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TRADE LIBERALIZATION WITH HETEROGENOUS FIRMS

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This paper is a substantially revised version of the Baldwin and Forslid (2004) working paper with an identical title. The present paper, however, also includes elements from Baldwin (2005), which has not and will not be submitted for publication anywhere. We are grateful for comments from Elhanan Helpman and Marc Melitz as well as other participants at ERWIT 2004. Any remaining errors are our own. Forslid thanks The Bank of Sweden Tercentenary Foundation (Reg. no. J2001-0684:1) for financial support. The views expressed herein are those of the author(s) and do not necessarily reflect the views of the National Bureau of Economic Research.

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Trade liberalisation with heterogeneous firms♣

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JEL: H32, P16.
Keywords: trade liberalisation, heterogeneous firms, anti-variety effect

1. INTRODUCTION

Empirical work over the past decade shows that the standard new trade theory assumption of identical firms glosses over many important aspects of reality. For instance, not all firms even trade in traded goods sectors, and productivity is also typically higher among exporting than non-exporting firms in a sector (Aw, Chung and Roberts 2000, Bernard and Jensen 1995, 1999a,b, 2001; Clerides, Lach and Tybout 1998; Eaton, Kortum and Kramarz 2004; see Tybout 2003 for a survey)

This empirical evidence has lead to the development of trade models which are modified to allow for a more sophisticated view of firms. The microeconomic link between trade liberalisation and firm productivity is explicitly modelled in Eaton and Kortum (2002), Bernard, Eaton, Jensen and

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Kortum (2003), Melitz (2003), and Yeaple (2005).[1] Extensions are found in Baldwin and Okubo (2005), Baldwin and Robert-Nicoud (2005), Bernard, Eaton, Jensen and Schott (2003), Bernard, Redding, and Schott (2004), Falvey, Greenaway and Yu (2004), Helpman, Melitz and Yeaple (2004), Melitz and Ottaviano (2005), and Yeaple (2004).

In this paper, we examine the various positive and normative aspects of trade liberalisation with heterogeneous firms using the Melitz (2003) model. The effect of lower trade costs as well as the effect of lower regulatory barriers to trade (beachhead costs) when countries are asymmetric in size are analysed. We find a number of novel results and effects including a Stolper-Samuelson like result and several results related to the volume of trade, which are empirically testable. Moreover we find what might be called an anti-variety effect, meaning that the consumed variety (the available range of product varieties) may fall in a country, as a result of trade liberalisation. Interestingly, the anti-variety effect resonates with the often voiced criticism from antiglobalists that globalisation leads the world to become more homogenous by eliminating local specialities. However, despite this anti-variety effect, we note that we find that trade liberalisation always leads to welfare gains in the model.

Though distinctly different, our paper is related to Melitz and Ottaviano (2005), who analyse trade liberalisation in a modified Melitz (2003) model, where a linear demand system à la Ottaviano, Tabuchi and Thisse (2002) is used. This specification allows them to study pro-competitive effects, which are absent in the standard model we use (as in all Dixit-Stiglitz based models). However, contrary to our paper, trade integration always produces an increased variety for consumers in their model specification. Another notable difference is that because Melitz and Ottaviano assume away the fixed beachhead costs in each market, they can not analyse regulatory liberalisation.

The rest of the paper is organised in four sections. The next section presents the model. Section 3 studies the positive effects of two types of trade liberalisation – the standard reduction in the marginal cost of trading goods and the reduction of fixed market-entry costs implied by so-called technical barriers to trade. The fourth section presents welfare results, and the final section concludes.

[1] Antecedents to these models are Montagna (2001) and Schmitt and Yu (2001).

2. THE MODEL

We work with the basic trade model with heterogeneous firms of Helpman, Melitz and Yeaple (2003). There are two nations that are identical in all respects except that they potentially differ by size. Each nation uses a single primary factor of production (labour L) to produce goods in two sectors (the T-sector and the M-sector). The T-sector (traditional sector) is a Walrasian, homogenous-goods sector with costless trade. The M-sector (manufactures) is marked by increasing returns, Dixit-Stiglitz monopolistic competition and iceberg trade costs. M-sector firms face constant marginal production costs and three types of fixed costs. The first fixed cost, F_I, is the standard Dixit-Stiglitz cost of developing a new variety (I is a mnemonic for innovation). The second and third fixed costs are what have been called 'beachhead' costs since they reflect the one-time expense of introducing a new variety into a market (i.e. establishing a beachhead). Specifically, the cost of introducing a new variety to domestic and non-domestic markets is F_D and F_X, respectively (D for domestic and X for exports).

Crucially, the model allows for heterogeneity with respect to firms' marginal production costs. Each Dixit-Stiglitz firm/variety is associated with a particular labour input coefficient – denoted as a_j for firm j. These a's are generated during a product innovation process that costs F_I units of labour. Specifically, just after paying F_I entry cost, the firm is randomly assigned an 'a_j' from the density function G[a], whose support is $0 \le a \le a_0$. Intuition may be served by thinking of the entry-cum-lottery as a single innovation process. That is to say, the innovation technology is stochastic since sinking F_I units of labour produces a 'blueprint' for a new variety with certainty, but the associated marginal cost is random.

Our analysis exclusively focuses on steady state equilibria and intertemporal discounting is ignored; the present value of firms is kept finite by assuming that firms face a constant probability of 'death' according to a Poisson process with the hazard rate δ.

Consumers in each nation have two-tier utility functions with the upper tier (Cobb-Douglas) determining the consumer's division of expenditure among sectors and the second tier (CES) dictating the consumer's preferences over the various differentiated varieties within the M-sector.

$$(1) \qquad U = \ln E - \ln P\,; \quad P \equiv \left(p_T\right)^{1-\mu}\left(\int_{i\in\Theta} p_i^{\,1-\sigma} di\right)^{\mu/(1-\sigma)}; \qquad 0 \le \mu \le 1 < \sigma\,,$$

where E is expenditure, p_T is the price of the homogenous traditional good, p_i is the consumer price of variety i, and Θ is the set of all varieties consumed; σ is the constant elasticity of substitution among varieties and μ is the Cobb-Douglas spending share on manufactures.

2.1. Equilibrium

As is well known, constant returns, perfect competition and zero trade costs in the T-sector equalise wages in the two nations. With a proper choice of units and numeraire, we have:

$$p_T = w = w^* = 1. \tag{2}$$

With nominal wages pinned down at unity, we can without ambiguity refer to a M-sector firm's 'a' as its marginal costs.

Although each firm has its own marginal cost, intuition is boosted by grouping firms into three types: firms that produce but only sell locally (D-types, short for domestic firms), firms that sell locally and also export (X-types, short for export firms), and firms that do not produce (N types, short for non-producers). Intuitively, firms whose F_I investment yields a very high marginal cost would sell very little if they produce and so, they will not find it worthwhile to sink the F_D; these become N-types. Firms that draw very low a's (marginal costs) will sell a great deal if they produce and so, they find it worthwhile to sink the beachhead costs in domestic and export markets; these become X-types. Firms with intermediate a's become D-types. We now turn to the formal cut-off conditions for the three types.

