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**ABSTRACT**

We show analytically that in a rational expectations present value model, an asset price manifests near random walk behavior if fundamentals are  $I(1)$  and the factor for discounting future fundamentals is near one. We argue that this result helps explain the well known puzzle that fundamental variables such as relative money supplies, outputs, inflation and interest rates provide little help in predicting changes in floating exchange rates. As well, we show that the data do exhibit a related link suggested by standard models - that the exchange rate helps predict these fundamentals. The implication is that exchange rates and fundamentals are linked in a way that is broadly consistent with asset pricing models of the exchange rate.

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A longstanding puzzle in international economics is the difficulty of tying floating exchange rates to macroeconomic fundamentals such as money supplies, outputs, and interest rates. Our theories state that the exchange rate is determined by such fundamental variables, but floating exchange rates between countries with roughly similar inflation rates are in fact well-approximated as random walks. Fundamental variables do not help predict future changes in exchange rates.

Meese and Rogoff (1983a, 1983b) first established this result. They evaluated the out-of-sample fit of several models of exchange rates, using data from the 1970s. They found that by standard measures of forecast accuracy, such as the mean-squared deviation between predicted and actual exchange rate, accuracy generally increased when one simply forecast the exchange rate to remain unchanged compared to when one used the predictions from the exchange rate models. While a large number of studies have subsequently claimed to find success for various versions of fundamentals-based models, sometimes at longer horizons, and over different time periods, the success of these models has not proven to be robust. A recent comprehensive study by Cheung, Chinn, and Pascual (2002) concludes, “the results do not point to any given model/specification combination as being very successful. On the other hand, it may be that one model will do well for one exchange rate, but not for another.”

In this paper, we take a new line of attack on the question of the link between exchange rates and fundamentals. We work with a conventional class of asset-pricing models, in which the exchange rate is the expected present discounted value of a linear combination of observable fundamentals and unobservable shocks. Linear driving processes are posited for fundamentals and shocks.

We first present a theorem concerning the behavior of an asset price determined in a present-value model. We show analytically that in the class of present value models we consider, asset prices will follow a process arbitrarily close to a random walk if (1) at least one forcing variable (observable fundamental or unobservable shock) has a unit autoregressive root, and (2) the discount factor is near unity. So, in the limit, as the discount factor approaches unity, the change in the time  $t$  asset price will be uncorrelated with information known at time  $t-1$ . We explain below that our result is *not* an application of the simple efficient markets model of Samuelson (1965) and others. When that model is applied to

exchange rates, it implies that cross-country interest rate differentials will predict exchange rate changes and thus that exchange rates will not follow a random walk.

Intuitively, as the discount factor approaches unity, the model puts relatively more weight on fundamentals far into the future in explaining the asset price. Transitory movements in the fundamentals become relatively less important compared to the permanent components. Imagine performing a Beveridge-Nelson decomposition on the linear combination of fundamentals that drive the asset price, expressing it as the sum of a random walk component and a transitory component. The class of theoretical models we are considering then expresses the asset price as the discounted sum of the current and expected future fundamentals. As the discount factor approaches one, the variance of the change of discounted sum of the random walk component approaches infinity, while the variance of the change of the stationary component approaches a constant. So the variance of the change of the asset price is dominated by the change of the random walk component as the discount factor approaches one.

We view as unexceptionable the assumption that a forcing variable has a unit root, at least as a working hypothesis for our study. The assumption about the discount factor is, however, open to debate. We note that in reasonable calibrations of some exchange rate models, this discount factor in fact is quite near unity.

Of course our analytical result is a limiting one. Whether a discount factor of .9 or .99 or .999 is required to deliver a process statistically indistinguishable a random walk depends on the sample size used to test for random walk behavior, and the entire set of parameters of the model. Hence we present some correlations calculated analytically in a simple stylized model. We assume a simple univariate process for fundamentals, with parameters chosen to reflect quarterly data from the recent floating period. We find that discount factors above 0.9 suffice to yield near zero correlations between the period  $t$  exchange rate and period  $t-1$  information. We do not attempt to verify our theoretical conclusion that large discount factors account for random walk behavior in exchange rates using any particular fundamentals model from the literature. That is, we do not pick specific models that we claim satisfy the conditions of our theorem, and then estimate them and verify that they produce random walks.

But if the present-value models of exchange rates imply random walk behavior, so that exchange rate changes are unpredictable, how then can we validate the models? We ask instead if these conventional models have implications for whether the exchange rate helps predict fundamentals. It is plausible to look in this direction. Surely much of the short-term fluctuations in exchange rates are driven by changes in expectations about the future. If the models are good approximations, and expectations reflect information about future fundamentals, the exchange rate changes will likely be useful in forecasting these fundamentals. So these models suggest that exchange rates Granger-cause the fundamentals. Using quarterly bilateral dollar exchange rates, 1974-2001, for the dollar versus the six other G7 countries, we find some evidence of such causality, especially for nominal variables.

The statistical significance of the predictability is not uniform, and suggests a link between exchange rates and fundamentals that perhaps is modest in comparison with the links between other sets of economic variables. But in our view, the statistical predictability is notable in light of the far weaker causality from fundamentals to exchange rates.

For countries and data series for which there is statistically significant evidence of Granger causality, we next gauge whether the Granger causality results are consistent with our models. We compare the correlation of exchange rate changes with two estimates of the change in the present discounted value of fundamentals. One estimate uses only the lagged value of fundamentals. The other uses both the exchange rate and own lags. We find that the correlation is substantially higher when the exchange rate is used in estimating the present discounted value.

To prevent confusion, we note that our finding that exchange rates predict fundamentals is distinct from our finding that large discount factors rationalize a random walk in exchange rates. It may be reasonable to link the two findings. When expectations of future fundamentals are very important in determining the exchange rate, it seems natural to pursue the question of whether exchange rates can forecast those fundamentals. But one can be persuaded that exchange rates Granger cause fundamentals, and still argue that the approximate random walk in exchange rates is not substantially attributable to a large discount factor. In the class of models we consider, all our empirical results are consistent with at

least one other explanation, namely, that exchange rate movements are dominated by unobserved shocks that follow a random walk. The plausibility of this explanation is underscored by the fact that we generally fail to find cointegration between the exchange rate and observable fundamentals, a failure that is rationalized in our class of models by the presence of an  $I(1)$  (though not necessarily random walk) shock. As well, the random walk also can arise in models that fall outside the class we consider. It does so in models with small sample biases, perhaps combined nonlinearities/threshold effects (see Taylor, Peel, and Sarno (2002), Kilian and Taylor (2003) and Rossi (2003).) Exchange rates will still predict fundamentals in such models, though a nonlinear forecasting process may be required.

Our suggestion that the exchange rate will nearly follow a random walk when the discount factor is close to unity means that forecasting changes in exchange rate is difficult, but perhaps still possible. Some recent studies have found success at forecasting changes in exchange rates at longer horizons, or using nonlinear methods, and further research along these lines may prove fruitful. Mark (1995), Chinn and Meese (1995), and MacDonald and Taylor (1994) have all found some success in forecasting exchange rates at longer horizons imposing long-run restrictions from monetary models. Groen (2000) and Mark and Sul (2001) find greater success using panel methods. Kilian and Taylor (2001) suggest that models that incorporate nonlinear mean-reversion can improve the forecasting accuracy of fundamentals models, though it will be difficult to detect the improvement in out-of-sample forecasting exercises.

The paper is organized as follow. Section 2 presents the theorem that the random walk in asset prices may result from a discount factor near one in a present value model. Section 3 demonstrates how the theorem applies to some models of exchange rates. Section 4 presents evidence that changes in exchange rates help predict fundamentals. Section 5 concludes. An Appendix has some algebraic details. An additional appendix containing empirical results omitted from the paper to save space is available on request.

## 2. RANDOM WALK IN ASSET PRICE AS DISCOUNT FACTOR GOES TO ONE

We consider models in which an asset price,  $s_t$ , can be expressed as a discounted sum of current and expected future “fundamentals.” We examine asset-pricing models of the form:

$$(2.1) \quad s_t = (1-b) \sum_{j=0}^{\infty} b^j E_t(a_1' x_{t+j}) + b \sum_{j=0}^{\infty} b^j E_t(a_2' x_{t+j}), \quad 0 < b < 1$$

where  $x_t$  is the  $(n \times 1)$  vector of fundamentals,  $b$  is a discount factor, and  $a_1$  and  $a_2$  are  $(n \times 1)$  vectors. For example, the model for stock prices considered by Campbell and Shiller (1987) and West (1988) is of this form, where  $s_t$  is the level of the stock price,  $x_t$  the dividend (a scalar),  $a_1 = 0$  and  $a_2 = 1$ . The log-linearized model of the stock price of Campbell and Shiller (1988) is also of this form, where  $s_t$  is the log of the stock price,  $x_t$  the log of the dividend,  $a_1 = 1$  and  $a_2 = 0$ . The term structure model of Campbell and Shiller (1987) also is a present-value model, where  $s_t$  is the yield on a consol,  $x_t$  the short-term rate,  $a_1 = 1$  and  $a_2 = 0$ . In section 3, we review models in which  $s_t$  is the log of the exchange rate, and  $x_t$  contains such variables as interest rates, and logs of prices, money supplies, and income.

We spell out here the sense in which the asset price should follow a random walk for a discount factor  $b$  that is near 1. Assume that at least one element of the vector  $x_t$  is an I(1) process, whose Wold innovation is the  $(n \times 1)$  vector  $\varepsilon_t$ . Our result requires that either (1)  $a_1' x_t \sim I(1)$ ,  $a_2 = 0$ , or (2)  $a_2' x_t \sim I(1)$ , with the order of integration of  $a_1' x_t$  essentially unrestricted (I(0), I(1) or identically 0). In either case, for  $b$  near 1,  $\Delta s_t$  will be well approximated by a linear combination of the elements of the unpredictable innovation  $\varepsilon_t$ . In a sense made precise in the Appendix, this approximation is arbitrarily good for  $b$  arbitrarily near 1. This means, for example, that all autocorrelations of  $\Delta s_t$  will be very near zero for  $b$  very near 1.

Of course, there is continuity in the autocorrelations in the following sense: for  $b$  near 1, the autocorrelations of  $\Delta s_t$  will be near zero if the previous paragraph's condition that certain variables are I(1) is replaced with the condition that those variables are I(0) but with an autoregressive root very near

one. For a given autoregressive root less than one, the autocorrelations will not converge to zero as  $b$  approaches 1. But they will be very small for  $b$  very near 1.

