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CAPITAL ALLOCATION IN MULTI-DIVISION FIRMS: HURDLE RATES VS. BUDGETS

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Abstract

It is common practice for firms to ration capital funds to their divisions, rather than set a price and let the divisions use as much as they want. This appears to be true even when the overall firm faces no rationing in the capital market. This paper offers an interpretation of this phenomenon based on Martin Weitzman's "Prices vs. Quantities" model. It is found that a rationing system is advantageous when division managers do not perceive the full consequences of their investment decisions for the firm as a whole. By contrast, a pricing system for allocating capital among divisions would be favored when the division managers possess valuable information that cannot be costlessly communicated to headquarters. It is then argued that actual capital budgeting practice in many firms reflects a mixture of these two systems and can thus be interpreted as an attempt to reap both kinds of benefits at once.

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Capital Allocation in Multi-Division Firms:

Hurdle Rates vs. Budgets

Finance theory offers a clear prescription for capital budgeting policy: accept all projects with positive net present values. In a multi-division firm, this prescription might be implemented as follows: First, central headquarters determines hurdle rates for each division. These rates correspond to the riskiness of each division's business and are based on the risk-return tradeoff implicit in market security prices.[1] Next, the division heads analyze potential projects, using their hurdle rates, and adopt all projects with positive net present values. Finally, central headquarters raises any external funds needed to undertake all such projects.

It is equally clear, however, that few, if any, corporations behave exactly in the prescribed manner. Instead of relying solely on an internal price mechanism to determine total capital expenditures, most corporations resort to some form of rationing, in which headquarters assigns budgets to the divisions without full knowledge of their opportunities.[2] In some companies, these budgets are inflexible.[3] In others, their use is part of a sequential process that still retains elements of a hurdle rate system. In such cases, preliminary budgets are set, divisions use hurdle rates to analyze projects within these budgets, and the analysis then serves as a basis for a certain amount of renegotiation of the final budgets.[4]

The prevalance of some form of capital rationing has been noted in the academic literature, and considerable discussion has been devoted to how capital budgeting decisions should be made in the presence of rationing.[5] However, most of these discussions offer no clear explanation for rationing. One possibility is that the external capital market constrains the quantity of funds that firms may raise, but this explanation is viewed with skepticism by most financial economists. A second possibility is that rationing is an internal constraint imposed by top management for reasons of organizational control. Brealey and Myers [2, p. 104], for example, offer some suggestions along these lines:

> Some ambitious divisional managers habitually overstate their investment opportunities. Rather than trying to distinguish which projects really are worthwhile, headquarters may find it simpler to impose an upper limit on divisional expenditure and thereby force the divisions to set their own priorities. In such instances budget limits are a rough but effective way of dealing with biased cash-flow forecasts. In other cases management may believe that very rapid corporate growth could impose intolerable strains on management and the organization. Since it is difficult to quantify such constraints explicitly, the budget limit may be used as a proxy.

The purpose of this paper is to study this second, or internal, form of capital rationing in the context of a more formal model. Weitzman [14, 15] has introduced a framework for assessing the relative merits of pricing and direct control as resource allocation mechanisms, and his analysis is applicable to the problem at hand. The ensuing sections of this paper proceed in two steps to analyze intracorporation rationing with a modified version of Weitzman's model.

In order to contrast their properties most sharply, Section I treats the internal pricing, or hurdle rate, system and the fixed divisional budget system as mutually exclusive. The hurdle rate system makes more efficient use of division managers' information and is thus favored when such information is both valuable and not easily communicated to headquarters. The rationing, or budget, system is advantageous when division managers' information is unreliable. That may occur either when divisional information contains an error component of high variance or when division managers fail to perceive the full impact of their actions on the firm as a whole. In general, though, firms need not rely exclusively on either system. Section II analyzes the merits of using mixed hurdle rate and budget systems and argues that many firms may be doing just that.

I. Hurdle Rates vs. Budgets

The model in this section is particularly intended to embody the problems of biased divisional forecasts and overall company growth constraints as described by Brealey and Myers [2] in the quotation above. As will be noted along the way, though, many of the model's specific details could be altered without changing its essential message.

In broad outline, the following situation is envisioned: A firm has a central headquarters, HQ, and J divisions. Investment levels, I_j, must be set for each of the divisions, and headquarters may either choose the I_j directly or assign each division a hurdle rate, $\hat{k}_j$, and let the division managers choose I_j. In either case, headquarters' objective is that the I_j be chosen so as to maximize net benefits for the whole firm.

