock of foreign claims never falls below $-(\bar{Y} + \bar{T})/r^*$.[21] The slope of the $\dot{F}=0$ locus depends on the sign of $\partial\dot{F}/\partial F$ [given by the southeast entry of the matrix in (4.10)]. Figure 4.1 shows the case we will discuss: that in which $\partial\dot{F}/\partial F < 0$, so that the $\dot{F}=0$ locus slopes downward.[22] Note that $\partial\dot{\ell}/\partial\ell$, given by the northwest entry of the matrix in (4.10), is always positive. As usual, the unique saddle path SS converging to long-run equilibrium at A is the rational-expectations equilibrium path of the economy provided no changes in exogenous variables are expected.

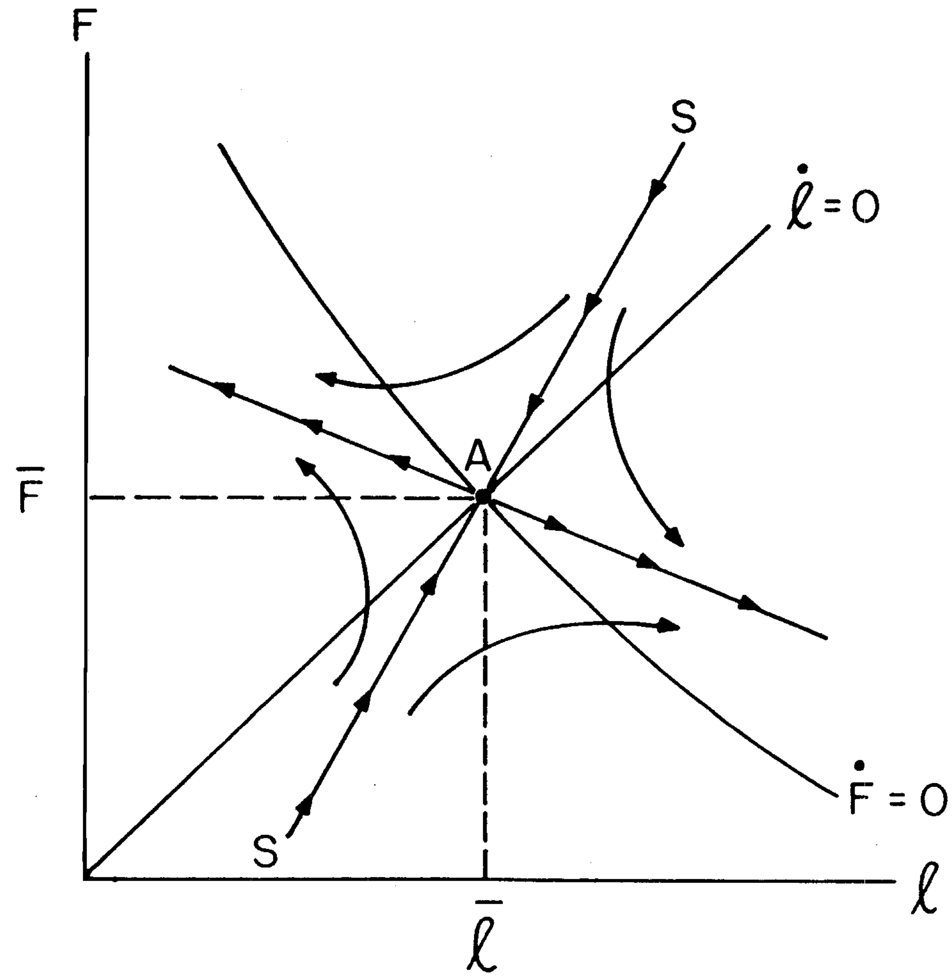


Figure 4.1

A salient feature of figure 4.1 is the positive association between real balances and foreign assets along the saddle path. This translates into a relationship between the exchange rate and the current account, for when the current account is in surplus, the currency depreciates more slowly than its trend depreciation rate $\bar{\mu}$. Intuitively, the growing wealth implied by a current surplus leads to a growing demand for real balances that prevents prices from rising to the full extent of the cumulative increase in the nominal money supply. We shall soon see that this surplus-appreciation, deficit-depreciation relationship, while characterizing the process of convergence to a fixed stationary state, need no longer hold once anticipated disturbances are considered. The relationship may break down also when non-monetary wealth can be held in the form of domestic capital as well as foreign claims.[23]

Assume now that the economy is initially in long-run equilibrium at point A. The first experiment to be considered is an unanticipated, permanent increase in the money stock — a discontinuous jump in M that leaves the growth rate of money unchanged. Equilibrium is restored if the currency depreciates immediately in proportion to the increase in money, leaving real balances at their original level $\bar{\ell}$ and real wealth at its long-run desired level. Because prices are fully flexible and the system is homogenous in all nominal variables, a monetary change of the type considered here has no real effects. In particular, it does not set into motion the intrinsic dynamics of the economy.[24]

An unanticipated permanent increase in the money growth rate $\bar{\mu}$, in contrast, can have a real impact on the economy. In other words, money is not superneutral, as it was in section 2. Figure 4.2 illustrates the effects of this policy shift when the $\dot{F}=0$ locus slopes downward.

As is easily verified, the $\dot{\ell}=0$ and $\dot{F}=0$ schedules both shift leftward near the steady state. While the long-run level of real balances falls unambiguously to $\bar{\ell}'$, the new long-run foreign asset stock $\bar{F}'$ may be greater or less than $\bar{F}$.[25]

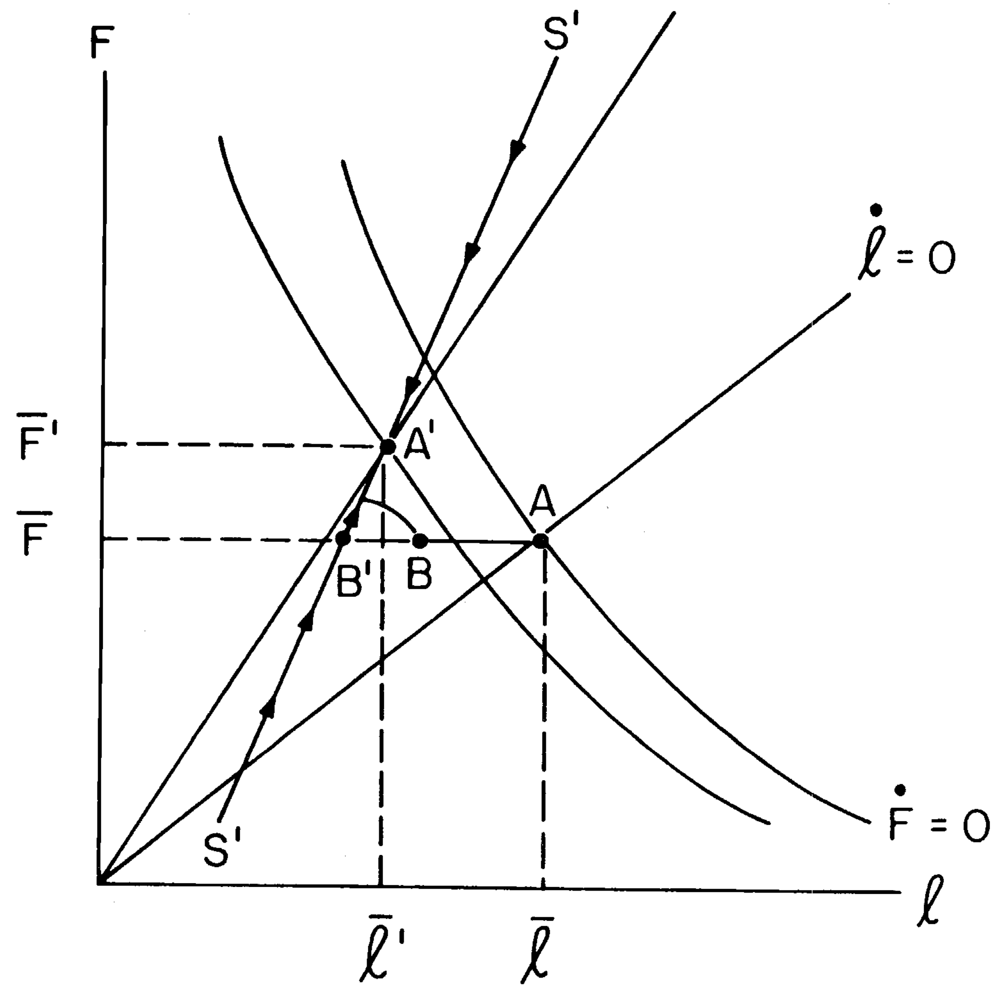


Figure 4.2

Figure 4.2 shows the case in which $\bar{F}' > \bar{F}$. When $\bar{\mu}$ is increased, the currency depreciates and a current surplus emerges as the economy jumps to point B' on the new saddle path $S'S'$. Real balances and foreign bonds subsequently rise together while the economy converges to its new long-run position A'. When $\bar{F}' < \bar{F}$, the increase in $\bar{\mu}$ naturally occasions a depreciation coupled with a current deficit.[26]

These real effects of a change in monetary growth come from three related sources, none of which was present in the flexible-price model of section 2. First, the concomitant increase in inflation reduces real wealth and hence private consumption by reducing desired real balances. Second, because real transfers $\bar{T}$ are held constant, there may be a change in the inflation tax that alters long-run disposable income. Third, a change in the product $\bar{\mu}\ell$ results in an equal change in government consumption, by (4.5). If the interest-rate elasticity of money demand exceeds 1, a rise in $\bar{\mu}$ lowers long-run government consumption and the long-run inflation tax. Because $C_{Y^d} < 1$, steady-state foreign assets must rise to ensure external balance. When money demand is inelastic with respect to the nominal interest rate, long-run foreign assets may fall.