2.1.1. The Cut-off Conditions

Given standard Dixit-Stiglitz results, the level of a firm's sales in its local market is related to its marginal cost and the marginal costs of its competitors according to:[2]

$$\frac{a^{1-\sigma}}{\Delta}\mu E\,; \qquad \Delta = n\int_0^{a_D} a^{1-\sigma} dG[a|a_D] + n^* \tau^{1-\sigma}\int_0^{a_X} a^{1-\sigma} dG[a|a_D]. \tag{3}$$

Here Δ (a mnemonic of the 'denominator' of the standard CES demand function) can be thought of as a weighted average of the marginal costs of all firms active in the market, n and n* are the masses of varieties produced in Home and Foreign, respectively, and τ is the iceberg trade cost; $G[a|a_D]$ is the conditional cumulative density function (only varieties with a's less than a_D are

[2] See guide to calculations.

produced so that we use the conditional cdf when considering the marginal cost of firm j relative to that of its competitors).

As usual with Dixit-Stiglitz monopolistic competition, the operating profit earned in this market will be $1/\sigma$ times the value of sales.[3] Since the beachhead costs are sunk, firms consider the present value of operating profit and the beachhead costs. Given the constant firm-death rate δ and the zero discount rate, the present value of a given firm is just π/δ, where π is the operating profit the firm would earn if it actually produces. For a Home-based firm, the cut-off levels of the marginal cost in the local and export markets are, respectively, defined by:[4]

$$a_D{}^{1-\sigma} B = f_D \; ; \qquad \phi a_X{}^{1-\sigma} B^* = f_X \; , \tag{4}$$

where

$$B \equiv \frac{\mu E}{\Delta}, \quad B^* \equiv \frac{\mu E^*}{\Delta^*}, \quad 0 \le \phi \equiv \tau^{1-\sigma} \le 1, \quad f_D \equiv \sigma \delta F_D, \quad f_X \equiv \sigma \delta F_X \, .$$

a_D and a_X are the cut-off marginal costs for entering the local market and the export market, respectively, and we have grouped σ δ and the F's for notational convenience; ϕ ranges from zero when trade is perfectly closed ($\tau=\infty$) to unity when trade is perfectly free ($\tau=1$); we refer to ϕ as the 'free-ness' of trade. The cut-off conditions for a Foreign-based firm are $a_D^{*\,1-\sigma} B^* = f_d$, and $a_X^{*\,1-\sigma} B = f_X$.

However, as shown by Helpman, Melitz and Yeaple (2003), $B=B^*$ in this model even if country size differs. The intuition for this is that free entry of firms will ensure that the expected operating profit (and therefore sales) must be the same in both markets in equilibrium, since fixed and variable costs are identical, trade costs are symmetric, and the distributions of a's are identical. Using that $B=B^*$ implies, from (4), that $a_D = a_D^*$, and $a_X = a_X^*$.

One fact that has been firmly established is that only a fraction of all firms that produce in a nation actually export (see Tybout 2003 for a survey of such findings). In terms of our model, this means that $a_X<a_D$ is a regularity condition. From (4), the necessary and sufficient condition for this is that:

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[3] See guide to calculations.
[4] See guide to calculations.

$$(5) \qquad \frac{a_X}{a_D} < 1 \qquad \Leftrightarrow \qquad \frac{F_X / \phi}{F_D} > 1\,.$$

We will take this as the base case.

2.1.2. The Free Entry Condition

A potential entrant pays F_I to develop a new variety with a randomly assigned 'a'. After developing the new variety and observing the associated 'a', the potential entrant decides whether to enter the local market only, or the local and export markets, or neither. For Home and Foreign, the free entry condition is that the ex ante (i.e. before 'a' is known) expected value of developing a variety equals the investment cost, F_I:

$$(6) \qquad \int_0^{a_D} \{a^{1-\sigma}B - f_D\} dG[a] + \int_0^{a_X} \{\phi a^{1-\sigma}B - f_X\} dG[a] \quad = \quad f_I\,.$$

The first and second integrals in (6) show the ex ante expected value of the operating profit net of beachhead costs arising from domestic sales and export sales (multiplied by σδ), respectively; F_I, which all firms must pay, is the sunk innovation cost. The free entry condition is identical for the two countries because $B=B^*$, $a_D = a_D^*$, and $a_X = a_X^*$.

An immediate implication from the free entry condition is that all active firms (D-types and X-types) except firms with a's exactly equal to a_D earn pure profits throughout their entire life in the sense of their revenue exceeding their variable costs by more than what would be needed to amortize their sunk costs. The reason is that the ex ante expected value of pure losses on N-types is balanced by the ex ante expected value of pure profits on D-types and X-types. These pure profits are not a payment to reward the foregone consumption wrapped up in the sunk costs (we have zero discounting), rather they are rents – rents earned for being lucky.

2.1.3. Solving the model with the Pareto distribution

All of the analysis up to this point has been conducted without resort to a functional form for G. Indeed some of the subsequent analysis can also be conducted in this manner, but the reasoning is clearer when we have explicit solutions, which requires an explicit G[a]. Following standard practice, we adopt the Pareto distribution:

$$(7) \qquad G[a] = \left(\frac{a}{a_0}\right)^k \qquad 0 \le a \le a_0 \equiv 1,$$

where k and a_0 are the 'shape' and 'scale' parameters, respectively. We normalise a_0 to unity without loss of generality (we are free to choose units of M-sector goods).

Given

(7) and the regularity condition $\beta \equiv k/(\sigma-1)>1$ (so that the integrals in Δ converge)[5]:

(8) $$\Delta = \frac{a_D^{1-\sigma}(1+\Omega)}{1-1/\beta}$$

where

$$0 \le \ \Omega \equiv \phi^{\beta} T^{1-\beta} \ \le 1, \qquad T \equiv \frac{F_X}{F_D}, \qquad \beta \equiv \frac{k}{\sigma-1} \ > \ 1 .$$

Here, Ω (a mnemonic for 'openness') summarises the impact of beachhead trade barriers and iceberg trade barriers. The variable Ω summarises the two types of trade barriers in the model, so it is worth pointing out four features of Ω that facilitate intuition and subsequent analysis: (1) Ω measures the combined protective effects of higher fixed and variable trade costs; (2) $\Omega=0$ with infinite τ and/or infinite F_X/F_D, (3) $\Omega=1$ with zero iceberg costs and $F_X=F_D$; (4) we can also express Ω as $\phi(F_X/F_D\phi)^{1-\beta}$ which tells us that as long as the inequality in **(5)** holds, Ω is bound between zero and unity.

Finally, with a zero discount rate, the foregone consumption necessary to create new varieties requires no compensation, so the only source of current income is labour and thus:[6]

(9) $$E = L .$$

Using this, **(3)**, the two cut-off conditions **(4)**, the free entry condition **(6)**, and **(8)**, we get explicit, closed form solutions for n, n*, a_D and a_X:[7]

(10)

$$n = \frac{(L-\Omega L^*)\mu(\beta-1)}{(1-\Omega^2)\beta f_D}; \quad n^* = \frac{(L^*-\Omega L)\mu(\beta-1)}{(1-\Omega^2)\beta f_D}; \quad a_D = \left(\frac{f_I(\beta-1)}{f_D(1+\Omega)}\right)^{\frac{1}{k}}; \quad a_X = \left(\frac{\Omega(\beta-1)f_I}{(1+\Omega)f_X}\right)^{\frac{1}{k}}.$$

Unlike in the standard Dixit-Stiglitz trade model – not all varieties are consumed by all agents since some are only sold locally. The number of consumed varieties in Home and Foreign are:[8]

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[5] See guide to calculations.

