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FINANCIAL MARKETS AND WAGES

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ABSTRACT

We study a labor market equilibrium model in which firms sign optimal long-term contracts with workers. Firms that are financially constrained offer an increasing wage profile: They pay lower wages today in exchange of higher wages once they become unconstrained and operate at a larger scale. In equilibrium, constrained firms are on average smaller and pay lower wages. In this way the model generates a positive relation between firm size and wages. Using data from the National Longitudinal Survey of Youth (NLSY) we show that the key dynamic properties of the model are supported by the data.

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1 Introduction

The fact that large firms pay higher wages is a well-known stylized fact. Brown and Medoff (1989) and Oi and Idson (1999) provide a review of the empirical studies. In this paper we ask whether financial factors—in addition to other considerations proposed in the theoretical literature—can contribute to explaining the dependence of wages on the size of the employer.

Our interest in understanding the importance of financial factors for the firm size-wage relation is motivated by a set of regularities about the link between the financial characteristics of firms and their size. In general, the view that emerges from the financial literature is that smaller and younger firms face tighter financial constraints, either in the form of lower ability to raise funds or in the form of higher cost of funds. In spite of these regularities, the role played by financial market imperfections in generating the firm size-wage relation has not been studied in the theoretical literature.

We develop a model in which firms sign optimal long-term (implicit) contracts with workers as in Harris and Holmstrom (1982) and Holmstrom (1983). Due to limited enforceability, external investors are willing to finance the firm only in exchange of collateralized capital. If the funds supplied by external investors are limited—that is, the firm is financially constrained—the optimal wage contracts offered by the firm to the workers will be characterized by increasing wage profiles. By paying lower wages today, the firm generates higher cash flows in the current period, *implicitly* borrowing from workers. Because firms with binding constraints operate at a sub-optimal scale—which then expand until they become unconstrained—small firms pay on average lower wages than large firms. Therefore, the model generates a positive relation between the size of the firm and the average wages paid to workers (the firm size-wage relation). At the same time, because constrained firms grow in size, the model also captures the empirical regularity that fast growing firms pay lower wages.

There are two features in the model that explain why firms are able to implicitly borrow from workers beyond what they can borrow from external investors. First, if a worker quits, the firm looses part of the accumulated capital. This could derive from recruiting costs, training expenses and/or enhanced worker's productivity through learning. The firm's loss of valuable capital endows the worker with a punishment tool which is not available to external investors. Second, a worker provides effort in the working place only if he or she believes that the effort will be rewarded by the firm. But when the firm reneges its wage promises, the worker looses its confidence and prefers to quit, since he or she expects the firm to renege the wage promises also in future periods. The treat of quitting guarantees that the firm does not renege the long-term wage contract.

We use data from the 1979-2002 National Longitudinal Survey of Youth (NLSY) to evaluate some of the properties of the model that are key to generate the firm size-wage relation. In particular, we test whether fast growing firms pay lower wages initially on the promise of higher future wages. We find that firm's growth rates have a negative effect on wages but positive effects on the return to tenure and labor market experience. This is what our model predicts.

The plan of the paper is as follows. In the next section we review the main empirical and theoretical contributions to the study of the firm size-wage relation. Section 3 describes the basic theoretical framework and characterizes the firm's dynamics. Section 4 extends the model to allow for firms' and workers' turnover, derives the labor market equilibrium and studies its properties numerically. Section 5 shows that the results are robust to the assumptions of job-to-job mobility and transferability of worker-specific capital. Section 6 describes how the long-term contract can be sustained as a sub-game perfect equilibrium of the strategic interaction between the firm and its worker. Section 7 conducts the empirical analysis and Section 8 concludes.

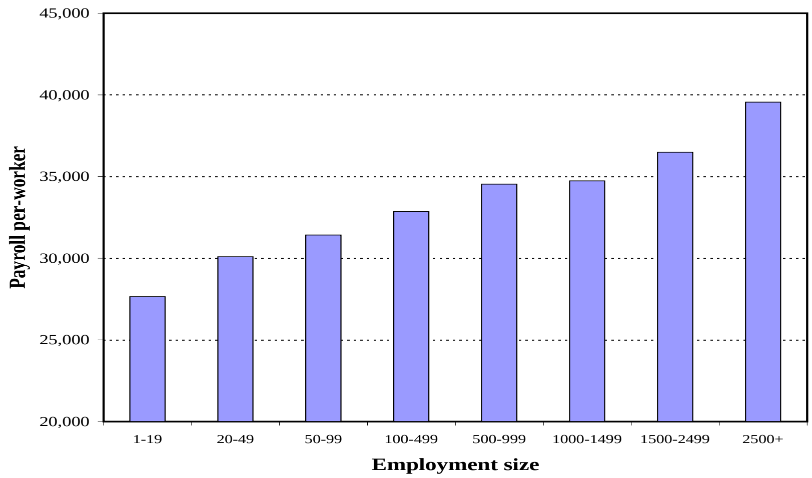
2 Empirical regularities and existing theories

Before describing our theoretical framework, we briefly review the main empirical regularities and theoretical contributions to the study of the firm size-wage relation. The review of the theoretical literature shows that the effect of firm size on wages is still an unresolved puzzle while some of the empirical findings suggest that financial factors could play an important role.

2.1 Empirical regularities

Figure 1 plots the payroll per-worker for different size classes of firms, which is increasing in the size of firms. This is the typical pattern in almost all industries and is robust to the introduction of several controls for worker's and firm's characteristics. See Brown and Medoff (1989) and Oi and Idson (1999).

Figure 1: Firm size and wages in 2001.



Source: U.S. Small Business Administration, Office of Advocacy

There are many factors that could generate the positive relation between firm size and wages. For instance, the fact that larger firms employ more skilled workers. However, using matched employer-employee data, recent studies have reached the conclusion that the effect of firm size on wages is mostly explained by variation in firms' characteristics rather than workers' characteristics. In particular Abowd and Kramarz (2000) report that both in France and in the US, variation in firms' characteristics explains about 70 per cent of the firm size-wage differential. In addition to this result, there are other important findings in the empirical literature that are relevant for our paper. We summarize them below.

1. *Fast growing firms pay lower wages.* Bronars and Famulari (2001) and Hanka (1998) report that firm's growth (in terms of employment and sales) has a negative effect on wages in a regression that controls for several workers' and firms' characteristics.

2. *Firms that are in financial distress have lower employment and pay lower wages.* Nickell and Wadhwani (1991) document the negative relation between debt and employment. Other studies provide some evidence that indicators of financial pressure are associated with lower wages. See Nickell and Nicolitsas (1999), Hanka (1998), Blanchflower, Oswald, and Garrett (1990).

3. *The link between firm age and wages is not clear-cut.* Doms, Dunne, and Troske (1997) find that the effect of firm age on wages is positive if we do not control for worker's characteristics but it becomes negative (albeit not significant) if we control for worker's experience. The same pattern is documented by Troske (1999) and Brown and Medoff (2003).

4. *Indirect indicators point out that small firms tend to be more financially constrained.* Small firms pay fewer dividends and have higher value of Tobin's q. They rely more on bank financing and their growth is sensitive to cash flows. See for example Fazzari, Hubbard, and Petersen (1988), Gilchrist and Himmelberg (1996), Ross, Westerfield, and Jordan (1993) and Smith (1977).

These empirical findings are important to evaluate our theoretical contribution in generating a firm size-wage relation. Before presenting the theoretical model, however, we summarize the existing theoretical contributions and how they relate to the above empirical findings.

2.2 Existing theories

There are several contributions in the theoretical literature that try to explain the firm size-wage relation. These contributions, however, are not fully successful in solving the puzzle. This view is clearly stated in Troske (1999) who concludes: "*After testing several possible explanations we are still left with the question: why do large firms pay higher wages?*". Following is a brief description of the main theoretical contributions.

1. *Sorting of high skilled workers in large firms.* This tends to occur if large firms either employ workers of higher quality or provide workers with higher incentives to accumulate general human capital, as in Zabojnik and Bernhardt (2001). If sorting was the only mechanism, then the firm size-wage relation should become insignificant after controlling for workers' skills. However, after controlling for several workers' characteristics, the effect of firm's characteristics remains large, see for example Brown and Medoff (1989) and Abowd and Kramarz (2000). The model studied in Kremer and Maskin (1996) emphasizes the complementarity that arises from matching high skilled workers in the same firm. In this way, the effect of sorting on wages could possibly translate into a firm's fixed effect that any single worker's characteristic fails to capture. Yet, the inclusion of measures of average workers' skill into a standard wage regression does not reduce significantly the size of the firm size-wage effect. See Bayard and Troske (1999).

2. *Efficiency wages.* In an efficiency wage model a la Shapiro and Stiglitz (1984), large firms may pay higher wages because detecting shirking is more difficult. Some empirical evidence is not fully consistent with this explanation. For example, there are no differences in the magnitude of the firm size-wage effect between production and non-production workers (see Brown and Medoff (1989)) or supervisory and non supervisory workers (see Troske (1999)). Moreover, the magnitude of the effect does not change after conditioning on the number of workers receiving incentive pay (again, see Brown and Medoff (1989)).

3. *Wage bargaining.* In bargaining models, wages increase with the net surplus generated by the job and with the bargaining power of workers. This theory can explain why wages are positively related to the size of the firm only if either the bargaining power of workers or the value of the job increases with the firm's size. However, the inclusion of variables that proxy for the bargaining power of workers, such as union-density or union-coverage, or the inclusion of variables that proxy for the value of the job such as firm's profit, firm's capital or severance payments, do not eliminate the significance of the firm size-wage effect. See Brown and Medoff (1989).

4. *Burdett and Mortensen's model.* In Burdett and Mortensen (1998) firms face a trade-off between paying high wages to attract and retain a large number of workers or paying low wages but with fewer workers hired and retained. In equilibrium there are firms that pay low wages and remain small and firms that pay high wages and become large. This model does not seem to capture the fact that fast growing firms tend to pay lower wages. In fact, firms that grow faster are the ones that pay higher wages. It should be pointed out, however, that this is only a conjecture since the firm dynamics generated by this model has not been fully explored. Similar considerations apply to the model studied in Burdett and Coles (2003).

The goal of our paper is to provide an additional explanation for the firm size-wage relation in which financial markets frictions play a central role. The importance of financial factors for the firm size-wage relation has not been previously studied in the theoretical literature.[1]

[1]Brown and Medoff (1989) and Oi and Idson (1999) hint a potential link. They conjecture that financial market imperfections increase the cost of capital for small firms and induce them to choose lower capital intensity. In a model with wage bargaining, the lower capital intensity implies that these firms pay lower wages. However, in empirical studies,

3 The basic model

We start describing a simple version of the model to illustrate the basic firm and wage dynamics that are key to generate the firm size-wage relation. In this model firms face a deterministic problem and they live forever. The analysis of the simple model will facilitate the understanding of the general model studied in Section 4.

Consider a risk-neutral infinitely lived entrepreneur with initial wealth a_0 and with lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t c_t$$

where β is the intertemporal discount factor and c_t is consumption.

The entrepreneur has the managerial skills to run an investment project that generates revenues $y = A \cdot N$. The variable N denotes the number of hired workers and A is a constant. The project is subject to the capacity constraint $N \leq \overline{N}$. In the general model studied in Section 4, the capacity constraint $\overline{N}$ is allowed to differ across entrepreneurs or firms.

The employment of each worker requires two types of fixed investment: fungible investment κ_f and worker-specific investment κ_w. The first type of investment, κ_f, has an external value and can be resold at no cost. The second type, κ_w, represents the cost incurred by the firm for recruiting and training a new worker. This is lost if the worker quits or is fired. We will denote by $\kappa = \kappa_f + \kappa_w$ the sum of the two components. The total capital accumulated at the end of time t by a firm created at time zero is $\kappa \sum_{\tau=0}^{t} n_\tau$, where n_τ is the number of workers hired at time τ (who start producing at time $\tau + 1$). The output produced by the firm at $t+1$ is $A \sum_{\tau=0}^{t} n_\tau$.

