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ABSTRACT

This paper develops an open-economy macroeconomic model which can be used to interpret the observed fluctuations in output, inventories, prices, and exchange rates in the medium-sized economies of the world. The model is consistent with the major empirical regularities that have been discovered in studies of business cycles as closed-economy phenomena and in empirical studies of prices and exchange rates. The empirical regularities are (i) changes in the nominal money supply cause real output fluctuations, (ii) deviations of output from a "natural rate" show persistence, (iii) exchange rates are more volatile than nominal prices of goods, and (iv) depreciations of the currency coincide with deteriorations of the terms of trade. A controversial aspect of the model is that only unperceived money has real effects. The channel through which these effects arise involves a misperception by rational maximizing firms of the true demand that they will face after having set prices. The firms learn about their environment from equilibrium asset prices, and the dynamics of the model reflect the optimal response of inventory-holding firms rather than ad hoc price dynamics.

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The purpose of this paper is to develop an open-economy model which can be used to interpret the observed fluctuations in output, inventories, prices, and exchange rates. We have constructed the model to be consistent with the empirical regularities that characterize fluctuations in these magnitudes discovered in studies of business and inventory cycles and with the empirical regularities concerning prices and exchange rates discovered in open-economy studies.[1]

At the center of our model is the optimization problem of domestic firms facing uncertain demand. The representative firm must set its price at the beginning of the period without knowledge of actual demand which occurs during the period. Although firms have less than full information about the current state of the economy, they do observe market clearing prices in asset markets, the government's preliminary announcement of the monetary aggregate, as well as prices being charged by other firms. Consequently, firms use this information to make inferences about what demand will actually occur, and they set their prices to maximize the present value of profits. It is crucial to our analysis of real effects of monetary disturbances that asset markets be informative about real aspects of the economy.[2] The firm's problem is nontrivially intertemporal because inventories are carried through time.

Each period firms make two sequential decisions. First, they set their prices based on incomplete information. Second, after they have received orders for their products, they decide how much of the orders to meet out of current production and how much out of inventories.

Our model is consistent with two major empirical regularities discovered in studies of business cycles. These two regularities are:

(i) changes in the money supply result in real output fluctuations and (ii) deviations of output from a "natural rate" of output show persistence. Fully perceived monetary shocks have no real effects in our model, but unperceived monetary shocks affect real variables in our framework because price-setting firms are unable to infer from asset prices the exact values of monetary disturbances and demand disturbances.[3]

Because the demand for money in our model depends on expenditure by domestic residents, a positive expenditure disturbance (for example) is also a positive money demand disturbance. In this structure either a positive money supply disturbance or a negative expenditure disturbance will elicit incipient excess supply in the money market. Since firms see only the equilibrating asset price responses to such incipient excess supply, there is a tendency for firms to misperceive a positive money supply shock as a negative disturbance to real demand for the home good. Firms are led to lower their relative prices below what would be optimal with full information. Since real demand actually has not shifted in this example, firms experience unexpectedly high demand once orders are processed. Because we assume that these orders must be filled at the posted prices, the typical firm's optimal response to the unexpected demand is to meet part of it by raising current production and part of it out of current stocks of inventories. The firm carries smaller inventory stocks into the future than previously intended. Deviations of inventories from their optimal levels are costly, and the avoidance of these costs causes persistence of deviations of output from its natural rate. In the example above, a monetary disturbance in this period causes inventories to be lower than previously intended next period. Hence, firms produce more in future periods than previously intended in order to make up the inventory shortfall.

A controversial aspect of our model is that real effects of money shocks depend on the part of those shocks that is unperceived rather than on the full unexpected shock. An unexpected shock is one which is unpredictable based on past information. An unperceived shock is one which cannot be inferred from current information. It is the distinction between unperceived money and unexpected money which separates the monetary business cycle models of the Lucas (1973) - Barro (1976, 1980) type (the island models) from those of the Gray (1976) - Fischer (1977) type (the wage indexing models). The well known empirical work of Barro (1977, 1978) examines only the effects of unexpected money. Since all unperceived money must also be unexpected, the Barro work does not clarify the type of monetary shock important for business cycles.

The more recent work of Barro and Hercowitz (1980) and Boschen and Grossman (1983) does attempt to disentangle these two types of shocks. Both sets of authors find evidence which they interpret as being unfavorable to the hypothesis that the monetary portion of the U.S. business cycle is due entirely to unperceived money. Interpreting the evidence as these authors have done requires the imposition of some strong exogeneity assumptions involving the money supply process and the behavior of the U. S. Federal Reserve Board as well as some additional arbitrary restrictions involving lack of correlation among disturbances. A more precise consideration of these issues will be given below in the context of our model.

The major empirical regularities confronting theories of exchange rate determination are the following closely related facts: (i) exchange rates are more volatile than nominal goods prices in the sense that for one period ahead, exchange rates are harder to predict than are goods prices, and (ii) countries' exchange rates are negatively correlated with their terms of trade, that is, a currency depreciation tends to coincide with a deterioration in the terms of trade. In light of these facts, the dominant model of exchange-rate determination has become the one developed by Dornbusch (1976).

In the Dornbusch model, markets in goods are in disequilibrium with prices increasing in response to excess demand, but asset markets are continually in equilibrium. In typical stochastic presentation of the framework, such as Mussa (1982), the domestic price of the home good is treated as a predetermined variable, unresponsive to current disturbances. Thus, a positive monetary shock increases real balances which requires a decrease in the nominal interest rate to reequilibrate the money market. The fall in the nominal interest rate is accomplished by an over depreciation of the domestic currency accompanied by the expectation that the currency will appreciate. The currency depreciation deteriorates the country's terms of trade increasing domestic and foreign demand for the home good, which slowly drives up its nominal and relative price.

Despite the large number of extensions of the basic Dornbusch model, several awkward aspects remain prevalent in the literature. First, as mentioned above, domestic prices of home goods are typically treated as predetermined variables with respect to current disturbances. In contrast, domestic prices of foreign goods are taken to be proportional to the exchange rate, and they jump with jumps in the exchange rate. Neither the predetermination of prices nor the assumed asymmetric treatment of home and foreign goods prices appears to characterize the observed macroeconomic price aggregates. In the Dornbusch model, predetermined prices are perfectly predictable given last period's information set. While it is difficult to imagine what macroeconomic price aggregate fulfills the condition of perfect predictability, presumably the predetermination of prices is intended as an abstraction which captures the stylized fact that prices do not appear to respond fully and immediately to monetary or other disturbances. Indeed, the essential aspects of the Dornbusch analysis are preserved in a model where price adjustment to past excess demand is not perfectly predictable. Price adjustment may have an unpredictable stochastic component which is uncorrelated with other monetary and real disturbances. It is not clear though how such a disturbance arises, and it is not obvious to what extent the results of the Dornbusch model depend on predetermination of prices per se versus some other form of incomplete current price adjustment. The second unappealing aspect of constant output versions of the Dornbusch model such as the presentations of Flood (1981) or Mussa (1982) concerns the reconciliation of the disequilibrium between demand and supply in the goods market. These models never specify how the disequilibrium is allocated across agents.

In the present paper both of these awkward aspects are confronted. Because firms are using asset market information when setting prices, domestic prices in our model are correlated with current disturbances. But, since the agents do not see and are unable to infer exactly the values of the actual disturbances affecting asset prices, domestic price adjustment to a money supply disturbance, for example, is less than its full information counterpart becoming complete only with the resolution of uncertainty. Further, the firms in our model are engaged in pricing, production, and inventory management. Any deviation between current demand and current production is accomodated by corresponding optimal inventory adjustment.

The exchange rate and price solutions of the model reflect the inherent dynamics induced by a current disturbance and propogated through time by optimal management of prices and inventories. The model was set up to match Dornbusch's with respect to initial effects of real and monetary disturbances. The dynamics of the Dornbusch model, though, result from slow price adjustment, while our dynamics result from slow inventory adjustment. This divergence implies quite different adjustment paths for exchange rates and relative prices following initial impact effects. For example, in the Dornbusch set up a one-time monetary disturbance causes the exchange rate to overshoot its long-run value and to be expected to approach that long-run value slowly but directly from above. In contrast, in our set up a one-time monetary disturbance causes the exchange rate to overshoot its long run initially and to be expected to undershoot it subsequently, approaching the long run from below. The crucial difference between these two response patterns centers on the real impact of a money surprise in our model. Such a surprise drives inventories down and output up. Inventories and output adjust directly to their long-run values. But, rising inventories are associated with a depreciating domestic currency, so the exchange rate must undershoot after overshooting and then depreciate to its long-run level. Consequently, the dynamics of our model imply even more wild gyrations of exchange rates than those implied by a Dornbusch-type model.

The model of price and inventory adjustment employed in this paper is a descendant from a long line of similar optimizing models. The early work on this type of model as a theory of the firm was done by Holt, Modigliani, Muth and Simon (1960) and by Lovell (1961) and was extended by Zabel (1972). More recently, Feldstein and Auerbach (1976) and Eichenbaum (1983) have implemented versions of the model to study inventory adjustment at the firm and industry level, while Blinder and Fischer (1982), Blinder (1982), Amihud and Mendelson (1982), Haraf (1981), and Brunner, Cukierman and Meltzer (1982) have worked with variations of the framework at the macroeconomic level. These latter models are designed to incorporate optimal price and inventory adjustment into rational expectations, closed economy macro models. While our model differs from each of the above in many details, two points stand out. First, the dimension of price aggregation is different across our work and the other models. Because the other models are constructed for hypothetical closed economy analysis, the only relative prices which concern firms are those arising in comparisons across firms producing the same product. In contrast, our open economy firms must be concerned with their relative prices compared to foreign good as well as their prices compared to other domestic firms. Second, our firms make price and output decisions sequentially, whereas the other models have simultaneous price and output decisions and often treat both variables as predetermined with respect to current period information. While our sequential decisions are slightly more difficult to anlayze, our results are consistent with three empirical regularities.

The first regularity, noted by Feldstein and Auerbach (p. 363, (1976), is that average absolute sales forecasts errors for durable goods are typically nine times larger than average absolute changes in inventories. This fact suggests that production bridges the gap between actual sales and forecasts of sales, indicating that production responds to unanticipated demand as in our framework. Since Feldstein and Auerbach are quite cautious about the quality of the sales forecast data used to establish the point, we are correspondingly reluctant to use their data as the only support for our modeling strategy.

The second and third regularities have been documented by Barro (1977, 1978). They concern the positive response of real output to monetary shocks and the lack of strong econometric evidence in support of "price surprise" terms in estimated aggregate supply functions. Our model produces a positive response of output to monetary shocks only when output is allowed to respond to actual demand. When output must be determined by the firm at the beginning of the period using its current information set, positive monetary shocks would induce firms to lower output because firms misperceive positive monetary shocks as decreases in real demand. Our sequential decision strategy produces a positive response of output to positive monetary shocks because positive monetary shocks are associated with unexpectedly high demand given the posted prices of firms derived in the first stage optimization. The final feature of our model which should be noted in this introduction concerns a difference across models in transmission of the real effects of monetary shocks. Since firms in our model have full knowledge of the prices charged by other firms, monetary shocks do not have real effects because of a "price surprise" term in the aggregate supply function, which is a common feature of open and closed economy models of the business cycle.[5]

While price surprise terms are crucial to many recent models of business cycles, it is disturbing that such shocks have not played an important role in econometric macro models. Indeed, Barro (1981, p. 71) writes, "Given the relatively minor role played by price surprises in the results of Sargent (1976) and Fair (1979) ..., it appears that monetary influences on output involve channels that have yet to be isolated." Although additional econometric work may show that price surprise terms are the primary transmission mechanism of monetary shocks, the response of output in our model is consistent with the current evidence.

