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ABSTRACT

The standard, representative agent, consumption-based asset pricing theory based on CRRA utility fails to explain the average returns of risky assets. When evaluated on cross- sections of stock returns, the model generates economically large unconditional Euler equation errors. Unlike the equity premium puzzle, these large Euler equation errors cannot be resolved with high values of risk aversion. To explain why the standard model fails, we need to develop alternative models that can rationalize its large pricing errors. We evaluate whether four newer theories at the vanguard of consumption-based asset pricing can explain the large Euler equation errors of the standard consumption-based model. In each case, we find that the alternative theory counterfactually implies that the standard model has negligible Euler equation errors. We show that the models miss on this dimension because they mischaracterize the joint behavior of consumption and asset returns in recessions, when aggregate consumption is falling. By contrast, a simple model in which aggregate consumption growth and stockholder consumption growth are highly correlated most of the time, but have low or negative correlation in severe recessions, produces violations of the standard model's Euler equations and departures from joint lognormality that are remarkably similar to those found in the data.

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1 Introduction

Previous research shows that the standard, representative agent, consumption-based asset pricing theory based on constant relative risk aversion utility fails to explain the average returns of risky assets.[1] One aspect of this failure, addressed here, is the large unconditional Euler equation errors that the model generates when evaluated on cross-sections of stock returns. We present evidence on the size of these errors and show that they remain economically large even when preference parameters are freely chosen to maximize the model's chances of fitting the data. Thus, unlike the equity premium puzzle of Mehra and Prescott (1985), the large Euler equation errors cannot be resolved with high values of risk aversion.

To explain why the standard model fails, we need to develop alternative models that can rationalize its large Euler equation errors. Yet surprisingly little research has been devoted to assessing the extent to which newer consumption-based asset pricing theories–those specifically developed to address empirical limitations of the standard consumption-based model–can explain its large Euler equation errors. Unconditional Euler equation errors can be interpreted economically as pricing errors; thus we use the terms "Euler equation error" and "pricing error" interchangeably.

This paper makes three contributions. First, we show that leading consumption-based asset pricing theories resoundingly fail to explain the mispricing of the standard consumption-based model. Specifically, we investigate four models at the vanguard of consumption-based asset pricing and show that the benchmark specification of each of these theories counterfactually implies that the standard model has negligible Euler equation errors when its parameters are freely chosen to fit the data. This anomaly is striking because early empirical evidence that the standard model's Euler equations were violated provided much of the original impetus for developing the newer models we investigate here.[2]

Second, we show that the leading asset pricing models we study fail to explain the mispricing of the standard model because they fundamentally mischaracterize the joint behavior of consumption and asset returns in recessions, when aggregate consumption is falling. In the model economies, realized excess returns on risky assets are negative when consumption is falling, whereas in the data they are often positive.

Our third contribution is to suggest one specific direction along which the current models can be improved, based on a time-varying, state-dependent correlation between stockholder and aggregate consumption growth. Specifically, we show that a stylized model in which aggregate consumption growth and stockholder consumption growth are highly correlated

[1] For example, Hansen and Singleton (1982); Ferson and Constantinides (1991); Hansen and Jagannathan (1991); Cochrane (1996); Kocherlakota (1996).

[2] For example, see the discussion in Chapter 8 of Campbell, Lo, and MacKinlay (1997).

most of the time, but have low or negative correlation in recessions, produces violations of the standard model's Euler equations and departures from joint lognormality of aggregate consumption growth and asset returns that are remarkably similar to those found in the data.

To motivate the importance of these findings for consumption-based asset pricing theory, it is helpful to consider, by way of analogy, the literature on the value premium puzzle in financial economics. In this literature, the classic Capital Asset Pricing Model (CAPM) resoundingly fails to explain the high average excess returns of value stocks, resulting in a value premium puzzle (Fama and French 1992, 1993). It is well accepted that a fully successful theoretical resolution to this puzzle must accomplish two things: (i) must provide an alternative theory to the CAPM that explains the high average returns of value stocks, and (ii) it must explain the failure of the CAPM to rationalize those high returns.

Analogously, the large empirical Euler equation errors of the standard consumption-based model place additional restrictions on new consumption-based models: not only must such models have zero pricing errors when the Euler equation is correctly specified according to the model, they must also produce large pricing errors when the Euler equation is incorrectly specified using power utility and aggregate consumption. A related point is made by Kocherlakota (1996), who emphasizes the importance of Euler equation errors for theoretical work seeking to explain the central empirical puzzles of the standard consumption-based model. To understand *why* the classic consumption-based model is wrong, alternative theories must generate the same large Euler equation errors that we observe in the data for this model.

Our analysis employs simulated data from several contemporary consumption-based asset pricing theories expressly developed to address empirical limitations of the standard consumption-based model. Clearly, it is not possible to study an exhaustive list of all models that fit this description; thus we limit our analysis to four that both represent a range of approaches to consumption-based asset pricing, and have received significant attention in the literature. These are: the representative agent external habit-persistence paradigms of (i) Campbell and Cochrane (1999) and (ii) Menzly, Santos, and Veronesi (2004), (iii) the representative agent long-run risk model based on recursive preferences of Bansal and Yaron (2004), and (iv) the limited participation model of Guvenen (2003). Each is an explicitly parameterized economic model calibrated to accord with the data, and each has proven remarkably successful in explaining a range of asset pricing phenomena that the standard model fails to explain.[3]

[3] The asset pricing literature has already demonstrated a set of theoretical propositions showing that any observed joint process of aggregate consumption and returns can be an equilibrium outcome if the second moments of the cross-sectional distribution of consumption growth and asset returns covary in the right way (Constantinides and Duffie (1996)). Such existence proofs, important in their own right, are not the

The rest of this paper is organized as follows. The next section lays out the empirical facts on the Euler equation errors of the standard model and shows that they are especially large for cross-sections that include a broad stock market index return, a short term Treasury bill rate, and the size and book-market sorted portfolio returns emphasized by Fama and French (1992, 1993). We then move on in Section 3 to investigate the extent to which the leading asset pricing models mentioned above explain the mispricing of the standard model. We show that some of these models *can* explain why we obtain implausibly high estimates of risk aversion and the subjective rate of time-preference when freely fitting aggregate data to the Euler equations of the standard consumption-based model. But, none can explain the large unconditional Euler equation errors associated with such estimates for plausibly calibrated sets of asset returns.[4] Indeed, the asset pricing models we consider counterfactually imply that parameter values can be found for which the unconditional Euler equations of the standard consumption-based model are exactly satisfied.

The next part of Section 3 helps to diagnose the result by showing that each of the four models studied satisfy sufficient conditions under which parameter values can always be found such that the Euler equations of the standard model will be exactly satisfied. The economically important condition satisfied by each model is that realized excess returns on risky assets are negative whenever consumption growth is sufficiently negative. We show that such a condition is violated in the data.

We then move on in Section 3 to address the question of measurement error in consumption. If aggregation theorems fail and per capita aggregate consumption is a poor measure of individual assetholder consumption or the consumption of stockholders, the standard model's large Euler equation errors could in principle be attributable to using the wrong measure of consumption in empirical work. To assess this possibility, we begin by considering a simple lognormal model of mismeasured consumption in which the aggregate consumption data used in Euler equation estimation is a poor measure of the consumption of stockholders (referred to hereafter as the *limited participation* hypothesis). We show that if the true pricing kernel based on stockholder consumption is jointly lognormally distributed with aggregate

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subject of this paper. Instead, we ask whether particular calibrated economies of leading consumption-based asset pricing models are quantitatively capable of matching the large pricing equation errors generated by the standard consumption-based model when fitted to historical data. This is important because it remains unclear whether fully specified models built on primitives of tastes, technology, and underlying shocks, and calibrated to accord with the data in plausible ways, can in practice generate the joint behavior of aggregate consumption and asset returns that we observe in the data.

[4] Campbell and Cochrane (2000) evaluate the pricing errors of the standard consumption-based model implied by the habit model of Campbell and Cochrane (1999), by looking at the pricing errors for the *most mispriced* portfolio. Their results suggest that there is scope for mispricing, but do not necessarily imply significant mispricing for the sets of stock portfolios we calibrate our models to match.

consumption and returns, then estimation of Euler equations using per capita aggregate consumption produces biased estimates of the stockholder's subjective discount factor and risk aversion parameters, but does not rationalize the large pricing errors generated by the standard model.

We close Section 3 by turning our attention back to stylized models in which the consumption used in our empirical tests is mismeasured (e.g., due to limited stock market participation), but we relax the assumption of joint lognormality. When limited participation is combined with specific departures from joint lognormality, such as those based on a time-varying, state-dependent correlation between stockholder and aggregate consumption, consumption-based asset pricing theories come much closer to rationalizing the large Euler equation errors of the standard paradigm that in large part motivated the search for newer models in the first place. Section 4 concludes.

2 Euler Equation Errors: Empirical Facts

In this section we consider the empirical properties of the standard consumption-based model. We begin by showing, using U.S. aggregate data, that there are no values of the risk-aversion parameter and subjective time discount factor for which violations of the standard model's unconditional Euler equations are not economically large.

Consider the intertemporal choice problem of a representative agent with constant relative risk-aversion (CRRA) utility over aggregate consumption, who maximizes the expectation of a time separable utility function:

$$\underset{C_t}{\text{Max}}\ E_t\left\{\sum_{k=0}^{\infty}\delta^k\frac{C_{t+k}^{1-\gamma}-1}{1-\gamma}\right\}, \qquad \gamma>0, \tag{1}$$

subject to an accumulation equation for wealth. C_{t+1} is per capita aggregate consumption, γ is the coefficient of relative risk-aversion and δ is a subjective time-discount factor. Agents have unrestricted access to financial markets and face no borrowing or short-sales constraints.

The asset pricing model comes from the first-order conditions for optimal consumption and portfolio choice, which, by the law of iterated expectations, can be expressed as as set of unconditional moment restrictions, or Euler equations, taking the form

$$E\left[M_{t+1}R_{t+1}^{j}\right]-1=0, \qquad M_{t+1}=\delta\left(C_{t+1}/C_t\right)^{-\gamma}, \tag{2}$$

where R_{t+1}^{j} denotes the gross raw return on any tradable asset. M_{t+1} is the intertemporal marginal rate of substitution (MRS) in consumption, which is the stochastic discount factor (SDF), or pricing kernel. Euler equations may also be expressed as a function of excess returns:

$$E\left[M_{t+1}\left(R^j_{t+1}-R^f_{t+1}\right)\right]=0, \tag{3}$$

where R^f_{t+1} is the return on any reference asset, here specified as the return on a one-period riskless bond. We refer to (2) and (3) as the *standard* consumption-based model.

Deviations from these two equations represent Euler equation errors. Define

$$e^j_R \equiv E\left[M_{t+1}R^j_{t+1}\right]-1, \qquad e^j_{R,t+1}\equiv M_{t+1}R^j_{t+1}-1 \tag{4}$$

$$e^j_X \equiv E\left[M_{t+1}\left(R^j_{t+1}-R^f_{t+1}\right)\right], \qquad e^j_{X,t+1}\equiv M_{t+1}\left(R^j_{t+1}-R^f_{t+1}\right). \tag{5}$$

We refer to either e^j_R or e^j_X as the *unconditional Euler equation error* for the jth asset return.

Euler equation errors can be interpreted economically as *pricing errors*, also commonly referred to as "alphas" in the language of financial economics. The pricing error of asset j is defined as the difference between its historical mean excess return over the risk-free rate and the risk-premium implied by the model with pricing kernel M_{t+1}. The risk premium implied by the model may be written as the product of the asset's beta for systematic risk times the price of systematic risk (Cochrane (2005) provides an exposition). Thus the pricing error of the jth return, α^j, is that part of the average excess return that cannot be explained by the asset's beta risk. It is straightforward to show that $\alpha^j=\frac{e^j_X}{E(M_{t+1})}$. Pricing errors are therefore proportional to Euler equation errors. Moreover, because the term $E\left(M_{t+1}\right)^{-1}$ is the mean of the risk-free rate and is close to unity for most models, pricing errors and Euler equation errors are almost identical quantities. If the standard model is true, both errors should be zero for any traded asset, for preference parameters δ and γ of the representative agent.

Given a set of test assets and data on aggregate consumption, (2) and (3) can be estimated using Generalized Method of Moments (GMM, Hansen (1982)). The parameters δ and γ are chosen to minimize a weighted sum of squared Euler equation errors:

$$\min_{\delta,\gamma}\; g_T\left(\gamma,\delta\right)\equiv \mathbf{w}'_T\left(\gamma,\delta\right)\mathbf{W}\mathbf{w}_T\left(\gamma,\delta\right), \tag{6}$$

where $\mathbf{W}$ is a positive semi-definite weighting matrix and $\mathbf{w}_T\left(\gamma,\delta\right)$ is the vector of Euler equation errors for each asset, with jth element $w_{jT}(\gamma,\delta)$ given either by

$$w_{jT}(\gamma)=\frac{1}{T}\sum_{t=1}^{T}e^j_{X,t},$$

in the case of excess returns, or

$$w_{jT}(\gamma,\delta)=\frac{1}{T}\sum_{t=1}^{T}e^j_{R,t},$$

in the case of raw returns. Let $\widehat{\delta}$ and $\widehat{\gamma}$ denote the $\arg\min g_T\left(\gamma,\delta\right)$.

For most of the results below, we use the identity matrix, $\mathbf{W} = \mathbf{I}$, to weight the GMM criterion function. We do so because this approach preserves the structure of the test assets, which were specifically chosen for their economically interesting characteristics and because they deliver a wide spread in cross-sectional average returns. Other matrices re-weight the Euler equations, so that the GMM procedure amounts to minimizing the pricing errors of re-weighted portfolios of the original test assets, destroying this structure. It should be noted, however, that other weighting matrixes such as the optimal weighting matrix of Hansen (1982) and the second moment matrix of Hansen and Jagannathan (1997) produce results very similar to those reported below and do not alter our main conclusions.

We focus our attention on the unconditional Euler equation errors for cross-sections of asset returns that include a broad stock market index return (measured as the CRSP value-weighted price index return and denoted R_t^s), a short term Treasury bill rate (measured as the three-month Treasury bill rate and denoted R_t^f), and six size and book-market sorted portfolio returns available from Kenneth French's Dartmouth web site. (A detailed description of the data is provided in the Appendix.) These returns are value-weighted portfolio returns of common stock sorted into two size (market equity) quantiles and three book value-market value quantiles. We use equity returns on size and book-to-market sorted portfolios because Fama and French (1992) show that these two characteristics provide a "simple and powerful characterization" of the cross-section of average stock returns, and absorb the roles of leverage, earnings-to-price ratio and many other factors governing cross-sectional variation in average stock returns. These returns are denoted as a vector $\mathbf{R}_t^{FF} \equiv (R_t^1, ...R_t^6)'$. We analyze the pricing errors for the eight assets $R_t^s, R_t^f, \mathbf{R}_t^{FF}$ as a group, as well as for the set of two assets comprised of only R_t^s and R_t^f. The latter is of interest because the standard model's inability to explain properties of these two returns has been central to the development of a consensus that the model is flawed. In addition, almost all asset pricing models seek to match the empirical properties of these two returns, whereas fewer generate implications for larger cross-sections of securities.

To measure consumption, we use quarterly United States data on per capita expenditures on nondurables and services, in 2000 dollars. The data span the period from the fourth quarter of 1951 to the fourth quarter of 2002. Returns are deflated by the implicit price deflator corresponding to this measure of consumption, C_t.

Table 1 and Figure 1 that follow present summary statistics from the GMM estimation of the Euler equations above. The square root of the average squared Euler equation errors (RMSE) is reported as a measure of the magnitude of mispricing. To give a sense of how the large pricing errors are relative to the returns being priced, the RMSE is often reported relative to RMSR, the square root of the average squared (mean) returns of the assets under consideration.[5]

Estimating the empirical counterpart of (2) and (3) by GMM demonstrates the dramatic failure of the standard model along several dimensions. Table 1 shows that when δ and γ are chosen to minimize (6) for R^s_{t+1} and R^f_{t+1} alone (using raw returns), the RMSE is 2.7% per annum, a magnitude that is 48% of the square root of the average squared returns on these two assets. Since there are just two moments in this case, this means that there are no values of δ and γ that set the two pricing errors to zero.[6] When δ and γ are chosen to minimize (2) for the eight asset returns, the RMSE is 3.05% per annum, a magnitude that is 33% of the square root of the average squared returns on the eight assets. The estimates $\widehat{\delta}$ and $\widehat{\gamma}$ (which are left unrestricted) are close to 1.4 and 90, respectively, regardless of which set of test assets are used. The final two columns of Table 1 report the results of statistical tests of the model, discussed below.

The same patterns are visible when estimation is conducted on the Euler equations using excess returns. Figure 1 displays the RMSE for the Euler equations in (3) over a range of values of γ. The solid line plots the case where the single excess return on the aggregate stock market, $R^s_{t+1} - R^f_{t+1}$, is priced; the dotted line plots the case for the seven excess returns $R^s_{t+1} - R^f_{t+1}$ and $\mathbf{R}^{FF}_t - R^f_{t+1}$. In the case of the single excess return for the aggregate stock market, the RMSE is just the Euler equation error itself. The figure shows that the pricing error for the excess return on the aggregate stock market cannot be driven to zero, for any value of γ. Moreover, the minimized pricing error is large. The lowest pricing error is 5.2% per annum, almost 60% of the average annual CRSP excess return. This result occurs at a value for risk aversion of $\gamma = 117$. At other values of γ, the error rises precipitously and reaches several times the average annual stock market return when γ is outside the ranges displayed in Figure 1.

Similar results hold when Euler equation errors are computed for the seven excess returns $R^s_{t+1} - R^f_{t+1}, \mathbf{R}^{FF}_t - R^f_{t+1}$. The minimum RMSE is about 60% of the square root of average squared returns being priced, which occurs at $\gamma = 118$. These results show that the degree of mispricing in the standard model is about the same regardless of whether we consider the

[5] For the Euler equations of raw returns, RMSE and RMSR are equal to

$$RMSE \equiv \sqrt{\frac{1}{N}\sum_{j=1}^{N}\left(e^j_R\right)^2}, \qquad RMSR = \sqrt{\frac{1}{N}\sum_{j=1}^{N}E\left(R^j_t\right)^2},$$

where N is the number of asset returns, and $E\left(R^j_t\right)$ is the (time-series) mean of the jth raw return. RMSE and RMSR are defined in an analogous fashion for excess returns.

[6] Note that the Euler equations are nonlinear functions of γ and δ. Thus, there is not necessarily a solution to the pair of equations. See Section 3.2 for sufficient conditions for the existence of a solution.

single excess return on the market or a larger cross-section of excess stock market returns.[7]

What drives the large Euler equation errors in the data? The lower panel of Table 1 provides an important clue: a significant part of the unconditional Euler equation errors generated by the standard model is associated with recessions, periods in which per capita aggregate consumption growth is negative. For example, when data points coinciding with the smallest six observations on consumption growth are removed from the sample, the root mean squared pricing errors are substantially reduced. The RMSE is just 0.73% per annum or 13% of the root mean squared returns for R^s_{t+1} and R^f_{t+1}, and 1.94% per annum or 21 percent of the root mean squared returns on the eight asset returns $R^s_{t+1}, R^f_{t+1}, \mathbf{R}^{FF}_t$. This result echoes the findings in Ferson and Merrick (1987) who report less evidence against the standard consumption-based model in non-recession periods.