Workers are infinitely lived with lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \Big[U(c_t) + \ell_t \Big], \qquad U(c_t) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

where β is the discount factor, σ is the coefficient of relative risk aversion, c_t is consumption, and $\ell_t \in \{0, \overline{\ell}\}$ denotes the utility of leisure which is forgone when the worker provides working effort. The assumption that there is some forgone utility is relevant only for the analysis of renegotiation studied in

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the firm size effect remains significant even if we control for the capital intensity and the productivity of the firm. As we will show in the next sections, the financial mechanism proposed in our paper does not rely on the capital intensity of the firm.

Section 6. In equilibrium workers provide effort and in the analysis that precedes Section 6 we impose $\ell_t = 0$.

Workers do not have any assets and can not borrow by pledging their future labor income. We also assume that workers cannot save, and therefore, consumption is simply equal to their wages.[2]

Funds are provided by *investors* who are risk-neutral and discount future payments at rate r. The individual supply is infinitesimal, but the aggregate number of investors is large enough to guarantee that the aggregate supply of funds is perfectly elastic at rate r. This implies that financial markets are perfectly competitive and the equilibrium interest rate is r. We assume that $\beta \leq 1/(1+r)$ so that internal financing does not dominate external financing.

The investment $\kappa = \kappa_f + \kappa_w$ necessary to employ a worker is what creates the financial need. Using the renegotiation idea of Hart and Moore (1994) and Kiyotaki and Moore (1997), the entrepreneur can borrow only the amount that can be collateralized, that is, the fungible capital κ_f. Since the collateral must also guarantee the interests on the loan, the firm can borrow at most $\bar{\kappa}_f = \kappa_f/(1+r)$, per each worker. The borrowing limit, then, can be written as $b_t \leq \bar{\kappa}_f \sum_{\tau=0}^{t} n_\tau$, where b_t denotes the debt contracted at time t. We will show in Section 6 that this is the only feasible contract with investors.

When a worker is hired, the firm signs a long-term contract that specifies the whole sequence of wages. By assuming that the labor market is *competitive*, the lifetime utility provided by the contract to the worker is equal to the utility earned by re-entering the labor market. This utility, denoted by q_{res}, is exogenous in the simple version of the model.

3.1 The firm's problem

We start analyzing the optimization problem assuming that firms and workers commit to the long-term contracts. In Section 6 we will describe the conditions under which the parties (firms and workers) never renege on their promises and the contract can be supported as a sub-game perfect equilibrium of the repeated game played by the firm with each individual worker.

Let $\{w_{t,t+j}\}_{j=1}^{\infty}$ be the sequence of wages that the firm promises to the workers hired at time t. Here $w_{t,t+j}$ denotes the wage paid at time $t+j$ to

[2]This is without loss of generality. Because we assume that the return from savings is smaller than $1/\beta - 1$ and wages do not decrease over time, the worker would not save even if he or she were allowed to. For the general model of Section 4, it is further required that β is sufficiently small. This condition is satisfied for the chosen parametrization.

workers hired at time t. Then the total wage payments at time $t+1$ are $\sum_{\tau=0}^{t} n_\tau w_{\tau,t+1}$. Let a_t denote the *net worth* at the end of period t—that is, after production and after the payment of wages and interests. The sum of the firm's net worth, a_t, and debt financing, b_t, equals the sum of firm's capital, $\kappa \sum_{\tau=0}^{t} n_\tau$, and dividend payments, d_t. Thus, $d_t = a_t + b_t - \kappa \sum_{\tau=0}^{t} n_\tau$.

Given the initial assets a_0, the firm maximizes the discounted value of the entrepreneur's consumption, which always equals dividends since the entrepreneur is at least as impatient as the market, $\beta \leq 1/(1+r)$. Thus, at time zero, the firm chooses the whole sequence of debt, employment and wages, $\left\{b_t, n_t, \{w_{t,t+j}\}_{j=1}^{\infty}\right\}_{t=0}^{\infty}$, to solve the problem:

$$V(a_0) \quad = \quad \max \sum_{t=0}^{\infty} \beta^t \left(a_t + b_t - \kappa \sum_{\tau=0}^{t} n_\tau \right) \tag{1}$$

subject to

$$a_t + b_t - \kappa \sum_{\tau=0}^{t} n_\tau \geq 0, \tag{2}$$

$$b_t \leq \bar{\kappa}_f \sum_{\tau=0}^{t} n_\tau, \tag{3}$$

$$\sum_{j=1}^{\infty} \beta^j U(w_{t,t+j}) \geq q_{res}, \tag{4}$$

$$a_{t+1} = (\kappa + A) \sum_{\tau=0}^{t} n_\tau - \sum_{\tau=0}^{t} n_\tau w_{\tau,t+1} - (1+r) b_t, \tag{5}$$

which all have to hold for any $t \geq 0$. Constraint (2) imposes the non-negativity of dividends. This results from the limited liability of the entrepreneur together with the non-negativity of consumption. Constraint (3) imposes the borrowing limit and (4) is the worker's participation constraint. This imposes that the sequence of wages offered to each cohort of new recruits cannot be smaller than the reservation value q_{res}. This constraint should be imposed not only when the worker is hired, but also in all future periods. However, as we will show below, wages never decrease. Therefore, if the participation constraint is satisfied when the worker is hired, it will also be satisfied at any future date. Finally, constraint (5) defines the law of motion for the end-of-period net worth.

Let γ_t and $\lambda_t n_t$ be the lagrange multipliers associated with the constraints (2) and (4), respectively. Then Appendix A shows that the first order conditions imply that

$$\lambda_\tau U_c(w_{\tau,t}) = 1 + \gamma_t, \tag{6}$$

where U_c denotes the marginal utility of consumption. The variable λ_τ is the marginal cost to the firm of providing one unit of utility to a worker hired at time τ. Thus, the term $\lambda_\tau U_c(w_{\tau,t})$ represents the marginal cost of reducing wages. The term $1 + \gamma_t$ is the value of one additional unit of internal funds. Therefore, equation (6) says that the optimal wage policy of the firm is such that the marginal cost of reducing wages is equal to the marginal value of internal funds. In other words, the firm 'borrows' from a worker until the cost of borrowing is equal to the marginal value of internal funds.

The multiplier γ_t captures the tightness of financial constraints and depends on the firm's net worth a_t. If a_t is small, the financial needs of the firm are high which imply that the value of an extra unit of internal funds is high. As the firm retains earnings, its assets increase over time and the variable γ_t converges to zero. Then, equation (6) implies that:

Property 1 *The wage received by each worker grows over time until the firm becomes unconstrained, that is,* $\gamma_t = 0$.

Equation (6) also implies that the ratio of marginal utilities between workers of different cohorts remains constant over time. If we consider (6) for two different cohorts indexed by τ_1 and τ_2, and we divide side by side we obtain that

$$\frac{U_c(w_{\tau_1,t})}{U_c(w_{\tau_2,t})} = \frac{\lambda_{\tau_2}}{\lambda_{\tau_1}}.$$

Since the right-hand-side does not depend on t, this condition implies that:

Property 2 *The ratios of marginal utilities between workers of different cohorts remain constant over time.*

In the next section we take advantage of this property to rewrite the problem recursively with a limited number of state variables. The recursive formulation will be convenient in the next section when we study the general model with entry and exit.

3.2 Recursive formulation of the firm's problem

Let $q_{\tau,t} = \sum_{j=1}^{\infty} \beta^j U(w_{\tau,t+j})$ be the lifetime utility promised at the end of time t to a worker hired at time τ, with $\tau \leq t$. Notice that $q_{\tau,t}$ follows the recursive form

$$q_{\tau,t} = \beta \Big[U(w_{\tau,t+1}) + q_{\tau,t+1} \Big] \tag{7}$$

with $q_{\tau,\tau} = q_{res}$.

With the utility function $U(c) = \left(c^{1-\sigma} - 1\right)/(1-\sigma)$, Property 2 implies that the ratios of wages paid to workers of different cohorts remain constant over time.[3] This property also implies that the ratios of lifetime utilities promised to different cohorts of workers remain constant over time. Thus, if we consider the last and the first cohort of workers, we have that, at any given point in time, their relative lifetime utilities and wages are linked by

$$\frac{q_{t,t}}{q_{0,t}} = \left(\frac{w_{t,t+1}}{w_{0,t+1}} \right)^{1-\sigma} = \frac{q_{res}}{q_{0,t}},$$

where the last equality uses the fact that $q_{t,t} = q_{res}$. Inverting the second equality provides an expression for the wage ratio between the cohort hired at time t and the cohort hired at time zero, which reads as

$$\frac{w_{t,t+1}}{w_{0,t+1}} = \left(\frac{q_{res}}{q_{0,t}} \right)^{\frac{1}{1-\sigma}} = \psi(q_{0,t}).$$

From now on we omit the zero subscript to identify the first cohort of workers. Therefore, w_t and q_t denote the time-t wage and promised utility of the first cohort of workers. The total wage payments paid by the firm at time t can be written as $H_t w_t$, where

$$H_t = \sum_{\tau=0}^{t-1} \psi(q_\tau) n_\tau,$$

which evolves recursively as

$$H_{t+1} = H_t + \psi(q_t) n_t. \tag{8}$$

[3] This implies that cohort of workers who earn more on entry maintain their advantage over time. The existence of these cohort effects in the wage policy of the firm is documented by Baker, Gibbs, and Holmstrom (1994)

Once we know H_t and the utility promised to the first cohort of workers, q_t, the determination of the whole wage structure paid by the firm at time $t+1$ only requires the determination of the wage for the first cohort of workers, that is w_{t+1}. This allows us to write the firm's problem recursively with a limited number of state variables as follows:

$$V(a, q, N, H) \;=\; \max_{b, w', q', N' \leq \overline{N}} \Big\{ a + b - \kappa N' + \beta V(a', q', N', H') \Big\} \tag{9}$$

subject to

$$a + b - \kappa N' \;\geq\; 0, \tag{10}$$

$$b \leq \bar{\kappa}_f N', \tag{11}$$

$$q = \beta \Big[U(w') + q' \Big], \tag{12}$$

$$a' = \kappa N' + AN' - H'w' - (1+r)b, \tag{13}$$

$$H' = H + \psi(q)(N' - N). \tag{14}$$

The variable N denotes the current employment of the firm and the prime denotes the next period value. Thus $N' - N$ is the change in employment, that is, the number of workers hired in the current period (who start producing in the next period). Constraints (10) and (11) impose the non-negativity of dividends and the borrowing limit, respectively. Equation (12) is the promise-keeping constraint for the first cohort of workers. Finally, equations (13) and (14) are the law of motion for the states a and H, respectively.

Let γ and $\lambda H'$ denote the lagrange multipliers associated with constraints (10) and (12), respectively. Appendix B shows that the first order conditions of the above problem imply that

$$\lambda U_{w'} \;=\; 1 + \gamma', \tag{15}$$

$$\lambda \;=\; \lambda'. \tag{16}$$

The first condition is analogous to (6) while the second says that the lagrange multiplier for the worker's participation constraint is constant over time.

These two conditions characterize the wage dynamics of the firm. As observed in the previous section, the lagrange multiplier γ decreases over time until it becomes zero. From equation (15) we can see that the wage paid to the first cohort of workers increases over time until $\gamma' = 0$. Because the wages paid to all other cohorts of workers are proportional to the wage paid to the first cohort, we also have that the average wages increase over time until $\gamma' = 0$. Wages differ across workers of different cohorts. In fact, because all workers start with $q = q_{res}$, after which the promised utility grows over time, older workers receive higher wages than younger workers. One of the predictions of the model is that the wage profile of constrained (young) firms is steeper than the wage profile of mature (old) firms.