Figure 4.2 may also be used to analyze an announced future increase in $\bar{\mu}$. As we have seen, the path of the economy between the announcement of the policy change and its implementation is described by an unstable trajectory of the autonomous system defined by (4.7) and (4.9). Further, there can be no discrete jump in E at the moment $\bar{\mu}$ rises. Accordingly, the economy jumps immediately to a point like B and reaches $S'S'$ at the moment $\bar{\mu}$ is increased. It is noteworthy that as the economy travels between the momentary equilibrium B and $S'S'$, the currency depreciates at a rate exceeding $\bar{\mu}$ even though the current account is in surplus. The example shows that the surplus-appreciation, deficit-depreciation pattern, while characterizing the intrinsic component of the system's dynamics, need not dominate the response to anticipated exogenous disturbances.

Non-monetary disturbances can be studied as well. A permanent, unanticipated increase in $\bar{Y}$ leads to a fall $\Delta\bar{Y}/r^*$ in long-run external assets but to no change in long-run real balances $\bar{\ell}$. Accordingly, the currency appreciates in the short run and a current deficit emerges. Real balances and foreign assets both fall in the transition to the new stationary state.

4.2 Investment and the Current Account

It was noted above that the presence of domestic investment opportunities may alter the simple relation between the exchange rate and the current account characterizing convergent paths. Money demand depends on wealth in a portfolio setting, but wealth and foreign assets can move in opposite directions in a model with capital accumulation, even as the economy converges to a fixed long-run equilibrium. It therefore becomes possible that a current deficit will be accompanied by rising real balances and a current surplus by falling real balances.

We illustrate these possibilities by incorporating investment into the portfolio-balance model developed above. (See Dornbusch, 1980, and Hodrick, 1980, for similar models.) The assumption of a single available consumption good is retained, but it is now assumed that the domestic supply of that good is endogenous and that the economy can produce, in addition, a non-traded investment good. The production technologies for the consumption and investment goods are described by constant-returns-to-scale, neo-classical production functions taking capital and labor as inputs. Because the investment good is not tradable, the output of the investment sector represents the (gross) capital accumulation of the economy. Labor is supplied inelastically at the fixed level N.

Claims on domestic capital cannot be held by foreigners, but capital and foreign bonds are perfect substitutes from the standpoint of home investors.[27] Let

ρ denote capital's real rental, the marginal product of capital in the consumption sector. If P^K is the price of a unit of capital in terms of the consumption good and ε is the rate of physical depreciation of capital, the perfect substitution assumption implies that

$$(4.12) \qquad r^* = \frac{\rho_t + \dot{P}_t^K}{P_t^K} - \varepsilon \ ,$$

so that the rate of physical return on capital plus the rate of capital gain equals the world bond rate.

On the assumptions that no factor-intensity reversals are possible, that the economy does not specialize in production, and that the investment good is relatively labor intensive, the Stolper-Samuelson reasoning allows us to write the rental ρ as a declining function of P^K and the real wage ω as an increasing function of P^K(see chapter 1, volume 1, by Jones and Neary). Arbitrage condition (4.12) then becomes a differential equation in the price of capital,

$$(4.13) \qquad \dot{P}_t^K/P_t^K = \varepsilon + r^* - \left[\rho(P_t^K)/P_t^K\right] , \qquad \rho' < 0.$$

The dynamic system in P^K described by (4.13) is unstable, with a single steady state at the unique capital price $\bar{P}^K$ such that $\varepsilon + r^* = \rho(\bar{P}^K)/\bar{P}^K$. The requirement of saddle-path stability allows us to conclude that the price of capital will always be constant at level $\bar{P}^K$, and, by implication, that the real rental and wage will also be constants. These are denoted by $\bar{\rho}$ and $\bar{\omega}$, respectively.

Given production possibilities (as summarized by the current capital stock K), output of the two goods depends on the relative price $\bar{P}^K$. The supply functions for the consumption good and the investment good may therefore be written as $Q^C(\bar{P}^K,K)$ and $Q^I(\bar{P}^K, K)$, respectively. By the Rybczynski theorem, $\partial Q^C/\partial K > 0$ and $\partial Q^I/\partial K < 0$ (chapter 1, volume 1).[28]

Three differential equations summarize the dynamics of the system. Real net investment is given by

$$(4.14) \quad \bar{P}^K \dot{K}_t = \bar{P}^K Q^I(\bar{P}^K, K_t) - \varepsilon \bar{P}^K K_t \, .$$

External asset accumulation equals the difference between the economy's endowment of the consumption good and consumption spending, so that

$$(4.15) \quad \dot{F}_t = Q^C(\bar{P}^K, K_t) + r^* F_t - C(Y_t^d, V_t) + \bar{T} - \bar{\mu} \ell_t \, ,$$

where now,

$$(4.16) \quad V_t = \ell_t + F_t + (\bar{\omega}N + \bar{T})/r^* + \bar{P}^K K_t \, .$$

The system's final equation is the analogue of (4.7),

$$(4.17) \quad \dot{\ell}_t = \{r^* + \bar{\mu} - \Phi[\ell_t/(V_t - \ell_t)]\} \ell_t \, .$$

The stationary values of asset stocks are denoted by $\bar{K}$, $\bar{F}$, and $\bar{\ell}$.

For the present purpose, it is convenient to work with a representation of the model that differs from the one given by equations (4.14), (4.15), and (4.17). To derive this alternative (but equivalent) representation, note that by (4.4), (4.12), and (4.17), we may write disposable income as $Y_t^d = \bar{\omega}N + [(\bar{\rho}/\bar{P}^K) - \varepsilon]\bar{P}^K K_t + r^* F_t + \bar{T} - \pi_t \ell_t = r^* (V_t - \ell_t) + \{r^* - \Phi[\ell_t/(V_t - \ell_t)]\}\ell_t$. Similarly, by adding (4.14) and (4.15), we obtain

$$(4.18) \quad \dot{V}_t - \dot{\ell}_t = r^*(V_t - \ell_t) - C[Y_t^d, (V_t - \ell_t) + \ell_t] - \bar{\mu} \ell_t \, .$$

Together, equations (4.17) and (4.18) constitute an autonomous differential-equation sub-system in real balances ℓ and non-monetary wealth $V-\ell$. As before, equation (4.14) describes the motion of domestic capital; but K does not appear explicitly in the dynamic sub-system defined by the two other equations.

The alternative representation implies that the economy's evolution does not depend on levels of capital and foreign claims separately, but only on their sum. (This would not be true if capital and bonds were imperfect substitutes in portfolios.) Thus, saving and equilibrium real balances are determined entirely by $V - \ell$: a lower capital stock, ceteris paribus, implies a higher investment level financed by an equal deterioration in the current account. What happens when the capital stock is initially at the stationary level $\bar{K}$ defined by $Q^I(\bar{P}^K, \bar{K}) = \varepsilon \bar{K}$? In this case capital and consumption-goods production remain constant through time, so that the model reduces to the simpler portfolio model discussed above in section 4.1.

It is easy to derive the phase diagram for the sub-system consisting of (4.17) and (4.19); indeed, we have already done the work. Note that the system in ℓ and $V - \ell$ described by (4.17) and (4.19) is identical to the system in ℓ and F that equations (4.7) and (4.9) describe when μ, T, and Y are constant. Qualitatively, the dynamics of the present model can therefore be illustrated by figure 4.1, with F replaced everywhere by $V - \ell$.[29]

The intrinsic dynamics of the extended model imply that increasing non-monetary wealth is accompanied by a rate of currency depreciation that falls short of the money growth rate $\bar{\mu}$, while decreasing wealth is accompanied by a depreciation rate exceeding $\bar{\mu}$. But no tight link between the exchange rate and the current account is implied, and a current deficit, say, may easily be accompanied by rising real balances. To see this, assume that non-monetary wealth is initially below its long-run level, that initial capital is lower than $\bar{K}$, and that initial foreign assets exceed $\bar{F}$. The stock of foreign claims cannot converge to its long-run level $\bar{F}$ unless the current account is in deficit along some portion of the subsequent transition path. As the diagram shows, however, real balances (and non-monetary wealth) will rise monotonically along that path, even during periods in which foreign assets are being run down.

We conclude that when the menu of assets is expanded, the current account, *per se*, may play no role in determining exchange-rate behavior along paths converging to a fixed long-run equilibrium. The linkage that does emerge is one between the exchange rate and the evolution of overall national wealth. Whether saving is external or internal is irrelevant in the present model.

