NBER WORKING PAPER SERIES

THE DYNAMIC EFFECTS OF TAX LAW ASYMMETRIES

Alan J. Auerbach

Working Paper No. 1152

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge MA 02138

June 1983

The research reported here is part of the NBER's research program in Taxation and project in Government Budget. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

NBER Working Paper #1152
June 1983

Abstract

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Under current U.S. tax law, a distinction is made between gains and losses by businesses. Losses that must be "carried forward" are subject to two penalties: a loss of interest, and expiration after fifteen years. Previous examinations have focused on the higher expected tax payments such a tax system without "full loss offset" imposes on risky projects.

This paper presents a dynamic analysis of the impact of taxation on investment when gains and losses are treated asymmetrically. The results provide a basis for analyzing recent tax changes, particularly the controversial "safe-harbor leasing" provisions of the 1981 tax legislation.

Alan J. Auerbach
National Bureau of Economic Research
1050 Massachusetts Avenue
Cambridge, MA 02138

(617) 868-3900
Extension 344

I. Introduction

Under current U.S. tax law, a distinction is made between gains and losses by businesses. For corporations, gains are taxed at what is essentially a flat rate of 46 percent, while losses do not necessarily qualify for a refund at the same rate. To obtain an immediate refund, the taxpayer must have had taxable income during the three previous years in excess of the current loss. Any losses that cannot be "carried back" in this fashion must be "carried forward," subject to two implicit penalties: such loss "carryforwards" earn no interest, and they expire after fifteen years. Hence, businesses investing in risky projects for which the probability of having to carry losses forward is nonzero can expect to pay higher taxes than they would under a system with full "loss offset," i.e. the immediate refund of taxes for losses incurred.

Previous examinations of the effects of the lack of a full loss offset have focused on risk-taking in a static model. Domar and Musgrave (1944) first pointed out the disincentive imposed when the government does not share in "downside" risks. Similar analysis was done by Stiglitz (1969) for a state preference model. Compared to a model with full loss offset, a tax system which refunds losses at a lower rate than the one at which gains are taxed causes risky assets to have both a lower expected return and a higher variance, while having no effect on the return to safe assets. Hence, risk taking is discouraged.

Many U.S. corporations experience the need to carry losses forward.[1] This problem became more acute with the acceleration of depreciation allowances under the Economic Recovery Tax Act of 1981, for two reasons. First, the acceleration resulted in a reduction in effective rates, to the point where, even for safe assets with positive returns, certain equity-financed investments faced negative effective taxes rates.[2] That is, under a hypothetical tax system with full loss offset, the present value of taxes associated with such investments would be negative. Moreover, the timing of depreciation allowances meant that, even for assets with positive expected tax payments in present value, the pattern over time would be tax refunds in the early years, caused by large depreciation allowances, followed by tax payments in later years. Firms with a high ratio of new assets to old could not expect actually to get such refunds, and by having to carry them forward would face higher total tax payments, in present value. Again, this result is for safe assets with positive annual income. Such problems would necessarily carry over and, indeed, probably be worse if one considered risky investments.

As a response to these problems, the 1981 legislation also included a liberalization of equipment leasing provisions. Under the new "safe harbor" leasing, transactions very similar to the outright sale of depreciation allowances and investment credits on new investments were legalized.[3] Aside from residual legal differences from a system permitting direct transactions in all such tax benefits, which would have been equivalent to one with a full loss offset, safe-harbor leasing also had the limitation of being permitted only with respect to new capital goods. While this may have been intended as a way of targeting the incentives toward new investment, some of the results were arguably perverse. Companies with tax loss carryforwards so large as to make them essentially tax exempt were provided with an opportunity to sell depreciation deductions and investment credits. While such treatment might actually have been appropriate under a system which, including interest deductibility, provides taxable investors with net refunds from investments in fixed assets,[4] it was not necessarily what was intended by the legislation, which appeared instead to be aimed at helping those firms that expected to be taxable in the near future and would otherwise have had to carry losses forward until such time.

The fact that safe-harbor leasing was scaled back by the 1982 tax legislation[5] demonstrates the ambivalence of legislators toward attempts at introducing elements of a loss offset into the tax system. Indeed, it is the perceived heterogeneity of firms incurring losses that helps explain this ambivalence. Are firms with tax losses "risk takers" with unfavorable current draws, or are they "losers," inefficiently managed companies with low expected earnings? There appears to be a perception that the tax law should penalize the latter type, perhaps to encourage a change of management, but not the former. One may interpret the current system of allowing losses to be carried forward as a compromise between these two objectives. Firms anticipating a high expected return, but with the prospect of occasional losses, may lose a year's interest on such losses by having to carry them forward, or perhaps (through carrying them back) none at all, while those anticipating runs of losses will suffer substantially more, in the limit recouping none of the losses carried forward. Hence, one would imagine the latter type of firm facing a greater discouragement to invest than the former.

Aside from the fact that penalizing losses is not necessarily the optimal way to distinguish between high-risk, high-return firms and low-return firms, this analysis becomes even less appropriate under the current tax system, where the tax base of a firm differs substantially from measures of its economic income. Even more fundamentally wrong with this approach is that it applies to _initial_ decisions firms make. While the high probability of a tax loss may discourage the low-return firm from investing initially, once the investment is sunk and, with some probability, the tax loss occurs, further investment decisions will be made taking account of the loss carryforward. Since such accumulated tax losses decay in value over time, firms may increase their investment to use them up. Thus, in analyzing the effect of such a tax system on a firm's behavior, we must account not only for the firm's decisions, given its current tax position, but we must also ascertain what this position is likely to be. A "loser" may suffer more from the absence of a loss offset, but may also be more likely in a position to accelerate investment to use up loss carryforwards.

The purpose of this paper is to present a dynamic analysis of the impact of taxation on investment when gains and losses are treated asymmetrically in the manner described above. For simplicity, our main focus is on two tax systems that, with a full loss offset, would both be completely neutral in the sense of not distorting investment decisions: an income tax with interest deductibility, and one without an interest deduction but with the immediate expensing of investment.[6] These tax systems both result in the taxation of pure economic rent, and differ only in the timing of tax payments. This distinction turns out to be very important in the current context. These tax systems are also of interest because the current U.S. tax system has elements of each: acceleration of depreciation allowances, and the partial deductibility (only to the extent that debt finance is used) of the opportunity cost of funds.

After introducing the model and its notation, we analyze the effects of these tax systems on the behavior of firms with different characteristics, first in a static, two-period context, and then in an infinite horizon model. We also simulate the stochastic steady states generated by the behavior of firms with an infinite horizon. Our results indicate that, for an income tax with interest deductibility or, more simply, an <u>income tax</u>, the intuition presented above is correct: firms with a greater probability of loss are less likely to invest, given a small loss carryforward, but also more likely to have a large loss carryforward. In the simulated stochastic steady states, the ultimate impact is perverse in the sense that, on average, such firms invest more than others. This result suggests that a potential improvement is available through the introduction of an option to "cash in" losses at a discount, for this encourages self-selection on the part of firms most likely to over invest. Simulations of such a tax system confirm this.

Finally, the behavior of firms under a system of income taxation with expensing or, more simply, a cash flow tax, is complicated by the timing of deductions, and it is less clear what the ultimate impact of the tax system is on the relative behavior of different types of firms.

II. The Model

We use the simplest type of model that allows us to study the problems of interest. Firms make investment decisions in each period, and these investments deliver a stochastic return in the following period only. Hence, depreciation is complete after one period. We also assume that each firm faces the same investment choices every period, and that the uncertainty is summarized by a multiplicative random variable that is independently and identically distributed over time, with unit mean. Thus, we have ruled out the possibility of previous investment decisions or return realizations affecting current behavior, except through any tax loss carried forward. If I_t is the investment at the beginning of period t, then the return at the end of period t, after depreciation, is

$$\Pi_t = x(I_t)\theta_t - B \tag{1}$$

where $x(\cdot)$ is concave, θ_t is the realized value of the random variable (not known to the firm when I_t is chosen) and B may be thought of as the firm's fixed operating costs. We assume $x(\cdot)$ is constant over firms and over time, and let B vary across firms to represent differences in overall operating efficiency. In the absence of taxation, B will not influence investment decisions. This is a very simple way of capturing differences among firms; one could imagine a model with differences being multiplicative, rather than additive, or a combination of the two.

We ignore limited liability, and assume that the firm's opportunity cost is the safe interest rate, r, and that the firm's objective is to maximize the present value of expected profits. This objective function is in contrast to the normal use of a concave utility function to study taxation and risk-taking. However, our interest here is not in the impact of risk-taking on the bearing of undiversifiable social risk, but rather on the effects of tax law asymmetries on the behavior of particular firms. One may imagine much of the current problem going away if all firms merged and pooled their risks; increased merger activity has, in fact, been cited as a potential response to the current tax law. For any individual firm in the present model, the asymmetric treatment of gains and losses imparts a concavity to the objective function in terms of before-tax returns, since a greater fraction of losses than gains is received after taxes.

