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MACROECONOMIC PLANNING AND DISEQUILIBRIUM ESTIMATES FOR POLAND, 1955-1980

Richard Portes

Richard Quandt

David Winter

Stephen Yeo

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ABSTRACT

This paper specifies and estimates a four-equation disequilibrium model of the consumption goods market in a centrally planned economy (CPE). The data are from Poland for the period 1955-1980, but the analysis is more general and will be applied to other CPEs as soon as the appropriate data sets are complete. This work is based on previous papers of Portes and Winter (P-W) and Charemza and Quandt (C-Q). P-W applied to each of four CPEs a discrete-switching disequilibrium model with a household demand equation for consumption goods, a planners' supply equation, and a "min" condition stating that the observed quantity transacted is the lesser of the quantities demanded and supplied. C-Q considered how an equation for the adjustment of planned quantities could be integrated into a CPE model with fixed prices and without the usual price adjustment equation. They made plan formation endogenous and permitted the resulting plan variables to enter the equations determining demand and supply. This paper implements the C-Q proposal in the P-W context. It uses a unique new data set of time series for plans for the major macroeconomic variables in Poland and other CPEs. The overall framework is applicable to any large organization which plans economic variables.

Richard Portes
Dept. of Economics
Birkbeck College
London University
7/15 Gresse Street
LONDON W1P 1PA

Tel: (01) 580 6622 x 412

Richard Quandt
Dept. of Economics, Princeton University

David Winter
Dept. of Economics, Bristol University

Stephen Yeo
Dept. of Economics, Birkbeck College

Macroeconomic Planning and Disequilibrium: Estimates for Poland, 1955-1980

Richard Portes, Richard Quandt, David Winter and Stephen Yeo†

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1. Introduction

This paper specifies and estimates a four-equation disequilibrium model of the consumption goods market in a centrally planned economy (CPE). The data are from Poland for the period 1955-1980, but the analysis is more general and will be applied to other CPEs as soon as the appropriate data sets are complete.

The work reported here is based on the previous papers of Portes and Winter (1980) and Charemza and Quandt (1982), referred to below as P-W and C-Q. The former applied to each of four CPEs a discrete-switching disequilibrium model with a household demand equation for consumption goods, a planners' supply equation, and a "min" condition stating that the observed quantity transacted is the lesser of the quantities demanded and supplied. C-Q considered how

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† Portes is at Birkbeck College, University of London, and Ecole des Hautes Etudes en Sciences Sociales, Paris, and is Research Associate, National Bureau of Economic Research; Quandt is at Princeton University; Winter is at the University of Bristol; and Yeo is at Birkbeck College. Quandt gratefully acknowledges support from the National Science Foundation under grant SES-8012592. Portes and Yeo thank the Social Science Research Council (U.K.) for support under grant B00230048. Portes has also benefitted from the assistance of the Maison des Sciences de l'Homme. We are especially indebted to T. Bauer, W. Charemza and M. Gronicki for assistance with time series for plan variables, and to I. Grosfeld for help with Polish data. I. Brunskill and A. Milne provided research assistance. We have received incisive and helpful comments from Hugh Davies and Guy Laroque.

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an equation for the adjustment of planned quantities could be integrated into a CPE model with fixed prices and without the usual price adjustment equation. They made plan formation endogenous and permitted the resulting plan variables to enter the equations determining demand and supply. Depending on the precise specification of the equation determining the plan, the model could adjust towards market clearing in a manner similar to that of disequilibrium models with price adjustment equations.

This paper implements the C-Q proposal in the P-W context. It differs from P-W in several respects: (i) the data are extended beyond 1975, up to 1980; (ii) the main series have been more or less substantially revised, using new information; (iii) a plan-adjustment equation determines the published plan for aggregate consumption by households; (iv) this plan enters the equation for the supply of consumption goods; (v) the variables constructed by P-W to measure deviations from plans for exogenous variables (output, investment, defence expenditure), which proxied the plan series by second-order quadratic trends, now use published plan data. The model here differs from C-Q in having a more general form of plan-adjustment equation than they propose.

The work reported here was possible only because we were able to assemble reliable time series for plans for the major macroeconomic variables in Poland and other CPEs. Using this new and unique data set, our empirical work can now go beyond the question posed by P-W, which concerned the existence of excess demand in the aggregate consumption goods markets of CPEs, to a range of important questions concerning the planning process and macroeconomic disequilibrium: Are the plans in a CPE properly represented as endogenous, determined by stable economic relationships rather than political caprice? How do plans so determined then influence the planners and the economy? Do the planners plan for macroeconomic equilibrium (i.e., does the plan refer to their planned supply or to their intention for the quantity transacted)? Is the disequilibrium macro framework appropriate and useful for the analysis of CPEs (see Portes, 1981a)? There are also interesting theoretical and econometric questions which arise, some of which will provide material for future work. The overall framework is applicable to any large organisation which plans economic variables.

2. The Simple Disequilibrium Model

The basic framework of our general model for Poland is taken from P-W with the modifications indicated above. Thus the consumption demand equation is identical to that in P-W, derived directly from the Houthakker-Taylor savings function:

$$CD_t = \alpha_1 DNFA_{t-1} + \alpha_2 DYD_t + \alpha_3 YD_{t-1} + u_{1t} \qquad (1)$$

where

CD = household desired expenditure on consumption goods and services

DNFA = household saving, measured as the change in net financial assets of households, NFA, during the period (NFA is the end-of-period net stock of financial assets); $DNFA_{t-1}$ was called S1 in P-W

DYD = change in disposable income from the previous to the current period

YD = disposable income

$u_1 \sim N(0, \sigma_1^2)$

Although it has a rather sophisticated theoretical rationale, this essentially just makes consumption depend on current and lagged income and on lagged consumption.†

The work of Houthakker and Taylor suggests the following a priori hypotheses:

$$-1 < \alpha_1 < -1/3, \quad 0 < \alpha_2 < 1, \quad \alpha_3 = 1$$

The modified supply equation is

$$CS_t = \beta_1 C^*_{t|t-1} + \beta_2 C^*Z_t + \beta_4 RNFA_{t-1} + \beta_5 CZXD_t + \beta_6 CZXI_t + u_{2t} \quad (2)$$

where

CS = planned supply of consumption goods and services

$C^*_{t|t-1}$ = plan for consumption in current period announced at end of previous period

(* denotes a plan throughout)

NMP = net material product

D = defence expenditure

I = investment expenditure

$C^*Z = (C^*/NMP^*)\cdot(NMP-NMP^*)$

$CZXD = [(D/NMP) - (D^*/NMP^*)]\cdot NMP$

$CZXI = [I/NMP) - (I^*/NMP^*)]\cdot NMP$

RNFA = deviation of current NFA from second-order exponential time trend fitted to observed values of NFA

$u_2 \sim N(0, \sigma_2^2)$

The annual plan for year t is formulated during the last quarter of year t-1 and announced during December of year t-1. These announced plans we denote by C*, NMP*, etc. More precisely, $C^*_{t|t-1}$ is the level of consumption planned for year t and announced at the

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† In more conventional form,

$$C_t = \alpha_2 Y_t + (\alpha_1 - \alpha_2 + \alpha_3)Y_{t-1} - \alpha_1 C_{t-1}$$

end of year t-1. The volume of consumer goods actually supplied to the population in year t is CS_t. It may differ from the previously announced plan $C^*_{t|t-1}$ - indeed, equation (2) is a model of how the planners depart from their previously announced plan $C^*_{t|t-1}$ to take into account unforeseen developments during year t.