Table 2 identifies these six observations as they are located throughout the sample. Each occur in the depths of recessions, as identified by the National Bureau of Economic Research. In these periods, aggregate per capita consumption growth is steeply negative but the aggregate stock return and Treasury-bill rate is, more often than not, steeply positive. Since the product of the marginal rate of substitution and the gross asset return must be unity on average, such negative comovement (positive comovement between M_{t+1} and returns) contributes to large pricing errors.[8] One can also reduce the pricing errors by using annual returns and year-over-year consumption growth.[9] This procedure averages out the worst quarters for consumption growth instead of removing them. Either way, a substantial proportion of the cyclical variation in consumption is eliminated. For example, on a quarterly basis the largest declines in consumption are about six times as large at an annual rate as those on a year-over-year basis. This explains why Kocherlakota (1996), who focuses on annual data, is able to locate parameter values for δ and γ that exactly satisfy the Euler equations of a stock return and Treasury-bill rate.

[7] In computing the pricing errors above, we use the standard timing convention that end-of-period returns dated in quarter t should be paired with consumption growth measured from $t-1$ to t. If, instead, returns at t are paired with consumption growth from t to $t+1$, a value for γ can be found that sets the pricing error to zero for the single excess return $R^s - R^f$. By contrast, the choice of timing convention has very little affect on the RMSE for the set of seven excess returns $R^s - R^f, \mathbf{R}^{FF}_t - R^f$. We use the former timing convention as it is standard empirical practice in estimation of Euler equations. We stress, however, that the timing convention itself is not important for the comparisons with theoretical models that follow, since those models always produce pricing errors that are close to zero *regardless* of which timing convention is used.

[8] Eliminating the recession periods, however, results in preference parameter estimates that are even more extreme than they are in the full sample; for example $\widehat{\gamma} = 225$. Therefore, if the criterion for success is reasonable preference parameter estimates, then the standard model does worse when recession periods are removed than when they are included.

[9] For a recent example along these lines, see Jagannathan and Wang (2005).

Of course, these quarterly recession episodes are not outliers to be ignored, but significant economic events to be explained. Indeed, we argue that such Euler equation errors, driven by periods of important economic change, are among the most damning pieces of evidence against the standard model. An important question is why the standard model performs so poorly in recessions relative to other times.

Although not reported above, we note that the pricing error of the Euler equation associated with the CRSP stock market return is always positive, implying a positive alpha in the expected return-beta representation of the model. This says that unconditional risk premia are too high to be explained by the stock market's covariance with the marginal rate of substitution of aggregate consumption, a result familiar from the equity premium literature (Mehra and Prescott (1985), Kocherlakota (1996)). Still, it is important to remember that unlike the equity premium puzzle, the large Euler equation errors cannot be resolved by high values of γ.

2.1 Sampling Error and Tests for Joint Normality

We use GMM distribution theory to ask whether the estimated pricing errors $\mathbf{w}_T(\gamma, \delta)$ are jointly more different from zero than what would be implied by sampling error alone. When there are more moments than parameters to be estimated, this amounts to a test of overidentifying restrictions. The last two columns of Table 1 report $p-$values from chi-squared tests of the model's overidentifying restrictions for estimation of the eight Euler equations for the raw returns R_t^s, R_t^f, and $\mathbf{R}_t^{FF}$. Although the results presented so far have used the identity weighting matrix, the last column in Table 1 presents the $p-$values from the same statistical test using an estimate of the optimal GMM weighting matrix (Hansen (1982)). The results from either weighting matrix are the same: we may strongly reject the hypothesis that the Euler equation errors are jointly statistically indistinguishable from zero; the $p-$values for this test are less than 0.0001.[10]

For the two-asset case, the model is just-identified, so the overidentifying tests above are not applicable. But note that the expectation in (3) is estimated using the sample means $e_{X,t+1}^j$. Fixing δ and γ, it is possible to compute the sampling variation in the sample mean of $e_{X,t+1}^j$, given as $\sigma^2 = \sigma_X^2/T$, where σ_X is the sample standard deviation of $e_{X,t+1}^j$ and T is the sample size.[11]

The sampling error of the mean of $e_{X,t+1}^j$ is large when evaluated at the estimated values

[10] Cochrane (2005), Chapter 11, explains how to apply Hansen's (1982) GMM results to compute p-values using an arbitrary fixed weighting matrix.

[11] We also calculated standard errors for the mean of $e_{X,t+1}^j$ using a nonparameteric correction for serial correlation. Since $e_{X,t+1}^j$ is close to serially uncorrelated, this correction has little affect on the error bands.

$\widehat{\delta} = 1.4$ and $\widehat{\gamma} = 117$. When $R^j_{t+1} = R^s_{t+1}$, a confidence interval formed by plus and minus two standard errors is $(-0.55\%, 11\%)$, in percent per annum. This large range is not surprising and arises partly for the same reason that it is difficult to estimate the equity premium accurately: excess returns are highly volatile. But the large error bands also arise because the data require a very high value for γ in an attempt to fit the equity premium. Such a high value of γ generates extreme volatility in the pricing kernel, making discounted returns even harder to estimate precisely than nondiscounted returns. Unless one views $\gamma = 117$ as plausible, however, such wide standard error bands for mean discounted returns serve only to provide further evidence of the model's empirical limitations, which even at $\gamma = 117$ leaves a pricing error that is more than half of the average annual stock return. If instead we restrict the value of risk aversion to lie in the range $0 \leq \gamma \leq 89$, the pricing errors are always statistically different from zero at the five percent level of significance. Accordingly, the sample mean of $e^j_{X,t+1}$ is statistically insignificant, not because the pricing errors are small–indeed they are economically large–but rather because discounted returns are so extremely noisy when $\gamma = 117$. Clearly the overidentifying restrictions deliver a much more powerful test of the model.

The results above are important for what they imply about the joint distribution of aggregate consumption and asset returns. If consumption and asset returns are jointly lognormally distributed, then GMM estimation of (2) on any two asset returns should produce estimates of δ and γ for which the population Euler equations are exactly satisfied. The results above therefore suggest that consumption and asset returns are not jointly lognormal. For this reason, it is natural to assess whether joint lognormality is a plausible description of our consumption and return data, once we account for sampling error. Although previous statistical studies suggest that stock returns are not lognormally distributed (see, for example, the studies discussed in Campbell, Lo, and MacKinlay (1997)), it is commonly held that consumption and stock returns may be approximately jointly lognormally distributed, especially in lower frequency data. We perform formal statistical tests of normality based on multivariate skewness and kurtosis[12] for the vector $\mathbf{Y}_t \equiv \left[\log\left(C_{t+1}/C_t\right), \log\left(R^s_{t+1}\right), \log\left(R^f_t\right)\right]'$, as well as for

[12] Multivariate skewness and kurtosis statistics are computed following Mardia (1970). Let $\mathbf{x}_t$ be a p-dimensional random variable with mean $\boldsymbol{\mu}$ and variance-covariance matrix $\mathbf{V}$ of sample size T. Multivariate skewness S and (excess) kurtosis K and asymptotic distributions are given by

$$\begin{aligned} S &= \left(\frac{1}{T^2}\sum_{t=1}^{T}\sum_{s=1}^{T} g_{ts}^3\right)^{1/2} \qquad \frac{TS^2}{6} \sim \chi^2_{p(p+1)(p+2)/6} \\ K &= \frac{1}{T}\sum_{t=1}^{T} g_{tt}^2 - p\,(p+2) \qquad \frac{\sqrt{T}K}{\sqrt{8p\,(p+2)}} \sim N\,(0,1)\,, \end{aligned}$$

where $g_{ts} = (\mathbf{x}_t - \hat{\boldsymbol{\mu}})'\hat{\mathbf{V}}^{-1}(\mathbf{x}_s - \hat{\boldsymbol{\mu}})$ and $\hat{\boldsymbol{\mu}}$ and $\hat{\mathbf{V}}$ are sample estimates of $\boldsymbol{\mu}$ and $\mathbf{V}$. S and K are zero if $\mathbf{x}$ is the larger set of variables $\mathbf{X}_t \equiv \left[\log\left(C_{t+1}/C_t\right), \log\left(R^s_{t+1}\right), \log\left(R^f_{t+1}\right), \log\left(\mathbf{R}^{FF}_t\right)\right]$.

Statistical tests based on multivariate skewness and kurtosis provide strong evidence against joint normality. For $\mathbf{Y}_t$ multivariate skewness is estimated to be 1.54 and multivariate excess kurtosis is 4.64, with $p-$values for the null hypothesis that these statistics are equal to those of a multivariate normal distribution less than 0.0001. Similarly for $\mathbf{X}_t$, multivariate skewness is 4.65 and multivariate kurtosis is 35.93, and the statistical rejections of normality are even stronger. The same conclusion arises from examining quantile-quantile plots (QQ plots) for the vector time-series $\mathbf{Y}_t$ and $\mathbf{X}_t$, given in Figure 2. This figure plots the sample quantiles for the data against those that would arise under the null of joint lognormality, along with pointwise standard errors bands.[13] The QQ plots show substantial departures from normality: a large number of quantiles lie far outside the standard error bands for joint normality. We come back to these results below.

3 Euler Equation Errors in Asset Pricing Models

This section of the paper investigates the extent to which newer consumption-based asset pricing theories–those specifically developed to address empirical limitations of the standard consumption-based model–can explain its large Euler equation errors. If leading asset pricing models are true, then in these models using (2) to price assets should generate large unconditional asset pricing errors, as in the data.

3.1 Leading Asset Pricing Models

We use simulated data from each of the leading asset pricing models mentioned above to study the extent to which these models explain the mispricing of the standard model. We show that some of these models *can* explain why an econometrician obtains implausibly high estimates of δ and γ when freely fitting aggregate data to (2). But, none can explain the large unconditional Euler equation errors associated with such estimates for plausibly calibrated sets of asset returns. Indeed, the asset pricing models we consider counterfactually imply that values of δ and γ can be found for which (2) satisfies the unconditional Euler equation restrictions just as well as the true pricing kernel, implying that the standard model generates negligible pricing errors for cross-sections of asset returns.

jointly normally distributed. If $\mathbf{x}$ is univariate S and K are equivalent to the standard univariate definitions of skewness and kurtosis.

[13] Pointwise standard error bands are computed by simulating from the multivariate normal distribution with length equal to the size of our data set.

3.1.1 Simulating the Models

To assess the extent to which the models above are capable of explaining the pricing errors of the standard model, we assume each model generates the asset pricing data, and then compute the pricing errors that would arise if an econometrician fit (2) to data generated by the models. This requires simulating the models and then computing pricing errors of the standard model using simulated data in precisely the same way that we did using historical data. Except where noted, our simulations use the baseline parameter values of each paper. It is important to emphasize that even though the primitive *shocks* in these theories are often specified as normally distributed, the pricing kernels are nonlinear, and thus both the marginal distribution of asset returns, and the joint distribution of consumption and returns—what matters for Euler equation errors—are endogenous features of the asset pricing model. It follows that the pricing kernels and returns in these models are not unconditionally jointly lognormally distributed with aggregate consumption growth as was presumed in the previous sections, a fact that can be verified by statistical tests on simulated data. The question posed here is whether these models can endogenously generate a return distribution sufficiently non-normal that it is capable of rationalizing the large Euler equation errors of the standard consumption-based model (2).[14] We briefly describe only the main features of each model, and refer the reader to the Appendix and the original articles for details.

3.1.2 Misspecified Preferences

We first consider theories that deviate from the standard consumption-based model (2) in their specification of investor preferences. These include the habit models of Campbell and Cochrane (1999) and Menzly, Santos, and Veronesi (2004), and the long-run risk model of Bansal and Yaron (2004). Since these are representative agent models, an econometrician who attempted to fit (2) to data generated by these models would err by using the wrong functional form for the marginal rate of substitution in consumption (misspecified preferences).

[14] For the three representative agents models, it is assumed that innovations in consumption growth are lognormally distributed. It is reasonable to ask whether the lognormality assumption for consumption is merely a convenient but inaccurate representation of the data that could be relaxed to generate the observed Euler equation errors. The difficulty with this scenario is that the distribution of aggregate consumption growth in the data appears to be well described by a lognormal process, while the distribution of stock returns displays higher kurtosis than lognormal. (Results available upon request.) Thus, the distributional assumptions made for consumption growth in these models are not only convenient, they are empirically reasonable.

The stochastic discount factor in the CC and MSV models takes the form

$$M_{t+1} = \delta \left(\frac{C_{t+1} - X_{t+1}}{C_t - X_t} \right)^{-\gamma},$$

where C_t is aggregate consumption and X_t is habit level (a function of current and past aggregate consumption), and δ is the subjective discount factor. The key innovation in each of these models concerns the specification of the habit process X_t, which in both cases evolves according to heteroskedastic autoregressive processes. However CC and MSV differ in their specification of X_t (see the Appendix). Let M_{t+1}^{CC} denote the specification of the SDF corresponding to the Campbell-Cochrane model of X_t, and M_{t+1}^{MSV} denote the specification of the SDF corresponding to the MSV model of X_t. Both CC and MSV assume that $\Delta c_t = \mu + \sigma v_t$,where v_t is a normally distributed, i.i.d. shock, and both models derive equilibrium returns for a risk-free asset and a risky equity claim that pays aggregate consumption as its dividend. As above, the returns to these assets are denoted R_{t+1}^f, and R_{t+1}^s, respectively. Campbell and Cochrane set $\gamma = 2$ and $\delta = 0.89$ under their baseline calibration, both at an annual rate. Menzly, Santos and Veronesi choose $\gamma = 1$ and $\delta = 0.96$. Notice that the curvature parameter γ, is no longer equal to relative risk-aversion in these models.

The MSV model is a multi-asset extension of the CC model that generates implications for multiple risky securities, thus we study the implications of the habit models for larger cross-sections of asset returns by applying the MSV framework. Each firm is distinguished by a distinct dividend process with dynamics characterized by fluctuations in the share s_t^j it represents in aggregate consumption, $s_t^j = \frac{D_t^j}{C_t}$. Cross-sectional variation in unconditional mean returns across risky securities is governed by cross-sectional variation in the covariance between shares s_t^j and aggregate consumption growth Δc_t.

Bansal and Yaron (2004) consider a representative agent who maximizes utility given by recursive preferences of Epstein and Zin (1989, 1991) and Weil (1989). The stochastic discount factor under Epstein-Zin-Weil utility used in BY takes the form

$$M_{t+1}^{BY} = \left(\delta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \right)^{\alpha} R_{w,t+1}^{\alpha-1}, \tag{7}$$

where $R_{w,t+1}$ is the simple gross return on the aggregate wealth portfolio, which pays a dividend equal to aggregate consumption, C_t, $\alpha \equiv (1-\gamma)/(1-1/\psi)$, ψ is the intertemporal elasticity of substitution in consumption (IES), γ is the coefficient of relative risk aversion, and δ is the subjective discount factor. The dynamics of consumption growth and stock market dividend growth, Δd_t, take the form

$$\Delta c_{t+1} = \mu + x_t + \sigma_t \eta_{t+1} \tag{8}$$

$$\Delta d_{t+1} = \mu_d + \phi x_t + \rho_d \sigma_t u_{t+1}, \tag{9}$$

$$x_{t+1} = \rho x_t + \rho_c \sigma_t e_{t+1}$$

$$\sigma_{t+1}^2 = \sigma^2 + \nu_1 \left(\sigma_t^2 - \sigma^2\right) + \sigma_w w_{t+1},$$

where σ_{t+1}^2 represents the time-varying stochastic volatility, σ^2 is its unconditional mean, and μ, μ_d, ϕ, ρ_d, ρ, ρ_c, ν_1 and σ_w are parameters, calibrated as in BY. Here, the stock market asset is the dividend claim, given by (9), rather than a claim to aggregate consumption, given by (8). We denote the return to this dividend claim R_{t+1}^s, since it corresponds the model's stock market return. BY calibrate the model so that x_t is very persistent, with a small unconditional variance. Thus, x_t captures long-run risk, since a small but persistent component in the aggregate endowment can lead to large fluctuations in the present discounted value of future dividends. Their favored specification sets $\delta = 0.998$, $\gamma = 10$ and $\psi = 1.5$.

We analyze the multi-asset implications of the BY model by considering risky securities, indexed by j, that are distinguished by their cash-flow processes:

$$\Delta d_{t+1}^j = \mu_d^j + \phi^j x_t + \rho_d^j \sigma_t u_{t+1}. \tag{10}$$

By considering a grid of values for ϕ^j, we create risky securities with different risk-premia, since this parameter governs the correlation of equilibrium returns with the stochastic discount factor. By altering ρ_d^j, we control the variance in the risky security returns, while μ_d^j controls the mean price-dividend ratio across risky assets.

For both the MSV and BY models, we choose parameters of the cash-flow processes to create a cross-section of asset returns that include a risk-free rate, an aggregate equity return, and six additional risky securities, or eight securities in total. For each model, we exactly replicate the authors' original calibration to obtain the same risk-free return and aggregate equity return studied there. For the six additional risky securities, we choose parameters of the individual cash-flow processes that allow us to come as close as possible to matching the spread in risk-premia found in the six size/book-market sorted portfolio returns in the data. For the BY model, we can generate a cross-section of returns that come very close to matching the historical spread in these returns. For example, the largest spread in average annualized returns is given by the difference between the portfolio in the smallest size and highest book-market category and the portfolio in the largest size and lowest book-market category, equal to about seven percent; thus we create six artificial returns for which the largest spread is 6.7 percent per annum. Constructing such returns for the MSV framework is more complicated, since the solutions for the multi-asset model hold only as an approximation (see the Appendix for the approximate relation). Unfortunately, we find that the approximation error in this model can be substantial under parameter values required to make the maximal spread as large as seven percent.[15] As a result, we restrict the parameter values to ranges that limit approximation error to reasonably small degrees. This still leaves us with a significant spread of 4.5 percent per annum in the returns of the six artificial securities created.

To study the implications of these representative-agent models, we simulate a large time-series (e.g., 20,000 periods) from each model and compute the pricing errors that would arise in equilibrium if $M_{t+1}^c = \delta_c \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma_c}$ were fitted to data generated by these models. Thus, we conduct precisely the same empirical estimation on model-generated data as was conducted on historical data, above. The parameters γ_c and δ_c are chosen by GMM to minimize the Euler equation errors $e_R^j = E[M_{t+1}^c R_{t+1}^j] - 1$. We denote the estimated parameters that minimize the GMM criterion as $\widehat{\delta}_c$ and $\widehat{\gamma}_c$. As in the historical data, we focus on the case of $N = 2$ asset returns (R_{t+1}^s and R_{t+1}^f), and the case of $N = 8$ asset returns, $(R_{t+1}^s, R_{t+1}^f, R_{t+1}^1, ..., R_{t+1}^6)$.

The main results, presented in Table 3, are as follows. For both habit models, we find the pricing errors that arise from fitting M_{t+1}^c to model-generated data are numerically zero, just as they are when the true habit pricing kernel is used. This result does not depend on the number of assets being priced; it is the same for the two-asset case and eight-asset case. Values of δ_c and γ_c can in each case be found that allow the standard consumption-based model to unconditionally price assets just as well as the true pricing kernel, as measured by the root mean-squared pricing error. The habit models *can* explain what many would consider the implausible estimates (Table 1) of time preference and risk aversion obtained when freely fitting aggregate data to (2). In the CC model, the values of δ_c and γ_c that minimize the GMM criterion for R_{t+1}^s and R_{t+1}^f are 1.28 and 57.48, respectively. The corresponding values in the MSV model are 1.71 and 30.64, respectively. This represents a significant distortion from the true values of these parameters. (Recall that the true preference parameters are $\gamma = 2$ and $\delta = 0.89$ in CC and $\gamma = 1$ and $\delta = 0.96$ in MSV.) But, it is in those parameters that all of the distortion from erroneously using M_{t+1}^c to price assets arises. No distortion appears in the Euler equation errors themselves.