Table 2.1 gives an indication of just how small “small” is. The table gives correlations of  $\Delta s_t$  with time  $t-1$  information when  $x_t$  follows a scalar univariate AR(2). (One can think of  $a_1 = 0$  and  $a_2 = 1$ , or  $a_1 = 1$  and  $a_2 = 0$ . One can consider these two possibilities interchangeably since for given  $b < 1$ , the autocorrelations of  $\Delta s_t$  are not affected by whether or not a factor of  $1-b$  multiplies the present value of fundamentals.) Lines (1)-(9) assume that  $x_t \sim I(1)$  – specifically,  $\Delta x_t \sim AR(1)$  with parameter  $\varphi$ . We see that for  $b = 0.5$  the autocorrelations in columns (4)-(6) and the cross-correlations in columns (7)-(9) are appreciable. Specifically, suppose that one uses the conventional standard error of  $1/\sqrt{T}$ . Then when  $\varphi = 0.5$ , a sample size larger than 55 will likely suffice to reject the null that the first autocorrelation of  $\Delta s_t$  is zero (since row (2), column (5) gives  $corr(\Delta s_t, \Delta s_{t-1}) = 0.269$ , and  $0.269/[1/\sqrt{55}] \approx 2.0$ ). (In this argument, we abstract from sampling error in estimation of the autocorrelation.) But for  $b = 0.9$ , the autocorrelations are dramatically smaller. For  $b = 0.9$ ,  $\varphi = 0.5$ , a sample size larger than 1600 will be required, since  $0.051/[1/\sqrt{1600}] \approx 2.0$ . Finally, in connection with the previous paragraph’s reference to autoregressive roots less than one, we see in lines (10)-(13) in the table that if the unit root in  $x_t$  is replaced by an autoregressive root of 0.9 or higher, the auto- and cross-correlations of  $\Delta s_t$  are not much changed.

To develop intuition on this result, consider the following example. Suppose the asset price is determined by a simple equation:

$$s_t = (1-b)m_t + b\rho_t + bE_t(s_{t+1}).$$

The “no-bubbles” solution to this expectational difference equation is a present-value model like (2.1):

$$s_t = (1-b)\sum_{j=0}^{\infty} b^j E_t m_{t+j} + b\sum_{j=0}^{\infty} b^j E_t \rho_{t+j}$$

Assume the first-differences of the fundamentals follow first order autoregressions:

$$\Delta m_t = \phi \Delta m_{t-1} + \varepsilon_{mt}; \quad \Delta \rho_t = \gamma \Delta \rho_{t-1} + \varepsilon_{\rho t}.$$

Then we can write the solution as:

$$(2.1) \quad \Delta s_t = \frac{\phi(1-b)}{1-b\phi} \Delta m_{t-1} + \frac{1}{1-b\phi} \varepsilon_{mt} + \frac{b\gamma}{1-b\gamma} \Delta \rho_{t-1} + \frac{b}{(1-b)(1-b\gamma)} \varepsilon_{\rho t}.$$

Consider first the special case of  $\rho_t = 0$ . Then as  $b \rightarrow 1$ ,  $\Delta s_t \approx \frac{1}{1-\phi} \varepsilon_{mt}$ . In this case, the variance of the change in the exchange rate is finite as  $b \rightarrow 1$ . If  $\rho_t \neq 0$ , then as  $b \rightarrow 1$ ,  $\Delta s_t \approx \text{constant} \times \varepsilon_{\rho t}$ . In this case, as  $b$  increases, the variance of the change in the exchange rate gets large, but the variance is dominated by the i.i.d. term  $\varepsilon_{\rho t}$ .

In section 3, we demonstrate the applicability of this result to exchange rates.

### 3. EXCHANGE RATE MODELS

Exchange rate models since the 1970s have emphasized that nominal exchange rates are asset prices, and are influenced by expectations about the future. The “asset-market approach to exchange rates” refers to models in which the exchange rate is driven by a present discounted sum of expected future fundamentals. Obstfeld and Rogoff (1996, p. 529) say, “One very important and quite robust insight is that *the nominal exchange rate must be viewed as an asset price*. Like other assets, the exchange rate depends on expectations of future variables.” [Italics in the original.] Frenkel and Mussa’s (1985) survey explains the asset-market approach (p. 726): “These facts suggest that exchange rates should be viewed as prices of durable assets determined in organized markets (like stock and commodity exchanges) in which current prices reflect the market’s expectations concerning present and future economic conditions relevant for determining the appropriate values of these durable assets, and in which price changes are largely unpredictable and reflect primarily new information that alters expectations concerning these present and future economic conditions.”

A variety of models relate the exchange rate to economic fundamentals and to the expected future exchange rate. We write this relationship as:

$$(3.1) \quad s_t = (1-b)(f_{1t} + z_{1t}) + b(f_{2t} + z_{2t}) + bE_t s_{t+1}.$$

Here, we define the exchange rate  $s_t$  as the log of the home currency price of foreign currency (dollars per unit of foreign currency, if the U.S. is the home country.)  $f_{it}$  and  $z_{it}$  ( $i=1,2$ ) are economic fundamentals that ultimately drive the exchange rate, such as money supplies, money demand shocks, productivity shocks, etc. We differentiate between fundamentals observable to the econometrician,  $f_{it}$ , and those that are not observable,  $z_{it}$ . One possibility is that the true fundamental is measured with error, so that  $f_{it}$  is the measured fundamental and the  $z_{it}$  include the measurement error; another is  $z_{it}$  is unobserved shocks.

Upon imposing the “no bubbles” condition that  $b^j E_t s_{t+j}$  goes to zero as  $j \rightarrow \infty$ , we have the present value relationship

$$(3.2) \quad s_t = (1-b) \sum_{j=0}^{\infty} b^j E_t (f_{1t+j} + z_{1t+j}) + b \sum_{j=0}^{\infty} b^j E_t (f_{2t+j} + z_{2t+j})$$

This equation is of the form of equation (2.1), where we have  $a_1' x_{t+j} = f_{1t+j} + z_{1t+j}$ , and  $a_2' x_{t+j} = f_{2t+j} + z_{2t+j}$ . We now outline some models that fit into this framework.

### A. Money-Income Model

Consider first the familiar monetary models of Frenkel (1976), Mussa (1976), and Bilson (1978); and their close cousins, the sticky-price monetary models of Dornbusch (1976) and Frankel (1979). Assume in the home country there is a money market relationship given by:

$$(3.3) \quad m_t = p_t + \gamma_t - \alpha_t + v_{mt}.$$

Here,  $m_t$  is the log of the home money supply,  $p_t$  is the log of the home price level,  $i_t$  is the level of the home interest rate,  $y_t$  is the log of output, and  $v_{mt}$  is a shock to money demand. Here and throughout we use the term “shock” in a somewhat unusual sense. Our “shocks” potentially include constant and trend terms, may be serially correlated, and may include omitted variables that in principle could be measured. Assume a similar equation holds in the foreign country. The analogous foreign variables are  $m_t^*$ ,  $p_t^*$ ,  $i_t^*$ ,  $y_t^*$ , and  $v_{mt}^*$ , and the parameters of the foreign money demand are identical to the home country’s parameters.

The nominal exchange rate equals its purchasing power parity value plus the real exchange rate:

$$(3.4) \quad s_t = p_t - p_t^* + q_t.$$

In financial markets, the interest parity relationship is

$$(3.5) \quad E_t s_{t+1} - s_t = i_t - i_t^* + \rho_t$$

Here  $\rho_t$  is the deviation from rational expectations uncovered interest parity. It can be interpreted as a risk premium or an expectational error.

Putting these equations together and rearranging,

$$(3.6) \quad s_t = \frac{1}{1+\alpha} \left[ m_t - m_t^* - \gamma(y_t - y_t^*) + q_t - (v_{mt} - v_{mt}^*) - \alpha \rho_t \right] + \frac{\alpha}{1+\alpha} E_t s_{t+1}.$$

This equation takes the form of equation (3.1) when the discount factor is given by  $b = \frac{\alpha}{1+\alpha}$ , the observable fundamentals are given by  $f_{1t} = m_t - m_t^* - \gamma(y_t - y_t^*)$ , and the unobservables are:  $z_{1t} = q_t - (v_{mt} - v_{mt}^*)$  and  $z_{2t} = -\rho_t$ . Following Mark (1995), our empirical work in section 4 sets  $\gamma = 1$ . We also investigate a version of this model setting  $f_{1t} = m_t - m_t^*$ , and moving  $y_t - y_t^*$  to  $z_{1t}$ . We do so largely because we wish to conduct a relatively unstructured investigation into the link between exchange rates and various measures of fundamentals. But we could argue that we focus on  $m_t - m_t^*$  because

financial innovation has made standard income measures poor proxies for the level of transactions. Similarly, we investigate the relationship between  $s_t$  and  $y_t - y_t^*$ .

Equation (3.6) is implied by both the flexible-price and sticky-price versions of the monetary model. In the flexible-price monetarist models of Frenkel (1976), Mussa (1976), and Bilson (1978), output,  $y_t$ , and the real exchange rate,  $q_t$ , are exogenous. In the sticky-price models of Dornbusch (1976) and Frankel (1979), these two variables are endogenous. Because nominal prices adjust slowly, the real exchange rate is influenced by changes in the nominal exchange rate. Output is demand determined, and may respond to changes in the real exchange rate, income and real interest rates. Nonetheless, since equation (3.3) (and its foreign counterpart), (3.4), and (3.5) hold in the Dornbusch-Frankel model, one can derive relationship (3.6) in those models. Dornbusch and Frankel each consider special cases for the exogenous monetary processes (in Dornbusch, all shocks to the money supply are permanent; Frankel considers permanent shocks to the level and to the growth rate of money.) As a result of their assumption that all shocks are permanent, they each can express the exchange rate purely in terms of current fundamentals, which may obscure the general implication that exchange rates depend on expected future fundamentals.

We note here that some recent exchange-rate models developed from the “new open economy macroeconomics” yield very similar relationships to the ones we describe in this section. For example, in Obstfeld and Rogoff (2002), the exchange rate is given by (their equation (30):

$$(3.7) \quad s_t = \sum_{j=0}^{\infty} b^j E_t \left[ (1-b)(m_{t+j} - m_{t+j}^*) - b\rho_{t+j} \right],$$

where we have translated their notation to be consistent with ours. Equation (3.7) is in fact the forward solution to a special case of equation (3.6) above. The discount factor,  $b$ , in Obstfeld and Rogoff (2002) is related to the semi-elasticity of money demand exactly as in equation (3.6). However, their money demand function is derived from a utility-maximizing framework in which real balances appear in the utility function, and their risk premium  $\rho_t$  is derived endogenously from first principles.

## B. Taylor-Rule Model

Here we draw on the burgeoning literature on Taylor rules. Let  $\pi_t = p_t - p_{t-1}$  denote the inflation rate, and  $y_t^g$  be the “output gap”. We assume that the home country (the U.S. in our empirical work) follows a Taylor rule of the form:

$$(3.8) \quad i_t = \beta_1 y_t^g + \beta_2 \pi_t + v_t.$$

In (3.8),  $\beta_1 > 0$ ,  $\beta_2 > 1$ , and the shock  $v_t$  contains omitted terms.<sup>1</sup>

The foreign country follows a Taylor rule that explicitly includes exchange rates:

$$(3.9) \quad i_t^* = -\beta_0 (s_t - \bar{s}_t^*) + \beta_1 y_t^{*g} + \beta_2 \pi_t^* + v_t^*.$$

In (3.9),  $0 < \beta_0 < 1$ , and  $\bar{s}_t^*$  is a target for the exchange rate. We will assume that monetary authorities target the PPP level of the exchange rate:

$$(3.10) \quad \bar{s}_t^* = p_t - p_t^*.$$

Since  $s_t$  is measured in dollars per unit of foreign currency, the rule indicates that *ceteris paribus* the foreign country raises interest rates when its currency depreciates relative to the target. Clarida, Gali and Gertler (1998) estimate monetary policy reaction functions for Germany and Japan (using data from 1979-1994) of a form similar to equation (3.9). They find that a one percent real depreciation of the mark relative to the dollar led the Bundesbank to increase interest rates (expressed in annualized terms) by five basis points, while the Bank of Japan increased rates by 9 basis points in response to a real yen depreciation relative to the dollar.