A planned supply function of this form is explained, justified and estimated in Portes and Winter (1977, 1980). The hypothesis is that consumption goods supply will be determined by the announced consumption plan and by deviations from plans of output, defence, investment and consumption, as well as deviations from trend of household financial assets. A coefficient β_3 for the lagged values of C*Z was considered in the general model of P-W but the corresponding term dropped out of their estimates for Poland and therefore has been excluded here, while their original numbering of coefficients has been retained to facilitate comparisons. On the other hand, in P-W, defence and investment expenditure were aggregated, with a single coefficient β_5. A-priori arguments here suggest $\beta_1 = 1$; β_2, $\beta_4 > 0$; β_5, $\beta_6 < 0$.

In both the demand and supply equations, we expect a priori that no constant term should appear. They were tried in initial estimates, however, and we could not reject the hypothesis that they were zero.

The simple disequilibrium model is completed by

$$C_t = \min\ (CD_t,\ CS_t) \qquad (3)$$

where C is the quantity observed.

3. The Plan-Adjustment Equation

We have worked previously (Portes et al., 1983) with a model of the form (1)-(3), supplemented with a plan-adjustment equation specified as

$$C^*_{t|t-1} = \delta_1 C^*_{t-1|t-2} + \delta_2 C_{t-1} + \delta_3 C_{t-2} + \delta_4 RNFA_{t-2}$$
$$+ \gamma(CD_t - CS_t) + u_{4t} \qquad (4)$$

where $u_4 \sim N(0,\sigma_4^2)$. †

We justified this by reference to the discussion of planners' behaviour in Gacs and Lacko (1973) and Kornai (1971). Single-equation models of plan formation involving only previous plans and realizations are discussed by Yeo (1983). Different schemes yield relationships including the first three terms of equation (4), with differing interpretations of (δ_1, δ_2, δ_3). We added responses to observed "excess" household liquidity (RNFA) and to excess demand. We justified the period t excess demand term in (4) with a "planners' rational expectations" argument, while recognizing that period (t-1) excess demand might have been preferable, were it not for the intractable likelihood function which it generates in the complete model of equations (1) - (4).

An alternative approach to the specification of the plan equation is to construct a model of optimizing behaviour by the planners. The full optimizing problem facing the planners of a CPE in drawing up a macroeconomic plan is extremely complicated. Planners' preferences would have to be optimized intertemporally over all macroeconomic

† We assume that the u's are jointly normally distributed, contemporaneously uncorrelated and serially independent.

aggregates; the constraints would include the planners' own macroeconomic model and the reaction functions of households, enterprises, the agricultural sector, and foreign demand and foreign suppliers. Here we consider the construction of the supply plan for consumption for one period ahead. The rest of the planning process is taken as given. The verbal interpretation is more natural if we consider planning in the current period for period (t + 1), so the left-hand side variable is $C^*_{t+1|t}$. We shall make a number of simplifying assumptions concerning the information available to the planners and how they form their expectations.

We represent the planners' objectives with a quadratic loss function, defined as follows:

$$L = \frac{1}{2} a_1 X_1^2 + a_2(X_1 X_2) + \frac{1}{2} a_3 X_3^2 + a_4(X_3 X_4) \qquad (9)$$

where

$$X_1 = C^*_{t+1|t} - \frac{(1+g)}{\mu_1+\mu_2} (\mu_1 C_t + \mu_2 C^*_{t|t-1})$$

$$X_2 = C_t - C^*_{t|t-1}$$

$$X_3 = CD_{t+1} - CS_{t+1}$$

$$X_4 = CD_t - CS_t$$

and a_1, a_3, $a_4 > 0$; $a_2 < 0$; $0 \leqslant \mu_1, \mu_2 \leqslant 1$; $\mu_1 + \mu_2 = 1$

The first argument of L embodies the steady growth objective, sometimes elevated to the status of the "Law of Planned Proportional Development". Planners give some weight to keeping the plan close to a long-run growth path of consumption. X_1 is the deviation of the plan for t+1 from this desired long-run growth path, where g is the long-run growth rate and μ1 and μ2 are weights. The supply plan for next period which corresponds to the long-run growth path is a "mark up" (by g) of a convex combination of this period's plan and actual consumption, which will contain new information about the consumption market to be incorporated into the planners' perceived optimum growth path. The planners may disregard that information - if $\mu_1 = 0$, $\mu_2 = 1$, it is only the sequence of plans that matters; $\mu_1 \neq 0$ is a concession to reality, insofar as they cannot implement their sequence of plans precisely. An alternative interpretation might recognize that C_t is not known with certainty in t, when the plan is formed, and the planners' best estimate might be a convex combination of preliminary data on C_t with the known $C^*_{t|t-1}$. Ideally X_1 should be zero, and in a quadratic specification planners will find both positive and negative values of X_1 equally costly.

The cost of a positive or negative deviation X may be reduced or increased depending on plan fulfilment in the current period. X_2 is the deviation of actual consumption from plan in period t.

Then the cost to the planners of a given X_1 will vary with the sign and size of X_2. Performance relative to plan in the current period gives the planners additional information about future performance, and they adjust accordingly the perceived costs of deviations from the long-run growth path. If $X_2 > 0$, the planners find that a plan for next period above the growth path is less costly. Similarly, if the plan in period t was underfulfilled ($X_2 < 0$), then the planners find a plan below the growth path again more acceptable than otherwise. On the other hand, when X_1 and X_2 have opposite signs this increases the cost of planning above or below the long-run growth path. The interaction term, $X_1 X_2$, therefore enters negatively into the loss function, so $a_2 < 0$. The effect of this term is similar to that of making the weights μ_1 and μ_2 a function of plan fulfilment.

The second main argument of the loss function is future excess demand for consumption goods. There is a large literature about the planners' attitude to excess demand and supply of consumption goods (e.g., Portes and Winter, 1980, and Portes, 1981a), and we see no further need to justify the inclusion of this term. Again we should be considering expected excess demand in period t+1, conditional on information available at period t. Again for simplicity, we use actual excess demand here and treat the problem of uncertainty about future demand below. We assume the planners want $X_3 = 0$ and will view positive and negative values of X_3 as equally costly.

As with the steady growth objective, the costs of future excess demand may be raised or lowered by an interaction term. X_4 is contemporaneous excess demand. If X_3 and X_4 have the same sign, i.e., if the plan implies either excess demand or excess supply for successive periods, the planners' perceived costs increase. If they are of opposite sign they decrease. Thus $a_4 > 0$. This is because the repercussions of both excess demand and excess supply cumulate from period to period. If excess demand persists households accumulate cash balances, their frustration increases and labour supply incentives diminish. Similarly, successive periods of excess supply entail the accumulation and wastage of unsold stocks.

Before deriving the first-order conditions, consider the partial derivative $\frac{\partial X_3}{\partial C^*_{t+1}}$. We will assume $\frac{\partial CD_{t+1}}{\partial C^*_{t+1}} = 0$. In practice, it would be rational to plan consumption goods supply in conjunction with plans for employment, earnings, social security benefits, etc. Data on these plans are not yet available to us, so we assume the supply planners take demand as given. Thus

$$\frac{\partial X_3}{\partial C^*_{t+1}} = -\frac{\partial CS_{t+1}}{\partial C^*_{t+1}} = -\beta_1 - \beta_2 \frac{[NMP_{t+1} - NMP^*_{t+1}]}{NMP^*_{t+1}} \tag{6}$$

from our supply equation (2).