In the absence of taxes, the representative firm's objective at time 1 is to maximize over $\{I_1, I_2, \ldots\}$ the expected value of total profits:

$$V = E_1\{\sum_{t=1}^{\infty}(1+r)^{-t}[x(I_t)\theta_t - rI_t - B]\} = E_1(1+r)^{-1}[W_1 + V_2] \qquad (2)$$

where $W_t = \Pi_t - rI_t$ is economic profit at the end of year t. Since W_{t+i}, $i > 0$, is not a function of I_t, this leads to the decision to maximize current profits. Since $E_t(\theta_t) = 1$, this yields the decision rule:

$$x_t' = r \tag{3}$$

i.e., that the marginal product should equal the interest rate.

III. Systems of Taxation

As stated above, either an income tax (with interest deductibility) or a cash flow tax would, with a full loss offset, result in a tax on economic rent. This is quite easy to see in the current model. Under an income tax, the tax base at the end of period t would be W_t, which equals economic profit. Under a cash flow tax, the firm would deduct its initial investment, I_t, at the end of period t-1, for an equivalent deduction of $(1+r)I_t$ at the end of period t, from the tax base of gross rents, before depreciation and interest, $\Pi_t + I_t$, again leading to W_t.

Once losses are not usable to obtain a tax refund, these results break down. To preserve simplicity, we ignore the possibility of carrying losses back, and assume that losses may be carried forward indefinitely. Neither of these assumptions should affect the qualitative nature of the results derived. To separate the indirect impact of lump-sum rent taxation from the effects of the tax law asymmetry itself, we assume that the tax system is one in which positive taxable income is taxed at a zero rate, while negative taxable income is taxed at a negative rate, $-\tau$. This is equivalent to combining a symmetric, nondistortionary subsidy at rate τ with a tax without loss offset at the same rate.

We first consider the problem facing a firm in some arbitrary period t, under an income tax, letting L_t be the accumulated loss carried forward from period t-1. The firm's decision at t affects the value of the loss carried forward into period t+1, and hence the present value of expected future after-tax returns, so one can no longer separate the problem into a series of independent one-period decisions. The firm seeks, at time t, to maximize:

$$V_t(L_t) = (1+r)^{-1}E_t[\tilde{W}_t + V_{t+1}(L_{t+1})] \tag{4}$$

subject to the choice of I_t, where $\tilde{W}_t$ is the current after-tax return. The expressions for $\tilde{W}_t$ and L_{t+1} depend on whether current period profits are sufficient to use up the entire tax loss carryforward. If they are, income is positive and hence taxable (at rate zero, in this case) at the margin, and the loss carried forward is not influenced by marginal changes in I_t. This will occur if θ_t is sufficiently large so that taxable profits, W_t, exceed L_t. Defining

$$\lambda_t = (L_t + B + rI_t)/x_t \tag{5}$$

we have

$$\tilde{W}_{t+1} = \begin{cases} W_t(1+\tau) & \text{if } \theta_t < \lambda_t \\ W_t + \tau L_t & \text{otherwise} \end{cases} \tag{6}$$

and

$$L_{t+1} = \begin{cases} L_t - W_t & \text{if } \theta_t < \lambda_t \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

Using (5) - (7), and letting $f(\cdot)$ be θ's probability density function, we may rewrite (4) as:

$$V_t(L_t) = (1+r)^{-1}\{ \int_{-\infty}^{\lambda_t} [W_t(1+\tau) + V_{t+1}(L_t - W_t)] f(\theta)d\theta$$

$$+ \int_{\lambda_t}^{\infty} [W_t + \tau L_t + V_{t+1}(0)] f(\theta)d\theta\} \tag{8}$$

The cash flow tax is more complicated to analyze because current decisions affect not only taxable income at the end of the period, at time t, but also at the beginning of the period, at time t-1. It is useful, therefore, to define the variable L_t as the tax loss carried forward from period t-1, <u>before</u> account is taken of period t's investment and potential expensing deductions. Moreover, period t investment can be expensed at t-1 only to the extent that the tax base is positive at t-1. Hence, we must include the period t-1 taxable profit as a state variable at time t, also. Fortunately, we can do this without introducing a new variable, since there cannot be both a loss carryforward from period t-1 and a taxable profit in period t-1. We simply let L_t take on a negative value equal to the firm's taxable profit at t-1, before period t decisions are accounted for, when this profit is nonnegative. Complications remain in that any positive excess profit, after deduction of investment costs at the beginning of period t, is not carried forward, while losses are. This will require certain additional notation.

There are three possible situations in which the firm may find itself with respect to its ability to expense period t investment. Letting I_t^* be the amount of investment immediately deductible (with $I_t - I_t^*$ carried forward), we have:

$$I_t^* = \begin{cases} 0 & \text{if } L_t > 0 \\ L_t + I_t & \text{if } L_t + I_t > 0 \geq L_t \\ I_t & \text{if } 0 \geq L_t + I_t \end{cases} \tag{9}$$

Letting $\phi_t = 1$ if $L_t < 0$ and 0 otherwise, and $\delta_t = 1$ if $L_t + I_t > 0$ and 0 if $L_t + I_t < 0$, we may rewrite (9) as:

$$I_t^* = I_t - \delta_t(I_t + \phi_t L_t) \tag{10}$$

Since losses incurred in period t-1, after accounting for additional deductions of investment expense, denoted L_t^*, are carried forward only if positive, we have, using the definition of δ,

$$L_t^* = \delta_t(L_t + I_t) \tag{11}$$

If we let ρ_t be taxable cash flow at time t, before accounting for decisions at period t+1, we have

$$\rho_t = x(I_t)\,\theta_t + I_t - B \tag{12}$$

which equals total investment returns, gross of depreciation, interest and taxes. As before, there are two possibilities concerning the state of the loss carried forward into period t+1, L_{t+1}. Either cash flow, ρ_t, exceeds the loss L_t^*, and a profit is carried forward ($L_{t+1} < 0$), or $\rho_t < L_t^*$ and a loss is carried forward. Defining

$$\alpha_t = \frac{L_t^* + B - I_t}{x(I_t)} \tag{13}$$

we have as the objective function in period t:

$$V_t(L_t) = -\tau(I_t - I_t^*) + (1+r)^{-1}\{\int_{-\infty}^{\alpha_t} [W_t + \tau\rho_t]f(\theta)d\theta$$

$$+ \int_{\alpha_t}^{\infty} [W_t + \tau L_t^*]f(\theta)d\theta + \int_{-\infty}^{\infty} V_{t+1}(L_t^* - \rho_t)f(\theta)d\theta\} \qquad (14)$$

which accounts for the immediate tax on additional losses if not all investment can be expensed, plus the after-tax receipts under the two regimes for net profits at the end of period t, plus the value of losses or profits carried forward. The important distinctions from the previous case of the income tax are that there is a direct effect of the firm's decision on taxes in two periods, and the value function is evaluated for negative as well as positive values of L_t, which here stands for the loss carried forward into period t before new investment.

The impact on firm behavior of these two tax systems, particularly the latter, is quite complicated. However, much that applies to the general case may be learned by considering a two-period model, in which investment occurs only in the first period, so that the next period's value function $V_{t+1}(\cdot)$ is uniformly zero.

III. Two-Period Analysis

Dropping subscripts for period t, and assuming $V_{t+1} \equiv 0$, we obtain the following condition for investment under the income tax from (8):

$$x' = r\,\frac{1 + \tau\int_{-\infty}^{\lambda} F(\theta)d\theta}{1 + \int_{-\infty}^{\lambda} \theta F(\theta)d\theta} = \frac{1 + \tau F(\lambda)}{1 + \tau F(\lambda)E(\theta/\theta < \lambda)} \tag{15}$$

where $F(\cdot)$ is the cumulative density function corresponding to $f(\cdot)$. Since $E(\theta/\theta < \lambda) \leq E(\theta) = 1$, with strict inequality holding as long as $F(\lambda) < 1$, expression (15) indicates that investment will be lower than would be true under symmetric taxation, or no taxation, with x' approaching r as λ, and hence L, become quite large. When this happens, the firm is essentially tax exempt, because the probability is small that it will use up its loss carryforward in the near future. What is, perhaps, somewhat surprising is that the relationship between I and L need not be monotonic. Total differentiation of (15) with respect to L yields:

$$\frac{dI}{dL} = -\frac{(x'\lambda - r)\tau F(\lambda)/x}{x''(1 + \tau F(\lambda)) - (x'\lambda - r)^2\tau F(\lambda)/x} \tag{16}$$