The conclusions for the Bansal-Yaron long-run risk model, also displayed in Table 3, are the same. Here we follow BY and simulate the model at monthly frequency, aggregate to annual frequency, and report the model's implications for pricing errors and parameter values. The monthly consumption data are time-aggregated to arrive at annual consumption, and monthly returns are continuously compounded to annual returns.[16] We find that δ_c is

[15] Menzly, Santos, and Veronesi (2004) state that the approximation error is small for the parameters they employ, but it is not small for our parameters, which were chosen to mimic returns of the Fama-French portfolios.

[16] The resulting Euler equation errors are unchanged if they are computed for quarterly time-aggregate

estimated to be close to the true value, but γ_c is estimated to be about five times as high as true risk aversion. As for the habit models, an econometrician will estimate high values of risk aversion when fitting the standard consumption-based model to the BY data, but the resulting Euler equation errors would be effectively zero.[17]

3.1.3 Misspecified Consumption

Next we consider the limited participation model of Guvenen (2003). The Guvenen economy has two types of consumers, stockholders and nonstockholders, and two assets, a stock return and a riskless bond. Nonstockholders are exogenously prevented from participating in the stock market. The stochastic discount factor in this model is denoted

$$M_{t+1}^{G} \equiv \delta_i \left(\frac{C_{t+1}^{i}}{C_t^{i}} \right)^{-\gamma_i},$$

where C_t^i is *stockholder* consumption, which by assumption is not the same as aggregate per capita consumption, δ_i is the subjective discount factor of the stockholder, and γ_i is the stockholder's relative risk aversion. Thus, an econometrician who attempted to fit (2) to aggregate data would err by using the wrong measure of consumption, aggregate consumption rather than stockholder consumption (misspecified consumption). In other respects, the model is a standard one-sector real business cycle model with adjustment costs in capital. Both stockholders and nonstockholders receive labor income with wages determined competitively by the marginal product of labor, and firms choose output by maximizing the present discounted value of expected future profits. Both agents have access to the riskless bond.

We follow the same procedure discussed above to quantify pricing errors in this model. We simulate a large time series of artificial data and use these data to quantify the magnitude of unconditional pricing errors that an econometrician would find if the misspecified MRS, $M_{t+1}^{c} = \delta_c \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma_c}$, based on aggregate consumption, were fitted to asset pricing data generated by M_{t+1}^{G}. Since cash-flows are endogenously determined by the properties of a general equilibrium setting in that model, the extension to multiple-assets is not straightforward. For this reason, we focus only on the implications of the Guvenen model for R^s and R^f.

The main results are presented in the bottom panel of Table 4. They show that the Guvenen model, like the habit models, generates effectively zero Euler equation errors when

consumption and quarterly returns rather than annual time-aggregated consumption and annual returns.

[17]For models based on recursive preferences, Kocherlakota (1990) shows that there is an observational equivalence to the standard model with power utility preferences, if the aggregate endowment growth is i.i.d. However, the endowment growth process in the BY model is not i.i.d., but instead serially correlated with stochastic volatility. Moreover, the annual consumption data are time-aggregated, which further distorts the time-series properties from those of the monthly endowment process.

M^c_{t+1} is used to price assets, but in this case estimates of the parameters show much less distortion from their true values. The table also reports the pricing errors using the true kernel M^G_{t+1} based on stockholder consumption, which are quite small (0.02% on an annual basis) but not exactly zero due to the rarely-binding borrowing constraints that apply to both stockholders and nonstockholders. Euler equation errors based on the misspecified M^c_{t+1} are tiny even when preference parameter values are not chosen to minimize those errors. For example, when δ_c and γ_c are set to their true values for stockholders, (in Guvenen's baseline specification, stockholders have risk aversion $\gamma_i = 2$ and subjective discount factor $\delta_i = 0.99$), the pricing errors using aggregate consumption are equal to about 0.4% at an annual rate for the stock return and -0.34% for the risk-free rate, small in magnitude compared to the data. When δ_c and γ_c are chosen to minimize the sum of squared pricing errors for these two asset returns, as in empirical practice, the Euler equation errors are, to numerical accuracy, zero for the stock return and risk-free return. Moreover, the estimated values for the subjective time-discount factor and risk aversion from such an estimation show minimal distortion from their true value, equal to $\widehat{\delta}_c = 0.99$ and $\widehat{\gamma}_c = 4.49$, respectively. These results imply that by increasing γ by a factor of 2.5–from 2 to 4.5–the Guvenen model delivers a power utility pricing kernel using aggregate consumption that explains the historical mean return on the stock market and risk-free (Treasury bill) return just as well as the true pricing kernel based on stockholder consumption. This model therefore does not explain the equity premium puzzle of Mehra and Prescott (1985), which is the puzzle that a high value of γ is required to explain the magnitude of the equity premium when the power utility model is fitted to aggregate consumption data.

To aid in understanding these results, the top panels of Table 4 provides summary statistics from the model. Panel A of Table 4 shows that stockholder consumption growth is about two and a half times as volatile as aggregate consumption growth, and perfectly correlated with it. Stockholder consumption is over four times as volatile as nonstockholder consumption growth, but the two are almost perfectly correlated, with correlation 0.99. This is not surprising since both types of consumers participate in the same labor market and bond markets; the agents differ only in their ability to hold equities and in their risk-aversion (nonstockholders have higher risk-aversion). As a consequence, the true pricing kernel based on the stockholder's marginal rate of substitution, M^{GUV}_{t+1}, is highly correlated with the misspecified aggregate consumption "pricing kernel" $M^c_{t+1} \equiv \delta_c (C_{t+1}/C_t)^{-\gamma_c}$, for a variety of values of δ_c and γ_c. Panel B of Table 4 shows this correlation for two combinations of these parameters, first with these parameters set at their true values $\delta_c = \delta_i = 0.99$ and $\gamma_c = \gamma_i = 2$, and second with δ_c and γ_c set to the values that minimize the equally-weighted sum of squared Euler equation errors when M^c_{t+1} is used to price assets. In both cases, the correlation between M^{GUV}_{t+1} and M^c_{t+1} is extremely high, 0.99. In addition, when $\gamma_c = 4.5$,

M_{t+1}^{GUV} and M_{t+1}^{c} have virtually identical volatilities, so their asset pricing implications are the same.

3.1.4 Additional Diagnostics

Misspecified Preferences and Misspecified Consumption One possible reaction to the results above, is that we should take the representative agent nature of the CC, MSV and BY models less literally and assume that they apply only to a representative stockholder, rather than to a representative household of all consumers. Would the results for these models be better reconciled with the data if we accounted for limited participation? Not necessarily. As an illustration, we consider a limited-participation version of the MSV model and show that the conclusions are unchanged from the representative agent setup.

Since the MSV model is a representative agent model, we modify it in order to study the role of limited participation. Assume that asset prices are determined by the framework above, where a valid stochastic discount factor is a function of any stockholder's consumption C_t^i and stockholder's habit X_t^i. The process for stockholder consumption is the same as in MSV, described above, but now with i subscripts:

$$\Delta c_t^i = \mu_i + \sigma_i v_t^i,$$

where v_t^i is a normally distributed i.i.d. shock. Aggregate consumption is assumed to follow a separate process given by

$$\Delta c_t = \mu_c + \sigma_c v_t^c,$$

with v_t^c a normally distributed i.i.d. shock. We analyze the results over a range of cases for the correlation between v_t^i and v_t^c, and their relative volatilities σ_i/σ_c.

Asset prices are determined by the stochastic discount factor of individual assetholders, denoted

$$M_{t+1}^{MSVi} \equiv \delta_i \left(\frac{C_{t+1}^i - X_{t+1}^i}{C_t^i - X_t^i} \right)^{-\gamma_i},$$

where X_{t+1}^i is the external habit modeled as in MSV, now a function of C_t^i (the Appendix provides an exact expression). We assume the data are generated by M_{t+1}^{MSVi} and compute the Euler equation errors that arise from fitting

$$M_{t+1}^c \equiv \delta_c \left(C_{t+1}/C_t \right)^{-\gamma_c}$$

to asset pricing data. We refer to this case as "misspecified preferences and misspecified consumption," since an econometrician who fit M_{t+1}^c to asset return data would be employing both the wrong model of preferences and the wrong consumption measure. The parameters,

δ_c and γ_c are chosen to minimize an equally-weighted sum of squared pricing errors of the assets under consideration, as with the historical data.

The results are presented in Table 5, where the Euler equation errors for a range of parameter values. The standard deviation of asset-holder consumption growth is allowed to range from one times to five times as volatile as that of aggregate consumption growth, the correlation from -1.0 to 1.0. The pricing errors (as measured by RMSE/RMSR) are reported in the bottom subpanels. The top panel reports these errors for the two-asset case where only R_{t+1}^s and, R_{t+1}^f are priced; the bottom panel reports for the eight-asset case with six additional risky securities. For each parameter configuration, we also report the values $\widehat{\delta}_c$ and $\widehat{\gamma}_c$ that minimize the quadratic form $g_T(\gamma_c, \delta_c)$, as above.

Table 5 shows that the pricing errors that arise from using M_{t+1}^c to price assets are always zero, even if assetholder consumption growth has very different properties from aggregate consumption growth. For example, aggregate consumption growth can be perfectly negatively correlated with stockholder consumption growth and five times as volatile, yet the pricing errors that arise from using C_t in place of C_t^i are still zero. Notice, however, that the parameters δ_c and γ_c can deviate substantially from the true preference parameters of stockholders. This is similar to the lognormal example in Section 3.1, in which the use of mismeasured consumption distorts preference parameters, but does not explain the large pricing errors generated by the standard consumption-based model.[18] Results for the multi-asset case are qualitatively the same as those for two-asset case. These findings reinforce the conclusion that changing the pricing kernel does not necessarily change the pricing implications.

The results reveal a striking implication of leading asset pricing models: the unconditional pricing errors of the standard consumption-based model can be virtually identical to those using the true pricing kernel, even when (i) the true kernel has preferences different from the CRRA form of the standard model, (ii) the consumption of marginal assetholders behaves differently from per capita aggregate consumption, and (iii) the number of assets exceeds the number of free parameters to be estimated. This implies that the explanation for the high average pricing errors produced by the standard model has to be something more than limited participation and/or nonstandard preferences per se, since in many models parameter values can be found that allow the standard model to price cross-sections of assets almost as well as the true pricing kernel that generated the data.

[18] Variation in σ_i/σ_c has little affect on the estimated value of the risk-aversion parameter γ_c. This happens because we adjust the parameter α in the MSV habit specification (see the Appendix) at the same time as we adjust σ_i/σ_c so that the mean excess return $R^s - R^f$ remains roughly what it is in MSV. Since the volatility of aggregate consumption is kept the same and α is adjusted to keep the returns of the same magnitude, γ_c doesn't change much.

Time Aggregated Consumption What if the decision interval of households is shorter than the data sampling interval, leading to time-aggregated consumption observations? We have repeated the same exercise for all the models above using time-aggregated consumption data, assuming that agents' decision intervals are shorter than the data sampling interval, for a variety of decision intervals. An example is provided in the Appendix. For all models the essential results for the Euler equation errors remain the same: values of δ_c and γ_c can always be found such that the unconditional pricing errors associated with using M^c_{t+1} to price assets are very small relative to the data.

Finite Sample Pricing Errors The results above are based on long samples of model-generated data, whereas the estimates using historical data are based on a finite sample of 204 observations. An analysis provided in the Appendix shows that that our main conclusions are robust to using samples equal in size to that of our historical dataset.

3.2 Diagnosing the Result

Why do leading asset pricing models counterfactually imply that the standard consumption-based model has negligible Euler equation errors? To explain these results, in this section we present sufficient conditions under which values for γ and δ can be found such that the Euler equations of the standard model in the just-identified two asset case are exactly satisfied, and show that each of the leading asset pricing models satisfies these conditions, while in the historical data such conditions are violated.[19] Specifically, the question we address is, in the model of interest, what sufficient conditions imply that there exists a γ such that

$$E\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(R^s_{t+1}-R^f_{t+1}\right)\right]=0. \tag{11}$$

If a γ can be found that satisfies the Euler equation above, we can always find a δ that satisfies the risk-free rate Euler equation.

Denote:

$$\begin{aligned} X_t &\equiv C_t/C_{t-1}. \\ Z_t &\equiv R_t - R^f_t. \\ f(X_t) &\equiv E[Z_t|X_t]. \end{aligned}$$

We make four assumptions.

Assumption 1

[19] We are grateful to Narayana Kocherlakota for suggesting this line of argument.

There is an $\bar{X} < 1$ and $\mu < 0$ such that $f(X_t) < \mu$ for all $X_t < \bar{X}$.

Assumption 2

$E[(X_t/\bar{X})^{-\gamma}|X_t < \bar{X}] \to \infty$ as $\gamma \to \infty$.

Assumption 3

$E[f(X_t)|X_t > \bar{X}]$ is finite.

Assumption 4

$E\left[(X_t)^{-\gamma} Z_t\right]$ is a continuous function of γ.

Proposition 1 *When the standard pricing kernel $M_{t+1} = \delta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}$ is fit to data generated by any model for which Assumptions 1 through 4 hold, values of δ and γ can be found that exactly satisfy the two Euler equations corresponding to a risky asset return and risk-free rate, or any two asset returns. Proof: See the Appendix.*

Assumptions 2, 3 and 4 are technical conditions that will be satisfied for most consumption-based asset pricing models with well behaved distributions, including those investigated above. Assumption 1 is the economically meaningful one. It that says that, for all realizations of consumption growth less than $\bar{X}$, the excess returns on the risky asset are negative on average. Intuitively, Assumption 1 says that realizations of the risky asset return that are low relative to the risk-free rate coincide with bad economic times.

The leading consumption-based asset pricing models we study all satisfy the four conditions listed above. In particular, negative realizations of the excess return on the risky asset coincide with bad economic times in the form of falling consumption growth, and the key economic Assumption 1 is satisfied. To illustrate, Figure 3 shows fitted values from nonparametric regressions of excess returns on consumption growth, for each of the four models:

$$R_t^j - R_t^f = m(C_t/C_{t-1}) + \epsilon_t, \tag{12}$$

where $m(\cdot)$ is a nonparametrically estimated function.[20] The estimates are formed from simulated data of each model's benchmark specification. Notice that in each model, for consumption realizations sufficiently negative, excess returns are also negative, satisfying Assumption 1. By contrast, this condition is violated in the historical data, because the lowest consumption realizations coincide with *positive* excess returns on average (Table 2). This property of the historical data is also visible in Figure 4, which presents the fitted values from the nonparametric regression (12) using quarterly historical consumption and return data. Conditional on consumption growth being sufficiently low, realized excess returns are on average positive in the historical data. However, as Figure 5 demonstrates, if we estimate the nonparametric regression (12) on historical data excluding all recession periods

[20] The regression uses a Gaussian kernel with optimally chosen bandwidth.

in our sample, Assumption 1 is now satisfied: excess return realizations are negative when consumption realizations low (top panel). Moreover, in the sample excluding recessions, a value for γ that satisfies the Euler equation (11) can now be found (Figure 5, bottom panel). These results show that the leading asset pricing models we consider fail to explain the mispricing of the standard model because they mischaracterize the joint behavior of consumption and asset returns in recessions.

Two points about these results deserve emphasis. First, the findings for the four consumption-based models we consider are unlikely to depend on the precise calibration of the models. Although this is impossible to verify in general for the models that require numerical solutions, it is straight forward to verify in the MSV model, for which closed-form solutions exist. Specifically, in the MSV model, the excess return on the market portfolio is an affine function of the innovation in consumption, where only the magnitude (but not the sign) of the parameters of the affine function depend on the calibration. Thus, sufficiently small realizations of consumption growth will always coincide with negative excess returns on the risky asset.

Second, it is straightforward to show that many simple models of limited stock market participation will also satisfy sufficient conditions of the type presented above regardless of the correlation between stockholder and aggregate consumption, as long as stockholder consumption is jointly lognormally distributed with aggregate consumption and returns. We analyze an example of this case in the next subsection.

3.2.1 A Limited Participation/Incomplete Markets Model With Joint Lognormality

Suppose that the consumption measure C_t, used by an econometrician to estimate the Euler equation (2) is mismeasured, perhaps because per capita aggregate consumption is a poor measure of individual assetholder consumption (markets are incomplete), or the consumption of stockholders (limited stock market participation). For our purposed here, a model of limited stock market participation is isomorphic to a model of incomplete markets, since what matters is the common implication that the consumption of the marginal assetholder may behave differently from per capita aggregate consumption.[21] Suppose also that aggregate consumption, stockholder or individual consumption, and asset returns are unconditionally jointly lognormally distributed. We use lowercase letters to denote log variables, e.g., $\Delta c_{t+1} \equiv \log\left(C_{t+1}/C_t\right)$.

[21] With limited stock market participation, the set of Euler equations of *stockholder* consumption imply that a representative stockholder's marginal rate of substitution is a valid stochastic discount factor. Similarly, with incomplete consumption insurance the set of Euler equations of *household* consumption imply that any household's marginal rate of substitution is a valid stochastic discount factor.

Denote the MRS of an individual stockholder as

$$M_{t+1}^{i} \equiv \delta_i \left(\frac{C_{t+1}^{i}}{C_t^{i}} \right)^{-\gamma_i}, \tag{13}$$

where C_t^i is the consumption of stockholder i, δ_i is the subjective time discount factor of this stockholder, and γ_i is the stockholder's coefficient of relative risk aversion. If agents have unrestricted access to financial markets, then M_{t+1}^i correctly prices any traded asset return held by the stockholder, implying that $E\left[M_{t+1}^i R_{t+1}^j\right] = 1$ for any traded asset return. The risk-free rate is defined as a one-period riskless bond, $R_{t+1}^f = 1/E_t\left[M_{t+1}^i\right]$.

We can interpret the MRS, M_{t+1}^i, either as that of a representative stockholder in a limited participation setting (in which case C_t^i is the consumption of a representative stockholder), or as that of an individual assetholder in an incomplete markets setting (in which case C_t^i is the consumption of any marginal assetholder, e.g., Constantinides and Duffie (1996)). For brevity, we hereafter refer to C_{t+1}^i simply as stockholder consumption, and to (13) simply as the limited participation model.

An econometrician who maintained the assumption of power utility but erroneously estimated Euler equations using data on per capita aggregate consumption, C_{t+1} in place of C_{t+1}^i, would use the misspecified "MRS" M_{t+1}^c:

$$M_{t+1}^{c} \equiv \delta_c \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma_c}, \tag{14}$$

where δ_c and γ_c are generic parameter values that do not necessarily correspond to the true preference parameters of stockholder i. Notice that while the Euler equation error associated with the true MRS, M_{t+1}^i, is zero by construction, the Euler equation error associated with the erroneous MRS, M_{t+1}^c, need not be zero.

Most economic models will satisfy the technical Assumptions 2-4 given above. In addition, many lognormal models of limited participation will satisfy Assumption 1. As one example, suppose the log difference in stockholder consumption, Δc_t^i, follows a stationary $ARMA(p,q)$ process and consider an orthogonal decomposition of aggregate consumption growth into a part that is correlated with asset-holder consumption and a part, ε_t^i, orthogonal to stockholder consumption,

$$\Delta c_t = \beta \Delta c_t^i + \varepsilon_t^i, \tag{15}$$

where $\beta = \frac{\mathrm{Cov}\left(\Delta c_t, \Delta c_t^i\right)}{\mathrm{Var}\left(\Delta c_t^i\right)} = \frac{\rho_{ci}\sigma_c}{\sigma_i}$. Here ρ_{ci} denotes the correlation between Δc_t and Δc_t^i. If the log difference in stockholder consumption has a Wold representation taking the form

$$\Delta c_t^i = k + \sum_{j=0}^{\infty} \theta_j \epsilon_{t-j},$$

where ϵ_t is an i.i.d. innovation.