We suppose that when constructing the consumption plan, the planners assume that the NMP plan will be exactly fulfilled. Thus

$$\frac{\partial X_3}{\partial C^*_{t+1}} = -\beta_1$$

Choosing C^*_{t+1} to minimize L then gives the first-order condition

$$a_1X_1 + a_2X_2 - \beta_1a_3X_3 - \beta_1a_4X_4 = 0 \tag{7}$$

In section 5, we argue for the restriction $\beta_1 = 1$, which is accepted by the data. Equation (7) then gives a plan equation

$$C^*_{t+1|t} = \frac{(1+g)}{\mu_1+\mu_2}(\mu_1C_t + \mu_2C^*_{t|t-1}) - \frac{a_2}{a_1}(C_t - C^*_{t|t-1})$$

$$+ \frac{a_3}{a_1}(CD_{t+1} - CS_{t+1}) + \frac{a_4}{a_1}(CD_t - CS_t) \tag{8}$$

If we now normalize on a_1 , setting $a_1 = 1$ w.l.o.g. (assuming $a_1 > 0$), and we also note that $\mu_1 + \mu_2 = 1$, we can simplify equation (8) to read:

$$C^*_{t+1|t} = (1+g)(\mu_1 C_t + \mu_2 C^*_{t|t-1}) - a_2(C_t - C^*_{t|t-1})$$
$$+ a_3(CD_{t+1} - CS_{t+1}) + a_4(CD_t - CS_t) \qquad (9)$$

The consumption plan is thus a linear function of the previous period's actual consumption, planned consumption and excess demand, as well as excess demand in the period being planned. The coefficients on C_t and C^*_t sum to (1+g). The coefficients of both excess demand terms are positive. Equation (9) is of course quite similar to equation (4) above. The chief difference is the absence from (9) of the second-order lag in actual consumption which appears in (4). Following our previous work (see beginning of this Section), this term has in fact been included in the estimated plan equation. The persistent insignificance of its coefficient (see below) supports the approach taken here.

The plan equation (9) contains terms in excess demand. To make this operational, it must be embedded in a model of the form of equations (1)-(3), where current excess demand is endogenously determined. Two different problems arise. First, although it is theoretically possible to derive the likelihood function of this type of model with lagged dependent variables (Quandt, 1981), it is still computationally intractable unless u_{4t} is identically zero. Thus only one of the excess demand terms in equation (9) can be included in the estimated equation. The other will have to be proxied by an alternative measure of excess demand. The second problem concerns the treatment of planners' expectations of future excess demand. We discuss this below and in Appendix B.

For estimation, we write the plan equation as

$$C^*_{t+1|t} = \delta_1 C^*_{t|t-1} + \delta_2 C_t + \delta_3 C_{t-1} + \delta_4 (CD_t - CS_t)$$
$$+ \gamma(CD_{t+1} - CS_{t+1}) + u_{4t} \qquad (10)$$

where

$$\delta_1 = (1+g)\mu_2 + a_2$$

$$\delta_2 = (1+g)\mu_1 - a_2 > 0,$$

$$\delta_4 = a_4 > 0, \ \gamma = a_3 > 0$$

$$\mu_1 + \mu_2 = 1$$

Thus we have $\delta_1 + \delta_2 = 1 + g$, from which we get $a_2 = (\delta_1 + \delta_2)\mu_1 - \delta_2$. If we take a range of hypothetical values for μ_1, we can identify a_2 as well as (a_3, a_4, g) from the estimated coefficients of equation (10).

Note also that long-run stability requires $|\delta_1|<1$ (Portes et al., 1983). There are several different ways of interpreting the excess demand terms in equation (10) so as to make it operational.

Model 1:

One relatively simple way of incorporating disequilibrium into the plan formation equation is to take account of liquid asset holdings of the population. In this model, to maintain comparability with our previous work, we consider the formation of the plan for year t at the end of year t-1. We therefore take a lagged version of equation (10), which is similar to equation (4). The variable $RNFA_{t-2}$ measures the deviation from trend of net financial assets at the end of period t-2 or the beginning of period t-1. The planners know this variable when they formulate the year t plan at the end of t-1. Our plan formation equation would therefore be

$$C^*_{t|t-1} = \delta_1 C^*_{t-1|t-2} + \delta_2 C_{t-1} + \delta_3 C_{t-2} + \delta_4 RNFA_{t-2} + u_{4t} \quad (11)$$

It is possible to estimate equation (11) together with equations (1)-(3), and we call this Model I. It should be noted, however, that since no current endogenous variables appear on the RHS of (11), simultaneous estimation of the plan equation is unnecessary and will yield the same results as estimation by OLS.

Model II:

We prefer to introduce disequilibrium explicitly into our model in a manner consistent with the optimizing model developed above. Taking again the formation of the plan for year t at the end of year t-1 we would have

$$\begin{aligned} C^*_{t|t-1} &= \delta_1 C^*_{t-1|t-2} + \delta_2 C_{t-1} + \delta_3 C_{t-2} \\ &\quad + \gamma(CD_t - CS_t) + \delta_4(CD_{t-1} - CS_{t-1}) + u_{4t} \end{aligned} \quad (12)$$

As remarked above, it is computationally infeasible to include both current and lagged excess demand terms, and we therefore seek proxies for the lagged excess demand term. One possible measure of excess demand in year t-1 is the behavior of financial assets held by the population, and we take the deviation from trend of net financial assets at the end of year t-2 or beginning of year t-1, $RNFA_{t-2}$. This gives the equation

$$C^*_{t|t-1} = \delta_1 C^*_{t-1|t-2} + \delta_2 C_{t-1} + \delta_3 C_{t-2}$$

$$+ \gamma(CD_t - CS_t) + \delta_4 RNFA_{t-2} + u_{4t} \qquad (13)$$

We call this Model IIa. A slightly more general version of (13) would allow CD_t and CS_t to have unequal effects on $C^*_{t|t-1}$, and we might enter this term as $\gamma_1 CD_t + \gamma_2 CS_t$. We denote this formulation as Model IIb. Model IIa is a special case of IIb with $\gamma_1 = -\gamma_2 = \gamma$, and Model I is a special case of Model IIa with $\gamma = 0$.

This formulation also has the advantage of nesting Model I inside Model II. The inclusion of actual or realized excess demand for year t, which was queried above, is discussed further in Appendix B.

Models III and IV:

There are several ways in which the very strong informational assumptions of Model II regarding $(CD_t - CS_t)$ might be relaxed. One possibility would be to replace $(CD_t - CS_t)$ by $E_{t-1}(CD_t - CS_t)$, the expectation of excess demand in year t, taken with respect to the information available in year t-1, i.e. the predetermined variables in the model. This variant is closer in spirit to the models which feature in the rational expectations literature.

$$C^*_{t|t-1} = \delta_1 C^*_{t-1|t-2} + \delta_2 C_{t-1} + \delta_3 C_{t-2} + \delta_4 RNFA_{t-2} + \gamma E_{t-1}(CD_t - CS_t)$$

$$+ u_{4t} \qquad (14)$$

In practice, one computes the likelihood function for this variant by substituting the equations for CD_t and CS_t in $E_{t-1}(CD_t - CS_t)$. Appendix B considers assumptions which can be made about the planners' expectations at time t-1 of the period t variables in $(CD_t - CS_t)$. If the planners' expectations of the deviations of NMP, D, etc., from their planned values in year t are taken to be what subsequently occurred in year t, we have Model III. If, as might be more plausible, the planners' expectations of these deviations are set to zero, we have Model IV. Both models introduce restrictions across the parameters of the equations of the full model.

Model V:

The strong informational requirements of Model I may be relaxed in another way as well - by shifting the timing of the plan formation equation one period forward relative to models I - IV. That is, we return to the original timing of the plan equation as the plan for t+1 formed at the end of year t. We still face the problem of the excess demand terms, and we seek use $RNFA_{t-1}$ as a proxy for $(CD_{t+1} - CS_{t+1})$. Our equation for Model Va is then

$$C^*_{t+1|t} = \delta_1 C^*_{t|t-1} + \delta_2 C_t + \delta_3 C_{t-1} + \gamma(CD_t - CS_t)$$

$$+ \delta_4 RNFA_{t-1} + u_{4t} \qquad (15)$$

In this model, the planners respond to current disequilibrium $(CD_t - CS_t)$ at the time they are making the plan for t+1. They also respond to the deviation from trend of assets at the beginning of period t, since this would be known at the beginning of year t. This formulation of the model is to some extent more natural in that it mimics the recursive nature of the planning process in which the announced plan $C^*_{t|t-1}$ is determined before CD_t and CS_t. It should be noted, however that the equation for $C^*_{t+1|t}$ cannot be estimated separately from (1)-(3) because of the presence of the endogenous C_t and $CD_t - CS_t$ (for details see appendix B).