Our investigation yields three principal results. First, monetary disturbances produce persistent real output effects. The impact effect is due to price-setting agents' confusion concerning the true nature of disturbances impinging on asset markets. The persistence is due to the effect of inventory movements on future production decisions. Second, our model predicts that output variance will be higher under fixed exchange rates than under floating rates unless the monetary authority announces the extent of intervention required to maintain the fixed rate. This result, which is similar to one derived by Kimbrough (1982), is due to the fact that fixing the exchange rate removes it as an information source firms may use to infer real disturbances. Unless that information source is replaced by intervention announcements, the quality of firms' information is poorer with fixed rates than with floating rates, and firms will have a tendency to react to disturbances with output changes rather than with price changes. Third, the model matches the open-economy empirical regularities the Dornbusch (1976) model was designed to explain even though prices in our model need not be predetermined. A special case of our model results in predetermined home goods prices. However, because our inventory-based dynamics are different from the Dornbusch-type price-based dynamics, our model does not generate a Dornbusch-type model as a special case.

Our analysis is presented in the next two sections. In Section II we develop the model by focussing first on the goods markets and second on the asset markets. At the end of Section II we summarize the model. Section III presents the full reduced form solution of the model, and Section IV is an analysis of the dynamic responses of the endogenous variables to the exogenous stochastic shocks that drive the model. The consistency of the predictions of the model with the various stylized facts is discussed in Section V. The volatility of output under alternative exchange rate regimes is discussed in Section VI which is followed by some concluding remarks.

II. The Open Economy Macro Model

This section presents an open economy macro model based in part on the decision problems of rational profit maximizing firms. Our presentation is in two parts. In the first part, we develop the equations of the goods markets which consist of demands for and supplies of the goods produced in the medium sized open economy being examined. The result of this part is a set of optimal decision rules governing pricing, inventory accumulation, and production. These decision rules are not reduced forms, however, since imbedded in the rules are beliefs concerning currently unobservable disturbances. The formation of those beliefs is based on information extracted from asset markets. In the second part of this section we provide the asset market structure. We emphasize that the goods markets determine only relative prices and the interaction of the goods and asset markets is required to determine nominal prices.

Our model is one in which some irrevocable decisions are made sequentially, and they are based on incomplete information.[6] At the beginning of the period, agents choose their asset portfolios for the period, and firms choose their prices for the period. These decisions are based on identical incomplete information concerning the state of the economy. Later in the period, firms and agents discover the actual level of demand facing the firms. Given the prices posted at the beginning of the period, firms respond to the actual quantities demanded by choosing profit maximizing levels of production and inventory accumulation.

IIA. Demand in the Goods Markets

There are k firms in the economy, each facing a demand curve of the form

$$D_t^j = \frac{1}{k}D_t - k\beta_4(R_t^j - \bar{R}_t) \qquad j = 1, 2, \ldots, k, \tag{1}$$

where D_t is economy-wide demand, $D_t = \Sigma_{j=1}^k D_t^j$, and R_t^j is the relative price charged by firm j which is equal to that firm's nominal price divided by the price level, a function of the domestic currency prices of domestic and foreign goods. $\bar{R}_t$ is the average economy-wide relative price, $\bar{R}_t$ = $(1/k)\ \Sigma_{j=1}^k R_t^j$. Economy-wide demand, D_t, is the sum of domestic demand, $\rho_0 - \rho_1\bar{R}_t + \rho_2 X_t$ and foreign demand, $\rho_0^* - \rho_1^*\bar{R}_t$, where ρ_i, ρ_i^* are positive parameters and X_t is the level of real expenditure by domestic residents. Real expenditure is assumed to depend positively on real income, $\bar{R}_t Y_t$, with the specification given by the following linearization:

$$X_t = \kappa_1\bar{R}_t + \kappa_2 Y_t + u_t, \qquad \kappa_1, \kappa_2 > 0 \tag{2}$$

where u_t is an aggregate expenditure disturbance. We assume that u_t is a white noise disturbance to the saving-spending decision.[7]

IIB. Pricing, Production and Inventory Holding

Firm j faces the demand curve given by (1). If the firm charges the average economy-wide relative price, $R_t^j = \bar{R}_t$, then its demand is its share of economy-wide demand, $(1/k)D_t$. If the firm charges a higher (lower) relative price than the average, its demand is reduced (increased) by the amount $k\beta_4(R_t^j - \bar{R}_t)$.[8] The larger is $k\beta_4$ the greater is the firm's sensitivity to deviations from the average relative price. In the limit $(k\beta_4 \to \infty)$ each firm will choose to charge the average price.

A firm produces output, Y_t^j, and holds inventories, N_t^j, such that

$$Y_t^j = D_t^j + N_t^j - N_{t-1}^j \tag{3}$$

describes the law of motion for end of period inventories. Firms hold inventories to smooth production costs which are assumed to be an increasing convex function of the firm's output, Y_t^j, and an increasing function of aggregate output, $Y_t = \Sigma_{j=1}^k Y_t^j$. We choose a specific functional form for firm production costs which is given by $\gamma_1 Y_t Y_t^j + (\gamma_2/2)(Y_t^j)^2$, γ_1, $\gamma_2 > 0$. Holding inventories is also costly. We allow negative inventories, interpreting them as a backlog of unfilled orders as in Blinder and Fischer (1981), Eichenbaum (1983) and Blinder (1982). Backlogged orders are costly to the firm because it must discount price to consumers to induce them to pay now and accept delivery in the future. Inventory costs are incurred on beginning of period inventories in accord with the cost function $\delta_1 N_{t-1} N_{t-1}^j + (\delta_2/2)(N_{t-1}^j)^2$, δ_1, $\delta_2 > 0$, where $N_{t-1} = \Sigma_{j=1}^k N_{t-1}^j$ is the aggregate inventory level.

We think of our cost functions as tractable approximations of more complex behavior. Our functions are nonstandard in that Y_t appears in the representative firm's production costs and N_{t-1} appears in inventory holding costs. The presence of Y_t is intended to capture the positive association of economy-wide real wages and aggregate output. The presence of N_{t-1} is intended to capture the positive association of the level of aggregate inventories and storage-space rents. Because we allow negative inventories, the term $\delta_1 N_{t-1} N^j_{t-1}$ could be negative. However, if each firm does not deviate by much from the average, then the effects of such aberations should be small.

The firms' first stage contingency plans are found from the following maximization problems:

$$\max_{\{R^j_{t+i},\ N^j_{t+i}\}} E_t \sum_{i=0}^{\infty} \{D^j_{t+i} R^j_{t+i} - \gamma_1 Y_{t+i} Y^j_{t+i} - (\gamma_2/2)(Y^j_{t+i})^2 - \delta_1 N_{t+i-1} N^j_{t+i-1} - (\delta_2/2)(N^j_{t+i-1})^2\}\sigma^i, \qquad j = 1, \ldots, k. \qquad (4)$$

The firm's maximization problem is subject to an initial stock of inventories, N^j_{t-1}, and to the relationships (1) and (3). The discount rate σ is a constant between zero and unity.[9] The operator E_t denotes the mathematical expectation conditional on the information available to the firm at the beginning of period t. All firms have identical information sets, so the operator is not specific to the firm.

In finding the firm's optimal plans we have assumed that k is sufficiently large that each firm takes the economy-wide variables $\bar{R}_{t+i}$, N_{t+i}, and Y_{t+i} as invariant to the firm's decisions. Such a strategy is exactly profit maximizing only when $k \to \infty$. There is nothing in our setup to preclude $k \to \infty$, and the reader may want to interpret out results in terms of this special case.[10]

The problem stated in (4) implies a pair of linear Euler equations for each of the k firms. These equations are recorded in Appendix A and must be used to find firm-specific decisions. Since our concern is with aggregates, we record here only the "aggregate Euler equations", which are obtained by summing the firm-specific Euler equations. These aggregate Euler equations are:

$$E_t\{D_{t+i} - k^2\beta_4\bar{R}_{t+i} + (k\gamma_1 + \gamma_2)k\beta_4 Y_{t+i}\} = 0, \tag{5a}$$

$$E_t\{Y_{t+i} - \sigma Y_{t+i+1} + \sigma\mu N_{t+i}\} = 0, \tag{5b}$$

where $\mu \equiv (\delta_1 k + \delta_2)/(\gamma_1 k + \gamma_2)$. Equation (5a) is obtained by summing across all the firm-specific Euler equations resulting from differentiating (4) with respect to R^j_{t+i}, j = 1, 2, ..., k, and (5b) results from summing across all the firm-specific Euler equations resulting from differentiating (4) with respect to N^j_{t+i}, j = 1, ..., k. We wish to solve (5a) and (5b) for aggregate contingency plans concerning $E_t\bar{R}_{t+i}$ and E_tN_{t+i}. Because prices are set based on beginning of the period information, the planned value $E_t\bar{R}_t$ and the actual magnitude $\bar{R}_t$ will coincide.

Before solving (5a) and (5b), it is convenient to define some demand-associated parameters. Use the definition of D_t, the aggregate law of motion $N_t = Y_t + N_{t-1} - D_t$, and the expenditure function in (2) to obtain

$$D_t = \beta_0 - \beta_1\bar{R}_t + \beta_2(N_t - N_{t-1}) + \beta_3 w_t, \tag{6}$$

where $\beta_0 = (\rho_0 + \rho_0^*)/(1 - \kappa_2\rho_2)$, $\beta_1 = (\rho_1 + \rho_1^* - \rho_2\kappa_1)/(1 - \rho_2\kappa_2)$, $\beta_2 = \rho_2\kappa_2/(1 - \kappa_2\rho_2)$, $\beta_3 = 1/(1 - \kappa_2\rho_2)$, and $w_t = \rho_2 u_t$. Since we assume $\rho_2\kappa_2 < 1$ and $\rho_1 + \rho_1^* - \rho_2\kappa_1 > 0$, we have $\beta_i > 0$, $i = 1, 2, 3$. Equation (6) gives the aggregate demand function we use when solving (5a) and (5b).

Since w_t is a white noise disturbance, we conjecture solutions for the Euler equations of the form

$$\bar{R}_t = \pi_{\bar{R}0} + \pi_{\bar{R}1}N_{t-1} + \pi_{\bar{R}2}E_t w_t \tag{7a}$$

and

$$E_t N_t = \pi_{N0} + \pi_{N1}N_{t-1} + \pi_{N2}E_t w_t. \tag{7b}$$

The values of the π coefficients in (7a) and (7b) are found by the method of the undetermined coefficients. These values are recorded in Table 1.