Once the firm becomes unconstrained, that is, $\gamma = 0$, the firm would like to increase employment beyond $\overline{N}$, but the capacity constraint binds.

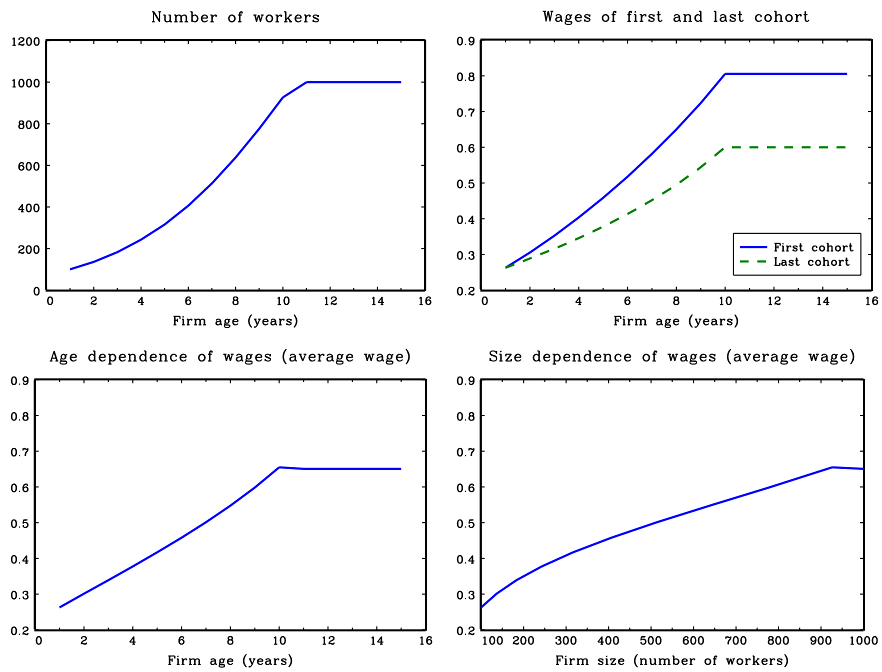
3.3 A numerical example

Figure 2 shows some of the properties of the model using a parameterized version of the model. The parameter values are as follows: $r = 0.03$, $\beta = 0.934$, $\sigma = 1$, $q_{res} = U(0.6)/(1-\beta)$, $\overline{N} = 1,000$, $A = 1$, $\kappa = 2.8$, $\kappa_f/\kappa = 0.3$ and a_0 is such that the initial size of the firm is 10 percent the maximum scale. This is obtained by setting $a_0 = 196$. The numerical example considered here is provided only for illustrative purposes. A formal calibration exercise will be conducted in Section 4.2, after the specification of the general model.

The first panel of Figure 2 plots the employment dynamics. The firm starts with an initial employment of 100 workers and then gradually grows over time until it reaches the optimal size $\overline{N} = 1,000$. The transition takes place in 11 periods. The second panel plots the wage profile of the first cohort of workers (those hired at time 0) and the initial wage paid to newly hired workers. The wage profile of the first cohort (continuous line) is increasing until the firm reaches the unconstrained status. The dashed line shows the wage earned in the first period of employment by workers of different cohorts. As the firm gets closer to the optimal scale, it offers higher initial wages, and therefore, the wage profile of newer workers is less steep overall.

The third panel plots the average wage paid by the firm as a function of its age and the fourth panel the average wage as a function of its size (measured by the number of employees). The average wage increases with the size and age of the firm. This is a direct consequence of the fact that, when the firm is young and constrained, it operates at a suboptimal scale and offers an increasing profile of wages.

Figure 2: Employment dynamics and wage patterns over age and size.



4 General model and simulated regressions

In the simple model described in the previous section, the profile of wages paid by the firm is fully captured by its age. Therefore, once we control for the age of the firm, its size becomes irrelevant because there is a one-to-one mapping between size and age. However, in a cross section of firms, size could have an independent effect. This is because firms may have different capacities $\overline{N}$ and they can start with different initial assets a_0. In order to capture the relation between firm size and wages in a cross-section of firms, we need to extend the model and specify the whole industry structure, including entrance and exit.

We extend the model by allowing: (i) firm heterogeneity in technology $\overline{N}$ and initial wealth a_0; (ii) firm entry and exit; and (iii) turnover of workers within the firm. The first extension allows us to generate a size distribution of firms close to the data. The second guarantees that at each point in time there is a fraction of firms that are financially constrained. The third is introduced for robustness.

We assume that there is a probability $1-p$ that an investment project becomes obsolete and the firm exits. Exiting firms are replaced by new entrant firms managed by new entrepreneurs. New entrepreneurs draw the project capacity $\overline{N}$ from the distribution $\Gamma(\overline{N})$. The mass of workers is L while the mass of firms (entrepreneurs) is normalized to 1.

The initial wealth of new entrepreneurs could be correlated with the project capacity. For instance, entrepreneurs with more promising projects may be able to raise more funds initially by pooling a larger number of founders. Alternatively, we can think that the probability of drawing large capacity projects increases with the ability of the entrepreneur, which in turn may be related to his initial wealth. To formalize this idea in a simple manner, we assume that there is a unique relation between the project capacity $\overline{N}$ and the initial wealth of the entrepreneur, given by $a_0 = \alpha \cdot \overline{N}^{\rho}$. The parameters α and ρ determine the degree of financial tightness for new firms, as a function of projects capacity. Given the linearity of the production function and the borrowing limit, the financial tightness of a new firm is captured by the ratio

$$FTI \equiv \frac{(\kappa - \bar{\kappa}_f) \cdot \overline{N}}{a_0} = \frac{(\kappa - \bar{\kappa}_f) \cdot \overline{N}^{1-\rho}}{\alpha},$$

where FTI stands for Financial Tightness Index. The numerator is the total capital that must be financed internally when the firm operates at the optimal scale $\overline{N}$. The denominator is the initial net worth. When this ratio is greater than 1 the firm is financially constrained. Lower values of α increases the financial tightness for all new firms while the parameter ρ differentiates the tightness across different types of firms. When $\rho = 1$, the tightness is independent of the firm's capacity. When $\rho < 1$, firms with larger capacity face tighter constraints.

The last assumption is that workers may die with some probability $1-\eta$. This feature implies that firms loose some workers at any point in time and there will be workers' turnover within the firm. To keep the model tractable we assume that $1-\eta$ is also the fraction of workers that the firm looses in every period, as if the firm employs a continuum of workers. Of course, this is a simplification but it is convenient to keep the firm's problem tractable. With this assumption, the only source of uncertainty for the firm is the technology obsolescence leading to the exit of the firm.

4.1 Optimization problem for the general model

Given the initial assets a_0 and the project capacity $\overline{N}$, the problem solved by an active entrepreneur is similar to the problem studied in the previous section, although now we have to specify what happens to the wage contracts when the project becomes obsolete.

When the investment project becomes obsolete, all workers lose their jobs and any claim toward the current employer. By re-entering the labor market, they will get the reservation utility q_{res}. This is the only equilibrium outcome of the strategic interaction between the worker and the firm as described in Section 6.[4] The promise-keeping constraint can then be written as:

$$q_{\tau,t} = \beta\Big[U(w_{\tau,t+1}) + \eta \cdot p \cdot q_{\tau,t+1} + \eta \cdot (1-p) \cdot q_{res}\Big].$$

Here the assumption is that the survival of the worker and the viability of the project is observed after paying the current wage (but before the new investment). Consequently, the current wage is not renegotiated.

For the analysis that follows it will be convenient to rescale the promised utility $q_{\tau,t}$ by the constant term $\eta\beta(1-p)q_{res}/(1-\eta p\beta)$. We then have

$$z_{\tau,t} = q_{\tau,t} - \frac{\eta\beta(1-p)q_{res}}{1-\eta p\beta}.$$

Using this rescaled variable, the promise-keeping constraint becomes:

$$z_{\tau,t} = \beta\Big[U(w_{\tau,t+1}) + p\, z_{\tau,t+1}\Big]. \tag{17}$$

Since the ratios of marginal utilities between different cohorts of workers is constant over time (i.e. Property 2 remains valid), the wage ratio between a new worker and the first cohort of workers satisfies:

$$\frac{w_{t,t+1}}{w_{0,t+1}} = \left(\frac{z_{t,t}}{z_{0,t}}\right)^{\frac{1}{1-\sigma}} = \psi(z_t)$$

which identifies the constant relative wage earned by the workers hired at time t. Notice that we maintain the convention of omitting the zero subscript to identify the first cohort of workers.

[4]The entrepreneur could promise extra payments to the worker if the firm is liquidated. However, these payments are not credible. Indeed, when the technology becomes obsolete, there is no cost for the firm from renegotiating because the sunk investment is lost.

The law of motion for the state variable H becomes

$$H' = \eta H + \psi(z)(N' - \eta N), \tag{18}$$

where $N' - \eta N$ is the number of workers hired in the current period.

Since only a fraction η of workers remain in the firm from one period to the next, the law of motion for the next period value of the firm's asset is:

$$a' = \kappa N' + AN' - H'w' - (1+r)b - \kappa_w(1-\eta)N', \tag{19}$$

where the last term accounts for the fact that a fraction $(1 - \eta)$ of workers exit the firm with consequent loss of worker's specific human capital.

The recursive representation is similar to that of section 3.2, once we use z as a state variable in place of q, and we use the laws of motion (17), (18) and (19) to characterize the evolution of z, H and a, respectively. The full description of the firm's problem and the derivation of the first order conditions are in Appendix C. We are now able to define a steady state labor market equilibrium.

Definition 1 *A steady state labor market equilibrium is defined by: (i) A distribution (measure) of firms $M(a, z, N, H, \overline{N})$; (ii) A reservation utility q_{res}; (iii) A transition function for the distribution of firms. Such that: (a) The transition function is consistent with the firm policies, the probability distribution of initial capacities, $\Gamma(\overline{N})$, and the initial distribution of wealth $a_0 = \alpha\overline{N}^\rho$; (b) The demand of labor $\int N \cdot dM(a, z, N, H, \overline{N})$ equals the fixed supply of workers L; (c) The next period distribution generated by the transition function is equal to the current distribution.*

Notice that, although the reservation value q_{res} is endogenously derived as the price which clears the labor market, the interest rate r is exogenous in the model (and equal to the subjective discount rate of investors).

4.2 Quantitative analysis

In this section we show that the model generates a positive firm size-wage relation by estimating wage regressions similar to those considered in the empirical literature but on model-generated data. We first describe the parametrization of the model and then we report the regression results.

Parametrization The interest rate on secured debt is set to $r = 0.03$ and the intertemporal discount factor to $\beta = 0.934$. This implies a discount rate for entrepreneurs equal to $1/\beta - 1 \approx 0.07$, which is close to the post-war stock market return in the U.S. economy. The risk-aversion parameter is set to $\sigma = 1$ (log-utility). We will conduct a sensitivity analysis with respect to this parameter. The per-worker investment κ is chosen to have a capital-output ratio of 2.8. With the normalization $A = 1$, this requires $\kappa = 2.8$. The non-sunk fraction of capital κ_f/κ determines the leverage of the firm. We set $\kappa_f/\kappa = 0.3$ which is consistent with the average leverage of Compustat companies. The probability of firms' death is set to $1-p = 0.0286$. This is the aggregate employment losses due to the death of firms observed in the 2001 data for the U.S. economy (see the footnote to Table 1 for the data source). The survival probability of workers is set to $\eta = 0.9778$. This corresponds to a working life duration of about 45 years, which is consistent with the calibration of explicit life-cycle models such as Auerbach and Kotlikoff (1987) and Rios-Rull (1996).[5]

Table 1: Size distribution of firms in the U.S. economy, 2001.