5. Exchange-Rate Models Based on Individual Intertemporal Optimization

The exchange-rate dynamics highlighted in section 4 were driven by external asset accumulation and domestic investment. Central to the analysis of that section were the assumed forms of the consumption function and the portfolio-balance schedule. We now turn to exchange-rate models in which the consumption function and asset demands are derived explicitly from individual preferences regarding alternative future expenditure paths. While the broad predictions of section 4 can be replicated in some optimizing models, the results are quite sensitive to the assumptions one makes about intertemporal tastes.

Models of optimal external borrowing developed by Bardhan (1967), Hamada (1969), and Bruno (1976) are the forerunners of optimizing exchange-rate models. While these models were concerned exclusively with real factors, the introduction of money yields a theory of exchange-rate dynamics in which the evolution of asset stocks results from optimal individual choices.

The proper role of money in an optimizing model is a controversial question, however, and results are sensitive to the way in which money is introduced. Why should maximizing agents hold money at all when it is dominated, in terms of both return and risk, by other assets? Below, we will discuss two methods of answering this question. The first, adopted by Sidrauski (1967) and Brock (1975), assumes that the level of real balances enters directly into agents' instantaneous utility functions. Thus, money offers a real "convenience yield" that may induce agents to hold it. The second device for introducing money, associated with Clower (1967), assumes that agents must acquire money and hold it for some time before purchasing consumption goods. In this sequential, "cash-in-advance" setup, money demand is closely linked to planned future purchases of consumption goods. It is clear that both approaches to money demand leave us far from a true theory of why money is held. Nonetheless, the models discussed below are both tractable and suggestive. Pending further developments in monetary theory, they represent the state of the art.[30]

5.1 A Small-Country, One-Good Model

We introduce individual optimization explicitly while retaining the assumptions and notation of section 4. To simplify, we abstract from domestic investment throughout this section, although investment could be introduced along the lines of section 4.2 above. Statements about the current account made below are predicated on the tacit assumption that the domestic capital stock is constant. If the assumption is relaxed, those statements must be interpreted as applying to the overall rate of accumulation of non-money assets. (Hodrick, 1982, and Greenwood, 1983, study versions of the present model. Sachs, 1983b, introduces investment into a related two-country simulation model that includes two consumption goods and an intermediate good.)

A representative agent is now assumed to derive instantaneous utility $U(C, \ell)$ from his expenditure on the single consumption good and his holdings of real money balances.[31] The consumer (who may also be thought of as a dynastic "family") is immortal and maximizes his lifetime welfare, W, subject to the lifetime budget constraint (4.1). It is assumed that W is a time-additive function of future instantaneous utilities,

$$(5.1) \qquad W(\{C_t\}_{t=0}^{\infty}, \{\ell_t\}_{t=0}^{\infty}) = \int_0^{\infty} \exp(-\delta t) U(C_t, \ell_t) dt ,$$

where δ is a constant rate of subjective time preference. (Alternative preference schemes are discussed later.) Let η_0 be the shadow price of real wealth at time 0. Given (5.1), the first-order conditions for the consumer's problem are

$$(5.2) \qquad U_C(C_t, \ell_t) = \eta_0 \exp[(\delta - r^*)t] ,$$

$$(5.3) \qquad U_\ell(C_t, \ell_t) = (r^* + \pi_t)\eta_0 \exp[(\delta - r^*)t] .$$

Conditions (5.2) and (5.3) define desired consumption and real balances as functions of the current inflation rate, the world interest rate, time, and η_0. The value of η_0 yielding an optimal program from the individual's standpoint is the unique value that allows the budget constraint (4.1) to hold with equality when planned consumption and real balances satisfy (5.2) and (5.3) at every point in the future.

It is instructive to compare the consumption and money demand functions implied by the constrained maximization of (5.1) with those assumed in section 4. Closed-form behavioral functions cannot be obtained unless we specify a particular functional form for the utility function $U(C,\ell)$, so we assume that it is a member of the constant relative risk aversion family $(C^{\alpha}\ell^{1-\alpha})^{1-R}/1-R$, where $R > 0$ and $0 < \alpha < 1$. (None of the results obtained below would be qualitatively altered if a wider class of utility functions was considered.) With this choice of utility function, (4.1), (5.2), and (5.3) imply that consumption and money demand are given by

$$(5.4) \qquad C_t = \frac{\alpha V_t\left[\dfrac{\alpha(r^*+\pi_t)}{1-\alpha}\right]^{-(1-\alpha)(1-R)/R}}{\displaystyle\int_t^{\infty}\exp\{[-r^*+(r^*-\delta)/R^*](s-t)\}\left[\dfrac{\alpha(r^*+\pi_s)}{1-\alpha}\right]^{-(1-\alpha)(1-R)/R} ds},$$

$$(5.5) \qquad \ell_t = \left[\frac{\alpha(r^*+\pi_t)}{1-\alpha}\right]^{-1} C_t ,$$

where it is assumed that $-r^* + (r^*-\delta)/R < 0$ and that the integral in (5.4) converges. (See Obstfeld, 1983, for the solution method.) As in (4.2) and (4.3), desired consumption and real balances are both increasing functions of wealth V, while money demand is a declining function of the current nominal interest rate $r^* + \pi$. But in addition, anticipated future inflation generally influences the demands for goods and assets; current inflation generally affects consumption; and current disposable income plays no role. An exception arises when R=1, so that the utility function takes the separable form $\alpha\ln(C) + (1-\alpha)\ln(\ell)$. In this special case, the consumption function takes the form given by (4.2) (with $C_{Yd} = 0$) and the demand for real balances takes the form given by (4.3).

One source of monetary non-neutrality in section 4's model was the assumption that government spending was a function of private real balances [see the discussion following equation (4.5)]. To better understand the possible sources of non-neutrality in optimizing models, we now depart further from the assumptions of section 4 and assume that it is the level of transfers, rather than government consumption, that adjusts passively to changes in inflation-tax revenue. With this assumption, transfers are given by

$$T_t = \mu_t \ell_t - G_t \,, \tag{5.6}$$

and the path of G is exogenous. Accordingly, changes in ℓ can no longer affect an important real variable, the portion of national income consumed by the government.

To study the economy's perfect foresight equilibrium, we assume initially that the money growth rate is a positive constant, $\bar{\mu}$. Logarithmic differentiation of (5.5) shows that

$$\frac{\dot{\ell}_t}{\ell_t} = \frac{\dot{V}_t}{V_t} + \left[\frac{\alpha(1-R)-1}{R}\right] \frac{\dot{\pi}_t}{(r^* + \pi_t)} + \frac{(r^*-\delta)}{R} - [r^* - (C_t/\alpha V_t)] \,, \tag{5.7}$$

where C is given by (5.4). Using the definition of V in (4.1) and equation (4.8), we find that the planned change in wealth, $\dot{V}$, is given by $r^*V - C - (r^*+\pi)\ell$. (This relationship also appeared in section 4.2.) Because $C + (r^*+\pi)\ell = C/\alpha$ (given the assumed form of the utility function), the first and last terms on the right-hand side of (5.7) cancel. Equation (4.4) then implies that the equilibrium inflation rate must satisfy the non-linear differential equation

$$\dot{\pi}_t = \frac{(r^* + \pi_t)}{[\alpha(1-R)-1]} [R(\bar{\mu} - \pi_t) + (\delta-r^*)]. \tag{5.8}$$

Figure 5.1 shows the phase diagram for (5.8). The equation has two stationary points, one stable (at $\pi = -r^*$) and one unstable (at $\pi = \bar{\mu} + (\delta - r^*)/R$). Because the marginal utility of money is always strictly positive, the stationary point at $\pi = -r^*$ is not an equilibrium, nor is any point to its left [cf. (5.3)]. Moreover, paths originating to the right of the unstable steady state imply that inflation explodes in spite of constant "fundamentals." Thus, the economy can reach its steady state equilibrium (the saddle path) only if inflation jumps immediately to the level

$$(5.9) \qquad \bar{\pi} = \bar{\mu} + (\delta - r^*)/R$$

and remains there forever.