Total net benefits depend on the values of the I_j and on two types of state-of-the-world variables. The first of these, η, reflects factors such as overall economic conditions that are common to all divisions. The second, θ_j, reflects factors impinging on division j but not necessarily on other divisions. These might include the state of demand and technology in the market served by division j.

Neither headquarters nor the division managers know the true value of η *ex ante*, and headquarters is likewise unaware of the true value of θ_j. Division j's manager, on the other hand, can use his specialized knowledge about divisional operations to make a forecast of θ_j. This forecast, $\hat{\theta}_j = \theta_j + \varepsilon_j$, consists of the true value of θ_j plus some error term, ε_j. The error term might arise either because division j's manager intentionally distorts his forecast in an effort to pursue personal objectives (e.g. maximize division j's size) that deviate from those of headquarters or simply because division j's manager has "noisy" information.

To simplify the analysis, it is assumed that all agents are risk neutral. Thus inter-division differences in project risk, which would complicate the model without adding any new insights, are ignored. Managers at both the headquarters and division levels simply maximize their own perceived measures of expected net benefits. A shortcoming of the risk-neutrality assumption is that differences in managers' attitudes toward risk are also ignored.

In the absence of risk aversion, three remaining problems condition the choice between a hurdle rate or a budget system for setting I_j. First, the division managers cannot communicate their information to headquarters. This may stem from the technical nature of the information or from credibility problems. Second, the division managers act as if $\hat{\theta}_j$, were the true value of θ_j. Again, this may arise either through intentional distortion or through division managers' inability to separate out the error component in their information. In the latter case, it is still rational under risk neutrality for division j's manager to act as if $\hat{\theta}_j$ were the truth as long as his expected value of ε_j is equal to zero. Finally, the division managers do not perceive that their own investment may affect the marginal benefits generated by other divisions' investment. Headquarters is aware of these interactions, but this knowledge cannot be communicated in any useful way to the division managers.

Before solving the optimal investment problem, it remains to specify the net benefits function. Following Weitzman [14], it is assumed that the net benefit from investment can be locally approximated by a quadratic form. As perceived by headquarters, the net benefit from all divisions' investment is given by[6]

$$B_{HQ}(I_1, \ldots, I_J; \eta; \theta_1, \ldots, \theta_J)$$

$$= \sum_j a_j(\eta, \theta_j) + \sum_j (h + b_j(\eta) + \theta_j)(I_j - \hat{I}_j) \tag{1}$$

$$- \sum_j \frac{f_j}{2} (I_j - \hat{I}_j)^2 - \frac{c}{2} (\sum_j I_j - \sum_j \hat{I}_j)^2 - k\sum_j I_j.$$

In this formulation, a_j and b_j are functions, while h, f_j, c, $\hat{I}_j$ and k are nonnegative constants. We can interpret k as the cost of capital, and since k is invariant to total investment, the firm faces no external rationing. The functions a_j represent gross benefits from divisional investment when $I_j = \hat{I}_j$ for all j. The $\hat{I}_j$ are in turn a set of investment levels around which total benefits exhibit some curvature.[7] The $\hat{I}_j$ might reflect both production technology and the technology of managing large-scale organizations.

Further insight into the $\hat{I}_j$ can be gained if we consider the marginal net benefit, $\partial B_{HQ}/\partial I_j$, for division j's investment. The set of terms $(h + b_j(\eta) + \theta_j - k)$ reflects the extent to which marginal net benefit is independent of I_j. However, the terms containing f_j and c indicate that marginal net benefit decreases as I_j departs from $\hat{I}_j$. The term containing f_j reflects that part of the decrease stemming from division-specific factors such as production technology. The term containing c indicates that marginal net benefit from I_j also declines as total investment, $\sum_j I_j$, departs from $\sum_j \hat{I}_j$. This latter effect might stem from the firm's capacity to absorb growth: if the firm as a whole invests too much (grows too fast) strains are imposed on the organization and the marginal benefit from investment is weakened for all the divisions; by the same token, if the firm as a whole invests too little, there is slack capacity for absorbing growth and some positive interactions among the different divisions' investment may be missed.