To relate consumption to returns, note that the innovation in log asset return surprises on any asset must be equal to the revision in expected future dividend growth on the asset (discounted at some constant rate ρ) minus the revision in expected future returns (also discounted at ρ, see Campbell (1991)). If the stock return is modeled as a claim to assetholder consumption, and if risk premia are constant, return surprises will then be linearly related to the consumption innovation, since both the revision in expected discounted dividend growth and the revision in expected discounted risk-free rates are functions of the consumption innovation ϵ_{t+1}. For example, if the risk-free rate is constant, the innovation in the log stock return is linear in the consumption innovation:

$$\begin{aligned} r^s_{t+1} - E_t r^s_{t+1} &= \epsilon_{t+1}\left(\theta_0 + \rho\theta_1 + \rho^2\theta_2 + ...\right) \\ &\equiv \omega\epsilon_{t+1}. \end{aligned}$$

Hence if a positive shock to stockholder consumption today is good news about future consumption ($\omega > 0$) and if stockholder consumption is positively correlated with aggregate consumption ($\beta > 0$), Assumption 1 is satisfied. This follows from the lognormality assumption (with its unbounded support for consumption growth) and the linear relation between return innovations and consumption innovations. Notice that, if stockholder consumption growth is negatively correlated with aggregate consumption growth ($\beta < 0$), the model will satisfy an analogous set of sufficient conditions to those given above, in which there exists an $\underline{X}$ such that, for all realizations of consumption growth greater than $\underline{X}$, the expected risk premium is negative. Either way, the assumption of lognormality implies that we can always find values for δ_c and γ_c such that the Euler equations of the standard model in the just-identified two asset case are exactly satisfied.

In addition, it is possible to derive explicit expressions for preference parameters that exactly satisfy the Euler equations of the standard model.[22] To do so, first note that, under joint lognormality, the pricing error may be written

$$e^j_R = E\left[R^j\right] E\left[M^c\right] \exp\left\{\text{Cov}\left(m^c, r^j\right)\right\} - 1. \tag{16}$$

Noting that the Euler equation error is identically zero under M^i, implying

$$E\left[R^j\right] E\left[M^i\right] \exp\left\{\text{Cov}\left(m^i, r^j\right)\right\} = 1,$$

and using $m = \log(\delta) - \gamma\Delta c$, we may write

$$e^j_R = \frac{E\left[M^c\right]}{E\left[M^i\right]} \exp\left\{-\gamma_c \text{Cov}\left(\Delta c, r^j\right) + \gamma_i \text{Cov}\left(\Delta c^i, r^j\right)\right\} - 1. \tag{17}$$

[22] The calculations below are similar in spirit to those in Vissing-Jorgensen (1999), who shows how limited stock market participation biases estimates of relative risk aversion based on aggregate consumption. Vissing-Jorgensen's calculations presume heterogenous households rather than a representative-stockholder, as below.

For a single risky asset return $R^s_{t+1}$and the risk-free return R^f_{t+1}, there are two equations in two unknowns, with analytical solutions given by

$$\widehat{\gamma}_c = \gamma_i \left(\frac{\sigma_{is} - \sigma_{if}}{\sigma_{cs} - \sigma_{cf}} \right), \tag{18}$$

$$\widehat{\delta}_c = \delta \exp \left[\gamma_c \mu_c - \frac{\gamma_c^2 \sigma_c^2}{2} - \gamma_i \mu_i + \frac{\gamma_i^2 \sigma_i^2}{2} + \gamma_c \sigma_{cs} - \gamma_i \sigma_{is} \right], \tag{19}$$

where $\sigma_{cs} \equiv \mathrm{Cov}(\Delta c, r^s)$, $\sigma_{if} \equiv \mathrm{Cov}\left(\Delta c^i, r^f\right)$, $\sigma_{cf} \equiv \mathrm{Cov}\left(\Delta c, r^f\right)$, $\sigma_c^2 \equiv Var\left(\Delta c\right)$, $\sigma_i^2 \equiv Var\left(\Delta c^i\right)$, μ_c is the mean growth rate of aggregate consumption, and μ_i is the mean growth rate of the consumption of asset-holder i.[23]

A more intuitively appealing expression for $\widehat{\gamma}_c$ can be obtained by assuming the risk-free rate is constant. While this is an approximation, it turns out to be well satisfied in the data, since most proxies for the risk-free rate (such as the Treasury-bill rate) are extremely stable. Using the orthogonal decomposition (15), (18) can be written

$$\widehat{\gamma}_c = \frac{\gamma_i}{\beta + \frac{\sigma_{\varepsilon^i s}}{\sigma_{is}}}, \tag{20}$$

where $\sigma_{\varepsilon^i s} = \mathrm{Cov}\left(\varepsilon_t^i, R^s_{t+1}\right)$. For assets that are uncorrelated with ε_t^i, (e.g., any risky asset that is on the log mean-variance efficient frontier), $\sigma_{\varepsilon^i s} = 0$ and (20) collapses to

$$\widehat{\gamma}_c = \frac{\gamma_i}{\beta} = \gamma_i \frac{\sigma_i}{\rho_{ci} \sigma_c}. \tag{21}$$

The above expression tells us that limited participation can in principal account for high estimated values of γ_c (and δ_c) obtained when fitting data to (14), if stockholder consumption is more volatile than aggregate consumption and/or very weakly correlated correlated with it.

It is important to emphasize, however, that, in the two asset case, the values of γ_c, and δ_c obtained when the model is estimated using the misspecified MRS based on aggregate consumption growth still insure that the log pricing errors for R^s_{t+1} and R^f_{t+1} are *identically* zero, $e_R^j = 0$. This follows because, under lognormality, the log model is linear and the problem collapses to solving two linear equations in two unknowns. This is demonstrated in Figure 6, for the two-asset cases using actual historical return data. The "data" line plots

[23] Notice that, in equilibrium, $\widehat{\gamma}_c$ and $\widehat{\delta}_c$ will take the same value regardless of the identity of the assetholder. This follows because any two households must in equilibrium agree on asset prices, so that the Euler equation holds for each individual household. Thus,

$$\gamma_c = \gamma_i \left(\frac{\sigma_{is} - \sigma_{if}}{\sigma_{cs} - \sigma_{cf}} \right) = \gamma_k \left(\frac{\sigma_{ks} - \sigma_{kf}}{\sigma_{cs} - \sigma_{cf}} \right)$$

for any two asset-holders i and k.

RMSE/RMSR over a range of values for γ_c, after choosing δ_c so as to minimize the sum of squared Euler equation errors $e^j_{R,t}$, which do not impose lognormality. The line labeled "lognormality" plots the RMSE/RMSR over a range of values for γ_c, after choosing δ_c to minimize the sum of squared pricing errors in (16), under the assumption that returns and consumption growth are jointly lognormal. Under lognormality, a value for γ_c can be found that exactly satisfies the Euler equation, in contrast to the data. Thus, the only consequence of using aggregate per capita consumption in this setting is a bias in the estimated parameters $\widehat{\gamma}_c$ and $\widehat{\delta}_c$; there is no consequence for the Euler equation errors themselves, which remain zero. It follows that any lognormal model of limited participation cannot explain the large empirical Euler equation errors of the standard model found in the data.

In the case of multiple risky assets, lognormality does not necessarily imply that such Euler equation errors will each be identically zero, since in this case there are more moment conditions than free parameters. Nevertheless, a lognormal model is unlikely to match the magnitude of the Euler equation errors found in the data. The Appendix discusses this case and shows that the lognormal model cannot match the magnitude of the Euler equation errors for the eight-asset case, especially in the empirically relevant region where γ_c is large.

It should be noted that the results in this subsection hold for *any* pricing kernel M^i_{t+1} that is jointly lognormally distributed with returns and aggregate consumption growth; it is not necessary that the pricing kernel take the form given in (13). As long as the true kernel M^i_{t+1} is jointly lognormally distributed with aggregate consumption and returns, values for the discount factor and risk aversion can be found for which the standard model generates identically zero unconditional Euler equation errors for any two asset returns.

3.3 Limited Participation/Incomplete Markets With State Dependent Correlations

We now revisit the potential role of limited participation in explaining the large Euler equation violations of the standard consumption-based model, with an eye toward the important role of recessions in these findings. This requires relaxing the assumption of unconditional joint lognormality.

3.3.1 Limited Participation with State Dependent Correlations

An intriguing feature of aggregate consumption and return data is that violations of Euler equations in (2) are especially large in recessions. For example, in the troughs of recessions in the 1950s, 1970s, early 1960s, 1980s and 1990s, as identified by the National Bureau of Economic Research, net aggregate consumption growth is negative but the aggregate stock return and Treasury-bill rate are, more often than not, positive (Table 2). These findings suggest that the link between the aggregate economy and asset returns is fundamentally different in economic downturns than in upturns. The findings above suggest that a complete description of the joint behavior of consumption and asset returns must be consistent with this state dependency.

As a preliminary step, we consider the following modification to the simple limited participation model motivated by the empirical findings above. Assume that both stockholder and aggregate consumption growth are i.i.d. processes, with normally distributed shocks. For simplicity, stockholders are presumed to have CRRA utility and, as above, stock prices are determined only by stockholder's consumption. We modify the previous framework, however, by assuming that the correlation between the growth rates of stockholder consumption and aggregate consumption is time-varying and depends on the state of the economy. In "normal" times, the correlation between consumption growth of stockholders and aggregate consumption growth is one. Normal times are modeled as any period in which aggregate consumption growth is not unusually low, say one standard deviation or more below its mean. In "bad" times, the correlation between consumption growth of stockholders and aggregate consumption growth is significantly less than one, even negative. Bad times are modeled as any period when aggregate consumption growth is more than one standard deviation below its mean. This changing correlation could be due to unemployment shocks that primarily affect less wealthy nonstockholders, to binding borrowing constraints that make it harder for nonstockholders to smooth consumption in recessions, or to cyclical shifts in the composition of income between labor and capital.

Interestingly, a time-varying correlation of this type between stockholder consumption and aggregate consumption growth generates deviations from joint log-normality of aggregate consumption growth and asset returns in the model that are remarkably similar to those found in the data. (Although the shocks to aggregate consumption and stockholder consumption growth are normally distributed, the time-varying correlation means that their joint distribution with endogenous returns is unconditionally nonnormal.) It also allows the model to rationalize the large Euler equation errors of the standard, representative agent, CRRA model. To illustrate, we model the equity claim as a claim to stockholder consumption, c_t^i, and model additional risky securities, indexed by j, as those with dividend processes taking the form $\Delta d^j = \lambda^j \Delta c_t^i + \varepsilon_t^j$,where ε_t^j is an i.i.d. shock uncorrelated with Δc_t^i. By varying λ^j across assets, we create a spread in the covariance of returns on these securities with stockholder consumption growth, and therefore a spread in risk premia. Values for λ^j and the standard deviation of ε_t^j are chosen to mimic the spread in returns in the 6 Fama-French portfolios for which we have historical data. For the results below, stockholder risk-aversion is set to $\gamma = 10$. Since we have assumed, for illustrative simplicity, that stockholders have CRRA utility, this stylized model has some important limitations. For example, with $\gamma = 10$, the model generates a mean risk-free rate that is much higher than in the data (Weil (1989)); thus we set $\delta = 1.2$ to obtain more reasonable values. Nevertheless, the simplicity of the model serves to illustrate an important point, namely that a state-dependent correlation between the consumption of stockholders and nonstockholders can help explain why the standard consumption-based model's Euler equations are violated by such large magnitudes.

Figure 7 shows QQ plots from model-simulated data, which are directly comparable to those using historical data in Figure 2. Note that the deviations from joint log-normality are concentrated in periods with observations that are in the tails of the joint distribution, both in the data and in the model. These deviations from log-normality are of the type necessary to generate large Euler equation errors for the misspecified SDF based on aggregate consumption and power utility. Table 6 shows that the state-dependent correlations model is able to generate pricing errors for the standard model that rival those in the data, both for the set of two asset returns that include the stock market return and the risk-free rate, as well as for a larger cross-section of returns that include the 6 additional risky securities. The table has a layout similar to that of Table 5, except that we vary the correlation in bad states at the top of each column, rather than the unconditional correlation. The calibrations that deliver the largest Euler equation errors are those for which the correlation between aggregate and stockholder consumption is unity most of the time (in good states), but is negative in bad states (defined as states in which aggregate consumption is more than one standard-deviation below its mean). For example, when the correlation in bad states is -0.5 and the standard deviation of stockholder consumption growth is twice that of aggregate consumption growth, this model implies Euler equation errors for the standard consumption-based model, as measured by RMSE/RMSR, of 0.47, a value that almost exactly replicates that found in the data when the standard model is fit to historical data on aggregate consumption, the stock market and Treasury-bill (Table 1). Finally, Figure 8 shows fitted values from nonparametric regressions of excess returns on consumption growth as in (12) using simulated data from this model. The fitted values have a pattern that is remarkably similar to those in the data: the lowest consumption realizations coincide with positive excess returns on average, violating Assumption 1. These results are promising because they go significantly in the direction required to explain why the standard model appears so misspecified.

We close this section by noting that limited participation combined with arbitrary departures from normality does not in general explain the mispricing of the standard model. The Appendix provides an analysis of this issue by considering a range of non-normal models based on Hermite expansions around the normal density. Most non-normal models we considered imply that the wrong pricing kernel based on aggregate consumption delivers tiny pricing errors even when the joint distribution of Δc_t, Δc_t^i, and returns are significantly non-normal. This suggests that the explanation for the large pricing errors of the standard representative agent model must be more than limited participation *per se*. The joint distribution of assetholder, aggregate consumption and returns has to be of a particular form, and it is that form that must be the central part of the story.

The examples in this section are designed to be illustrative and are not meant to be taken as realistic models. Nevertheless, they are useful for building intuition about *why* the leading models fail to match the empirical properties of the standard model's Euler equations found in the data. The previous section showed that a very low or even negative unconditional correlation between stockholder and nonstockholder consumption is not by itself enough to explain why the standard model fails: when the MSV model is modified to have limited participation, a low unconditional correlation between stockholder and nonstockholder consumption does not generate non-negligible pricing errors. Instead, what is needed is a state-dependent correlation, of the type explored above. It is straightforward to introduce the same state-dependent correlation between stockholder and aggregate consumption into the limited participation version of the MSV habit model. Doing so, we obtain results very similar to those reported above. This is encouraging because it suggests that leading consumption-based models can be modified to fit the Euler equation facts, while at the same time preserving their favorable implications for a range of other asset pricing phenomena.

4 Conclusion

It is well understood that the standard, representative agent, consumption-based asset pricing theory based on constant relative risk aversion utility fails to explain the behavior of risky assets. Some aspects of this failure have been famously pointed out by authors like Mehra and Prescott (1985), who argue that the model is incapable of rationalizing the equity premium for reasonable levels of risk aversion. Other researchers have estimated the Euler equations of the model using GMM, and found that the model is formally rejected in statistical tests (e.g., Hansen and Singleton (1982)). This paper points out that the puzzle with this model runs much deeper: the unconditional Euler equation errors for the standard consumption-based model cannot be driven to zero–indeed they remain economically large–for *any* value of risk aversion or the subjective rate of time-preference.

The empirical failure of the standard consumption-based model (including its rejection in GMM tests of the model's Euler equations) has driven the search for new consumption-based models. Many of these theories have delivered important insights into financial market behavior. We show here, however, that none explain why the standard model is so soundly rejected in basic GMM tests of its Euler equations. If the data on asset returns and consumption were generated by any of the leading models considered in the previous section, an econometrician would estimate zero Euler equation errors and the consequence of using the wrong pricing kernel would simply be incorrect estimates of δ and γ. Assets could be priced just as well using the misspecified standard consumption-based model as they could using anyone of the newer models. This is true both for explaining the behavior of the market return and risk-free rate generated by the models' own baseline calibrations, and for explaining larger cross-sections of risky returns. Moreover, some leading models imply that the standard consumption-based has negligible asset pricing errors even when it is based both on the wrong consumption measure (aggregate consumption instead of individual assetholder consumption) and on the wrong model of underlying preferences (CRRA instead of habit or recursive preferences).

We show that the leading asset pricing models we study fail to explain the mispricing of the standard model because they fundamentally mischaracterize the joint behavior of consumption and asset returns in periods of significant economic change, namely in economic downturns, when aggregate consumption is falling. An important question is why the models we study perform so poorly in recessions relative to other times.

We suggested one specific direction along which the current models can be improved, based on a time-varying, state-dependent correlation between stockholder and aggregate consumption growth. But our preliminary analysis leaves room for much future work. Ultimately it will be important to model the primitive technological sources of any state-dependent correlation between the consumption of stockholders and that of the rest of the economy. The theoretical results also raise tantalizing empirical questions. Is there any direct evidence that the correlation of stockholder and non-stockholder consumption is state-dependent? If so, can this time-variation be linked to asset returns and cyclical variation in the economy? Unfortunately, these questions are difficult to answer because of the dearth of time-series data on household consumption.

A number of alternative research directions could prove fruitful for explaining the misspricing of the standard consumption-based model. Possibilities include classes of economic models with endogenously distorted beliefs, as surveyed in the work of Hansen and Sargent (2000) or illustrated in the learning model of Cogley and Sargent (2004). In such models, beliefs are distorted away from what a model of rational expectations would impose, so asset return volatility can be driven by fluctuations in beliefs not necessarily highly correlated with consumption. Other candidates include any modifications to the standard model that would make unconditional Euler equations more difficult to satisfy, especially in recessions, such as binding restrictions on the ability to trade and smooth consumption, short-sales constraints, and transactions costs (e.g., Luttmer (1996); He and Modest (1995); Heaton and

Lucas (1996, 1997); Ludvigson (1999); Guo (2004)) or infrequent adjustment in consumption (Gabaix and Laibson (2002); Jagannathan and Wang (2005)). An important area for future research will be to determine whether such modifications are capable of delivering the empirical facts, once introduced into plausibly calibrated economic models with empirically credible frictions.

5 Appendix

5.1 Data Description

This appendix describes the data. The sources and description of each data series we use are listed below.

CONSUMPTION
Consumption is measured in per capita terms as expenditures on nondurables and services, excluding shoes and clothing. The quarterly data are seasonally adjusted at annual rates, in billions of chain- weighted 1996 dollars. The components are chain-weighted together, and this series is scaled up so that the sample mean matches the sample mean of total personal consumption expenditures. We exclude shoes and clothing expenditure from this series since they are partly durable and are therefore inappropriate in a measure of the service flow of consumption. Our source is the U.S. Department of Commerce, Bureau of Economic Analysis.

POPULATION
A measure of population is created by dividing real total disposable income by real per capita disposable income. Our source is the Bureau of Economic Analysis.

PRICE DEFLATOR
Real asset returns are deflated by the implicit chain-type price deflator (1996=100) given for the consumption measure described above. Our source is the U.S. Department of Commerce, Bureau of Economic Analysis.

ASSET RETURNS

- Three-Month Treasury Bill Rate: secondary market, averages of business days, discount basis%; Source: H.15 Release – Federal Reserve Board of Governors.

- Six size/book-market returns: Six portfolios, monthly returns from July 1926-December 2003. The portfolios, which are constructed at the end of each June, are the intersections of 2 portfolios formed on size (market equity, ME) and 3 portfolios formed on the ratio of book equity to market equity (BE/ME). The size breakpoint for year t is the median NYSE market equity at the end of June of year t. BE/ME for June of year t is the book equity for the last fiscal year end in t-1 divided by ME for December of t-1. The BE/ME breakpoints are the 30th and 70th NYSE percentiles. Source: Kenneth French's homepage, http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

- The stock market return is the Center for Research and Security Prices (CRSP) value-weighted stock market return. Our source is the Center for Research in Security Prices.