As the next equation makes clear, our argument still follows if the U.S. were also to target exchange rates. We omit the exchange rate target in (3.8) on the interpretation that U.S. monetary policy has virtually ignored exchange rates except, perhaps, as an indicator.

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<sup>1</sup> Much of the Taylor rule literature—wisely, in our view—puts expected inflation in the monetary policy rule. Among other benefits, this facilitates thinking of the monetary authority as setting an ex-ante real rate. We use actual inflation for notational simplicity. If expected inflation is in the monetary rule, then inflation in the formulas below is replaced by expected inflation.

Subtracting the foreign from the home money rule, we obtain

$$(3.11) \quad i_t - i_t^* = \beta_0(s_t - \bar{s}_t^*) + \beta_1(y_t^g - y_t^{*g}) + \beta_2(\pi_t - \pi_t^*) + v_t - v_t^*$$

Use interest parity (3.5) to substitute out for  $i_t - i_t^*$ , and (3.10) to substitute out for the exchange rate target:

$$(3.12) \quad s_t = \frac{\beta_0}{1 + \beta_0}(p_t - p_t^*) - \frac{1}{1 + \beta_0}[\beta_1(y_t^g - y_t^{*g}) + \beta_2(\pi_t - \pi_t^*) + v_t - v_t^* + \rho_t] + \frac{1}{1 + \beta_0}E_t s_{t+1}.$$

This equation is of the general form (3.1) of the expected discounted present value models. The discount factor is equal to  $\frac{1}{1 + \beta_0}$ . We have  $f_{1t} = p_t - p_t^*$ . In our empirical work (in section 4), we will treat the

remaining variables as unobservable, so we have  $z_{2t} = -[\beta_1(y_t^g - y_t^{*g}) + \beta_2(\pi_t - \pi_t^*) + v_t - v_t^* + \rho_t]$ .

Equation (3.11) can be expressed another way, again using interest parity (3.5), and the equation for the target exchange rate, (3.10):

$$(3.13) \quad s_t = \beta_0(i_t - i_t^*) + \beta_0(p_t - p_t^*) - \beta_1(y_t^g - y_t^{*g}) - \beta_2(\pi_t - \pi_t^*) - v_t + v_t^* - (1 - \beta_0)\rho_t + (1 - \beta_0)E_t s_{t+1}$$

This equation is very much like (3.12), except that it incorporates the interest differential,  $i_t - i_t^*$ , as a “fundamental”. The discount factor in this formulation is given by  $1 - \beta_0$ . The observed fundamental is given by  $f_{1t} = i_t - i_t^* + p_t - p_t^*$ . In our empirical work, we treat the remaining period  $t$  variables in equation (3.13) as unobserved.

### C. Discussion

We begin by noting that the classic efficient markets model of Samuelson (1965) and others does *not* predict a random walk in exchange rates. The essence of this model is that there are no predictable profit opportunities for a risk-neutral investor to exploit. If the U.S. interest rate  $i_t$  is higher than foreign interest rate  $i_t^*$  by  $x\%$ , then the U.S. dollar must be expected to fall by  $x\%$  over the period of the

investment if there is to be no such opportunities. In terms of equation (3.5), then, the classic efficient markets model says that the risk premium  $\rho_t$  is zero, and that a population regression of  $\Delta s_{t+1}$  on  $i_t - i_t^*$  will yield a coefficient of 1. (For equities, the parallel prediction is that the day a stock goes ex-dividend its price should fall by the amount of the dividend (e.g., Elton and Gruber (1970).)

Our explanation yields a random walk approximation even when, as in the previous paragraph, uncovered interest parity holds. The reader may wonder how the data can simultaneously satisfy: (1) a regression of  $\Delta s_{t+1}$  on  $i_t - i_t^*$  yields a nonzero coefficient, and (2)  $s_t$  is arbitrarily well approximated as a random walk (i.e.,  $\Delta s_{t+1}$  is arbitrarily well approximated as white noise). The answer is that when  $b$  is arbitrarily close to 1, the  $R^2$  of the regression of  $\Delta s_{t+1}$  on  $i_t - i_t^*$  will be arbitrarily close to zero, and the correlation of  $\Delta s_{t+1}$  with  $i_t - i_t^*$  will be arbitrarily small. It is in those senses that the random walk approximation will be arbitrarily good.

The key question is not the logic of our result but its empirical validity. The result does not require uncovered interest parity, which was maintained in the previous two paragraphs merely to clarify the relation of our result to the standard efficient markets result. Instead, two conditions are required. The first is that fundamentals variables be very persistent – I(1) or nearly so. This is arguably the case with our data on the observed fundamentals. We will present evidence in section 4 that we cannot reject the null of a unit root in any of our data. Further, there is evidence in other research that the unobservable variables are very persistent. For the money-income model (equation (3.6)), this is suggested for  $v_{mt}$ ,  $q_t$ , and  $\rho_t$  by the literature on money demand, e.g., Sriram (2000); purchasing power parity, e.g., Rogoff (1996); and, interest parity, e.g., Engel, (1996). (We recognize that theory suggests that a risk premium like  $\rho_t$  is I(0); our interpretation is that if  $\rho_t$  is I(0), it has a very large autoregressive root.) We are not concerned if  $\rho_t$  or other variables are highly persistent I(0) variables rather than I(1) variables, for we saw in lines (10)-(13) of Table 2.1 that a near random walk can result for such processes.

A second condition for  $s_t$  to follow an approximate random walk is that  $b$  is sufficiently close to 1. The evidence we present below in Table 4.1 on the first-order autocorrelations for the exchange-rate fundamentals suggests that the lines in Table 2.1 most relevant to our data are those with  $\varphi = 0.3$  or  $\varphi = 0.5$ . If so, Table 2.1 suggests that if  $b$  is around 0.9 or above, the asset price appears to be nearly indistinguishable from a random walk.

In the money-income models,  $b$  is related to the interest semi-elasticity of money demand:

$$b = \frac{\alpha}{1 + \alpha}. \text{ Bilson (1978) estimates } \alpha \approx 60 \text{ in the monetary model, while Frankel (1979) finds } \alpha \approx 29.$$

The estimates from Stock and Watson (1993, Table 2, panel I, page 802) give us  $\alpha \approx 40$ .<sup>2</sup> They imply a range for  $b$  of 0.97 to 0.98 for quarterly data.

To get a sense of the plausibility of this discount factor, compare it to the discount factor implied in a theoretical model in which optimal real balance holdings are derived from a money-in-the-utility-function framework. Obstfeld and Rogoff (2002) derive a money demand function that is very similar to equation (3.3), when utility is separable over consumption and real balances, and money enters the utility

function as a power function:  $\frac{1}{1 - \varepsilon} \left( \frac{M_t}{P_t} \right)^{1 - \varepsilon}$ . They show that  $\alpha \approx 1/\bar{\varepsilon}$ , where  $\bar{\varepsilon}$  is the steady-state

nominal interest rate in their model. They state (p. 27), “Assuming time is measured in years, then a value between 0.04 and 0.08 seems reasonable for  $\bar{\varepsilon}$ . It is usually thought that  $\varepsilon$  is higher than one, though not necessarily by a large margin. Thus, based on a priori reasoning, it is not implausible to assume  $1/\bar{\varepsilon} = 15$ .” For our quarterly data, the value of  $\alpha$  would be 60, which is right in line with the estimate from Bilson cited above.

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<sup>2</sup> Bilson uses quarterly interest rates that are annualized and multiplied by 100 in his empirical study. So his actual estimate of  $\alpha = 0.15$  should be multiplied by 400 to construct a quarterly discount rate. MacDonald and Taylor (1993) estimate a discounted sum of fundamentals and test for equality with the actual exchange rate – following the methods of Campbell and Shiller (1987) for equity prices. MacDonald and Taylor rely on the estimates of Bilson to calibrate their discount factor, but mistakenly use 0.15 instead of 60 as the estimate of  $\alpha$ . Stock and Watson’s data estimates also use annualized interest rates multiplied by 100, so we have multiplied their estimate by 400.

In the Taylor-rule model, the discount factor is large when the degree of intervention by the monetary authorities to target the exchange rate is small. The strength of intervention is given by the parameter  $\beta_0$  from (3.11), and the discount factor is either  $\frac{1}{1+\beta_0}$  in the formulation of (3.12), or  $1-\beta_0$  in the representation in (3.13). In practice, it seems as though foreign exchange intervention within the G7 has not been very active. For example, if the exchange rate were 10 percent above its PPP value, it is probably an upper bound to guess that a central bank would increase the short-term interest rate by one percentage point (expressed on an annualized basis.) With quarterly data, this would imply a value of  $b$  of about 0.975, which is consistent with the discount factors we imputed in the monetary models. Clarida, Gali and Gertler's (1998) estimates of the monetary policy reaction functions for Germany and Japan over the 1979-1994 period find that a 10 percent real depreciation of the currency led the central banks to increase annualized interest rates by 50 and 90 basis points, respectively. This translates to quarterly discount factors of 0.988 and 0.978.

Our result does not require that the fundamentals evolve exogenously to the exchange rate. The result is not, however, consistent with a thought experiment that allows the stochastic process for the fundamentals to change as  $b$  gets near to 1. But we can answer the question: with given data for fundamentals, and plausible values for  $b$ , will a present value model yield an approximate random walk? For the values of  $b$  taken from the literature (which we have just discussed), and for serial correlation plausible for exchange rate fundamentals (reported in Table 4.1 below), the figures in Table 2.1 indicate near random walk behavior.

We note that the presence of persistent deviations from uncovered interest parity, in the form of a risk premium or expectational error, could potentially play a large role in accounting for movements in exchange rates. Equation (3.2) draws a distinction between fundamentals that are multiplied by the discount factor,  $b$ , ( $f_{2t}$  and  $z_{2t}$ ), and fundamentals that are multiplied by  $1-b$  ( $f_{1t}$  and  $z_{1t}$ ). As  $b \rightarrow 1$ , the former become increasingly dominant in determining exchange rate movements. In both the money-income model and the Taylor-rule model, the deviation from interest parity is like a  $z_{2t}$  variable – an

unobservable fundamental multiplied by  $b$  in equation (3.2). This analysis alone cannot determine whether deviations from interest parity are very important. A more detailed model would determine the size of these deviations. (For example, in a particular model, it may be that the deviation from interest parity depends on the discount factor in such a way that as  $b \rightarrow 1$ , the deviation gets smaller.) We note one model in which a theoretical risk premium is derived – that of Obstfeld and Rogoff (2002). They refer to the effect of the risk premium on the level of the exchange rate – the discounted present value of the risk premium – as the “level risk premium.” They explicitly note that in their model the discount factor  $b$  is large, and that in turn means that a volatile deviation from interest parity has a large impact on the variance of exchange rate changes. (See equation (3.7).)

#### 4. EMPIRICAL FINDINGS

We have argued that when standard exchange rate models are plausibly calibrated, they have the property that the exchange rate should nearly follow a random walk. Evidence that the exchange rate change is not predictable is an implication of the models, not evidence against the models. But merely observing that exchange rates follow random walks is not a very complete validation of the models.