Perceptions of the benefit from investment differ between headquarters and the division managers. As perceived by the manager of division j, the net benefit, B_j, from division j's investment is given by

$$B_j(I_j;\ \eta;\ \hat{\theta}_j) = a_j(\eta,\ \hat{\theta}_j) + (h + b_j(\eta) + \hat{\theta}_j)(I_j - \hat{I}_j) - \frac{f_j}{2}(I_j - \hat{I}_j)^2 - \hat{k}_j I_j. \tag{2}$$

Expression (2) differs from (1) in three respects. First, the division manager acts as if his forecast, $\hat{\theta}_j$, were the true value of θ_j. Second, the division manager does not perceive the effect of his own investment on the company's capacity to absorb growth; hence, $c = 0$ in (2). Finally, to the extent that the division manager perceives a cost of capital, its value, $\hat{k}_j$, is dictated to him by headquarters.

A. Optimal Budgets

Headquarters must now decide whether to determine the I_j directly (the budget system) or to assign divisional hurdle rates, $\hat{k}_j$, and let the division managers choose the I_j themselves. If headquarters sets budgets it will choose the I_j to maximize expected net benefits as perceived by itself. From (1), the optimization condition for I_j is

$$h + E_{HQ}(b_j(\eta)) + E_{HQ}(\theta_j) - f_j(I_j - \hat{I}_j) - c(\sum_j I_j - \sum_j \hat{I}_j) = k. \tag{3}$$

Simpler expressions, but no loss of generality are entailed if we assume in all that follows that $E_{HQ}(\theta_j) = E_{HQ}(b_j(\eta)) = 0$ for all j, and h = k. In that case a solution to the problem is for headquarters to set $I_j = \hat{I}_j$ for all j.

B. Optimal Hurdle Rates

Alternatively, if headquarters sets hurdle rates, $\hat{k}_j$, the division managers will choose the I_j to maximize their expected value of (2).[8] This results in

$$h + E_j(b_j(\eta)) + \hat{\theta}_j - f_j(I_j - \hat{I}_j) = \hat{k}_j. \tag{4}$$

Or, assuming $E_j(b_j(\eta)) = 0$ and h = k, the investment level, I_j^*, that division j's manager perceives as optimal is

$$I_j^* = \hat{I}_j + \frac{k + \hat{\theta}_j - \hat{k}_j}{f_j}. \tag{5}$$

Expression (5) can be thought of as division j's response function to $\hat{k}_j$, and headquarters would want to set $\hat{k}_j$ so that, given this response function, the levels of investment chosen will maximize $E_{HQ}(B_{HQ})$. That is, the optimal $\hat{k}_j$ must satisfy

$$E_{HQ}\left[\frac{\partial B_{HQ}}{\partial I_j^*} \frac{dI_j^*}{d\hat{k}_j} \right] = 0. \tag{6}$$

Using (1) and (5), this implies setting k_j so that

$$E_{HQ}(I_j^* - \hat{I}_j) + \frac{c}{f_j} E_{HQ}(\sum_j I_j^* - \sum_j \hat{I}_j) = 0, \tag{7}$$

or so that $E_{HQ}(I_j^*) = \hat{I}_j$ for all j. Let $E_{HQ}(\hat{\theta}_j) = E_{HQ}(\theta_j + \varepsilon_j) = E_{HQ}(\varepsilon_j) = e_j$.[9] It is then optimal for headquarters to set $\hat{k}_j = k + e_j$, in which case

$$I_j^* = \hat{I}_j + \frac{\hat{\theta}_j - e_j}{f_j} . \tag{8}$$

Given (8) headquarters expects division j's investment to be equal to $\hat{I}_j$ and thus (7) is satisfied. However division j's actual investment will differ from $\hat{I}_j$ to the extent that its actual forecast, $\hat{\theta}_j$, differs from headquarters' expectation of that forecast (that is, to the extent that $\theta_j + \varepsilon_j$ differs from e_j).

C. The Relative Merits of Hurdle Rates and Budgets

Headquarters will want to use the capital allocation mechanism that has the largest expected net benefits. Headquarters' expectation of the difference, Δ, between the net benefits from a hurdle rate system and those from a budget system is given by

$$\begin{aligned} \Delta = E_{HQ}[B_{HQ}(I_1^*, \ldots, I_j^*; \eta; \theta_1, \ldots, \theta_j) \\ - B_{HQ}(\hat{I}_1, \ldots, \hat{I}_j; \eta; \theta_1, \ldots, \theta_j)]. \end{aligned} \tag{9}$$