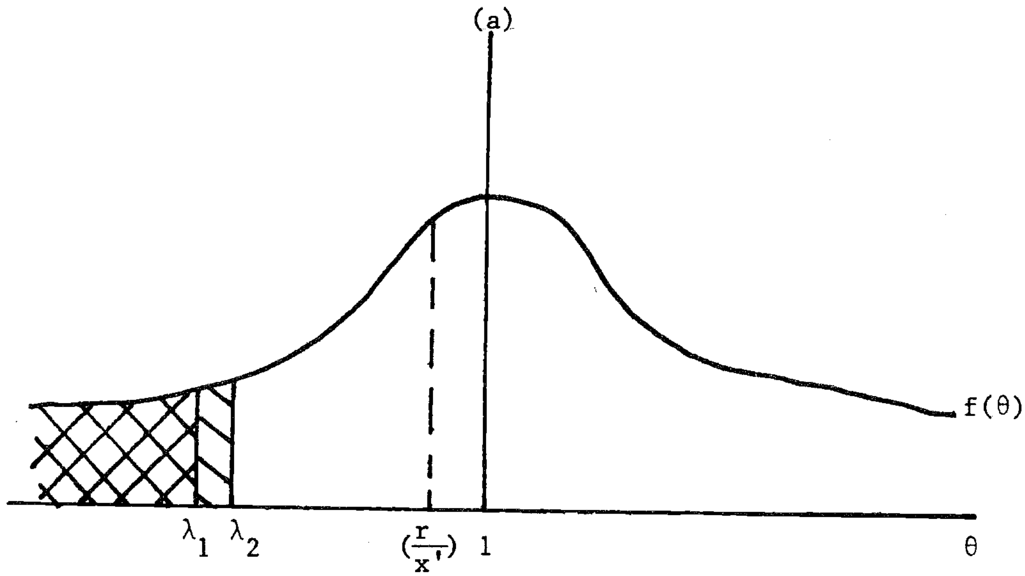
The denomination of (16) is equal to $\partial^2 V/\partial I^2$ and, as required for an optimum, is negative. Thus, $\frac{dI}{dL} > 0$ if and only if $x'\lambda > r$. A sufficient condition for this is that $\lambda = 1$, since $x' > r$. By the definition of λ in (5), $\lambda > 1$ if $(L + B)$ exceeds $x - Ir$ (the maximum value of which occurs when $x' = r$). Thus, a sufficient condition for investment to increase with the size of the tax loss carryforward is that the loss exceed the expected value of profits, before tax. For the general case, an increase in L increases the value of θ at which profits from current investment become taxable, λ. Since marginal profits at this value, $x'\lambda - r$, may be negative if L is small and there is a substantial amount of pure rent being collected, an increase in λ may actually subject more expected losses to taxation, rather than shielding profits, at the margin. This suggests that if only a small amount of risk is present, the observed range of L may be sufficiently small that I is non-increasing over a large part of it. As risk increases, the effect of L in shielding profits dominates the investment decision.

These two possibilities are illustrated in panels (a) and (b) of Figure 1, which graph $f(\theta)$ versus θ, with a dotted line at $\theta = (\frac{r}{x'})$, the point at which the firm earns a profit at the margin, *ex post*. This value of θ must lie to the left of the mean of the distribution at $\theta = 1$. In panel (a), rents are sufficiently large, and L sufficiently small, that the firm is in a net loss position only when it loses money at the margin. Hence an increase in L, holding I fixed, means that more of its marginal losses are subject to the discriminating treatment of the tax system. Once $\lambda > (\frac{r}{x'})$, however, all marginal losses are taxed, and some marginal gains are subsidized. Increases in L, holding I fixed, increase the amount of gains receiving the subsidy.

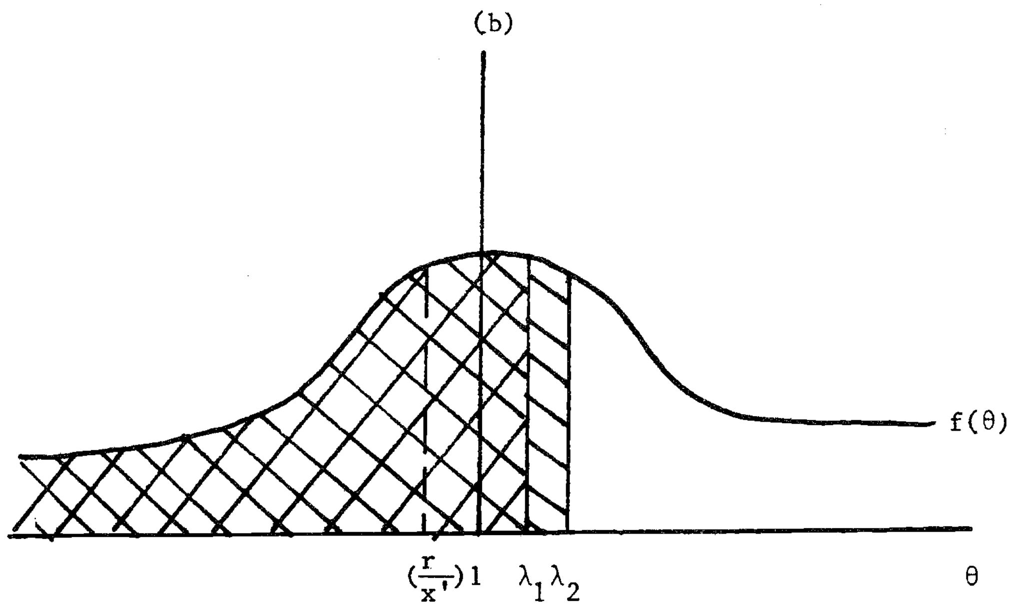
To understand how firms with different values of B will behave under the income tax, we simply note that B and L enter symmetrically into (15) through λ. Hence, (16) may also be interpreted as the value of dI/dB, given L. For values of L satisfying $\lambda > 1$, increases in B will increase investment. With small losses carried forward, increases in B may decrease investment, by the

Figure 1

The Effect of an Increased Loss Carryforward on the Incentive to Invest Under an Income Tax



low λ: increase in nondeductible losses



high λ: increase in shielded gains

same argument given above for small values of L. It is in this range that we are correct in viewing the tax law as discouraging investment by inefficient firms.

Behavior under a cash flow tax depends on whether current investment is deductible at the margin, against period t-1 income. The first-order condition derived from the objective function (14) is:

$$x' = \frac{r(1+\tau\delta) - (1-\delta)\tau \int_{-\infty}^{\alpha} f(\theta)d\theta}{1 + \tau\int_{-\infty}^{\alpha}\theta f(\theta)d\theta} = \frac{r(1+\tau\delta) - (1-\delta)\tau F(\alpha)}{1 + \tau F(\alpha)E(\theta/\theta < \alpha)} \tag{17}$$

where, as defined above, $\delta = 1$ if $L + I > 0$, and 0 otherwise. Aside from this first-order condition, the value of investment must satisfy the inequality consistent with the value of δ. An intermediate solution for I is also possible, at neither margin, with $L + I = 0$. Here, δ will lie between zero and one and may be interpreted as the Lagrange multiplier of the constraint that $L + I = 0$. We review these three cases in turn.

<u>Case 1</u>: $\delta = 0$ $(L + I < 0)$

Here, firms expense at the margin. The first order condition becomes

$$x' = \frac{r - \tau F(\alpha)}{1 + \tau F(\alpha)E(\theta/\theta < \alpha)} \tag{17.1}$$

where $\alpha = \frac{B - I}{x}$. Since α is not a function of L, this solution is invariant with respect to L, for all values of $L < L$, where $I + L = 0$. In addition, investment here will likely <u>exceed</u> the value at which $x' = r$. From (17.1), it follows that $x' < r$ if and only if

$$1 + x'E(\theta/\theta < \alpha) > 0 \tag{18}$$

which says that expected gross marginal returns in the state where a loss is carried into period t+1 are positive. A sufficient condition for this is the requirement that gross returns be nonnegative, a rather weak requirement.

This result says that firms able to expense current investment do better under the asymmetric tax system. They receive their expensing deduction, but since gross rents may be negative, some of the firms' marginal gross rents (which by assumption are always positive) will be sheltered.

It is straightforward to show that the right-hand side of (17.1) must stay the same or increase with B, holding I fixed, provided that θ cannot be lower than $-1/r$. Hence, provided the second-order conditions for I are met, I will increase with B. The situation always corresponds to that for an income tax with sufficiently high losses. This is because marginal gross rents cannot be negative.

Case 2: $\delta = 1 \quad (L + I > 0)$

Here, firms must carry forward marginal expensing deductions. At best, they can deduct them one period later, as they could under an income tax without interest deductibility. Hence, in this regime they are even worse off than under the income tax with a deduction for interest. The first-order condition (17) becomes:

$$x' = \frac{r(1 + \tau)}{1 + \tau F(\alpha)E(\theta/\theta < \alpha)} \tag{17.2}$$

where $\alpha = \frac{B + L}{x}$. Not only does the right-hand side of (17.2) exceed r, but it equals or exceeds the right-hand side of (15) for all values of L and I. This follows directly from the fact that $\alpha > \lambda$. Moreover, the effect of increases in B or L on I are clear, since the right-hand side of (17.2) increases (decreases) with either if $\alpha < (>) 0$. The intuition is similar to that applying under the income tax. An increase in the value of α brings marginal profits at $\theta = \alpha$ into the loss category. These profits (with the expensing deduction carried forward and subtracted) are $x'\alpha$ (rather than $x'\alpha - r$, since interest is not deductible). Hence, if $\alpha > 0$, more profits are shielded from taxation. If $\alpha < 0$, more tax losses are suffered.

Case 3: $0 < \delta < 1$ $(L + I = 0)$

The derivative of the right-hand side of (17) with respect to δ, holding L and I fixed, is strictly positive. Thus, for the appropriate second-order condition on I satisfied,[7] I will decrease with δ and there will be a unique solution for I, given L. One of three outcomes will occur:

(1) $I + L < 0$ at $\delta = 1$ (Case 1 above)

(2) $I + L > 0$ at $\delta = 0$ (Case 2 above)

(3) $I + L = 0$ at $0 < \delta < 1$ (Case 3)

We may view δ as the Lagrange multiplier on the constraint that $I + L = 0$. It has the interpretation of the fraction of their losses that firms cannot deduct immediately but must carryforward.

Over the intermediate range of L (for which L must be negative as long as $x' \to \infty$ as $I \to 0$), investment drops quite sharply, as firms go from a regime of expensing to one with an income tax.