To find the equilibrium exchange rate, note that by (4.4) and (5.9),

$$(5.10) \qquad \ell_s = \ell_t \exp\left[(r^* - \delta)(s-t)/R\right]$$

for $s > t$. In equilibrium, therefore, the present value of future government transfer payments is

$$(5.11) \qquad \tilde{T}_t = \int_t^\infty (\bar{\mu}\ell_s - G_s) \exp[-r^*(s-t)]ds = \frac{\bar{\mu}\ell_t}{[r^* + (\delta - r^*)/R]} - \tilde{G}_t,$$

where $\tilde{G}_t = \int_t^\infty G_s \exp[-r^*(s-t)]ds$. When combined, (5.4), (5.5), (5.9), and (5.11) imply that equilibrium real balances are

$$(5.12) \qquad \ell_t = \frac{(1-\alpha)}{\alpha} \left[\frac{r^* + (\delta - r^*)/R}{\bar{\mu} + r^* + (\delta - r^*)/R}\right] (F_t + \tilde{Y}_t - \tilde{G}_t)$$

and that M_t/ℓ_t is the equilibrium exchange rate. Equilibrium consumption is

$$(5.13) \qquad C_t = [r^* + (\delta - r^*)/R](F_t + \tilde{Y}_t - \tilde{G}_t).$$

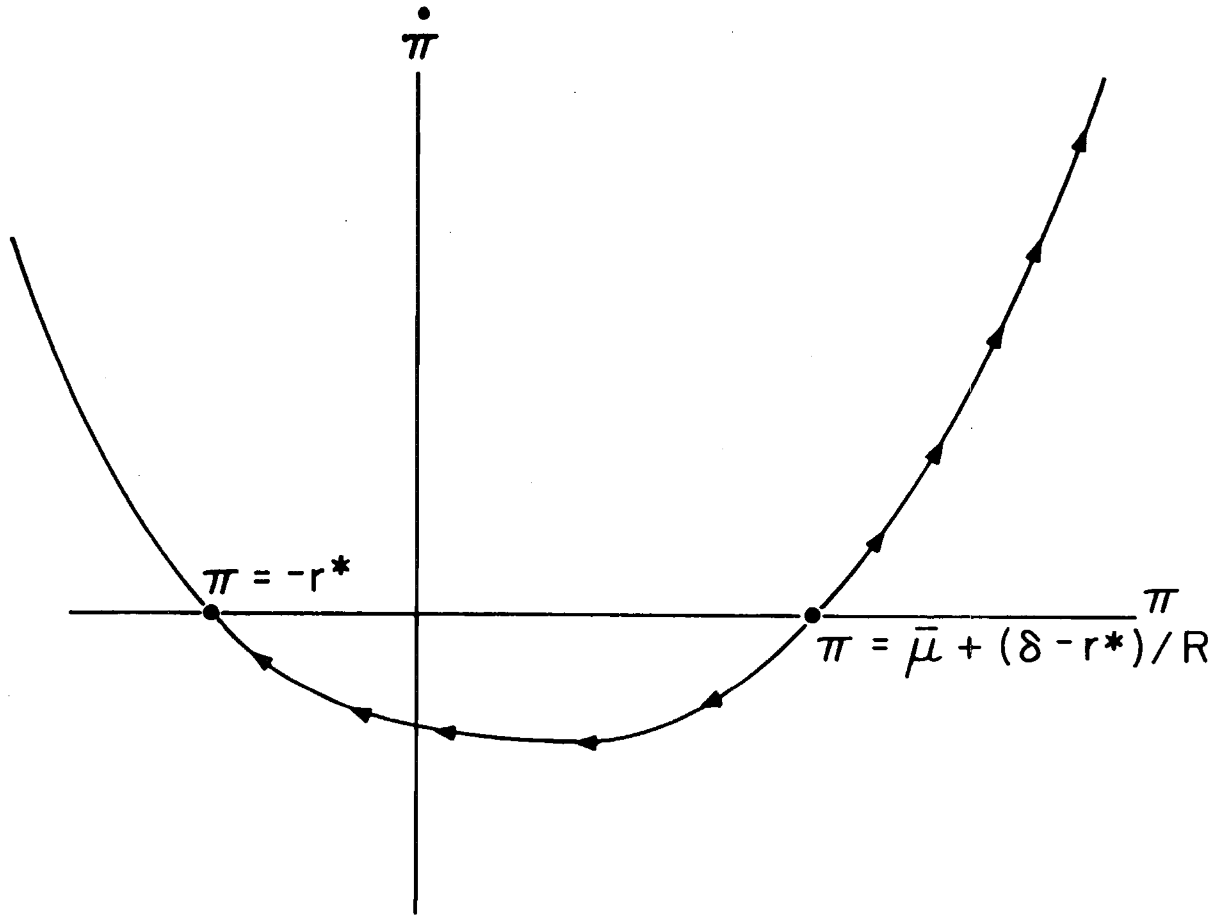


Figure 5.1

To interpret this equilibrium, return to the individual's lifetime budget constraint. By substituting (4.4) and (5.6) into (4.1) and integrating by parts, we obtain [32].

$$(5.14)\qquad \int_t^\infty C_s \exp[-r^*(s-t)]ds = F_t + \tilde{Y}_t - \tilde{G}_t.$$

Because money is a non-traded asset, the present value of domestic private consumption must equal the economy's non-monetary wealth, $F + \tilde{Y}$, net of the present value of future public consumption, $\tilde{G}$. In equilibrium, therefore, current consumption equals a fixed fraction of the present value of planned future consumption. If $\delta = r^*$, consumption equals "permanent" income, i.e.

$$C_t = r^*(F_t+\tilde{Y}_t-\tilde{G}_t) = r^* \int_t^\infty C_s \exp[-r^*(s-t)]ds,$$

so that $\dot{C} = 0$. If, however, the consumer is less patient than the rest of the world ($\delta > r^*$), consumption will fall over time; and if he is more patient ($\delta < r^*$), consumption will rise. The current account is given by

$$(5.15)\qquad \dot{F}_t = Y_t + r^*F_t - C_t - G_t$$

$$= \left[(Y_t - G_t) - r^* (\tilde{Y}_t-\tilde{G}_t)\right] - \left[(\delta-r^*)(F_t+\tilde{Y}_t-\tilde{G}_t)/R\right] .$$

The first term on the right-hand side of (5.15) shows that external borrowing and lending is used to smooth consumption in the face of deviations between disposable output, $Y - G$, and its "permanent" level, $r^*(\tilde{Y}-\tilde{G})$; this term is zero when the paths of Y and G are flat. The second term on the right-hand side of (5.15) reflects the discrepancy between the domestic and foreign time preference rates. If $\delta > r^*$, for example, the current account is perpetually in deficit if the paths of Y and G are flat.

The intrinsic dynamics of the economy are driven entirely by the discrepancy between δ and r*. It is clear from (5.9) that the currency's depreciation rate exceeds or falls short of $\bar{\mu}$ as consumption falls or rises over time (see also Sachs, 1983a). If $\delta > r^*$, however, the economy's wealth shrinks to zero asymptotically, while if $\delta < r^*$, the economy must grow until the small-country assumption is violated. Only if $\delta = r^*$ does the economy have a well-behaved steady state of the type assumed in section 4. But that steady state is not unique: in equilibrium, the private sector chooses the highest constant consumption level consistent with the economy's non-monetary wealth, given the future path of government consumption. The currency depreciates at rate $\bar{\mu}$ in this case [by (5.9)], regardless of the current account's position. The system has no intrinsic dynamics.

Any previously unexpected increase in output or fall in government spending -- whether permanent, transitory, or anticipated -- causes a once-and-for-all rise in consumption and appreciation of the currency. An unanticipated increase in $\bar{\mu}$ or M, similarly, occasions a once-and-for-all depreciation, but does not influence consumption. The exchange rate response to an anticipated, permanent rise in $\bar{\mu}$ or M, however, is similar to those studied in section 2: the exchange rate takes an initial upward jump, then rises at an accelerating rate, but does not jump when the monetary change occurs. It is noteworthy that anticipated monetary shocks need not be neutral, as (5.4) shows. Only when R = 1 (so that $U_{C\ell} = 0$) does anticipated mone-tary expansion have no impact on consumption and the current account. In general, the direction of the current-account effect is negative or positive as consumption and real balances are complements ($U_{C\ell} > 0$) or substitutes ($U_{C\ell} < 0$). The model of section 4 also predicted a real dynamic response to an anticipated monetary shock, but the dynamics of the present system are entirely extrinsic when $\delta = r^*$.

The intrinsic dynamics caused by a divergence between the world interest rate and a constant time-preference rate are inconsistent with the existence of a well-behaved small-country steady state. Obstfeld (1981) studies a model in which the time-preference rate δ is endogenous and temporary discrepancies between δ and r^* drive the economy toward a conventional long-run equilibrium with $\delta = r^*$. Following Uzawa (1968), Obstfeld assumes that the subjective time preference rate is a monotonic function $\delta(U)$ of contemporaneous utility. The steady state is then characterized by a unique long-run utility level $\bar{U}$ satisfying $\delta(\bar{U}) = r^*$. Because expenditure is rising when $\delta(U) < r^*$, $\delta(U)$ can converge to r^* only if domestic residents become more impatient as utility increases, i.e., only if $\delta'(U) > 0$. This increasing impatience assumption plays the same role here that stability condition (4.11) played in section 4: it ensures that when the current account is in surplus, say, consumption increases rapidly enough to eventually drive the surplus to zero.[33]

The resulting model is similar to that of section 4. In particular, there is a unique small-country steady state with positive wealth, and a unique convergent saddle path along which foreign assets and real money balances rise together. Further, permanent changes in monetary growth cause movements along the economy's long-run utility contour $\bar{U}$ and thus alter the steady-state stock of foreign claims. This occurs even when the instantaneous utility function is separable in consumption and real balances ($U_{C\ell} = 0$).

5.2 Models with Two Countries and Two Goods

The previous sections of this chapter have studied small countries facing at least some prices that are determined outside the economy. We now turn to models of the world economy in which all prices are endogenously determined. As in sections 2 and 3, it is assumed that two distinct consumption goods are available.

Lucas' (1982) model is a useful benchmark because there can be no intercountry wealth redistribution and all goods are traded. In addition, the "cash-in-advance" framework utilized in that model yields monetary equilibrium conditions that reduce to a simple quantity theory.