5.2 Detailed Description of Models

The utility function in the CC and MSV models take the form

$$U = E\left\{\sum_{t=0}^{\infty}\delta^t\frac{\left(C_t^i - X_t^i\right)^{1-\gamma} - 1}{1-\gamma}\right\}, \qquad \gamma > 0 \tag{22}$$

where C_t^i is individual consumption and X_t is habit level which they assume to be a function of aggregate consumption, and δ is the subjective discount factor. In equilibrium, identical agents choose the same level of consumption, so C_t^i is equal to aggregate consumption, C_t. CC define the surplus consumption ratio

$$S_t \equiv \frac{C_t - X_t}{C_t} < 1,$$

and model its log process as evolving according to a heteroskedastic first-order autoregressive process (where as before lowercase letters denote log variables):

$$s_{t+1} = (1-\phi)\,\overline{s} + \phi s_t + \lambda\,(s_t)\,(c_{t+1} - c_t - g)\,,$$

where ϕ, g, and $\overline{s}$ are parameters. $\lambda\,(s_t)$ is the so-called sensitivity function that CC choose to satisfy three conditions: (1) the risk-free rate is constant, (2) habit is predetermined at steady state, and (3) habit moves nonnegatively with consumption everywhere. We refer the reader to the CC paper for the specific functional form of $\lambda\,(s_t)$. The stochastic discount factor in the CC model is given by

$$M_{t+1}^{CC} = \delta\left(\frac{C_{t+1}}{C_t}\frac{S_{t+1}}{S_t}\right)^{-\gamma}.$$

In all of the models considered here, the return on a risk-free asset whose value is known with certainty at time t is given by

$$R_{t+1}^f \equiv \left(E_t\left[M_{t+1}\right]\right)^{-1},$$

where M_{t+1} is the pricing kernel of whichever model we are considering.

MSV model the behavior of Y_t, the inverse surplus consumption ratio:

$$Y_t = \frac{1}{1-(X_t/C_t)} > 1.$$

Following Campbell and Cochrane (1999), MSV assume that Y_t follows a mean-reverting process, perfectly negatively correlated with innovations in consumption growth:

$$\Delta Y_t = k\left(\overline{Y} - Y\right) - \alpha\,(Y_t - \lambda)\,(\Delta c_t - E_{t-1}\Delta c_t)\,,$$

where $\overline{Y}$ is the long-run mean of Y and k, α, and λ are parameters, calibrated as in MSV. Here $\Delta c_t \equiv \log(C_{t+1}/C_t)$, which they assume it follows an i.i.d. process

$$\Delta c_t = \mu + \sigma v_t,$$

where v_t is a normally distributed i.i.d. shock. The stochastic discount factor in the MSV model is

$$M_{t+1}^{MSV} = \delta \left(\frac{C_{t+1}}{C_t} \frac{Y_t}{Y_{t+1}} \right)^{-\gamma}.$$

Since the MSV model is a representative agent model, we modify it in order to study the role of limited participation. Assume that asset prices are determined by the framework above, where a valid stochastic discount factor is a function of any stockholder's consumption C_t^i and stockholder's habit X_t^i. The process for stockholder consumption is the same as in MSV, described above, but now with i subscripts:

$$\Delta c_t^i = \mu_i + \sigma_i v_t^i,$$

where v_t^i is a normally distributed i.i.d. shock. Aggregate consumption is assumed to follow a separate process given by

$$\Delta c_t = \mu_c + \sigma_c v_t^c,$$

with v_t^c a normally distributed i.i.d. shock. We analyze the results over a range of cases for the correlation between v_t^i and v_t^c, and their relative volatilities σ_i/σ_c.

For the representative stockholder, we model the first difference of Y_t^i as in MSV:

$$\Delta Y_t^i = k\left(\overline{Y}^i - Y^i\right) - \alpha\left(Y_t^i - \lambda\right)\left(\Delta c_t^i - E_{t-1}\Delta c_t^i\right),$$

and compute equilibrium asset returns based on the stochastic discount factor $M_{t+1}^{MSVi} = \delta\left(C_{t+1}^i/C_t^i\right)^{-\gamma}\left(Y_t^i/Y_{t+1}^i\right)^{-\gamma}$. As before, this is straightforward using the analytical solutions provided in MSV.

Next, we compute two types of unconditional pricing errors. First, we compute the pricing errors generated from erroneously using aggregate consumption in the pricing kernel in place of assetholder consumption. That is, we compute the pricing errors that arise from using $M_{t+1}^{ch} \equiv \delta_c\left(C_{t+1}/C_t\right)^{-\gamma_c}\left(Y_t^c/Y_{t+1}^c\right)^{-\gamma_c}$ in place of M_{t+1}^{MSVi} to price assets, where δ_c and γ_c are chosen freely to fit the data, and where Y_t^c follows the process

$$\Delta Y_t^c = k\left(\overline{Y}^c - Y^c\right) - \alpha\left(Y_t^c - \lambda\right)\left(\Delta c_t - E_{t-1}\Delta c_t\right).$$

With the exception of α, all parameters are set as in MSV. The parameter α is set to keep the mean return on the aggregate wealth portfolio the same as in MSV. Thus, if $\sigma_i/\sigma_c = 2$, the value of α in MSV is divided by two.

To model multiple risky securities, MSV model the share of aggregate consumption that each asset produces,

$$s_t^j = \frac{D_t^j}{C_t} \quad \text{for } j = 1, ..., n,$$

where n represents the total number of risky financial assets paying a dividend D. MSV assume that these shares are bounded, mean-reverting and evolve according to

$$\Delta s_t^j = \phi^j \left(\overline{s}^j - s_t^j \right) + s_t^j \boldsymbol{\sigma} \left(s_i \right) \boldsymbol{\epsilon}_t,$$

where $\boldsymbol{\sigma}(s_j)$ is an $N-$dimensional row vector of volatilities and $\boldsymbol{\epsilon}_t$ is an $N-$dimensional column vector of standard normal random variables, and ϕ^j and $\overline{s}^j$ are parameters. ($N \leq n+1$ because MSV allow for other sources of income, e.g., labor income, that support consumption.) Cross-sectional variation in unconditional mean returns across risky securities in this model is governed by cross-sectional variation in the covariance between shares and aggregate consumption growth: $\text{Cov}\left(\frac{\Delta s_t^j}{s_t^j}, \frac{\Delta c_t}{c_t} \right)$, for $j = 1, ..., n$. This in turn is determined by cross-sectional variation in ϕ^j, $\overline{s}^j$ and $\boldsymbol{\sigma}(s_j)$. We create n artificial risky securities using an evenly spaced grid of values for these parameters. The values of ϕ^j lie on a grid between 0 and 1, and the values of $\overline{s}^j \in [0, 1)$ lie on a grid such that the sum over all j is unity. The parametric process for $\boldsymbol{\sigma}(s_j)$ follows the specification in MSV in which the volatilities depend on a N-dimensional vector of parameters $\boldsymbol{v}^j$ as well as the individual share processes

$$\boldsymbol{\sigma}(s_j) = \boldsymbol{v}^j - \sum_{k=0}^{n} s_t^k \boldsymbol{v}^k.$$

We choose the parameters ϕ^j, $\overline{s}^j$, and $\boldsymbol{v}^j$, to generate a spread in average returns across assets. In analogy to the empirical exercise (Panel B of Table 1), we do this for $n = 6$ risky securities plus the aggregate wealth portfolio return and the risk-free for a total of 8 asset returns.

Closed-form solutions are not available for the individual risky securities, but MSV show that equilibrium price-dividend ratios on the risky assets are given by the approximate relation

$$\frac{P_t^j}{D_t^j} \approx a_0^j + a_1^j S_t + a_2^j \frac{\overline{s}^j}{s_t^j} + a_3^j \frac{\overline{s}^j}{s_t^i} S_t, \tag{23}$$

where $S_t \equiv 1/Y_t^i$ and where Y_t^i again denotes the inverse surplus ratio of an individual assetholder indexed by i, which should not be confused with the indexation by j, which denotes a security. The parameters a_0^j, a_1^j, a_2^j, and a_3^j are all defined in terms of the other parameters above. Using these solutions for individual price-dividend ratios, we create a cross-section of equilibrium risky securities using

$$R_{t+1}^i = \left(\frac{P_{t+1}^j / D_{t+1}^j + 1}{P_t^j / D_t^j} \right) \exp\left(\Delta d_{t+1}^j \right). \tag{24}$$

Bansal and Yaron (2004) consider a representative agent who maximizes utility given by recursive preferences of Epstein and Zin (1989, 1991) and Weil (1989). The utility function to be maximized takes the form

$$U = E\left\{\sum_{t=0}^{\infty} \delta^t \left\{(1-\delta)\, C_t^{\frac{1-\gamma}{\alpha}} + \delta\left(E_t U_{t+1}^{1-\gamma}\right)^{\frac{1}{\alpha}}\right\}^{\frac{\alpha}{1-\gamma}}\right\}, \tag{25}$$

where $\alpha \equiv (1-\gamma)\,/\,(1-1/\psi)$, ψ is the intertemporal elasticity of substitution in consumption (IES), γ is the coefficient of relative risk aversion, and δ is the subjective discount factor. The stochastic discount factor under Epstein-Zin-Weil utility takes the form given in (7).

5.3 Proof of Proposition 1

For $\gamma = 0$: $E[X_t^{-\gamma} Z_t] > 0$ since $E[Z_t] > 0$ (i.e., the unconditional risk premium is positive). Given continuity (Assumption 4), the question then becomes, is there a γ so that $E[X^{-\gamma}Z] < 0$?

$$\begin{aligned} E[X_t^{-\gamma} Z_t] &= E[X_t^{-\gamma} E[Z_t|X_t]] = E[X_t^{-\gamma} f(X_t)] \\ &= P(X_t < \bar{X}) E[X_t^{-\gamma} f(X_t)|X_t < \bar{X}] + P(X_t \geq \bar{X}) E[X_t^{-\gamma} f(X_t)|X_t \geq \bar{X}] \end{aligned}$$

Since $X_t^{-\gamma}$ is decreasing in X if $\gamma > 0$, the second conditional expectation has the following property:

$$E[X_t^{-\gamma} f(X_t)|X_t > \bar{X}] < \bar{X}_t^{-\gamma} E[f(X_t)|X_t \geq \bar{X}]$$

The first conditional expectation has the following property:

$$\begin{aligned} E[X_t^{-\gamma} f(X_t)|X_t < \bar{X}] &< E[X_t^{-\gamma} \mu|X_t < \bar{X}] \\ &= \mu \bar{X}^{-\gamma} E[(X_t/\bar{X})^{-\gamma}|X_t < \bar{X}] \end{aligned}$$

Thus,

$$E[X_t^{-\gamma} Z_t] < \bar{X}^{-\gamma}\left(P(X_t < \bar{X}) \mu E[(X_t/\bar{X})^{-\gamma}|X_t < \bar{X}] + P(X_t \geq \bar{X}) E[f(X_t)|X_t \geq \bar{X}]\right)$$

As γ goes to infinity the right hand side goes to minus infinity since

$$\bar{X}^{-\gamma} \to \infty \text{ since } \bar{X} < 1$$
$$E[f(X_t)|X_t \geq \bar{X}] \text{ is finite by Assumption 3,}$$
$$\mu E[(X_t/\bar{X})^{-\gamma}|X_t < \bar{X}] \to -\infty \text{ by Assumption 2 and since } \mu < 0$$

Q.E.D.

5.4 A Simple Limited Participation/Incomplete Markets Model with Joint Lognormality: Multiple Risky Assets Case

This appendix discusses the distortion in parameter values and Euler equation errors that arise in the simple lognormal model with multiple risky asset returns. In this case, lognormality does not necessarily imply that such Euler equation errors will each be identically zero, since in this case there are more moment conditions than free parameters. For $N > 2$ asset returns, it is not possible to give a intuitively appealing analytical expression for the distortion in parameter values, although values can be obtained numerically. It is, however, possible to illustrate analytically the distortion in γ_c to a very close approximation, by focusing on log pricing errors and assuming that the risk-free rate is constant. In this case we can choose δ_c so that $E\left[M^i\right] = E\left[M^c\right]$, which insures that the pricing error for the risk-free rate is zero. Note that this does not imply that the risk-free rate puzzle is trivial, since δ_c is unrestricted and in particular can be chosen to be greater than unity if required to set the pricing error to zero. While this is an approximation, it turns out to be well satisfied in the data, since the Treasury-bill rate is extremely stable.[24] We maintain this approximation purely for expositional purposes; the reader should be aware that exact results are very close.

With this approximation in hand, the value of γ_c that minimizes the sum of squared log errors, $\log\left(1 + e_R^j\right)$, is given by

$$\widehat{\gamma}_c = \gamma_i \left(\frac{\sum_j \sigma_{cj}\sigma_{ij}}{\sum_j \sigma_{cj}^2} \right), \tag{26}$$

where $\sigma_{cj} \equiv \text{Cov}(\Delta c, r^j)$, $\sigma_{ij} \equiv \text{Cov}(\Delta c^i, r^j)$, and "$\sum_j$" indicates summation over all asset returns j being priced. "Hats" indicate parameter values estimated by minimizing the sum of squared Euler equation errors (using GMM with the identity weighting matrix), as above. A more complicated expression can be obtained for $\widehat{\delta}_c$. Note that the estimates of δ_c and γ_c are biased, and do not correspond to any marginal investor's true risk aversion parameter.

Figure A1 shows that a lognormal model is unlikely to match the magnitude of the Euler equation errors found in the data. As above, the "data" line plots RMSE/RMSR over a range of values for γ_c, after choosing δ_c so as to minimize the sum of squared Euler equation errors $e_{R,t}^j$, which do not impose lognormality. The line labeled "lognormality" plots the RMSE/RMSR over a range of values for γ_c, after choosing δ_c to minimize the sum of squared pricing errors in (16), under the assumption that returns and consumption

[24] If M_t^i is the true pricing kernel, then $E\left[M_t^i\right] = E\left[1/R_t^f\right]$. Since we assume $E\left[M_t^i\right] = E\left[M_t^c\right]$, our assumption implies $E\left[M_t^c\right] = E\left[1/R_t^f\right]$, which prices the risk-free rate exactly if R_t^f is constant. It follows that the approximation error in pricing the risk-free rate is $E\left[1/R_t^f\right] - 1/E\left[R_t^f\right]$, which is -0.01 percent per annum.

growth are jointly lognormal. One way to interpret the "lognormal" line is to note that, under joint lognormality, we can always find a pricing kernel $M_{t+1}^i = \exp\{\log(\delta) - \gamma \Delta c_{t+1}^i\}$ that generates a set of log returns taking the form $r_t^j = \theta^j \Delta c_t^i + \eta_t^j$, for some constant θ^j and i.i.d. innovation η_t^j, that have the same means, variances and covariances with Δc_t as those in the historical data, and prices those asset exactly. This is done by choosing θ^j to match the mean excess return for each asset, choosing $\text{var}(\eta^j)$ to match the volatility of each return, and choosing $\text{cov}(\eta, \varepsilon^i)$ to match the $\text{cov}(r^j, \Delta c)$ from the data. The dashed line labeled "lognormality" then gives the pricing errors that would arise from fitting M_{t+1}^c to data generated from this lognormal model. Figure A1 shows that no lognormal model can explain the magnitude of the pricing errors in the data.

5.5 Hermite Expansions Around the Normal Density

Gallant and Tauchen (1989) show that the Hermite expansion can be put in tractable form by specifying the density as

$$h(y) = \frac{a(y)^2 f(y)}{\int\int\int a(u)^2 f(u)\, du_1 du_2 du_3}.$$

Here, $a(y)$ is the sum of polynomial basis functions of the variables in y; it is squared to insure positivity and divided by the integral over $\mathbb{R}^3$ to insure the density integrates to unity. We set $a(y)^2 = (a_0 + a_1 y_{1,t} + a_2 y_{2,t} + a_3 y_{3,t})^2$, a first-order expansion but one that can nonetheless accommodate quite significant departures from normality. We investigate a large number of possible joint distributions by varying the parameters a_0,...,a_3. When $a_0 = 1$ and $a_1 = a_2 = a_3 = 0$, $h(y)$ collapses to the Gaussian joint distribution, $f(y)$.

Under the assumptions above, the equilibrium price-dividend ratio is a constant, P/D, that satisfies

$$\frac{P/D}{P/D+1} = \int\int \delta^i \exp\left(-\gamma^i y_2\right) \exp(y_3)\, h(y_2, y_3)\, dy_2 dy_3.$$

5.5.1 Expansions Around Normality

We employ first-order Hermite expansions around the multivariate normal distribution, and consider the Euler equation errors associated with two assets, a stock market return and a risk-free rate. Let $y_t = (\Delta c_t, \Delta c_t^i, \Delta d_t)' \equiv (y_{1,t}, y_{2,t}, y_{3,t})'$, where Δc_t is aggregate consumption growth, Δc_t^i is individual asset-holder consumption growth, and Δd_t is dividend growth of an aggregate stock market claim. We will consider asset pricing models in which these variables are i.i.d., but not necessarily jointly lognormally distributed.

Ideally, the unconditional joint density of y_t would be estimated. Unfortunately, this density must be calibrated because a lack of sufficiently long time-series data on stockholder consumption prohibits estimation. Let the joint density of y_t be denoted $h(y)$.

The MRS of individual assetholder consumption, $M^i_{t+1} \equiv \delta \left(C^i_{t+1}/C^i_t\right)^{-\gamma}$, is a valid stochastic discount factor. Under the assumptions above, the equilibrium price-dividend ratio is a constant, P/D. Given a distribution $h(y)$ and the equilibrium value for P/D, it is straightforward to compute the pricing errors associated with erroneously using $M^c_{t+1} \equiv \delta_c \left(C_{t+1}/C_t\right)^{-\gamma_c}$ to price assets. As above, we assume the asset return data are generated by M^i_{t+1} and solve numerically for the values of δ_c and γ_c that minimize an equally-weighted sum of squared pricing errors e^j_R that arise from using M^c_{t+1} to price assets.

Parameters of the leading normal density are calibrated to match data on aggregate consumption growth and dividend growth for the CRSP value-weighted stock market index, on an annual basis.[25] The parameters for asset-holder consumption and assetholder preferences are somewhat arbitrary since there is insufficient data available to measure these empirically. We therefore consider a range for γ, δ, σ_i/σ_c, μ_i/μ_c, ρ_{ci}, and ρ_{id}, where $\mu_i \equiv E\left(\Delta c^i_t\right)$, $\mu_c \equiv E\left(\Delta c_t\right)$, $\sigma_i \equiv \sqrt{\mathrm{Var}\left(\Delta c^i_t\right)}$, $\sigma_c \equiv \sqrt{\mathrm{Var}\left(\Delta c_t\right)}$, and $\rho_{id} \equiv$Cov$(\Delta c^i_t, \Delta c_t)$ are parameters of $f(y)$. Because our calibration corresponds to an annual frequency, the Euler equation errors we compute are comparable to the annualized errors from U.S. data reported in Table 1.