Table 1

$$\pi_{N0} = \text{constant}$$

$$\pi_{N1} = 1/2[A - (A^2 - 4/\sigma)^{1/2}], \qquad 0 < \pi_{N1} < 1$$

$$\pi_{N2} = [\frac{1}{(\pi_{N1} - A)\sigma}] \frac{k^2\beta_3\beta_4}{\beta_1 + k^2\beta_3\beta_4}, \qquad \pi_{N2} < 0$$

$$\pi_{\bar{R}0} = \text{constant}$$

$$\pi_{\bar{R}1} = \frac{\Delta_2}{\Delta_1}(\pi_{N1} - 1), \qquad \pi_{R1} < 0$$

$$\pi_{\bar{R}2} = \frac{\Delta_2}{\Delta_1}\pi_{N2} + \frac{\Delta_3}{\Delta_1}, \qquad \pi_{R2} > 0$$

where

$$A = 1 + \frac{1}{\sigma} + \frac{\mu\Delta_1}{\beta_1 + k^2\beta_3\beta_4}$$

$$\Delta_1 = (1 + k^2\gamma_1\beta_4 + k\gamma_2\beta_4)\beta_1 + k^2\beta_4$$

$$\Delta_2 = (1 + k^2\gamma_1\beta_4 + k\gamma_2\beta_4)\beta_2 + k^2\gamma_1\beta_4 + k\gamma_2\beta_4$$

$$\Delta_3 = (1 + k^2\gamma_1\beta_4 + k\gamma_2\beta_4)\beta_3$$

Equation (7a) gives both the contingency plan for $\bar{R}_t$ and actual $\bar{R}_t$ since these values are identical. However, equation (7b) does not give actual N_t, only expected N_t, since actual inventories will not be determined until the second stage of optimization when actual demand is revealed to the firms. Equations (7a) and (7b) make intuitive sense. Since $\pi_{\bar{R}1} < 0$ and $\pi_{\bar{R}2} > 0$, $\bar{R}_t$ responds negatively to beginning of period inventories and responds positively to expected demand disturbances. Since $0 < \pi_{N1} < 1$, expected inventories obey a stable autoregression; and since $\pi_{N2} < 0$, inventories are expected to fall in response to a positive demand disturbance.

After the firms set prices, they are confronted with actual demand. We assume that the firms may not alter their posted prices after they see demand. However, the firms need not follow their contingency plans for inventory accumulation. Instead, upon seeing demand the firms satisfy the demand with an optimal combination of current production and inventory change. This is the second stage of optimization, and in this stage each firm takes as given its own price, the economy-wide average price, beginning of period inventory, and actual demand for the current period. The economy-wide Euler equations for this stage of the optimization are obtained in a manner similar to (5a) and (5b) except that R_t^j is now not a decision variable and the information set relevant to the optimization now includes the actual value of demand at time t.

The inventory decision may be derived from the following aggregate Euler equation:

$$E_t'\{Y_t - \sigma Y_{t+1} + \sigma\mu N_t\} = 0, \tag{8}$$

where E_t' is the expectation operator conditional on full information for period t which includes w_t.

Since actual inventories will differ from contingency plans only due to differences of w_t from $E_t w_t$, we express the solution for inventories as

$$N_t = \pi_{N0} + \pi_{N1}N_{t-1} + \pi_{N2}E_t w_t + \pi_{N3}(w_t - E_t w_t). \tag{9}$$

Using (8) and our previous results we find

$$\pi_{N3} = \frac{-\beta_3}{\beta_3 + [\sigma(1-\pi_{N1})(\beta_1+k^2\beta_3\beta_4)/\Delta_1] + \sigma\mu}, \tag{10}$$

where

$$-1 < \pi_{N3} < \pi_{N2} < 0.$$

In (10) note that $\pi_{N3} < \pi_{N2}$ implies a stronger response of inventories to unexpected demand than to expected demand. Firms respond to expected demand shocks with their relative prices and an expected response in inventories and production. When actual demand occurs, the firm responds optimally given its set price. Consequently, the response of inventories and production to unexpected demand under the constraint of no price change is greater than the response to expected demand.

Aggregate output is given by $Y_t = D_t + N_t - N_{t-1}$. Using our previous results we derive

$$Y_t = \pi_{Y0} + \pi_{Y1}N_{t-1} + \pi_{Y2}E_tw_t + \pi_{Y3}(w_t - E_tw_t) \tag{11}$$

where the coefficients are given in Table 2. Because $\pi_{Y1} < 0$, larger beginning of period inventories result in a lower output. Since $\pi_{Y2} > 0$, increased expected demand increases expected output.

Table 2

π_{Y0} = constant	
$\pi_{Y1} = \dfrac{(\pi_{N1} - 1)(\beta_1 + k^2\beta_3\beta_4)}{\Delta_1}$,	$\pi_{Y1} < 0$
$\pi_{Y2} = \dfrac{k^2\beta_3\beta_4}{\Delta_1}[1 + \dfrac{1}{(\pi_{N1} - A)\sigma}]$	$\pi_{Y2} > 0$
$\pi_{Y3} = \beta_3(1 - \pi_{N3})$	$\pi_{Y3} > \pi_{Y2}$

An increase in E_tw_t produces an increase in $\bar{R}_t$ and a higher expected quantity demanded along the shifted demand curve. Firms plan on meeting this increase in demand partly out of current production and partly by drawing down current inventory stocks. Because $\pi_{y3} > \pi_{y2}$, unexpected demand has a larger output effect than does expected demand, since expected demand is reflected in increases in relative prices while unexpected demand is not.

This completes our development of the goods markets. We have not yet obtained reduced forms for relative prices, inventories or output because our expressions for these magnitudes all contain the expectation $E_t w_t$. To determine this magnitude agents use their knowledge of the entire economy which consists of both the goods markets and the asset markets. We turn now to the development of the asset markets.

IIc. The Asset Markets

The economy we are portraying is one which is small in the world securities markets, where all securities are perfect substitutes, and small in the markets for foreign produced goods. However, the country is large in the markets for domestically produced goods and for domestic money. Thus, foreign interest rates and foreign goods prices are exogenous to our economy. The principal equations describing the asset markets are the following:

$$m_t - p_t = -\alpha_1 i_t + \alpha_2 \tilde{X}_t, \qquad \alpha_1, \alpha_2 > 0 \tag{12a}$$

$$i_t = i_t^* + E_t s_{t+1} - s_t \tag{12b}$$

$$p_t = \theta \bar{h}_t + (1 - \theta)(\bar{h}_t^* + s_t), \qquad 0 < \theta < 1 \tag{12c}$$

Equation (12a) expresses money market equilibrium and states that the real money supply, $m_t - p_t$, equals real money demand, $-\alpha_1 i_t + \alpha_2 \tilde{X}_t$. In (12a), m_t is the logarithm of the supply of nominal transactions balances and p_t is the logarithm of the nominal price level. According to (12c), p_t is a weighted average of the logarithms of the average domestic currency price of domestic goods, $\bar{h}_t$, and the average domestic currency price of imported goods, $\bar{h}_t^* + s_t$, where $\bar{h}_t^*$ is the logarithm of the average foreign currency price of imported goods and s_t is the logarithm of the exchange rate quoted as the home currency price of foreign currency. The relative price of home goods, $\bar{R}_t$, may be approximated as $\bar{R}_t = \bar{h}_t - p_t + 1$.

Money demand is specified in the spirit of cash-in-advance models such as those of Clower (1967), Lucas (1980) and Kohn (1981). The opportunity cost of holding cash balances in excess of planned expenditure is i_t, the level of the domestic rate of interest. According to (12b), i_t obeys the uncovered interest rate parity condition, with i^*_t being the level of the foreign interest rate.[11] The scale variable in money demand, $\tilde{X}$, is the sum of agents' expected expenditures, $\tilde{X}_t = \Sigma^n_{i=1} E^i_t X^i_t$, where n is the number of agents in the economy, E^i_t is agent i's expectation operator at the beginning of period t and X^i_t is agent i's expenditure during period t. We assume X^i_t obeys

$$X^i_t = \frac{\kappa_1}{n}\bar{R}_t + \frac{\kappa_2}{n}Y_t + u^i_t \qquad (14)$$

where u^i_t is the individuals saving-expenditure disturbance at time t. We allow each agent to see his own u^i_t at the beginning of the period. However, we assume that u^i_t is composed of two uncorrelated white noise components, e^i_t and a^i_t, $u^i_t = e^i_t + a^i_t$. Further, we impose $\Sigma^n_{i=1} e^i_t = 0$. Thus, u^i_t contains an individual-specific component, e^i_t, and the individual's contribution to the aggregate disturbance, $\Sigma u^i_t = \Sigma a^i_t = u_t$. We assume that the variance of e^i_t is sufficiently large compared to the variance of a^i_t such that even though each agent sees his own expenditure disturbance, u^i_t, he always thinks that disturbance to be dominated by the individual-specific component, e^i_t. Hence, the agent cannot use his observation of u^i_t to form useful inferences concerning u_t or other aggregate disturbances. Thus, when $\tilde{X}_t$ is formed, one obtains

$$\tilde{X}_t = \kappa_1 \bar{R}_t + \kappa_2 E_t Y_t + u_t. \tag{14}$$

$\bar{R}_t$ appears in (14) because $\bar{R}_t$ is in each agent's information set. $E_t Y_t$ appears because $E_t^i Y_t = E_t Y_t$ since knowing u_t^i provides the agent with no information concerning the aggregates. u_t appears in (14) because $E_t^i u_t^i = u_t^i$ and by construction $\Sigma_{i=1}^n u_t^i = u_t$.

The logarithm of actual transactions balances is assumed to follow a random walk $m_t = m_{t-1} + v_t$ where v_t is white noise. At the beginning of the period agents do not know v_t, but it is assumed that they know m_{t-1}. Also, in keeping with the practices of many countries, we assume that at the beginning of the period agents observe a preliminary indicator of the nominal money supply, the "money number", $m_t^{\#}$, which we assume to be equal to actual money plus a white noise disturbance, z_t: $m_t^{\#} = m_t + z_t$. The three white noise disturbances, u_t, v_t, and z_t, are assumed to be mutually orthogonal.[12]

The aggregate information available at the beginning of the period is the conditioning information for the operator E_t, appearing both in the asset markets and in the goods markets. That information set contains the realized values of all lagged variables and current values of all goods and asset market prices, i_t, i_t^*, s_t, R_t^j ($j = 1, 2, \ldots, k$), the money number, $m_t^{\#}$, as well as foreign goods prices and the structure of the model. Notably, the beginning of period information set does <u>not</u> contain current values of the disturbance terms $w_t = \rho_2 u_t$, v_t or z_t.

For simplicity, we complete the model by assuming that the average price of foreign goods is constant, $\bar{h}_t^* = \bar{h}^*$, and the foreign interest rate is also constant, $i_t^* = i^*$.

For convenience we list the entire model including a glossary of notation in table 3 along with the model's important derived and assumed restrictions.