Consider a world of two countries, two goods, and two monies. All consumers in the world economy have identical tastes, and, as in section 5.1, are risk-averse and infinitely lived. A resident of the home country receives an exogenous stochastic endowment, Y, of a non-storable "home good" that can be traded with zero transport costs; a resident of the foreign country receives an exogenous stochastic endowment Y* of a "foreign good" that can also be traded costlessly. The money supplies of each country, M and M*, are determined exogenously by the respective governments. Monetary growth rates are stochastic: M increases via lump-sum transfer payments to domestic residents at the beginning of each period, and M^* increases via lump-sum transfers to foreign residents at the beginning of each period. Output levels and the growth rates of the two money supplies follow a joint first-order Markov process.

Let C and C* denote consumption levels of the home and foreign goods. Consumers maximize the welfare criterion

$$W(\{C_t\}_{t=0}^{\infty}, \{C_t^*\}_{t=0}^{\infty}) = \underline{E}_0\{\sum_{t=0}^{\infty}\beta^t U(C_t, C_t^*)\}, \quad 0 < \beta < 1, \tag{5.16}$$

where $\underline{E}_0$ is an expected value conditional on t = 0 information, β is a constant subjective discount factor, and U is bounded. The maximization is subject to a budget constraint and to cash-in-advance constraints that provide an alternative to placing money directly into the instantaneous utility function. A typical consumer begins each trading "period" with a portfolio of assets that can include: domestic money, foreign money, claims to delivery of either money in any future period, and claims to shares of the nominal proceeds from future sales of either good (equities).

The sequence of events within each period is as follows. First, realized values of the stochastic endowments are revealed and the consumer receives a transfer payment of his country's money. After observing all current-period prices (including P, the home-currency price of the home good, and P^*, the foreign-currency price of the foreign good), the consumer visits an asset market in which monies and the other available assets are traded. Finally, the consumer visits a goods market where the two monies are used to purchase the consumption goods and endowments are sold. Only money held at the close of the current period's asset trading may be used to purchase current consumption. Money earned through the sale of endowments cannot be used in the same period, and thus enters the following period's pre-trade portfolio. Further, it is assumed that all goods-market transactions involve the seller's money, implying that a domestic consumer receives PY units of domestic money in exchange for his endowment Y. (Helpman and Razin, 1981, use a one-good framework to compare the dynamics of exchange rates when buyers' rather than sellers' currencies are used in transactions.) The consumer's choices are therefore subject to the cash-in-advance constraints

$$(5.17) \quad M_t^d \geq P_t C_t,$$

$$(5.18) \quad M_t^{*d} \geq P_t^* C_t^*,$$

where M^d and M^{*d} denote the quantities of the domestic and foreign monies that the consumer holds at the close of asset trading.

Although all consumers have the same tastes, the equilibrium of the model depends on the initial distribution of wealth. Lucas investigates a perfectly pooled, stationary equilibrium in which all consumers have the same wealth. Because tastes are identical and markets are complete, all choose the same portfolio and all consume the per capita world endowment of each good. As section 5.1 showed, however, this perfectly-pooled equilibrium, while easy to analyze, need never be attained when the domestic and foreign time preference rates are fixed constants.

Lucas assumes that monetary policies are such that nominal interest rates are strictly positive. Because consumers would forego interest payments by holding money balances exceeding planned consumptions, the monetary equilibrium conditions are

$$(5.19)\qquad M_t = P_t Y_t,$$

$$(5.20)\qquad M_t^* = P_t^* Y_t^*,$$

where quantities are now expressed in world per-capita terms. The necessary conditions for utility maximization include the standard requirement that the marginal rate of substitution between domestic and foreign goods equal their relative price:

$$(5.21)\qquad P_t/E_t P_t^* = U_C(Y_t, Y_t^*)/U_{C*}(Y_t, Y_t^*).$$

Together, (5.19)-(5.21) imply the exchange-rate equation

$$(5.22)\qquad E_t = (M_t/M_t^*)(Y_t^*/Y_t)[U_{C*}(Y_t, Y_t^*)/U_C(Y_t, Y_t^*)].$$

Using (5.22) and the joint probability distribution for the exogenous variables Y, Y^*, M, and M^*, one can derive the probability distribution of the exchange rate.

This solution has several important characteristics. First, both changes in money supplies and changes in outputs of goods affect the exchange rate. Changes in tastes for goods (changes in the marginal rate of substitution function) also move the exchange rate. An increase in the output of the domestic good causes a fall in E, unless U_{C*}/U_C rises with an elasticity greater than one when Y rises. The latter possibility would correspond to the usual condition for immiserizing growth, but immiserization can never occur here because the assumed perfect pooling prevents any agent's utility from falling when one country's endowment rises. The exchange rate is affected both by factors emphasized in the monetary approach to exchange rates and by factors emphasized in the elasticities approach.

Second, only current values of money supplies and outputs affect the exchange rate, even though prices of claims to future deliveries of goods or monies depend on the probability distributions of future money supplies and outputs. In a sense, therefore, the exchange rate is not really an "asset price" in this model, although the prices of claims clearly are. This characteristic of the model is not surprising given (a) the fixed velocity of money, (b) the intertemporally separable utility function that limits substitution over time in the goods market and prevents future or past levels of output from affecting the current marginal rate of substitution in equilibrium, and (c) the absence of real investment opportunities. (Barro and King, 1983, discuss the roles of assumptions (b) and (c) in a closed-economy equilibrium.) These features yield a model resembling that of section 2 in the special case $\lambda = \sigma = 0$. The condition $\lambda = 0$ has an exact analogue in the present setup because velocity is fixed. But consumption demands do respond to intertemporal relative prices here, so the condition $\sigma = 0$ has no counterpart. Rather, the insensitivity of the terms of trade to future real disturbances is a characteristic of the model's equilibrium.

Third (although Lucas does not discuss this), the volatility of exchange rates and price levels can differ in the model. A higher realized value of domestic output has an exchange-rate effect given by

$$(5.23) \qquad \frac{de_t}{dy_t} = -\left[1 + \frac{(U_{CC}U_{C*} - U_{CC*}U_C)}{U_C U_{C*}} Y_t\right]$$

(where small letters, as always, denote logs). P falls in proportion to the rise in Y. If demand is sufficiently elastic that (5.23) is negative, then E and P both fall in response to an increase in Y, and the percentage change in E is smaller than that in P. If (5.23) is positive (the immiserizing-growth case), then E rises while P falls, and if demand is so inelastic that (5.23) exceeds unity, then the percentage change in E exceeds the percentage change in P. In this last case, real disturbances cause the exchange rate to have greater volatility than the price level.

The result that the exchange rate is unaffected by the probability distribution of future money supplies and outputs is eliminated if the model is altered so that velocity is variable. In the model of Stockman (1980), velocity is variable because households, when they visit asset markets, do not observe the nominal prices that they will subsequently face in the goods market. In Lucas' model, positive nominal interest rates lead households to leave the asset market with just enough money to finance planned consumption (and never more, since that would involve sacrificing interest): (5.17) and (5.18) hold as equalities, and aggregation yields (5.19) and (5.20) in equilibrium. But if goods prices are uncertain when households choose their portfolios, they must trade off foregone interest against the utility cost of the consumption foregone in the event that they have insufficient cash to finance desired consumption. This results in a "precautionary" demand for money as well as a "transactions" demand. Because velocities depend on interest rates, constraints (5.17) and (5.18) are not necessarily binding in equilibrium.

Consider a model in which households visit the goods-market at the beginning of each period and the asset market at the end of each period, as in Stockman (1980). The sequence of events now requires households to use money obtained in period t-1, plus transfer payments at the beginning of period t, to buy goods in period t. The prices of goods in period t, however, are uncertain when portfolios are allocated among assets in period t-1. Suppose also that the only assets available are the two monies and two one-period nominal bonds, B and F, which pay $1 + r$ and $1 + r^*$ units of domestic and foreign currency (respectively) after a period. The limited menu of assets implies that it is no longer feasible to perfectly pool risk and that the current account of the balance of payments is no longer identically zero, as in Lucas' model.

To simplify the analysis, we now assume that the representative agent's planning horizon is finite. A domestic consumer maximizes

$$(5.24)\qquad W(\{C_t\}_{t=0}^{H}, \{C_t^*\}_{t=0}^{H}) = \underline{E}_0\{\sum_{t=0}^{H} \beta^t U(C_t, C_t^*)\}$$

subject to the budget constraint

$$(5.25)\qquad P_tY_t + M_{t-1} + P_tT_t + E_t M_{t-1}^* + (1+r_{t-1})B_{t-1} + (1+r_{t-1}^*)E_tF_{t-1}$$

$$- P_tC_t - E_tP_t^*C_t^* - B_t - E_tF_t - M_t - E_tM_t^* = 0,$$

the cash-in-advance constraints

$$(5.26)\qquad M_{t-1} + P_tT_t \geq P_tC_t, \quad E_tM_{t-1}^* \geq E_tP_t^*C_t^*,$$

initial conditions on asset stocks, and terminal conditions preventing debt at time $t = H$. A foreign representative household solves a similar problem but with income EP^*Y^* from selling the foreign good (instead of PY) and transfer payments EP^*T^* from the foreign government (instead of PT). Domestic and foreign outputs, Y and Y^*, and money supplies, M and M^*, are stochastic. Equilibrium requires that markets for all goods and assets clear.

