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ON LOW-FREQUENCY ESTIMATES OF "LONG RUN" RELATIONSHIPS IN MACROECONOMICS

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Abstract

A number of recent studies have attempted to test propositions concerning "long run" economic relationships by means of frequency-domain time series techniques that concentrate attention on low frequency co-movements of variables. The present paper emphasizes that many of these propositions involve expectational relationships that are not inherently related to specific frequencies or periodicities. Thus the association of low-frequency time series test statistics with long-run economic propositions is not generally warranted. That such an association can be misleading is demonstrated by analysis of examples taken from notable papers by Geweke, Lucas, and Summers.

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I. Introduction

A number of recent studies have attempted to test propositions concerning "long run relationships" by means of frequency-domain time series techniques that permit attention to be concentrated on low-frequency comovements in variables. For example, notable papers by Geweke (1982b), Lucas (1980), and Summers (1983) have featured tests of important macroeconomic propositions by reliance on statistics that pertain to low frequency--or, equivalently, long periodicity--aspects of time series data on money, prices, output, and/or interest rates. Apparently the general idea is that by focussing attention upon frequencies that correspond to cycle lengths of more than (say) five years, the investigator may be able to obtain results that "provide an empirical counterpart for the elusive 'long run' of economic theory" (Geweke, 1982b, p.1).

The purpose of the present paper is to discuss these studies and argue, by way of example, that the association of low-frequency time series statistics with "long run" economic propositions is not generally warranted. Instead, many so-called long run propositions involve <u>expectational</u> relationships which have little or nothing to do with low frequencies per se, so that such an association is inappropriate in a fundamental sense. Some of the test results reported in the three papers cited above, for example, fall into this category.

The arguments presented below are not new in terms of the basic principles involved, which were developed and expounded in a time-domain context by Lucas (1972a) and Sargent (1971) (1973) (1976a)(1979). Evidence provided by the literature suggests, however, that a new exposition--one that emphasizes frequency-domain aspects of the principles--is warranted.

II. First Example: The Fisher Effect

The study by Summers (1983) is concerned with the Fisher effect, i.e., the relationship between interest rates and inflation. As a result of various empirical investigations, Summers concludes that U.S. time series evidence is unfavorable to the proposition that interest rates respond to inflation rates point for point as suggested by classical theory.[1/] The specific investigation to be discussed here -- and the one featured by Summers--attempts to treat the proposition as one that pertains to the long run by proceeding as follows. Letting x_t and y_t denote the inflation rate and some nominal interest rate, respectively, Summers tests the hypothesis $\beta_1 = 1$ in a relationship of the form

$$(1) \qquad y_t = \beta_0 + \beta_1 x_t + u_t$$

where u_t is an unobserved stochastic term. To respect the long-run qualification, however, this relationship is required to hold only at low frequencies. In particular, β_1 is estimated by means of the band-spectrum regression technique developed by Engle (1974). Thus the estimator is of the form

$$(2) \qquad \hat{\hat{\beta}}_1 = [\Sigma \hat{f}_x(\theta_k)]^{-1} \Sigma \hat{f}_{yx}(\theta_k)$$

where $\hat{f}_x$ is the periodogram of x, $\hat{f}_{yx}$ is the cross periodogram between x and y, and where the summation includes only those values of k that correspond to low values of the frequency index θ_k.[2/]

Equation (2) refers to implementation of the band spectrum estimator in practice, i.e., with actual finite samples and the attendant difficulties that arise with frequency-domain techniques. But as our concern is not with estimation, but with the more fundamental problem of the correspondence between theoretical constructs and time series models, the discussion can be simplified and clarified if we restrict it to population concepts. From that perspective, the relevant low-frequency estimator of β_1 can be represented as

$$(3) \qquad \hat{\beta}_1 = [\int f_x(\omega)\, d\omega]^{-1} \int f_{yx}(\omega)\, d\omega$$

where f_x and f_{yx} are spectral and cross spectral density functions and where the integrals are taken over a restricted interval of frequency values close to zero. Indeed, the principles at issue will stand out most clearly if the estimator is viewed as pertaining to the single frequency $\omega = 0$, in which case we have

$$(4) \qquad \hat{\beta}_1(0) = [f_x(0)]^{-1} f_{yx}(0).$$

The following discussion will proceed mainly in terms of this zero-frequency estimator.