Firm size (Employees)	*Firms*	*Employees*	*Employees / Firms*
New firms			
1-19	95.37%	53.28%	3.3
20-499	4.58%	37.66%	48.0
500+	0.05%	9.06%	1,022.7
Total	100.00%	100.00%	5.8
All firms			
1-19	87.46%	17.90%	4.7
20-49	7.94%	10.27%	30.0
50-99	2.53%	7.43%	68.4
100-499	1.72%	14.26%	192.4
500-999	0.17%	5.13%	689.0
1,000-1,499	0.06%	3.02%	1,217.4
1,500-2,499	0.05%	3.84%	1,915.8
2,500+	0.07%	38.13%	12,074.1
Total	100.00%	100.00%	23.2

Source: Small Business Administration, Office of Advocacy, data from U.S. Census Bureau, Statistics of U.S. Businesses. http://www.sba.gov/advo/stats/data.html.

The employment capacity $\overline{N}$ can take eight values. These values and the

[5]The typical assumption is that agents start working at age 20 and retire at 65.

corresponding probabilities $\Gamma(\overline{N})$ are determined jointly with the parameters α and ρ. These are the parameters of the function $a_0 = \alpha \cdot \overline{N}^{\rho}$ determining the initial assets of new firms. We use a simulated method of moments to pin down these parameters. More specifically, we minimize the square errors between specific moments generated by the model and the ones observed in the data. The moments are the size distribution of new and incumbent firms as reported in Table 1, plus a capital income share of 40 percent.[6] Table 2 reports the estimated distribution of new projects and their initial financial tightness. The estimated parameters imply that firms with larger projects face higher initial tightness. This is a consequence of the fact that the distribution of new firms shown in Table 1 is much more concentrated toward small firms than the distribution of incumbent firms. The values of the other two parameters are $\alpha = 1.860$ and $\rho = 0.716$.

Table 2: Distribution of new projects and financial tightness.

$\overline{N}$	$\Gamma(\overline{N})$	FTI
5.9	0.81887	*1.74*
31.7	0.11367	*2.81*
53.0	0.03634	*3.26*
189.8	0.02671	*4.68*
602.0	0.00237	*6.50*
1,148.5	0.00074	*7.81*
1,866.4	0.00058	*8.96*
17,875.6	0.00071	*17.04*

Simulated regression Using the steady state distribution of firms, we estimate the following regression:

$$\begin{aligned}\ln(\text{Wage}_{i,j}) &= \bar{\alpha} + \alpha_T \cdot \text{WorkerTenure}_{i,j} + \alpha_{T^2} \cdot \text{WorkerTenure}^2_{i,j} + \\ &\qquad \alpha_A \cdot \text{FirmAge}_j + \alpha_S \cdot \ln(\text{FirmSize}_j) + \alpha_G \cdot \text{FirmGrowth}_j\end{aligned}$$

[6]The size distribution reported in Table 1 gives us 20 independent moments. With the addition of the capital income share we have 21 moments to match but only 17 parameters: eight values of $\overline{N}$, seven probabilities $\Gamma(\overline{N})$, plus α and ρ. Once we have the values of these parameters we also have the labor supply. The implied value is $L = 27.2$.

The index i identifies the worker and j the firm where the worker is employed. This specification is similar to the one used in the empirical literature although we include a smaller set of control variables consistent with the structure of our model. The estimation results are reported in Table 3 with t-statistics in parenthesis.

Table 3: Wage equation estimation from model-generated data.

Description	*(1)*	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*
Constant	-0.5583	-0.5771	-0.5159	-0.5360	-0.6272	-0.6316
	(-174.7)	*(-165.5)*	*(-181.5)*	*(-217.0)*	*(-206.7)*	*(-196.6)*
Worker tenure	0.0068	0.0031	-	-	0.0104	0.0067
	(30.2)	*(13.5)*	-	-	*(45.8)*	*(31.4)*
Worker tenure^2/1,000	-0.0343	-0.0330	-	-	-0.0869	-0.0770
	(-10.9)	*(-9.6)*	-	-	*(-27.5)*	*(-23.1)*
Firm age	-0.0031	-	-0.0006	-	-0.0025	-
	(-45.9)	-	*(-13.9)*	-	*(-35.1)*	-
Firm log-size	0.0105	0.0073	0.0084	0.0077	0.0095	0.0070
	(31.8)	*(20.7)*	*(23.5)*	*(21.4)*	(26.4)	*(18.7)*
Firm growth	-0.6720	-0.5382	-0.7788	-0.6869	-	-
	(-43.7)	*(-32.4)*	*(-49.7)*	*(-47.9)*	-	-
R-square	0.372	0.239	0.231	0.216	0.252	0.160
Observations	10,005	10,005	10,005	10,005	10,005	10,005

Notes: t-statistics in parenthesis.

The first column reports the coefficient estimates when all variables are included in the regression. All the estimates are statistically significant. Of special interest are the coefficients of firm's size and growth. The estimates for these two parameters are consistent with the findings of the empirical literature. In particular, while the size of the firm has a positive impact on wages, the effect of firm's grow is negative. We discuss in details each of the coefficient estimates.

The firm size effect: The largest firms are those that experienced tight financial constraints in the past, when they were operating at a suboptimal (smaller) scale. In order to accelerate their grow, these firms paid low wages in exchange of higher future wages. Now that they are unconstrained (and large), they pay higher wages in fulfillment of their promises. This generates a positive correlation between firm's size and wages. In quantitative terms the effect of the firm's size is important and comparable to those found in the empirical literature. Brown and Medoff (1989) survey the empirical literature and report estimates of log-firm-size coefficient that ranges from 0.01 to 0.03. Similar results are reported by Bronars and Famulari (2001). The findings of Bronars and Famulari are particularly relevant for us since, as in our simulated regression, they include firm growth. If we compare firms that are in the size class 1-19 (whose average size is 4.7) with firms that employ more than 2,500 employees (whose average size is 12,074), our estimates imply that the average wage paid by the second group of firms is about 8 percent higher than the average wage paid by the first group of firms.

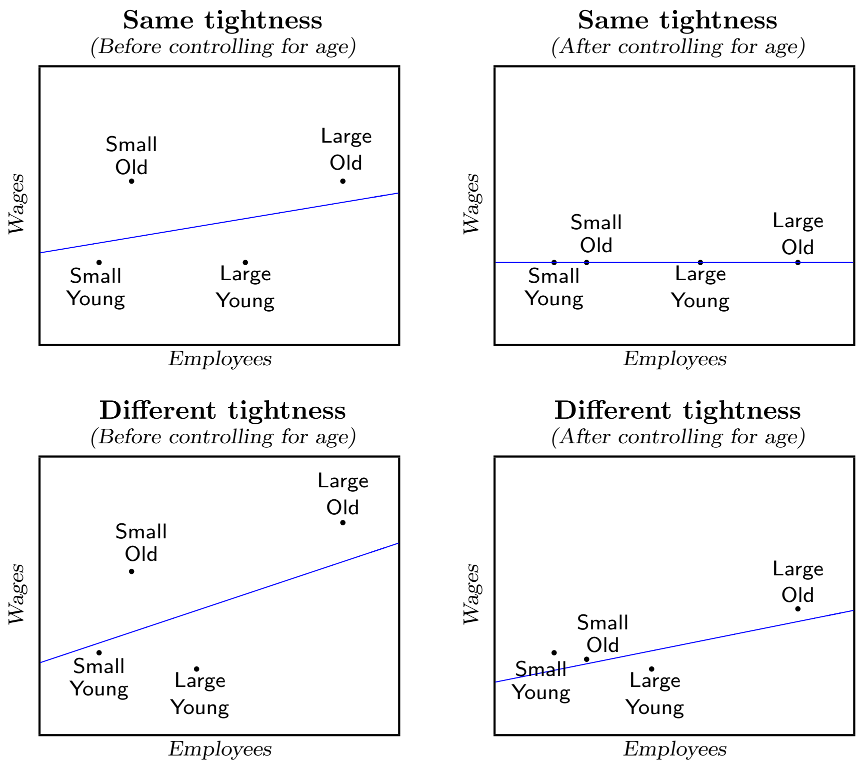
It is important to emphasize that the presence of financial constraints is not enough to generate the positive firm size-wage relation. What is key is that these constraints are tighter for high capacity firms. Our estimated value of ρ is 0.716. This implies that the financial tightness of new firms with the largest $\overline{N}$ is almost 10 times the tightness of firms with the smallest $\overline{N}$ (see Table 2). If ρ was equal to 1—implying that all new firms face the same financial tightness—then the differences in wages would be fully captured by the age of the firm.[7]

To further illustrate the intuition behind this result, consider the following example. Suppose that there are only two types of firms: low capacity and high capacity firms. We refer to the first type of firms as "Small" and to the second type as "Large". Suppose that firms live for two periods. When young they are financially constrained. When old they are unconstrained and operate at the optimal scale. This implies that young firms pay lower wages and operate at a smaller scale. Figure 3 plots the wages and size for these two types of firms, when they are young and old. The top panels are for the case in which all firms face the same financial tightness when young, that is, $\rho = 1$. The bottom panels are for the case in which high capacity firms face tighter constraints when young, that is, $\rho < 1$.

When ρ is equal to one (top panels), the differential in wages between young and old firms is the same for Small and Large firms. Therefore, a dummy variable that differentiates young firms from old firms would be suf-

[7] Indeed, if we constrain ρ to be one and we control for firm age, the estimated coefficient for the size of the firm becomes insignificant. On the other hand, the sign and significance of the coefficient for size is not affected by α. This is important for the growth coefficient.

Figure 3: Financial tightness and firm size-wage relation.



ficient to account for the wage differential. In other words, after conditioning on age, there is no relation between firm size and wages. This is shown in the right-hand-side panel of Figure 3. When instead ρ is less than one, the wage profile is steeper in Large firms than in Small firms (see the bottom panels of Figure 3). In this case, an age dummy is unable to fully capture the wage differential and there still remains a positive correlation between firm size and wages, even after controlling for the age of the firm.

The firm growth effect: The second important result is the negative effect of firm growth on wages. The intuition for this result arises naturally from the discussion above: firms that grow are those with binding financing constraints. Because of these constraints, growing firms pay lower wages today in exchange of higher future wages when they will be able to operate at the optimal scale. Quantitatively, the estimates of this coefficient is not very different from those in the empirical literature. Bronars and Famulari (2001) report a coefficient of firm growth that ranges from -0.4 to -0.35.

Tenure and firm age: The other two variables included in the regression is the worker's tenure and the age of the firm. The positive effect of the worker's tenure derives from the fact that the wages paid by constrained firms increase over time, and therefore, with the tenure of workers. The return to tenure is smaller than the one estimated by Topel (1991), but comparable to the effect estimated by Altonji and Shakotko (1987). The estimated coefficient for firm's age is negative. However, the sign and magnitude of this coefficient depends on the variables we include in the regression. For instance, if we exclude worker's tenure, the coefficient of firm's age decreases significantly and it becomes positive if we also exclude firm size from the regressors. In brief, the unconditional correlation between wage and firm age is positive while it becomes negative after controlling for some workers and firms characteristics. The fact that the relation between firm age and wages depends on the variables included in the regression is consistent with the empirical findings that the effect of age is not clear cut (see Section 2.1).