Finally, we also allow for the possibility that the planners respond in an asymmetric fashion to excess supply and to excess demand. In Va we permit $\gamma = \gamma_1$ when $CD_t > CS_t$ and $\gamma = \gamma_2$ when $CD_t < CS_t$, and we call this Model Vb.†

4. The Plan Data

We believe this to be the first study of the macroeconomic behaviour of a CPE which includes consistent and comparable time-series data on plans drawn from original sources. The data themselves throw light on the abilities of the planners, events in Poland, and planning as a process of prediction.

We have used plan data from the mid-1950s to 1980 on four time series for this paper: consumption, NMP, gross investment and defence. All variables are defined in constant zloties. The data on both plans and realizations (actuals), together with detailed discussion of methods and sources, are given in Portes et al. (1983). Table 1 shows the absolute deviations (in percent) of planned growth rates from actual growth rates. Viewed in this way, the planners' performance varies markedly over the period. The worst year in terms of accuracy is 1972, when investment, which was planned to grow at 9.6%, in fact grew at 23.01%. Thereafter actual investment was

† Our Model V is a more general version of what C-Q (p. 112) call "Model 3". We include in our plan adjustment equation the terms in both their (4b) and their (4d), and we allow their coefficients to differ from unity.

above plan until 1980. But the planners maintained a fair degree of control. The 1978 plan reduced investment by 5.3%. Actual investment rose slightly that year but fell in both 1979 and 1980.

The 1972 plan substantially underpredicted consumption as well. Then, starting with the plan for 1976, the consumption plans were underfulfilled for five successive years, with a remarkably large shortfall in 1978. The NMP plans, too, were all underfulfilled from 1975 onwards, with a progressive deterioration in the performance both of the planners and of the economy itself.

Of the 19 observations 1957-75, 9 consumption plans were underfulfilled and 10 exceeded; for NMP, 7 and 12 fell in these respective categories. Thus there was no clear pattern of excessive pressure on the economy by the planners (or overoptimism), nor of underestimating performance. From 1976 onwards, however, all consumption and NMP plans were consistently underfulfilled, reflecting the continuous deterioration of performance (as Mr. Gierek's economic strategy disintegrated) and the planners' inability to come to grips with it (Portes, 1981b). Investment and defence expenditure plans show quite different pictures. The former were consistently overfulfilled during the period (19 out of 24). The latter were underfulfilled in all but two of the years 1957-67, overfulfilled in all but one thereafter (through 1980)!

In relation to the mean planned growth rates, the average absolute deviations given in Table 1 show a fair degree of planning inaccuracy, if the plans are treated as predictions. Perhaps a better measure of the predictive power of the planners is in terms of levels. Table 2 gives some comparisons. If the standard errors of the deviations of planned levels from actuals are compared with the standard deviations of the residuals from second-order autoregressive processes fitted by OLS to the whole sample, we find the planners out-perform the time-series regression for investment and defence but not for consumption and NMP. Our consumption function [equation (1)] estimated by OLS over the whole sample has a standard error of the residuals under half that of the AR2 process and a third of the "standard error of the plan".

5. Results

The likelihood functions for the models specified in Section 3 are derived in Appendix B. We used the Davidon-Fletcher-Powell algorithm to obtain maximum likelihood estimates, with numerical first derivatives.

We tried variants of each of the models discussed. Model IV performed poorly, so we do not report estimates from it. Model IIb (unequal γs) gives no significant improvement in the likelihood over IIa (which is nested in it), whereas Va is rejected against Vb. Thus we report results from Models I, IIa, III and Vb.

All the results we give are from estimation with the restriction $\beta_1 = 1$. Our theory regards C* as the ex ante supply plan, with equation (2) representing the planners' adjustment of actual supply away from that plan in response to new data. The planners may consciously intend excess demand or excess supply. The restriction assumes only that when they announce the plan, that is what they do intend to offer to get their intended outcome. The restriction $\beta_1 = 1$ is accepted by the data for the three models cited and for most other runs.

The estimates are given in Table 3. We see immediately that a likelihood ratio test rejects Model I against IIa, in which it is nested. On the whole, the correspondence with prior beliefs about the coefficients is good. The demand equations for Model I and III satisfied all conditions; those for Models IIa and Vb give α_1 the right sign, but somewhat too small in absolute value (though not significantly so), and both also make α_3 slightly but significantly less than its theoretical value of unity. The long-run savings ratio implied by the estimates for Model I is 2.1% in an economy growing at 5%.

The supply equation is less satisfactory; although the equation's standard error is reasonable, the plan must be doing most of the work in explaining supply (which is of course not inconsistent with the deviations of <u>actual</u> from plan shown in Table 1). The NMP deviation term is significant only for Models I and III, with the correct sign. The defence expenditure term works well in all models. But both β_4 and β_6 appear to take the wrong sign consistently. We discuss this below.

The plan equation performs well. The second-order lag on actual consumption, which we included because of previous work by others and ourselves but does not appear in the theory of Section 3, also does not appear in the data: δ_3 is insignificant throughout. The signs of other coefficients are as predicted, and the results seem fairly stable; this equation is probably "best" in Model Vb. Note that in Model I, the plan equation is "decoupled" from the rest of the model, and results obtained for it by OLS are identical within the limits of numerical approximation to those shown in Table 3, which confirms that our optimization programme is working properly. As in our previous paper, we can calculate what constant growth rate of C would be consistent with exact realization of plans in the estimated version of equation (4). The answer ranges from 5.7% p.a. in Model Vb to 7.4% in Model I, quite close to the observed 6.3% *p.a.* growth of actual consumption in 1957-80.

As noted in Section 3, we can identify the parameters of our loss function from the estimated plan-adjustment equation. The presence of δ_3 is a nuisance here, so we re-estimated subject to the restriction that $\delta_3 = 0$, which was accepted for three of the four models. With the normalization $a_1 = 1$, we then find a_3 not far from 0.5, a_4 in the range 1.4-1.9, and $-1.8 < a_2 < -0.7$ for $0.25 \leq \mu_1 \leq 0.75$. The smallest absolute values for a_2 come in Model Va when μ is large, giving higher weight to C_t than to $C^*_{t|t-1}$ in defining the path on which the planners are trying to keep the economy. The growth rate on that path is implied to lie between 5.7% and 7.2%. The signs of all these parameters accord with our hypothesis, and the magnitudes seem reasonable.

In addition to examining the point estimates, it is useful to evaluate the model's ability to forecast actual consumption, C. Ideally forecasts both from within and outside the sample are of interest. In an ordinary linear regression model, it is natural to compare the point estimate of a forecast with its confidence interval. Some procedures, such as that of Chow, take into account the sampling errors of the estimated coefficients. Others, such as that of Davidson *et al.* (1978), confine themselves to the ratio of forecast variance to residual variance.

In the present models, since the predicted value of C is the output of a min. function, it is not straightforward to calculate its confidence interval. It appears that the simplest method of evaluating the standard error of the predicted value of C is by stochastic simulations (for details see Appendix A).

Table 4 shows the predictive performance of the disequilibrium models. We use Model Vb, estimated through 1978, to give an ex post forecast for 1979, and Models I, IIa, and III, estimated through 1979, to give an ex post forecast for 1980. In each case, the prediction is the minimum of the supply and the demand forecasts. For 1979, the disequilibrium model is less than one standard deviation from the actual (see the notes to the Table for interpretation of these standard deviations). Indeed, it is closer (from below) to the actual than was the plan, the forecast from an AR2 process on C, or that from an OLS estimate of equation (1), all of which substantially overpredicted. For 1980, Model I significantly underpredicted, but Model IIa came within two standard deviations and Model III within one. This was a creditable showing in view of the extraordinary circumstances, in which the output plan proved wildly overoptimistic (though both the AR2 process and equation (1) do as well as disequilibrium Model III and somewhat better than I and IIa.)