In order to see that a study of the Fisher effect based on such a procedure is in principle inappropriate, consider an imaginary economy in which inflation and interest rate values are generated as follows:

$$(5) \qquad y_t = \rho + E_t x_{t+1} + v_t$$

$$(6) \qquad x_t = \mu_0 + \mu_1 x_{t-1} + e_t \qquad\qquad |\mu_1|<1$$

Here $E_t x_{t+1}$ is the conditional expectation of x_{t+1} given values of all relevant variables in period t and before; expectations concerning future inflation are rational. Thus (5) expresses a case in which the Fisher relationship holds in full, period by period, and in which the real interest rate fluctuates randomly (as a result of the stochastic disturbance v_t) around a constant mean value of ρ.[3/] Furthermore, the inflation rate is assumed to be exogenous and generated by a stable first-order autoregressive process; e_t is white noise and independent of v_s for all s.

In this simple case it is clear that the expected inflation rate $E_t x_{t+1}$ equals $\mu_0 + \mu_1 x_t$ so the true relationship between interest and inflation is

(7) $\quad y_t = (\rho + \mu_0) + \mu_1 x_t + v_t.$

And it is also clear that if the latter were estimated by ordinary least squares (OLS), the slope coefficient corresponding to β_1 in (1) would (in large samples) take on the value μ_1. A researcher following this strategy would conclude that the Fisher relationship does not hold, even though it is built into the economy under investigation.[4/]

The point of this example is, of course, that use of a band-spectrum or other low-frequency estimator would not eliminate the problem. In the particular case at hand, the relationship between inflation and interest is the same at all frequencies; the population value of $\hat{\beta}_1(0)$ is precisely μ_1.[5/]

To make the point somewhat more generally, suppose that instead of (6) we have an autoregression of order K generating the inflation rate:

(6') $\quad x_t = \mu_0 + \mu_1 x_{t-1} + \dots + \mu_K x_{t-K} + e_t$

Then $E_t x_{t+1} = \mu_0 + \mu_1 x_t + \dots + \mu_K x_{t-K+1}$ so in place of (7) we have

(7') $\quad y_t = (\rho + \mu_0) + \mu_1 x_t + \dots + \mu_K x_{t-K+1} + v_t.$

And suppose that again the test of the long run Fisher effect is based on the estimator $\hat{\beta}_1(0) = f_{yx}(0)/f_x(0)$. But with this specification, the spectral density functions are related to the parameters in (7') according to[6/]

(8) $\quad f_{yx}(\omega) = \tilde{\mu}(\omega)\ f_x(\omega),$

where $\tilde{\mu}(\omega)$ is the Fourier transform of the μ_k sequence, $\tilde{\mu}(\omega) = \sum_{k=1}^{K} \mu_k e^{-i\omega(k-1)}$.

Consequently, the estimator in (4) tends to yield the value

(9) $\quad \hat{\beta}_1(0) = \tilde{\mu}(0) = \mu_1 + \mu_2 + \dots + \mu_K.$

Again, the resulting estimate reflects the time-series behavior of the inflation process instead of--or, more generally, in addition to--the effect of expected inflation on interest rates. The problem is essentially the same as that described in Sargent's (1973) discussion of the Fisher effect or his earlier (1971) remarks on the Phillips curve.

It is necessary to recognize that the values taken on by the estimator $\hat{\beta}_1(0)$ in the foregoing cases are dependent upon the maintained assumption that x_t is exogenous to y_t. In actuality, of course, one would not expect inflation to be exogenous with respect to interest rates for reasons discussed by Summers (1983) and many others. But that does not affect the validity of the present argument. If, for example, lagged values of y_t as well as x_t appeared in equation (6'), making x_t non-exogenous, then the magnitude of $\hat{\beta}_1(0)$ would depend upon the coefficients attached to these lagged y_ts as well as upon the μ_ks.