Sensitivity analysis: Table 4 reports the estimates for alternative values of the coefficient of risk aversion σ. When $\sigma = 0.5$ (low concavity), the firm-size wage effect increases more than 20 percent. In this case, the wages of firms with more than 2,500 employees are about 10 percent higher than the wages paid by firms in the size class 1-19. This derives from the fact that the cost of offering an increasing wage profile is smaller when the intertemporal elasticity of substitution is high. Consequently, firms offer a steeper wage profile and the effects of firm size and growth on wages are stronger. The opposite is true when $\sigma = 2.0$. In the limit case in which $\sigma = \infty$, all firms would pay a constant wage and the model would not generate any wage differential.[8]

[8] There is a limit to how small σ can be. If this parameter is very small, the wage profile becomes so steep that large-unconstrained firms pay much higher wages than the ones offered to new workers. This implies that the gains from replacing an existing worker (and paying lower wages) exceed the loss in sunk capital. With $\sigma = 0.5$ the non-renegotiation condition is no longer satisfied, as we will show in Section 6.

Table 4: Sensitivity analysis

Description	*(1)* $\sigma = 0.5$	*(2)* $\sigma = 1.0$	*(3)* $\sigma = 2.0$
Constant	-0.5548	-0.5583	-0.5488
	(-120.6)	*(-174.7)*	*(-301.8)*
Worker tenure	0.0071	0.0068	0.0043
	(21.8)	*(30.2)*	*(33.5)*
Worker tenure^2/1,000	-0.0257	-0.0343	-0.0239
	(-5.7)	*(-10.9)*	*(-13.2)*
Firm age	-0.0039	-0.0031	-0.0018
	(-40.0)	*(-45.9)*	*(-46.6)*
Firm log-size	0.0130	0.0105	0.0069
	(27.3)	*(31.8)*	*(36.1)*
Firm growth	-1.2324	-0.6720	-0.2821
	(-61.3)	*(-43.7)*	*(-28.7)*
R-square	0.412	0.372	0.325
Observations	10,045	10,005	10,133

Notes: t-statistics in parenthesis.

5 Job-to-job flows and employer vs. occupational tenure

In this section we extend the model to overcome two apparent shortcomings of our analysis. First, the turnover of workers generated by the model tends to be too small, since it is just the result of workers and firms death. In reality, a substantial fraction of workers switch their occupation from one employer to the other without unemployment spells. See, for example, Akerlof, Rose, and Yellen (1988) and Fallick and Fleischman (2001). Another important feature of our model is that *employer tenure* is an important determinant of wages. However, a recent paper by Kambourov and Manovski (2002) argues that the tenure of a worker with an employer is not very important for the determination of wages. What matters is the *occupational tenure*—*i.e.*, the experience in a particular occupation even if with multiple employers. To address these issues, we extend the model to allow for occupation-to-occupation flows.

We make the following assumptions. First, in each period a firm is able to contact a measure m of workers, who already hold a job. We interpret these workers as holding jobs in the same occupation as that offered by the new employer. Because the worker does not change occupation, he or she can transfer the (occupation) specific human capital to the new employer. This allows the new employer to save on the investment cost κ_w.

Second, the firm is able to attract the worker simply by offering the utility earned with the current employer. As in Burdett and Mortensen (1998) and Burdett and Coles (2003), the worker is unable to let the current and new employers compete over his services, and the poaching firm has all the bargaining power. The microfundations for these assumptions, based on the existence of some renegotiation costs, are developed in Hashimoto (1981) and Anderlini and Felli (2001).[9] Notice that the new employer is willing to offer a utility greater than q_{res} because it saves on the training cost k_w.

To keep the model tractable, we also make two additional simplifying assumptions. First, the matching technology is balanced as in Burdett and Vishwanath (1988), in the sense that the number of workers contacted by the firm is proportional to its size, that is, $m = \chi N$. This implies that each employed worker has a probability χ of being contacted by another employer offering a job in the same occupation. Notice that the mass of workers contacted by a firm and the workers who get contacted by other firms are not stochastic. Consequently, the mass of workers poached by a firm is equal to the mass of workers leaving the firm. Here we are proceeding as if we can apply some law of large numbers. Second, we assume that each firm contacts workers that are employed in firms with the same capacity and age. The idea is that workers employed in firms of the same type are more likely to have transferable skills. This implies that the promised utilities of the workers who quit the firm are exactly equal to the utilities of the new hired workers. As a result, neither the state variables of the firm nor their law of motion change.

The previous assumptions are simple abstractions that allow us to keep

[9] The idea goes as follows. The poaching firm makes a *take-it or leave-it* offer to the contacted worker. The offer is private information and in order to make the offer verifiable to the employer, the worker needs to exercise some effort. The current employer would match the external offer if the worker demands to renegotiate the contract. However, because to renegotiate the contract the worker has to face an effort cost, the utility from renegotiating is smaller than the utility from accepting the external offer. This generates an hold-up problem and the worker never tries to renegotiate the contract. Anticipating this, the poaching firm offers an expected utility slightly higher than the utility that the worker earns by staying with the current employer.

the model tractable because the firm faces exactly the same problem it was facing before. However, we can now distinguish between employer tenure and occupational tenure. We then have that employer tenure is no longer relevant for the determination of wages once we control for occupational tenure.

Even though the firm's problem does not change with the addition of job-to-job flows, the tenure of workers with the same employer is shorter on average. Consequently, the coefficient estimates of the wage equation may change. Using a value of $\chi = 0.15$, the new estimates are:[10]

$$
\begin{aligned}
\ln(\text{Wage}) = \; & \underset{(-110.8)}{-0.8614} + \underset{(27.1)}{0.0476} \cdot \text{WorkerTenure} \underset{(-10.7)}{-0.0016} \cdot \text{WorkerTenure}^2 \\
& \underset{(-29.4)}{-0.0024} \cdot \text{FirmAge} + \underset{(27.4)}{0.0091} \cdot \ln(\text{FirmSize}) \underset{(-28.7)}{-0.496} \cdot \text{FirmGrowth}
\end{aligned}
$$

As can be verified, these numbers are not very different from the case in which there are no job-to-job flows as reported in Table 3.

6 Contracts implementation

In the analysis of the long-term contract we have assumed that the firm commits to the long-term wage contracts. Commitment could be problematic because the promised utilities increase over time until the firm becomes unconstrained. More specifically, a new worker starts with $q_t = q_{res}$ and receives $q_{t+j} \geq q_{res}$, for all $j > 0$. Because new workers can be hired with initial utility q_{res}, the firm may have an incentive to renege promises that exceed q_{res}. The goal of this section is to discuss the conditions that prevent the firm from renegotiating the long-term contract. We then discuss why collateralized debt is the only form of external financing for the firm.

Before continuing, it will be convenient to summarize the timing of the model. First workers decide whether to provide effort—which has a cost $\bar{\ell}$ in forgone utility—and whether to quit the firm. Then production takes place and the firm observes whether the worker has provided effort. At this point the firm could renege its wage promises. Afterwards, the firm decides

[10]Together with $\eta = 0.9778$ and $p = 0.9714$ used in the calibration, $\chi = 0.15$ implies that about 80 percent of workers have more than one year of tenure with the same employer. This is the number reported for the U.S. economy by Farber (1999).

whether to renegotiate the debt. Renegotiation entitles the investors to seize the firm's assets. After the payment of the wages and the repayment of the debt, the survival of the firm and the workers are observed.

6.1 Worker-firm relationship

If both the worker and the entrepreneur cooperate (the worker by exerting effort and the entrepreneur by paying the promised wage), output is produced and the worker earns the promised wage. The only Nash Equilibrium of each period sub-game is the one in which the firm reneges its promises and pays zero wages. Anticipating that, the worker withdraws effort and quits. In the repeated game, however, cooperation can be sustained through trigger strategies, provided that replacing the worker is sufficiently costly for the firm. Suppose that the worker and the firm follow these strategies (which for simplicity are specified independently of the investors' past history):

- **Worker**: The worker provides effort as long as the firm pays the contracted wages. If one of the two parties has reneged sometimes in the past (either the worker has shirked or the firm has paid a wage different from the one contracted), the worker withdraws effort and quits.

- **Firm**: The firm pays the contracted wages as long as the worker provides effort. If one of the two parties has reneged sometimes in the past (either the worker has shirked or the firm has paid a wage different from the one contracted), it sets the wage to zero.

The equilibrium associated with these strategies is sub-game perfect. To see this, let's consider first the worker. Providing low effort would trigger a wage cut which forces the worker to quit the firm and be left with the reservation value q_{res} starting from the next period. But the utility from doing so, $U(0)+\bar{\ell}+\eta q_{res}$, is not bigger than the utility obtained from providing effort, that is, $U(w_t)+\eta p q_t+\eta(1-p)q_{res}$. Thus, along the equilibrium path, the worker never shirks and quits. If the firm has sometimes paid a different wage from the one contracted, quitting is optimal since the firm would pay a zero wage both today and in the future.

Consider now the firm. When the firm expects the worker to quit tomorrow, setting the wage to zero today is always the firm's best response. Thus, given each worker's strategy, paying zero wages is optimal when the worker has sometimes shirked. Along the equilibrium path, the firm never finds optimal to deviate from the promised long-term contract because, if the firm reneges its wage promises, the worker quits and the firm looses the sunk investment κ_w. Therefore, the assumptions that part of the investment is worker-specific, is key to prevent the firm from renegotiating the contract.

The fact that the replacement of an existing worker is costly for the firm, creates an indirect form of "collateral" for workers. This allows the firm to borrow from the workers beyond what it can borrow from external investors. Of course, there is a limit to this. If the worker's utility becomes very large, the loss of sunk investment could be smaller than the gains from reducing the wage obligations (by reneging the long-term contract and hiring a new worker). This may happen if $\bar{\kappa}_f/\kappa$ is close to 1 and the initial assets of the firm, a_0, are small. In this paper we have implicitly assumed that $\bar{\kappa}_f/\kappa$ is sufficiently small and a_0 sufficiently large so that this never arises in equilibrium.

To show that the non-renegotiation condition is satisfied in the numerical exercises conducted in the paper, Table 5 reports the maximum gains that can be obtained by replacing an existing worker (and paying lower wages afterwards). The maximum gain can be achieved by firms with the largest capacity $\overline{N}$ once they become unconstrained.[11] These firms are paying the highest wages to the first cohort of workers. Denote the wage paid to this cohort by w_{max}. A firm could replace these workers with new workers receiving a constant wage w_{res}. This is the wage that gives the reservation utility $q_{res} = \beta U(w_{res})/(1-\eta\beta)$. By doing so, the firm would save $w_{max} - w_{res}$ in wage payments in each period, with expected discounted value given by

$$RG(\mathbf{P}) \equiv \frac{\beta(w_{max} - w_{res})}{1 - \beta(1-\chi)\eta p},$$

where RG stands for Renegotiation Gains and $\mathbf{P}$ are the model's parameters. Notice that the term $\beta(1-\chi)\eta p$ becomes the discount factor of the gains for the firm because the worker remains in the firm with probability $(1-\chi)\eta p$: the worker does not quit the firm with probability $1-\chi$, he survives with probability η, and the firm remains in operation with probability p.

Table 5 reports the renegotiation gains for different curvatures of the utility function. In computing these numbers we have used $\chi = 0.15$. As

[11] It can be shown that the maximal promised utility for which the firm does not renegotiate is decreasing in the age of the firm. This together with the fact that the promised utility of workers increases with tenure (till the firm becomes unconstrained), proves that the incentive to renegotiate is the highest when the firm is unconstrained.

expected from the theoretical analysis, the renegotiation gains increase as we reduce the curvature of the utility function σ. This is because with a lower σ it is cheaper to borrow from workers and the profile of wages is steeper.

Table 5: Renegotiation gains for different curvatures of the utility function.