The model can be solved by working backwards from time $t = H$. The necessary conditions for the optimization problem and the equilibrium conditions can be used to obtain the expression

$$(5.27)\qquad E_t = \underline{E}_t[U_{C^*}(C_{t+1}, C_{t+1}^*)/P_{t+1}^*] \,/\, \underline{E}_t[U_C(C_{t+1}, C_{t+1}^*)/P_{t+1}] \quad (t < H),$$

where C and C^* are now the equilibrium levels of consumption in the domestic country (which depend on the international distribution of wealth). While it is not possible to obtain a simple reduced-form expression for the exchange rate in the general case, (5.27) restricts the relation between the exchange rate and other endogenous variables in a manner similar to consumption-based models of asset pricing (Hansen and Singleton, 1983). This restriction does not depend heavily on the set of assets available to households. Svensson (1983), for example, obtains a similar result in a modified version of Lucas' model that permits variable velocities of money. As in the model of section 2, with $\lambda > 0$ and $\sigma > 0$, anticipated future outputs and money supplies affect the exchange rate: (5.27) relates E_t to the probability distributions of Y_{t+1}, Y^*_{t+1}, P_{t+1}, and P^*_{t+1}.

Money is neutral in this model if nominal transfers are distributed in proportion to initial net nominal asset stocks.[34] But money is not superneutral because inflation in either currency acts as a tax on goods purchased with that currency. Thus, anticipated inflation affects the terms of trade and so the exchange rate.

If rates of time preference differ across countries, the model has no steady state in which all agents have positive wealth and consumption. Helpman and Razin (1982), assuming perfect foresight and an infinite horizon, discuss the exchange rate changes that occur in this case as wealth is redistributed across countries. The results are very similar to those derived in the model of section 5.1.

5.3 The Role of Non-traded Goods

The previous sections of this chapter have focused mainly on models in which all goods are traded. But the exchange-rate effects of disturbances in the market for non-traded goods differ from those of disturbances in markets for traded goods. In all the models we have considered, an increase in domestic output raises the demand for money and tends to reduce all nominal prices including E. As we have seen, however, the reduction in E is mitigated or reversed if the rise in domestic output causes a fall in the terms of trade. Lucas' model implies that the exchange rate falls unless the condition for immiserizing growth is met. In contrast, the exchange-rate effect of an increase in the supply of non-tradables depends on the parameters of the demand for money. This will be illustrated by a two-country, finite-horizon model with a single traded good and a non-traded good in each country.

Assume that a representative domestic household maximizes

$$(5.28)\quad W(\{C_t^T\}_{t=0}^H, \{C_t^N\}_{t=0}^H, \{\ell_t\}_{t=0}^H) = \underline{E}_0 \{\sum_{t=0}^H \beta^t U(C_t^T, C_t^N, \ell_t)\}$$

where C^T and C^N denote consumptions of traded and non-traded goods and real balances ℓ are defined as nominal money M deflated by a price index Π. The latter is a weighted average of the money prices of traded and non-traded goods, P^T and P^N. Maximization of (5.28) is subject to initial and terminal conditions[35] and budget constraints of the form,

$$(5.29)\quad P_t^T(Y_t^T-C_t^T) + P_t^N(Y_t^N-C_t^N) + P_t^T(B_{t-1} - P_t^B B_t) + M_{t-1} + P_t T_t - M_t = 0.$$

Above, Y^T and Y^N are endowments of traded and nontraded goods; B_{t-1} is the number of real bonds (claims to one unit of the traded good delivered at date t) purchased at date t-1 at price P_{t-1}^B; M_{t-1} denotes nominal money held before the period t transfer payment of money, $P_t T_t$; and M_t is the nominal money holding chosen by the household at date t. There is a similar maximization problem for the representative foreign household. (Foreign variables are marked with an asterisk.) We assume that rates of time preference are constant and equal in the two countries and that U and U^* are separable in their three arguments. Implicit in (5.29) is the assumption that national monies are not traded between countries.

The properties of the model can be analyzed by working backwards from the final period.

Equilibrium conditions require that the world market for tradables clear; that the two markets for non-tradables clear; and that all asset markets clear. These conditions can be combined with the necessary conditions for utility maximization to show the effect of changes in traded and non-traded outputs on the final period's exchange rate. Abstracting from any changes in money supplies or foreign output of either good, we can write

(5.30) $E_H = g^H(Y^T, Y^N)$.

Let Π_1 and Π_2 be the partial derivatives of the price index $\Pi = \Pi(P^T, P^N)$, let J' be the derivative of $J(\cdot) = U_{C^T}^{-1}(\cdot)$, and define

$$K \equiv \frac{\Pi_1 P^N}{\Pi} [U_\ell + U_{\ell\ell}\ell]/[U_\ell(1 - \frac{\Pi_2 P^N}{\Pi}) - U_{\ell\ell}\ell \frac{\Pi_2 P^N}{\Pi}] .$$

The partial derivatives of the function $g^H(\cdot,\cdot)$ appearing in (5.30) are

$$(5.31) \quad g^H_{Y^T} = [\frac{J' g^H}{\Pi P^T} \{\frac{P^T}{\Pi} [U_\ell + U_{\ell\ell}\ell](\Pi_1 + \Pi_2 K) - U_\ell\}]^{-1},$$

$$(5.32) \quad g^H_{Y^N} = g^H_{Y^T}(\frac{J' P^T}{\Pi^2})[U_\ell + U_{\ell\ell}\ell](\Pi_1 + \Pi_2 K) .$$

Since Π is a price index, it is homogeneous of degree one in P^T and P^N, so the denominator of K is positive. Because $J' < 0$ and (as is easily verified) $\Pi_1 + \Pi_2 K > 0$, $g^H_{Y^T}$ is negative. An increase in Y^T thus causes an appreciation of the domestic currency. But the sign of $g^H_{Y^N}$ depends on the sign of the term $U_\ell + U_{\ell\ell}\ell$. Denote the elasticity of the marginal utility of money by $\chi \equiv (MU_{\ell\ell}/\Pi U_\ell)$. Then an increase in the output of the non-traded good lowers (raises) the exchange rate, E, as $\chi \lessgtr -1$. If the marginal utility of money is elastic, then an increase in Y^N causes the domestic currency to appreciate, just as an increase in Y^T does. But if the marginal utility of money is inelastic, then an increase in Y^N depreciates the domestic currency.[36]

This result illustrates the different rules for obtaining a probability distribution on the exchange rate from the probability densities on outputs of traded and non-traded goods. Although the specific rule derived above applies only to the final period of the model, a recursive solution of the model shows how the dynamics of exchange rate changes over time are affected differently by disturbances in the two sectors. To conserve space, we discuss here the backward recursion in the intermediate case in which χ is constant and equal to -1.

The optimization problem of the representative domestic household in an (H+1)-period model takes the form of maximizing

$$(5.33)\qquad U(C_t^T, C_t^N, \ell_t) + \beta \underline{E}_t V(M_t, B_t, H - t)$$

subject to (5.29), where V is the value function or indirect utility function that shows the maximized value of utility from periods t + 1 through H. Let η_t be the time-t shadow value of a unit of domestic money. Standard techniques can be used to obtain the necessary conditions,

$$(5.34)\qquad \eta_t = \frac{U_{C^T}}{P_t^T} = \frac{U_{C^N}}{P_t^N} = \frac{U_\ell}{\Pi_t} + \beta \underline{E}_t(\eta_{t+1}) = \beta \frac{1}{P_t^T P_t^B} \underline{E}_t\left[\eta_{t+1} P_{t+1}^T\right] .$$

Similar conditions are obtained for the analogous maximization problem in the foreign country.