Before moving to the next example, it will be useful for what follows to note that low-frequency estimators such as (2) or (3) can be interpreted as OLS estimators computed with time series observations on variables that are <u>filtered</u> versions of x_t and y_t. The specific filter in this interpretation is one that precisely eliminates all of the signal in each series for frequencies outside the chosen band around $\omega = 0$. To obtain the filtered variables, one would first calculate Fourier transforms of the x_t and y_t series, set the values of the transform functions equal to zero outside the chosen band of frequencies, and apply the inverse Fourier transform. That OLS applied to the resulting variables x_t^* and y_t^* is equivalent to (2) in the finite-sample case is demonstrated by Engle (1978); for the population case, we proceed as follows.

First, let us note that in the population the counterpart of the finite-sample OLS estimator $(\Sigma x_t^* x_t^*)^{-1} \Sigma x_t^* y_t^*$ is $(Ex_t^* x_t^*)^{-1} \; Ex_t^* y_t^*$. Thus our task is to show that $(Ex_t^* x_t^*)^{-1} Ex_t^* y_t^*$ is equivalent to the low-frequency estimator defined in (3) when x_t^* and y_t^* are formed as described in the last paragraph. Formally, then, the filtered series x_t^* is defined as

$$x_t^* = \int_{-\pi}^{\pi} \tilde{B}(\omega)\ \tilde{X}(\omega)e^{i\omega t}\ d\omega \qquad\qquad t = 0,\pm1,\pm2,\ldots$$

where $\tilde{X}(\omega) = \sum_{t=-\infty}^{\infty} x_t e^{-i\omega t}$ denotes the Fourier transform of the x_t series and $\tilde{B}(\omega)$ is the frequency-domain representation of a filter with the property that $\tilde{B}(\omega) = 1$ for $-\bar{\omega} < \omega < \bar{\omega}$, $0 < \bar{\omega} < \pi$, and $\tilde{B}(\omega) = 0$ elsewhere.[7/] The series y_t^* is of course defined analogously.

To establish the desired equivalence it will suffice to prove that $Ex_t^* y_t^* = \int f_{yx}(\omega)d\omega$ with the integral taken over the restricted interval $-\bar{\omega} < \omega < \bar{\omega}$; that implies $Ex_t^* x_t^* = \int f_x(\omega)d\omega$ as a special case. Now it is well-known that, with y_t^* and x_t^* constructed in this way,

$$f_{y^*x^*}(\omega) = \tilde{B}(\omega)\ \tilde{B}(-\omega)f_{yx}(\omega) = |\tilde{B}(\omega)|^2\ f_{yx}(\omega);$$

see Fishman (1969, p.71). And of course the covariance of y_t^* and x_t^* is simply the integral of $f_{y^*x^*}(\omega)$ from $-\pi$ to π. So we have

$$Ey_t^* x_t^* = \int_{-\pi}^{\pi} |\tilde{B}(\omega)|^2\ f_{yx}(\omega)d\omega = \int_{-\bar{\omega}}^{\bar{\omega}} f_{yx}(\omega)d\omega,$$

which reproduces the appropriate term in (3). Making the special-case application to $Ex_t^* x_t^*$ then completes the demonstration.

III. Second Example: The Quantity Theory

Let us now turn to Lucas's (1980) study of the quantity-theory proposition that a given change in an economy's money growth rate will induce an equal change in its inflation rate. In order to conduct a statistical examination that treats this relationship as applicable to "long-run average behavior," Lucas obtains estimates of a slope parameter using time series values after subjecting the raw inflation and money growth series to a filter that "retains power at very low frequencies, while sharply reducing power at high frequencies" (1980, p. 1008). In effect, then, he estimates the parameter β_1 in a relationship such as

$$(10) \qquad y_t^* = \beta_1 x_t^* + u_t,$$

where y_t^* and x_t^* are the filtered observations on inflation rate and money growth rate variables, respectively. Although the estimation procedure in Lucas's paper is implemented graphically, it is in principle similar to use of the OLS estimator $\left(\sum x_t^* x_t^*\right)^{-1} \sum x_t^* y_t^*$.[8]