Table 3

The Complete Model

Goods Market

$$\bar{R}_t = \pi_{\bar{R}0} + \pi_{\bar{R}1}N_{t-1} + \pi_{\bar{R}2}E_t w_t;\ \pi_{\bar{R}1} < 0,\ \pi_{\bar{R}2} > 0 \tag{15a}$$

$$N_t = \pi_{N0} + \pi_{N1}N_{t-1} + \pi_{N2}E_t w_t + \pi_{N3}(w_t - E_t w_t);\quad 0 < \pi_{N1} < 1,\ \pi_{N3} < \pi_{N2} < 0 \tag{15b}$$

$$Y_t = \pi_{Y0} + \pi_{Y1}N_{t-1} + \pi_{Y2}E_t w_t + \pi_{Y3}(w_t - E_t w_t);\quad \pi_{Y1} < 0,\ \pi_{Y3} > \pi_{Y2} > 0 \tag{15c}$$

Asset Markets

$$m_t - p_t = -\alpha_1 i_t + \alpha_2 \tilde{X}_t;\ \alpha_1,\ \alpha_2 > 0 \tag{15d}$$

$$i_t = i^*_t + E_t s_{t+1} - s_t \tag{15e}$$

$$p_t = \theta\bar{h}_t + (1 - \theta)(\bar{h}^*_t + s_t);\ 0 < \theta < 1 \tag{15f}$$

$$\tilde{X}_t = \kappa_1\bar{R}_t + \kappa_2 E_t Y_t + u_t;\ \kappa_1,\ \kappa_2 > 0 \tag{15g}$$

Exogenous Processes

$$m^{\#}_t = m_t + z_t \tag{15h}$$

$$m_t = m_{t-1} + v_t \tag{15i}$$

$$i^*_t = i^* \text{ (constant)} \tag{15j}$$

$$h^*_t = \bar{h}^* \text{ (constant)} \tag{15k}$$

Glossary of Variables

$\bar{h}_t$ = logarithm of average price of domestic goods in units of domestic currency

$\bar{h}^*_t$ = logarithm of average price of foreign goods in units of foreign currency

i_t = level of domestic interest rate

i^*_t = level of foreign interest rate

m_t = logarithm of domestic money, transactions balances

$m^{\#}_t$ = logarithm of the money number

N_t = level of aggregate domestic inventories

p_t = logarithm of domestic price index

$\bar{R}_t$ = level of average relative price of domestic goods in terms of the price level

s_t = logarithm of exchange rate quoted as domestic currency price of foreign currency

u_t = white noise disturbance to expenditure

v_t = white noise disturbance to m_t

X_t = level of real spending by domestic residents

Y_t = level of aggregate domestic output

z_t = white noise measurement error in $m_t^{\#}$

v_t, $w_t = \rho_2 u_t$, z_t are mutually orthogonal white noise disturbances having variances σ_v^2, σ_w^2 and σ_z^2 respectively.

Before turning to the formal solution of the model, it is useful to summarize informally the working of the model. At the beginning of each perioa prices are set, and the exchange rate and interest rate are determined. However, agents do not know, at this stage, the actual values of the disturbances v_t, w_t and z_t. The agents see all prices, the exchange rate, the current money number, and both the domestic and foreign interest rates. From these data the agents form inferences concerning the values of the disturbances. It is the inferred value of w_t, $E_t w_t$, which feeds into the pricing decision. After prices are set, the actual value of the demand disturbance and the other disturbances are revealed to the agents. The firms then choose optimal production and inventory accumulation based on the actual quantity demanaed which is determined in part by the prices set under partial inforation and in part by the demand distrubance, w_t.

III. The Solution

In this section we will provide our model's reduced form solutions for the level of output, imventories, the exchange rate, the average relative price of the domestic good, and the nominal price of the domestic good. The first step required in obtaining a solution is to extract information from the clearing of the asset markets and from the money number to form agents' perceptions about current disturbances. Agents will be able to observe two signals of the three underlying aggregate disturbances.

IIIA. Information and the Asset Markets

At the beginning of the period each agent has the information set I_t, which contains the values of s_t, $\bar{h}_t$, $\bar{h}^*_t$, i_t, i^*_t, $R^j_t(j = 1, \ldots, k)$, $m^{\#}_t$, and full information concerning all variables dated t - 1 or earlier as well as complete information concerning the structure of the model. I_t does not contain the current distrubances v_t, w_t or z_t. Since agents' decisions at the beginning of the period in both price setting and in the asset markets depend on their perceptions of these distrubances, they will use the information in I_t to draw inferences about the disturbances. We assume that $E_t v_t$, $E_t w_t$, and $E_t z_t$ denote the linear least squares projections of the respective disturbances onto the information set I_t.[13]

To find the values of these linear least squares estimates we must isolate the new information entering I_t concerning the disturbances at the beginning of the period.[14] Two of the disturbances impinge directly on the asset markets, and it is from these markets that agents extract one signal concerning the disturbances. Using international capital market equilibrium, (12b), money market equilibrium may be written as

$$m_{t-1} + v_t - p_t = -\alpha_1(i^*_t + E_t s_{t+1} - s_t) + \alpha_2(\kappa_1 \bar{R}_t + \kappa_2 E_t Y_t) + \alpha_3 w_t \quad (16)$$

where $\alpha_3 \equiv \alpha_2/\rho_2$. I_t contains m_{t-1}, p_t, i^*_t, s_t and $\bar{R}_t$ as well as the parameters α_1, α_2, and α_3. In addition, I_t is used to form $E_t s_{t+1}$ and $E_t Y_t$ which implies that I_t contains the following variable:

$$g_{1t} = v_t - \alpha_3 w_t. \tag{17}$$

The variable g_{1t} carries the asset markets' information concerning the underlying disturbances. The second signal is contained in the money number, $m_t^{\#} = m_t + z_t = m_{t-1} + v_t + z_t$. The beginning of period information set contains m_{t-1}, so the new information in $m_t^{\#}$ is

$$g_{2t} = v_t + z_t. \tag{18}$$

The variables g_{1t} and g_{2t} contain the current-period information about v_t, w_t, and z_t available to agents at the beginning of the period. Agents use these two pieces of information to form $E_t v_t$ and $E_t w_t$ as linear least squares projections of v_t and w_t onto g_{1t} and g_{2t}. Hence,

$$E_t v_t = \phi_{v1} g_{1t} + \phi_{v2} g_{2t} \tag{19a}$$

and

$$E_t w_t = \phi_{w1} g_{1t} + \phi_{w2} g_{2t} \tag{19b}$$

where

$$\phi_{v1} = \Delta^{-1} \sigma_v^2 \sigma_z^2 > 0$$

$$\phi_{v2} = \Delta^{-1} \alpha_3^2 \sigma_w^2 \sigma_v^2 > 0$$

$$\phi_{w1} = -\Delta^{-1} \alpha_3 \sigma_w^2 (\sigma_v^2 + \sigma_z^2) < 0$$

$$\phi_{w2} = \Delta^{-1}\alpha_3\sigma_w^2\sigma_v^2 > 0$$

and

$$\Delta^{-1} = [\sigma_v^2\sigma_z^2 + \alpha_3^2\sigma_w^2\sigma_v^2 + \alpha_3^2\sigma_w^2\sigma_z^2]^{-1}.$$

Using these projections we can derive the full reduced form solution of the model. The reduced-form solutions for the real sector of the model, $\bar{R}_t$, N_t, and Y_t, can be found by substituting $E_t w_t$ in (19b) into (7a), (9), and (11). Reduced-form solutions for the exchange rate and the domestic price are found from the money market equilibrium in (16) and from the approximation $\bar{R}_t = \bar{h}_t - p_t + 1$. Given the assumed time series properties of the exogenous stochastic processes and ignoring constant terms, reduced-form equations have the form

$$J_t = \lambda_{JN}N_{t-1} + \lambda_{Jm}m_{t-1} + \lambda_{Jv}v_t + \lambda_{Jw}w_t + \lambda_{Jz}z_t \tag{20}$$

for $J = N_t$, Y_t, $\bar{R}_t$, s_t, $\bar{h}_t$. To solve for the exchange rate in (16), recognize that $p_t = [\theta/(1-\theta)]\bar{R}_t + s_t$, substitute the appropriate expression for s_t in (20) into the equation, use $E_t s_{t+1} = \lambda_{sN}E_t N_t + \lambda_{sm}(m_{t-1} + E_t v_t)$, and substitute for $\bar{R}_t$, $E_t N_t$, $E_t Y_t$, and $E_t v_t$. Then, equate the values of the coefficients of the state variables on both sides of the equation.

The algebraic signs of the λ coefficients of the full reduced form are recorded in Table 4, and the actual values of the coefficients are listed in Appendix B. The dynamics of the model are described in the next section with the aid of Figure 1.

Table 4

Endogenous Variable	Signs of Reduced Form Coefficients on N_{t-1}	m_{t-1}	v_t	w_t	z_t
N_t	$0 < \lambda_{NN} < 1$	$\lambda_{Nm} = 0$	$\lambda_{Nv} < 0$	$\lambda_{Nw} < 0$	$\lambda_{Nz} > 0$
Y_t	$\lambda_{YN} < 0$	$\lambda_{Ym} = 0$	$\lambda_{Yv} > 0$	$\lambda_{Yw} > 0$	$\lambda_{Yz} < 0$
$\bar{R}_t$	$\lambda_{\bar{R}N} < 0$	$\lambda_{\bar{R}m} = 0$	$\lambda_{\bar{R}v} < 0$	$\lambda_{\bar{R}w} > 0$	$\lambda_{\bar{R}z} > 0$
s_t	$\lambda_{sN} > 0$	$\lambda_{sm} = 1$	$\lambda_{sv} > 0$	$\lambda_{sw} < 0$	$\lambda_{sz} \gtrless 0$
$\bar{h}_t$	$\lambda_{\bar{h}N} \gtrless 0$	$\lambda_{\bar{h}m} = 1$	$\lambda_{\bar{h}v} \gtrless 0$	$\lambda_{\bar{h}w} \gtrless 0$	$\lambda_{\bar{h}z} \gtrless 0$

IV. The Dynamics of the Model

As the reduced-form equations (20) indicate, the beginning of period inventory stock, N_{t-1}, the actual money supply from the previous period, m_{t-1}, and the stochastic disturbances v_t, w_t, and z_t are the state variables of the system. We assume that actual money is known with a one period lag. Therefore, the lagged nominal money stock does not influence the real sector of the model, and since the logarithm of the actual nominal money supply is assumed to follow a random walk, the exchange rate and domestic price change equiproportionality to known changes in m_{t-1}. Consequently, $\lambda_{Nm} = \lambda_{ym} = \lambda_{\bar{R}m} = 0$, and $\lambda_{sm} = \lambda_{\bar{h}m} = 1$.