There are various criteria by which we can assign each observation to an excess demand or excess supply regime. In past work, we have used the estimated marginal or conditional (on the observed C_t) probability π_t that $CD_t > CS_t$ (Burkett, 1981 showed that the marginal and conditional probabilities were very close for Poland in the original P-W study). Here we report in Table 5 the average simulated excess demands generated by each model for each year. The simulation procedure is described in Appendix A. The average excess demand for a given year is in practice positive if and only if more than 50 of the 100 simulations for that year show excess demand, but the correlation with the estimated conditional probabilities is not perfect.† A more discriminating standard is set by classifying an observation as excess demand or supply only if the absolute value of the difference between the average simulated demand and average simulated supply exceeds twice the standard deviation of the simulated transacted quantity; in practice, this occurred only when the proportion of cases in the favoured regime exceeded 75%.

A fairly consistent pattern averaged from applying all these criteria with the three selected models. 1959, 1961, 1968, 1971-72, 1975, and 1979 were selected as years of clear excess demand. On the other hand, 1958, 1960, 1962-64, 1967, and 1976-78 appeared to be periods of excess supply in the consumption goods market. An overall (though not unambiguous) judgment suggests 1965-66 were also excess supply years, while 1974 and 1980 were probably excess demand. This leaves 1957, 1969-70, and 1973 as impossible to classify. The models themselves do not often disagree, though Vb differs distinctly from the other three for 1957 and 1965-66, while Models I and III differ from the other two for 1973-74.

This pattern is broadly consistent with the development of the Polish economy since the mid-1950s, and it is undoubtedly more plausible than the classification (using estimated probabilities) in Portes and Winter (1980). The dominance of excess supply in the earlier years and excess demand in the 1970s accords with the dominance of tight money wage control until Gierek replaced Gomulka at

† The estimated conditional π_t suggest excess demand in 1963 and 1965 under Model I, in 1965-67 under Model IIa, and in 1967 under Model Vb, the end of 1970 (see the discussion in Portes, 1981b). 1959 saw a all of which show negative average simulated excess demand.

tremendous investment boom, 1968 political disquiet. The excess supply shown in 1976-78 may reflect the planners' efforts to satisfy consumer pressures after the political explosion of mid-1976, while tightening up their control of money wages and letting inflation accelerate to soak up purchasing power.

The large estimated excess supply in 1978 appears to be the consequence of a wildly overoptimistic plan, which our models do not scale down sufficiently in response to the considerable shortfall of NMP from its plan in 1978. This points out the relative weakness of the supply equation, which may in part be due to the absence here of any treatment of foreign trade and borrowing. For example, the consistently "wrong" sign on β_6 was explained in our earlier paper by a supposed structural change around 1972, when the foreign borrowing constraint was relaxed. Then investment need no longer have crowded out consumption, and indeed the planners might have allowed for some multiplier effects of investment on consumer demand and accommodated them with additional imports.

The surprising but small negative coefficient on $RNFA_{t-1}$ in the supply equation must be viewed in the light of the rather large positive coefficient on the RNFA term in the plan-adjustment equation. A sustained departure of NFA from trend will give a total effect on supply, acting through C* as well as directly, of

$$\frac{\partial CS}{\partial RNFA} = \frac{\beta_4 + (\beta_1+\beta_2\theta)\delta_4}{1+\gamma},$$

where $\theta = (NMP - NMP^*)/NMP^*$. With $-0.05 < \theta < 0.05$, our estimates give a range between 0.72 (in Model Vb) and 1.06 (in Model IIa).

These suggest the planners do indeed seek market clearing: in response to a sustained increase in household NFA, they would increase the supply of consumption goods by roughly the same amount.

Judging only by the equation variance, the supply equation appears to perform better than the demand equation in Models I and III, whereas we have the converse in Models IIa and Vb. This would suggest that Models I and III would tend to put a higher proportion of observations on or near the supply curve - i.e., more excess demand in those models (see P-W, p. 151). Yet in fact the opposite is true, which may suggest that the classification of observations between regimes is not merely the consequence of the relative strength of the specification of the demand and supply equations.

6. Conclusions

We believe we have taken substantial steps towards answering the questions posed in Section 1 and demonstrating the applicability of the C-Q model. Estimation has shown that it is both feasible and informative to use plan data, and to model the regularities in the process of plan construction. The plan depends upon planned and actual consumption and excess demand. These announced plans are embodied in a supply function which reflects, in addition, unforeseen subsequent developments in the economy. The planners do appear to try to adjust announced plans and actual supply in order to reduce excess demand. The disequilibrium macroeconomic framework, with fixed prices and planned quantities, can be estimated for centrally planned economies and seems to provide insight into their behaviour. The pattern of excess demands revealed by the data appears broadly consistent with economic events in Poland.

We have data sets which permit application of the model to at least two other countries, and there are various extensions of the analysis which we shall explore in future work. Moreover, we intend to apply a similar approach to other macro variables and markets - e.g., investment, the labour market or NMP itself.

TABLE 1

DEVIATION OF ACTUAL FROM PLANNED GROWTH RATE
(percent)

Year	C	NMP	I	D
1957	-2.16	2.66	2.10	-.30
1958	-.87	-.37	1.93	-9.36
1959	.88	-.91	4.64	3.21
1960	-3.03	-1.32	-.53	-6.92
1961	1.60	3.14	-1.92	.57
1962	-1.23	-4.91	.09	-6.81
1963	.60	1.74	-5.65	-6.76
1964	-.05	3.13	3.02	-3.04
1965	2.65	1.80	.90	-2.15
1966	1.40	3.41	2.57	-1.89
1967	-.53	2.31	3.22	-1.67
1968	1.38	4.20	2.96	4.85
1969	-1.74	-2.10	-.64	.73
1970	-1.19	-.59	1.56	.97
1971	1.33	2.71	.20	.37
1972	9.01	4.47	13.41	-2.43
1973	.19	2.91	12.45	3.31
1974	-.83	.94	9.95	2.82
1975	.25	-.82	4.70	.73
1976	-5.08	-1.50	1.00	2.44
1977	-2.97	-.70	1.90	6.87
1978	-8.29	-2.41	7.40	1.70
1979	-1.16	-5.10	1.10	4.68
1980	-2.83	-7.59	-4.30	1.73
Average deviation	-0.53	0.21	2.59	-0.26
Average absolute deviation	2.14	2.57	3.67	3.18
Average planned growth	7.32	5.71	5.33	5.00

The deviation for a variable X is defined as $100(X_t - X^*_t)/X_{t-1}$, where X_t is the actual and $X^*_t = X^*_{t|t-1}$ is the planned level of the variable X for period t.

TABLE 2

Performance of Plans, 1957-80

(billion zloties, constant prices of 1971)

	C	NMP	I	D
Sample mean	448.88	836.73	270.85	31.87
Standard deviation of plan from actual	17.72	31.80	14.78	1.09
Standard error of residual from AR2 process	12.38	21.86	17.91	1.95
Standard error of residual from OLS estimates:				
CD [equation (1)]	5.26	-	-	-
CS [equation (2)]	11.61	-	-	-

Notes:

A second-order autoregressive (AR 2) process was fitted to the series of actual data, and we cite above the standard error of the residuals from this form of "explanation".

All estimates are ML estimates. There are no small sample adjustments.