	$\sigma = 0.5$	$\sigma = 1.0$	$\sigma = 2.0$
$RG(\mathbf{P})$	2.605	1.820	1.045

The renegotiation gains are compared with the loss of workers specific capital κ_w, which in the parameterized model takes the value of 1.935. For the baseline parametrization with $\sigma = 1$, the non-renegotiation condition is satisfied. However, for smaller values of σ this is no longer the case.

6.2 Investors-firm relationship

Suppose that when the entrepreneur renegotiates (defaults on) the debt contract, investors have the right to liquidate the assets of the firm but cannot exclude the entrepreneur from participating in financial markets. In other words, the entrepreneur can get new financing from other investors. Furthermore, when the firm refinances investment, it can retain the hired workers. This implies that the investment in recruitment and training is not lost.

Under the above conditions, collateralized debt is the only type of financing that the firm can get from investors. To see this, suppose that the firm could borrow above the value of the collateral. After receiving the loan, the entrepreneur would renegotiate down the part of the debt in excess of the collateral and obtain a new (identical) financial contract from other investors. Anticipating this, only secured loans will be offered.

7 Empirical analysis

We have seen that our model is consistent with several empirical findings. The model also generates some predictions about the interaction between individual wages, worker's tenure, employer's size and firm's growth that, as far as we know, are yet to be tested. These predictions can be summarized as follows:

1. The initial size of the employer, when the worker is hired, has a positive effect on wages. This is because, on average, small firms have tighter constraints and pay lower wages initially.

2. The growth rate of the firm has a negative effect on wages. In fact, the growth rate of the firm is an indicator of its financial tightness and when firms are constrained they temporarily pay lower wages.

3. The wage tenure profile of workers in fast growing firms is steeper. Again, fast growing firms pay lower wages initially in the promise of higher future wages. Therefore, the wage tenure profile of these workers is steeper.

4. If we consider the extended model with transferability of worker-specific capital, then the return to (labor market) experience should be positively correlated with the growth rate of the firm.

To test these predictions, we use data from the National Longitudinal Survey of Youth, started in 1979 (NLSY79). This is a nationally representative sample of 12,686 young men and women who were 14-22 years old when they were first interviewed in 1979. We focus on a sample of 6,111 individuals designed to be representative of the non-institutionalized civilian segment of the U.S. young population. These individuals have been interviewed annually through 1994 and starting in 1996 every two years. Because of missing data concerning information on the size of the employer, we use only the 13 more recent waves, from 1986 to 2002. The initial total sample with non missing observations on key variables includes 18,570 observations.

We make some further selection. First, we restrict the sample to full time workers (working a minimum of 35 hours per week) with reliable data on wages and with positive labor market experience. Firms' growth rates are self reported and thus likely to be subject to substantial measurement error. To eliminate outliers that are likely to be the result of measurement errors and/or exceptional events in the life-cycle of the firm, we restricted the average annual growth rate of the employer over the tenure of the worker to be greater than -10 percent and smaller than 100 percent.[12] This leads to our final sample of 11,651 observations for 2,447 individuals (11,616 observations for 2,446 individuals when considering information about industries and geographical location). Further details are provided in Appendix E.

[12] For robustness, we also repeated the estimations after imposing a minimum average growth rate of -50 percent and a maximum growth rate of 150 percent. The results do not change substantially.

Table 6 reports some descriptive statistics for the variables used as regressors in the wage equations below—including the usual Mincer regressors. One important variable is the size of the firm. Because the NLSY79 reports only the employment size of the establishment where the respondent works, we use this variable to proxy for the *firm size*. The survey also asks whether the employer has more than one establishment. We use this information to better approximate the whole size of the employer.

Table 6: Sample Statistics

Description	*Mean*	*Std. Dev.*	*Min*	*Max*
Hourly wage	13.38	14.08	2	500
Male	0.57	0.49	0	1
Black	0.10	0.30	0	1
Hispanic	0.06	0.24	0	1
White	.83	.37	0	1
Years of schooling	13.5	2.47	4	20
Experience	12.49	5.31	0	32
Tenure	3.48	3.19	.02	16.40
Initial firm size	273.71	802.15	1	15,000
Initial multiple establishment	0.64	0.48	0	1
Average yearly firm's growth rate	0.10	0.19	-0.1	1
No. of observations	11,651			
No. of individuals	2,447			

Notes: Tenure, years of schooling and labor market experience are in years. The original measure of tenure in weeks is converted into years dividing by 52. Hourly wages are in dollars. White refers to individuals that are neither black nor hispanics. Initial firm's size is the number of employees at the location of the current job at the time when the worker was hired. Initial multiple establishment is a dummy variable that equals one if the employer had more than one establishment at the time when the worker was hired. Firm's growth rates are averages of the yearly firm's growth rate during the tenure of the worker with the current employer. See Appendix E for more details.

We start investigating the correlation between the average annual firm's growth rate during the tenure of the worker (*Firm growth*) and the (logged) number of employees when the worker was hired (*Initial firm size*). We find a statistically significant negative correlation of about 1 percent. This is in line with the previous findings that small firms grow faster. We also investigated the magnitude of the firm's size effect in our sample. In a standard OLS regression that includes all the variables in Table 6 plus a full set of year dummies, twelve industry dummies and four regional dummies, we find a coefficient for the current logged size of the firm of about of 3 percent. This is consistent with the values reported by Brown and Medoff (1989). After considering a fixed effects (within) estimator to control for workers' unobserved heterogeneity, the size coefficient falls to about 1.5 percent, but it is still highly significant.

Empirical results: To test properties 1-4 described above, we estimate the following wage equation:

$$\ln W_{it} = \mu_i + \beta_x \cdot X_{it} + \alpha_S \cdot \ln S_{it} + \alpha_G \cdot G_{it} + \alpha_T \cdot G_{it} \cdot T_{it} + \alpha_E \cdot G_{it} \cdot E_{it}$$

where W_{it} is the hourly wage earned by worker i in the primary job at time t; μ_i is the individual fixed effect; X_{it} is a set of controls for the worker; S_{it} is the size (number of employees) of the current employer when the worker was hired; G_{it} is the average yearly growth rate of the firm during the worker tenure with the firm; T_{it} is the tenure in the current job; E_{it} is the working experience (current age minus years of schooling minus six). In the controls X_{it} we include a square polynomial in tenure, a square polynomial in experience, a full set of year dummies, and a dummy variable which takes the value of one if the firm had more than one establishment at the time when the worker was hired. We also consider a specification that includes twelve industry dummies, four regional dummies and a dummy variable for living in a metropolitan area to control for a possible spurious correlation between firms' growth rates and sectoral and geographical characteristics, that may affect wages. To to control for workers' heterogeneity, we use a fixed effects (within) estimator.

We are interested in the sign of the coefficients α's. The coefficient α_S is expected to be positive while α_G should be negative. The coefficients α_T and α_E capture the interaction of firm's growth with tenure and experience. These coefficients tell us whether the slope of the wage profile changes with the growth rate of the firm. According to the considerations made above, we expect these coefficients to be positive.

The top section of Table 7 reports the estimation results. The basic estimations are reported in columns 1-3. The regressions in columns 4-6 add to the basic regressions twelve industry dummies, four regional dummies, and a dummy for working in a metropolitan area. The coefficient estimates have the expected sign and are statistically significant at the conventional levels. In particular, workers earn higher wages on average when they start working with larger firms. Fast growing firms initially pay lower wages and offer steeper wage profiles. These are the patterns predicted by our model and are key to generate a positive relation between firm size and wages.

In order to show that these results are also generated by our model, we repeat the same regressions using simulated data from the model specification discussed in Section 5. In order to be as close as possible to the characteristics of the NLSY79 sample, we simulate 2,500 workers for 15 years. These are the approximate number of years covered by the NLSY79. The estimation results are reported in columns 1-3, in the second section of Table 7. The estimated coefficients have the same sign as those estimated from the NLSY79 data and are statistically significant.

Although we obtain the same signs, the magnitude of the coefficients for the firm's growth and its interactions with tenure and experience are bigger than the estimates from the data. It should be noted, however, that in the NLSY79 the growth rates of the firm are self-reported. This implies that measurement errors are likely to be quite important. We then ask how the estimations from the artificial data would change if there were measurement errors also in the simulation.

In columns 4-6 we repeat the estimations after adding artificial measurement errors to the size of the firm. In particular, we add a noise that is normally distributed with a mean of zero and a standard deviation of 40 percent to the true logged size of the firm. As can be seen in columns 4-6, the addition of the noise reduces (in absolute value) the estimates of the coefficients of the growth rate and its interaction with tenure and experience while the coefficient of the initial firm size increases. Now they are much closer to the parameter estimated from the NLSY79. In general, a bigger measurement error leads to smaller (in absolute value) estimates of α_G, α_T and α_E and to a larger estimate of α_S. Of course, it is not possible to say how big the measurement errors are in the data. But the numbers reported in Table 7 show that our model can generate similar estimates for measurement

Table 7: Wage equation estimation with NLSY79 and simulated data.

A) NLSY79 Data

Description	*(1)*	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*
Initial (log) firm size when the worker was hired	.017**	.0162**	.0168**	.0151**	.0139**	.0145**
	(4.38)	*(4.05)*	*(4.18)*	*(3.70)*	*(3.40)*	*(3.54)*
Average (yearly) firm growth during worker's tenure	-.083**	-.201**	-.165**	-.095**	-.210**	-.172**
	(-2.66)	*(-3.68)*	*(-2.83)*	*(-3.00)*	*(-3.84)*	*(-2.94)*
Interaction between firm growth and tenure	.023**		.015*	.024**		.015*
	(3.46)		*(1.75)*	*(3.51)*		*(1.78)*
Interaction between firm growth and experience		.0148**	.009*		.015**	.009*
		(3.44)	*(1.69)*		*(3.41)*	*(1.68)*
No. of Observations	*11,651*	*11,651*	*11,651*	*11,616*	*11,616*	*11,616*
No. of Individuals	*2,447*	*2,447*	*2,447*	*2,446*	*2,446*	*2,446*

B) Simulated Data

Description	*(1)*	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*
Initial (log) firm size when the worker was hired	.004**	.001**	.003**	.011**	.009**	.011**
	(11.40)	*(2.36)*	*(7.52)*	*(18.71)*	*(15.77)*	*(18.62)*
Average (yearly) firm growth during worker's tenure	-1.620**	-1.707**	-1.897**	-.246**	-.300**	-.353**
	(-153.18)	*(-134.41)*	*(-165.01)*	*(-37.7)*	*(-36.05)*	*(-42.44)*
Interaction between firm growth rate and tenure	.306**		.232**	.088**		.070**
	(117.33)		*(80.54)*	*(42.25)*		*(30.87)*
Interaction between firm growth and experience		.155**	.085**		.036**	.023**
		(93.81)	*(49.87)*		*(35.20)*	*(20.67)*
No. of Observations	*26,171*	*26,171*	*26,171*	*22,754*	*22,754*	*22,754*
No. of Individuals	*2,438*	*2,438*	*2,438*	*2,328*	*2,328*	*2,328*

Panel A: The dependant variable is the log hourly wage of workers with at least 35 working hours per week. The regressions in columns 1-3 include experience in level and squared; tenure in level and squared; twelve year dummies. The regressions in columns 4-6 add four region dummies; a dummy for working in a metropolitan area; twelve industry dummies. All regressions include a dummy variable for multi-establishment firms when the worker was hired.

Panel B: Data is obtained by simulating a cohort of 2,500 individuals for 15 years. The model also includes job-to-job flows as discussed in Section 5. All regressions include age in level and squared and tenure in level and squared. The regressions in columns 4-6 are estimated on the simulated data after adding a measurement error to the size of the firm. The measurement error is normally distributed with a mean of zero and a standard deviation of 0.4. As in Panel A we also restrict the average annual growth rate of the firm to be between -10 percent and 100 percent.