[6] In essence, all operating profit is spent on creating 'knowledge capital' in the form of the three types of sunk cost for the flow of new N, D and X types; labour income is spent on producing consumption goods.

$$(11) \qquad n_C = \frac{L(1-\Omega\psi) - L^*(\Omega-\psi)}{f_D\beta(1-\Omega^2)/(\beta-1)\mu}, \qquad n_C^* = \frac{L^*(1-\Omega\psi) - L(\Omega-\psi)}{f_D\beta(1-\Omega^2)/(\beta-1)\mu}; \qquad \psi \equiv (\frac{\phi}{T})^{\beta} \leq 1,$$

where n_C and n_C^* are the number of varieties consumed in Home and Foreign, respectively, and ψ is the ratio of X-types to D-types, $\left(\frac{a_X}{a_D}\right)^k$, in each nation.

2.1.4. Trade volume and pattern

One of the stark differences to the standard homogenous-firms trade model concerns the export pattern. In particular, only a fraction of firms export their goods. This model displays standard intra-industry trade in differentiated varieties produced by X-types, but the varieties of D-types are non-traded even though they would be classified as being in a 'traded goods' sector.

The value of the exports of a typical X-type firm tends to infinity as 'a' approaches zero, and equals f_X for $a=a_X$. For X-firms with intermediate a's, the value of exports is: $a^{1-\sigma}\phi\mu L/\Delta\sigma$, but the export cut-off condition tells us $\phi\mu L/\Delta\sigma = f_X/a_X^{1-\sigma}$, thus:

$$(12) \qquad v[a] = (\frac{a}{a_X})^{1-\sigma} f_X, \qquad V = \frac{\mu\Omega(L-\Omega L^*)}{(1-\Omega^2)},$$

where v[a] is the per firm export. Integrating over all X-types (weighting by frequency) and using **(10)**, we have V, which is defined as the total value of exports in terms of the numeraire, i.e. $nv[a]dG[a \mid a_D]$ integrated from 0 to a_X.

It is interesting to note that the standard approach to 'horizontal' intra-industry trade (IIT) and vertical IIT fails in this model.[9] Many studies use unit value indices to deduce product quality – with higher prices indicating higher quality; the underlying assumption is that the trade classifications are too broad and thus, group together goods that are fundamentally different. In this model, the goods are absolutely symmetric in terms of product characteristics but nevertheless, they have very different prices.

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[7] See guide to calculations.
[8] n_C equals n plus n* times the fraction of Foreign varieties exported, which equals $(a_X/a_D)^k$; using **(10)** yields the expression.
[9] An example is the study by Greenaway, Hine and Milner (1995).

3. POSITIVE EFFECTS OF LIBERALISATION

There are two natural definitions of trade liberalisation in this model, one concerns the variable cost of trade ϕ and the other concerns the differential beachhead cost for local and imported varieties F_X/F_D. As we shall see, the two types of trade barriers usually affect variables in an isomorphic manner, since they are combined into the aggregate measure of openness, Ω.

3.1. Lower marginal cost of trade; symmetric countries

We begin by considering reducing marginal trade costs, i.e. freer trade in the sense of $d\phi>0$, when nations are symmetric in size.

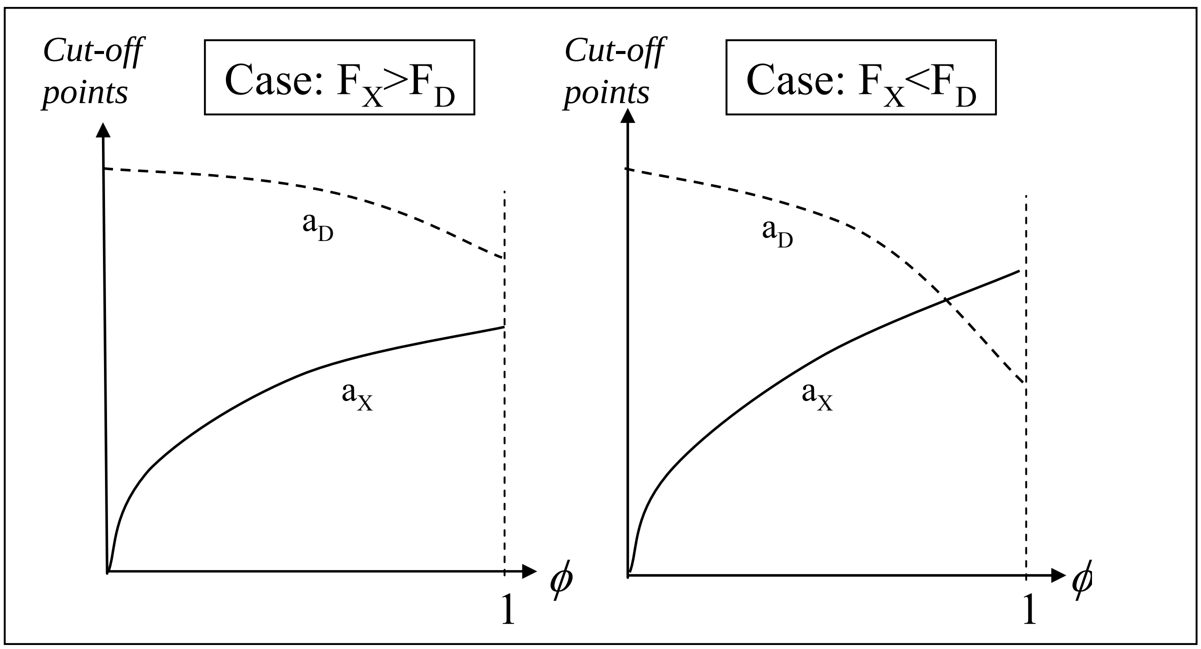

Figure 1: Cut-off points

Inspection of **(10**) confirms the finding of Melitz (2003) that lower marginal trade costs (i.e. $d\phi>0$) lowers a_D and raises a_X in both nations. When trade is at zero freeness (i.e. infinite trade costs) $a_X=0$, so that even a firm with zero marginal cost does not export. Greater openness lifts a_X while lowering a_D. Following Melitz (2003), we have $F_X/(\phi F_D)>1$ as our base case. However, for this to hold for all $\phi\in[0,1]$, it must be that $F_X > F_D$.

Figure 1 shows two cases. When $F_X<F_D$, the cut-off points cross at some level of openness less than free trade. When $a_X>a_D$, the model predicts that the most efficient firms sell in both markets, while the most inefficient firms only sell to the export market. This prediction does not agree with the facts and therefore, we take $F_X>F_D$ as the focal case. [10]

3.1.1. Anti-variety production and consumption effect

Turning to 'n' (the mass of produced varieties in a typical nation), an inspection of **(10)** with L=L* shows that freer trade reduces n in each nation. The proportional change is:

$$\hat{n} = -\beta \frac{\Omega}{1+\Omega}\hat{\phi} \tag{13}$$

where we have used the standard 'hat' notation for proportional changes (e.g. $\hat{x}$ equals dx/x), and $\Omega/(1+\Omega)$ is the import share (i.e. the expenditure share on all imported varieties in a typical market).[11] Since the import share rises with openness, this expression tells us that the proportional decline in 'n' is magnified, as trade becomes progressively freer, as shown in Figure 2.