There are other possible explanations of the random walk behavior of exchange rates. The exchange rate may be dominated by unobservable shocks that are well-approximated by random walks – that is, that the  $z_{it}$  from equation (3.1) are well-approximated by a random walk, and the variance of  $\Delta s_t$  is dominated by the changes in  $z_{it}$  rather than by changes in  $f_{it}$ . The standard set of fundamentals (money, income, prices, interest rates) may not be important determinants of exchange rates, and instead there may be some other variable that models have not captured or which is unobserved that drives the exchange rate.

In this section, we consider an implication of asset pricing models: that the asset price might help to predict the fundamentals. This basic insight led Campbell and Shiller (1987) to develop a test of present value models of asset prices. We do not follow their method here, because we acknowledge the

possibility of unobserved fundamentals (the  $z_{it}$ ), which make the exact method of Campbell and Shiller inapplicable. However, our approach to model validation is inspired by the Campbell-Shiller methodology.

#### A. Data and Basic Statistics

We use quarterly data, usually 1974:1-2001:3 (with exceptions noted below). With one observation lost to differencing, the sample size is  $T = 110$ .

We study bilateral US exchange rates versus the other six members of the G7: Canada, France, Germany, Italy, Japan and the United Kingdom. The *International Financial Statistics* (IFS) CD-ROM is the source for the end of quarter exchange rate  $s_t$  and consumer prices  $p_t$ . The OECD's *Main Economic Indicators* CD-ROM is the source for our data on the seasonally adjusted money supply,  $m_t$  (M4 in the U.K., M1 in all other countries; 1978:1-1998:4 for France, 1974:1-1998:4 for Germany, 1975:1-1998:4 for Italy). The OECD is also the source for real, seasonally adjusted GDP,  $y_t$ , for all countries but Germany, which we obtain by combining IFS (1974:1-2001:1) and OECD (2001:2-2001:3) data, and Japan, which combines data from the OECD (1974:1-2002) with 2002:3 data from the web site of the Japanese Government's Economic and Social Research Institute. Datastream is the source for the interest rates,  $i_t$ , which are 3 month Euro rates (1975:1-2001:3 for Canada, 1978:3-2001:3 for Italy and Japan). We convert all data but interest rates by taking logs and multiplying by 100. Through the rest of the paper, the symbols defined in this paragraph ( $s_t, m_t, y_t, p_t$ ) refer to the transformed data.

We focus on the bivariate relationship between  $s_t$  and the following five measures of fundamentals:  $m_t, p_t, i_t, y_t, m_t - y_t$ . We briefly discuss results when we look at full systems of variables suggested by particular versions of the models sketched in section 3. As noted in that section, we focus on the simple bivariate relationships because we wish to conduct a relatively unstructured investigation.

Let  $f_t$  denote a measure of “fundamentals” in the U.S. relative to abroad (for example,  $f_t = m_t - m_t^*$ .) Using Dickey-Fuller tests with a time trend included, we were generally unable to reject the null of a unit root in any of the five measures of  $f_t$  (i.e., in  $m_t$ ,  $p_t$ ,  $i_t$ ,  $y_t$ , and  $m_t - y_t$ ). Hence our analysis presents statistics on  $\Delta f_t$  for all measures of fundamentals. Even though we fail to reject unit roots for interest differentials, we are uneasy using interest differentials only in differenced form. So we present statistics for both levels and differences of interest rates.

Some basic statistics are presented in Table 4.1. Row 1 is consistent with much evidence that changes in exchange rates are serially uncorrelated, and quite volatile. The standard deviation of the quarterly change is over 5 percentage points for all except the Canadian dollar exchange rate. First order autocorrelations are small, under 0.15 in absolute value. Under the null of no serial correlation, the standard error on the estimator of the autocorrelation is approximately  $1/\sqrt{T} \approx 0.1$ , so none of the estimates are significant at even the 10 percent level.

Rows 2 through 7 present statistics on our measures of fundamentals. A positive value for the mean indicates that the variable has been growing faster in the U.S. than abroad. For example, the figure of -0.92 for the mean value of the U.S.- Italy inflation differential means that quarterly inflation was, on average, 0.92 percentage points lower in the U.S. than in Italy during the 1974-2001 period. Of particular note is that the vast majority of estimates of first order autocorrelation coefficients suggest a rejection of the null of no serial correlation at the 10% level, and most do at the 5% level as well (again using an approximate standard error of 0.1). (An exception to this pattern is in output differentials in row (7). None of the autocorrelations are significant at the 5% level, and only one (France, for which the estimate is 0.19) at the 10% level.) The magnitude of the autocorrelations—less than 0.5 for virtually all differenced series—suggests that the for calibrating an exchange rate model, the relevant entries in Table 2.1 are those with  $\varphi = 0.3$  or  $\varphi = 0.5$  but not  $\varphi = 0.9$ .

For each country we conducted five cointegration tests, between  $s_t$  and each of our measures of fundamentals,  $m_t - m_t^*$ ,  $p_t - p_t^*$ ,  $i_t - i_t^*$ ,  $y_t - y_t^*$  and  $m_t - y_t - (m_t^* - y_t^*)$ . We used Johansen's (1991) trace and maximum eigenvalue statistics, with critical values from Osterwald-Lenum (1992). Each bivariate VAR contained four lags. Of the 30 tests (6 countries, 5 fundamentals), we rejected the null of no cointegration at the 5 percent level in 5 instances using the trace statistic. These were for  $m_t - m_t^*$ ,  $p_t - p_t^*$ , and  $i_t - i_t^*$  for Italy, and,  $p_t - p_t^*$ , and  $i_t - i_t^*$  for the U.K. Of the 30 tests using the maximum eigenvalue statistic, the null was rejected only once, for the U.K. for  $p_t - p_t^*$ . We conclude that it will probably not do great violence to assume lack of cointegration, recognizing that a complementary analysis using cointegration would be useful.

We take the lack of cointegration to be evidence that unobserved variables such as real demand shocks, real money demand shocks, or possibly even interest parity deviations have a permanent component, or at least are very persistent. Alternatively, it may be that the data we use to measure the economic fundamentals of our model have some errors with permanent or very persistent components. For example, it may be that the appropriate measure of the money supply has permanently changed because of numerous financial innovations over our sample, so that the M1 money supply series vary from the "true" money supply by some I(1) errors.

## B. Granger-Causality Tests

Campbell and Shiller (1987) observe that when a variable  $s_t$  is the present value of a variable  $x_t$ , then either (1)  $s_t$  Granger causes  $x_t$  relative to the bivariate information set consisting of lags of  $s_t$  and  $x_t$ , or (2),  $s_t$  is an exact distributed lag of current and past values of  $x_t$ . That is, as long as  $s_t$  embodies *some* information in addition to that included in past values of  $x_t$ ,  $s_t$  Granger causes  $x_t$ .<sup>3</sup> As was emphasized in the previous section, however, exchange rate models must allow for unobservable

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<sup>3</sup> In the appendix, this additional information is formalized as additional random variables that are used by private agents in forecasting future fundamentals.

fundamentals – the possibility that  $x_t$  is a linear combination of unobservable as well as observable variables, and thus  $x_t$  itself is unobservable. Failure to find Granger causality from  $s_t$  to observable variables no longer implies an obviously untenable restriction that the exchange rate is an exact distributed lag of observables. It is clear, though, that a finding of Granger causality is supportive of a view that exchange rates are determined as a present value that depends in part on observable fundamentals.

Table 4.2 summarizes the results of our Granger causality tests on the full sample. We include a constant and four lags of each variable in all causality tests reported in this and all other tables. For all tests of no causality we use likelihood-ratio statistics using the degrees of freedom correction suggested in Sims (1980).

We see in panel A that at the five percent level of significance, the null that that  $\Delta s_t$  fails to Granger cause  $\Delta(m_t - m_t^*), \Delta(p_t - p_t^*), i_t - i_t^*, \Delta(i_t - i_t^*), \Delta(y_t - y_t^*),$  and  $\Delta[m_t - y_t - (m_t^* - y_t^*)]$ , can be rejected in 9 cases at the 5 percent level, and 3 more cases at the 10 percent level. There are no rejections for Canada and the U.K., but rejections in 12 of the 24 tests for the other four countries. The strongest rejections are for prices, where the null is rejected in three cases at the one percent level.<sup>4</sup>

In a sense, this is not particularly strong evidence that exchange rates predict fundamentals.<sup>5</sup> After all, even if there were zero predictability, one would expect a handful of significant statistics just by chance. We accordingly are cautious in asserting that the posited link is well established. But one

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<sup>4</sup> The overall level of predictability, though not the pattern, is consistent with the point estimates in Stock and Watson (2003). Using inflation and output from the G7 countries (rather than for six countries relative to the U.S.), and a 1985-99 sample, Stock and Watson (2003) examine the ability of the exchange rate (and many other financial variables) to forecast out-of-sample. They find that the exchange rate lowers the mean squared prediction error for inflation in one country (Canada), for GDP in four countries (Canada, Germany, Italy and Japan). Thus the overall rate of success (five out of fourteen data series) is comparable to ours, though the pattern (more success with real than nominal) is not. We have not investigated the extent to which results differ because different series are being fit or because of in- versus out-of-sample.

<sup>5</sup> A referee has pointed out that for series other than interest rates, seasonal adjustment may be lead to spurious findings of causality. We were not able to collect a complete set of not seasonally adjusted data. But we did repeat our Granger-causality tests using money supply data that was not seasonally adjusted for the U.S., France, and Japan from *International Financial Statistics*. Our findings were not affected by the use of n.s.a. money supply data: we reject no Granger causality at the 10% level for France, and at the 5% level for Japan. (We were only able to obtain n.s.a. M2 money supply for Italy. The p-value for the test of no causality was 20%.)

statistical (as opposed to economic) indication that the results are noteworthy comes from contrasting these results with ones for Granger causality tests running in the opposite direction. We see in panel B of Table 3.2 that the null that the fundamentals fail to Granger cause  $\Delta s_t$  can be rejected at the 5 percent level in only one test, and at the 10 percent level in only one more test. So, however modest is the evidence that exchange rates help to predict fundamentals, the evidence is distinctly stronger than that on the ability of fundamentals to predict exchange rates.

There were some major economic and non-economic developments during our sample that warrant investigation of sub-samples. Several of the European countries' exchange rates and monetary policies became more tightly linked in the 1990s because of the evolution of the European Monetary Union. Germany's economy was transformed dramatically in 1990 because of reunification. We therefore look at causality results for two subsamples. Table 4.3 presents results for 1974:1-1990:2, and Table 4.4 for the remaining part of the sample (1990:3-2001:2).

The results generally go the same direction as for the whole sample. In Table 4.3A, we see that for the first part of the sample, we reject the null of no Granger causality from exchange rates to fundamentals at the one or five percent level in 10 cases, and at the ten percent level in 2 more cases. Table 4.3 B indicates that there are no cases in which we can reject the null of no Granger causality from fundamentals to exchange rates at the five percent level, and only 2 cases at the ten percent level.