Summary

The effects of taxation on investment are summarized graphically in

Figure 2. The dotted lines show the effects of increases in B. The different segments of the cash flow tax may be summarized by the values of δ and ϕ. When $\delta = 0$ (Case 1) the firm invests more than in a no-tax world, and expenses at the margin. As L increases, a transition during with investment is just entirely expensed and $\delta > 0$ occurs, with I decreasing (Regime 3). Then, a loss is carried forward into period t, as $I + L > 0$ and $\delta = 1$, with investment continuing to decrease with the increase in L until $L = 0$, i.e., as long as $\phi = 1$ (Regime 2a) and then increases with L (Regime 2b). When L is positive, investment is always higher under an income tax although it may decrease with L initially. Ultimately, as $L \to \infty$, investment approaches an asymtote at the no-tax value.

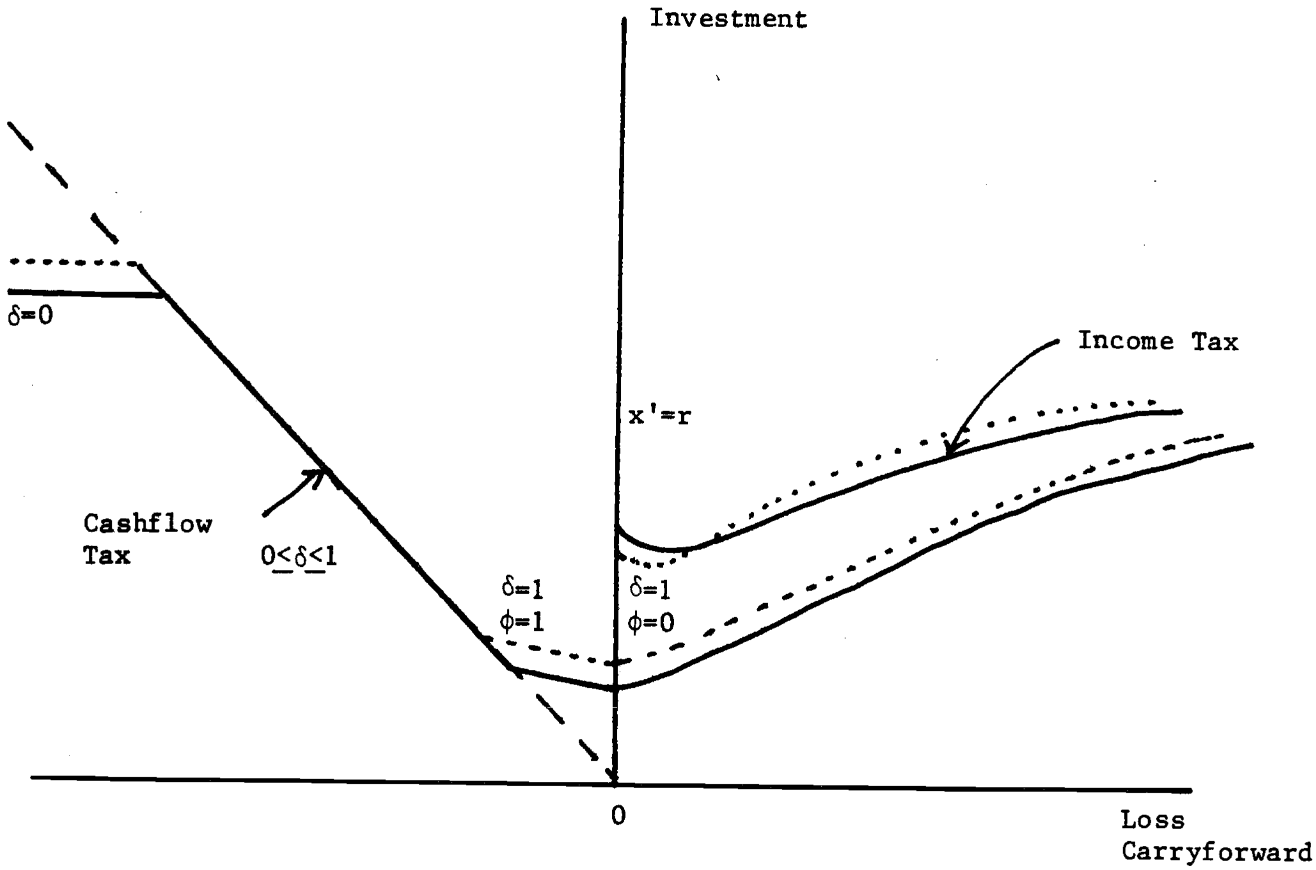
In both tax systems, a higher value of B leads to generally higher levels of investment, for a given value of L. The one exception is for small values of L under an income tax. Thus, the logic suggesting that inefficient firms will be discriminated against by tax systems with a loss offset is only correct under limited circumstances. Moreover, we have yet to determine the effects of such tax systems when more than one period of investment is possible. Here, further differences among firms may arise, since tax losses carried forward from the current period are not simply lost, and the extent to which they are recouped depends on future decisions.

IV. Response to Taxation in a Multi-Period Model

Once there is a future beyond the present period t, changes in V_{t+1} influence the current investment decision. The first-order condition for the income tax based on (8) becomes:

Figure 2

Investment versus Tax Loss Carryforward
Two-Period Model



$$x' = r \cdot \frac{1 + \tau \int_{-\infty}^{\lambda_t} f(\theta)d\theta - \int_{-\infty}^{\lambda_t} V'_{t+1}(L_t + I_t r + B - x_t\theta) f(\theta) d\theta}{1 + \tau \int_{-\infty}^{\lambda_t} \theta f(\theta)d\theta - \int_{-\infty}^{\lambda_t} \theta V'_{t+1}(L_t + I_t r + B - x_t\theta) f(\theta) d\theta}$$

$$= r \cdot \frac{1 + F(\lambda_t)(\tau - E(V'_{t+1}/\theta < \lambda_t))}{1 + F(\lambda_t)E(\theta/\theta < \lambda_t)(\tau - E(V'_{t+1}/\theta < \lambda_t)) - F(\lambda_t)C(V'_{t+1}\theta/\theta < \lambda_t)} \tag{19}$$

When $V'_{t+1} \equiv 0$, (19) reduces to (15). More generally, there are two new effects on the choice of I_t caused by the presence of future decisions:

(1) Since the losses carried forward may be used to offset future profits, they are not totally lost. The average value of such losses is $E(V'_{t+1}/\theta < \lambda_t)$: the average marginal increase in the value function at t+1 with an increase of a dollar in loss carried forward. The value is bounded above by $\frac{\tau}{1+r}$, since, at best, the loss carried forward can be used up entirely in the next period. It is the difference between τ and $E(V'_{t+1}/\theta < \lambda_t)$ that represents the penalty for average losses carried forward. We would expect this value to be larger for firms with lesser prospects for future profits, since they must wait longer, on average, to use up their loss carryforwards.

(2) An additional effect relates to the shape of $V_{t+1}(\cdot)$. As shown in the Appendix, $V_{t+1}(\cdot)$ is strictly concave for arbitrary t. Thus, the covariance between V'_{t+1} and θ is positive: when θ is larger, a smaller loss is carried forward, with a greater prospect of early recoupment. The impact of this term is to raise the right-hand

side of (19), serving as a correction for the use of the simple average of V'_{t+1} in the denominator. Average recoupment of losses is less than recoupment of average losses.

Taken together, these two effects still do not change the outcome that x'_t exceeds r, and that in the limit as $L \to \infty$, $x'_t = r$.

For the cash flow tax, the first-order condition based on (14) is:

$$x' = \frac{r(1+\tau\delta) - (1-\delta)[\tau \int_{-\infty}^{\alpha_t} f(\theta)d\theta - \int_{-\infty}^{\infty} V'_{t+1}(L^*_t+B-I_t-x_t\theta)f(\theta)d\theta]}{1 + \tau\int_{-\infty}^{\alpha_t} \theta f(\theta)d\theta + \int_{-\infty}^{\infty} \theta V'_{t+1}(L^*_t+B-I_t-x_t\theta)f(\theta)d\theta} \tag{20}$$

$$= \frac{r(1+\tau\delta) - (1-\delta)[\tau F(\alpha_t) - E(V'_{t+1})]}{1 + \tau F(\alpha_t)E(\theta/\theta < \alpha_t) - E(\theta V'_{t+1})}$$

which, when $V'_{t+1} \equiv 0$, collapses to (17).