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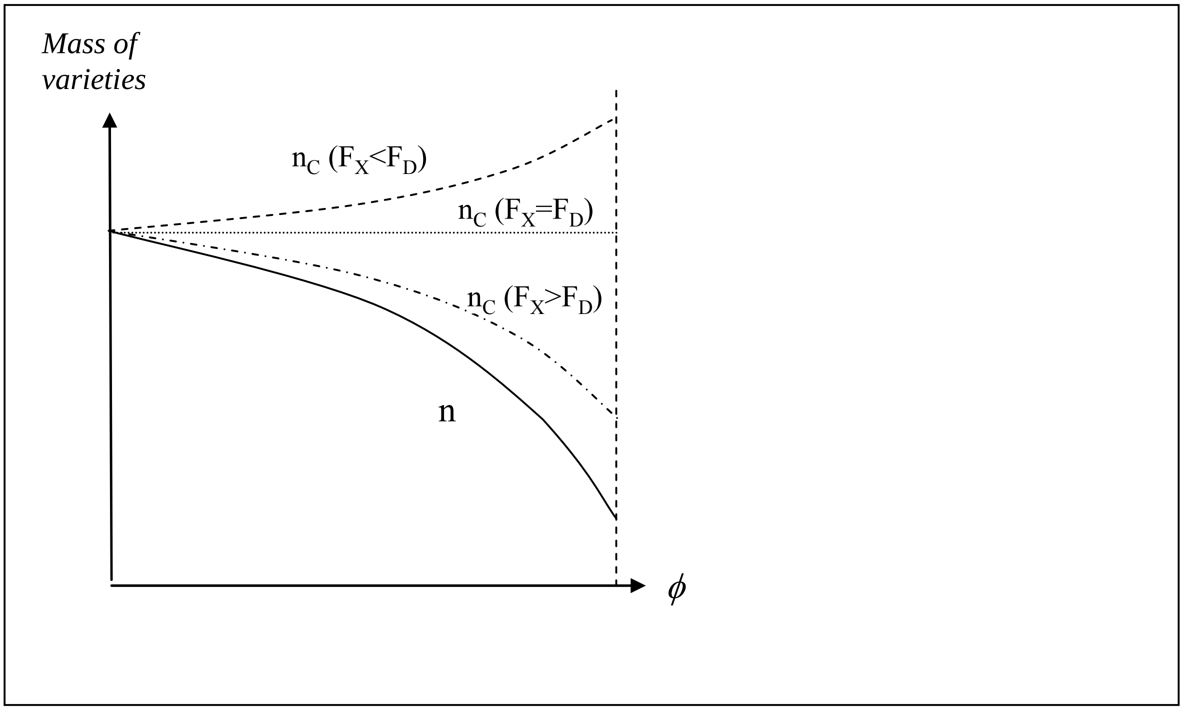
[10] One could imagine a modification of the standard model where F_X is an additional cost on top of F_D. In particular, an addition decision making stage for potential firms could be added. After sinking F_I, they would have to decide whether to sink the cost of a factory, say F_F, and then they would decide whether to sink F_D and F_X. The domestic cut-off condition would be $a_D^{1-\sigma}B=F_F+F_D$, while the export cut off would be $\phi a_D^{1-\sigma}B=F_X$. Naturally, the standard equations – cut-offs and free entry – would only be valid for levels of openness where $a_X<a_D$; for the level of openness in the neighbourhood of free trade, a different set of equations would apply. However, in this article, we are considering the standard model.

[11] The market share of imported variety j is $\phi a_j^{1-\sigma}/\Delta$, so integrating over all foreign varieties $a_j \in [0..a_X]$, the import share equals $\Omega/(1+\Omega)$.

Figure 2: Varieties produced and consumed



One of the great novelties of the monopolistic competition trade model was the fact the 'varieties effect' i.e. the fact that an autarky-to-free-trade liberalisation could raise the number of varieties available to consumers. In this model, that need not be the case.[12] The potential ambiguity stems from the fact that greater openness raises a_X and thus, raises the fraction of Foreign-made varieties that are imported to Home while, at the same time, there is a drop in locally produced varieties. It is simple to characterise the ambiguity. Continuing to assume that L=L* and using (11) gives

$$n_C = \frac{(1-1/\beta)\mu L(1+\phi^\beta T^{-\beta})}{f_D(1+\phi^\beta T^{1-\beta})}, \tag{14}$$

which shows that the number of varieties bought by a typical consumer falls monotonically as the freeness of trade rises – as long as $T \equiv F_X/F_D > 1$. Thus, a lower variable cost of trade will produce an 'anti-variety' effect, i.e. the range of consumed varieties falls as trade becomes freer. If $T \equiv F_X/F_D < 1$, then freer trade results in the more standard pro-variety effect. The two cases, and the knife-edge T=1 case, are shown in Figure 2.

The basic intuition for these results flows most easily by first examining the knife-edge case of no fixed-cost protection, T=1, where an inspection of (14) shows that n_C is constant with respect to ϕ.

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[12] Melitz (2003) did note that the impact on the range of varieties available for a typical consumer could be ambiguous, but n_c was not calculated.

When T=1, there is no intrinsic difference between local and imported varieties, so that changes such as trade liberalisation which introduce more imported varieties will produce a one-for-one reduction in local varieties. More generally, when the ratio of beachhead costs F_X/F_D exceeds unity, n_C falls as ϕ rises because imported varieties have systematically lower prices than the domestic varieties they displace. Restoring zero profits thus requires more than one D-type variety to be displaced by each additional X-type variety that is imported. Conversely, if beachhead costs are lower for imported varieties, the relationship is reversed and freer trade means a wider range of varieties available to consumers.[13]

3.2. Lower marginal cost of trade; asymmetric countries

The symmetric country case is useful for fixing ideas, but assumes away a range of interesting interactions between country size and trade liberalisation. In particular, once asymmetric country sizes are allowed, the model is marked by a modified version of the well-known 'Home Market Effect' (HME)[14].

3.2.1. Home Market Effect, HME Magnification and Delocation effects

A convenient way of expressing the HME is that a nation's share of industrial firms grows more than proportionally as its share of world expenditure on industrial goods grows. Note that from **(10)**, Home's share of the worldwide mass of M-sector firms and the total mass of firms worldwide are:

(15) $$s_n = \frac{s_E - (1-s_E)\Omega}{(1-\Omega)}, \quad n^w = \frac{\mu(\beta-1)(L+L^*)}{(1+\Omega) f_D \beta}; \quad s_n \equiv \frac{n}{n^w}, \quad n^w \equiv n + n^*, \quad s_E \equiv \frac{L}{L+L^*}.$$

Using s_E to denote Home's share of world expenditure and s_n to denote its share of the world's M-sector firms, log differentiation of n in **(10)** implies:

(16) $$\hat{s}_n = \frac{s_E}{s_E - \frac{\Omega}{(1+\Omega)}} \hat{s}_E .$$

An inspection of this shows that the HME does hold since the coefficient on $\hat{s}_E$ is greater than unity. We can also see that the HME is subject to the usual HME magnification effect (the shift in firms to Home as Home's expenditure share rises becomes stronger as trade becomes freer).

