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ON CHOOSING A FLAT-RATE INCOME TAX SCHEDULE

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ABSTRACT

This paper applies a numerical optimization technique using microunit tax data to the problem of choosing the parameters of a flat-rate tax system, should one be desired. Our approach is to first formulate explicit objectives that a flat-rate tax might reasonably be designed to meet, such as minimizing the extent of changes in households' tax burdens and minimizing the efficiency cost of the tax system. The next step uses an optimization algorithm to calculate the flat-rate schedule which comes closest to meeting the objectives, subject to the constraint that it raise the same revenue as the current income tax system. The calculations are carried out using a sample of 947 tax returns randomly drawn from the Treasury Tax File for 1977 which are updated to reproduce the pattern of tax returns that would be filed in 1982.

The analysis shows that the flat-rate system which minimizes the sum of the absolute deviations in tax liabilities features a marginal tax rate between 0.204 and 0.254, though a different definition of tax burden changes which puts more emphasis on reproducing the tax burdens of high-income households has an optimal marginal tax rate of 0.382. We also derive the optimal flat-rate schedules when another objective is to minimize the efficiency cost of the tax system.

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On Choosing a Flat-Rate Income Tax Schedule

This paper is an exercise in applied optimal tax reform. It applies a numerical optimization technique using microunit tax data to the problem of choosing a flat-rate income tax system, should one be desired. Although analytic models of the optimal linear income tax system abound in the literature[1], these models make simplifying assumptions about the distribution of income and do not consider the distribution of the array of potential deductions and exclusions that exist in the current U.S. income tax system. The virtue of the optimization technique, first introduced by Yitzhaki (1982, 1983) is that it can solve a much more general problem using more realistic and detailed information about incomes and the tax system. This research is an exercise in optimal tax reform rather than optimal tax design because we maintain the assumption that one of the desirable characteristics of a flat-rate system is a minimum of change in the existing distribution of tax burdens.[2]

The essence of the flat-rate tax is simple. Each taxpaying unit is allowed some level of exemption, probably associated with the number of persons in the unit, and perhaps dependent on the age or marital status of the unit. Income above the exemption level is subject to a cnstant tax rate. Although recent flat-rate tax proposals differ with regard to the extent to which deductions from gross income in addition to the exemption level mentioned above will be permitted, they all tend to reduce the number of deductions from what is currently allowed.

Our goal here is not to assess the desirability of substituting a flat-rate income tax for the present tax system. Rather, our more limited objective is to shed light on some of the issues involved in choosing the rate structure of a flat-rate tax system, should one be desired. This is not a trivial question since an infinite number of different flat-rate tax systems can be constructed which will yield the same amount of total revenue. There are three aspects which can be altered: (i) the level of personal exemptions, (ii) the (constant) marginal tax rate and

(iii) the extent of deductions from taxable income allowed. The lower is the exemption level and the extent of deductions allowed, the lower is the marginal tax rate that is needed to raise a given amount of total revenue. It is the tradeoff between these aspects of a flat-rate tax that will be explored here.

Although our purpose is not to assess the merits of a flat-rate tax system, nevertheless it will be helpful for discriminating among alternative systems to briefly discuss the issues that have been raised in connection with moving to such a system. These issues may be grouped into three categories: administrative/compliance costs, efficiency/incentive effects, and the distribution of tax burdens.

It has been argued that a flat-rate tax would be simpler for the government to administer and less costly for taxpayers to comply with. Even if the often-heard claim that the tax form could fit on a postcard is not true, a significant decrease in administrative and compliance costs could be expected to the extent that the range of exclusions, deductions, and credits is reduced. The cost savings do not seem to be related to the rate schedule itself, as this is a small step in the calculation of tax liability.

To the extent that a flat-rate tax system is accompanied by a lower marginal tax rate, it is argued that there will be increased work effort and saving, as well as less activity in the underground (untaxed) economy and tax shelter investments which are desirable only because of their peculiar tax treatment. One must be careful here, though, because as will be clear later in this paper, many flat-rate tax plans entail increased marginal tax rates for middle-income individuals. In this case, incentive effects at the ends of the income scale must be balanced against disincentive effects at the middle.

Changing to a flat-rate tax system would cause some households' tax burdens to rise, and others' to fall. Who would gain and who would lose would depend on the particular flat-rate system chosen, the household's income level and the extent to which the household took advantage of the current tax system's deductions, exemptions, and exclusions. The changes in tax liability would affect people in the same income group differently. For example, a disallowance of the interest paid deduction would affect homeowners more severely than renters. Whether such "horizontal" changes in tax burden are desirable depends to some extent on whether the current system of preferences is judged to be desirable.

The changes in tax liability would also depend systematically on the income level of the household. Depending on the parameters of the flat-rate tax system, the burden of raising revenue could be shifted from higher-income to lower-income households, from both higher-income and lower-income households to the middle class, or according to some other pattern. As was true for horizontal changes in tax burden, one's judgment on these "vertical" changes in the distribution of tax burdens depends on how equitable the distribution of burdens under the replaced tax system is seen to be.