To analyze the impact of this tax system on investment, we must know the characteristics of the value function, $V_{t+1}(\cdot)$. In the Appendix, we derive the following characteristics for arbitrary t:

(1) For $L_{t+1} > 0$, $0 < V'_{t+1} < \tau/(1+r)$

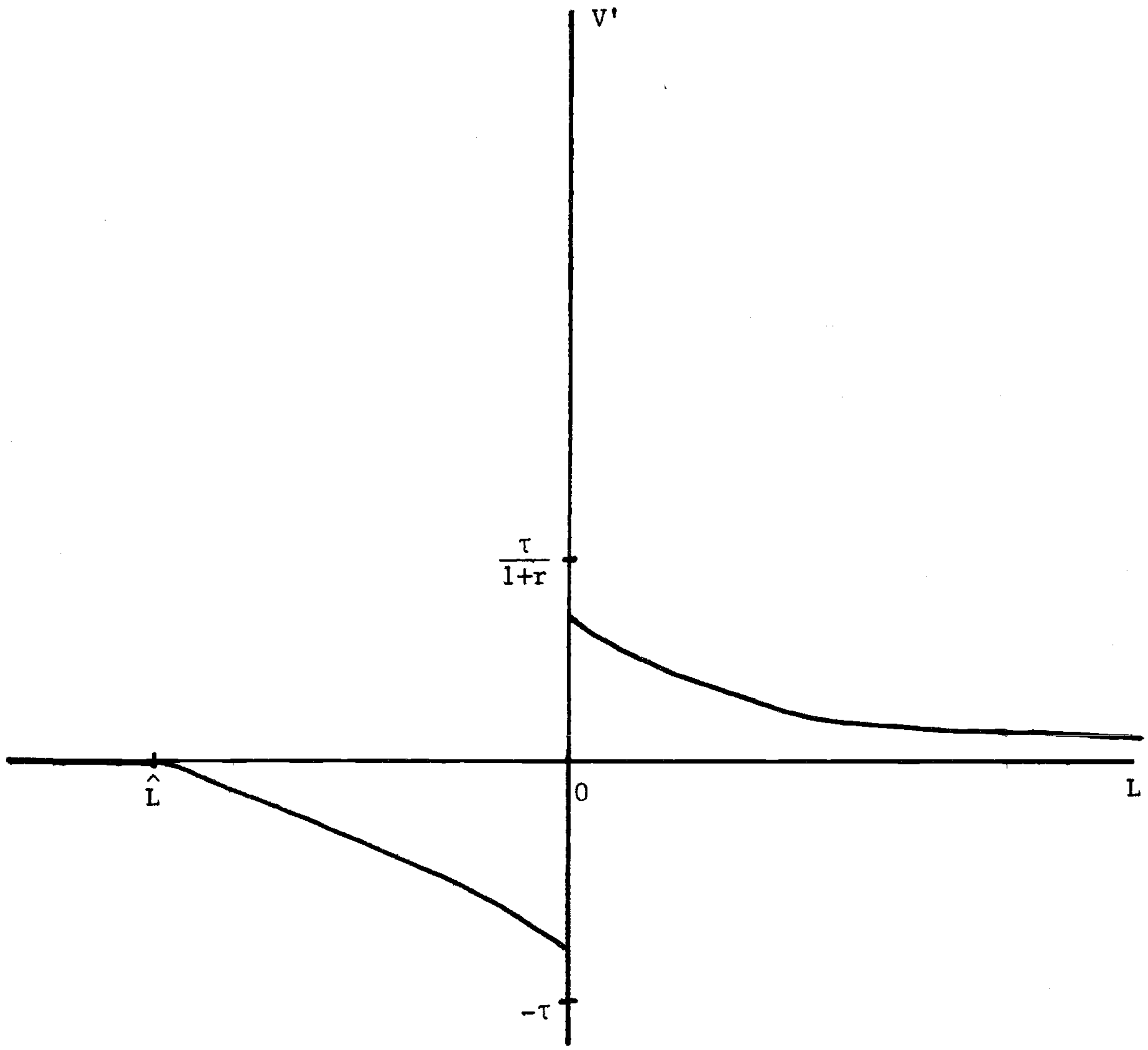
(2) For $L_{t+1} < 0$, $-\tau < V'_{t+1} < 0$

(3) There exists some value of L_{t+1}, $\hat{L}_{t+1}$, such that if $L_{t+1} < \hat{L}_{t+1}$, then $V'_{t+1} = 0$.

An example of what $V'_{t+1}(\cdot)$ might look like is given in Figure 3. The possible discontinuity at L = 0 is not surprising, in light of the fact that L is really a combination of two separate variables, the loss carried forward, if positive, and minus the current taxable profit, if negative. Values of L below

Figure 3

Derivatives of the Value Function
Under a Cash Flow Tax



$\hat{L}$ correspond to those for which all new investment will be expensed with profits to spare. Since a small change in profits has no effect on this outcome, nor on the loss carried forward to the end of the next period, $V' = 0$. The range of L for which $V' < 0$ corresponds to cases where increases in L decrease the amount of expensing done, either when $I + L = 0$ and I declines until for unit with increases in L (Case 3 above) or when $I + L > 0$ but $L < 0$, so that some inframarginal investment is expensed (Case 2a above). Once $L > 0$, increases in the loss carried forward increase the value of the firm. This is not entirely clear from intuition alone. Although the losses carried forward provide a shield against future taxes, they also increase the possibility that firms won't be able to expense future investment. However, the first effect always dominates the second.

Little more of a qualitative nature can be said about V. However, these results do allow us to draw certain inferences about investment behavior under a cash flow tax. For Case 1, in which $L + 1 < 0$ ($\delta = 0$), investment still exceeds the no-tax level if gross returns must be nonnegative. We now demonstrate this.

$$x_t' = \frac{r - \tau F(\alpha_t) + E(V_{t+1}')}{1 + \tau F(\alpha_t)E(\theta/\theta < \alpha_t) - E(\theta V_{t+1}')} \tag{20.1}$$

Thus, $x_t' < r$ if and only if:

$$\tau F(\alpha_t)[1 + x'E(\theta/\theta < \alpha_t)] > E(V_{t+1}') + x'E(V_{t+1}') \tag{21}$$

Equation (21) may be written:

$$F(\alpha_t)E[(\tau - V_{t+1}')(1+x'\theta)/\theta < \alpha_t] > (1-F(\alpha_t))E[V_{t+1}'(1+x'\theta)/\theta > \alpha_t] \tag{22}$$

Assuming, as before, that $1+x_t'\theta > 0$ for all θ, we know that the left-hand side of (22) is positive since $V_{t+1}' < \frac{\tau}{1+r}$ for $\theta < \alpha$ ($L_{t+1} > 0$), and that the right-hand side is less than or equal to zero, since $V_{t+1}' < 0$ for $\theta > \alpha$ ($L_{t+1} < 0$).

None of the terms on the right-hand side of (20.1) depend on L_t. Thus, again, the solution for I_t given that $\delta = 0$ is constant, and is an equilibrium as long as the constraint that $L_t + I_t < 0$ is satisfied. Above the value of L_t, $\hat{L}_t$, for which this constraint is just satisfied, the only equilibrium can be with $I_t + L_t > 0$. As before, we can find the equilibrium by letting δ increase until either $L_t + I_t = 0$, or $\delta = 1$ with $L_t + I_t$ still greater than zero. This procedure requires that the right-hand side of (20.1) increases with δ, so that, if the second-order condition for I_t is met, I_t will decrease as δ increases. Letting χ equal the right-hand side of (20.1), we obtain:

$$\frac{d\chi}{d\delta} = r + \tau F(\alpha_t) - E(V_{t+1}') = r + F(\alpha_t)\left[\tau - E(V_{t+1}'\theta < \alpha_t)\right] + (1-F(\alpha_t))E(V_{t+1}'/\theta > \alpha_t) \tag{23}$$

Since $V_{t+1}' < \tau/(1+r)$ for $\theta < \alpha_t$, and $V_{t+1}' < 0$ for $\theta > \alpha_t$, $\frac{d\chi}{d\delta}$ is positive, as required.

Thus, we again have a range over which I_t declines, starting from a value exceeding the no-tax level. When $\delta = 1$, (20) becomes:

$$x' = \frac{r(1+\tau)}{1 + \tau F(\alpha_t)E(\theta/\theta < \alpha_t) - E(\theta V_{t+1}')} \tag{20.2}$$

It is difficult to characterize the behavior of investment in this regime, except to say that, as $L \to \infty$, investment still converges to the no-tax level, as $F(\alpha) \rightarrow 1$ and $V'_{t+1} \rightarrow 0$. We cannot, for example, rule out values of investment in excess of this level. To gain further insights, simulation analysis will be helpful.

V. Simulation Results

In this section, we present simulation results for firms subject to each of the tax systems analyzed above. We also study a variant of the income tax that permits self-selection by firms. Our focus is on the behavior of firms with an infinite horizon, and we consider the properties of the stochastic steady states these firms converge to in their behavior. In particular, after we have solved for the level of investment a firm will choose in response to each level of the loss carryforward, we use this decision rule to generate a steady state probability distribution for the firm's loss carryforward. This, in turn, allows us to calculate the expected value of the firm's investment level in the steady state, a measure of the long-run impact of taxation on the firm. In the absence of the asymmetric treatment of gains and losses, both the income tax and the cash flow tax would have no effect on investment, which would be the same across firms and across states of nature.