[13] It is also possible that the anti-variety effect could be reversed for some non-Pareto distributions.

Another feature of the monopolistic competition trade model is the so-called delocation effect, namely the tendency of freer trade to 'shift' more industry to the large region. Log differentiation of Home's share of world M-sector firms with respect to 'openness' Ω yields:

(17) $$\hat{s}_n = \frac{(2s_E - 1)\Omega/(1-\Omega)}{s_E - (1 - s_E)\Omega}\hat{\Omega}.$$

An inspection of this shows that if Home is larger (i.e. L>L* so s_E>½), then freer trade will increase Home's share of M-sector firms. Moreover, there is also a magnification effect since the strength of the shift in M-sector production to the larger market becomes stronger as trade becomes freer. From **(15**), it can be seen that the so-called sustain point when the entire M-sector is located at home is reached when $\Omega = \Omega^S = L^*/L$.

Non-monotonic production shifting effects

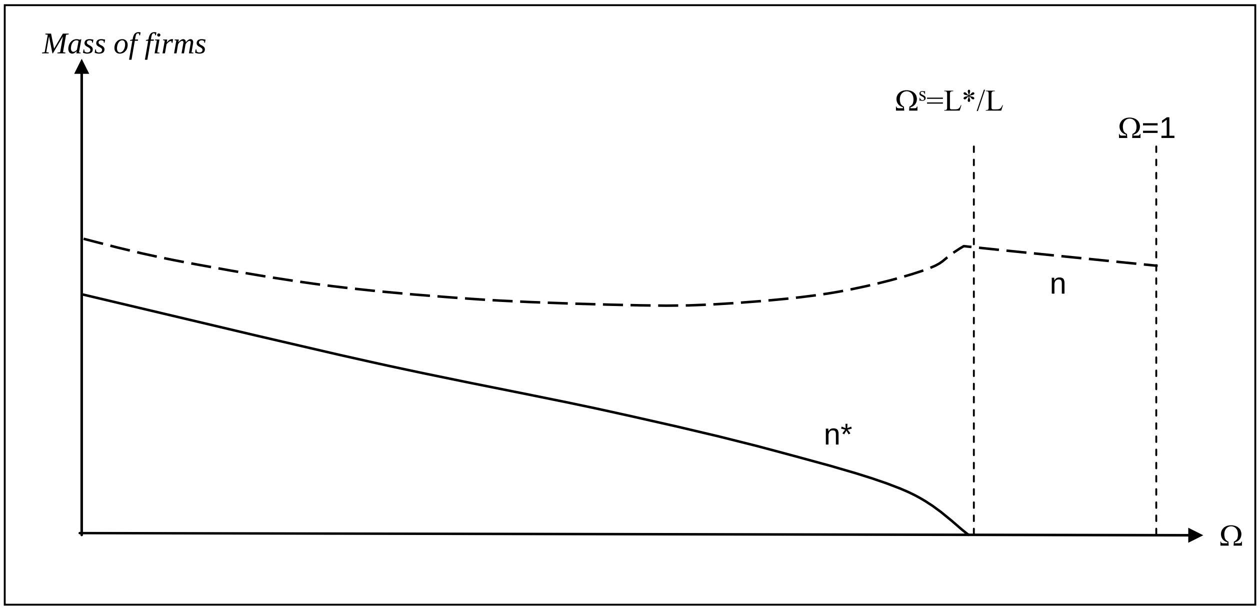
The actual mass of firms in Home varies very non-monotonically with openness since the HME magnification, which tends to raise n, interacts with the overall drop in n^W. The derivative of n with respect to openness Ω is:

(18) $$\frac{dn}{d\Omega} = \mu(1 - 1/\beta)\frac{2\Omega L - L^*(1+\Omega^2)}{f_D(\Omega^2 - 1)^2}.$$

The sign of this is ambiguous, e.g. when Ω=0, the derivative is unambiguously negative, but when $\Omega=\Omega^S$, the derivative is unambiguously positive. Naturally, the impact on the mass of firms in the small region is unambiguously negative since the delocation effect and the drop in n_w work in the same direction. This is illustrated in Figure 3, where it may be noted that $n=n^w$ beyond Ω^s.

[14] See e.g. Helpman and Krugman (1985).

Figure 3: Non-monotonic shifts in n and n*



3.2.2. Anti-variety effects; asymmetric nations

The fact that production is marked by non-monotonic relationships suggests that the number of consumed varieties will also be marked by non-monotonicities. To study this, it proves useful to re-write n_C from **(11)** in terms of Ω and T using Ω≡ψT, namely:

(19)

$$n_C = \upsilon\left(\frac{1-\Omega^2/T}{1-\Omega^2}L - \frac{\Omega(T-1)}{T\left(1-\Omega^2\right)}L^*\right), \quad n_C^* = \upsilon\left(\frac{1-\Omega^2/T}{1-\Omega^2}L^* - \frac{\Omega(T-1)}{T\left(1-\Omega^2\right)}L\right); \qquad \upsilon \equiv \frac{\mu\left(1-1/\beta\right)}{f_D} > 0.$$

By inspection, we see that the first term of n_C tends to increase with openness Ω when T>1, but the second term tends to decrease with Ω; the magnitude of both effects depends upon T and Ω, as well as on the relative size of the nations. Differentiation of n_C with respective to Ω gives:

(20) $$\frac{dn_C}{d\Omega} = \upsilon\frac{(T-1)\left(2\Omega L-(1+\Omega^2)L^*\right)}{T(\Omega^2-1)^2}.$$

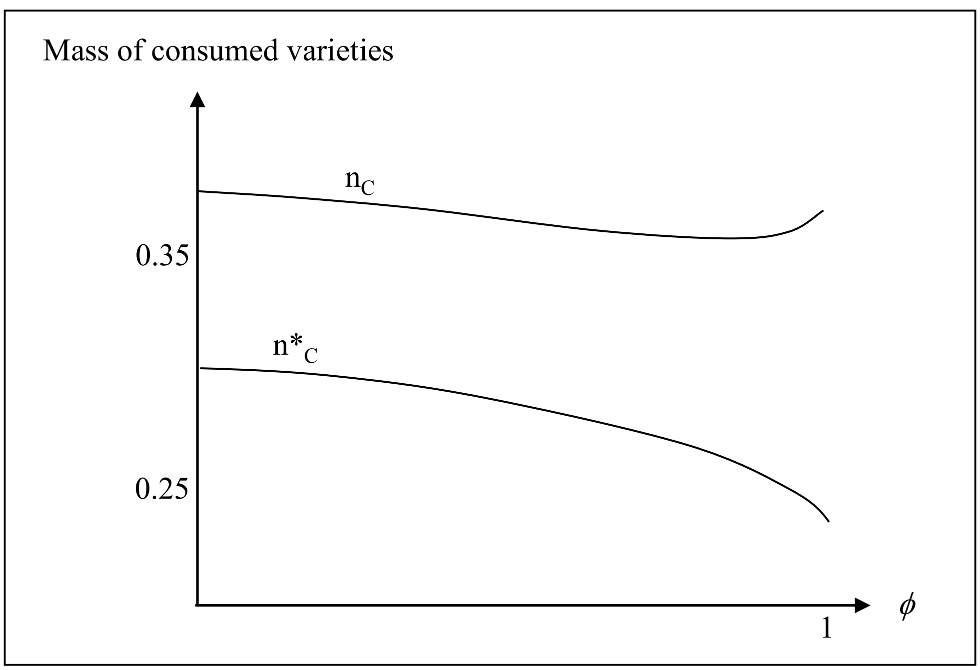
Evaluating this derivative in the base case with T>1 at no-trade, i.e. Ω=0, and at the sustain point, i.e. Ω=L*/L gives:

(21) $$\left.\frac{dn_C}{d\Omega}\right|_{\Omega=0} = \upsilon\frac{-(T-1)L^*}{T} < 0, \qquad \left.\frac{dn_C}{d\Omega}\right|_{\Omega=\Omega^S} = \upsilon(T-1)L^*\frac{2-\left(1+\left(\frac{L^*}{L}\right)^2\right)}{T\left(\left(\frac{L^*}{L}\right)^2-1\right)^2} \;>\; 0\,.$$

Thus, openness is anti-variety for the large nation when trade barriers are initially high, but openness is pro-variety when trade is close to the sustain point level of openness (i.e. when almost all M-sector production is in the large region). Naturally, when trade is more open than Ω^s, further opening is anti-variety since all varieties are in the Home nation and we know that openness unambiguously monotonically reduces the worldwide number of varieties.

For the small nation, the derivative of n_C* with respect to openness is identical to that of the large nation but with L and L* swapped. It is easily shown that $d(n_C^*)/d\Omega$ is negative for all permissible values of Ω, given that T>1.[15] The consumption variety effect, shown in Figure 4 (when $\Omega(\phi=1) < \Omega^S$), thus resembles the production variety effect shown in Figure 3.

Figure 4: Mass of consumed varieties



Trade integration implies fewer domestically produced varieties and, for a small country, also a significant drop in total consumed variety. Nevertheless, as shown below, welfare always increases with trade freeness, irrespective of country size. However, if some individuals have a very high valuation of variety, or if one supposed there to be some intrinsic cultural or nationalist value in the availability of traditional varieties, this unambiguous impact on individual welfare might be mitigated or reversed in a social welfare evaluation. Indeed, many nations spend taxpayers' money on keeping old ways and goods alive.

3.2.3. Trade Volume and Pattern

From (12) we have that

$$(22)\qquad \hat{V} = \frac{s_E\left(1+\Omega^2\right)-\left(1-s_E\right)2\Omega}{\left(s_E-\left(1-s_E\right)\Omega\right)\left(\Omega^2-1\right)}\hat{\Omega}\,; \qquad \hat{\Omega} = \beta\hat{\phi} + (1-\beta)\hat{T}$$

As inspection of this expression shows that, for large countries, M-sector export increases with the level of trade freeness, ϕ. However, for sufficiently small countries, the inverse is true. This is due to the fact that even though, from (8), a_X and therefore the share of exporting firms increase, the mass of firms decline in the small country.

There are a couple of more subtle points related to the trade volume that suggest empirically testable hypotheses. First, the exports of each existing X-type firm expand in proportion. From (12), the change in firm-level export as a function of 'a' (and thus the size of firms) is:

$$(23)\qquad \frac{dv[a]}{d\Omega} = v[a]\left(\frac{\sigma-1}{a_X}\right)\frac{da_X}{d\Omega}.$$

Second, every new exporter should be smaller (in the sense of the value of domestic sales) than every existing exporter, since the drop in a_X affects firms with a's that were just below the cut-off before the liberalisation.

The model also makes a number of predictions concerning 'zeros' in the trade matrix. Empirically oriented trade economists have long know there to be many zero bilateral trade flows in the world.[16] The facts have been more recently documented in a systematic fashion by Feenstra and Rose (1997)

[15] See guide to calculations.

[16] For example, the old gravity equation literature pondered on the best solution to this with some authors dropping these observations, others performing Tobit regressions and others plugging in small positive values. See, e.g. Wang and Winters (1992).

and Helpman, Melitz and Rubinstein (2004). Stepping slightly outside our two-nation model, one simple empirically testable implication of the model concerns the pattern of zeros.

First, as has already been indirectly pointed out by Helpman, Melitz and Rubinstein (2004), the pattern of zeros in bilateral aggregate trade flows should follow a geographical pattern, assuming that trade costs increase with distance. This is easily testable and indeed this is confirmed by the first stage of the Helpman-Melitz-Rubinstein results. One could also test for zeros at the very finely defined commodity level using the time dimension of the data. For example, taking the US's very finely disaggregated export data, the likelihood of a zero (controlling for the usual gravity equation issues like economic size of the importing nation) increasing with distance should be found. Likewise, as a result of any well defined liberalisation exercise, such as the phase in of Uruguay Round tariff cuts, it should be found that the impact of distance on the zeros diminishes as tariffs are cut.

Second, some forms of trade liberalisations are more likely to reduce beachhead costs than variable trade costs. One common source of beachhead costs is known as technical barriers to trade (TBTs), many of which involve health, safety and environment certification of new products (see Baldwin 2000 for details). One way in which such measures are reduced is via international agreements – e.g. Mutual Recognition Agreements either on testing (US-EU MRA) or product norms (New Approach Directives in the EU). These agreements should diminish the probability of observing a zero in any given bilateral trade flow that is affected. A simple difference-in-difference approach should pick this up on aggregate or disaggregate data. Notice that a reciprocal MRA predicts that the effects should be two-way in the affected sectors.

Third, and more to the heart of the model logic, one should find a pattern in firm-level zeros in the data. Specifically, there should be a positive correlation between a firm's domestic market share (which varies with its marginal cost) and the number of markets to which it exports, or the likelihood that it has a zero in any given market, controlling for standard market-specific factors. If one expands the model to allow for the standard proximity-versus-scale FDI à la Helpman-Melitz-Yeaple, the prediction is still true but it is for the number of markets in which the firm sells (via location production is it is going for proximity; by exporting it is going for scale economies).

3.3. Lower beachhead costs

The classical notion of liberalisation is a reduction of marginal trading costs, however many of the trade barriers remaining among industrialised nations that are related to standards and regulations that make it difficult to introduce foreign-produced varieties into a market. These barriers, called technical barriers to trade in industrial goods and sanitary and phyto-sanitary measures in food trade, are some of the few remaining barriers to trade in manufactures among the US, Canada, the EU and Japan. Moreover, since classic trade barriers were eliminated in Western Europe by the mid-1970s, the last four decades of trade liberalisation in Western European nations have been mainly concerned with TBTs. This suggests that it is important to analyse the positive and normative implications of reducing the gap between the beachhead cost facing local and imported varieties. In particular, in this section, we assume these regulatory barriers to be reflected in the beachhead costs and that this explains why $F_X>F_D$, so the liberalisation involves a moving $T=F_X/F_D$ towards unity. Naturally, it is not possible to change T without changing F_D and/or F_X, so we must be explicit about how T falls. To be concrete, consider a ceteris paribus reduction in F_X as a fixed-cost trade liberalisation, while a reduction of F_D and F_X in tandem (such that T is unaffected) is a domestic de-regulation.