The dynamic path of the economy is induced by innovations in the exogenous stochastic processes, the innovation in the actual money stock, v_t, the domestic demand disturbance, w_t, and the error in the money number, z_t. These contemporaneously unobservable disturbances shock the system away from its steady state which is labeled with an F subscript in

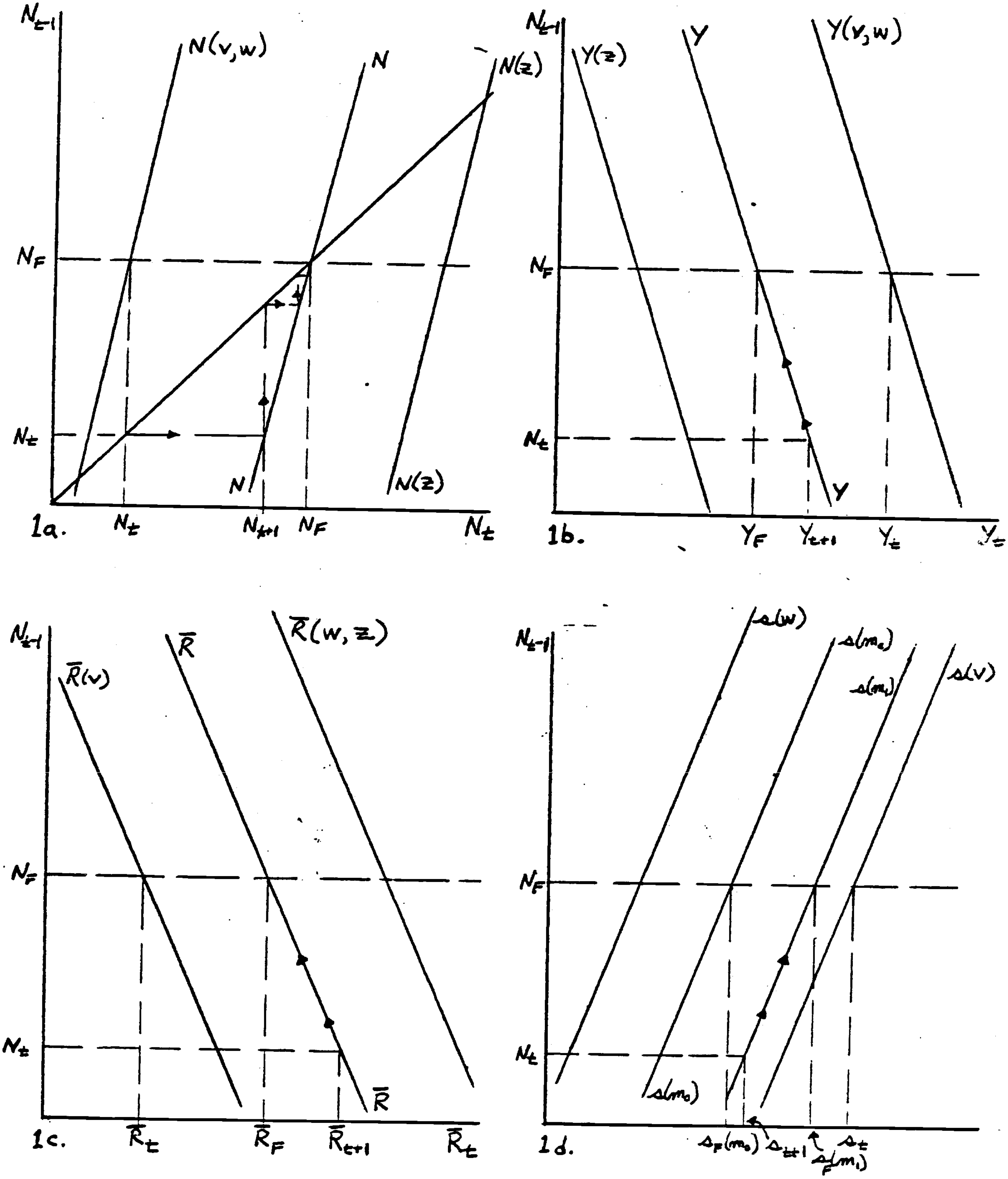

Figure 1: Dynamic Adjustment Paths

Figure 1. In that figure, the NN, YY, $\bar{R}\bar{R}$, and $s(m_0)s(m_0)$ loci indicate the values of N_t, Y_t, $\bar{R}_t$, and s_t that are consistent with any particular value of N_{t-1} given a level of money, $m_{t-1} = m_0$, and no new shocks to the system.[15] Figure 1a demonstrates that when inventories are away from their steady state, they converge over time in a stable autoregression toward the full equlibrium N_F. As YY indicates, output is above Y_F when inventories are below N_F. Along the adjustment path firms set their relative prices higher when inventories are low as indicated by $\bar{R}\bar{R}$, and for $N_{t-1} < N_F$ the exchange rate is expected to depreciate, as $\overline{ss}$ indicates, as the economy moves toward full equilibrium. This is consistent with asset market equilibrium and with the expected fall in $\bar{R}$. We turn now to consideration of how the economy responds to the stochastic disturbances.

Consider how the economy responds to an unobservable stochastic increase in the money supply, v_t, given that it begins in full equilibrium and given that $w_t = z_t = 0$. From (19b), notice that $E_t w_t = (\phi_{w1} + \phi_{w2})v_t < 0$ indicating that agents misperceive the increase in the money supply as a reduction in real goods demand. This occurs because the information provided by the equilibrium values of prices, the interest rates and the exchange rate obtained by firms in observing g_{1t} is consistent with an increase in the money supply and with a reduction in expenditure. As (19a) indicates, combining g_{1t} with the information in the money number allows firms to infer that v_t has increased, but $E_t v_t = (\phi_{v1} + \phi_{v2})v_t$ which is positive and smaller than v_t. Since firms expect a fall in real demand, they lower their relative price to $\bar{R}_t$ in Figure 1c which is the intersection of the locus R(v) and N_F. At this point firms are anticipating an increase in inventories and a reduction in output along a shifted aggregate demand curve. When real demand is actually realized, it occurs along the unshifted demand curve because we are discussing the influence of a monetary shock and are holding $w_t = 0$. Since firms have set low relative prices, demand is unexpectedly high. Firms respond with an optimal combination of increased production, at the intersection of the locus $Y(v,w)$ and N_F in Figure 1b, and inventory depletion, at the intersection of the locus $N(v,w)$ and N_F in Figure 1a. The domestic currency depreciates in response to the v_t shock for two reasons. First, to the extent that the monetary shock is perceived, all nominal prices including the exchange rate rise equiproportionately. Second, part of the deterioration in the terms of trade, i.e. the decrease in $\bar{R}_t$, is accomplished by a depreciation of the currency, the exchange rate rises. In Figure 1d, the exchange rate is determined by the intersection of the locus $s(v)$ with N_F.

A fundamental insight of Dornbusch (1976) was that monetary shocks would cause exchange rate overshooting if goods prices were fixed and the money market was in equilibrium. In this model overshooting is not a necessary result although it is more likely the smaller is α_1, the semi-elasticity of the demand for money with respect to the interest rate. To demonstrate this result, notice that λ_{sv}, the initial response of the exchange rate to a money shock, can be written as its full information response plus an additional term:

$$\lambda_{sv} = 1 + \left(\frac{1}{1+\alpha_1}\right)\left(\frac{\alpha_3\sigma_w^2\sigma_z^2}{\Delta}\right)\{[(\frac{\theta}{1-\theta} + \alpha_2\kappa_1)\pi_{\bar{R}2} - \alpha_1\lambda_{sN}\pi_{N2} + \alpha_2\kappa_2\pi_{Y2}] - \alpha_1\alpha_3\} \quad (21)$$

In (21) the exchange overshoots if the positive term in square brackets is larger than $\alpha_1\alpha_3$. Figure 1d is drawn under that supposition.

After the initial response to the monetary shock, the economy adjusts over time back to its unconditional equilibrium unless new stochastic disturbances alter its path. Inventories begin to accumulate since firms produce output in excess of demand which firms constrain by raising the relative price of their product above its unconditional equilibrium value. The exchange rate falls in period t+1 to facilitate the improvement in the terms of trade. As Figure 1d indicates, the exchange rate is then expected to depreciate over time to its unconditional equilibrium value $s_F(m_1)$. Rather than approach its new steady state from above as in the Dornbusch (1976) framework, the approach is from below.

Now consider the influence of a positive shock to real demand which we normalize to have the same effect on inventories and output as the previously discussed money shock in Figure 1. This shock is considered in isolation from other shocks, i.e., $v_t = z_t = 0$. When a positive but unobservable real shock occurs, $w_t > E_t w_t > 0$, since $E_t w_t = -\alpha_3 \phi_{w1} w_t$ and $0 < -\alpha_3 \phi_{w1} < 1$. Firms expect an increase in demand and raise their relative prices. In Figure 1c, $\bar{R}_t$ is given by the intersection of the locus $\bar{R}(w,z)$ and N_F. Firms expect to draw down inventories and to increase production, but they are surprised by the magnitude of actual demand. Output for period t occurs at the intersection of the locus $Y(v,w)$ and N_F in Figure 1b, and N_t is given by the intersection of N_F and the locus $N(v,w)$ in Figure 1a. The exchange rate falls as the currency appreciates for two reasons. First, the currency appreciates to facilitate the improvement in the terms of trade. Second, agents think that the supply of money has fallen since $E_t v_t = -\alpha_3 \phi_{v1} w_t < 0$. Consequently, the exchange rate falls to reflect the perceived decrease in the money supply.

The dynamic adjustment in period t+1 and afterward is exactly as in the case of the positive monetary disturbance except that the exchange rate in period t+i+1, i = 0, 1, 2, ..., is given by the intersection of N_{t+i} and the locus $s(m_0)$ in Figure 1d.

Next, consider the response of the economy to z_t, the reporting error between the logarithms of the measured nominal money supply and the actual money supply, given $v_t = w_t = 0$. The money number, $m_t^{\#}$, is a source of "news" about the actual money supply. Knowing $m_t^{\#}$ allows agents to make inferences about the state of the economy. Frenkel (1981) and Frenkel and Mussa (1980) have stressed that modern asset theories of the determination of the exchange rate predict that much of the observed changes in exchange rates will be in response to new information or news. If that news is measured with error, such as $m_t^{\#}$ is, then the noise in the news will be a fundamental determinant of all of the endogenous variables of the economy including the exchange rate.

Given the stochastic structure of the economy, agents misinterpret positive z_t disturbances as positive real demand disturbances, since $E_t w_t = \phi_{w2} z_t > 0$, and as positive money supply disturbances, since $E_t v_t = \phi_{v2} z_t > 0$. Consequently, for the real sector of the economy, a positive money supply reporting error operates exactly like a negative disturbance to the actual money supply. Firms expect an increase in demand, raise their relative prices, and expect to increase production and decumulate inventories. When actual demand is realized, it is lower than expected, and firms must cut back on production and increase inventories. Because the positive z_t is misinterpreted as a positive increase in the actual money supply, the effect of z_t on the exchange rate is ambiguous without further assumptions. Under the assumption that produces exchange rate overshooting with respect to a v disturbance, $\lambda_{sz} < 0$ since

$$\lambda_{sz} = \frac{-\alpha_3 \sigma_w^2 \sigma_v^2}{(1 + \alpha_1)\Delta}\left[\left(\frac{\theta}{1-\theta} + \alpha_2 \kappa_1\right)\pi_{\bar{R}2} - \alpha_1 \lambda_{sN} \pi_{N2} + \alpha_2 \kappa_2 \pi_{Y2} - \alpha_1 \alpha_3\right]. \qquad (22)$$

The effects of the disturbances on the nominal price of the domestic good are also generally indeterminate in algebraic sign which is why we have not discussed the effects of the shocks on this endogenous variable.

The next section discusses the consistency of the model with this various stylized facts mentioned in the introduction of this paper.

V. Consistency with Empirical Regularities

Several empirical regularities were mentioned in the introduction, and this section discusses the consistency of the implications of our model with these regularities.

The first regularity addressed is that nominal monetary disturbances must have persistent real effects. This is true in our model since v_t affects all real variables and because the explicit modeling of inventories induces persistent dynamics. A potential criticism of the model is that the real effects of money are caused only by unperceived money.