The method of solution for a firm's infinite horizon behavior comes from the fact that we can interpret the expressions for firm value in (4) and (14) as functional equations mapping one element of the space of value functions defined on L into another. For example, for the income tax equation (4) defines the mapping:

$$TV(L) = \max_{I}\ (1+r)^{-1} E_{\theta}[\tilde{W}(L,\theta,I) + V(\max(0,L-W(\theta,I)))] \tag{24}$$

from the space $S = \{V(\cdot)\}$ into itself. The class of functions T is bounded, since the firm cannot have value greater than it would with sufficient carryforwards to offset all future profits, and cannot be worth less than its value if it simply chose not to invest. Further, they satisfy Blackwell's sufficient conditions for a contraction mapping, namely[8]

<u>Monotonicity</u>: For V, $V^* \in S$, $V(L) < V^*(L)\ \forall\ L \rightarrow TV(L) < TV^*(L)$

This follows from the fact that we may fix the investment function $I(L)$ at the value defined by the mapping T of $V(L)$, $\tilde{I}(L)$, and consider the mapping $\tilde{T}$ from S into itself holding $I(L) = \tilde{I}(L)$. Clearly, $TV(L) = \tilde{T}V(L)$ Moreover, since $\tilde{W}(L,\theta,\tilde{I}(L))$ is the same, given any L and θ, and $V^*(\max(0,L-W(0,\tilde{I}(L)))) > V(\max(0,L-W(\theta,\tilde{I}(L))))$ for any L and θ, $\tilde{T}V(L) < \tilde{T}V^*(L)$. Finally, since $I(L)$ is not fixed under T, $\tilde{T}V^*(L) < TV^*(L)$.

<u>Discounting</u>: $T(V(L) + a) = TV(L) + \beta a$, for $\beta < 1$. Here, $\beta = (1+r)^{-1}$.

Because T is a contraction mapping, of modulus $(1+r)^{-1}$, it has a unique fixed point $T: \bar{V} \rightarrow \bar{V}$, that can be solved for recursively using the fact that, for the norm $||\ ||$ defined as the maximum difference among elements of two vectors,

$$||TV - \bar{V}|| = ||TV - T\bar{V}|| < \frac{1}{1+r}\ ||V - \bar{V}|| \tag{25}$$

That is, the greatest distance from the equilibrium decreases at least by a factor of $(1+r)$ with each iteration. We may solve for $\bar{V}(\cdot)$ by beginning with the value function obtained from the two-period model above, and working backward. The economic interpretation of this is that we are solving for the value function of a problem with a T period horizon, and letting $T \to \infty$. (A similar analysis holds for the cash flow tax.)

For our actual simulations, we solved in each period for the function $V(\cdot)$ over a grid of size 200 for L, with I also taking one of 200 possible values. The parameters for each grid were based on different upper and lower bounds for each, depending on the problem. In practice, we assumed a fixed point had been reached when the entire investment function I(L) remained constant over an iteration, since very small changes in $V(\cdot)$ continued to occur for many more iterations.[9]

The values for τ and r in all simulations were .5 and .3, respectively, the former chosen as a realistic value, the latter because by our assumption of immediate capital decay after one period, a period should be thought of as lasting longer than one year. In addition, of course, larger values of r lead to faster convergence of the algorithm.

The payoff function used was $x(I) = 5I^{1/2}$ while the distribution of θ was assumed to be uniform, ranging from (2-C) to C, with $f(\theta) = \frac{1}{2(1-C)}$. For all results reported in the paper, a value of C = 5 was used. Finally, to represent firm differences, we performed simulations for B = 5 and B = 15. Obviously, these values are arbitrary, and are intended simply to illustrate the patterns possible for the infinite horizon model.

The results of these simulations for both income and cash flow taxes are graphed in Figure 4. Qualitatively, they are exactly like those shown in Figure 2 for the two-period model. For positive values of L, the income tax always leads to more investment, with a higher value of B leading to more investment under each system, except where I decreases with L, for small positive values of L. For the cash-flow tax, with L negative, investment is initially substantially above the no-tax investment level of 69.4, dropping sharply once the constraint on expensing becomes binding, and eventually rising but lower than under the income tax. It is reassuring how similar those findings are to those predicted by the static model.

With the results of these simulations, we can also solve for the probability distribution of L in the stochastic steady state. The method used is to solve recursively for the probability distribution over the grid defined for L by taking an initial guess for some period and solving for the resulting distribution in the next period, doing so until a stationary probability distribution is reached, i.e., where the probability distribution for L is the same from one period to the next. This, along with the decision rule for investment, defines the stochastic steady state.

These steady state probability distributions for L under each of the four simulations graphed in Figure 4 are shown in Figure 5.[10] For the income tax simulations, L cannot be negative and there is a probability mass at L = 0. Not surprisingly, the means of the distributions for the inefficient firm (B = 15) lie to the right of those for the efficient firm (B = 5). From a comparison of Figures 4 and 5, we may draw a number of conclusions. First,

Figure 4
Investment versus Loss Carryforward: Simulation Results

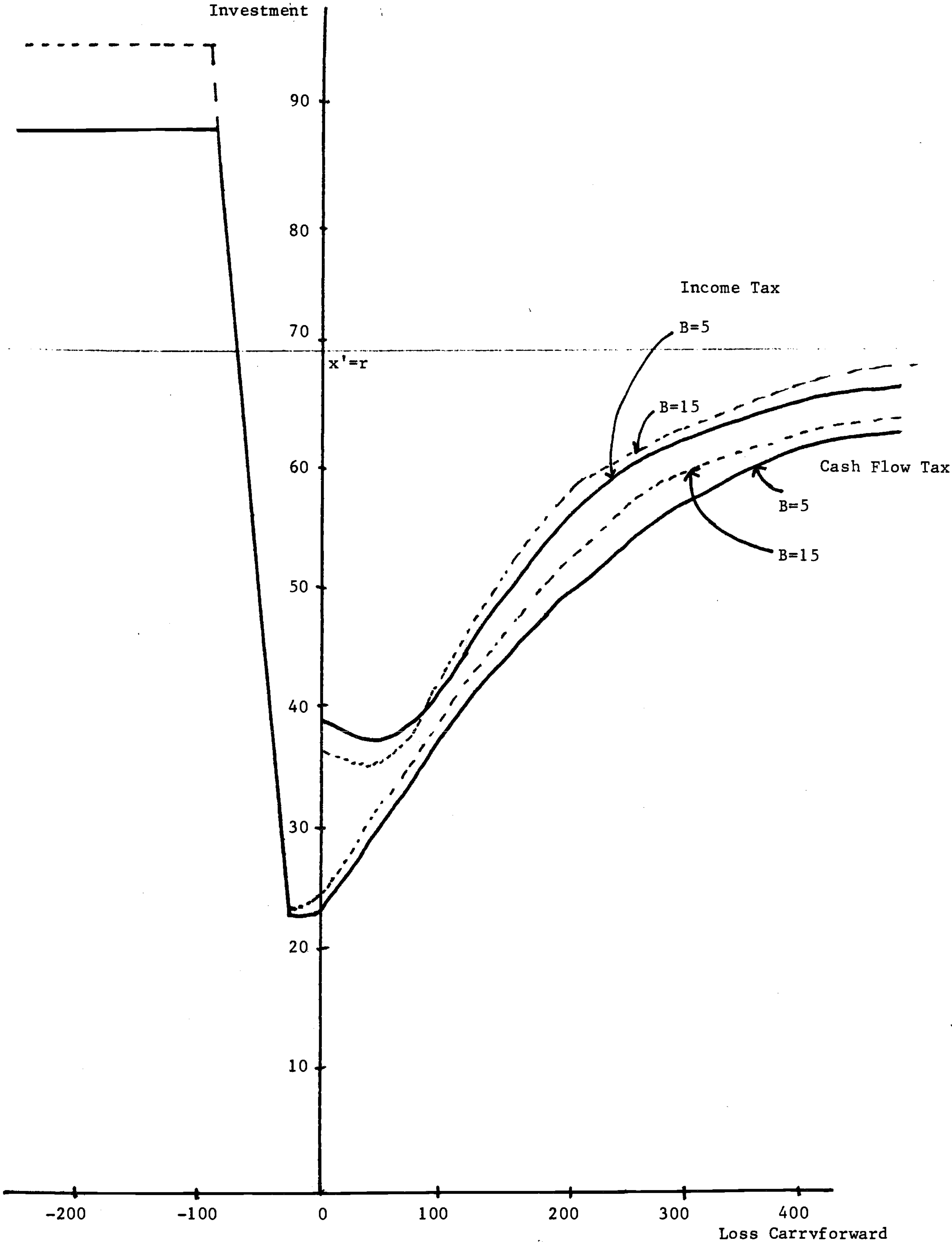
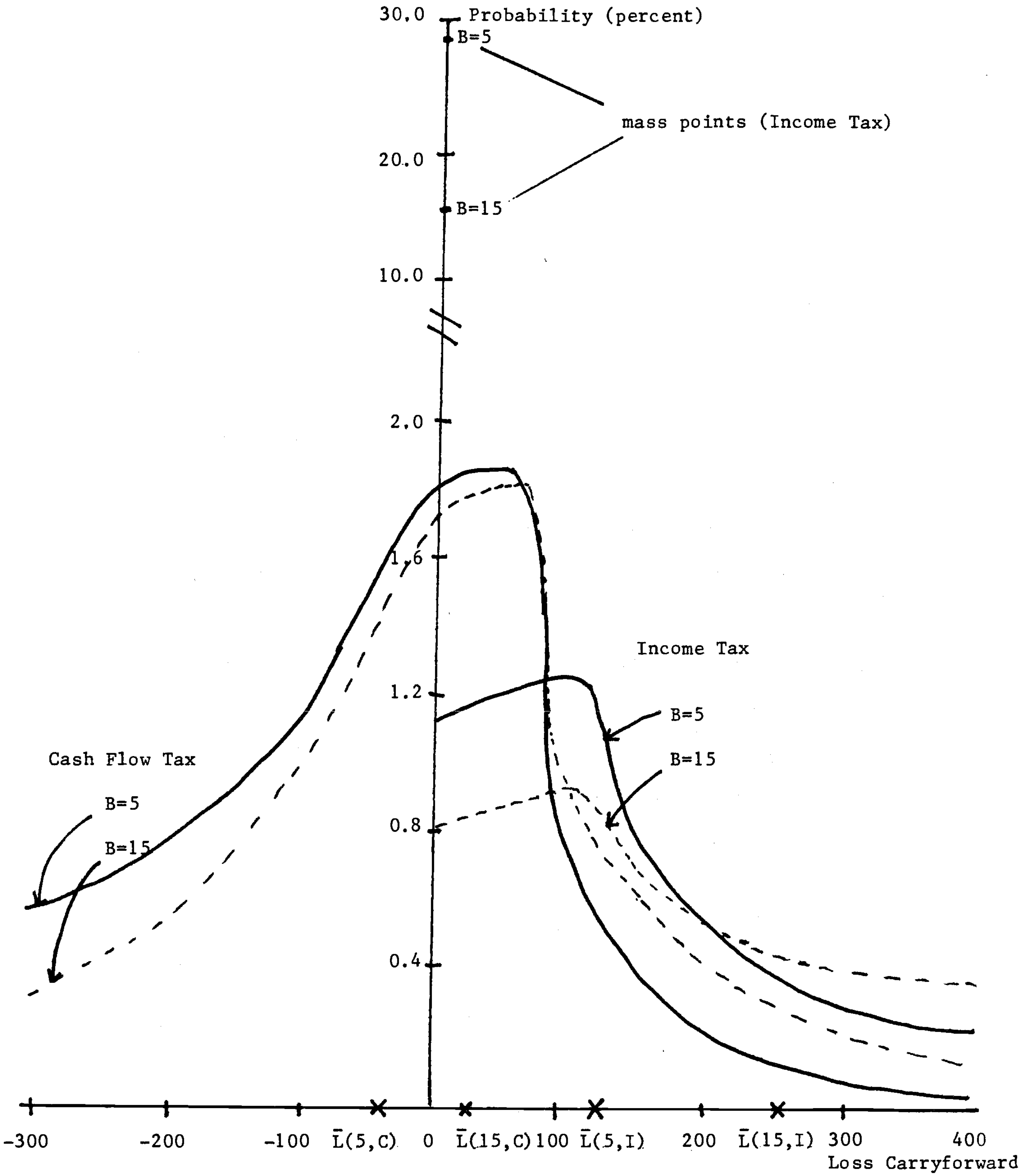