Substituting (8) and (1) into (9), Δ may in turn be expressed as

$$\begin{aligned} \Delta = \sum_j \frac{\mathrm{cov}(b_j(\eta), \hat{\theta}_j)}{f_j} + \sum_j \frac{\sigma^2(\theta_j)}{2f_j} - \sum_j \frac{\sigma^2(\varepsilon_j)}{2f_j} \\ - \frac{c}{2} [\sum_j \frac{\sigma^2(\hat{\theta}_j)}{f_j^2} + 2\sum_{i \neq j} \sum \frac{\mathrm{cov}(\hat{\theta}_j, \hat{\theta}_i)}{f_j f_i}] . \end{aligned} \tag{10}$$

Neither the hurdle rate nor budget system is unambiguously optimal, since Δ may be either positive or negative. Factors that push Δ in the positive direction would weigh in favor of the hurdle rate system, while those with a negative impact on Δ would favor the budget system. Interpretation of the different terms in (10) gives some indication of what these factors might be.

When headquarters dictates investment budgets, it does so on the basis of expected values of η and θ_j, and in some cases the divisions may be able to improve on headquarters' selection. From the first term in (10), the division's forecast, $\hat{\theta}_j$, may be related to η. If the covariance between $\hat{\theta}_j$ and $b_j(\eta)$ is positive, then when division j makes decisions based on its analysis of its own business, these decisions are also right for the firm in terms of general business conditions. Headquarters stands to gain in this case by setting hurdle rates and letting the divisions make use of their own information. Conversely, a negative covariance favors the budget system. Suppose, for example, that η represents the inflation rate and that division j's true cash flows from investment are positively related to inflation (i.e. $b_j'(\eta) > 0$). Suppose further that division j's manager systematically underestimates these cash flows by an amount that is directly proportional to the true inflation rate. There is then a negative covariance between $b_j(\eta)$ and the error component of $\hat{\theta}_j$, and, other things equal, the budget system will be preferred.

The second term in (10) indicates that variability in θ_j favors the hurdle rate system. The higher is the variance of θ_j the more valuable is the division's information about the true θ_j, and the greater is the advantage to headquarters of decentralizing the investment decision. On the other hand, when

θ_j varies little around its mean, then headquarters, acting on the basis of $E_{HQ}(\theta_j)$, cannot go too far wrong by making the choice itself.

The third term in (10) implies that the more variable is the error component in the division's forecasts, the more dangerous it is for headquarters to rely on hurdle rates. Note that headquarters' expectation of the size of the error appears nowhere in (10). It is uncertainty about divisional biases, rather than their size, that weighs against the hurdle rate system. The same insight explains the inflation example above: since headquarters is uncertain about the true inflation rate it is also uncertain about the size of division j's inflationary bias, and thus reliance on the division's forecasts is dangerous in periods of highly variable inflation. By contrast, if division j's error is proportional to the inflation rate, but headquarters knows the inflation rate with certainty, it can simply assign division j a commensurately lower hurdle rate. Likewise, if division j consistently exaggerates its investment opportunities by a known amount, it could be given a higher hurdle rate.[10] In either case, the expedient of adjusting the hurdle rate corrects a known bias more efficiently than would budgeting, since it still allows the division to exploit its useful information.[11]

The choice between hurdle rates and budgets is also influenced by relative perceptions of the curvature in benefits. All of the terms in (10) become small, for example, as the f_j increase. The f_j reflect that part of the curvature that is perceived identically by both headquarters and the division managers, and the larger are the f_j, the more likely are the division managers to set $I_j = \hat{I}_j$ of their own accord. Hence, the choice between hurdle rate and budget systems becomes immaterial.

That part of the curvature, c, that is recognized by headquarters but not by the division managers has the opposite effect. Since the term in square brackets in (10) is the variance of a sum and therefore positive, increases in c tend to favor the budget system. This might correspond to the situation described by Brealey and Myers [2] in which the budget system is preferred as the firm's capacity to absorb growth diminishes.

The terms in square brackets further indicate that the effect of c is conditioned by the extent of variations in the divisions' forecasts. As the variances of $\hat{\theta}_j$ grow larger, the pricing mechanism becomes more dangerous, because there is a greater chance that each division's information will lead its investment further away from $\hat{I}_j$, with deleterious consequences for the firm as a whole. Since $\hat{\theta}_j = \theta_j + \varepsilon_j$, part of the variation in $\hat{\theta}_j$ consists of variation in the true value of θ_j. Contrasting the second and fourth terms in equation (10), then, we see that uncertainty about θ_j confronts the firm with a tradeoff. On the one hand, the more uncertain is headquarters about the true value of θ_j, the greater are the potential benefits from fully exploiting the divisions' information. On the other, wider variations in θ_j make it more likely that I_j^* will diverge from $\hat{I}_j$, particularly since the divisions do not perceive the total effect of their investment on the firm's net benefits. The detrimental effect is magnified if each division's forecast is positively related (as indicated by $\mathrm{cov}(\hat{\theta}_j, \hat{\theta}_i)$) to that of other divisions.