Our approach to choosing a flat-rate income tax schedule is to first formulate explicit objectives that a flat-rate tax might reasonably be designed to meet. The next step uses an optimization algorithm to calculate the flat-rate schedule which comes closest to meeting the objectives, subject to the constraint that it raise the same revenue as the current income tax system. The calculations are carried out using a sample of 947 tax returns randomly drawn from the Treasury Tax File for 1977.[3] Though the data refer to tax year 1977, they have been updated to reproduce the pattern of tax returns that would be filed in 1982.[4] The analysis also uses a procedure that can apply the 1982 tax law and any alternative tax rules, such as a flat-rate tax, to individual records like the Treasury Tax File.[5] The primary advantage of the optimization procedure is that it finds the tax system that best meets any particular definition of its objectives. Its principal disadvantage is that the answers it produces are only as good as the objectives that are proposed. The calculated optimum will reflect the extent that certain objectives are not readily quantifiable or easily comparable to other possible objectives.

I. Which Flat-Rate Tax System Most Closely Approximates the Current Distribution of Tax Burden?

One concern about a flat-rate tax system is that it might significantly redistribute the burden of tax payments among households. Our methodology is well-suited to calculate precisely which flat-rate tax schedule would minimize such changes in tax burden.[6] We require an explicit expression for the degree of redistribution of tax liability. A natural class of candidates is given by

$$(1) \qquad \sum_i Q_i \left|T_i^C - T_i^F\right|^\alpha$$

$$\text{where} \quad Q_i = \begin{cases} Q^D & \text{if } T_i^F \leq T_i^C \\ Q^I & \text{if } T_i^F > T_i^C \end{cases}$$

where T_i^C and T_i^F refer to household i's current tax liability and liability under a flat-rate tax, respectively. T_i^C is calculated by using the existing tax law while T_i^F depends on the particular form of the flat-rate tax system to be investigated, especially what level and what form deductions, exclusions, and exemptions take. An alternative class of measures of the change in tax liabilities is

$$(2) \qquad \sum_i Q_i \left|(T_i^C - T_i^F)/Y_i\right|^\alpha$$

where Y_i is income.

Several natural possibilities can be illustrated by expressions (1) and (2). Consider first the case where Q^D equals Q^I and α is unity. Then expression (1) is equal to Q^D multiplied by the sum of the absolute values of the changes in tax liability incurred by households due to instituting a flat-rate tax system. Expression (2) measures the sum of the changes in the average tax rates incurred by households. Note that when α is one and Q^D equals Q^I, it can be shown that the optimal flat-rate system will always result in exactly half the taxpayers facing a tax increase and half facing a tax decrease. Furthermore, exactly half of the total absolute deviation in total or average tax liabilities will come from tax increases and the other half will be due to tax reductions.

Setting Q^I to be greater than Q^D means that increases in a household's tax liability or average tax rate are considered more serious than decreases. In the extreme case where Q^D is zero, the objective is to minimize the sum of the increases in tax liabilities, or in expression (2), average tax rates. A value of α greater than one penalizes large changes in tax liability more than proportionately greater than small changes. For example, if α is two and Q^D is zero, expression (1) implies that a situation where a household faces a tax increase of \$200 should be considered four times as serious as a case where a household is faced with a \$100 tax increase.

Given a particular form of expression (1) or (2), the optimizing procedure finds the flat-rate tax system parameters that minimize the objective function while at the same time raising the same amount of revenue that would have been raised in 1982 under the current income tax system.[7] We have to specify the class of flat-rate tax systems we want to optimize over. The three general forms we consider are

$$(3) \qquad T^F = \max(-a - bX_i + tY_i \, , 0)$$

$$(4) \qquad T_i^F = \max(-\ a + tY_i\ ,\ 0)$$

$$(5) \qquad T_i^F = \max(-\ bX_i + tY_i\ ,\ 0)$$

where X_i refers to the number of personal exemptions currently allowed to the taxpaying unit and Y_i is a measure of income. Form (3) allows a credit of value a to each taxpaying unit, plus an additional credit of b for each exemption. The flat-rate tax represented by (4) eliminates the per-exemption credit, allowing only a credit for each taxpaying unit. Type (5) does away with the per-unit credit, leaving only a credit for each exemption. This latter system has the advantage of completely eliminating any marriage tax or subsidy, since the value of the exemptions are the same regardless of whether one or two returns are filed. Although these three systems feature credits, they are exactly equivalent to systems which have deductions and exemptions from taxable income. For example, the tax liability under system (3) can also be written as $\max\{t(Y_i - (a + bX_i)/t),\ 0\}$. In all cases the equivalent deduction can be obtained by dividing the credit by t. The tax liability is zero until a break-even level of income is reached, which is $(a + bX_i)/t$ in system (3), a/t in system (4), and bX_i/t in system (5). Once the break-even level is reached, all income is taxed at rate t.