Figure 5

Steady State Probability Distributions



under each tax system, the probability is concentrated where investment is lowest. For the income tax, the shift in the distribution as B increases seems clearly to favor investment by the inefficient firm. The decline in I for small values of L is very small compared to increases thereafter. Indeed, the expected values of investment for the high-B and low-B firms are 52.7 and 46.3, respectively, compared to a no-tax investment level of 69.4. Under the cash flow tax, on the other hand, the effect of the distribution shift as B changes is much less clear, since the investment function is much more nonmonotonic in the relevant range. The firm with high B is more likely to have a big loss carryforward, which encourages investment. This is comparable to the effect of the income tax. In addition, however, such a firm is much less likely to be able to expense all or part of its investment, which discourages investment. In the end, these two effects nearly cancel, as investment averages 56.1 when B = 15 and 56.0 when B = 5.

The rather perverse result that inefficient firms contract less than efficient ones in response to income taxation is due to the fact that they are more likely to have big loss carryforwards, and hence the incentive to increase investment to use these losses up before they decay. A revision of the tax system (suggested in Auerbach, 1982b) that might change this outcome without having to rely on information about the firm's characteristics would be the introduction of an option for firms to cash in their loss carryforward at any time, for a discount. The logic is that, once losses are cashed in, firms lose the incentive to increase investment. Moreover, one would expect inefficient firms to cash in at lower values of L, since they would expect to have to wait longer to use their accumulated losses up. Thus, we would expect the system to have a greater impact on the firms with high B. A further impact on all firms would be the increase, at low values of L, in the amount of investment, since losses incurred would now have a lower bound on the extent to which they could be recouped. Thus, we would expected a flattening of the investment schedules shown in Figure 4.

All of these predictions are satisfied in our simulations. Letting p equal the value at which all losses can immediately be taken ($p < (1+r)^{-1}$), we simulate optimal behavior for infinite horizon firms and the resulting probability distributions for L. This involves calculating the value of L at which cashing in becomes worthwhile. Since cashing in involves a reduction in the loss carried forward from L to zero, the value of L at which cashing in becomes worthwhile is defined by the equality:

$$p\tau L = V(L) - V(0) \tag{26}$$

where, of course, $V(\cdot)$ differs from earlier simulations in reflection of the change in the tax system. We have assumed uniqueness for the values of L so defined, in our simulations. Table 1 shows the effect of changes in p for B = 5 and B = 15.

As p increases, each type of firm chooses to cash in its loss carryforward at a lower value of L. This is because the cost of doing so becomes sufficiently small that only relatively small tax losses, which the

Table 1

Income Tax with Option

	B = 5				B = 15			
p	I(0)	$\bar{\bar{I}}$	Pr (L=0)	Lmax	I(0)	$\bar{\bar{I}}$	Pr (L=0)	Lmax
0	38.9	46.3	.28	∞	36.1	52.7	.16	∞
.25	38.9	43.6	.35	433	36.1	44.2	.31	367
.40	39.6	42.9	.44	229	38.2	43.1	.43	200
.55	44.4	45.6	.61	108	44.4	45.3	.69	88

firm can expect to work off quickly, are carried forward. The high-B firm always chooses a lower cash-in value for L, denoted Lmax, than the low-B firm, and the probability of its having no loss to carry forward increases more rapidly with p, until it is more likely to have a zero carryforward for $p = .55$. As p increases, there are offsetting effects on the average value of I for each firm, denoted $\bar{I}$. The distribution of L shifts toward zero, but the amount of investment for low values of L, such as the value at $L = 0$ shown in the table, increases. The net effect is not monotonic, as $\bar{I}$ first falls then, as the tax system approaches one with full loss offset, rises. As suggested, the difference between average investment for low-B and high-B firms declines until, at $p = .55$, the efficient firms invest more, on average.

VI. Conclusions

The results in this paper, both analytical and from simulation, demonstrate the importance of the asymmetric treatment of gains and losses under otherwise neutral tax systems, and emphasize how tax systems that are similar in the presence of a loss offset become quite dissimilar in the absence of one. Under an income tax with interest deductibility, the lack of loss offset discourages investment, with the problem most severe for small positive values of a tax loss carryforward. As the tax loss carryforward increases, the problem disappears, as firms become, essentially, tax exempt. The presence of high fixed costs, by making firms more likely to have large tax losses carried forward, leads to greater expected investment in the simulated steady states.

Under a system of expensing, similar analysis applies to situations in which firms must carry losses forward. However, when they have sufficient profits to offset the deduction for current investment, firms actually invest <u>more</u> than they would in the absence of taxation or with a perfect loss offset. This is because while current costs are deductible, future gross rents, in expected value, are not necessarily taxable, at the margin. The firms most likely to be in this position are those with low fixed costs. Hence, there is an offset to the encouragement of high fixed cost firms to invest relatively more, and in our simulation results these two effects are essentially offsetting.

The pattern of investment under a system of expensing is helpful in explaining some of the ambivalence and confusion surrounding the recent debate over safe-harbor leasing. In terms of the current analysis, the problem lay in giving firms the ability to expense current investment, while still carrying pre-existing losses forward. For companies with losses caused only by the investment deductions, this would lead to their being like companies for which, in our model, $\delta = 0$ $(L + I < 0)$. However, for companies already essentially "tax-exempt," this would permit more investment than in the absence of taxation. In our model, this would lead to a marginal product $x' = r(1-\tau)$ in the limit and, for the payoff function used in our simulations, to a level of investment <u>four times</u> as big as in the no-tax situation. Hence, transferability of investment incentives for such firms would cause them to invest more than profitable firms. It must be noted, however, that the coexistence of interest deductibility and the expensing deduction makes this less clear, since profitable firms get further potential tax benefits, while those with large losses do not.[11]

The fact that firms with high fixed costs will generally invest more under an income tax relates, in part, to their lower probability of using up loss carryforwards in the near future. These losses have lower value to the firm, presenting a lower opportunity cost to trying to use them up. This intuition suggests that giving all firms a choice to exchange their accumulated losses for a discounted payment will encourage the less efficient firms to "cash in" at lower values of accumulated loss carryforwards, and our simulations support this conjecture. Roughly speaking, the choice mechanism serves to separate firms by type, with those who would increase investment more in response to losses being more likely to cash the losses in, instead. The outcome in the simulations is that, for a sufficiently generous exchange offer, firms with low fixed costs will actually be expected to invest more than high fixed cost firms in the steady state. If the ultimate aim of the tax law's asymmetry is to drive inefficient firms out of business or, in the current context, reduce their scale of operations (and, perhaps, encourage a takeover by the management of the more efficient firm), this type of provision seems helpful, although the problem is less apparent when the tax law provides for expensing.

If it is not the intention of those who make the tax laws to penalize inefficient firms, then it is not clear why the current system is to be preferred to one with a loss offset or, equivalently, one with a provision allowing losses to be carried forward indefinitely with interest.