TABLE 3

Estimates of Disequilibrium Models, Poland 1957-80

	Model I	Model IIa	Model III	Model Vb
α_1	-0.685 (0.300)	-0.222 (0.097)	-0.492 (0.242)	-0.201 (0.091)
α_2	0.852 (0.165)	0.899 (0.044)	0.775 (0.136)	0.898 (0.136)
α_3	1.000 (0.007)	0.989 (0.002)	0.999 (0.007)	0.988 (0.003)
σ_1^2	38.99 (15.79)	2.685 (1.665)	32.98 (10.06)	2.742 (1.465)
β_2	0.372 (0.116)	0.026 (0.213)	0.391 (0.159)	-0.023 (0.256)
β_4	-0.396 (0.089)	-0.272 (0.148)	-0.387 (0.136)	-0.190 (0.170)
β_5	-3.945 (1.195)	-5.171 (2.239)	-3.760 (1.479)	-5.637 (0.425)
β_6	0.736 (0.075)	0.304 (0.127)	0.692 (0.106)	0.352 (0.132)
σ_2^2	7.861 (4.585)	47.676 (17.816)	11.36 (10.75)	52.949 (21.32)
δ_1	-0.741 (0.205)	-1.029 (0.278)	-1.095 (0.297)	-0.445 (0.192)
δ_2	2.032 (0.146)	2.033 (0.182)	2.391 (0.266)	1.477 (0.176)
δ_3	-0.233 (0.198)	0.069 (0.276)	-0.228 (0.255)	0.026 (0.164)
δ_4	1.203 (0.252)	1.943 (0.447)	1.935 (0.482)	1.385 (0.199)
γ_1	-	0.619 (0.269)	0.900 (0.362)	1.055 (0.286)
γ_2	-	$-\gamma_1$	$-\gamma_1$	0.282 (0.168)
σ_4^2	85.297 (24.623)	123.5 (52.153)	75.77 (7.725)	39.072 (13.129)
logL	-155.8	-144.6	-152.8	-137.8

Notes

Asymptotic standard errors in parentheses. The sample mean of C was 448.88.

TABLE 4

Comparative Predictions

	1979		1980
Actual	794.82		817.97
Plan	803.87		840.44
AR2	812.61		815.44
CD function (OLS)	800.93		815.29
Disequilibrium estimates	790.32	(I)	799.11
[min (CD, CS)]		(IIa)	812.97
		(III)	815.51
Standard deviations of disequilibrium estimates in simulations	6.66	(I)	6.99
		(IIa)	2.69
		(III)	3.50

Notes

The second-order autoregressive process (AR2) in C and the consumption function (our CD, estimated by OLS) are estimated on 1957-78 for the 1979 prediction and 1957-79 for the 1980 prediction. The disequilibrium model prediction for 1979 is from model Vb run through 1978, while those for 1980 are from models I, IIa and III, respectively, each run through 1979. The last line is the standard deviation of C_t^s from the stochastic simulations (see Appendix A). Normally, we would compare the forecast error with the simulated error of the equation; here, however, we do not have values of the simulated prediction errors, but we do have the standard error of the transacted quantity.

TABLE 5

Average Simulated Excess Demands (Per Cent)

	Model I	Model IIa	Model III	Model Vb
1957	3.0	3.7	0.7	-3.6
1958	-4.1	-1.8	-2.5	-3.9
1959	0.8	0.7	-0.8	1.0
1960	-6.8	-5.5	-5.2	-4.4
1961	5.1	2.2	1.7	1.4
1962	-4.1	-3.1	-2.8	-2.0
1963	-0.3	-1.5	-0.8	-0.6
1964	-4.2	-2.5	-3.1	-1.6
1965	-1.1	-0.8	-1.0	1.8
1966	-3.5	-2.0	-3.0	1.0
1967	-3.0	-1.1	-2.1	-0.7
1968	0.4	1.8	0.5	2.5
1969	-0.9	1.6	-0.8	0.2
1970	-0.2	0.9	0.1	-1.0
1971	4.5	3.8	2.2	2.5
1972	0.7	5.0	0.9	7.0
1973	-3.0	1.8	-2.3	2.7
1974	-0.9	3.6	-0.3	1.2
1975	1.0	2.2	-0.4	2.8
1976	-1.0	-0.8	-2.4	-2.9
1977	-1.5	-1.3	-2.6	-0.1
1978	-10.7	-7.1	-7.2	-8.7
1979	1.5	0.4	0.4	1.2
1980	1.2	-0.1	0.5	1.2

References

Burkett, J., 1981, Marginal and conditional probabilities of excess demand, Economics Letters 8, 159-162.

Charemza, W., and R. Quandt, 1982, Models and estimation of disequilibrium for centrally planned economies, Review of Economic Studies 49, 109-116.

Davidson, J., D. Hendry, F. Srba and S. Yeo, 1978, Econometric modelling of the aggregate time-series relationship between consumer's expenditure and income in the United Kingdom, Economic Journal 88, 661-692.

Gacs, L. and M. Lacko, 1973, A study of planning behaviour on the national-economic level, Economics of Planning 13, 91-119.

Kornai, J., 1971, Anti-equilibrium (Amsterdam, North-Holland).

Portes, R., 1981a, Macroeconomic equilibrium and disequilibrium in centrally planned economies, Economic Inquiry 19, 559-578.

Portes, R., 1981b, The Polish Crisis (London, RIIA).

Portes, R., R. Quandt, D. Winter, and S. Yeo, 1983, Planning the consumption goods market: Preliminary disequilibrium estimates for Poland 1955-80, NBER Working Paper no. 1076, forthcoming in a conference volume edited by P. Malgrange and P.-A. Muet (Blackwell).

Portes, R., and D. Winter, 1977, The supply of consumption goods in centrally planned economies, Journal of Comparative Economics 1, 351-365.

Portes, R., and D. Winter, 1980, Disequilibrium estimates for consumption goods markets in centrally planned economies, Review of Economic Studies 47, 137-159.

Quandt, R., 1981, Autocorrelated errors in simple disequilbrium models, Economics Letters 7, 55-61.

Yeo, S., 1983, Some simple models of plan adjustment, Birkbeck Discussion Paper in Economics (forthcoming).

Appendix A: Description of the Stochastic Simulations

We performed stochastic simulations for several of the models. The basic steps in these simulations are as follows:

1) Define the vector $y'_t = (y_{1t}, y_{2t}, y_{4t})$ as (CD_t, CS_t, C^*_t) for models I to IV and as (CD_t, CS_t, C^*_{t+1}) for model V. We then can write the structural equations (excluding the min condition) as

$$Ay_t = w_t + u_t \qquad (A.1)$$

where $w'_t = (w_{1t}, w_{2t}, w_{4t})$ represents the exogenous and predetermined variables and their coefficients from the right hand sides of these equations, $u'_t = (u_{1t}, u_{2t}, u_{4t})$ the corresponding error terms and where A is a square matrix of coefficients.

2) For each observation, t = 1, ..., T and for each simulation i substitute in A the values of the estimated coefficients and substitute in w_t the estimated coefficients of the predetermined and exogenous variables and the values of these variables for the t^{th} observation. Note that A and w_t do not vary across the simulations i.

3) For each observation t = 1, ..., T, and for each simulation i = 1, ... 100 generate three independent normal variables with mean zero and variances $\hat{\sigma}_1^2$, $\hat{\sigma}_2^2$, $\hat{\sigma}_4^2$, where the latter are the estimated error variances; denote these simulated normal errors by u^s_{ti} for simulation i.

4) Obtain simulated solutions y^s_{ti} by solving

$$y^s_{ti} = A^{-1}(w_t + u^s_{ti}) \qquad (A.2)$$

5) Obtain simulated values of the actual level of consumption and the simulated forecast error for simulation i from

$$C^s_{ti} = \min(y^s_{1ti}, y^s_{2ti}) \qquad (A.3)$$

$$f^s_{ti} = C_t - C^s_{ti} \qquad (A.4)$$

6) Repeat steps 2 through 5 for simulations i = 1, ... 100

7) For each t = 1, ... T compute the arithmetic means and standard deviations of y_t^s , C_t^s and f_t^s across the simulations i.