Consequently, from the discussion in the last paragraph in Section II, we see that Lucas's procedure is of the same general type as that utilized by Summers. The main issue, then, is whether this sort of procedure will in principle yield appropriate results concerning the validity of the quantity-theory relationship in question. But, to the extent that the filter employed to emphasize low-frequency fluctuations succeeds in doing so, it will produce an estimator that is closely related to $\hat{\beta}_1(0)$, the one that obtains in the pure zero-frequency case. And, as we know from the discussion leading to equation (9) above, this estimator measures the sum of the coefficients in a distributed-lag regression of inflation on money growth rates. Thus the issue reduces to whether this sum provides evidence concerning the validity of the quantity theory--i.e., whether the quantity theory obtains if and only if $\sum_{j=1}^{\infty} \beta_j$ equals 1.0 in a regression such as

$$(11) \quad y_t = \beta_0 + \beta_1 x_t + \beta_2 x_{t-1} + \ldots + u_t,$$

with x_t = money growth and y_t = inflation.

At this point it might as a matter of logic suffice to say that careful readers of Sargent's textbook (1979, pp. 295-6) already know the answer to this question--which is "no." But the framework used there for the demonstration is slightly ad hoc, so there may be some interest in an analysis based on a model with (perhaps) somewhat better justification. 9/

Consequently, let us consider a linearized version of the model in Lucas's famous "Expectations and the Neutrality of Money" (1972b). Following the development in McCallum (1983), we therefore specify the demand and supply of money in market One with the following equations:

$$(12) \quad \lambda_t - p_t = a_0 + a_1 E_t[m_{t+1} - m_t - p_{t+1} + p_t] \qquad a_1 > 0$$

$$(13) \quad m_t = \rho m_{t-1} + e_t \qquad |\rho| < 1$$

Here λ_t and m_t are logarithms of money demand per young person and money supply per old person, respectively, with p_t the log of the local price. Values of m_t are not observed until t+1 so $E_t(.)$ is defined as $E(.|p_t, \Omega_{t-1})$ where Ω_{t-1} includes $p_{t-1}, p_{t-2}, \ldots, m_{t-1}, m_{t-2}, \ldots$. Because of the population allocation shock θ_t, which designates the fraction of young agents allocated to market One, market clearing is described by

$$(14) \quad \lambda_t = m_t - \theta_t.$$

Under this specification, the price in market One obeys

$$(15) \quad p_t = \pi_0 + \pi_1 m_{t-1} + \pi_2 e_t + \pi_3 \theta_t$$

so we have

(16) $E_t p_{t+1} = \pi_0 + \pi_1 E_t m_t = \pi_0 + \pi_1[\rho m_{t-1} + \beta(e_t - \theta_t)]$

since $E_t e_t = \beta(e_t - \theta_t)$ with $\beta = \sigma_e^2/(\sigma_e^2 + \sigma_\theta^2)$. Similarly, $E_t m_t = \rho m_{t-1} + \beta(e_t - \theta_t)$ and $E_t m_{t+1} = \rho E_t m_t$. Consequently, substitution into (12) gives

(17) $m_t - \theta_t - (1 + a_1)[\pi_0 + \pi_1 m_{t-1} + \pi_2 e_t + \pi_3 \theta_t] = a_0 +$

$a_1[(\rho - 1)(\rho m_{t-1} + \beta e_t - \beta\theta_t) - \pi_0 - \pi_1(\rho m_{t-1} + \beta e_t - \beta\theta_t)]$.

Equating coefficients and solving for $\pi_0, \ldots, \pi_3$ results in

(18) $p_t = -a_0 + \rho m_{t-1} + \emptyset(e_t - \theta_t)$

where $\emptyset = (1 + \beta a_1)/(1 + a_1)$, $0 < \emptyset < 1$. In market Two, the expression is the same except that the coefficient of θ_t is $+\emptyset$, so the economy-wide average value of p_t equals $-a_0 + \rho m_{t-1} + \emptyset e_t$. But that is equivalent to

(19) $p_t = -a_0 + \emptyset m_t + \rho(1 - \emptyset)m_{t-1}$

so a regression of p_t on m_t, $m_{t-1}, \ldots$, would yield a sum of coefficients equal to $\emptyset + \rho(1 - \emptyset) = 1 - a_1(1 - \beta)(1 - \rho)/(1 + a_1)$, which differs from 1.0 except in the special case $\rho = 1$. The same may be said, clearly, for a regression of Δp_t on Δm_t, $\Delta m_{t-1}, \ldots$.