There is at least a rough analogy between this last set of implications and the conclusions reached in recent discussions of the strategic aspects of capital budgeting. Kester [10] and Myers [11], for example, have pointed out that naive applications of discounted cash flow analysis may fail to capture the value of future growth opportunities inherent in today's investment. In effect, they argue that a useful capital budgeting system must deal with any externalities between projects undertaken at different times.

In a similar vein, the term $\frac{c}{2}(\sum_j I_j - \sum_j \hat{I}_j)^2$ in expression (1) represents a variety of these externalities that might be missed at the divisional level of analysis. Thus far, these have been interpreted as externalities between the investment of different divisions at a given time. The summations could, however, be thought of as extending over projects within a given division or over projects through time. The term $\sum_j \hat{I}_j$ could represent a "critical mass" of investment that is needed to sustain a competitive strategy in a given line of business. Total firm benefits may be sharply kinked around this critical mass, and if this fact is not appreciated at the division level, investment budgets may have to be imposed by a centralized strategic planning group. The variables $\hat{\theta}_j$ could be interpreted as those aspects of the divisions' information that tend to lead them away from the critical mass of investment. The more variable is this information or the more positive is its covariance across divisions or across time periods, the greater will be the need for centralized budgeting.[12]

The model could be subjected to numerous other variations or reinterpretations, but the essential message would remain the same: Budgeting is favored when there are divisional blind spots that cannot be overcome through price signals from headquarters. These situations include divisional biases of uncertain and highly variable magnitude and divisions' failure to correctly perceive the second derivative of the net benefits function. Budgeting should be most prevalent, therefore, for firms with a high degree of strategic interaction between divisions or with a large number of potential projects having implicit growth options. Research and development expenditures, for example, might come under these headings. In addition, it is sometimes argued that expenditures like R&D are "difficult to quantify" and therefore not susceptible to hurdle rate analysis. While this argument may be true for other reasons (see Myers [11]), it may also reflect a judgment that the error component in divisional analyses of R&D expenditures is highly uncertain.

By contrast, a hurdle rate system would be favored when there are few interactions among divisions and when the division managers possess valuable information that gives them a comparative advantage in setting investment. A conglomerate firm made up of unrelated businesses, for example, might find that a hurdle rate system makes best use of its division managers' information, while at the same time interactions between divisions are sufficiently small that a budget system would yield no special advantages. Indeed, in the absence of other considerations, there may be no compelling economic rationale for keeping such divisions together as part of a single firm.

II. Mixed Systems

Since both hurdle rate and budget systems have their advantages and disadvantages, it is natural to ask whether some combination of the two might be preferable to using either system alone. There are at least two ways of effecting such a combination, both of which rely on increasing the flow of information between headquarters and the divisions. The first of these is for headquarters to transmit a more complex signal to the divisions, consisting of both a hurdle rate and a penalty function for investment levels that deviate from $\hat{I}_j$. This mechanism has been considered by Weitzman [15], and it is discussed in Section II.A. The second method is for headquarters to try to learn the division managers' information. This method is considered briefly, but without formal analysis, in Section II.B.

A. Hurdle Rates Plus Penalties

In the face of linear marginal net benefit schedules, such as those assumed in Section I, Weitzman [15] has shown that a mixed system is optimal. Specifically, headquarters presents each division with a cost function,

$$\hat{C}_j(I_j) = \hat{k}_j I_j + \frac{\hat{q}_j}{2}(I_j - \hat{I}_j)^2 \qquad (11)$$

where $\hat{q}_j$ is a penalty for deviations from $\hat{I}_j$.[13] This cost function can be thought of as a mixed pricing and budget mechanism, with $\hat{k}_j$ representing the pricing component. The heavier is the penalty $\hat{q}_j$, the closer this mechanism comes to simply fixing $I_j = \hat{I}_j$.