We evaluated pricing errors obtained from a wide grid (over 20,000 parameter combinations) for the Hermite parameters a_0 through a_3. To conserve space, we report a limited number of results. Table A1 reports results for which γ is set to 5, δ to 0.99, $\sigma_i/\sigma_c = 1, 2, 4$, $\mu_i/\mu_c = 0.85, 1.5$, $\rho_{ci} = 0.1$, $\rho_{id} = 0.9$. The point of this table is that there are a wide range of cases in which the joint distribution of y_t deviates considerably from normality (often producing bimodal marginal density shapes) and yet the pricing errors associated with erroneously using M^c_{t+1} to price assets in place of M^i_{t+1} are, to numerical accuracy, zero. For example, the kurtosis of the marginal distribution of Δc_t is often greater than 11, and the skewness greater than 4, but still the Euler equation errors from using a representative agent pricing kernel are zero. The parameter estimates are biased, however, echoing the lognormal results. The parameter γ_c is larger than the true γ when asset-holder consumption growth is more volatile than aggregate consumption growth or when it is not highly correlated with it, as suggested by (21). When Cov$(\Delta c, \Delta d) = \sigma_{cd}$ is negative, γ_c is negative, as also suggested

[25] From annual post-war data used in Lettau and Ludvigson (2005), we take the $E(\Delta c)$ to be 2% annually and $E(\Delta d)$ to be 4% annually; the standard deviation Δc is $\sigma_c = 1.14\%$ and the standard deviation of Δd is $\sigma_d = 12.2\%$. The covariance σ_{cd} between Δc and Δd is notoriously hard to measure. It is estimated to be negative, equal to -0.000177 in the annual post-war data used by Lettau and Ludvigson (2005), but others have estimated a positive correlation (e.g., Campbell (2003)). We therefore consider both small negative values for this covariance (equal to the point estimate from Lettau and Ludvigson (2005)), and small positive values of the same order of magnitude, e.g., 0.000177.

by (21).

We reach similar conclusions when evaluating the Euler equation errors for a larger cross-section of returns. These results have been omitted to conserve space, but can be summarized as follows. As in the two-asset case, we find that the average pricing errors from using M_{t+1}^c to price assets are often very small, indeed close to zero, even for significant perturbations from joint lognormality. A small number of cases provided larger pricing errors, but these cases were relatively rare, occurring in less than 0.2% of the parameter permutations.

5.6 Additional Diagnostics

5.6.1 Time Aggregated Consumption

To explore how time aggregation of aggregate data is likely to affect our results, we assume that agents make decisions quarterly but that the data sampling interval is annual. We also allow for the possibility that aggregate consumption is a misspecified measure of assetholder consumption. For all models the essential results for the Euler equation errors remain: values of δ_c and γ_c can always be found such that the unconditional pricing errors associated with using M_{t+1}^c to price assets are very small relative to the data, even when using time-averaged data. As one example, Table A.1 shows results for the MSV model with limited participation. To conserve space, we report only the results for this model, since the conclusion is unchanged for the other models, although note that the results above for the BY model are already based on time-aggregate data. The table shows that the pricing errors are again small, even when data is time-aggregated. Most values of RMSE/RMSR are close to zero. The largest occurs for the eight asset case and is equal to 0.07, far smaller than the value of 0.33 found in the data, which happens only if we assume stockholder consumption growth is negatively correlated with aggregate consumption growth. Since time-averaging changes both the serial dependence of the consumption data and its unconditional correlation with returns, this suggests that the exact time-series properties of consumption growth are not crucial for explaining the large pricing errors of the standard model.

5.6.2 Finite Sample Pricing Errors

To investigate how finite sample considerations are likely to affect our conclusions, we redo the simulation exercises reported on above using samples of the size employed in our empirical application. Table A.2 reports the *maximum* RMSE/RMSR over 1,000 samples of size 204 that arises from fitting M_{t+1}^c to data generated from the relevant model. We do not report small-sample results for the eight-asset MSV model. The small sample behavior of the MSV model is problematic because the model is solved in continuous time and moreover holds only as an approximation for multiple risky securities. As a result, we find that small amounts of approximation error are compounded by discretization error in small samples and it is not possible to reduce these errors to reasonable levels unless the number of decisions within the period is almost infinite. Nevertheless, we are able to report the results for the two-asset case, since the solutions for the aggregate consumption claim and risk-free rate in the MSV model are not approximate. Table A.2 shows that, for the three representative agent models, CC, MSV, and BY, the maximum Euler equation errors that arise from fitting M_{t+1}^c to data are numerically zero, for both the two-asset and eight-asset specifications. The Guvenen model produces a slightly higher maximum RMSE/RMSR in finite samples, equal to about 0.87% at an annual rate, but still well below the value of almost 50% found in historical data (Table 1).

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Table 1: Euler Equation Errors with CRRA Preferences

Assets	$\widehat{\delta}$	$\widehat{\gamma}$	RMSE (in %)	RMSE/RMSR	p $(W = I)$	p $(W = S^{-1})$
R^s, R^f	1.41	89.78	2.71	0.48	N/A	N/A
R^s, R^f, 6 FF	1.39	87.18	3.05	0.33	0.00	0.00
Excluding Periods with low Consumption Growth						
R^s, R^f	2.55	326.11	0.73	0.13	N/A	N/A
R^s, R^f, 6 FF	2.58	356.07	1.94	0.21	0.00	0.00

Notes: This table reports the minimized annualized postwar data Euler Equation errors for CRRA preferences. The preference parameters $\widehat{\delta}_c$ and $\widehat{\gamma}_c$ are chosen to minimize the mean square pricing error for different sets of returns: $min_{\delta_c,\gamma_c}\left[g(\delta_c,\gamma_c)'Wg(\delta_c,\gamma_c)\right]$ where $g(\delta_c,\gamma_c) = E[\delta_c(C_t/C_{t-1})^{-\gamma_c}\boldsymbol{R}_t - 1]$. R^s is the CRSP-VW stock returns, R^f is the 3-month T-bill rate and C_t is real per-capita consumption of nondurables and services excluding shoes and clothing. The table also reports results when the periods with the lowest six consumption growth rates are eliminated. The table reports estimated $\widehat{\delta}, \widehat{\gamma}$ and the minimized value of RMSR/RMSRR where RMSE is the square root of the average squared Euler Equation error and RMSR is the square root of the averaged squared returns of the assets under consideration for $W = I$. The last two colums report χ^2 p-values for tests for the null hypothesis that Euler Equation errors are jointly zero for $W = I$ and $W = S^{-1}$ where S is the spectral density matrix at frequency zero. The data span the period 1951Q4 to 2002Q4.

Table 2: Low Consumption Growth Periods

Quarter	NBER Recession Dates	$C_t/C_{t-1}-1$	R_t^s	R_t^f
1980Q02	80Q1-80Q3	-1.28	16.08	3.59
1990Q04	90Q3-91Q1	-0.87	8.75	2.16
1974Q01	73Q4-75Q1	-0.85	-1.26	2.37
1958Q01	57Q3-58Q2	-0.84	7.03	0.65
1960Q03	60Q2-61Q1	-0.64	-4.93	0.67
1953Q04	53Q1-54Q2	-0.60	7.87	0.47

Notes: This table reports consumption growth, the return of the CRSP-VW stock returns R^s and the 3-month T-bill rate R^f (all in in percent per quarter) in the six quarters of our sample with the lowest consumption growth rates. The consumption measure is real per-capita expenditures on nondurables and services excluding shoes and clothing. The data span the period 1951Q4 to 2002Q4.

Table 3: Euler Equation Errors

Model	$\widehat{\delta}_c$	$\widehat{\gamma}_c$	RMSE/RMSR (R^s, R^f)	RMSE/RMSR (8 assets)
Data			0.48	0.33
CC Habit	1.28	57.48	0.00	N/A
MSV Habit	1.71	30.64	0.00	0.00
BY LR Risk	0.93	48.97	0.00	0.00

Notes: This table reports the annualized Euler Equation errors for stock returns R^s and the riskfree rate R^f from simulated data from Campbell and Cochrane's habit model (CC Habit), Menzly, Santos and Veronesi's habit model (MSV Habit) and Bansal and Yaron's long run risk model (BY LR Risk) for CRRA preferences. The preference parameters $\widehat{\delta}_c$ and $\widehat{\gamma}_c$ are chosen to minimize the mean square Euler Equation error: $min_{\delta_c,\gamma_c}\left[g(\delta_c,\gamma_c)'g(\delta_c,\gamma_c)\right]$ where $g(\delta_c,\gamma_c) = E[\delta_c(C_t/C_{t-1})^{-\gamma_c}\boldsymbol{R}_t - 1]$. RMSR is the square root of the averaged squared returns of the assets under consideration. RMSE is the square root of the average squared Euler Equation error. Euler Equation errors are computed from simulations with 10,000 observations.

Table 4: Properties of Guvenen's Model

Panel A: Consumption Growth

	$C_t/C_{t-1}-1$	C_t^i/C_{t-1}^i-1	C_t^n/C_{t-1}^n-1	R_t^s	R_t^f
Mean	0.01	0.02	0.00	1.31	0.64
Std. Dev.	2.04	4.53	0.83	7.30	1.69
Correlation	1.00	1.00	0.99	1.00	0.17
	1.00	1.00	0.98	0.99	0.17
	0.99	0.98	1.00	0.99	0.16
	1.00	0.99	0.99	1.00	0.19
	0.17	0.17	0.16	0.19	1.00

Panel B: Stochastic Discount Factors

	$M_t^i(0.99, 2.00)$	$M_t^c(0.99, 2.00)$	$M_t^c(0.99, 4.49)$
Mean	0.99	0.99	0.99
Std. Dev.	0.09	0.04	0.09
Correlation	1.00	1.00	1.00
	1.00	1.00	1.00
	1.00	1.00	1.00

Panel C: Euler Equation Errors

Consumption	(δ, γ)	$E[M_t(\delta, \gamma)\ R_t^s - 1]$	$E[M_t(\delta, \gamma)\ R_t^f - 1]$
SH	(0.99, 2.00)	0.02%	0.02%
AC	(0.99, 2.00)	0.39%	-0.34%
AC	(0.99, 4.49)	0.00%	0.01%

Notes: This table reports properties of Guvenen's model. Panel A reports the properties of consumption growth rates of aggregate consumption C_t/C_{t-1}, stockholders consumption C_t^i/C_{t-1}^i, nonstockholders consumption C_t^n/C_{t-1}^n, stock returns R_t^s and the riskfree rate R_t^f in Guvenen's model. Panel B reports properties of stochastic discount factors. The first row reports properties of the SDF for stockholders consumption. The remaining rows report SDF properties for total consumption and different preference parameters. The stochastic discount factors are of the CRRA form $M_t = \delta(C_t/C_{t-1})^{-\gamma}$. The first parameter in parenthesis is δ, the second one is γ. Panel C reports the annual Euler Equation error Guvenen's model. The preference parameters δ and γ are chosen to minimize the equally weighted sum of Euler Equation errors for the stock returns R^s and the riskfree rate R^f. The first row labelled "SH" reports the Euler Equation errors for stockholders consumption. The remaining rows labelled "AC" report Euler Equation errors for aggregate consumption and different preference parameters. All statistics are quarterly.

Table 5: Properties of a Limited Participation Habit Model

σ_i/σ_c	$\rho(C_t^i/C_{t-1}^i, C_t/C_{t-1})$					
	-1.0	-0.5	-0.25	0.25	0.5	1.0
			2 Assets: R^s, R^f			
			$\widehat{\delta}_c$			
1	0.51	0.24	0.03	5.27	2.69	1.61
2	0.52	0.24	0.03	5.20	2.75	1.83
5	0.48	0.23	0.03	4.94	2.81	1.79
			$\widehat{\gamma}_c$			
1	-30.71	-60.15	-128.80	127.03	58.59	27.93
2	-29.22	-61.24	-132.02	117.99	61.69	33.28
5	-33.48	-64.30	-131.01	117.94	64.43	32.56
			RMSE/RMSR			
1	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.00	0.00	0.00
			8 Assets			
			$\widehat{\delta}_c$			
1	0.50	0.24	0.04	5.44	2.76	1.74
2	0.50	0.23	0.04	5.60	2.80	1.74
5	0.48	0.21	0.03	5.74	2.94	1.85
			$\widehat{\gamma}_c$			
1	-30.83	-61.99	-123.23	124.24	61.51	31.21
2	-31.69	-62.76	-124.21	126.92	62.34	31.22
5	-33.73	-67.43	-134.53	133.41	65.50	34.11
			RMSE/RMSR			
1	0.03	0.03	0.03	0.03	0.04	0.03
2	0.04	0.03	0.03	0.03	0.03	0.03
5	0.03	0.03	0.04	0.03	0.03	0.04

Notes: This table reports preference parameters and Euler Equation errors in Menzly, Santos and Veronesi's (2004) habit model. Consumption growth of stockholders is assumed to follow a random walk with a mean of 2% and standard deviation of 1%. All parameters are as in Menzly, Santos and Veronesi except α, which is set obtain the same average stock return as in Menzly-Santos-Veronesi. σ_i and σ_c are the standard deviations of stockholder's and aggregate consumption growth, respectively, and $\rho(C_t^i/C_{t-1}^i, C_t/C_{t-1})$ is their correlation. The preference parameters $\widehat{\delta}_c$ and $\widehat{\gamma}_c$ are chosen to minimize the mean square Euler Equation error $min_{\delta_c,\gamma_c}\left[g(\delta_c,\gamma_c)'Wg(\delta_c,\gamma_c)\right]$ where $g(\delta_c,\gamma_c) = E[M_t^c \boldsymbol{R}_t - 1], M_t^c = \delta_c(\frac{C_t}{C_{t-1}})^{-\gamma_c}$. C_t is aggregate consumption, R^s is the return of equity, R^f is the riskfree rate, and $W = I$. $\boldsymbol{R}$ includes the return of the market R^s, the riskfree rate R^f and the returns of six individual assets. RMSR is the square root of the averaged squared returns of the assets under consideration. RMSE is the square root of the average squared Euler Equation error. The weighting matrix W is the identity matrix.

Table 6: Limited Participation CRRA Model and State-Dependent Correlation Estimated with Aggregate Consumption CRRA SDF

σ_i/σ_c	$\rho^-(C_t^i/C_{t-1}^i, C_t/C_{t-1})$					
	-1.0	-0.5	-0.25	0.25	0.5	1.0
			2 Assets: R^s, R^f			
			$\widehat{\delta}_c$			
1	1.39	2.27	3.33	2.16	1.67	1.26
2	2.11	3.04	4.27	6.67	3.75	2.19
5	4.42	4.30	5.15	0.00	2.55	5.15
			$\widehat{\gamma}_c$			
1	19.30	47.77	72.82	44.27	29.26	14.07
2	47.37	68.58	90.13	162.03	83.06	45.21
5	107.78	93.29	95.69	142.34	193.52	101.68
			RMSE/RMSR			
1	0.55	0.43	0.27	0.00	0.00	0.00
2	0.53	0.47	0.40	0.00	0.00	0.00
5	0.33	0.41	0.41	0.00	0.00	0.00
			8 Assets			
			$\widehat{\delta}_c$			
1	1.33	2.22	3.23	2.19	1.71	1.30
2	2.00	2.81	3.75	6.95	3.80	2.25
5	3.86	3.10	3.17	5.12	6.69	4.79
			$\widehat{\gamma}_c$			
1	19.30	47.77	72.82	44.27	29.26	14.07
2	47.37	68.58	90.13	162.03	83.06	45.21
5	107.78	93.29	95.69	142.34	193.52	101.68
			RMSE/RMSR			
1	0.31	0.25	0.16	0.00	0.00	0.00
2	0.29	0.26	0.22	0.01	0.00	0.00
5	0.17	0.22	0.22	0.19	0.12	0.02

Notes: This table reports preference parameters and Euler Equation errors in a CRRA model with state-dependent correlation of stockholder's and aggregate consumption growth rates. Aggregate consumption growth is assumed to follow a random walk with a mean of 2% and standard deviation σ_c of 1% (annually). The standard deviation of stockholders is σ_i. Aggregate consumption growth and stockholders consumption growth is perfectly correlation unless aggregate consumption growth is more than one standard deviation below its mean. In such periods, the correlation is $\rho^-(C_t^i/C_{t-1}^i, C_t/C_{t-1})$. Risk aversion of stockholders is 10 and their time discount factor is 1.2. Equity is modelled as levered claims to stockholders consumption. The Euler eqution is estimated using aggregate consumption growth. The preference parameters $\widehat{\delta}_c$ and $\widehat{\gamma}_c$ are chosen to minimize the mean square Euler Equation error $min_{\delta_c,\gamma_c}[g(\delta_c,\gamma_c)'Wg(\delta_c,\gamma_c)]$ where $g(\delta_c,\gamma_c) = E[M_t^c \boldsymbol{R}_t - 1], M_t^c = \delta_c(\frac{C_t}{C_{t-1}})^{-\gamma_c}$. C_t is aggregate consumption, R^s is the return of equity, R^f is the riskfree rate, and $W = I$. $\boldsymbol{R}$ includes the return of the market R^s, the riskfree rate R^f and the returns of six individual assets. RMSR is the square root of the averaged squared returns of the assets under consideration. RMSE is the square root of the average squared Euler Equation error. The weighting matrix W is the identity matrix.

Table A.1: Lim. Partic./Inc. Markets Euler Equation Errors for Stock Return and Risk-Free Rate: Hermite Densities

$Cov(\Delta c,\Delta d)=0.00017$

γ	δ	$\rho(\Delta c,\Delta c^i)$	$\rho(\Delta c^i,\Delta d)$	$\sigma(i)/\sigma(c)$	$\mu(\Delta c^i)/\mu(\Delta c)$	γ_c	δ_c	e(Rs)	e(Rf)	Sk[c]	Ku[c]	Sk[i]	Ku[i]	Sk[d]	Ku[d]
5	0.99	0.1	0.9	1	0.85	36.211	2.5613	4.95E-10	4.90E-10	4.0917	11.195	-0.0042337	3	0.036111	3.0009
5	0.99	0.1	0.9	1	1.5	36.217	2.3988	2.12E-10	2.10E-10	4.0899	11.181	-0.004228	3	0.036063	3.0009
5	0.99	0.1	0.9	2	0.85	71.495	6.0675	1.14E-09	1.12E-09	4.0952	11.207	0.0078509	3	0.04699	3.0015
5	0.99	0.1	0.9	2	1.5	71.53	5.6869	1.50E-09	1.49E-09	4.0934	11.193	0.0078403	3	0.046927	3.0015
5	0.99	0.1	0.9	4	0.85	129.08	14.235	9.75E-08	9.67E-08	4.1018	11.229	0.032021	3.0007	0.068751	3.0032
5	0.99	0.1	0.9	4	1.5	129.22	13.395	-9.01E-08	-8.50E-08	4.1	11.215	0.031977	3.0007	0.068658	3.0031

$Cov(\Delta c,\Delta d)=-0.00017$

γ	δ	$\rho(\Delta c,\Delta c^i)$	$\rho(\Delta c^i,\Delta d)$	$\sigma(i)/\sigma(c)$	$\mu(\Delta c^i)/\mu(\Delta c)$	γ_c	δ_c	e(Rs)	e(Rf)	Sk[c]	Ku[c]	Sk[i]	Ku[i]	Sk[d]	Ku[d]
5	0.99	0.1	0.9	1	0.85	-64.06	0.1052	-1.41E-14	-1.40E-14	4.1037	11.313	-0.0041596	3	-0.66437	3.2899
5	0.99	0.1	0.9	1	1.5	-64.05	0.0987	-1.27E-08	-1.27E-08	4.1034	11.303	-0.0041543	3	-0.66356	3.2891
5	0.99	0.1	0.9	2	0.85	-118.7	0.0121	-6.34E-14	-6.22E-14	4.1071	11.324	0.0077134	3	-0.65304	3.2802
5	0.99	0.1	0.9	2	1.5	-118.6	0.0113	-8.05E-11	-8.07E-11	4.1067	11.314	0.0077036	3	-0.65225	3.2795
5	0.99	0.1	0.9	4	0.85	-210.4	0.0002	-6.58E-13	-6.60E-13	4.1134	11.346	0.031459	3.0007	-0.63043	3.2614
5	0.99	0.1	0.9	4	1.5	-210.2	0.0001	-2.31E-13	-2.31E-13	4.1131	11.336	0.03142	3.0007	-0.62966	3.2607