Two empirical papers, one by Barro and Hercowitz (1980), hereafter referred to as B-H, and one by Boschen and Grossman (1983), hereafter B-G, address this issue. It is important to discuss the relationship between the present structure of our model and the regressions used to test hypotheses in B-H and B-G because the results of these studies provide some evidence against the hypothesis that unperceived money is the primary channel through which nominal money affects real variables.

For clarity of presentation, the stochastic processes v_t, w_t and z_t were specified as being jointly orthogonal as well as being independently and identically distributed. It turns out that the predictions of our model in the jointly orthogonal case are inconsistent with the evidence presented in B-G and B-H. In each case, though, significant covariance between v_t and z_t in one case and z_t and w_t in the other is enough to overturn the apparent inconsistencies between the model and the data.

The two empirical propositions of B-G and B-H are the following: (i) the measurement error between actual money and reported money should have a significant effect on real output, and (ii) reported money, since it is fully perceived, should have no real effort. The first hypothesis is tested and rejected in B-H with U.S. annual average data from 1950-75 and in B-G with U. S. quarterly average data for 1953-78 while the second hypothesis is rejected in B-G.

The hypotheses are most easily discussed in terms of the reduced form for output which may be written as

$$Y_t = \pi_{Y1}N_{t-1} + (\pi_{Y2} - \pi_{Y3})[\phi_{w1}(v_t-\alpha_3 w_t) + \phi_{w2}(v_t+z_t)] + \pi_{Y3}w_t \qquad (23)$$

where $(\pi_{Y2} - \pi_{Y3}) < 0$. Define $\tilde{Y}_t = Y_t - \pi_{y1}N_{t-1}$ where $\tilde{Y}_t$ is the innovation in Y_t. The first empirical proposition is that z_t, the noise in the news about the money supply, should have a significant coefficient in ordinary least squares regressions (OLS) of $\tilde{Y}_t$ on z_t. Consider the estimation of equation (24),

$$\tilde{Y}_t = \beta_z z_t + \nu_{1t} \tag{24}$$

by OLS. The estimated parameter, $\hat{\beta}_z$, is

$$\hat{\beta}_z = \frac{E[z_t(\beta_z z_t + \nu_{1t})]}{E(z_t^2)} = \beta_z + \frac{E(z_t \nu_{1t})}{E(z_t^2)} . \tag{25}$$

If z_t and ν_{1t} are uncorrelated, $\hat{\beta}_z$ is an unbiased estimate of the true influence of z_t on $\tilde{Y}_t$. In the present form of our model the population parameter $\beta_z = (\pi_{Y2} - \pi_{Y3})\ \phi_{w2} < 0$, and $\nu_{1t} = (\pi_{Y2} - \pi_{Y3})(\phi_{w1} + \phi_{w2})v_t + [(\pi_{Y2} - \pi_{Y3})(-\alpha_3\phi_{w1}) + \pi_{Y3}]w_t$ which is orthogonal to z_t making OLS appropriate. Since B-G and B-H estimate $\hat{\beta}_z$ to be insignificantly different from zero, this specification of our model is suspect. Relaxing the restriction that v_t, w_t, and z_t are mutually orthogonal, though, implies that the OLS estimates of β_z given in (25) is not an unbiased estimate. A sufficient condition to bias the coefficient toward zero is a negative covariance between v_t and z_t. In a more complicated framework with a complete covariance matrix, presumably other combinations of covariances would bias the OLS estimate $\hat{\beta}_z$ toward zero as well.

Now consider the second empirical hypothesis of B-G. In OLS regressions of output on perceived money, the OLS estimate should be zero, but it is estimated to be significantly different from zero by B-G. This is inconsistent with the present version of the model because OLS regression of $\tilde{Y}_t$ on $m_t^{\#} - m_{t-1}$ would produce a zero coefficient if the covariances between v_t and z_t with w_t are zero. Notice from (23) that

$$\tilde{Y}_t = \beta_m(m_t^{\#} - m_{t-1}) + \nu_{2t} \tag{26}$$

where $\beta_m = \phi_{w2}(\pi_{Y2} - \pi_{Y3})$ and $\nu_{2t} = \phi_{w1}(\pi_{Y2} - \pi_{Y3})v_t + [\pi_{Y3} - \alpha_3(\pi_{Y2} - \pi_{Y3})\phi_{w1}]w_t$. Ordinary least squares estimation of β_m gives

$$\begin{aligned}\hat{\beta}_m &= \frac{E\{[\beta_m(m_m^{\#} - m_{t-1}) + \nu_{2t}](m_t^{\#} - m_{t-1})\}}{E(m_t^{\#} - m_{t-1})^2} \\ &= \beta_m + \frac{E[(m_t^{\#} - m_{t-1})(\nu_{2t})]}{E(m_t^{\#} - m_{t-1})^2} \qquad (27) \\ &= (\pi_{Y2} - \pi_{Y3})[(\phi_{w1} + \phi_{w2})\sigma_v^2 + \phi_{w2}\sigma_z^2]/(\sigma_v^2 + \sigma_z^2) = 0\end{aligned}$$

from the definitions of ϕ_{w1} and ϕ_{w2}. Intuitively, $m_t^{\#} - m_{t-1} = v_t + z_t$ is uncorrelated with w_t, hence it provides no information about $E_t w_t$ and consequently cannot affect anything real in the model. Clearly, this would not be the case if the covariance of w_t with z_t or v_t was nonzero in which case the money number would provide direct evidence about the shock to aggregate demand for the home good.[16]

The second empirical regularity that was mentioned in the introduction was that the exchange rate and the relative price of the export good of the country were negatively correlated. Thus, depreciations of the currency and deteriorations of the term of trade tend to coincide. Let $C'_{t-1}(A_t;B_t) = E'_{t-1}(A_t - E'_{t-1}A_t)(B_t - E'_{t-1}B_t)$ be the conditional covariance of two random variables A_t and B_t conditional on full information about variables dated t-1 and earlier. Then the formal requirement on the model is that $C'_{t-1}(\bar{R}_t;s_t) < 0$. This condition is satisfied for our model since

$$C'_{t-1}(\bar{R}_t;s_t) = \lambda_{\bar{R}v}\lambda_{sv}\sigma_v^2 + \lambda_{\bar{R}w}\lambda_{sw}\sigma_w^2 + \lambda_{\bar{R}z}\lambda_{sz}\sigma_z^2 < 0 \qquad (28)$$

from examination of the algebraic signs of the λ coefficients in Table 4 and by imposition of the argument that $\lambda_{sz} < 0$.

The third empirical regularity discussed in the introduction requires exchange rates to be more volatile than domestic price indexes where by volatility is meant one-step-ahead predictability. Let $V'_{t-1}(A_t) = E'_{t-1}(A_t - E'_{t-1}A_t)^2$ be the definition of volatility for any random variable A_t, and recognize that $p_t = [\theta/(1-\theta)]\bar{R}_t + s_t$. Then, the volatility definition implies that

$$V'_{t-1}(p_t) = (\frac{\theta}{1-\theta})^2 V'_{t-1}(R_t) + 2(\frac{\theta}{1-\theta})C'_{t-1}(\bar{R}_t;s_t) + V'_{t-1}(s_t) \qquad (29)$$

which is smaller than $V'_{t-1}(s_t)$ when $|2C'_{t-1}(\bar{R}_t;s_t)| > [\theta/(1-\theta)]V'_{t-1}(\bar{R}_t)$. For this condition to be true,

$$(\frac{\theta}{1-\theta})\{\lambda_{\bar{R}v}[(\frac{\theta}{1-\theta})\lambda_{\bar{R}v} + 2\lambda_{sv}]\sigma_v^2 - \lambda_{\bar{R}w}[-(\frac{\theta}{1-\theta})\lambda_{\bar{R}w} - 2\lambda_{sw}]\sigma_w^2$$
$$- \lambda_{\bar{R}z}[-(\frac{\theta}{1-\theta})\lambda_{\bar{R}z} - 2\lambda_{sz}]\sigma_z^2 < 0. \qquad (30)$$

In (30) each term multiplying the terms in square brackets is negative. Hence, if each term in square brackets is positive, the condition is satisfied. A sufficient condition for each of the terms in square brackets to be positive is that $[2/(1+\alpha_1)] > 1$. This is only a sufficient condition and is not necessary. The point is that the model allows domestic prices of domestic goods to be determined within the period as opposed to assuming them to be predetermined variables, yet it remains consistent with the empirical regularity for at least some values of free parameters of the model.

VI. A Positive Aspect of Exchange-Rate Regime Choice

In this section we investigate the predictions of our model regarding the extent to which unsterilized intervention in the foreign exchange market influences the conditional variance of output. The conditional variance measure is

$$V_t(Y_t) = E_t[Y_t - E_t(Y_t)]^2 \qquad (31)$$

which captures the variability of output relative to the level planned by firms at the beginning of the period.[17]

Implicitly, the analysis so far has been based on the assumption of freely floating exchange rates. Discussion of unsterilized intervention requires that we introduce high-powered money created by purchases of domestic credit or international reserves and recognize that monetary transactions balances are a multiple of high-powered money. As a linear approximation, let

$$m_t = \omega d_t + (1-\omega)b_t + mm_t, \ 0 < \omega < 1, \qquad (32)$$

where d_t is the natural logarithm of domestic credit, b_t is the natural logarithm of the book value of international reserves and mm_t is the logarithm of the money multiplier. We assume that the logarithms of domestic credit and the money multiplier follow random walks: $d_t = d_{t-1} + q_t$ where q_t is independently and identically distributed, and $mm_t = mm_{t-1} + c_t$ where c_t is independently and identically distributed. Under flexible exchange rates, international reserves are constant, $b_t = \bar{b}$; while unsterilized intervention is captured by the following policy rule:

$$b_t = \bar{b} - \eta s_t. \tag{33}$$

When $\eta = 0$ as was assumed in preceding section of the paper, the innovation in monetary transactions balances is $v_t = v_{1t} \equiv \omega q_t + c_t$. When η is finite and nonzero, the innovation in transaction balances can be written as $v_t = v_{1t} - (1-\omega)\eta(s_t - E_t^{\prime}s_t)$.

The first point to notice is that the choice of any <u>finite</u> η does not affect the conditional distribution of output. In this case the new information in the asset markets regarding the underlying disturbances remains $g_{1t} = v_{1t} - \alpha_3 w_t$. Output volatility in this case is

$$V_t(Y_t) = \pi_{Y3}^2 \; (\sigma_{v1}^2 \sigma_w^2 \sigma_z^2/\Delta) \tag{34}$$

where $\Delta = [\sigma_{v1}^2\sigma_z^2 + \alpha_3^2\sigma_w^2(\sigma_{v1}^2 + \sigma_z^2)]$ and σ_{v1}^2 represents the variance of v_{1t}.

The only choice of η that does influence the conditional distribution of output is $\eta = \infty$. In this case the exchange rate is fixed at some constant level $\bar{s}$, which is not expected to change. This choice of the monetary authority may affect the information available to firms regarding the underlying disturbances driving the economy.