Table 4.4 reports results for the second part of the sample. Panel A shows we reject the null of no Granger causality from exchange rates to fundamentals in 9 cases at the one or five percent level, and five more cases at the 10 percent level. But for the test of no causality from fundamentals to exchange rates, Panel B shows we reject nine times at the one or five percent level, once at the 10 percent level. In the 1990s, then, there appears to be more evidence of exchange-rate predictability. This perhaps is not entirely surprising given the effort by the European countries to stabilize exchange rates. We note, however, that several of the rejections of the null are for the yen/dollar rate.

In addition to the causality tests we report from bivariate VARs, we also performed cointegration and causality tests based on some multivariate VARs. We chose several different combinations of variables to include in these VARs, based on the models outlined in Section 3. There are five groupings:  $(\Delta s_t, \Delta(y_t - y_t^*), \Delta(p_t - p_t^*), i_t - i_t^*)'$ ,  $(\Delta s_t, \Delta(m_t - m_t^*), \Delta(y_t - y_t^*))'$ ,  $(\Delta s_t, \Delta(p_t - p_t^*), \Delta(y_t - y_t^*))'$ ,  $(\Delta s_t, \Delta(m_t - m_t^*), \Delta(y_t - y_t^*), \Delta(p_t - p_t^*))'$ , and  $(\Delta s_t, \Delta(y_t - y_t^*), \Delta(p_t - p_t^*), \Delta(i_t - i_t^*))'$ . All variables were entered in differences because of results of tests for cointegration.<sup>6</sup> We performed causality tests for the null that  $\Delta s_t$  does not Granger cause for each of the fundamentals or the fundamentals as a group, and conversely. For example, in the first grouping (i.e.,  $(\Delta s_t, \Delta(y_t - y_t^*), \Delta(p_t - p_t^*), i_t - i_t^*)'$ ), there were four tests of Granger causality from  $\Delta s_t$ , to each of the three fundamentals and to the block of fundamentals as a whole. There was also the corresponding set of four tests from fundamentals to  $\Delta s_t$ . Across the six countries, this yielded 24 tests of causality in each direction for this grouping. Across all five groupings, 108 test statistics were computed in each direction.

The results are very much like the results from the bivariate VARs. There is almost no evidence of causality from the fundamentals to the exchange rate. Of the 108 tests we performed, there are no cases in which we could reject at the 5 percent level the hypothesis of no causality from fundamentals to exchange rates, and only four cases where that hypothesis is rejected at the 10 percent level. In contrast, in 35 tests (out of 108 performed) we rejected the null of no causality from exchange rates to fundamentals at the 10 percent level, and these were significant at the 5 percent level in 16 cases. We present details for the Granger causality tests on the fundamentals as a group in Table 4.5, relegating to the additional appendix details on the other tests. As Table 4.5 demonstrates, there were no cases in which we rejected the joint null of no causality from the group of fundamentals to the exchange rate. Notable are the tests for whether the exchange rate does not Granger cause any of the economic fundamentals. Table 4.5 reports that we reject the null of no causation in 16 of the 30 tests performed at

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<sup>6</sup> According to Johansen's (1991) trace and maximum eigenvalue statistics, there were only three cases in which we were able to reject the null of no cointegration (one for Canada, and two for Italy), so for uniformity we treated all variables as if they were not cointegrated.

the 10 percent level, and 12 of those were significant rejections at the 5 percent level. Nonetheless, there were many more cases in which the exchange rate could not help predict fundamentals. The exchange rate was found to be useful in forecasting real output in only two cases.

To summarize, while the evidence is far from overwhelming, there does appear to be a link from exchange rates to fundamentals, going in the direction that exchange rates help forecast fundamentals.

### C. Correlation between $\Delta s$ and the Present Value of Fundamentals

The previous subsection established a statistically significant link between exchange rates and certain fundamentals. We now examine such links to ask whether the signs of the regression coefficients are in some sense right. The statistic we propose is broadly similar to one developed in Campbell and Shiller (1987). The modification of the Campbell-Shiller statistic is necessary for two reasons. First is that, unlike Campbell and Shiller, our variables are not well approximated as cointegrated. Second is that we allow for unobservable forcing variables, again in contrast to Campbell and Shiller.

Write the present value relationship for exchange rates as

$$(4.1) \quad s_t = \sum_{j=0}^{\infty} b^j E_t f_{t+j} + \sum_{j=0}^{\infty} b^j E_t z_{t+j} \equiv F_t + U_t .$$

Now  $\sum_{j=0}^{\infty} b^j E_t f_{t+j} = \frac{1}{1-b} (f_{t-1} + \sum_{j=0}^{\infty} b^j E_t \Delta f_{t+j})$ . Thus

$$(4.2) \quad s_t - \frac{1}{1-b} f_{t-1} = \frac{1}{1-b} \sum_{j=0}^{\infty} b^j E_t \Delta f_{t+j} + U_t .$$

Our unit root tests indicate that  $\Delta f_t$ , and hence  $\sum_{j=0}^{\infty} b^j E_t \Delta f_{t+j}$  are  $I(0)$ , and that  $s_t$  and  $f_t$  are not cointegrated. For (3.2) to be consistent with lack of cointegration between  $s_t$  and  $f_t$ , we must have  $U_t \sim I(1)$ . A stationary version of (4.1) is then

$$(4.3) \quad \Delta s_t = \Delta F_t + \Delta U_t .$$

Let  $F_{it}$  be the present value of future  $f$ 's computed relative to an information set indexed by the  $i$  subscript. The two information sets we use are univariate and bivariate:

$$(4.4) \quad F_{1t} \equiv E\left(\sum_{j=0}^{\infty} b^j f_{t+j} \mid f_t, f_{t-1}, \dots\right),$$

$$(4.5) \quad F_{2t} \equiv E\left(\sum_{j=0}^{\infty} b^j f_{t+j} \mid s_t, f_t, s_{t-1}, f_{t-1}, \dots\right).$$

We hope to get a feel for whether either of these information sets yield economically meaningful present values by estimating  $\text{corr}(\Delta F_{it}, \Delta s_t)$ , the correlation between  $\Delta F_{it}$  and  $\Delta s_t$ . The finding of Granger causality from exchange rates to observable fundamentals supports the view that exchange rates are determined as a present value that depends in part on these observables. A more demanding verification of the relationship between exchange rates and observed fundamentals implied by the model is that  $\text{corr}(\Delta F_{it}, \Delta s_t)$  be high.<sup>7</sup>

We estimate  $\text{corr}(\Delta F_{it}, \Delta s_t)$  using estimates of  $\Delta F_{it}$  constructed from univariate autoregressions ( $F_{1t}$ ) or bivariate vector autoregressions ( $F_{2t}$ ). If the estimated correlation is substantially stronger using the bivariate estimate, we take that as evidence that the coefficients of  $\Delta s_t$  in the VAR equation for  $\Delta f_t$  are economically reasonable and important. We limit our analysis to the variables in which there is a statistically significant relationship between  $\Delta f_t$  and  $\Delta s_t$ , as indicated by the Granger causality tests in Table 4.2.

Note that a low value of the correlation is not necessarily an indication that  $s_t$  is little affected by the present value of  $f_t$ . A low correlation will result from a small covariance between  $\Delta F_{it}$  and  $\Delta s_t$ . But since  $\text{cov}(\Delta F_{it}, \Delta s_t) = \text{cov}(\Delta F_{it}, \Delta F_t) + \text{cov}(\Delta F_{it}, \Delta U_t)$ , this covariance might be small because a sharply negative covariance between  $\Delta F_{it}$  and  $\Delta U_t$  offsets a positive covariance between  $\Delta F_{it}$  and  $\Delta F_t$ .

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<sup>7</sup> Engel and West (2004) propose a method for calculating the variance of  $\Delta F_t$  (from equation (4.3)) relative to the variance of  $\Delta s_t$ .

Conversely, of course, a high correlation might reflect a tight relationship between  $\Delta F_{it}$  and  $\Delta U_t$  with little connection between  $\Delta F_{it}$  and  $\Delta F_t$ .<sup>8</sup>

We do, however, take as reasonable the notion that if the correlation is higher for the bivariate than for the univariate information set, the coefficients on lags of  $\Delta s_t$  in the  $\Delta f_t$  equation are economically meaningful.

We construct  $\hat{F}_{1t}$  from estimates of univariate autoregressions, and  $\hat{F}_{2t}$  from bivariate VARs, imposing a value of the discount factor  $b$ . The lag length is four in both the univariate and bivariate estimates. We then estimate the correlations  $corr(\Delta F_{it}, \Delta s_t)$  using these estimated  $\hat{F}_{it}$ . We report results only for data that show Granger causality from  $\Delta s_t$  to  $\Delta f_t$  at the 10 percent level or higher in the whole sample (Table 4.2, panel A). We construct confidence intervals using the percentile method and a non-parametric bootstrap. We sample with replacement from the bivariate VAR residuals, with actual data used as initial conditions. We use 5000 replications. For  $\hat{F}_{1t}$  and  $\hat{F}_{2t}$ , we construct 90 percent confidence intervals using the .05 and .95 quantiles. For  $\hat{F}_{2t} - \hat{F}_{1t}$ , we use the .10 and 1.0 quantiles. We do not attempt to control for the data dependent fact that we only study samples in which the previous subsection found Granger causality.

We tried three values of the discount factor,  $b = 0.5$ ,  $b = 0.9$ , and  $b = 0.98$ . Results were strongest for  $b = 0.98$ . So to be conservative we report results only for  $b = 0.5$ ,  $b = 0.9$ . See Panels A and B, respectively, of Table 4.6. For the univariate information set ( $F_{1t}$ ), the three discount factors give very similar results. Of the 10 estimated correlations, only two are positive for each value of  $b$ . (All of the relations should be positive for the four variables reported in Table 4.6 --  $\Delta(m_t - m_t^*)$ ,  $\Delta(p_t - p_t^*)$ ,

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<sup>8</sup> Since  $s_t$  is an element of the bivariate information set, projection of both sides of (3.1) onto this information set yields  $s_t = F_{2t} + E(U_t | s_t, f_t, s_{t-1}, f_{t-1}, \dots)$ . It may help readers familiar with Campbell and Shiller (1987) to note that because our models include unobserved forcing variables (i.e., because  $U_t$  is present), we may not have  $s_t = F_{2t} = F_t$ . These equalities hold only if  $E(U_t | s_t, f_t, s_{t-1}, f_{t-1}, \dots) = 0$ .

$\Delta(i_t - i_t^*)$ , and  $\Delta[m_t - y_t - (m_t^* - y_t^*)]$  -- according to the models of section 3, if the contribution of  $\Delta U_t$  is sufficiently small.) So if one relies on univariate estimates of the present value, one would find little support for the notion that changes in exchange rates reflect changes in the present value of fundamentals.