As an aside, we note that the formulation of the model makes F_D and F_X entirely distinct. That is, we cannot think of F_D being part of the cost of, say, establishing a product's safety in the home market, and F_X as the extra (lower) cost of using the basic domestic results to establish the product's safety in the foreign market. If this were the case, the export and local market entry conditions would be linked in the sense that the cost of entering the foreign market would be higher for firms that had not entered the domestic market.

From **(10**), and (12), it can be seen that fixed cost trade liberalisation has qualitatively identical effects as $d\phi>1$ when it comes n, a_D, a_X and the volume of trade. The impact on the number of varieties consumed differs and is, in principle, ambiguous. However, differentiating n_C from (19) with respect to F_x and evaluating this differential at $F_X=F_D$ gives

$$\left.\frac{dn_C}{-dF_X}\right|_{F_X=F_D} = \upsilon \frac{\left(\phi^{\beta}L - L^*\right)\left(\phi^{\beta} - \phi^{3\beta}\right)}{\left(\phi^{2\beta}-1\right)^2 F_D} > 0 \quad for \quad \Omega < \Omega^S,$$

which indicates that at least for very low TBTs, further liberalisation will increase n_C in both small and large countries.

4. WELFARE EFFECTS OF LIBERALISATION

Next, we turn to the welfare effects of liberalisation in this model, focusing on the aggregate and re-distributive impact of greater openness.

4.1.1. Aggregate Gains from Trade

As noted above, the utility of a typical agent in this model can be described by the indirect utility function E/P, where from **(1)**, $P = p_T^{1-\mu}\Delta^{\mu/(1-\sigma)}$, and $P^* = p_T^{1-\mu}(\Delta^*)^{\mu/(1-\sigma)}$, and where the Δ's are defined in **(3)**. Plugging in the equilibrium values for p_T, n and n* and using the distribution for G in **(7)**, the price indices simplify to:

$$(24) \qquad P = a_D{}^{\mu}\left(\frac{\mu L}{f_D}\right)^{\frac{\mu}{1-\sigma}}, \qquad P^* = a_D{}^{\mu}\left(\frac{\mu L^*}{f_D}\right)^{\frac{\mu}{1-\sigma}}.$$

Since we showed that a_D always falls as trade becomes more open (in a variable or fixed trade cost sense) we see that both nations gain from integration regardless of size differences. Moreover, an inspection of (10) shows that greater openness raises a_D at an accelerating rate, as Ω approaches unity (i.e. free trade).

4.1.2. Stopler-Samuelson-like result

Income distribution effects can also be easily worked out. Indeed, this model displays classical Stolper-Samuelson-like behaviour.[17] There is only one primary factor in this model; however we can think of firm owners as owning 'knowledge capital'. In particular, we can consider there to be three types of capital in this model: D-type capital, X-type capital and N-type capital, where the reward to D-type and X-type capital is the operating profit on D-type and X-type firms, respectively. Recall that although the average reward to the three types of capital must be zero (zero profit condition), this average consists of pure losses for some balanced by pure profits for others; D-type and X-type firm owners earn pure profits while drawers of 'losing' varieties earn the flow equivalent of minus F_I.

[17] Our earlier draft Baldwin and Forslid (2004) derived the impact on nominal rewards to labour and the three types of capital, while here we derive the real rewards, which is closer to the spirit of the original Stolper-Samuelson theorem; this requires an additional assumption on σ.

Of course the owner of a unit of X-type knowledge capital does not hold this forever, since her capital will eventually depreciate. But if the depreciation rate is chosen to give reasonable churning rates, each of the owners of D-type and X-type knowledge capital will care about openness for a very long time. As we shall see, under certain conditions, X-type capital owners win from reciprocal liberalisation while D-type firms lose from it. Thus, X-type capital owners – the large, efficient exporting firms – will support reciprocal trade liberalisation, while it will be opposed by D-type capital owners. Assuming that the durability of capital is high compared to the electoral cycle, such effects could be important in determining firm-level political support for multilateral trade negotiations.

The reward to capital is a firm's Ricardian surplus; its sales times the Dixit-Stiglitz operating profit margin 1/σ. Thus

$$(25) \qquad r_X[a] = \left(\frac{a}{a_D}\right)^{1-\sigma}(1+\phi)\frac{f_D}{\sigma}; \qquad r_D[a] = \left(\frac{a}{a_D}\right)^{1-\sigma}\frac{f_D}{\sigma},$$

where we define r_D and r_X as the Ricardian surplus of typical D and X type firms.

Turning to the impact of openness on nominal factor rewards, the easiest is labour. Labour is the numeraire, so that freer trade has no impact on the wage in terms of the numeraire good. The impact on the rental rates on D-type and X-type capital is also as simple to derive. As noted above, a firm's total operating profit is proportional to its sales. Using **(10**) and (25), we get:

$$(26) \qquad \frac{\hat{r}_D}{\hat{\phi}} = \frac{-\Omega}{1+\Omega} \le 0, \qquad \frac{\hat{r}_X}{\hat{\phi}} = \frac{\phi-\Omega}{(1+\phi)(1+\Omega)} \ge 0, \qquad \hat{w} = 0.$$

Next, turning to real factor rewards, it is easily shown that if the elasticity of substitution among varieties is sufficiently high (specifically, σ>1+μ), we get a Stolper-Samuelson chain:[18]

$$(27) \qquad \hat{r}_D - \hat{P} \;<\; 0 \;<\; \hat{w} - \hat{P} \;<\; \hat{r}_X - \hat{P}.$$

Note that this holds for countries irrespectively of size, since $\hat{P} = \hat{P}^*$ from **(24**), and that even if σ violates the condition σ>1+μ, we still have that the real gain to X-types exceeds that of D-types.

An interesting implication of (27) when combined with the fact that rental rates are inversely proportional to a's is that the income distribution among active-firm owners follows a fractals-like pattern. That is, capital rental rates will follow a Pareto distribution with the shape parameter ρ+1-

σ. Thus if, for example, y% of the gains from liberalisation accrue to the top x% of the income distribution, the same is true for the top x% of the top x%. This fractal-like income distribution has received some empirical support from income distribution studies.

It is important to recall that each D-type and each X-type earns pure profits throughout its life time, so that these Stolper-Samuelson results are not transitory. They are permanent, firm by firm. Naturally, new firms that become active will not experience an increase in their Ricardian surplus, but they will earn a reward that is higher than it would have been without the liberalisation.

The Stolper-Samuelson-like result in (26) should be testable via stock market data for large and small firms. The impact of a clearly defined liberalisation 'treatment' should be asymmetric for large and small firms. Simply put, the rise in a firm's stock market price in reaction to a reciprocal trade liberalisation should be positive for firms that are sufficiently large and negative for firms that are sufficiently small.

4.1.3. Fixed cost liberalisation

An inspection of (25) and (10) shows that the income distribution impact of fixed cost trade barrier liberalisation will be quite different compared to variable trade cost liberalisation, at least for X-type firms. In particular, reducing F_X reduces a_D and this, as per (25), reduces the reward to both X-type and D-type capital. In other words, greater regulatory liberalisation reduces both operating profits by the same proportion, but does not affect the wage. The intuition for this result is clear. The beachhead costs create barriers to enter that must, in equilibrium, be compensated by higher operating profits. Reducing the beachhead costs thus reduces the flow reward to active firms.