All regressions use a fixed effects (within) estimator. t-statistic in parenthesis; * Significant at 10 percent level; ** Significant at 5 percent level.

errors that are quite reasonable.

8 Conclusion

In this paper we have studied how financial constraints affect the compensation structure of workers. Firms that are financially constrained find optimal to offer an upward profile of wages in order to alleviate their financial restrictions. Because large firms are more likely to have experienced a history of financial tightness with low wage payments, they have to pay high wages after becoming unconstrained. This mechanism can generate a positive correlation between firm size and wages. We test the key properties of the model that generate this relation using data from the 1979-2002 National Longitudinal Survey of Youth (NLSY79). The estimation results support our theory.

By offering an upward profile of wages, firms implicitly borrow from workers. This rises the question of why firms are able to borrow from workers beyond what they can borrow from external investors. In our model this is possible because workers can use punishment mechanisms that are not available to external investors. An external investor can punish the debtor only by confiscating the firm's physical assets, which represents the only collateral that the firm can use to raise funds in financial markets. But the firm can expand its debt capacity by using another form of implicit "collateral" in the hands of workers. If a worker quits, the firm looses the job-specific investment. This gives the worker a credible punishment tool in the event of repudiation that is not available to investors. The cost of replacing the worker—due to the sunk nature of the investment—guarantees that the long-term wage contract between the worker and the firm is never reneged and allows the firm to use the wage policy to finance its growth.

Indeed, there is both direct and indirect evidence that firms borrow from their employees. In some cases, the borrowing is explicit.[13] In others, the loan is implicit in the compensation structure of employees, as in our model. For example, the widespread use of stock options and/or stock grants to ordinary workers, such as middle-run managers, secretaries and clerks—whose effort,

[13] An example is Energy Services Group International, an energy-services engineering and construction company in Williamsburg, VA. The company got a major new contract from an electric utility in Florida but it could not persuade banks to lend any more money. Only employees came forward with investments that ranged from \$200 to \$74,000 in exchange of promissory notes. See Inc. Magazine, January 1992, http://www.inc.com/magazine/19920101/3886.html.

when individually considered, is likely to have a negligible effect on the overall value of the firm—can hardly be justified as a way to provide incentives. This view is also expressed in Hall and Murphy (2003). Most likely, stock options are used to delay the cash compensation of employees and retain more funds in the firm. In accordance with this interpretation, Blasi, Kruse, and Bernstein (2003) find that stock options were especially rewarding for workers hired before their companies went public—*i.e.*, companies that were likely to be financially constrained when they awarded the options. Also consistent with this interpretation is the finding of Core and Guay (2001) for which the use of stock options is more common in firms that are financially constrained.

A Characterization of the firm's problem

Let γ_t, μ_t, $\lambda_t n_t$ and θ_t denote the lagrange multipliers associated with constraints (2), (3), (4) and (5) respectively. Then the Lagrangian can be written as:

$$\begin{aligned}\mathcal{L} = \sum_{t=0}^{\infty}\beta^t \Bigg\{ & \left(a_t + b_t - \kappa\sum_{\tau=0}^{t} n_\tau\right) + \\ & \gamma_t\left[a_t + b_t - \kappa\sum_{\tau=0}^{t} n_\tau\right] + \\ & \mu_t\left[\bar{\kappa}_f\sum_{\tau=0}^{t} n_\tau - b_t\right] + \\ & \lambda_t n_t\left[\sum_{j=1}^{\infty}\beta^j U(w_{t,t+j}) - q_{res}\right] + \\ & \theta_t\left[\sum_{\tau=0}^{t}(\kappa + A - w_{\tau,t+1})n_\tau - (1+r)b_t - a_{t+1}\right] \Bigg\}.\end{aligned}$$

The first order conditions with respect to $w_{\tau,t}$ and a_t, for $t \geq 1$, are

$$\beta\lambda_\tau U_c(w_{\tau,t}) = \theta_{t-1}, \quad \forall\tau \leq t \tag{20}$$

and

$$\theta_{t-1} = \beta(1+\gamma_t), \tag{21}$$

respectively. Using (21) to substitute for θ_{t-1} in (20) yields (6) in the text.

B First order conditions for the recursive formulation

The Lagrangian can be written as:

$$\begin{aligned}\mathcal{L} &= a + b - \kappa N' + \beta V(a', q', N', H') \\ &+ \gamma\Big[a + b - \kappa N'\Big] \\ &+ \mu\Big[\bar{\kappa}_f N' - b_t\Big] \\ &+ \lambda H'\Big[\beta(U(w') + q') - q\Big]\end{aligned}$$

where γ, μ and $\lambda H'$ are lagrange multipliers. The problem is also subject to the laws of motion for the next period value of a and H, that is, constraints (13) and (14), respectively.

The first order conditions are:

$$b: \quad 1+\gamma-\mu=\beta(1+r)V_{a'} \tag{22}$$

$$w': \quad V_{a'}=\lambda U_{c'} \tag{23}$$

$$q': \quad V_{q'}+\lambda H'=0 \tag{24}$$

$$N': \quad \beta\Big[\Big(\kappa+A-\psi(q)w'\Big)V_{a'}+V_{N'}+\psi(q)V_{H'}\Big]\geq(1+\gamma)\kappa-\mu\bar{\kappa}_f \tag{25}$$

where the last condition is satisfied with equality if $N'<\overline{N}$. The envelope conditions are:

$$V_a = 1+\gamma \tag{26}$$

$$V_q = -\beta\psi_q(N'-N)\Big[w'V_{a'}-V_{H'}\Big]-\lambda H' \tag{27}$$

$$V_N = \beta\psi(q)\Big[w'V_{a'}-V_{H'}\Big] \tag{28}$$

$$V_H = -\beta\Big[w'V_{a'}-V_{H'}\Big] \tag{29}$$

Equation (15) in the text comes from using (26) to substitute for V_a in (23). We now show that the above conditions also imply that $\lambda=\lambda'$.

By substituting (26) in (29) we get:

$$-V_H=\beta\Big[(1+\gamma')w'-V_{H'}\Big]. \tag{30}$$

From (23) we have that $(1+\gamma')w'=\lambda(w')^{1-\sigma}=\lambda(1-\sigma)U(w')$, which substituted in (30) yields

$$-V_H=\beta\Big[(1-\sigma)\lambda U(w')-V_{H'}\Big]. \tag{31}$$

Now consider the promise-keeping constraint $q=\beta[U(w')+q']$. Multiplying the left and right-hand side by $(1-\sigma)\lambda$ we get:

$$(1-\sigma)\lambda q=\beta\Big[(1-\sigma)\lambda U(w')+(1-\sigma)\lambda q'\Big]. \tag{32}$$

Equations (31) and (32) imply:

$$-V_H = (1-\sigma)\lambda q \tag{33}$$

$$-V_{H'} = (1-\sigma)\lambda q' \tag{34}$$

Updating the first term we also have that:

$$-V_{H'} = (1-\sigma)\lambda' q' \tag{35}$$

Condition (34) and (35) then imply that $\lambda = \lambda'$.

C Recursive formulation of the general model

The problem solved by a firm with capacity $\overline{N}$ can be written recursively as follows:

$$V(a,z,N,H) = \max_{\substack{b,w',z',\\ N'\leq \overline{N}}} \left\{ d + \beta\Big[p\cdot V(a',z',N',H') + (1-p)\cdot L'\Big]\right\} \tag{36}$$

subject to

$$
\begin{aligned}
d &= a + b - \kappa N' \geq 0 && (37)\\
b &\leq \bar{\kappa}_f N' && (38)\\
z &= \beta\Big[U(w') + \eta p\, z'\Big] && (39)\\
a' &= \kappa N' + AN' - H'w' - (1+r)b - \kappa_w(1-\eta)N' && (40)\\
H' &= \eta H + \psi(z)(N' - \eta N) && (41)\\
L' &= \kappa_f N' + AN' - H'w' - (1+r)b && (42)
\end{aligned}
$$

where L is the liquidation value of the firm, which consists of the sum of its physical capital and its current profits minus the value of debt.

Let γ, μ and $\lambda H'$ be the lagrange multipliers associated with the constraints (37), (38), and (39), respectively. Following the same steps as in Appendix B we obtain the first order conditions:

$$b: \quad 1 + \gamma - \mu = \beta(1+r)(1+p\gamma') \tag{43}$$

$$w': \quad 1 + p\gamma' = \lambda U_{c'} \tag{44}$$

$$z': \quad V_{z'} + \eta\lambda H' = 0 \tag{45}$$

$$N': \quad \beta\Big[(1+p\gamma')\Big(\kappa + A - \psi(z)w' - (1-\eta)\kappa_w\Big) \tag{46}$$

$$+p\Big(V_{N'} + \psi(z)V_{H'}\Big) - \eta(1-p)\kappa_w\Big] \;\geq\; (1+\gamma)\kappa - \mu\bar{\kappa}_f$$

where the last equation is satisfied with equality if $N' < \overline{N}$. Notice that (43), (44) and (46) make use of the envelope condition $V_a = 1 + \gamma$. The remaining envelope conditions are:

$$V_z = \beta\psi_z(N' - \eta N)\Big[pV_{H'} - (1 + p\gamma')w'\Big] - \lambda H' \tag{47}$$

$$V_N = -\eta\beta\psi(z)\Big[pV_{H'} - (1 + p\gamma')w'\Big] \tag{48}$$

$$V_H = \eta\beta\Big[pV_{H'} - (1 + p\gamma')w'\Big] \tag{49}$$

D Computation of the equilibrium

Solving for the firm's problem: For given $\overline{N}$ and q_{res}, the firm problem is solved backward starting from the state in which the firm is unconstrained. Let's assume that the firm takes T periods to become unconstrained. Therefore, we know that $N_{T+1} = \overline{N}$ and $\gamma_T = \gamma_{T+1} = 0$.

We start by guessing the value of w_{T+1} and H_{T+1}. Using the first order condition $1 = \lambda U_c(w_{T+1})$, we determine the lagrange multiplier λ. Using the promise-keeping constraint $z_T = \beta[U(w_{T+1}) + \eta p z_{T+1}]$, and imposing $z_T = z_{T+1}$, we determine the (transformed) promised utility at time $T+1$. Using condition (47) with the terminal condition $V_{H,T} = V_{H,T+1}$, we determine the partial derivative of the value function with respect to H. Finally, we determine b_T using the borrowing limit $b_T = \bar{\kappa}_f N_{T+1}$ and μ_T using the first order condition $\mu_T = 1 + \gamma_T - \beta(1 + r)(1 + p\gamma_{T+1})$. At this point we have all the terminal conditions to solve the problem backward at each point $t = T, T-1, ..., 0$. The solution at each point t is determined as follows:

1. Using the budget constraint with $d_t = 0$, we determine the firm's assets:

$$a_t = \kappa N_{t+1} - b_t$$

2. The wage w_t is determined using the first order condition:

$$1 + p\gamma_t = \lambda U_c(w_t)$$

3. We now determine the variables N_t, H_t and b_{t-1} using the laws of motion for a_t, H_{t+1}, and the borrowing limit:

$$\begin{aligned} a_t &= (\kappa + A)N_t - H_t w_t - (1 + r)b_{t-1} - \kappa_w(1 - \eta)N_t \\ H_{t+1} &= \eta H_t + \psi(z_t)(N_{t+1} - \eta N_t) \\ b_{t-1} &= \bar{\kappa}_f N_t \end{aligned}$$

4. The values of $V_{H,t}$ and z_{t-1} are determined using condition (47) and the promise-keeping constraint, that is:

$$\begin{aligned} V_{H,t} &= -\eta\beta(1+p\gamma_{t+1})w_{t+1} + \eta\beta p V_{H,t+1} \\ z_{t-1} &= \beta[U(w_t) + \eta p z_t] \end{aligned}$$

5. The values of μ_{t-1} and γ_{t-1} are then determined using the first order conditions for debt and employment, that is:

$$1 + \gamma_{t-1} - \mu_{t-1} = \beta(1+r)(1+p\gamma_t)$$

$$\beta\Big[(1+p\gamma_t)\Big(\kappa + A - \psi(z_{t-1})w_t - (1-\eta)\kappa_w\Big) + p\Big(\psi(z_{t-1}) - \psi(z_t)\Big)V_{H,t} \\ -\eta(1-p)\kappa_w\Big] = (1+\gamma_{t-1})\kappa - \mu_{t-1}\bar{\kappa}_f$$

After solving for all $t = T, T-1, ..., 0$, we check whether $z_0 = z_{res}$ and $H_1 = N_1$. The condition $H_1 = N_1$ implies that $N_0 = H_0 = 0$. If the two conditions are not satisfied, we change the guesses for w_{T+1} and H_{T+1} until convergence.