The foregoing example admittedly does not reflect one aspect of Lucas's (1980) discussion, namely, the possibility of occasional stochastic "structural changes"--shifts in the monetary authority's policy rule. Instead, equation (13) depicts a single unchanging stochastic policy rule. But our intention is not to argue that the low-frequency procedure will never work--nor, certainly, that Lucas's substantive conclusion is incorrect--but to show that it will fail under a rather broad and plausible set of conditions.

IV. Third Example: Neutrality of Money

Geweke (1982a) has recently devised an ambitious and elegant scheme for the measurement of (linear) dependence and feedback among time series variables. A special feature of this scheme is its emphasis on the decomposition of the feedback measures by frequency. In particular, for a bivariate time series $\{x_t, y_t\}$, Geweke defines a measure of linear feedback from x to y at frequency ω, denoted $f_{x \to y}(\omega)$, as follows. Assuming that the bivariate system permits autoregressive representations, consider the particular representation that attributes all contemporaneous correlation to an apparent dependence of x_t on y_t:

(20a) $$x_t = A(L)y_t + B(L)x_{t-1} + \xi_t.$$

(20b) $$y_t = C(L)y_{t-1} + D(L)x_{t-1} + \eta_t.$$

Here A(L), B(L), C(L), and D(L) are polynomials in the lag operator defined by $L^n Z_t = Z_{t-n}$ while ξ_t and η_t are white noise disturbances that are uncorrelated with past values of x_t and y_t and are by construction contemporaneously uncorrelated.

Next, with ξ_t and η_t defined by the representation (20), consider the related moving average representation

(21a) $$x_t = E(L)\xi_t + F(L)\eta_t$$

(21b) $$y_t = H(L)\xi_t + G(L)\eta_t.$$

Then Geweke's measure $f_{x \to y}(\omega)$ is defined as

(22) $$f_{x \to y}(\omega) = \log[\,|f_y(\omega)|/\sigma_\eta^2 \; |\tilde{G}(\omega)|^2]$$

where $\tilde{G}(\omega) = \sum_{j=0}^{\infty} g_j e^{-i\omega j}$ is the Fourier transform of the sequence of coefficients on $\eta_t, \eta_{t-1},\ldots$ in (21b). For reference below, we note that Geweke (1982a, p. 308) demonstrates that $f_{x\rightarrow y}(\omega)$ equals zero at frequency ω if and only if $\tilde{D}(\omega) = 0$.

In both the cited paper and in a related application to macroeconomic time series (1982b), Geweke has suggested that values of $f_{x\rightarrow y}(\omega)$ at high and low frequencies could be useful in evaluating the strength of short-run and long-run effects of x on y. In particular, Geweke (1982b) proposes to test the classical hypothesis of long-run monetary neutrality by determining whether $f_{x\rightarrow y}(\omega)$ is significantly different from zero at $\omega = 0$, with x and y defined as annual growth rates of the money stock and real GNP, respectively. 10/

In order to investigate the validity of this procedure, let us consider an extremely simple macroeconomic structure of the general type described by Barro (1977) and Sargent (1976b). For concreteness, let u_t and e_t be independent white noises and imagine an economy in which the log of aggregate supply is expressible as

$$(23) \qquad y_t = \alpha_0 + \alpha_1(m_t - E_{t-1}m_t) + \alpha_2(m_{t-1} - E_{t-2}m_{t-1}) + u_t$$

while the money supply is governed by

$$(24) \qquad m_t = \mu_0 + \mu_1 m_{t-1} + e_t.$$

In such an economy output is obviously determined as

$$(25) \qquad y_t = \alpha_0 + \alpha_1 e_t + \alpha_2 e_{t-1} + u_t.$$

Thus monetary policy is neutral in the sense that policy choices of the μ_0 and μ_1 parameters have no effect on the characteristics of the y_t process. 11/