When the exchange rate is fixed, the supply of money is entirely demand determined. If the change in the supply of domestic credit plus the change in the money multiplier produces an incipient amount of nominal money different than that demanded by individuals, international reserves must adjust to bring the actual nominal amount of money supplied into equality with the nominal amount of money demanded. Consequently, the nominal supply can be written as

$$m_t = p_t + \alpha_2(\kappa_1 \bar{R}_t + \kappa_2 E_t Y_t) + \alpha_3 w_t. \tag{35}$$

Knowing that the money and capital markets are in equilibrium now provides no new information regarding the underlying disturbances. Nevertheless, agents obtain information about the disturbances from the money number which now conveys

$$g_{3t} = \alpha_3 w_t + z_t \tag{36}$$

which is an exact linear combination of the information received under flexible exchange rates or with finite intervention since $g_{3t} = g_{2t} - g_{1t}$.

Output volatility under a fixed exchange rate regime is

$$V_t^{\bar{s}}(Y_t) = \pi_{Y3}^2[\sigma_w^2\sigma_z^2/(\alpha_3^2\sigma_w^2 + \sigma_z^2)] \tag{37}$$

which is necessarily greater than $V_t(Y_t)$ in (34) since

$$V_t(Y_t) = [\sigma_{v1}^2(\alpha_3^2\sigma_w^2 + \sigma_z^2)/\Delta]V_t^{\bar{s}}(Y_t), \tag{38}$$

and the term in square brackets in (38) is a positive fraction. Intuitively, the volatility of output under either regime is produced by the inability of firms to forecast demand precisely. Under flexible exchange rates, firms receive two sources of information about the underlying disturbances while under fixed exchange rates, they receive a linear combination of the two information sources which is a smaller information set. Since the forecast of demand, $E_t w_t$, is calculated to minimize the forecast error variance, forecast errors must be larger under fixed exchange rates in which the forecasts are made with a strictly smaller information set. These larger forecast errors increase the volatility of output.[18]

This result depends critically on our assumption about the rate of release of money supply information. If high-powered money information is available at the beginning of the period, then high-powered money does not influence aggregate output regardless of exchange rate regime choice. Furthermore, if under fixed rates, the monetary authority releases the extent of current intervention, $b_t - b_{t-1}$, at the beginning of the period, then b_t will be in agents' information sets and the asset markets will convey the information variable g_{1t}, as under floating rates. Complete information that is made available at the beginning of the period on the quantity of intervention under fixed rates is equivalent to the information carried by the exchange rate under floating rates, and if such information is revealed, the stochastic distribution of output will be invariant to exchange-rate regime choice.

An apparently important restrictive aspect of our model is that our demand disturbance, w_t, is uncorrelated with any foreign variable. Less restrictive would be an assumption that the disturbance to demand for domestic goods consists of two components, w_t as in our set up, plus w_t^*, a foreign-based disturbance. The complete demand disturbance would then be $w_t + w_t^*$. The w_t^* portion of the disturbance would be correlated with other foreign variables known to domestic agents. In particular we would expect i_t^* and $\bar{h}_t^*$ to be correlated with w_t^*, and we would drop the assumptions that i_t^* and $\bar{h}_t^*$ are constant. In this circumstance adopting a fixed exchange rate or a floating one will be irrelevant to agents' inferences concerning w_t^*. However, if i_t^* and $\bar{h}_t^*$ remain uncorrelated with w_t, fixing the exchange rate, without revealing intervention, still removes an information source relevant to inferring that disturbance. Consequently, the prediction errors of $w_t + w_t^*$ will be larger under fixed rates than with any finite degree of intervention, and output variance also will be larger under fixed than under floating rates.

The result that setting s_t at a constant $\bar{s}$ results is higher output variance than that available under floating rates when intervention is not announced immediately is not standard and seem to us to be at odds with traditional sticky price models of the open economy. In the traditional model, the domestic currency price of domestic output is predetermined with respect to current shocks. Thus, when these shocks disturb the exchange rate, they alter the relative price of domestic output, which thereby affects the quantity of domestic output demanded. With domestic output demand determined, a current shock therefore alters domestic production. Our model does not presume that the domestic price is predetermined. Price setting agents see the exchange rate and the nominal interest rate and set their relative prices using the information from the asset market. This is the crucial difference between our model and previous models. Our agents use the information content of the exchange rate in setting prices and when that information is removed, as when the exchange rate is fixed, output becomes more variable. In the traditional model, prices are set without the use of exchange rate information so movements of the exchange rate contribute to relative price and demand variability rather than conveying information concerning demand disturbances.

The decisiveness of our result concerning $V(Y_t)$, when intervention is not revealed, is due largely to our assumption that the country being studied is small in world capital markets. If this assumption were relaxed, then i_t^* would be correlated with w_t, and i_t^* would convey information concerning w_t even with fixed exchange rates. Consequently, it would no longer be true under fixed rates that forecasts of w_t necessarily have a larger variance than under flexible rates. Since we have not yet modeled the correlation of i_t^* and w_t, we do not know the results for output in such a case.

VII. Concluding Remarks

Our model was constructed to be consistent with the major empirical regularities discovered in studies of business cycles and those discovered in studies of prices and exchange rates. Unexpected monetary disturbances are not neutral in our model because price setting agents do not observe money directly. They see only indicators of the underlying disturbances, and they tend to confuse positive (negative) monetary shocks with negative (positive) demand shocks. Business cycles are propagated through time via optimal inventory adjustment.

Prices in our model are set at the beginning of the period, prior to the revelation of actual values of the underlying disturbances. Thus, our prices are sticky in the sense that they do not respond as quickly to monetary disturbances as they would if pricing were based on full information. Our model is consistent with the observations that exchange rates are more difficult to predict than are commodity prices and that countries' exchange rates and terms of trade are negatively correlated.

In presenting our results we worked with unrealistically simple time series processes governing the supply of money and the level of real expenditure. These processes were chosen for clarity of presentation and none of the results we have emphasized concerning the effects of unperceived monetary disturbances on output depend on our choice of processes. These effects stem only from innovations to the money supply and to money demand. Consequently, these results will be robust to any stationary time series processes for the money supply. What will change with change in the time series process for the money supply are the time series properties of nominal prices.

We recognize that much work remains to be done on our framework. In particular, the linkages between firm level outcomes and the levels of aggregate domestic expenditure and aggregate money demand need to be incorporated into the maximizing framework. However, we conjecture that the crucial analytic feature of our model in this area, which is the correlation between the scale variable in money demand and the scale variable in goods demand, will appear in a wide variety of sensible specifications.

FOOTNOTES

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1. Barro (1981) and Lucas (1976) summarize the business cycle empirical regulatities, while Mussa (1979) and Flood (1981) have discussed regularities for prices and exchange rates.

2. The information content of asset prices has been emphasized by Barro (1980), King (1982), and Grossman and Weiss (1982) in the context of business cycle models and by Grossman and Stiglitz (1976) and many others in the context of microeconomic financial markets.

3. That only unexpected changes in money result in real output effect is a controversial aspect of our model. Some non-supportive results are presented by Makin (1982) and Mishkin (1982). The work of Barro (1977, 1978), Barro and Rush (1980), Leiderman (1980), and Wogin (1980), is supportive of the proposition. In our model unexpected money is unperceived. The empirical support for such disturbances has been challenged by Boschen and Grossman (1983) and by Barro and Hercovitz (1980), and we discuss their results in Section V in the context of our model.

4. In contrast, the dynamics in the Dornbusch-type model are rationalized in terms of slow and costly price adjustments.

5. Price surprises typically enter aggregate supply either through the information confusion channel of Lucas (1973) or through the incomplete wage indexing channel of Gray (1976) and Fischer (1977). The Gray-Fischer framework has been particularly amenable to open economy analysis such as that of Flood and Marion (1982). Marston (1982) surveys recent open economy models that generate aggregate supply curves containing price surprise terms generated via wage indexation.

6. This seems to be a popular method for introducing monetary induced business cycles into models. The wage indexing models of Gray (1976) and Fischer (1977) follow this pattern. An alternative method for introducing monetary influences into business cycles is that of Lucas (1973) where decisions are not importantly sequential but important information channels are assumed not to exist. Both approaches assume some curcial transactions costs prohibit otherwise profitable transactions, and thus both are somewhat unsatisfactory.

It is notable that not all business cycle models depend on price surprise terms. In particular, the simple Keynesian model, popular in undergraduate texts, postulates demand-determined output with all prices known to agents. Further, Grossman and Weiss (1982) and McCallum (1982) present models designed to avoid price surprise terms in aggregate supply.

7. For now we make foreign demand non-stochastic. In section VI we will discuss stochastic foreign demand.

8. We adopt the functional form $k\beta_4$ for concreteness. An arbitrary function $\beta_4(k)$ with $\partial\beta_4(k)/\partial k > 0$ and $\lim_{k\to\infty}\beta_4(k) \to \infty$, would have served equally well. None of our macro results would be altered if we set $\beta_4 = 0$

and maximized the objective with respect to $\bar{R}$, as in Blinder (1982). We include $\beta_4 > 0$ to allow some price and inventory dispersion at the macro level. Allowing idiosyncratic shocks to the firm demands would add additional micro dispersion without changing the macroeconomic paradigm.

9. Our specification abstracts from possibly important aspects of a time-varying discount rate.

10. In a set up much like ours, Eichenbaum (1983) has shown the observational equivalence (i.e., equivalence of decision rules at the industry level) of an unknown number, n, of firms acting as (i) perfect competitors, (ii) an n-plant monopolist, and (iii) Nash competitors. Consequently, we expect the qualitative properties of the aggregate decision rules we derive to be robust to a wide variety of firm-level specifications.

11. We view equation (12) as a useful simplifying assumption that allows us to focus directly on production, exchange rates and prices without complicating the theory with a model of a time-varying risk premium. The evidence in Hansen and Hodrick (1983) suggests that statistically significant risk premia may characterize the relationship between forward exchange rates and the expected future spot rates. However, their evidence also suggests that if risk premia exist, they are small in comparison to unexpected changes in exchange rates.

12. The assumption that the three disturbances are mutually orthogonal is made strictly for clarity of presentation. The implications of relaxing the assumption are investigated in Section V.

13. If the underlying disturbances were normally distributed, maximum likelihood estimates of the disturbances would be obtained by linear least squares projections of the disturbances onto the information set. While

it is plausible to assume that v_t and z_t are normally distributed, such an assumption about $w_t = \rho_2 u_t$ would be absurd because u_t is a disturbance to the level of real expenditure. A normal distribution for u_t would imply that we sometimes would observe negative aggregate expenditure. See Sargent (1979) for a discussion of such projections.

14. Canzoneri, Henderson and Rogoff (1982) discuss the information extraction method we use. The method tractably separates information extraction from dynamics and is equivalent to extracting information directly from exchange rates or interest rates.

15. In discussing the dynamics of the economy it is important to remember that each period new stochastic shocks will buffet the economy. Hence, the discussion of the path following a shock is only a discussion of the conditional path given that the new disturbances have expected value of zero.

16. Consider, for instance, the case in which w_t and z_t are correlated and v_t and z_t are correlated. In this case $\tilde{Y}_t = \beta_m'(m_t^{\#} - m_{t-1}) + \nu_{2t}'$ where $\beta_m' = (\pi_{y2} - \pi_{y3})\phi_{w2}'$, $\nu_{2t}' = (\pi_{y2} - \pi_{y3}\phi_{w1}'(v_t - \alpha_3 w_t) + \pi_{y3}w_t$ and ϕ_{w1}' and ϕ_{w2}' are the new OLS regression coefficients in the linear prediction of w_t using g_{1t} and g_{2t}. An OLS regression of $\tilde{Y}_t$ on $m_t^{\#} - m_{t-1}$ produces the estimate $\hat{\beta}_m' = [\sigma_{wz}/(\sigma_v^2 + \sigma_z^2 + 2\sigma_{vz})]\pi_{y2}$. In this case, neither the true β_m' not its OLS estimate is nonzero.