$Cov(\Delta c,\Delta d)=0.00017$

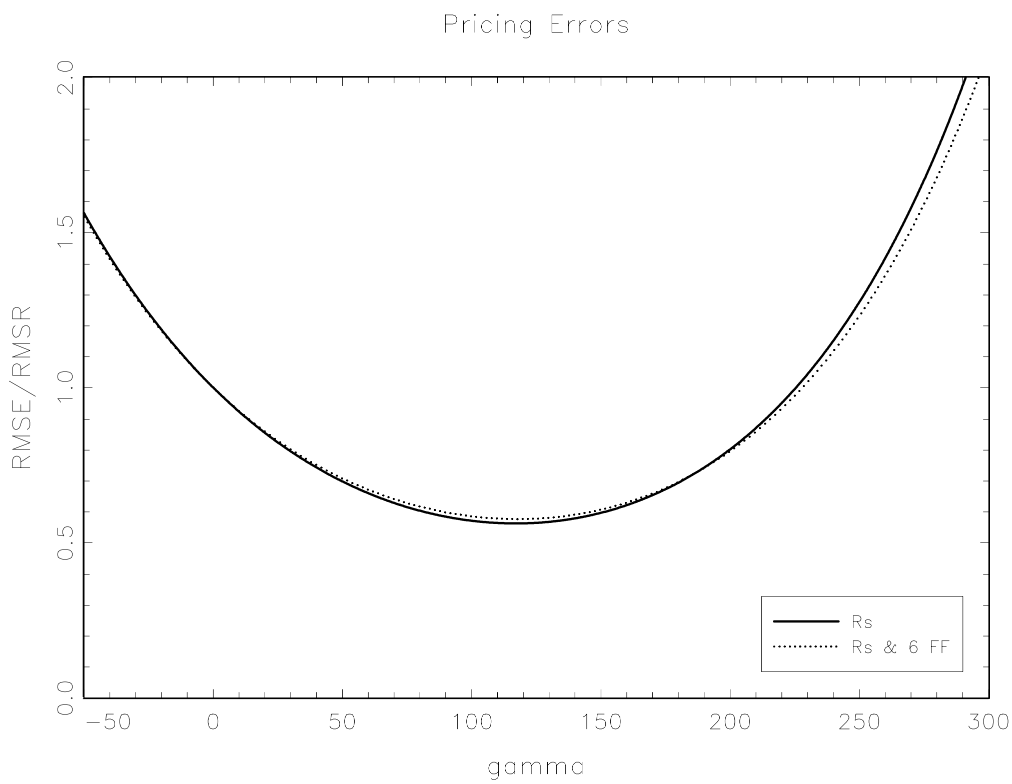
γ	δ	$\rho(\Delta c,\Delta c^i)$	$\rho(\Delta c^i,\Delta d)$	$\sigma(i)/\sigma(c)$	$\mu(\Delta c^i)/\mu(\Delta c)$	γ_c	δ_c	e(Rs)	e(Rf)	Sk[c]	Ku[c]	Sk[i]	Ku[i]	Sk[d]	Ku[d]
5	0.99	0.1	0.9	1	0.85	35.488	1.7114	-7.93E-09	-8.00E-09	0.2175	3.0307	0.49692	2.4804	0.46153	2.2946
5	0.99	0.1	0.9	1	1.5	35.488	1.6032	-7.94E-09	-7.99E-09	0.2175	3.0307	0.49691	2.4804	0.46153	2.2946
5	0.99	0.1	0.9	2	0.85	70.978	2.7445	9.82E-09	9.64E-09	0.2175	3.0307	0.49692	2.4804	0.46154	2.2946
5	0.99	0.1	0.9	2	1.5	70.978	2.571	9.82E-09	9.64E-09	0.2175	3.0307	0.49691	2.4804	0.46153	2.2946
5	0.99	0.1	0.9	4	0.85	141.96	4.3612	2.26E-07	2.25E-07	0.2175	3.0307	0.49692	2.4804	0.46154	2.2946
5	0.99	0.1	0.9	4	1.5	141.96	4.0855	2.26E-07	2.25E-07	0.2175	3.0307	0.49692	2.4804	0.46153	2.2946

$Cov(\Delta c,\Delta d)=-0.00017$

γ	δ	$\rho(\Delta c,\Delta c^i)$	$\rho(\Delta c^i,\Delta d)$	$\sigma(i)/\sigma(c)$	$\mu(\Delta c^i)/\mu(\Delta c)$	γ_c	δ_c	e(Rs)	e(Rf)	Sk[c]	Ku[c]	Sk[i]	Ku[i]	Sk[d]	Ku[d]
5	0.99	0.1	0.9	1	0.85	-35.46	0.4115	-4.87E-08	-4.88E-08	-0.218	3.0308	0.49691	2.4804	0.46152	2.2946
5	0.99	0.1	0.9	1	1.5	-35.46	0.3855	-4.88E-08	-4.88E-08	-0.218	3.0308	0.4969	2.4804	0.46151	2.2946
5	0.99	0.1	0.9	2	0.85	-70.92	0.1587	4.66E-15	4.66E-15	-0.218	3.0308	0.49691	2.4804	0.46152	2.2946
5	0.99	0.1	0.9	2	1.5	-70.92	0.1487	4.22E-15	5.33E-15	-0.218	3.0308	0.4969	2.4804	0.46151	2.2946
5	0.99	0.1	0.9	4	0.85	-141.8	0.0146	1.21E-13	1.22E-13	-0.218	3.0308	0.49692	2.4805	0.46153	2.2946
5	0.99	0.1	0.9	4	1.5	-141.8	0.0137	1.15E-13	1.18E-13	-0.218	3.0308	0.49691	2.4804	0.46152	2.2946

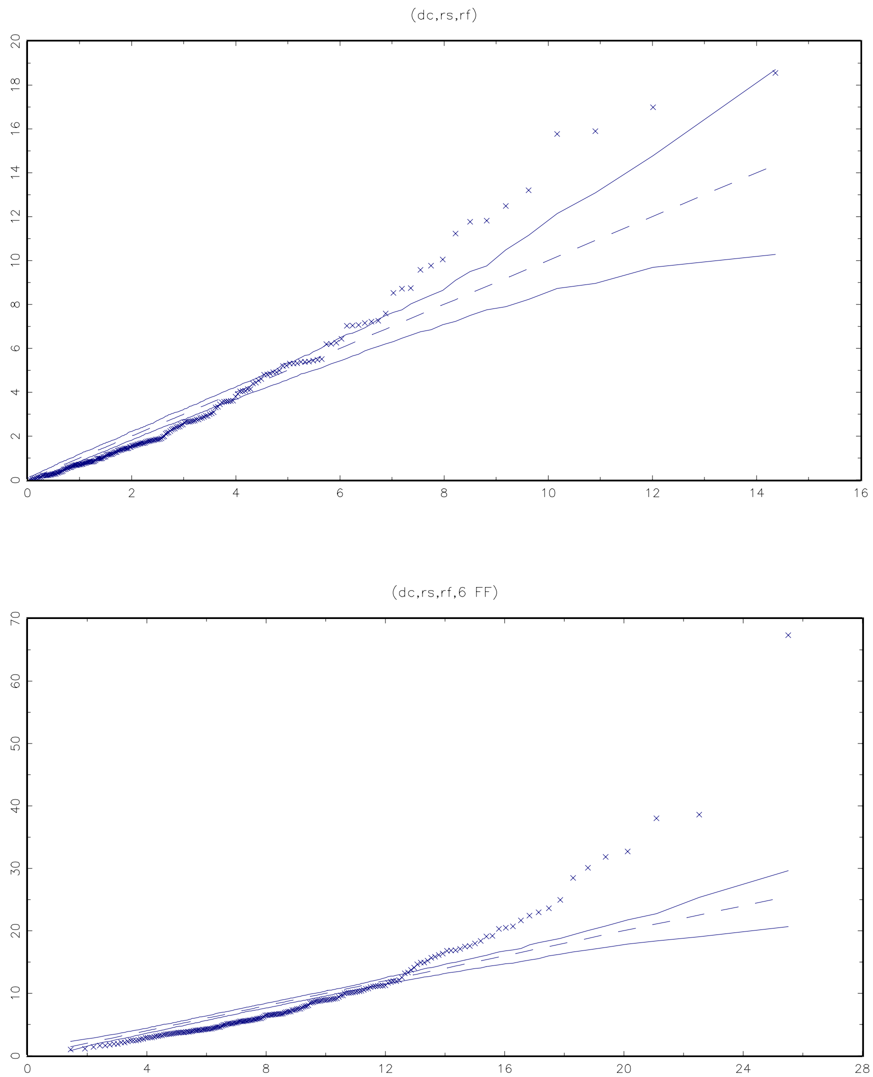
Notes: This table reports output on the pricing error associated with erroneously using aggregate consumption in place of asset-holder consumption, for a range of parameter values and joint distributions. γ^i is the persumed value of asset-holder risk-aversion; δ^i is the presumed value of the asset-holder's subjective discount rate; $\rho(\Delta c,\Delta c^i)$ denotes the correlation between aggregate consumption growth and asset-holder consumption growth in the leading normal; $\rho(\Delta c^i,\Delta d)$ denotes the correlation between asset-holder consumption growth and dividend growth in the leading normal; $\sigma(\Delta c^i)/\sigma(\Delta c)$ denotes the standard deviation of asset-holder consumption growth divided by the standard deviation of aggregate consumption growth in the leading normal; $\mu(\Delta c^i)/\mu(\Delta c)$ denotes the mean of asset-holder consumption growth divided by the mean of aggregate consumption growth in the leading normal; γ^c and δ^c are the values of γ and δ that minimize the pricing errors using aggregate consumption; e(Rs) is the error for the Euler equation associated with the stock return; e(Rf) is the pricing error of the Euler equation associated with the risk-free rate, and Sk[ ], Ku[ ] refer to the skewness and kurtosis of aggregate consumption (c), asset-holder consumption (i), and dividends (d).

Figure 1: Euler Equation Errors for CRRA Preferences: Excess Returns



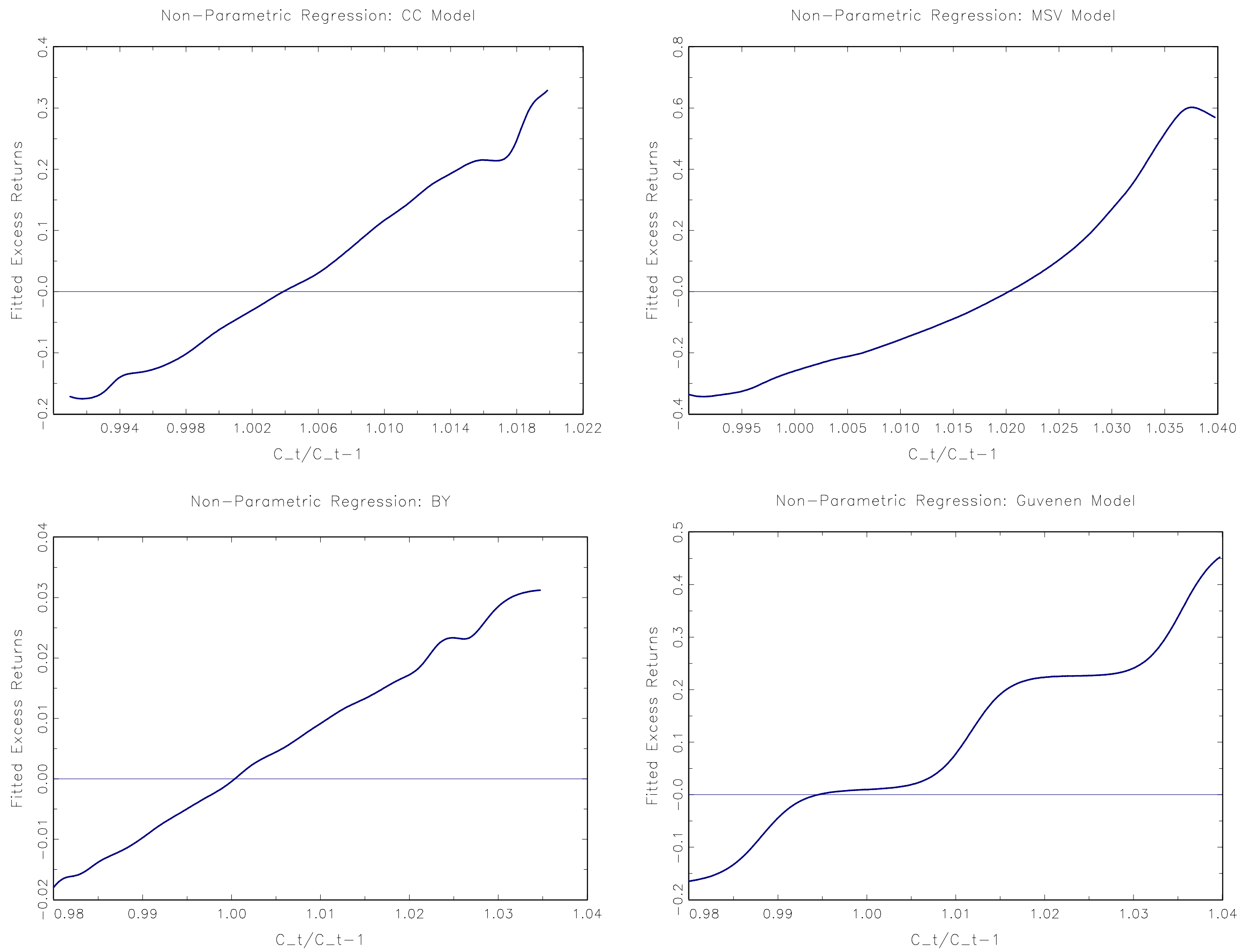
Notes: The figure plots RMSE/RMSR as a function of γ for excess returns. The Euler equation errors are $\boldsymbol{e}_X = E\left[\delta(C_{t+1}/C_t)^{-\gamma}(\boldsymbol{R}_{t+1} - R^f_{t+1})\right]$. The solid line shows RMSE/RMSR for $\boldsymbol{R} = R^s$, the dotted line shows RMSE/RMSR for $\boldsymbol{R} = (R^s, 6\ \text{FF})$. For each value of γ, δ is chosen to minimize the Euler equation error for the risk-free rate.

Figure 2: QQ Plots – Data



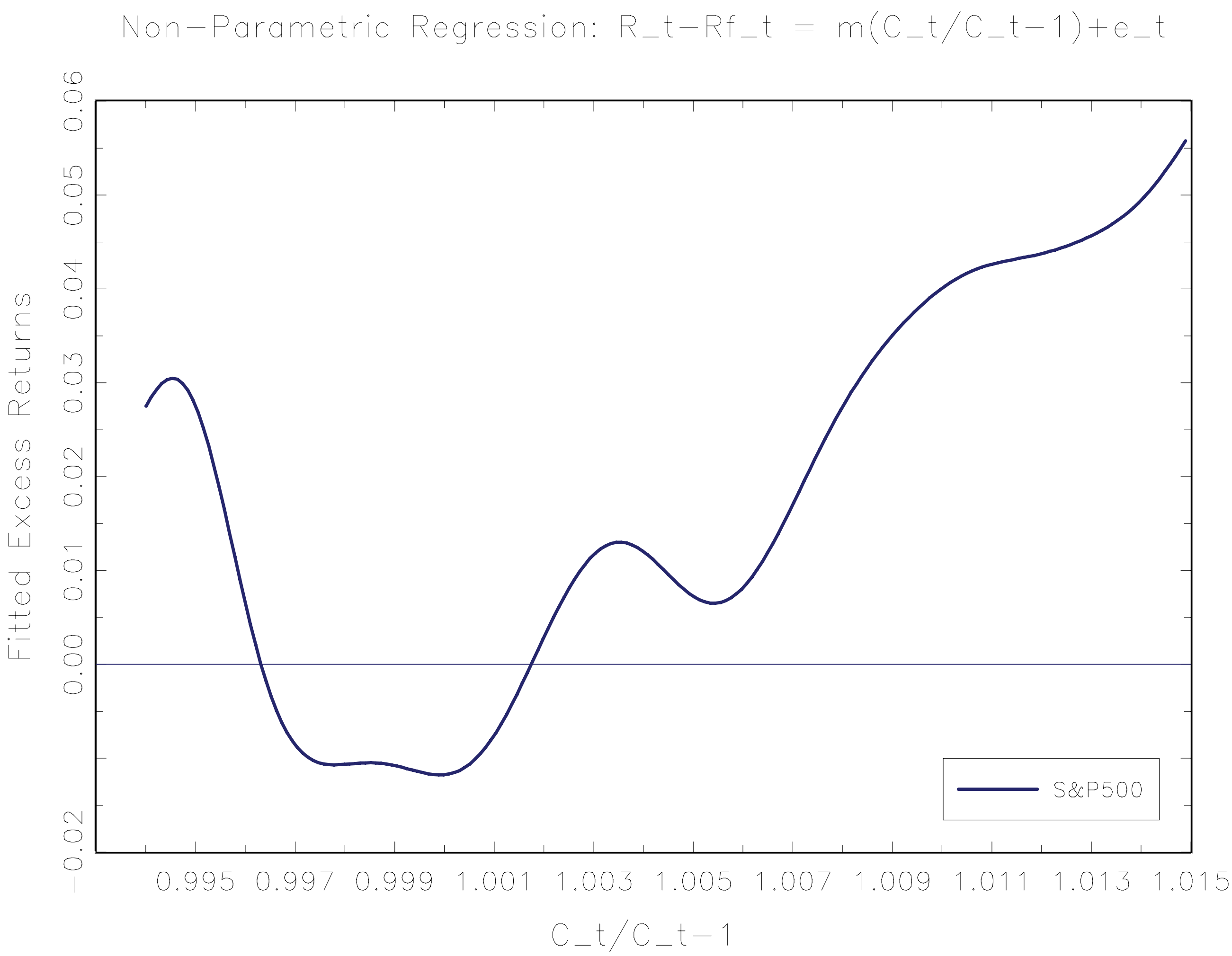
Notes: This figure shows multivariate quantile-quantile (QQ) plots of log consumption growth and asset returns. Each panel plots the sample quantiles (on the y-axis) against the quantiles of a given distribution (on the x-axis) as well pointwise 5% and 95% bands. The top panel shows the QQ plot for the joint distribution of $\Delta c, r_s$ and r_f, i.e. the quantiles of the squared Mahalanobis distances against those of a χ^2_3 distribution. The bottom panel shows the QQ plot for the joint distribution of $\Delta c, r_s, r_f$ and 6 FF portfolios, i.e. the quantiles of the squared Mahalanobis distances against those of a χ^2_9 distribution. The squared Mahalanobis distance D_t for a p-dimensional multivariate distribution $\boldsymbol{x}_t$ with mean $\boldsymbol{\mu}_x$ and variance-covariance matrix $\boldsymbol{V}$ is defined as $D_t = (\boldsymbol{x}_t - \boldsymbol{\mu}_x)'\boldsymbol{V}^{-1}(\boldsymbol{x}_s - \boldsymbol{\mu}_x)$. Under the null hypothesis that $\Delta c, r_s$ and r_f are jointly normally distributed, D_t has a χ^2_p distribution.