In the solution of the model we also solve for the initial assets a_0. If a_0 is bigger than the initial assets, we increase T. This takes advantage of the fact that smaller are the initial assets of the entrepreneur and longer is the transition to the unconstrained status.

Labor market equilibrium: To compute the labor market equilibrium we start by guessing the equilibrium value of z_{res}. Given this value we solve for the firm's problem for all values of $\overline{N}$. The procedure to solve for the firm's problem has been described above. After finding the invariant distribution of firms, we find the aggregate demand of labor and we check the clearing condition in the labor market. We update z_{res} until the labor market clears.

E Data appendix

In the NLSY79, information on the number of employees at working location in the current or most recent job (the so called CPS job) is collected in all survey years except 1981-1985. Since to construct firms' growth rates we need to link information between consecutive surveys, we focus on the 13 more recent waves. They include the 9 annual waves from 1986 to 1994 and 4 bi-annual waves from 1996 to 2002. Following is the description of the main variables used in the estimation.

Regional Dummies. There are four regional dummies constructed from the variable "Region of current residence".

Schooling. This is the variable "Highest grade completed as of May 1 survey year".

Experience. This is calculated as the age of the worker at interview date, minus years of schooling, minus six.

Working Hours. Until 1993 the number of working hours per week is obtained from the variable "Hours per week usually worked at current/most recent job". Starting from 1994, job 1 always coincides with the CPS job and information about working hours is obtained from the variable "Hours per week worked at job 1".

Metropolitan Area. This is obtained from the question "Is Respondent current residence Urban/Rural?".

Multiple establishments. Until 1993, information about whether the firm has multiple establishments is obtained from the question "Does employer at current job have greater-than-one location?". Starting in 1994, we use the question "Does employer at job 1 have greater-than-one location?"

Firm's Size. The NLSY79 provides information on the total employment size of the employer by only reporting the size of the establishment where the respondent works. Until 1993, this is equal to "Number of employees at location of current job". Starting in 1994 we use "Number of employees at location of job 1". In order to account for firms with more than one establishment, we also include a multi-establishment dummy. We set to missing value observations with a reported value of either 99995 or 99996.

Industry Dummies. Until 1993 the industry dummies were constructed by using the variable "Type of business or industry of most recent job (Census 3 digit)". Starting in 1994 we used the variable "Type of business or industry job 1 (Census 3 digit)". From these variables we constructed twelve industry dummies: 1) Agriculture, Forestry and Fisheries; 2) Mining; 3) Construction; 4) Manufacturing; 5) Transportation, Communication and Public Utilities; 6) Wholesale and Retail Trade; 7) Finance, Insurance and Retail Estate; 8) Business and Repair Services; 9) Personal Services; 10) Entertainment and Recreation Services; 11) Professional and Related Services; and 12) Public Administration.

Hourly wage. Until 1993 the hourly wage in dollar is obtained from the variable "Hourly rate of pay current job". Starting in 1994 we used the variable "Hourly rate of pay of job 1". To eliminate obvious data entry errors we drop observations whose hourly wage is greater than $500 or it is less than $2. We also investigated the robustness of our results by imposing a maximum hourly wage of $100 and a minimum of $5.

Employer Tenure. This is obtained from the five variables "Total Tenure in weeks with employer job 1 (2, 3, 4, 5)". We then identify whether job 1, 2, 3, 4 or 5 corresponds to the CPS job by using the questions "Internal Check: Is job 1 (2, 3, 4, 5) the same as current job". After 1993 the CPS job corresponds to job 1. The tenure variable is originally expressed in weeks and it is converted into years dividing by 52.

New vs. Continuing Jobs. To identify whether the current CPS job is a *new* or a *continuing* job, we follow the procedure detailed in Appendix 9 of the user's guide to NLSY79. In brief, we first identify the number of the job (1 to 5) corresponding to the CPS job in the current survey. Then we identify what is the number of the job in the previous survey that corresponds to the CPS job in the current survey. For the 1979-1992 surveys, the NLSY79 contains two variables that allows to do the match. After 1992, the match is obtained with just one variable. A job is classified as *new* if either variables report a valid missing code. If at least one of these variables contains a valid number (1 or greater), this is the number of the job in the previous survey that is the same as the CPS job in the current survey. Then the CPS job in the current survey is a *continuing* job if in the previous survey the reported job number was also a CPS job. To eliminate possible data entry errors we also require that a continuing job should have a tenure greater than 30 weeks, while a new job should have a tenure smaller than 52 weeks.

Average Firm's Growth Rates. To calculate the average firm's yearly growth rate over the tenure of the worker, we first calculate the annual growth rate of the current employer for all years in which the worker stayed with the same employer (*continuing* job). Then we take the average of these rates. To eliminate possible data entry errors we drop from the sample any observation whose average annual growth rate is bigger than 100% and smaller than -10%.

References

Abowd, J. M. and F. Kramarz (2000). Inter-industry and firm-size wage differentials: New evidence from linked employer-employee data. Unpublished manuscript, Cornell University.

Akerlof, G., A. Rose, and J. Yellen (1988). Job switching and job satisfaction in the US labour market. *Brookings Papers on Economic Activity 2*, 495–582.

Altonji, J. and R. Shakotko (1987). Do wages rise with job seniority? *Review of Economic Studies 54*(3), 437–59.

Anderlini, L. and L. Felli (2001). Costly bargaining and renegotiation. *Econometrica 69*(2), 377–411.

Auerbach, A. and L. Kotlikoff (1987). *Dynamic Fiscal Policy*. New York: Cambridge Uinversity Press.

Baker, G., M. Gibbs, and B. Holmstrom (1994). The wage policy of a firm. *Quarterly Journal of Economics 109*(4), 921–950.

Bayard, K. and K. Troske (1999). Examining the employer-size wage premium in the manufacturing, retail trade, and service industries using employer-employee matched data. *American Economic Review, Papers and Proceedings 89*(2), 99–103.

Blanchflower, D. G., A. J. Oswald, and M. D. Garrett (1990). Insider power in wage determination. *Economica 57*(226), 143–70.

Blasi, J., D. Kruse, and A. Bernstein (2003). *In the Company of Owners: The Truth About Stock Options (And Why Every Employee Should Have Them)*. Basic Books.

Bronars, S. and M. Famulari (2001). Shareholder wealth and wages: Evidence for white-collars workers. *Journal of Political Economy 109*(2), 328–54.

Brown, C. and J. L. Medoff (1989). The employer size-wage effect. *Journal of Political Economy 97*(5), 1027–59.

Brown, C. and J. L. Medoff (2003). Firm age and wages. *Journal of Labour Economics 21*(3), 677–697.

Burdett, K. and M. Coles (2003). Equilibrium wage-tenure contracts. *Econometrica 71*(5), 1377–1404.

Burdett, K. and D. Mortensen (1998). Wage differentials, employer size, and unemployment. *International Economic Review 39*(2), 257–73.

Burdett, K. and T. Vishwanath (1988). Balanced matching and labor market equilibrium. *Journal of Political Economy 96*(5), 1048–1065.

Core, J. E. and W. R. Guay (2001). Stock option plans for non-executive employees. *Journal of Financial Economics 61*, 253–287.

Doms, M., T. Dunne, and K. Troske (1997). Workers, wages, and technology. *Quarterly Journal of Economics 112*(1), 253–90.

Fallick, B. and C. Fleischman (2001). The importance of employer-to-employer flows in the US labour market. Federal Reserve Board Finance and Economics Discussion Paper # 2001-18.

Farber, H. (1999). Mobility and stability: the dynamics of job change in labour markets. In O. Ashenfelter and D. Card (Eds.), *Hanbook of Labor Economics*, pp. 2439–83. New York and Oxford: Elsevier Science.

Fazzari, S. M., G. R. Hubbard, and B. C. Petersen (1988). Financing constraints and corporate investment. *Brookings Papers on Economic Activity 88*(1), 141–95.

Gilchrist, S. and C. P. Himmelberg (1996). Evidence on the role of cash flow for investment. *Journal of Monetary Economics 36*(3), 541–72.

Hall, B. J. and K. J. Murphy (2003). The trouble with stock options. *Journal of Economic Perspectives 17*(3), 49–70.

Hanka, G. (1998). Debt and the terms of employment. *Journal of Financial Economics 48*(3), 245–82.

Harris, M. and B. Holmstrom (1982). A theory of wage dynamics. *Review of Economic Studies 49*(3), 315–33.

Hart, O. and J. Moore (1994). A theory of debt based on the inalienability of human capital. *Quarterly Journal of Economics 109*, 841–79.

Hashimoto, M. (1981). Firm-specific human capital as a shared investment. *American Economic Review 71*(3), 475–482.

Holmstrom, B. (1983). Equilibrium long-term contracts. *Quarterly Journal of Economics 98*(Supplement), 23–54.

Kambourov, G. and I. Manovski (2002). Occupational specificity of human capital. Unpublished manuscript, University of Toronto and University of Pennsylvania.

Kiyotaki, N. and J. Moore (1997). Credit cycles. *Journal of Political Economy 105*(2), 211–48.

Kremer, M. and E. Maskin (1996). Wage inequality and segregation by skill. NBER Working Paper # 5718.

Nickell, S. and D. Nicolitsas (1999). How does financial pressure affect firms? *European Economic Review 43*(8), 1435–56.

Nickell, S. and S. Wadhwani (1991). Employment determination in British industry: Investigations using micro-data. *Review of Economic Studies 58*(5), 955–69.

Oi, W. Y. and T. L. Idson (1999). Firm size and wages. In O. Ashenfelter and D. Card (Eds.), *Handbook of Labour Economics*, pp. Volume 3B. Amsterdam: North Holland.

Rios-Rull, J. (1996). Life-cycle economies and aggregate fluctuations. *Review of Economic Studies 63*(3), 465–89.

Ross, S. A., R. W. Westerfield, and B. D. Jordan (1993). *Fundamentals of Corporate Finance*. Irwin Press, 2nd Edition.

Shapiro, C. and J. E. Stiglitz (1984). Equilibrium unemployment as a worker discipline device. *American Economic Review 74*(3), 433–44.

Smith, C. (1977). Alternative methods for raising capital: Rights versus underwritten offerings. *Journal of Financial Economics 5*(3), 273–307.

Topel, R. (1991). Specific capital, mobility and wages: Wage rise with seniority. *Journal of Political Economy 99*(1), 145–76.

Troske, K. (1999). Evidence on the employer size-wage premium from worker-establishment matched data. *Review of Economics and Statistics 81*(1), 15–26.

Zabojnik, J. and D. Bernhardt (2001). Corporate tournaments, human capital acquisition, and the firm size-wage relation. *Review of Economic Studies 68*(3), 693–716.