Division j is then instructed to set I_j so as to equate its own perception of gross marginal benefit with the marginal cost measure derived from (11). Division j's optimality condition is thus

$$k + \hat{\theta}_j - f_j(I_j - \hat{I}_j) = \hat{k}_j + \hat{q}_j(I_j - \hat{I}_j),$$

and its investment level is

$$I_j^* = \hat{I}_j + \frac{k + \hat{\theta}_j - \hat{k}_j}{\hat{q}_j + f_j}.$$

Headquarters will want to set the cost function (11) in an optimal manner, and as we saw in the preceding section, one requirement for this is $E_{HQ}(I_j^*) =$

$\hat{I}_j$. Thus headquarters will set $\hat{k}_j = k + e_j$ as in Section I.B., and the division's optimal investment will be

$$I_j^* = \hat{I}_j + \frac{\hat{\theta}_j - e_j}{\hat{q}_j + f_j} . \tag{12}$$

When $\hat{k}_j$ is set in this way, headquarters' unconditional expectation of total net benefits is maximized. The role of the second part of the cost function, the penalty term, is to ensure that expected net benefit is also maximized conditional on the divisions' information. Suppose, for example, that headquarters knew when division j's forecast, $\hat{\theta}_j$, diverged from its expected value, e_j. Headquarters knows from (12) that every unit increase in $\hat{\theta}_j$ will increase division j's investment by $1/(q_j + f_j)$. From (1), therefore, headquarters' expectation of marginal net benefit from I_j would change by

$$\begin{aligned} E_{HQ}\left(d\left(\frac{\partial B_{HQ}}{\partial I_j}\right)/d\hat{\theta}_j\right) &= E_{HQ}(b_j(\eta)|\hat{\theta}_j) + E_{HQ}(\theta_j|\hat{\theta}_j) \\ &- \frac{f_j}{\hat{q}_j + f_j} - \sum_i \frac{c}{\hat{q}_i + f_i} (E_{HQ}(\hat{\theta}_i - e_i|\hat{\theta}_j)). \end{aligned} \tag{14}$$

There are four separate effects in (13). First, if headquarters knew that $\hat{\theta}_j$ were higher than expected, it might revise its expectation about the effect, $b_j(\eta)$, of the overall state-of-nature variable. Second, based on its knowledge of the usual accuracy of division j's information, headquarters might revise its estimate of the true θ_j. Third, headquarters would know that a change in division j's forecast changes division j's investment and hence changes marginal benefits through f_j. Finally, overall firm investment will change marginal benefits through c. Not only will division j's investment affect total firm investment, but to the extent that headquarters knows other divisions' information to be correlated with j's information it will expect investment in the other divisions to change as well.

Since headquarters wants expected net benefits to be maximized at all times, it will set $\hat{q}_j$ so that (13) is equal to zero, or

$$E_{HQ}(b_j(\eta) | \hat{\theta}_j) + E_{HQ}(\theta_j | \hat{\theta}_j) = \frac{f_j}{\hat{q}_j + f_j} + \sum_i \frac{c}{\hat{q}_i + f_i} (E_{HQ}(\hat{\theta}_i - e_i | \hat{\theta}_j). \tag{14}$$

The optimal $\hat{q}_j$ must therefore satisfy a set of simultaneous equations and their interpretation in the most general case is complicated. Some additional insights may be gained, however, by examining special cases. Suppose, for example, that $E_{HQ}(\hat{\theta}_i - e_i | \hat{\theta}_j)$ is zero for all $i \neq j$. In that event, (14) can be rearranged to read

$$\hat{q}_j = -f_j + \frac{(c + f_j)}{E_{HQ}(b_j(\eta) | \hat{\theta}_j) + E_{HQ}(\theta_j | \hat{\theta}_j)} . \tag{15}$$

If the denominator of the fraction in (15) approaches unity, indicating that headquarters believes division j's information to give a completely accurate picture of the effects of $b_j(\eta)$ and θ_j on the marginal benefits of investment, then (15) becomes $\hat{q}_j = c$. That is, the only factor that division j doesn't properly consider of its own accord is the "growth capacity" effect, c, so departures from the pure hurdle rate system are based solely on c. On the other hand, if the denominator approaches zero, indicating that division j's information is useless, $\hat{q}_j$ becomes infinite. This means that division j is effectively proscribed from choosing an investment level other than $\hat{I}_j$, and it amounts to an abandonment of the hurdle rate system in favor of a pure budget system. The reverse occurs when $f_j = c = 0$ or when $c = 0$ and the denominator of the fraction approaches unity. In this case $\hat{q}_j = 0$ and the firm relies on a pure hurdle rate system, as there is something to lose but nothing to be gained from budgeting. In the general case, we would expect the $\hat{q}_j$ to be neither zero nor infinite and the firm would thus use a combination of the two systems.