To determine whether $f_{m\rightarrow y}(0)$ is zero for this economy, our first step is to rewrite (23) and (24) in a form in which the contemporaneous relation between m and y appears (in this case, misleadingly) to result from a dependence of m_t on y_t and in which the disturbances are uncorrelated. This is accomplished by writing (23) as

$$(26) \qquad y_t = \alpha_0 - \alpha_2\mu_0 + \alpha_2 m_{t-1} - \alpha_2\mu_1 m_{t-2} + \eta_t,$$

where $\eta_t = u_t + \alpha_1 e_t$, and then subtracting θy_t from each side of (24) with $\theta = \alpha_1\sigma_e^2/(\sigma_u^2 + \alpha_1^2\sigma_e^2)$:

$$(27) \qquad m_t = [\mu_0 - \theta(\alpha_0 - \alpha_2\mu_0)] + \theta y_t + (\mu_1 - \theta\alpha_2)m_{t-1} - \theta\alpha_2\mu_1 m_{t-2} + \xi_t.$$

Here $\xi_t = e_t - \theta(u_t + \alpha_1 e_t)$ and $E\xi_t\eta_t = 0$. Now, since (26) and (27) constitute the autoregressive representation of the form (20) for the model at hand, we can determine whether $f_{m \rightarrow y}(0)$ equals zero by evaluating the counterpart of $\tilde{D}(\omega)$. Inspection of (26) shows that to be

$$(28) \qquad \alpha_2 e^{-i\omega} - \alpha_2\mu_1 e^{-i\omega 2}$$

so for $\omega = 0$ we have $\alpha_2(1 - \mu_1)$. Except in the special cases in which $\mu_1 = 1$ or $\alpha_2 = 0$, then, the zero-frequency feedback measure misleadingly indicates that long-run monetary neutrality does not prevail.

An objection that might be raised to the foregoing example is that it seems implausible--as argued in McCallum (1979)--that lagged values of monetary innovations such as e_{t-1} could directly affect y_t as is presumed in (23). But that objection is not compelling for the same sort of result would hold if the lagged innovation were deleted from (23)--i.e., if $\alpha_2 = 0$--provided that u_t is autocorrelated.

V. Conclusions

The foregoing examples should be sufficient to demonstrate that it is not generally appropriate to rely upon low-frequency measures of relationships among variables as indicators of the validity of propositions concerning "long run" effects or relationships. The reason for this failure is basically the same in all of the examples: the low-frequency measures in question are simply not designed to reflect the distinction between anticipated and unanticipated fluctuations that is crucial for accurately characterizing intervariable relationships in many dynamic models.

More generally, it might be said, most of the familiar propositions of traditional economic analysis concerning long run relationships are based, implicitly or explicitly, upon comparisons of alternative steady-state magnitudes in deterministic (i.e., non-stochastic) models. Consequently, these propositions refer to relationships that obtain in settings in which there are no fluctuations that are less than perfectly anticipated. But in models designed for empirical implementation--as in reality--stochastic ingredients and expectational errors are necessarily involved. As a result, the propositions under discussion must be reformulated in a manner appropriate to a stochastic context to make them amenable to empirical analysis. Some such reformulation, which will typically involve the distinction between anticipated and unanticipated components, is an essential prelude to the design of any valid empirical test.

References

Barro, Robert J., "Unanticipated Money Growth and Unemployment in the United States," American Economic Review 67 (March 1977), 101-115.

Engle, Robert F., "Band Spectrum Regression," International Economic Review 15 (February 1974), 1-11.

________________, "Testing Price Equations for Stability Across Spectral Frequency Bands," Econometrica 46 (July 1978), 869-881.

Fishman, George S., Spectral Methods in Econometrics (Cambridge, MA: Harvard University Press, 1969).

Geweke, John, "The Measurement of Linear Dependence and Feedback between Multiple Time Series," Journal of the American Statistical Association 78 (June 1982), 304-313 (a).

____________, "The Neutrality of Money in the United States, 1872-1980: An Interpretation of the Evidence," Working Paper, (November 1982) (b).