It is possible to introduce three sources of random variation in a stochastic simulation: variations in the estimated coefficients, in the exogenous variables and in the equation error terms. We have only introduced variation in the error terms u_{ti}^s in the work reported here, but we hope to investigate full stochastic simulations in later work.

Appendix B: The Likelihood Functions

Models I and II:

It is convenient to derive the likelihood function for Model IIb first, since the results for models I and IIa then follow as special cases.

Model IIb can be written

$$CD_t = z_{1t} + u_{1t} \tag{B.1}$$

$$CS_t = (\beta_1+\beta_2 z_{4t})C^*_{t|t-1} + z_{2t} + u_{2t} \tag{B.2}$$

$$C_t = \min\,(CD_t, CS_t) \tag{B.3}$$

$$C^*_{t|t-1} = z_{3t} + \gamma_1 CD_t + \gamma_2 CS_t + u_{4t} \tag{B.4}$$

where $C^*_{t|t-1}$ represents the plan for period t formulated in t-1. For convenience of notation this is written below as C^*_t . We have also

$$z_{1t} = \alpha_1 DNFA_{t-1} + \alpha_2 DYD_t + \alpha_3 YD_{t-1}$$

$$z_{2t} = \beta_4 RNFA_{t-1} + \beta_5 CZXD_t + \beta_6 CZXI_t$$

$$z_{3t} = \delta_1 C^*_{t-1} + \delta_2 C_{t-1} + \delta_3 C_{t-2} + \delta_4 RNFA_{t-2}$$

$$z_{4t} = (NMP_t - NMP^*_t)/NMP^*_t$$

We assume that u_{1t}, u_{2t} and u_{4t} are serially independent normal variates with a diagonal covariance matrix. The pdf of CD_t, CS_t, C^*_t is immediate from (B.1) to (B.4):

$$f(CD_t, CS_t, C^*_t) = \frac{|1-\gamma_2(\beta_1+\beta_2 z_{4t})|}{(2\pi)^{3/2}\sigma_1\sigma_2\sigma_4} \exp\left(-\frac{1}{2}\left[\frac{(CD_t - z_{1t})^2}{\sigma_1^2} + \frac{(CS_t-(\beta_1+\beta_2 z_{4t})C^*_t - z_{2t})^2}{\sigma_2^2} + \frac{(-\gamma_1 CD_t - \gamma_2 CS_t + C^*_t - z_{3t})^2}{\sigma_4^2}\right]\right) \tag{B.5}$$

The pdf of the observable random variables C_t, C^*_t is

$$h(C_t, C^*_t) = \int_{C_t}^{\infty} f(C_t, CS_t, C^*_t)\,dCS_t + \int_{C_t}^{\infty} f(CD_t, C_t, C^*_t)\,dCD_t \tag{B.6}$$

It is easy to show by completing the square that the integrals in (B.6) can be obtained as

$$\int_{C_t}^{\infty} f(C_t, CS_t, C_t^*)dCS_t = \frac{|1 - \gamma_2(\beta_1+\beta_2 z_{4t})|}{2\pi\sigma_1\sqrt{(\sigma_4^2+\gamma_2^2\sigma_2^2)}} \times \tag{B.7}$$

$$\exp\left(-\frac{1}{2}\left[\frac{(C_t-z_{1t})^2}{\sigma_1^2} + \frac{B_t - A_t^2}{\omega_1^2}\right]\right) \times \left(1 - \Phi\left[\frac{C_t - A_t}{\omega_1}\right]\right)$$

$$\int_{C_t}^{\infty} f(CD_t, C_t, C_t^*)dCD_t = \frac{|1 - \gamma_2(\beta_1+\beta_2 z_{4t})|}{2\pi\sigma_2\sqrt{(\sigma_4^2+\gamma_1^2\sigma_1^2)}} \times \tag{B.8}$$

$$\exp\left(-\frac{1}{2}\left[\frac{(C_t-z_{5t})^2}{\sigma_2^2} + \frac{G_t - F_t^2}{\omega_2^2}\right]\right) \times \left(1 - \Phi\left[\frac{C_t - F_t}{\omega_2}\right]\right)$$

where

$$z_{5t} = z_{2t} + (\beta_1+\beta_2 z_{4t})C_t^*$$

$$z_{6t} = C_t^* - \gamma_1 C_t - z_{3t}$$

$$z_{7t} = C_t^* - \gamma_2 C_t - z_{3t} \tag{B.9}$$

$$A_t = \frac{\sigma_4^2 z_{5t} + \sigma_2^2\gamma_2 z_{6t}}{\sigma_4^2 + \gamma_2^2\sigma_2^2} \qquad F_t = \frac{\sigma_4^2 z_{1t} + \sigma_1^2\gamma_1 z_{7t}}{\sigma_4^2 + \gamma_1^2\sigma_1^2}$$

$$B_t = \frac{\sigma_4^2 z_{5t}^2 + \sigma_2^2 z_{6t}^2}{\sigma_4^2 + \gamma_2^2\sigma_2^2} \qquad G_t = \frac{\sigma_4^2 z_{1t}^2 + \sigma_1^2 z_{7t}^2}{\sigma_4^2 + \gamma_1^2\sigma_1^2}$$

$$\omega_1^2 = \frac{\sigma_2^2\sigma_4^2}{\sigma_4^2 + \gamma_2^2\sigma_2^2} \qquad \omega_2^2 = \frac{\sigma_1^2\sigma_4^2}{\sigma_4^2 + \gamma_1^2\sigma_1^2}$$

and where $\Phi(\ )$ is the standard normal distribution function. The log-likelihood for model IIb is then $L = \sum_t \log h(C_t, C_t^*)$.

The likelihood function for model I can be obtained by taking $\gamma_1 = \gamma_2 = 0$, and the likelihood function for model IIa by taking $\gamma_1 = -\gamma_2 = \gamma$.

Models Involving Expected Excess Demand (III and IV):

The likelihood functions for the models involving expected excess demand in the plan adjustment equation can be obtained as follows:

The structural equations include (B.1), (B.2), (B.3), but the plan equation is now

$$C^*_{t|t-1} = z_{3t} + \gamma E_{t-1}(CD_t - CS_t) + u_{4t} \tag{B.10}$$

where z_{1t}, z_{2t}, z_{3t}, z_{4t} and z_{5t} are as defined for Model II. For simplicity we again denote $C^*_{t|t-1}$ by C^*_t and $E_{t-1}(CD_t - CS_t)$ by E_{t-1} in what follows. Because E_{t-1} itself depends only on predetermined and exogenous variables, the density function of the observable random variables is the product of the pdf for a simple disequilibrium model corresponding to (B.1), (B.2) and (B.3), and the pdf of the single equation model given by (B.10):

$$f(C_t, C^*_t) = \frac{1}{\sqrt{2\pi}\,\sigma_4} \exp\left(-\frac{1}{2}\frac{(C^*_t - z_{3t} - \gamma E_{t-1})^2}{\sigma_4^2}\right) \times \tag{B.11}$$

$$\left\langle \frac{1}{\sqrt{2\pi}\,\sigma_2} \exp\left(-\frac{1}{2}\frac{(C_t - z_{5t})^2}{\sigma_2^2}\right) \times \left(1 - \Phi\left[\frac{C_t - z_{1t}}{\sigma_1}\right]\right) + \right.$$

$$\left. \frac{1}{\sqrt{2\pi}\,\sigma_1} \exp\left(-\frac{1}{2}\frac{(C_t - z_{1t})^2}{\sigma_1^2}\right) \times \left(1 - \Phi\left[\frac{C_t - z_{5t}}{\sigma_2}\right]\right) \right\rangle$$

It should be noted that the model does not decouple into two independent sub-models because in general, E_{t-1} will depend on the parameters in the other equations, thus introducing cross-equation restrictions.