B. Revelation of the Divisions' Information

A second way that headquarters might try to reap the advantages of both systems at once would be to have the divisions reveal their information. If this could be done costlessly, of course, the whole allocation problem would become trivial. Headquarters would simply gather the information, analyze potential projects using market-based hurdle rates, and tell the divisions how much to invest.

In a more realistic setting, as discussed by Flaherty [5], this communication process is costly and will be undertaken only to the extent that it is believed worthwhile. As has been noted earlier, headquarters may not fully understand the information, and, more seriously perhaps, it may have no way to ensure that the divisions' information is truthfully revealed. In addition, headquarters may become overloaded if the divisions must communicate all their information about every investment project. Presumably this costly communication process is justified only for those cases in which the divisions' information is potentially very valuable but at the same time divisional perceptions of net benefits are subject to serious errors.

C. Capital Budgeting Practice in Large Firms

Descriptions of the capital allocation process in large firms provide evidence that some sort of mixed system is in fact frequently used.[14] First, top management in such firms commonly specifies preliminary budgets for its divisions as well as approval levels for projects. Smaller projects can typically be approved by division heads, and thus, within the budget limits, decision-making for these projects is completely decentralized as in a pure hurdle rate system. Larger projects, by contrast, must be approved by a central investment committee or even the board of directors. Division managers must submit formal proposals which include cash flow projections, estimated rates of return and other information. When more is at stake, in other words, top management finds it worthwhile to try to learn the divisions' information through a costly communication mechanism. The divisions analyze projects using hurdle rates assigned by headquarters, but for large projects the details of the analysis must be shared with headquarters.

Moreover, the investment levels that are ultimately chosen for each division will, according to Bower and Lessard [1], "sum approximately but not exactly to the planned capital budget." If a division's worthwhile projects exceed its budget, top management may be willing to renegotiate. The strength of the resistance that division heads encounter in this renegotiation process can be thought of as performing much the same function as the penalty coefficients, $\hat{q}_j$, in Section II.A.

While corporate capital budgeting procedures have often appeared strange to finance theorists, many of their features may be similar to the mixed price and penalty system analyzed by Weitzman [15] or the costly communication mechanism described by Flaherty [5]. Many firms apparently try to make efficient use of decentralized information but at the same time to provide checks on division managers' decisions that correct for divisional blind spots and overall firm objectives.

III. Conclusion

The primary purpose of this paper has been to show that budgeting, or capital rationing, can make sense in a multi-division firm, even in the context of a value-maximization model in which the firm faces no external rationing. Among the factors favoring this intra-firm rationing are sources of curvature in the net benefit function (growth capacity constraints, for example) that are not perceived at the division level and a high variance in the error components of the divisions' information. The primary factor favoring the hurdle rate system is the divisions' possession of valuable information about investment projects that cannot be communicated at low cost to headquarters. Since, in general, companies would face some trade-off between these different types of factors, the optimal capital allocation system for most companies probably entails some mixture of budgets and hurdle rates.

Although the preceding analysis suggests that a direct application of Weitzman's models is capable of yielding insights into corporate capital allocation procedures, several questions are in need of further work. First, the analysis is primarily concerned with finding optimal operating rules for headquarters to convey to the divisions. The problem of enforcing these rules has not been addressed, nor has the question of incentive compensation schemes. If the divisions receive credit for the benefits from their investment and are "charged" for the capital they employ, a full analysis of division decisions can only be made if the mapping from this accounting scheme to division managers' compensation is specified.[15] In addition, the possibility of gaming behavior on the part of the division managers poses a number of problems for headquarters as it tries to gather the information necessary to implement optimal capital allocation procedures.

Footnotes

1. To the extent that a division chooses among projects of differing risk, it would be assigned a schedule of hurdle rates corresponding to these risks.

2. Surveys of capital budgeting practice have found rationing to be quite common. See, for example, Fremgen [6] and Gitman and Forrester [7].

3. Gitman and Forrester [7] report that 52 percent of the respondents to their questionnaire allocated a fixed annual budget among competing projects.

4. See Bower and Lessard [1] and Hastie [9] for descriptions of this process.

5. Weingartner [13] provides a detailed critical survey.

6. In a perpetuity context, this benefit could be interpreted as annual net operating cash flow minus annual capital cost. For simplicity, we will not deal with cases where cash flows differ from year to year.

7. Because of the quadratic form, this curvature is symmetric around $\hat{I}_j$. The algebra would be somewhat messier but similar results would hold if asymmetries were introduced.