Appendix

In this appendix, we demonstrate results about the shape of the value function $V(\cdot)$ under both income and cash flow taxes that are asserted in the text.

Income Tax

The value function under an income tax is concave. This is proved by backwards induction.

By the envelope theorem, we obtain from (8):[12]

$$V_t' = \frac{dV_t}{dL_t} = (1+r)^{-1}\left[\tau\int_{\lambda_t}^{\infty} f(\theta)d\theta + \int_{-\infty}^{\lambda_t} V_{t+1}' f(\theta)d\theta\right] \qquad \text{(A1)}$$

Differentiation of (A1) yields:

$$V_t'' \cdot (1+r) = \int_{-\infty}^{\lambda_t} V_{t+1}''\left(1 - (x_t'\theta - r)\frac{dI_t}{dL_t}\right)f(\theta)d\theta + (V_{t+1}'(0)-\tau)f(\lambda_t)\frac{d\lambda_t}{dL_t} \qquad \text{(A2)}$$

where λ_t is as defined in equation (5). After a few steps, (A2) may be rewritten as:

$$V_t'' \cdot (1+r) = \left\{(x_t\lambda_t - r)\frac{f(\lambda_t)}{x_t}(\tau - V_{t+1}'(0)) - \int_{-\infty}^{\lambda_t}(x_t'\theta - r)V_{t+1}'' f(\theta)d\theta\right\}\frac{dI}{dL_t}$$

$$+ \int_{-\infty}^{\lambda_t} V_{t+1}'' f(\theta)d\theta - \frac{f(\lambda_t)}{x_t}(\tau - V_{t+1}'(0)) \qquad \text{(A3)}$$

or $\quad V''(1+r) = \left[ab - \int_{-\infty}^{\lambda_t}(x_t'\theta - r)V_{t+1}'' f(\theta)d\theta\right]\frac{dI}{dL_t} + \int_{-\infty}^{\lambda_t} V_{t+1}'' f(\theta)d\theta - b$

where

$$a = (x'\lambda_t - r); \; b = \frac{f(\lambda_t)}{x_t}(\tau - V_{t+1}'(0)) \qquad \text{(A4)}$$

Total differentiation of the first-order condition (19) with respect to L_t leads, after a few steps, to:

$$\frac{dI_t}{dL_t} = - \frac{[ab - \int_{-\infty}^{\lambda_t}(x_t'\theta - r)V''_{t+1}f(\theta)d\theta]}{x_t''(1 + \int_{-\infty}^{\lambda_t}(\tau - V'_{t+1})\theta f(\theta)d\theta) - a^2 b + \int_{-\infty}^{\lambda_t}(x_t'\theta - r)^2 V''_{t+1}f(\theta)d\theta} \quad \text{(A5)}$$

The denominator of (A5) equals $\partial^2 V_{-t}/\partial I^2$. Under the assumption that $V''_{t+1} < 0$, it too must be negative, since $\tau > V'_{t+1}$, $b > 0$, and since the term multiplied by x_t'' equals the denominator of the right-hand side of (19) (the numerator of which is positive), which must be positive since $x_t' > 0$.

Combining (A3) and (A5), we obtain, after a few steps:

$$V'' \cdot D = x_t''(1 + \int_{-\infty}^{\lambda_t}(\tau - V'_{t+1})\theta f(\theta)d\theta)(\int_{-\infty}^{\lambda_t} V''_{t+1}f(\theta)d\theta - b)$$

$$- b\int_{-\infty}^{\lambda_t} V''_{t+1}(a - (x'\theta - r))^2 f(\theta)d\theta$$

$$+ \int_{-\infty}^{\lambda_t} V''_{t+1}f(\theta)d\theta \cdot \int_{-\infty}^{\lambda_t}(x_t'\theta - r)^2 V''_{t+1}f(\theta)d\theta - (\int_{-\infty}^{\lambda_t}(x_t'\theta - r)V''_{t+1}f(\theta)d\theta)^2$$

where $D < 0$ is the denominator of (A5). Hence, $V_t'' < 0$ if the right-hand side of (A6) is positive. The first two terms clearly are. Dividing the third term through by the positive term $(\int_{-\infty}^{\lambda_t} V''_{t+1}f(\theta)d\theta)^2$ yields a term that may be interpreted as the variance of the variable $(x_t'\theta - r)$ with probability density function $\frac{V''_{t+1}f(\theta)}{\int_{-\infty}^{\lambda_t} V''_{t+1}F(\theta)d\theta}$ defined over the interval $(-\infty, \lambda_t)$. Hence, it too must be positive, and $V''_{t+1} < 0 \rightarrow V_t'' < 0$.

To show that $V''_t < 0$, where $V_{t+1} \equiv 0$, it is sufficient to note that the foregoing arguments hold for $V''_{t+1} < 0$, and hence for $V''_{t+1} = 0$.

Cash Flow Tax

Here, we desire to show that

(1) For $L_t > 0$, $0 < V'_t < \tau/(1+r)$

(2) For $L_t < 0$, $-\tau < V'_t < 0$

(3) For some value of L_t, $\hat{L}_t < 0$, $L_t < \hat{L}_t + V'_t = 0$

We again proceed by induction. The general formula for V'_t is, by the envelope theorem, from (14):

$$V'_t = \delta[-\tau\phi + (1+r)^{-1}(\int_{\alpha_t}^{\infty}\tau f(\theta)d\theta + \int_{-\infty}^{\infty}V'_{t+1}\ f(\theta)d\theta)]$$

$$= \delta[-\tau\phi + (1+r)^{-1}(\tau - \int_{-\infty}^{\alpha_t}(\tau - V'_{t+1})f(\theta)d\theta + \int_{\alpha_t}^{\infty}V'_{t+1}\ f(\theta)d\theta)] \qquad (A7)$$

Clearly, if L_t is less than that value at which the solution for I_t, given $\delta = 0$, satisfies $I_t + L_t < 0$, then $\delta = 0$ and $V'_t = 0$. If $\delta \neq 0$, the maximum value of the term multiplied by $(1+r)^{-1}$ in (A7) occurs when $V'_{t+1} = 0$ for $\theta > \alpha_t$ (since $L_{t+1} < 0$ and $V'_{t+1} < 0$) and $V'_{t+1} = \frac{\tau}{1+r}$ for $\theta < \alpha$. The minimum value occurs if $V'_{t+1} = -\tau$ for $\theta > \alpha$, and $V'_{t+1} = 0$ for $\theta < \alpha$. Thus, for $L_t < 0$ ($\phi = 1$, $0 < \delta < 1$), the maximum value of V'_{t+1} is

$$\delta[-\tau + (1+r)^{-1}(\tau - \tau\int_{-\infty}^{\alpha_t}(1 - \frac{1}{1+r})f(\theta)d\theta)] < 0 \qquad (A8)$$

while the minimum value is

$$\delta[-\tau + (1+r)^{-1}(\tau - \int_{-\infty}^{\alpha_t}\tau f(\theta)d\theta - \int_{\alpha_t}^{\infty}\tau f(\theta)d\theta)] > -\tau \qquad (A9)$$

For $L_t > 0$, the maximum value of V'_t is (since $\delta = 1$):

$$(1+r)^{-1}(\tau - \tau \int_{-\infty}^{\alpha_t}(1 - \frac{1}{1+r})f(\theta)d\theta) < \frac{\tau}{1+r} \tag{A10}$$

while the minimum value is

$$(1+r)^{-1}(\tau - \int_{-\infty}^{\alpha_t}\tau f(\theta)d\theta - \int_{\alpha_t}^{\infty}\tau f(\theta)d\theta) = 0 \tag{A11}$$

For the final period, $V_t' = \delta[-\tau\phi + (1+r)^{-1} \int_{\alpha_t}^{\infty}\tau f(\theta)d\theta]$. This is clearly zero if $\delta = 0$, negative but greater than $-\tau$ if δ is less than or equal to 1 and $\phi = 1$, and positive but less than $\frac{\tau}{1+r}$ if $L_t > 0$ and $\phi = 0$.

Footnotes

1. See Cordes and Sheffrin (1981).

2. This resulted from a combination of depreciation allowances and investment tax credits that were more generous than immediate expensing.

   See Auerbach (1982a).

3. See the descriptions in Auerbach (1982a) and Warren and Auerbach (1982).

4. See footnote 11 and the related text.

5. See Warren and Auerbach (1983).

6. These results are well-known for the certainty case (see King (1975), for example) and are easily extended to the case of uncertainty. By interest deductibility, we mean a deduction for the opportunity cost of funds, regardless of the form of finance.

7. This condition is that $\frac{\partial^2 V}{\partial I^2}$ is negative in the relevant range, not just locally at the optimum. We assume this condition holds throughout the following analysis.

8. See the discussion in Lucas, Prescott and Stokey (1983).

9. Further details available from the author on request.

10. The graphs in Figure 5 are smoothed versions of the actual solutions, which, because of the discrete grid, have small wiggles in them. The means shown are based on the actual solutions.

11. Calculations in Warren and Auerbach (1982) suggest that in the absence of uncertainty, the transfer of incentives through safe-harbor leasing

fell short of providing "tax exempt" firms with the same incentive to invest as fully profitable one, assuming complete debt finance, i.e. the deductibility of interest.

12. Note that it is immediately clear by induction on (A1) that $V'_t > 0$.

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