The likelihood function in (B.11) cannot be used directly, since we have not specified how the term E_{t-1} is to be evaluated. Substituting from (B.1) and (B.2) into the expression for E_{t-1} yields

$$E_{t-1}(CD_t - CS_t) = E(z_{1t}) - \beta_1 E(z_{3t}) - \beta_2 E(z_{4t} z_{3t}) - \tag{B.12}$$

$$\beta_1 \gamma E(CD_t - CS_t) - \beta_2 \gamma E(CD_t - CS_t) E(z_{4t}) - \beta_2 E(z_{4t} u_{4t}) - E(z_{2t}) +$$

$$E(u_{1t}) - E(u_{2t}) - \beta_1 E(u_{4t})$$

where the expectations E are all taken at time t-1.

We assume that $E(u_{it}) = 0$ in model III and model IV. This is consistent with our assumption that the disturbances are serially uncorrelated, which implies that the conditional mean at t-1 will equal the unconditional mean.

Model III:

One very simple assumption is that the z variables in the expression for E_{t-1} are all known with certainty at time t-1 and are thus effectively non-random. If we replace each expectation involving z's with the corresponding realized value, and if we take $E(z_{4t}u_{4t}) = z_{4t}E(u_{4t}) = 0$ we have from (B.12)

$$E_{t-1} = \frac{z_{1t} - z_{2t} - (\beta_1+\beta_2 z_{4t})z_{3t}}{1 + \gamma(\beta_1+\beta_2 z_{4t})} \tag{B.13}$$

This assumes that the planners know the realizations of these variables when they draw up the plan in period t-1. Since some of these are period t realizations, this is a fairly strong assumption.

Model IV:

An alternative method is to evaluate this expectation based on an information set more likely to be available to planners at t-1. We proceed as follows:

We assume $E(u_{1t}) = 0$ as before. We also assume that the expectations of the deviations from plan are zero, giving $E(z_{4t}) = 0$ and $E(z_{2t}) = \beta_4 RNFA_{t-1}$ which we take to be zero.

We then have to evaluate the following:

(a) $E(z_{1t})$. This variable consists of current and lagged income and lagged savings. The logic of our model suggests that we should use the realizations of lagged income and savings and the plan for income for period t. We have not yet been able to construct a suitable series of income plans and we therefore make the assumption that the planners know at time t-1 what income at time t will be. We therefore assume $E(z_{1t}) = z_{1t}$.

(b) $E(z_{3t})$: We set this equal to z_{3t} since it is known (or nearly known) at time t-1.

(c) $E(z_{4t}z_{3t})$: This is the correlation between the (proportional) deviation of

NMP from plan and the non-endogenous variables in the plan equation. Since z_{3t} is known when the NMP plan is formed, an assumption in the spirit of the expectations literature would be that deviations from such plans represent "surprises" which are orthogonal to the information set on which the plan is based. Thus we assume z_{3t} to be orthogonal to z_{4t} .

(d) $E(z_{4t}u_{4t})$: This is the correlation between the deviation of NMP from plan and the residual in the consumption plan equation. We will assume this to be zero. If is not z_{4t} is essentially an endogenous variable. We recognize that while there may be no feedback from actual consumption to actual NMP within one period, consumption plans and NMP plans (like all plans of macro variables) will be mutually dependent. We could, however, assume a two-stage process in which the planners plan NMP, investment and defence and then plan consumption.

These assumptions give us

$$E(CD_t - CS_t) = \frac{z_{1t} - \beta_1 z_{3t}}{1 + \beta_1\gamma} \tag{B.14}$$

so that the "reduced form" plan equation for model IV would be

$$C^*_t = \frac{\gamma}{1+\beta_1\gamma} z_{1t} + \frac{1-\beta_1\gamma}{1+\beta_1\gamma} z_{3t} + u_{4t}$$

<u>Models Involving Plans for Period T+1 (Va and Vb)</u>:

The structural equations of model Va are:

$$CD_t = z_{1t} + u_{1t} \tag{B.15}$$

$$CS_t = (\beta_1+\beta_2 z_{4t})C^*_{t|t-1} + z_{2t} + u_{2t} \tag{B.16}$$

$$C_t = \min\,(CD_t, CS_t) \tag{B.17}$$

$$C^*_{t+1|t} = \delta_2 C_t + \gamma(CD_t - CS_t) + z_{3t} + u_{4t} \tag{B.18}$$

where

$$z_{3t} = \delta_1 C^*_{t|t-1} + \delta_3 C_{t-1} + \delta_4 RNFA_{t-1} \tag{B.19}$$

Because current C appears on the right hand side of (B.18), the system has two regimes:

(a) $CD_t < CS_t$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\delta_2-\gamma & \gamma & 1 \end{bmatrix} \begin{bmatrix} CD_t \\ CS_t \\ C^*_{t+1} \end{bmatrix} = \begin{bmatrix} z_{1t} + u_{1t} \\ (\beta_1+\beta_2 z_{4t})C^*_t + z_{2t} + u_{2t} \\ z_{3t} + u_{4t} \end{bmatrix} \qquad \text{(B.20)}$$

(b) $CD_t > CS_t$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\gamma & \gamma-\delta_2 & 1 \end{bmatrix} \begin{bmatrix} CD_t \\ CS_t \\ C^*_{t+1} \end{bmatrix} = \begin{bmatrix} z_{1t} + u_{1t} \\ (\beta_1+\beta_2 z_{4t})C^*_t + z_{2t} + u_{2t} \\ z_{3t} + u_{4t} \end{bmatrix} \qquad \text{(B.21)}$$

Since the determinants of the matrices on the left-hand sides of (B.20) and (B.21) have the same sign, we can establish that the model is coherent. From (B.20) and (B.21) we can immediately obtain the pdf's of CD_t,CS_t and C^*_{t+1}. Completing the square, integrating out CS_t for (a) and CD_t for (b) and adding we obtain the pdf of C_t, C^*_{t+1} as

$$f(C_t,C^*_{t+1}) = \qquad \text{(B.22)}$$

$$\frac{1}{2\pi\sigma_1\sqrt{(\sigma_4^2+\gamma^2\sigma_2^2)}} \exp\left(-\frac{1}{2}\left[\frac{(C_t-z_{1t})^2}{\sigma_1^2} + \frac{B_t - A_t^2}{\omega_1^2}\right]\right) \times \left(1 - \Phi\left[\frac{C_t - A_t}{\omega_1}\right]\right) +$$

$$\frac{1}{2\pi\sigma_2\sqrt{(\sigma_4^2+\gamma^2\sigma_1^2)}} \exp\left(-\frac{1}{2}\left[\frac{(C_t-z_{5t})^2}{\sigma_2^2} + \frac{G_t - F_t^2}{\omega_2^2}\right]\right) \times \left(1 - \Phi\left[\frac{C_t - F_t}{\omega_2}\right]\right)$$

where A_t,B_t,F_t,G_t, ω_1, ω_2 and z_{5t} are as given in (B.9), γ_1 and γ_2 in (B.9) are set equal to γ and $-\gamma$ respectively in (B.22), z_{3t} is as given in (B.19), and the previously defined z_{6t} and z_{7t} in (B.9) are now given by:

$$z_{6t} = C^*_{t+1} - (\delta_2+\gamma)C_t - z_{3t}$$

$$z_{7t} = C^*_{t+1} - (\delta_2-\gamma)C_t - z_{3t}$$

For model Vb, the γ's are asymmetric: $\gamma = \gamma_1$ if $CD_t > CS_t$ and $\gamma = \gamma_2$ otherwise. In this case the appropriate terms (corresponding to the two regimes) contain γ_1 and γ_2 respectively.