If the elasticity of the marginal utility of money is one, then $\eta_H \propto M_H^{-1}$, $\eta_{H-1} \propto M_{H-1}^{-1} + \beta \underline{E}_{H-1}(1/M_H)$ and η_t is independent of the output of non-traded goods for all t (as long as the output of non-traded goods in t does not affect $\underline{E}_t(\frac{1}{M_{t+s}})$ for s > 0, i.e. as long as it does not signal future changes in the money supply -- a possibility from which we abstract in this discussion). Since $\eta_H P_H^T = U_{C^T}(Y_H^T + B_{H-1})$ and $\eta_H^* P_H^{T*} = U_{C^T}^*(Y_H^{T*} - B_{H-1})$, the conditions

$$(5.35)\qquad \eta_t = \frac{\beta}{P_t^T P_t^B} \underline{E}_t(\eta_{t+1} P_{t+1}^T), \quad \eta_t^* = \frac{\beta}{P_t^{T*} P_t^B} \underline{E}_t(\eta_{t+1}^* P_{t+1}^{T*}) ,$$

can be used to solve for P_{H-1}^B and B_{H-1}. The important property of the solution (used below) is that P^B and B are functions of P^T and P^{T*} (as well as of η and η^*) but not (at least directly) functions of Y^N or Y^{N*}. Let $J^*(\cdot) = U_{C^T}^{*-1}(\cdot)$. The equations

$$(5.36) \qquad Y_t^T - J(\eta_t P_t^T) = -B_{t-1} + P_t^B B_t , \qquad Y_t^{T*} - J^*(\eta_t^* P_t^{T*}) = B_{t-1} - P_t^B B_t,$$

allow us to define the functions

$$(5.37) \qquad P_t^T = h(B_{t-1}, \eta_t, \eta_t^*, Y_t^T, Y_t^{T*}, H-t), \; P_t^{T*} = h^*(B_{t-1}, \eta_t, \eta_t^*, Y_t^T, Y_t^{T*}, H-t).$$

Then (5.35) and (5.37) constitute a system of four equations that can be solved for P^B, B, P^T, and P^{T*} as functions of Y^T, Y^{T*}, η, and η^*. But it was already demonstrated that η and η^* are independent of current and expected future outputs. Thus the exchange rate $E = P^T/P^{T*}$ depends on (current) outputs of traded goods in each country, but is independent of the outputs of non-traded goods when $\chi = -1$. By varying χ, one can change the rule that translates the dynamics of the output of both traded and non-traded goods into the dynamics of the exchange rate. The probability distribution on the exchange rate can be independent of the probability distribution on outputs of non-traded goods, as when $\chi = -1$, or (as in the analysis of $t = H$) the probability distribution induced on the exchange rate can respond in similar or in very different ways to the probability distributions on outputs of traded and non-traded goods.

This result that an increase in output of non-traded goods can push the value of the domestic currency upward or downward should not be surprising even if the precise condition was not initially obvious. On the one hand, an increase in Y^N reduces P^N and, for any given demand for money, requires a higher exchange rate to keep a weighted average of P^N and P^T fixed, as required for money-market equilibrium. On the other hand, an increase in Y^N raises aggregate output at the initial relative price and, given P^N, raises the demand for money and requires a lower exchange rate. The relative strengths of these effects turn on the elasticity of the marginal utility of money.

6. Conclusion

This chapter has reviewed a variety of dynamic exchange-rate models. These models have been developed to explain certain facts about floating rates, but they have other testable implications that can, perhaps be used in the future to further limit the set of models that are consistent with the data. Existing empirical research on the models is inconclusive, however.

A common feature of all the models we have discussed is the assumption of rational expectations: individuals know the structure of the economy, and use all available information to make optimal forecasts of future variables. Most of the models reviewed can be analyzed under alternative expectational assumptions, as in Kouri (1976). But while the informational requirements of the rational-expectations hypothesis may appear extreme, we see two principal reasons for basing exchange-rate models on the assumption of rationality. First, the assumption yields results that arise entirely from the inherent logic of a model, not from arbitrary expectation-formation mechanisms that have been grafted onto it. Second, the assumption is probably much closer to the truth than simple alternatives like "static" or "adaptive" expectations. Exchange rates clearly do respond to anticipated future events, and while the rationality hypothesis may be incorrect in a literal sense, the qualitative correctness of its implications is difficult to deny. As the chapter has illustrated, expectations play a key role in exchange-rate determination, and little can be said about short- and medium-run exchange rate behavior unless some stand on the process generating expectations is taken. It is unfortunate, therefore, that formal empirical tests are unlikely to provide decisive evidence for or against rational expectations. As Levich argues in chapter 18, any test of rationality is a joint test of rationality <u>and</u> an underlying exchange-rate model which may itself be inappropriate.

The additional assumption of saddle-path stability was invoked repeatedly in the models studied above. That assumption requires more than just the efficient forecasting of future prices. There must also be market forces that prevent the emergence of self-fulfilling speculative bubbles, so that the exchange rate is tied to its fundamental determinants. Several simple theoretical models show how bubbles can be ruled out through considerations of intertemporal arbitrage or possible government interventions (see, e.g., Obstfeld and Rogoff, 1983). Casual empiricism reinforces these theoretical results, for it suggests that protracted bubbles have not been a feature of the recent experience with floating rates. Unfortunately, identification problems similar to those involved in testing rational expectations plague any attempt to detect speculative bubbles in actual data.

The question of which models and types of implications will be most useful in future attempts to understand exchange rates is open, and leads to deep philosophical and statistical questions that we will not try to resolve here. Nonetheless, it seems likely to us that as more data become available, progress will be made in serious attempts to develop and test new implications of models similar to those discussed above.

Footnotes

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1. The economy is small in the sense that it faces a given foreign interest rate and a given foreign price of imports. However, the economy faces a downward-sloping demand curve for its export good. An alternative assumption, which yields a similar model, is that the economy produces both a traded good priced in the world market and a non-traded good priced at home (Dornbusch, 1976a). Sections 3 and 5 below study models with non-traded goods.

2. A number of recent econometric studies reject this hypothesis (see chapter 18 by Levich). No alternative model of nominal interest differentials has received much statistical support, however, so it seems reasonable to entertain the perfect substitution hypothesis as a close empirical approximation for some applications.

3. This assumption implies very strong restrictions on the form of any underlying optimization problem, as it severely limits the allowable substitutions between goods at different points in time. Note that the domestic real interest rate defined here can differ from the world real interest rate $r^* - \dot{p}^*$.

4. Sargent and Wallace (1973) proposed this convention for monetary perfect-foresight models. While the exclusion of bubbles typically results in a unique equilibrium (as in the present case), there exist "badly-behaved" models with multiple convergent equilibrium paths. (See Calvo, 1979, for an example.) Such models possess multiple rational-expectations equilibria.

5. Obstfeld and Rogoff (1983), building on work by Brock (1975), describe conditions under which the saddle path can be identified as the unique equilibrium path in an economy of infinitely-lived maximizing agents similar to the one studied in section 5.1 below.

6. It can be verified that if r^* and $\dot{p}^*$ rise permanently by equal amounts, the paths of p and r are unaffected and the level of e does not change immediately. However, the currency begins to appreciate over time so as to offset rising foreign prices and the higher level of r^*. This result may seem strange: a rise in foreign trend inflation should be associated with a rise in foreign monetary growth and hence an appreciation of the domestic currency. The reason e does not jump immediately in the present model is the small country assumption, which artificially holds p^* constant when foreign inflation rises. In a two-country model, an increase in foreign money growth would move the exchange rate on impact.

7. Does the discrete, anticipated fall in the interest rate r at time T represents a violation of this principle? The answer is no, essentially because the short-term interest rate r is the nominal return on a bond of instantaneous maturity. What cannot jump, in the present setting, is the price of a long-term bond or consol. If P^C represents the price of a consol and C is the coupon payment, then, under perfect substitution, the short-term interest rate must satisfy the arbitrage condition $r = (C + \dot{P}^C)/P^C$ equating instantaneous returns on short- and long-term assets. The saddle-path solution for P^C as a function of the coupon C and expected future short rates is:

$$P_t^C = \int_t^{\infty} C\cdot\exp(-\int_t^s r_\tau d\tau)ds.$$

Thus, P^C does not jump when an anticipated discrete jump in r occurs. Rogoff (1979) studies the impact of anticipated monetary disturbances on the term structure of interest rates. For a more detailed discussion of the asset-price continuity condition, see Calvo (1977).

It is apparent from (2.10) and the definitions of the forcing functions x and z that anticipated discrete jumps in the foreign price level p^* have no impact on the exchange rate until the moment they occur (provided the expected paths of the other exogenous variables are not simultaneously affected). This fact does appear to contradict the asset-price continuity condition, since an anticipated discrete jump in p^* then implies an equal and opposite anticipated discrete jump in e. The small-country assumption is again responsible for this rather artificial violation of the continuity condition (cf. footnote 6, above). Because anticipated discrete jumps in p^* would generally be impossible in an explicit two-country model, the problem would not arise; and we therefore assume in what follows that the path of p^* is expected to be continuous.

8. This is also a property of the Lucas (1982) model discussed in section 5.2. When $\lambda = \sigma = 0$, $\tilde{e}$ and $\tilde{p}$ are given by $\tilde{e}_t = m_t + [(\alpha-\psi\phi)/\phi]\bar{y}_t - (\alpha/\phi)g_t - p^*_t$ and $\tilde{p}_t = m_t - [(1-\alpha+\psi\phi)/\phi]\bar{y}_t + [(1-\alpha)/\phi]g_t$. It is important to keep in mind that many of the anticipated discrete price jumps possible in the small-country case when $\sigma = 0$ or $\lambda = 0$ would be impossible in a two-country model if the corresponding foreign interest elasticities differed from zero.

9. The analysis here differs from that in Flood and Garber (1983) because we assume agents know the date at which the return to fixed rates will take place. Flood and Garber allow that date to be endogenously determined as the date of the exchange rate's first passage through its new peg, but they are able to obtain a determinate solution to their problem only through the tacit assumption that central-bank foreign reserves are not expected to jump at the moment of pegging. No theoretical justification for such an assumption has been suggested, however.

10. Other exogenous variables are unaffected, as always. We also assume that the monetary and fiscal policies pursued both before and after pegging do not result in a speculative attack on the central bank's reserves.