5. CONCLUDING REMARKS

Trade models with heterogeneous firms and beachhead costs constitute an important new instrument in the toolbox of international trade theorists. Of course, nothing under the sun is entirely new – many, many trade theorists have published models where firms have different marginal costs, and the late 1980s saw a flourishing of papers and books on models with beachhead costs. Nevertheless, the assumption of continuous marginal-cost heterogeneity in a monopolistic competition setting teamed with beachhead costs constitutes more than just an incremental improvement on existing models, since it allows us to consider a broad range of real-world facts –

[18] See guide to calculations.

those concerning firm-size and trade issues – that the standard monopolistic competition trade models had to assume away.

This paper works out the impact of great openness, in terms of variable and fixed trade costs, and develops a sequence of testable hypotheses. The paper also studies the impact of greater openness at both the firmlevel and the aggregate level, focusing on changes in the numbers and types of firms, trade volumes and trade prices. Contrary to the standard Dixit-Stiglitz trade model, global variety falls as the result of trade liberalisation, and it may be that consumed variety falls in a country. The normative effects of liberalisation are also studied and here, the paper focuses on aggregate gains from trade, and income redistribution effects, showing inter alia that the model is marked by a Stolper-Samuelson-like effect.

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GUIDE TO CALCULATIONS FOR REFEREES AND READERS

Calculations refered to in 2.

The standard CES demand function is:

$$c_j=\mu E(p_j)^{-\sigma}/(\int_{i\in\Theta} p_i^{1-\sigma} di)$$

multiplying both sides by p_j, we get:

$$p_j c_j=\mu E(p_j)^{1-\sigma}/(\int_{i\in\Theta} p_i^{1-\sigma} di)$$

for variety j, where the integral is over all competing varieties (i.e. $i\in\Theta$), and μE is total expenditure on all varieties in the market, since μ is the Cobb-Douglas share of expenditure on M-goods. Given the well-known Dixit-Stiglitz feature called mill price, the price-marginal cost mark-ups are all identical and thus cancel out, so

$$p_j c_j=\mu E(p_j)^{1-\sigma} / (\{ n\int_0^{a_D} a^{1-\sigma} dG[a| a_D] + n^* \tau^{1-\sigma} \int_0^{a_X} a^{1-\sigma} dG[a| a_D] \},$$

since $ndG[a|a_D]$ gives the mass of varieties with marginal cost 'a'. That is, $G[a|a_D]$ is the condition density of a, given that the variety is actually produced, and there is a mass of 'n' on each 'a'; for a proof that this is the conditional density in equilibrium, see Melitz (2003).

Note that we have here included all D-type varieties produced in the local market in the first integral and all varieties that are imported from the other market (i.e. varieties with a's between zero and a_X) in the second integral. Here, $\phi\equiv\tau^{1-\sigma}$ measures the iceberg trade costs that are passed on by foreign firms. Multiplying by a_j yields the expression in the text.

Calculations referred to in footnote 3:

A typical Dixit-Stiglitz first-order condition is $p(1-1/\sigma)=wa$, where w is wage and p is price; rearranging, the operating profit, (p-wa)c, equals pc/σ with c defined as consumption. Thus, operating profit, (p-wa)c, is proportional to revenue, pc; the factor of proportionality is $1/\sigma$.

Calculations referred to in footnote 4:

Defining $\pi[a]$ as the steady-state operating profit earned by a firm with marginal cost 'a', the present value with a discount rate ρ and the Poisson firm-death process assumed is: $\pi[a]$

$\int_0^\infty e^{-\rho t} e^{-\delta t} dt$ since the probability of the firm still being alive at t is $e^{-\delta t}$. Setting ρ=0 and solving the integral yields the expression in the text.

Calculations referred to in footnote 5:

The expression for Δ follows directly from (4) and

(**7**); specifically

$$\Delta=\{k/(1-\sigma+k)\}(a_D{}^{1-\sigma})(1+\phi(a_X/a_D)^{1-\sigma+k}).$$

Using the ratio of the cut-offs, $\beta\equiv k/(\sigma-1)$ and our definition of Ω,

$$\Delta=\{1/(1-1/\beta)\}(a_D{}^{1-\sigma})(1+\Omega).$$

Calculations referred to in footnote 7.

To find n, plug Δ from (**8**) into the D-type cut-off condition. To find a_D, plug the closed form solution for n into the free entry condition using the ratio of cut-off conditions to evaluate $G[a_X]/G[a_D]$. a_X then follows from this the expression for a_D and the ratio of the cut-off conditions.

Using the solution for Δ (and the corresponding expression for Δ*) in the domestic cut-off condition for the Home nation, we see that the a_D drops out to leave:

$$\frac{\mu L(1-\beta)/\beta}{n+n^*\Omega} = f_D.$$

Doing the same for the Home export cut-off condition, we get a similar expression where the a_D does not drop out, but the expression involves the ratio of a_D and a_X:

$$(\frac{a_X}{a_D})^{1-\sigma}\frac{\mu L^*(1-\beta)/\beta}{n\Omega+n^*} = f_X/\phi.$$

We eliminate the ratio of a_D and a_X by using the ratio of the cut-off conditions, and simplify to obtain:

$$\frac{\mu L^*(1-\beta)/\beta}{n\Omega+n^*} = f_D.$$

Solving these two expressions for n and n* yields the result in the text.

Calculations referred to in footnote 15.

The derivative is:

$\frac{dn_C^*}{d\Omega} = \upsilon \frac{2\Omega(T-1)L^* - (1+\Omega^2)(T-1)L}{T(1-\Omega^2)^2}$ and naturally, the entire action is in the numerator, which can be re-written as: $\left(2 - \frac{1+\Omega^2}{\Omega}\frac{L}{L^*}\right)\Omega(T-1)L^*$. Since Ω is bound by zero and unity, and L/L*>1, the term in the large parentheses is always negative. This proves the assertion in the text.

Calculations referred to in footnote 18.

Noting that a firm's Ricardian surplus is 1/σ times its sales, the proportional change in the r's is identical to the proportional changes in sales. Thus, we know that $\hat{r}_D < 0 = \hat{w} < \hat{r}_X$. To establish real factor reward changes, we must compare these to the proportional change in P shown in **(24)**. The real changes are: $\hat{r}_D - \hat{P} = (\sigma - 1 - \mu)\hat{a}_D$, $\hat{r}_X - \hat{P} = (\sigma - 1 - \mu)\hat{a}_D + \frac{\phi}{1+\phi}\hat{\phi}$ and $\hat{w} - \hat{P} = 0 - \hat{a}_D$.

Since $\hat{a}_D$ is negative when trade freeness rises (dϕ>0), we know that workers always gain, and σ>1+μ is a necessary and sufficient condition for D-type firm owners to lose. The X-type firm gains in the foreign market but loses at home. Using **(10)** $\hat{a}_D = -\frac{1}{k}\frac{\Omega}{1+\Omega}\beta\hat{\phi}$, we get that

$\hat{r}_X - \hat{P} = -\frac{(\sigma-1-\mu)}{1-\sigma}\frac{\Omega}{1+\Omega}\hat{\phi} + \frac{\phi}{1+\phi}\hat{\phi}$. Since $\phi > \Omega$ for T>1, we have that $\hat{r}_X - \hat{P} > 0$.