17. Another popular measure of output variance is $\hat{V}(Y_t) = E(Y_t - \hat{Y}_t)^2$, where $\hat{Y}_t$ is the full information value of output. To find $\hat{Y}_t$ replace $E_t w_t$ with w_t in (15c) resulting in the expression $\hat{Y}_t = \pi_{Y0} + \pi_{Y1}N_{t-1} + \pi_{Y2}w_t$. Subtracting this expression from (15c) yields $Y_t - \hat{Y}_t = \pi_{Y2}(E_t w_t - w_t)$. Thus $\hat{V}(Y_t) = \pi_{Y2}^2 E(w_t - E_t w_t)^2$, which is proportional to the result in (34)

with a constant factor of proportionality $(\pi_{Y3}/\pi_{Y2})^2$. It follows that our results concerning (34) will hold exactly for $\hat{V}(Y_t)$.

18. Kimbrough (1982) derives a similar result on the volatility of output across exchange rate regimes for the same type of reason.

APPENDIX A

This appendix provides the solutions to the representative firm's maximization problem given by (4) in the text, demonstrates that summing these solutions produces the aggregate Euler equations (5a,b), and explains the steps necessary to obtain the solutions in Table 1.

The firm's problem is rewritten here as (A1):

$$\max_{\{R^j_{t+i},N^j_{t+i}\}} E_t \sum_{i=0}^{\infty} \{D^j_{t+i}R^j_{t+i} - \gamma_1 Y_{t+i}Y^j_{t+i} - (\gamma_2/2)(Y^j_{t+i})^2 - \delta_1 N_{t+i-1}N^j_{t+i-1} - (\delta_2/2)(N^j_{t+i-1})^2\}\sigma^i \qquad j = 1, 2, \ldots, k \qquad (A1)$$

subject to

$$Y^j_t = D^j_t + N^j_t - N^j_{t-1}, \qquad (A2)$$

$$D^j_t = \tfrac{1}{k}D_t - k\beta_4(R^j_t - \bar{R}_t), \qquad (A3)$$

and an initial level of inventories, N^j_{t-1}. In (A3), $D_t = \beta_0 - \beta_1\bar{R}_t + \beta_2(N_t - N_{t-1}) + \beta_3 w_t$ is the quasi reduced form for aggregate demand derived in (6). A solution to the problem is found by differentiating (A1) with respect to R^j_{t+i} and N^j_{t+i}, $i = 0, 1, 2, \ldots$, setting the derivative equal to zero, and imposing the transversality condition. In the maximization it is assumed that each firm is small enough such that it takes as given the economy-wide average price, $\bar{R}_t \equiv \Sigma^k_{j=1}R^j_t/k$, the economy-wide output, $Y_t \equiv \Sigma^k_{j=1}Y^j_t$, and economy-wide level of inventories, $N_t \equiv \Sigma^k_{j=1}N^j_t$.

Differentiating with respect to R^j_{t+i} and N^j_{t+i} gives the following system of stochastic Euler equations:

$$E_t\{(1/k)D_{t+i} - 2k\beta_4 R^j_{t+i} + k\beta_4 \bar{R}_{t+i} + \gamma_1 Y_{t+i} k\beta_4 + 2k\beta_4 \gamma_2 [N^j_{t+i} - N^j_{t+i-1} + (1/k)D_{t+i} - k\beta_4 (R^j_{t+i} - \bar{R}_{t+i})]\} = 0 \qquad \text{(A4a)}$$

$$E_t\{-\gamma_1 Y_{t+i} - \gamma_2[(1/k)D_{t+i} - k\beta_4(R^j_{t+i} - \bar{R}_{t+i}) + N^j_{t+i} - N^j_{t+i-1}] + \sigma\gamma_1 Y_{t+i+1} + \sigma\gamma_2[(1/k)D_{t+i+1} - k\beta_4(R^j_{t+i+1} - R_{t+i+1}) + N^j_{t+i+1} - N^j_{t+i}] - \sigma[\delta_1 N_{t+i} + \delta_2 N^j_{t+i}]\} = 0 \quad i = 0, 1, 2, \ldots \qquad \text{(A4b)}$$

Following Sargent (1979), the appropriate transversality condition is found by examining the finite horizon problem, differentiating with respect to the final inventory stock, N^j_{t+T}, and taking the limit as T goes to infinity. The transversality condition is

$$\lim_{T\to\infty} E_t[-\gamma_1 Y_{t+T} - \gamma_2(D^j_{t+T} + N^j_{t+T} - N^j_{t+T-1})]\sigma^T = 0. \qquad \text{(A5)}$$

In a finite horizon world, the firm would be tempted to meet entire final period demand out of its negative inventory stock since it would accept payment today but never deliver the goods and never incur any inventory carrying cost. This tendency remains in the infinite horizon problem, but the transversality condition prevents inventories from growing faster than the rate $1/\sigma$.

The aggregate Euler equations in the text, (5a,b), are found by summing (A4a,b) across the k firm in the economy using the definitions of Y_t, N_t, and $\bar{R}_t$.

A solution to the two Euler equations requires several steps. First, solve (5a) for $E_t\bar{R}_{t+i}$ and substitute this into (5b). The resulting equation is a second order difference equation in $E_t N_{t+i}$. The solution to the equation is readily obtained by the method of undetermined coefficients. Conjecture that the solution will have the form of (7b). Substitute this into the equation to eliminate terms in E_tN_t and E_tN_{t+1}. The resulting coefficients can then be equated and solved for the π coefficients. There are two possible values for π_{N1}, one less than 1 and one greater than $1/\sigma$. We choose the stable value since otherwise aggregate inventories would grow at a rate greater than $1/\sigma$. The transversality condition derived above prevents each firm from following an inventory accumulation path that grows faster than $1/\sigma$, hence the choice of the stable root is the only choice consistent with the optimizing strategies of the firms.

Rather than treating R^j_{t+i} and N^j_{t+i} as the choice variables in (A1), we could have chosen D^j_{t+i} and N^j_{t+i} as the choice variables in which case the same firm behavior would have been indicated, but it would have been clearer that firms choose their relative price to pick an expected level of sales which equates expected marginal revenue to expected marginal cost of production even though sales and production are not equal. The second first-order condition requires equality between the marginal cost of production today plus the expected marginal inventory holding costs and the discounted expected marginal cost of producing in the future.

APPENDIX B

This appendix records the actual values of the reduced form parameters, the λ coefficients, whose algebraic signs were given in Table 4. For a typical variable $J_t = Y_t, N_t, \bar{R}_t, s_t, \bar{h}_t$, the reduced form equation is the following:

$$J_t = \lambda_{JN}N_{t-1} + N_{Jm}m_{t-1} + \lambda_{Jv}v_t + \lambda_{Jw}w_t + \lambda_{Jz}z_t. \quad \text{(B1)}$$

Coefficients in N_t equation

$$\lambda_{NN} = \pi_{N1} = (1/2)[A - (A^2 - 4/\sigma)^{1/2}], \qquad 0 < \lambda_{NN} < 1$$

$$\lambda_{Nm} = 0$$

$$\lambda_{Nv} = (\pi_{N2} - \pi_{N3})(\phi_{w1} + \phi_{w2}) < 0$$

$$\lambda_{Nw} = (\pi_{N2} - \pi_{N3})(-\alpha_3\phi_{w1}) + \pi_{N3} < 0$$

$$\lambda_{Nz} = (\pi_{N2} - \pi_{N2})\phi_{w2} > 0$$

Coefficients in Y_t equation

$$\lambda_{YN} = \pi_{Y1} < 0$$

$$\lambda_{Ym} = 0$$

$$\lambda_{Yv} = (\pi_{Y2} - \pi_{Y3})(\phi_{w1} + \phi_{w2}) > 0$$

$$\lambda_{Yw} = (\pi_{Y2} - \pi_{Y3})(-\alpha_3\phi_{w1}) + \pi_{Y3} > 0$$

$$\lambda_{Yz} = (\pi_{Y2} - \pi_{Y3})\phi_{w2} < 0$$

Coefficients in $\bar{R}_t$ equation

$$\lambda_{\bar{R}N} = \pi_{\bar{R}1} < 0$$

$$\lambda_{\bar{R}m} = 0$$

$$\lambda_{\bar{R}v} = \pi_{\bar{R}2}(\phi_{w1} + \phi_{w2}) < 0$$

$$\lambda_{\bar{R}w} = \pi_{\bar{R}2}(-\alpha_3\phi_{w1}) > 0$$

$$\lambda_{\bar{R}z} = \pi_{\bar{R}2}\phi_{w2} > 0$$

Coefficients in s_t equation

$$\lambda_{sN} = \frac{1}{[1 + \alpha_1(1 - \pi_{N1})]}[-(\frac{\theta}{1-\theta} + \alpha_2\kappa_1)\pi_{\bar{R}1} - \alpha_2\kappa_2\pi_{Y1}] > 0$$

$$\lambda_{sm} = 1$$

$$\lambda_{sv} = \frac{1}{1 + \alpha_1}\{1 - (\phi_{w1} + \phi_{w2})[(\frac{\theta}{1-\theta} + \alpha_2\kappa_1)\pi_{\bar{R}2} - \alpha_1\lambda_{sN}\pi_{N2}$$

$$+ \alpha_2\kappa_2\pi_{Y2}] + \alpha_1(\phi_{v1} + \phi_{v2})\} > 0$$

$$\lambda_{sw} = \frac{-\alpha_3}{1 + \alpha_1}\{1 - \phi_{w1}[(\frac{\theta}{1-\theta} + \alpha_2\kappa_1)\pi_{\bar{R}2} - \alpha_1\lambda_{sN}\pi_{N2} + \alpha_2\kappa_2\pi_{Y2}]$$

$$+ \alpha_1\phi_{v1}\} < 0$$

$$\lambda_{sz} = \frac{1}{1 + \alpha_1}\{-\phi_{w2}\{(\frac{\theta}{1-\theta} + \alpha_2\kappa_1)\pi_{\bar{R}2} - \alpha_1\lambda_{sN}\pi_{N2} + \alpha_2\kappa_2\pi_{Y2} + \alpha_1\phi_{v2}\} < 0$$

Coefficients in $\bar{h}_t$ equation

$$\lambda_{\bar{h}N} = (\frac{1}{1-\theta})\pi_{\bar{R}1} + \lambda_{sN} \gtrless 0$$

$$\lambda_{\bar{h}m} = 1$$

$$\lambda_{\bar{h}v} = (\frac{1}{1-\theta})\pi_{\bar{R}2}(\phi_{w1} + \phi_{w2}) + \lambda_{sv} \gtrless 0$$

$$\lambda_{\bar{h}w} = (\frac{1}{1-\theta})\pi_{\bar{R}2}(-\alpha_3\phi_{w1}) + \lambda_{sw} \gtrless 0$$

$$\lambda_{\bar{h}z} = (\frac{1}{1-\theta})\pi_{\bar{R}2}\phi_{w2} + \lambda_{sz} \gtrless 0$$

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