The bivariate estimates lend rather more support for this notion, especially for  $b = 0.9$ . The estimated correlation between  $\Delta F_{2t}$  and  $\Delta s_t$  is positive in 6 of the 10 cases for  $b = 0.5$  (though significantly different from zero at the 90% level in only one case [Japan,  $\Delta(m - m^*)$ ]); it is positive in 7 of the 10 cases for  $b = 0.9$  and significant in 4 of these (all three inflation series, and  $\Delta(i - i^*)$  in Japan). The sharpest result is that the correlation is higher for  $\Delta F_{2t}$  than for  $\Delta F_{1t}$ : the difference between the two is positive and significant in 8 cases for  $b=0.5$ , positive in 9 cases and significant in 7 for  $b=0.9$ .<sup>9</sup> The median correlations can be summarized as:

Information set	$b = 0.5$	$b = 0.9$
(4.6) $F_{1t}$	-0.04	-0.05
$F_{2t}$	0.10	0.24

It is clear that using lags of  $\Delta s_t$  to estimate the present value of fundamentals results in an estimate that is more closely tied to  $\Delta s_t$  itself than when the present value of fundamentals is based on univariate estimates. But even limiting ourselves to data in which there is Granger causality from  $\Delta s_t$  to  $\Delta f_t$ , the largest single correlation in the table is 0.51 (Germany, for  $\Delta(p_t - p_t^*)$ , when  $b=0.9$ .) A correlation less than one may be due to omitted forcing variables,  $U_t$ . In addition, we base our present values on the expected present discounted value of fundamental variables one at a time, instead of trying to find the appropriate linear combination (except when we use  $m - y$  as a fundamental.) So we should not be surprised that the correlations are still substantially below one.

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<sup>9</sup> Here it is advisable to recall that we only examine series that display Granger causality. So the statistical significance of the difference is unsurprising. On the other hand, the sign of the difference (positive) was not foretold by our Granger causality tests.

The long literature on random walks in exchange rates causes us to interpret the correlations in Table 4.6 as new evidence that exchange rates are tied to fundamentals. We recognize, however, that these estimates leave a vast part of the movements in exchange rates not tied to fundamentals. The results may suggest a direction for future research into the link between exchange rates and fundamentals – looking for improvements in the definition of fundamentals used to construct  $F_{2t}$ .

## 5. CONCLUSIONS

We view the results of this paper as providing some counterbalance to the bleak view of the usefulness (especially in the short run) of rational expectations present value models of exchange rates that has become predominant since Meese and Rogoff (1983a, 1983b). We find that exchange rates may incorporate information about future fundamentals, a finding consistent with the present value models. We also show theoretically that under some empirically plausible circumstances the inability to forecast exchange rates is a natural implication of the models. The models do suggest that innovations in the exchange rate ought to be highly correlated with news about future fundamentals – a link that seems to garner support from the recent study of Andersen, Bollerslev, Diebold, and Vega (2003), who find strong evidence of exchange-rate reaction to news (and in a direction consistent with standard models) in intraday data.

On the other hand, our findings certainly do not provide strong direct support for these models, and indeed there are several caveats that deserve mention. First, while our Granger causality results are consistent with the implications of the present value models – that exchange rates should be useful in forecasting future economic variables such as money, income, prices and interest rates – there are other possible explanations for these findings. It may be, for example, that exchange rates Granger cause the domestic consumer price level simply because exchange rates are passed on to prices of imported consumer goods with a lag. Exchange rates might Granger cause money supplies because monetary policy-makers react to the exchange rate in setting the money supply. In other words, the present value

models are not the only models that imply Granger causality from exchange rates to other economic variables. Table 4.6, which concerns the correlation of exchange rate changes with the change in the expected discounted fundamentals, provides some evidence that the Granger causality results are generated by the present value models, but it is far from conclusive.

Second, the empirical results are not uniformly strong. As well, it remains to be seen how well they hold upon, for example, use of panel data or out of sample techniques such as in Groen (2002), Mark and Sul (2001), or Stock and Watson (2003).

Third, while we read the exchange rate literature as agreeing with us that there is a role for “unobserved” fundamentals – money demand shocks, real exchange rate shocks, risk premiums – we recognize that others might view such a role as evidence of a failure of the model. We do not find much evidence that the exchange rate is explained only by the “observable” fundamentals. Our bivariate cointegration tests generally fail to find cointegration between exchange rates and fundamentals. Moreover, we know from Mark (2001) that actual exchange rates are likely to have a much lower variance than a discounted sum of observable fundamentals. Our view is that it is perhaps unrealistic to believe that only fundamentals that are observable by the econometrician should affect exchange rates, but it is nonetheless important to note that observables do not obviously dominate exchange rate changes.

But perhaps our findings shift the terms of the exchange rate debate. We have shown analytically that if discount factors are large (and fundamentals are  $I(1)$ ), then it may not be surprising that present value models cannot outforecast the random walk model of exchange rates. We have found some support for the link between fundamentals and the exchange rate is in the other direction – exchange rates can help forecast the fundamentals. We tentatively conclude that exchange rates and fundamentals are linked in a way that is broadly consistent with asset pricing models of the exchange rate.

Finally, our analytical results may also help explain the near random walk behavior of other asset prices. It is well known that as a theoretical matter, asset prices follow random walks only under very special circumstances. A priority for future research investigating the power of our results to explain the time series behavior of a variety of asset prices.

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## APPENDIX

In this Appendix, we prove the statement in the text concerning random walk behavior in  $s_t$  as the discount factor  $b \rightarrow 1$ .

We suppose there is an  $(n \times 1)$  vector of fundamentals  $x_t$ . This vector includes all variables, observable as well as unobservable (to the economist), that private agents use to forecast  $f_{1t}, f_{2t}, z_{1t}$ , and  $z_{2t}$ . For example, we may have  $n = 9$ ,  $x_t = (m_t, m_t^*, y_t, y_t^*, v_{mt}, v_{mt}^*, q_t, \rho_t, u_t)'$ ,  $f_t = m_t - m_t^* - (y_t - y_t^*)$ , with  $u_t$  a variable that helps predict one or more of  $m_t, m_t^*, y_t, y_t^*, v_{mt}, v_{mt}^*, q_t$ , and  $\rho_t$ . We assume that  $u_t$  is a scalar only as an example; there may be a set of variables like  $u_t$ . We assume that  $\Delta x_t$  follows a stationary finite order ARMA process (possibly with one or more unit moving average roots – we allow  $x_t$  to include stationary variables, as well as cointegrated I(1) variables.) Let  $\varepsilon_t$  denote the  $(n \times 1)$  innovation in  $\Delta x_t$ , and  $L$  the lag operator,  $Lx_t = x_{t-1}$ . For notational simplicity we assume tentatively that  $\Delta x_t$  has zero mean. Write the Wold representation of  $\Delta x_t$  as

$$(A.1) \quad \Delta x_t = \theta(L)\varepsilon_t = \sum_{j=0}^{\infty} \theta_j \varepsilon_{t-j}, \quad \theta_0 \equiv I.$$

We define  $E_t \Delta x_{t+j}$  as  $E(\Delta x_{t+j} | \varepsilon_t, \varepsilon_{t-1}, \dots)$ , and assume that mathematical expectations and linear projections coincide.

Define the  $(n \times 1)$  vectors  $w_{1t}$  and  $w_{2t}$  as

$$(A.2) \quad w_{1t} = (1-b) \sum_{j=0}^{\infty} b^j E_t x_{t+j}, \quad w_{2t} = b \sum_{j=0}^{\infty} b^j E_t x_{t+j}, \quad w_t = (w'_{1t}, w'_{2t})'.$$

Then  $s_t$  is a linear combination of the elements of the elements of  $w_{1t}$  and  $w_{2t}$ , say

$$(A.3) \quad s_t = a'_1 w_{1t} + a'_2 w_{2t}.$$

for suitable  $(n \times 1)$   $a_1$  and  $a_2$ . We assume that either (a)  $a'_1 w_{1t} \sim I(1)$  and  $a_2 \equiv 0$ , or (b) if  $a_2 \neq 0$ ,  $a'_2 w_{2t} \sim I(1)$  with  $a'_1 w_{1t}$  essentially unrestricted (stationary,  $I(1)$  or identically zero).

We show the following below.

1. Suppose that  $a_2 \equiv 0$  (that is,  $\rho_t = 0$  in the monetary model). Then

$$(A.4) \quad \text{plim}_{b \rightarrow 1} [\Delta s_t - a'_1 \theta(1) \varepsilon_t] = 0.$$

Here,  $\theta(1)$  is an  $(n \times n)$  matrix of constants,  $\theta(1) = \sum_{j=0}^{\infty} \theta_j$ , for  $\theta_j$  defined in (A.1). We note that if  $a'_1 x_t$  were stationary (contrary to what we assume when  $a_2 = 0$ ), then  $a'_1 \theta(1) = 0$ , and (A.3) states that as  $b$  approaches 1,  $s_t$  approaches a constant. But if  $a'_1 x_t$  is  $I(1)$ , as is arguably the case in our data, we have the claimed result: for  $b$  very near 1,  $\Delta s_t$  will behave very much like the unpredictable sequence  $a'_1 \theta(1) \varepsilon_t$ .

2. Suppose that  $a_1 \equiv 0$ ,  $a_2 \neq 0$ . Then

$$(A.5) \quad \text{plim}_{b \rightarrow 1} \{[(1-b)\Delta s_t] - b a'_2 \theta(1) \varepsilon_t\} = 0.$$

By assumption,  $a'_2 x_t \sim I(1)$ , so  $a'_2 \theta(1) \neq 0$ . Then for  $b$  near one,  $(1-b)\Delta s_t$  will behave very much like the unpredictable sequence  $a'_2 \theta(1) \varepsilon_t$ . This means in particular that the correlation of  $(1-b)\Delta s_t$  with any information known at time  $t-1$  will be very near zero. Since the correlation of  $\Delta s_t$  with such information is identical to that of  $(1-b)\Delta s_t$ ,  $\Delta s_t$  will also be almost uncorrelated with such information.

Let us combine (A.4) and (A.5). Then for  $b$  near 1,  $\Delta s_t$  will be approximately uncorrelated with information known at  $t-1$ , since for  $b$  near 1

$$(A.6) \quad \Delta s_t \approx \{a_1 + [ba_2 / (1 - b)]\}' \theta(1) \varepsilon_t .$$

Two comments. First, for any given  $b < 1$ , the correlation of  $\Delta s_t$  with period  $t-1$  information will be very similar for (1)  $x_t$  processes that are stationary, but barely so, in the sense of having autoregressive unit roots near 1, and (2)  $x_t$  processes that are I(1). This is illustrated in the calculations in Table 4.1.

Second, suppose that  $\Delta x_t$  has non-zero mean  $\mu$  ( $n \times 1$ ). Then (A.6) becomes

$$(A.7) \quad \Delta s_t \approx \{a_1 + [ba_2 / (1 - b)]\}' [\mu + \theta(1)] \varepsilon_t .$$

Thus the exchange rate approximately follows a random walk with drift  $\{a_1 + [ba_2 / (1 - b)]\}' \mu$ , if

$$\{a_1 + [ba_2 / (1 - b)]\}' \mu \neq 0 .$$

Proof of A.4:

With elementary rearrangement, we have

$$(A.8) \quad w_{1t} = x_{t-1} + \sum_{j=0}^{\infty} b^j E_t \Delta x_{t+j} .$$

Project (A.8) on period  $t-1$  information and subtract from (A.8). Since  $w_{1t} - E_{t-1} w_{1t} = \Delta w_{1t} - E_{t-1} \Delta w_{1t}$

and  $x_{t-1} - E_{t-1} x_{t-1} = 0$ , we get

$$(A.9) \quad \Delta w_{1t} - E_{t-1} \Delta w_{1t} = \sum_{j=0}^{\infty} b^j (E_t \Delta x_{t+j} - E_{t-1} \Delta x_{t+j}) = \theta(b) \varepsilon_t ,$$

the last equality following from Hansen and Sargent (1981). Next, difference (A.8). Upon rearranging

the right hand side, we get  $\Delta w_{1t} = \sum_{j=0}^{\infty} b^j (E_t \Delta x_{t+j} - b E_{t-1} \Delta x_{t+j})$ . Project upon period  $t-1$  information

and rearrange to get

$$(A.10) \quad E_{t-1}\Delta w_{1t} = (1-b)\sum_{j=0}^{\infty} b^j E_{t-1}\Delta x_{t+j}.$$

From (A.3) (with  $a_2 = 0$ , by assumption), (A.8) and (A.9),

$$(A.11) \quad \Delta s_t = a'_1\theta(b)\varepsilon_t + a'_1(1-b)\sum_{j=0}^{\infty} b^j E_{t-1}\Delta x_{t+j}.$$

Since  $a'_1\Delta x_t$  is stationary,  $a'_1\sum_{j=0}^{\infty} b^j E_{t-1}\Delta x_{t+j}$  converges in probability to a stationary variable as

$b \rightarrow 1$ . Since  $\lim_{b \rightarrow 1} (b-1) = 0$ ,  $(1-b)a'_1\sum_{j=0}^{\infty} b^j E_{t-1}\Delta x_{t+j}$  converges in probability to zero as  $b \rightarrow 1$ .