8. Division managers are assumed to obediently follow the instruction to maximize expected net benefits given $\hat{k}_j$. Gaming behavior can be thought of as entering the problem if the division managers make strategic mis-estimates of θ_j.

9. The possibility that e_j might take on nonzero values is included for generality. In the Brealey and Myers case, for example, in which "divisional managers habitually overstate their investment opportunities," we would expect $e_j > 0$. When divisional forecast errors arise from noisy information, on the other hand, we have seen above that division j will rationally behave as if $\hat{\theta}_j$ is the true value of θ_j only when $E_j(\varepsilon_j) = 0$. In that event, symmetric rationality would suggest that $E_{HQ}(\varepsilon_j) = e_j = 0$ as well. None of the conclusions that follow would change if $e_j = 0$. The only difference would be that headquarters would set $\hat{k}_j = k$.

10. In cases where division managers are deliberately trying to deceive headquarters, this analysis captures only the first round of the game. If division j anticipates headquarters' hurdle rate adjustment, it may simply increase the size of its bias. Thus an initially known bias may eventually become an uncertain one.

11. In an explicitly multiperiod context, this point would have to be qualified considerably. Changes in the hurdle rate would not affect short-term and long-term projects equally and may therefore introduce a new kind of bias. I am grateful to Roger Bohn for bringing this point to my attention.

12. As above, the variability of the divisions' tendency to go astray is the primary factor that favors budgets. Kester [10] discusses the attempts of some firms to adjust hurdle rates so as to account for strategic interaction between projects. A division whose business the firm wished to emphasize for strategic reasons, for example, might be assigned a lower hurdle rate. This procedure will work when the size of

the division's investment bias is perfectly predictable, but uncertainty about the bias will favor budgets.

13. Conceivably the "penalty", $\hat{q}_j$, could be negative, in which case deviations from $\hat{I}_j$ are encouraged.

14. In addition to Bower and Lessard [1] and Hastie [9], cited earlier, useful descriptive material can be found in Kester [10], Prendergast [12] and in the "Super Project" and "MRC (A)" cases in Butters _et_. _al_ [3].

15. See Harris, Kriebel and Raviv [8] for a model incorporating these features.

References

1. Richard S. Bower and Donald R. Lessard. "An Operational Approach to Risk-Screening." Journal of Finance 28 (May 1973),

2. Richard Brealey and Stewart Myers. Principles of Corporate Finance. New York: McGraw-Hill, 1981.

3. J. Keith Butters, William E. Fruhan, Jr., David W. Mullins, Jr. and Thomas R. Piper. Case Problems in Finance. Eigth edition. Homewood Illinois: Irwin, 1981.

4. Willard T. Carleton. "Linear Programming and Capital Budgeting Models: A New Interpretation." Journal of Finance 24 (December 1969), 825-33.

5. M. Therese Flaherty. "Prices Versus Quantities and Vertical Financial Integration." Bell Journal of Economics 12 (Autumn 1981), 507-525.

6. James M. Fremgen. "Capital Budgeting Practices: A Survey." Management Accounting 54 (May 1973), 19-25.

7. Lawrence J. Gitman and John R. Forrester, Jr. "A Survey of Capital Budgeting Techniques Used by Major U.S. Firms." Financial Management 6 (Fall 1977), 66-71.

8. Milton Harris, C.H. Kriebel and Artur Raviv. "Asymmetric Information, Incentives and Intrafirm Resource Allocation." Management Science 28 (June 1982), 604-620.

9. K. Larry Hastie. "One Businessman's View of Capital Budgeting." Financial Management 3 (Winter 1974), 36-44.

10. W. Carl Kester. "Evaluating Growth Options: A New Approach to Strategic Capital Budgeting." Working Paper 83-38, Harvard University, Graduate School of Business Administration.

11. Stewart C. Myers. "Finance Theory and Financial Strategy." Interfaces, forthcoming.

12. George J. Prendergast, Jr., "Strategic Planning in DuPont." Paper presented at TIMS/ORSA Special Interest Conference on Corporate Strategic Planning, New Orleans, Louisiana, March 1977.

13. H. Martin Weingartner. "Capital Rationing: n Authors in Search of a Plot." Journal of Finance 32 (December 1977), 1403-31.

14. Martin L. Weitzman. "Prices vs. Quantities." Review of Economic Studies 41 (October 1974), 477-91.

15. ____________. "Optimal Rewards for Economic Regulation." American Economic Review 68 (September 1978), 683-91.