Figure 3: Nonparametric Regressions – Models



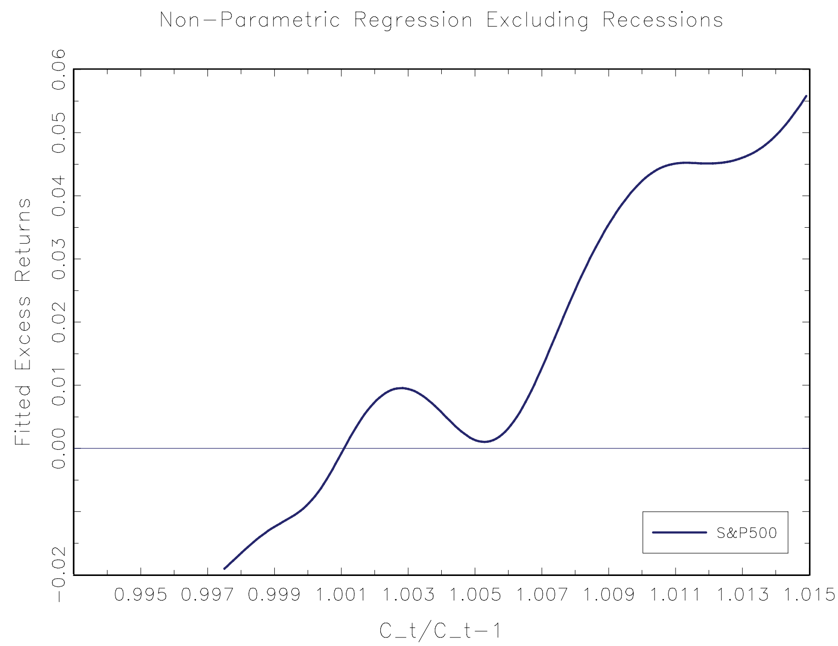
Notes: The figure shows fitted values for nonparametric regressions $R_t = m(C_t/C_{t-1}) + e_t$ using a Gaussian kernel with optimally chosen bandwidth for simulated data generated by the asset pricing models of Campbell-Cochrane, Menzly-Santos-Veronesi, Bansal-Yaron and Guvenen.

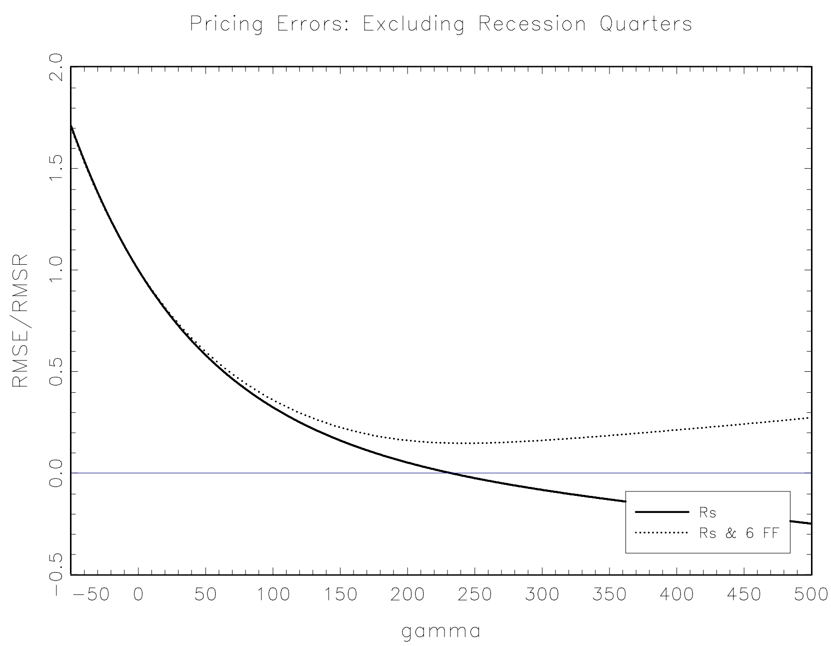
Figure 4: Nonparametric Regressions – Data



Notes: The figures shows fitted values of nonparametric regression $R_t = m(C_t/C_{t-1}) + e_t$ using a Gaussian kernel with optimally chosen bandwidth.

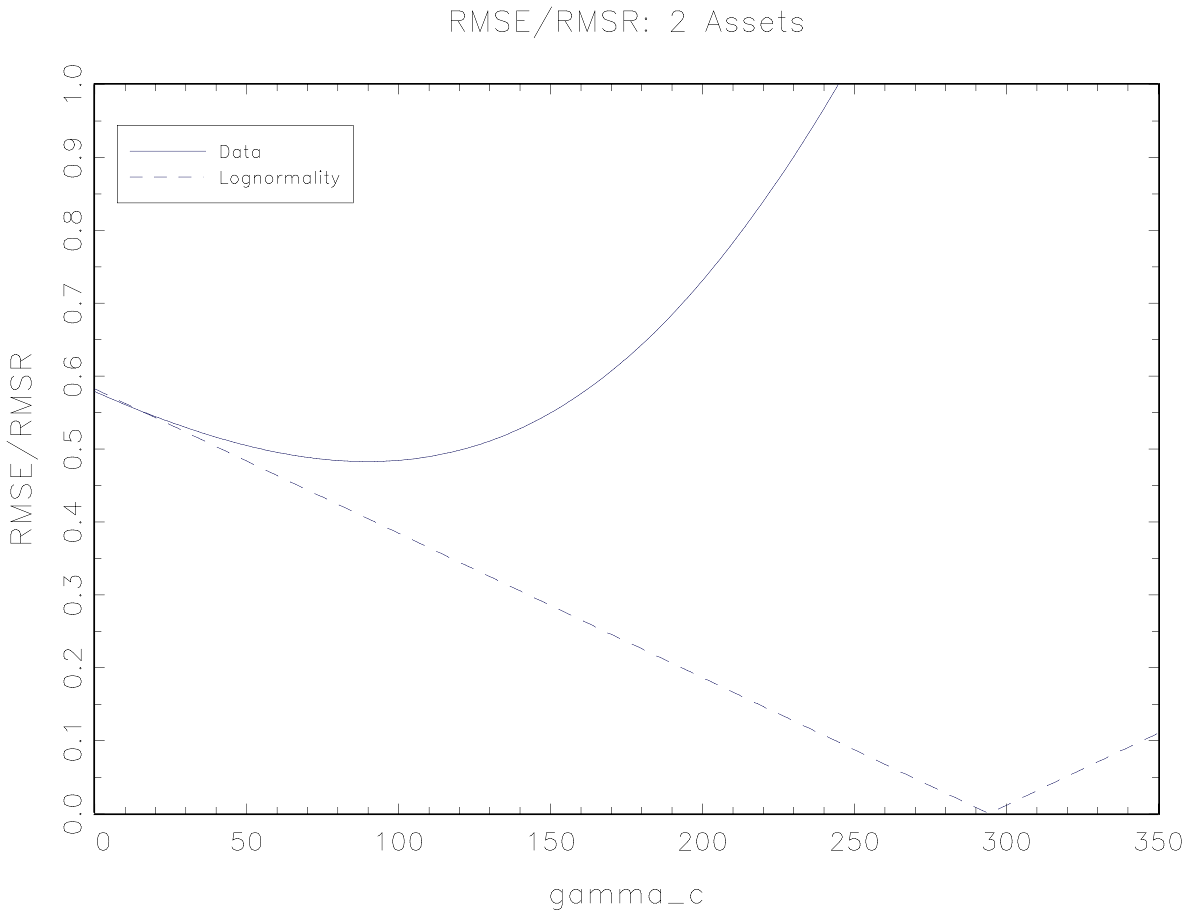
Figure 5: Nonparametric Regressions Excluding Recessions





Notes: The top panel shows fitted values of nonparametric regression $R_t = m(C_t/C_{t-1}) + e_t$ where quarters designated as recessions by the NBER are excluded. The bottom panel plots pricing errors as in Figure 1 but when quarters designated as recessions by the NBER are excluded.

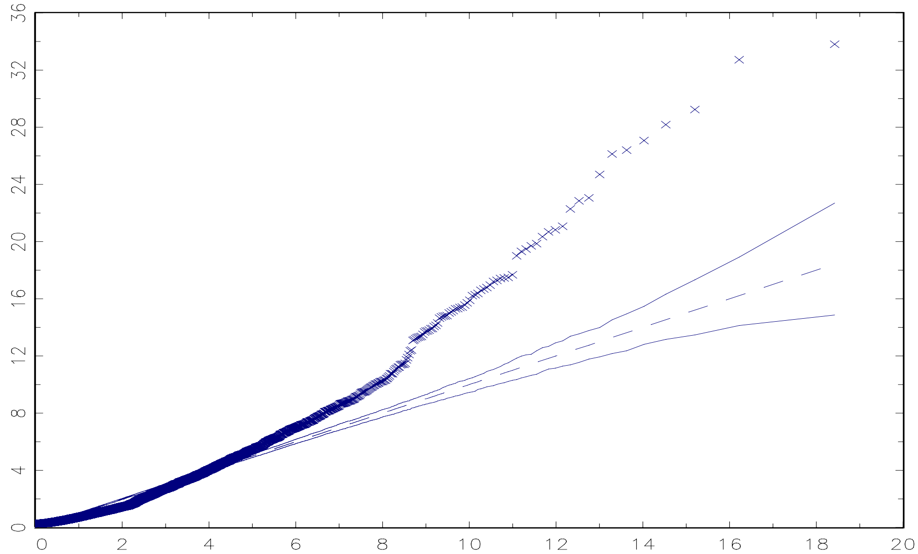
Figure 6: Euler Equation Errors with and without Lognormality – 2 Assets



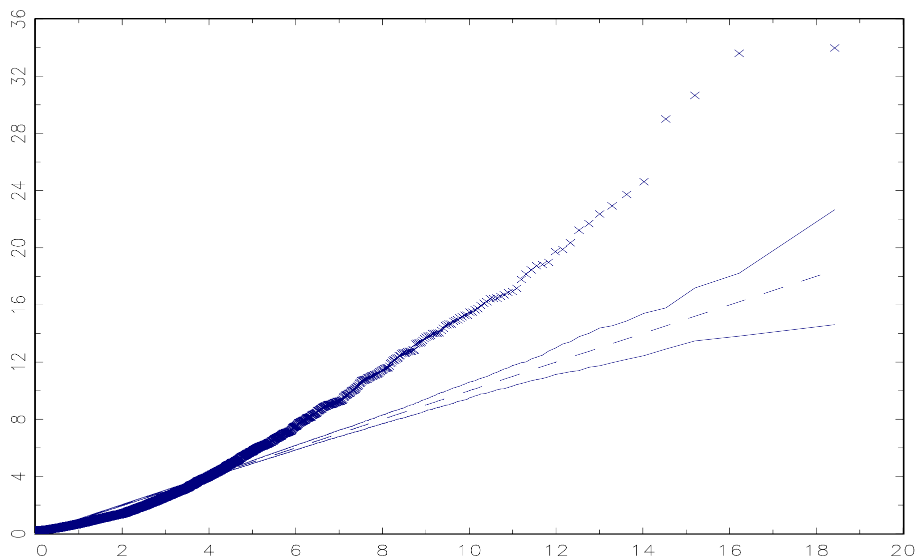
Notes: This figure plots RMSE/RMSR with and without the assumption of joint lognormality as a function of γ_c. δ_c is chosen to minimize the RMSE for each value of γ_c for $\boldsymbol{R} = (R^s, R^f)$. The Euler equation error for asset j without assuming lognormality is $e_R^j = \delta_c E\left[\exp\left\{-\gamma_c \Delta c + r^j\right\}\right] - 1$. Under the assumption of joint lognormality, the Euler equation error is $e_R^j = \delta_c \exp\left\{-\gamma_c E\Delta c + \gamma_c^2 \sigma_c^2/2 + Er^j + \sigma_r^2/2 - \gamma_c \mathrm{Cov}(\Delta c, r^j)\right\} - 1$.

Figure 7: QQ Plots – Model with State-Dependent Correlation

Panel A: 2 Assets

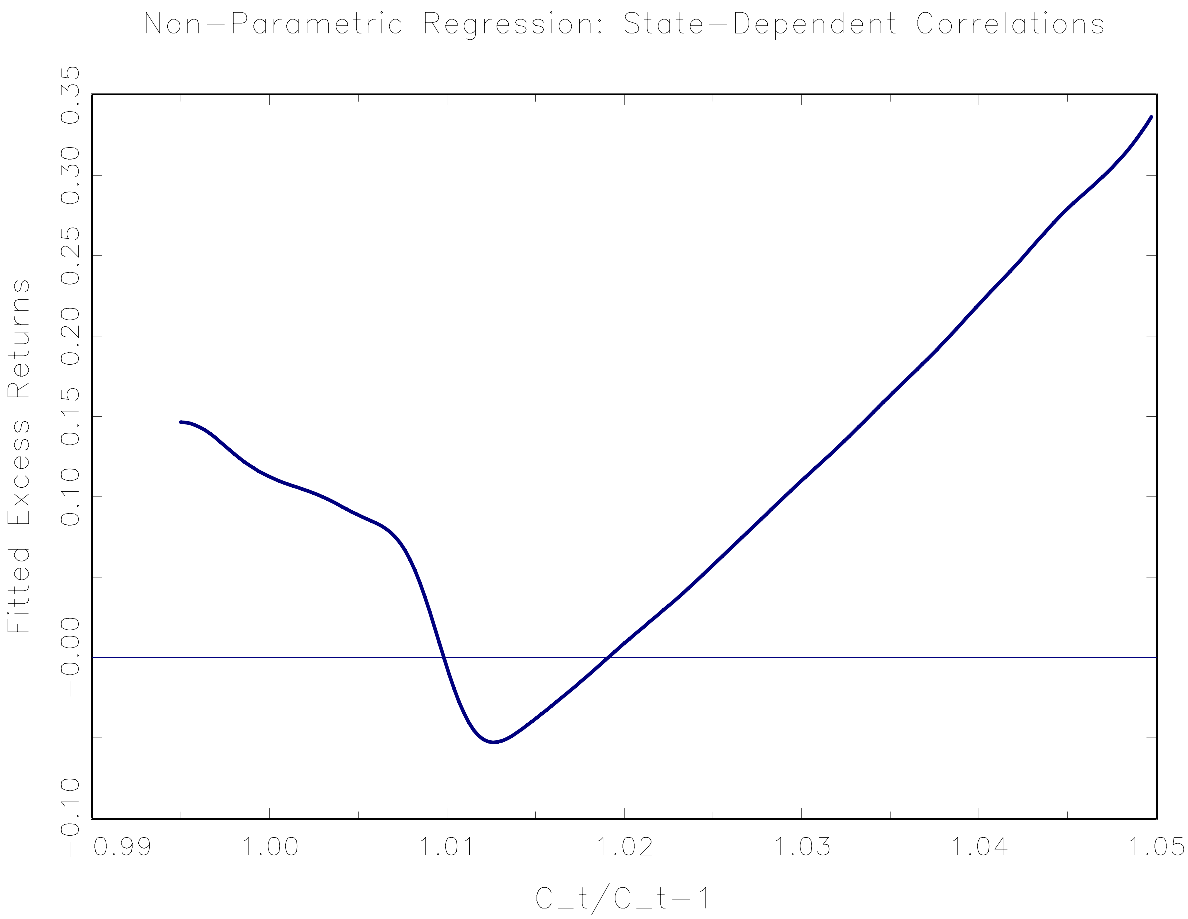


Panel B: 8 Assets



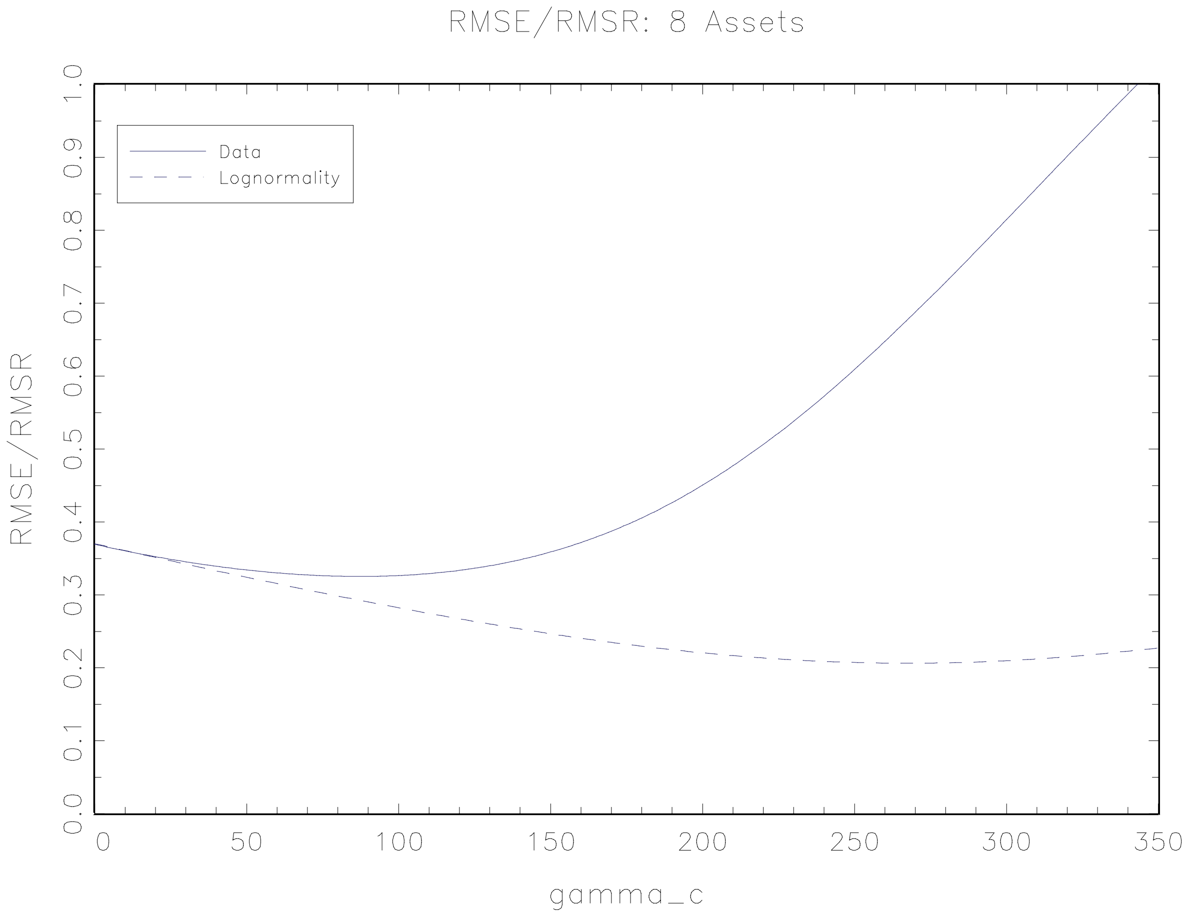
Notes: This figure shows multivariate quantile-quantile (QQ) plots of log consumption growth and asset returns for data generated by the CRRA model with state-dependent correlation. See the notes to figure 3 for a description of the QQ plots and the notes to table 7 for a description of the model and values for the parameters. $\rho^{-}(C_t^i/C_{t-1}^i, C_t/C_{t-1}) is -0.5$ and σ_i/σ_c is 2. Panel A shows the case of 2 assets, Panel B presents the 8 asset case.

Figure 8: Nonparametric Regressions: State-Dependant CRRA Model



Notes: The figure shows fitted values for nonparametric regressions for simulated data generated by the CRRA asset pricing model with state dependant correlation. The regression specification is the same as in Figure 4.

Figure A.1: Euler Equation Errors with and without Lognormality – 8 Assets



Notes: This figure plots RMSE/RMSR with and without the assumption of joint lognormality as a function of γ_c. δ_c is chosen to minimize the RMSE for each value of γ_c for $\boldsymbol{R} = (R^s, R^f, 6 \text{ FF})$. The Euler equation error for asset j without assuming lognormality is $e_R^j = \delta_c E\left[\exp\left\{-\gamma_c \Delta c + r^j\right\}\right] - 1$. Under the assumption of joint lognormality, the Euler equation error is $e_R^j = \delta_c \exp\left\{-\gamma_c E\Delta c + \gamma_c^2 \sigma_c^2/2 + Er^j + \sigma_r^2/2 - \gamma_c \mathrm{Cov}(\Delta c, r^j)\right\} - 1$.