Hence  $[\Delta s_t - a'_1\theta(b)\varepsilon_t]$  converges in probability to zero, from which (A.2) follows.

Result (A.5) results simply by noting that when  $a_1 \equiv 0$ ,  $(1-b)s_t = a'_2(1-b)b\sum_{j=0}^{\infty} b^j E_t x_{t+j}$ ,

and the argument for (A.2) may be applied to  $(1-b)s_t$ .

Table 2.1

Population Auto- and Cross-correlations of  $\Delta s_t$ 

	(1)	(2)	(3)		(4)	(5)	(6)		(7)	(8)	(9)
					----- Correlation of $\Delta s_t$ with: -----						
	$b$	$\varphi_1$	$\varphi$		$\Delta s_{t-1}$	$\Delta s_{t-2}$	$\Delta s_{t-3}$		$\Delta x_{t-1}$	$\Delta x_{t-2}$	$\Delta x_{t-3}$
(1)	0.50	1.0	0.3		0.15	0.05	0.01		0.16	0.05	0.01
(2)			0.5		0.27	0.14	0.07		0.28	0.14	0.07
(3)			0.8		0.52	0.42	0.34		0.56	0.44	0.36
(4)	0.90	1.0	0.3		0.03	0.01	0.00		0.03	0.01	0.00
(5)			0.5		0.05	0.03	0.01		0.06	0.03	0.01
(6)			0.8		0.09	0.07	0.06		0.13	0.11	0.09
(7)	0.95	1.0	0.3		0.02	0.01	0.00		0.02	0.01	0.00
(8)			0.5		0.03	0.01	0.01		0.03	0.01	0.01
(9)			0.8		0.04	0.04	0.03		0.07	0.05	0.04
(10)	0.90	0.90	0.5		0.04	-0.01	-0.03		0.02	-0.03	-0.05
(11)	0.90	0.95	0.5		0.05	0.01	-0.01		0.04	-0.00	-0.02
(12)	0.95	0.95	0.5		0.02	-0.00	-0.01		0.01	-0.02	-0.03
(13)	0.95	0.99	0.5		0.02	0.01	0.00		0.03	0.01	-0.00

Notes:

1. The model is  $s_t = (1-b) \sum_{j=0}^{\infty} b^j E_t x_{t+j}$  or  $s_t = b \sum_{j=0}^{\infty} b^j E_t x_{t+j}$ . The scalar variable  $x_t$  follows an AR(2) process with autoregressive roots  $\varphi_1$  and  $\varphi$ . When  $\varphi_1 = 1.0$ ,  $\Delta x_t \sim \text{AR}(1)$  with parameter  $\varphi$ .
2. The correlations in columns (4)-(9) were computed analytically. If  $\varphi_1 = 1.0$ , as in rows (1) to (9), then in the limit, as  $b \rightarrow 1$ , each of these correlations approaches zero.

Table 4.1  
Basic Statistics

	(1)	(2)		(3)		(4)		(5)		(6)		(7)	
		Canada		France		Germany		Italy		Japan		U.K.	
		mean (s.d.)	$\rho_1$	mean (s.d.)	$\rho_1$	mean (s.d.)	$\rho_1$	mean (s.d.)	$\rho_1$	mean (s.d.)	$\rho_1$	mean (s.d.)	$\rho_1$
(1):	$\Delta s$	-0.44 (2.20)	-0.03	-0.35 (5.83)	0.10	0.15 (6.06)	0.07	-1.11 (5.79)	0.14	0.76 (6.22)	0.13	-0.44 (5.26)	0.15
(2):	$\Delta(m - m^*)$	-0.56 (2.59)	0.19	0.03 (2.41)	0.25	-0.55 (2.38)	0.28	-1.19 (2.24)	0.28	-0.39 (2.18)	0.46	-1.34 (1.94)	0.54
(3):	$\Delta(p - p^*)$	-0.04 (0.58)	0.47	-0.13 (0.68)	0.62	0.49 (0.77)	0.42	-0.92 (1.17)	0.62	0.50 (0.86)	0.16	-0.54 (1.29)	0.27
(4):	$i - i^*$	-0.92 (1.72)	0.75	-1.89 (3.70)	0.62	2.02 (3.01)	0.84	-4.33 (4.25)	0.66	3.64 (2.78)	0.78	-2.40 (2.88)	0.76
(5):	$\Delta(i - i^*)$	-0.01 (1.21)	-0.39	0.06 (3.23)	-0.37	-0.01 (1.70)	-0.34	0.06 (3.51)	-0.35	-0.04 (1.83)	-0.15	0.06 (2.00)	-0.13
(6):	$\Delta(m - m^*)$ $-\Delta(y - y^*)$	-0.60 (2.65)	0.17	-0.24 (2.59)	0.17	-0.72 (2.92)	0.13	-1.42 (2.35)	0.24	-0.43 (2.54)	0.35	-1.53 (2.19)	0.41
(7):	$\Delta(y - y^*)$	0.04 (0.79)	-0.08	0.21 (0.88)	0.19	0.17 (1.47)	0.08	0.20 (1.01)	0.14	0.04 (1.21)	0.06	0.19 (1.06)	-0.04

Notes:

1. Variable definitions:  $\Delta s$  = percentage change in dollar exchange rate (higher value indicates depreciation). In other variables a “\*” indicates a non-U.S. value, absence of “\*” a U.S. value:  $\Delta m$  = percentage change in M1 (M2 for U.K.);  $\Delta y$  = percentage change in real GDP;  $\Delta p$  = percentage change in consumer prices;  $i$  = short-term rate on government debt. Money and output are seasonally adjusted.

2. Data are quarterly, generally 1974:2-2001:3. Exceptions include an end date of 1998:4 for  $m - m^*$  for France, Germany and Italy, start dates for  $m - m^*$  of 1978:1 for France, 1974:1 for Germany and 1975:1 for Italy, and start dates for  $i - i^*$  of 1975:1 for Canada and 1978:3 for Italy and Japan. See the text.

3. s.d. refers to the standard deviation of the indicated variable.  $\rho_1$  is the first-order autocorrelation coefficient of the indicated variable.

Table 4.2

Bivariate Granger Causality Tests, Different Measures of  $\Delta f_t$   
 Full Sample: 1974:1-2001:3

A. Rejections at 1%(\*\*\*), 5% (\*\*), and 10% (\*) level of  $H_0: \Delta s_t$  fails to cause  $\Delta f_t$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
		Canada	France	Germany	Italy	Japan	U.K.
(1)	$\Delta(m - m^*)$		*		**	**	
(2)	$\Delta(p - p^*)$			***	***	***	
(3)	$i - i^*$		**			**	
(4)	$\Delta(i - i^*)$		**			***	
(5)	$\Delta(m - m^*)$ $-\Delta(y - y^*)$		*		*		
(6)	$\Delta(y - y^*)$						

B. Rejections at 1%(\*\*\*), 5% (\*\*), and 10% (\*) level of  $H_0: \Delta f_t$  fails to cause  $\Delta s_t$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
		Canada	France	Germany	Italy	Japan	U.K.
(1)	$\Delta(m - m^*)$						
(2)	$\Delta(p - p^*)$	*					
(3)	$i - i^*$					**	
(4)	$\Delta(i - i^*)$						
(5)	$\Delta(m - m^*)$ $-\Delta(y - y^*)$						
(6)	$\Delta(y - y^*)$						

Notes:

1. See notes to earlier tables for variable definitions.

2. Statistics are computed from fourth order bivariate vector autoregressions in  $(\Delta s_t, \Delta f_t)'$ . Because four observations were lost to initial conditions, the sample generally is 1975:2-2001:3, with exceptions as indicated in the notes to Table 3.1.

Table 4.3

Bivariate Granger Causality Tests, Different Measures of  $\Delta f_t$   
 Early Part of Sample: 1974:1-1990:2

A. Rejections at 1%(\*\*\*), 5% (\*\*), and 10% (\*) level of  $H_0: \Delta s_t$  fails to cause  $\Delta f_t$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
		Canada	France	Germany	Italy	Japan	U.K.
(1)	$\Delta(m - m^*)$		**		*		
(2)	$\Delta(p - p^*)$			**	***	**	
(3)	$i - i^*$		***				*
(4)	$\Delta(i - i^*)$		***			**	**
(5)	$\Delta(m - m^*)$ $-\Delta(y - y^*)$		**		**		
(6)	$\Delta(y - y^*)$					**	

B. Rejections at 1%(\*\*\*), 5% (\*\*), and 10% (\*) level of  $H_0: \Delta f_t$  fails to cause  $\Delta s_t$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
		Canada	France	Germany	Italy	Japan	U.K.
(1)	$\Delta(m - m^*)$						
(2)	$\Delta(p - p^*)$	*		*			
(3)	$i - i^*$						
(4)	$\Delta(i - i^*)$						
(5)	$\Delta(m - m^*)$ $-\Delta(y - y^*)$						
(6)	$\Delta(y - y^*)$						

Notes:

1. See notes to earlier tables for variable definitions.

Table 4.4

Bivariate Granger Causality Tests, Different Measures of  $\Delta f_t$   
 Later Part of Sample: 1990:3-2001:3

A. Rejections at 1%(\*\*\*), 5% (\*\*), and 10% (\*) level of  $H_0: \Delta s_t$  fails to cause  $\Delta f_t$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
		Canada	France	Germany	Italy	Japan	U.K.
(1)	$\Delta(m - m^*)$		**			***	
(2)	$\Delta(p - p^*)$	*	***	*			
(3)	$i - i^*$			*		**	**
(4)	$\Delta(i - i^*)$			**		**	**
(5)	$\Delta(m - m^*)$ $-\Delta(y - y^*)$		*				**
(6)	$\Delta(y - y^*)$					*	

B. Rejections at 1%(\*\*\*), 5% (\*\*), and 10% (\*) level of  $H_0: \Delta f_t$  fails to cause  $\Delta s_t$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
		Canada	France	Germany	Italy	Japan	U.K.
(1)	$\Delta(m - m^*)$			**		**	
(2)	$\Delta(p - p^*)$		***		**		
(3)	$i - i^*$					***	
(4)	$\Delta(i - i^*)$		**			***	
(5)	$\Delta(m - m^*)$ $-\Delta(y - y^*)$			**		**	
(6)	$\Delta(y - y^*)$		*				