Lucas, Robert E., Jr., "Econometric Testing of the Natural Rate Hypothesis," in O. Eckstein, ed., The Econometrics of Price Determination Conference (Washington: Board of Governors of the Federal Reserve System, 1972) (a).

____________________, "Expectations and the Neutrality of Money," Journal of Economic Theory 4 (April 1972), 103-124 (b).

____________________, "Two Illustrations of the Quantity Theory of Money," American Economic Review 70 (December 1980), 1005-1014.

McCallum, Bennett T., "A Linearized Version of Lucas's Neutrality Model," Working Paper, February 1983.

____________________, "On the Observational Inequivalence of Classical and Keynesian Models," Journal of Political Economy 87 (April 1979), 395-402.

Sargent, Thomas J., "A Note on the Accelerationist Controversy," Journal of Money, Credit, and Banking 3 (August 1971), 50-60.

__________________, "Rational Expectations, the Real Rate of Interest, and the Natural Rate of Unemployment," Brookings Papers on Economic Activity (No. 2, 1973), 429-472.

__________________, "Testing for Neutrality and Rationality," in P. Miller, ed., A Prescription for Monetary Policy: Proceedings from a Seminar Series (Minneapolis: Federal Reserve Bank of Minneapolis, 1976) (a).

__________________, "On the Observational Equivalence of Natural and Unnatural Rate Theories of Macroeconomics," Journal of Political Economy 84 (June 1976), 631-640. (b)

__________________, Macroeconomic Theory (New York: Academic Press, 1979).

Summers, Lawrence H., "The Non-Adjustment of Nominal Interest Rates: A Study of the Fisher Effect," in Macroeconomics, Prices, and Quantities: Essays in Memory of Arthur M. Okun, edited by James Tobin (Washington: Brookings Institution, 1983).

Whiteman, Charles H., Moving Average Representations in Rational Expectations Models Ph.D. Thesis, University of Minnesota, 1981.

Footnotes

1. In the presence of taxes on nominal interest earnings, the response should actually be greater than point-for-point--a complication that is not highly relevant to the issues of interest here.

2. The latter is, in Engle's notation, $\theta_k = 2\pi k/T$ where T is sample size.

3. Alternatively, one could view the real interest rate as strictly constant and v_t as a disturbance to the Fisher relationship.

4. Clearly, if (5) were written as $y_t = \rho + \beta_1 E_t x_{t+1} + u_t$, then the approach would yield an estimate of $\mu_1\beta_1$.

5. It might parenthetically be noted that the difficulty does not arise from data misalignment, as a comparison of equations (1) and (5) has led some readers to suggest. If the band spectrum regression used x_{t+1} in place of x_t, the value of the resulting $\hat{\beta}_1(0)$ would still be μ_1, as can be readily verified from equation (9) below.

6. See Fishman (1969, p.64) or Sargent (1979, p.242).

7. Alternatively, $x_t^* \equiv \sum_{j=-\infty}^{\infty} b_j x_{t-j}$. Then the b_js are related to the function $\tilde{B}(\omega)$ by the inverse Fourier transform $b_j = \int_{-\pi}^{\pi} \tilde{B}(\omega)e^{i\omega j}\, d\omega$ and we find that $b_j = (2/j) \sin j\bar{\omega}$.

8. It is similar to that particular OLS estimator, rather than the inverse of $(\Sigma y_t^* y_t^*)^{-1}\, \Sigma y_t^* x_t^*$, in that y_t appears on the vertical axis in the diagrams.

9. A much more extensive discussion of Lucas's procedure is provided by Whiteman (1981, pp. 156-227). Although Whiteman does not interpret Lucas's procedure in the same way, and uses a different model for illustrative purposes, his conclusions are very similar to the ones presented below.

10. To be more precise, Geweke proposes $f_{x \rightarrow y}(0) = 0$ as the defining characteristic of "dynamic neutrality," a concept that he distinguishes from "stochastic neutrality." Since the latter is a more stringent concept (Geweke, 1982b, pp. 18-20), it also requires $f_{x \rightarrow y}(0) = 0$.

11. This definition of neutrality corresponds to Sargent's (1979, pp. 357-360) use of the term and to Geweke's (1982b) concept of stochastic neutrality.