11. The price $\bar{\bar{p}}$ can differ from $\tilde{p}$, the output price clearing the goods market in a full flexible-price equilibrium. Obstfeld and Rogoff (1984) show that the alternative pricing rule $\dot{p}_t = \zeta (y_t - \bar{y}_t) + \dot{\tilde{p}}_t$ leads to an observationally equivalent exchange-rate model if ζ is chosen suitably. Dornbusch (1976b) implicitly adopts the latter pricing scheme. Frankel (1979), Liviatan (1980), and Buiter and Miller (1982) allow for an inflationary steady state by appending to the Phillips curve an expectations term equal to the current monetary growth rate. This formulation is consistent with rational expectations only if there are no anticipated changes in money growth or other exogenous variables (Obstfeld and Rogoff, 1984).

12. In general, one can say that the exchange rate overshoots in response to a shock if its impact change exceeds the change that would be necessary if all predetermined variables could move immediately to their long-run levels. Flood (1979) offers this definition of overshooting. In the present context, the "long-run" level of the predetermined domestic output price is its flexible-price value. But, as we shall see below, overshooting can arise even with flexible prices if behavior is influenced by predetermined asset stocks that adjust over time toward some long-run target.

13. The case shown in figure 3.1 implies exchange-rate overshooting in response to an unanticipated money supply change. There are two other cases. If the first inequality in (3.18) is reversed, the same disturbance causes undershooting [cf. (3.13)]. If only the second inequality in (3.18) is reversed, there is overshooting and the $\dot{p} = 0$ locus is negatively, rather than positively,

sloped. In all cases the long-run equilibrium is a saddle point, but when $\alpha - \psi\phi(1+\theta\sigma\gamma) < 0$, the saddle path slopes upward.

14. Marston's chapter includes extensive references to the literature in this area.

15. The possibility of a temporary appreciation is due entirely to the effect of money on expenditure: if the relative price of non-traded goods is pushed sufficiently high, a currency appreciation may be required to restore money-market equilibrium in the short run. A similar result is derived by Kind (1982) in a sticky-price model incorporating external asset accumulation.

16. In the present model, as in that of Flood and Hodrick (1983), only unperceived (or unanticipated) money has real effects. As we shall see in the next section, however, a fully understood change in trend inflation can have real effects in models with external asset accumulation. These real effects are absent in this particular model because money demand is insensitive to the nominal interest rate [equation (3.24)] and expenditure depends, not on real money balances, but on the unperceived component of the nominal money supply.

17. A satisfactory treatment of portfolio diversification naturally requires specification of both investors' preferences and the stochastic processes generating real returns. For a discussion of these topics, see chapter 15 by Branson and Henderson.

18. This model is more general than it seems to be, for it would be easy to introduce a domestic-currency bond paying an interest rate linked to the foreign rate by interest parity. Under perfect substitution, however, the fraction of domestic wealth held in the form of foreign-currency bonds is indeterminate, and so unanticipated shocks causing exchange-rate changes have indeterminate wealth effects. The problem does not arise in portfolio models assuming that bonds of different

currency denomination are imperfect substitutes. In chapter 15, Branson and Henderson discuss portfolio-balance models that include imperfectly substitutable interest-bearing assets.

19. In principle, consumption is also a function of the real interest rate r^*, as in previous sections. Because r^* is held constant, it does not appear explicitly in (4.2).

20. There is an implicit assumption that the central bank does not hold interest-bearing foreign reserves. If these were held, the income they yielded could be used to help finance government outlays.

21. Henderson and Rogoff (1982) study the stability properties of a two-country portfolio-balance model and allow for the possibility of negative net foreign asset stocks. Kouri (1983, appendix 3) discusses a small-country case. These authors conclude that saddle-path stability must always obtain under rational expectations. However, this result follows from their assumption that interest earnings on foreign assets do not affect the current account. As expression (4.11) shows, the present model always has the saddle-path property in the special case $r^*=0$; but if $r^*>0$, an otherwise well-behaved model can become completely unstable once the possibility that $1 - L < 0$ is admitted. Fortunately, this is never a problem in a model that incorporates an appropriate definition of wealth. As section 5, below, shows, the private intertemporal budget constraint (4.1) and the government constraint (4.5) imply that the present value of future private consumption is bounded from above by $F + (\bar{Y} + \bar{T})/r^*$ in equilibrium [see equation (5.14)]. Accordingly, that quantity will normally be positive.

22. In the case where $\partial\dot{F}/\partial F > 0$, the $\dot{F} = 0$ locus is positively sloped but steeper than the $\dot{\ell} = 0$ locus. The saddle path SS lies between these two loci and thus slopes upward as in figure 4.1.

23. We elaborate on this point in section 4.2; see also Kouri and Macedo (1978). Another exception can occur when there are more than two countries (Krugman, 1983). Two distinct alternative mechanisms can give rise to the familiar correlation between the exchange rate and current account along a convergent path. The pattern arises in models assuming imperfect asset substitutability when domestic residents have a greater marginal propensity to hold wealth in the form of domestic-currency bonds than do foreigners (see chapter 15 by Branson and Henderson). Even when all bonds are perfect substitutes and wealth does not enter the money-demand function, the pattern will arise when domestic and foreign goods are imperfect substitutes in consumption, the terms of trade are endogenous, domestic residents have a relative preference for domestic goods, and the home goods market is stable in the Walrasian sense (see Calvo and Rodriguez, 1977; Dornbusch and Fischer, 1980; and Obstfeld, 1980). This second case reflects the usual transfer mechanism whereby a current-account induced transfer of wealth from abroad raises demand for home goods, improving the terms of trade.

24. An anticipated increase in the money stock can induce current-account adjustment, however (Dornbusch and Fischer, 1980). We have considered a "helicopter" monetary expansion rather than an expansion achieved through a central-bank purchase of bonds. The latter operation has the same effect as the helicopter expansion if individuals fully capitalize expected future transfer payments from the government (as they do here). Because the interest earnings on bonds purchased from the public are merely returned to the public in the form of transfers, there are no real effects (Obstfeld, 1981, and Stockman, 1983). If capitalization is incomplete, however, an official bond purchase will induce a current-account surplus, as in Kouri (1976, 1983).

25. When $\partial\dot{F}/\partial F > 0$ (the case discussed in footnote 22), it is possible that long-run real balances rise in response to an increase in $\bar{\mu}$. Long-run foreign assets

must also rise in this case. All one can say in general is that the ratio of real balances to other wealth must fall.

26. In the case shown in figure 4.2, the exchange rate overshoots (in the sense discussed in footnote 12).

27. The model would not be altered if trade in equities were introduced, but the assumption in the text avoids some additional notation. If trade in capital goods were allowed, however, the rate of domestic investment would become indeterminate.

28. The factor intensity assumption is crucial, as it yields both the uniqueness of P^K [from equation (4.13)] and the stability of the capital-accumulation process. It is also important that capital depreciates at a positive rate. If ε were zero, the economy would be specialized at the steady state and the Stolper-Samuelson and Rybczynski arguments would therefore not apply.

29. As footnote 22 suggests, there is an alternative configuration in which the $\dot{V} - \dot{\ell} = 0$ locus slopes upward.

30. Another class of models introduces money through the assumption of finitely-lived, overlapping generations. See, for example, Helpman and Razin (1979), Kareken and Wallace (1981), Clarida (1982), Eaton (1982), and Lapan and Enders (1983). Limited space precludes an adequate discussion of the interesting questions raised by these models.

31. The utility function is strictly concave and twice continuously differentiable. Also assumed are the standard Inada conditions. Both consumption and money services are normal goods.

32. The calculation leads to the equation

$$\int_t^{\infty} C_s \exp[-r^*(s-t)]ds = F_t + \tilde{Y}_t - \tilde{G}_t + \lim_{s\to\infty} \ell_s \exp[-r^*(s-t)].$$

By (5.10), however, $\lim_{s\to\infty} \ell_s \exp[-r^*(s-t)] = \ell_t(\lim_{s\to\infty} \exp\{[-r^* + (r^*-\delta)/R](s-t)\})$. The last limit is zero because of the assumption that $-r^* + (r^*-\delta)/R < 0$.

33. The intertemporal welfare criterion with an endogenous time preference rate is no longer time additive, unlike the criterion W given by equation (5.1). Lucas and Stokey (1982) study a general optimal growth model with heterogenous consumers whose intertemporal preferences are not time additive. They, too, find that "the hypothesis of increasing marginal impatience ... appears to be an essential component that any theory within the class considered in this paper must possess if it is to generate dynamics under which wealth distributions converge to determinate, stationary equilibria in which all agents have positive wealth and consumption levels." See also Epstein and Hynes (1983). If the constant time preference hypothesis is retained, the assumption that real bonds (as well as real money balances) yield direct utility leads to a model with a unique small-country steady state (Liviatan, 1981).

34. Helpman and Razin (1982) emphasize the real wealth-redistribution effects of unanticipated increases in money supplies when there are unindexed nominal bonds. Note that equation (5.27) would still hold if bonds were indexed, as assumed in section 5.3, below.

35. The terminal condition used here requires that all debts be paid at the end of the final period.

36. Obviously a Cobb-Douglas utility function leads to the result that the exchange rate in the final period is independent of the supply of non-traded goods (though it is not independent of the supply of traded goods). See Stockman (1983).

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