Notes:

1. See notes to earlier tables for variable definitions.

Table 4.5

VAR Causality Tests  
Full Sample: 1974:1-2001:3

Rejections at 1%(\*\*\*), 5% (\*\*), and 10% (\*)  
Null Hypothesis A:  $\Delta s_t$  fails to cause  $\Delta f_t$  jointly  
Null Hypothesis B:  $\Delta f_t$  jointly fail to cause  $\Delta s_t$

VAR	Variables in VAR		Canada	France	Germany	Italy	Japan	U.K.
1	$\Delta(y - y^*), \Delta(p - p^*), i - i^*$	A		*	**	***	***	
		B						
2	$\Delta(y - y^*), \Delta(p - p^*), \Delta(i - i^*)$	A		**	*	***	***	
		B						
3	$\Delta(m - m^*), \Delta(y - y^*)$	A		**				
		B						
4	$\Delta(m - m^*), \Delta(y - y^*), \Delta(p - p^*)$	A		**	*	***	*	
		B						
5	$\Delta(y - y^*), \Delta(p - p^*)$	A			**	***		
		B						

Notes:

1. See notes to earlier tables for variable definitions

Table 4.6

Correlation between  $\Delta s_t$  and  $\Delta F_t$ A. Discount factor  $b = 0.5$ 

	(1)	(2)	(3)	(4)	(5)	(6)
		Info Set	France	Germany	Italy	Japan
(1)	$\Delta(m - m^*)$	(a) $F_{1t}$	-0.02 (-0.24, 0.16)		-0.13 (-0.32, 0.06)	0.24 (0.08, 0.37)
		(b) $F_{2t}$	0.10 (-0.14, 0.31)		-0.05 (-0.26, 0.17)	0.23 (0.05, 0.38)
		(c) $F_{2t} - F_{1t}$	0.13 (0.05, 0.36)		0.08 (0.02, 0.29)	-0.01 (-0.08, 0.16)
(2)	$\Delta(p - p^*)$	(a) $F_{1t}$		-0.03 (-0.20, 0.14)	0.19 (-0.03, 0.34)	-0.21 (-0.35, -0.06)
		(b) $F_{2t}$		0.10 (-0.09, 0.28)	0.27 (0.04, 0.44)	-0.13 (-0.29, 0.06)
		(c) $F_{2t} - F_{1t}$		0.13 (0.07, 0.28)	0.08 (0.02, 0.24)	0.09 (0.03, 0.25)
(3)	$\Delta(i - i^*)$	(a) $F_{1t}$	-0.21 (-0.38, -0.03)			-0.05 (-0.25, 0.13)
		(b) $F_{2t}$	-0.07 (-0.27, 0.13)			0.13 (-0.09, 0.34)
		(c) $F_{2t} - F_{1t}$	0.14 (0.07, 0.34)			0.18 (0.11, 0.42)
(4)	$\Delta(m - m^*)$ $-\Delta(y - y^*)$	(a) $F_{1t}$	-0.01 (-0.23, 0.17)		-0.10 (-0.28, 0.07)	
		(b) $F_{2t}$	0.10 (-0.15, 0.31)		-0.05 (-0.25, 0.16)	
		(c) $F_{2t} - F_{1t}$	0.11 (0.03, 0.33)		0.05 (-0.01, 0.31)	

B. Discount factor  $b = 0.9$

	(1)	(2)	(3)	(4)	(5)	(6)
		Info Set	France	Germany	Italy	Japan
(1)	$\Delta(m - m^*)$	(a) $F_{1t}$	-0.05 (-0.24, 0.12)		-0.13 (-0.31, 0.05)	0.19 (0.04, 0.33)
		(b) $F_{2t}$	0.25 (-0.15, 0.55)		-0.03 (-0.33, 0.34)	-0.05 (-0.32, 0.24)
		(c) $F_{2t} - F_{1t}$	0.30 (0.05, 0.89)		0.10 (-0.07, 0.69)	-0.24 (-0.41, 0.30)
(2)	$\Delta(p - p^*)$	(a) $F_{1t}$		-0.01 (-0.18, 0.16)	0.17 (-0.03, 0.34)	-0.17 (-0.31, -0.02)
		(b) $F_{2t}$		0.49 (0.19, 0.68)	0.51 (0.18, 0.71)	0.31 (0.00, 0.53)
		(c) $F_{2t} - F_{1t}$		0.50 (0.32, 0.81)	0.34 (0.16, 0.71)	0.47 (0.28, 0.84)
(3)	$\Delta(i - i^*)$	(a) $F_{1t}$	-0.21 (-0.39, -0.03)			-0.06 (-0.27, 0.12)
		(b) $F_{2t}$	0.15 (-0.19, 0.45)			0.54 (0.19, 0.75)
		(c) $F_{2t} - F_{1t}$	0.37 (0.15, 0.86)			0.60 (0.41, 0.95)
(4)	$\Delta(m - m^*)$ $-\Delta(y - y^*)$	(a) $F_{1t}$	-0.04 (-0.23, 0.14)		-0.10 (-0.28, 0.06)	
		(b) $F_{2t}$	0.23 (-0.17, 0.53)		-0.04 (-0.34, 0.31)	
		(c) $F_{2t} - F_{1t}$	0.27 (0.02, 0.78)		0.06 (-0.11, 0.67)	

Notes

<sup>1</sup>  $F_{1t}$  and  $F_{2t}$  are the expected discounted value of fundamentals, computed using lagged fundamentals alone ( $F_{1t}$ ) or lagged fundamentals and lagged exchange rates ( $F_{2t}$ ).

<sup>2</sup> The point estimates are the correlation between the change in the estimates of the expected present discounted values and the change in the actual exchange rate. They may be interpreted as correlations between fitted and actual values.

<sup>3</sup>The numbers in parentheses are 90 percent confidence intervals, computed from a nonparametric bootstrap

APPENDIX: VAR Causality Tests

(Not intended for publication)

Multivariate Granger Causality Tests, Different Measures of  $\Delta f_t$   
Full Sample: 1974:1-2001:3

Table A.1

A. Rejections at 1% (\*\*\*) , 5% (\*\*), and 10% (\*) level of  $H_0$ :  $\Delta s_t$  fails to cause  $\Delta f_t$

B. Rejections at 1% (\*\*\*) , 5% (\*\*), and 10% (\*) level of  $H_0$ :  $\Delta f_t$  fails to cause  $\Delta s_t$

	(1)	(2)	(2)	(3)	(4)	(5)	(6)	(7)
		Test Performed	Canada	France	Germany	Italy	Japan	U.K.
(1)	$\Delta(y - y^*)$	A. B.						
(2)	$\Delta(p - p^*)$	A. B.	*		***	**		
(3)	$i - i^*$	A. B.		**			** *	
(4)	All variables	A. B.		*	**	***	***	

Notes:

1. See notes to earlier tables for variable definitions.
2. Statistics are computed from 4th order vector autoregressions in  $(\Delta s_t, \Delta(y_t - y_t^*), \Delta(p_t - p_t^*), i_t - i_t^*)'$ .
3. "All variables" refers to the hypothesis that  $(\Delta(y_t - y_t^*), \Delta(p_t - p_t^*), i_t - i_t^*)'$  jointly fail to cause  $\Delta s_t$ .

Table A.2

A. Rejections at 1% (\*\*\*) , 5% (\*\*), and 10% (\*) level of  $H_0: \Delta s_t$  fails to cause  $\Delta f_t$

B. Rejections at 1% (\*\*\*) , 5% (\*\*), and 10% (\*) level of  $H_0: \Delta f_t$  fails to cause  $\Delta s_t$

	(1)	(2)	(2)	(3)	(4)	(5)	(6)	(7)
		Test Performed	Canada	France	Germany	Italy	Japan	U.K.
(1)	$\Delta(y - y^*)$	A. B.						
(2)	$\Delta(p - p^*)$	A. B.			***	**		
(3)	$\Delta(i - i^*)$	A. B.		**		*	***	
(4)	All variables	A. B.		**	*	***	***	

Notes:

1. See notes to earlier tables for variable definitions.

2. Statistics computed from 4th order vector autoregressions in  $(\Delta s_t, \Delta(y_t - y_t^*), \Delta(p_t - p_t^*), \Delta(i_t - i_t^*))'$ .

Table A.3

A. Rejections at 1% (\*\*\*) , 5% (\*\*), and 10% (\*) level of  $H_0: \Delta s_t$  fails to cause  $\Delta f_t$

B. Rejections at 1% (\*\*\*) , 5% (\*\*), and 10% (\*) level of  $H_0: \Delta f_t$  fails to cause  $\Delta s_t$

	(1)	(2)	(2)	(3)	(4)	(5)	(6)	(7)
		Test Performed	Canada	France	Germany	Italy	Japan	U.K.
(1)	$\Delta(m - m^*)$	A. B.				*	**	
(2)	$\Delta(y - y^*)$	A. B.	*					
(4)	All variables	A. B.		**				

Notes:

1. See notes to earlier tables for variable definitions.

2. Statistics computed from 4th order vector autoregressions in  $(\Delta s_t, \Delta(m_t - m_t^*), \Delta(y_t - y_t^*))'$ .

Table A.4

A. Rejections at 1% (\*\*\*) , 5% (\*\*), and 10% (\*) level of  $H_0: \Delta s_t$  fails to cause  $\Delta f_t$

B. Rejections at 1% (\*\*\*) , 5% (\*\*), and 10% (\*) level of  $H_0: \Delta f_t$  fails to cause  $\Delta s_t$

	(1)	(2)	(2)	(3)	(4)	(5)	(6)	(7)
		Test Performed	Canada	France	Germany	Italy	Japan	U.K.
(1)	$\Delta(m - m^*)$	A. B.				**	*	
(2)	$\Delta(y - y^*)$	A. B.	*					
(3)	$\Delta(p - p^*)$	A. B.	*		***	***	**	
(4)	All variables	A. B.		**	*	***	*	

Notes:

1. See notes to earlier tables for variable definitions.

2. Statistics computed from 4th order VAR in  $(\Delta s_t, \Delta(m_t - m_t^*), \Delta(y_t - y_t^*), \Delta(p_t - p_t^*))'$ .

Table A.5

A. Rejections at 1% (\*\*\*) , 5% (\*\*), and 10% (\*) level of  $H_0: \Delta s_t$  fails to cause  $\Delta f_t$

B. Rejections at 1% (\*\*\*) , 5% (\*\*), and 10% (\*) level of  $H_0: \Delta f_t$  fails to cause  $\Delta s_t$

	(1)	(2)	(2)	(3)	(4)	(5)	(6)	(7)
		Test Performed	Canada	France	Germany	Italy	Japan	U.K.
(1)	$\Delta(p - p^*)$	A. B.			***	***	**	
(2)	$\Delta(y - y^*)$	A. B.						
(4)	All variables	A. B.			**	***		

Notes:

1. See notes to earlier tables for variable definitions.

2. Statistics computed from 4th order vector autoregressions in  $(\Delta s_t, \Delta(p_t - p_t^*), \Delta(y_t - y_t^*))'$ .