Most flat-rate tax proposals include a broadening of the tax base, though they differ on the extent of the broadening. In these exercises we consider two possible tax bases, adjusted gross income and what we call extended income, which is adjusted gross income plus the adjustments to income and the excluded portion of realized capital gains.

Each optimization exercise must specify a specific objective function to minimize and a specific flat-rate tax system, which includes a choice of form (3), (4), or (5) and one of two definitions of the tax base. Because there are so many possible combinations of assumptions, we have chosen to concentrate our attention on the cases where α is one and Q^I equals Q^D. For convenience we set Q^I to be one. Thus we find the flat-rate tax schedule which minimizes either the sum of the changes in the tax liabilities or else the sum of changes in average tax rates. With two different targets, three different flat-rate systems, and two bases, there are twelve different optimization problems. An example of the exact problem that is solved is

$$\underset{a,b,t}{\text{Minimize}} \sum_i \left| T_i^C - \max(-a - bX_i + tY_i , 0)\right|$$

$$\text{subject to } \sum_i T_i^C = \sum_i \max(-a - bX_i + tY_i , 0) .$$

The results of several of these optimizations are presented in Table 1. Except for case 8, which we will discuss later, the flat-rate schedule which minimizes changes in current tax burdens has a marginal rate between 0.204 and 0.254, regardless of the minimand definition, type of flat-rate system, or tax base. The zero-tax level of income does vary somewhat for different systems, depending on the number of allowable exemptions. For example, a family of four with no special exemptions has a zero-tax income level of $11,417 in Case 1, $7,318 in Case 3, and $11,900 in Case 4. In Cases 1 and 4 the zero-tax level increases with family size, so it favors large families; the system of Case 3, with a $7,318 zero-tax income level for all taxpaying units favors single households and small families.

The effect of expanding the tax base can be seen by comparing Case 1 with Case 2. A large part of the additional base in extended income is the excluded part of realized capital gains, which accrue disproportionately to upper-income households. With an expanded base, the optimal tax table is apparently less progressive, featuring a lower marginal tax rate.

A comparison of Cases 1, 3, and 4 and also Cases 5, 6, and 7 highlights the relevance of the tax system chosen. Flat-rate system (3), which allows both a credit for each taxpaying unit and a credit for each allowable exemption dominates either system (4) or system (5), which have one credit or the other, in terms of minimizing changes in tax burden. This should not be surprising, because system (3) has three instruments compared to two for the other systems. If one had to choose between a credit per taxpaying unit or a credit per exemption, the results of Table 1 offer ambiguous counsel. If the minimand is the sum of absolute deviations, then the credit per exemption is superior to the credit per taxpaying unit (compare Cases 3 and 4). If, however, the minimand is the sum of deviations of tax rates, then the credit per taxpaying unit is slightly superior (compare Cases 6 and 7).

Finally, by comparing Case 1 with Case 5, Case 3 with Case 6, and Case 4 with Case 7, we can assess the impact of different definitions of the changes in tax burden resulting from instituting a flat-rate income tax.[8] In the first two situations, measuring the change in tax burdens using average tax rates instead of absolute tax liabilities implies that the optimal schedule is less progressive; in the third case, it is slightly more progressive. In no case does the optimal marginal income tax rate change more than 0.032 when the minimand changes. This relative insensitivity to the choice of target definition does not apply generally, as Case 8 makes clear. If the measure of tax burden changes is the sum of squared differences in tax liability ($\alpha=2$), then the best flat-rate system in the case of system (3) with a tax base of AGI is characterized by a marginal tax rate of 0.382. The much higher rate results because this measure of tax changes penalizes the large reductions in tax liability that many high-income individuals would get. Thus the optimal flat-rate tax in this case is more successful in reproducing the current tax liabilities of the high-income taxpayers, which requires a more progressive flat-rate tax.

Just looking at the sum of the absolute or squared deviations from current tax burdens does not tell us how the tax burden changes are distributed through the population. To see this, we must look at a disaggregated breakdown of tax liabilities. As an example, we will focus on Case 1 of Table 1. This exercise finds the flat-rate tax system with both a unit credit and a credit per exemption, which uses AGI as the tax base, and which minimizes the sum of absolute deviations in tax liabilities. The best such schedule has a credit per taxpaying unit of $944, a credit per exemption of $489, and a marginal tax rate of 0.254.

Table 2 shows the changes in the average tax rate that would result from shifting to this flat-rate tax schedule, arranged by AGI class.[9] Each cell is the fraction of the taxpaying units in the AGI class that would experience a particular change in their average tax rate. First of all, notice that fifty-two percent of all taxpaying units would have a minimal change in their average tax liability, meaning no more than a two percentage point change one way or the other. The percentage of households that benefit significantly (a decrease of more than two percentage points) is relatively small for the lowest income group, because most households in this group don't pay much tax under either tax system. From then on the percentage of significant gainers is U-shaped: forty-seven percent of the $5,000 - $10,000 class gain significantly, twenty-six percent of the $10,000 - $15,000 group gain significantly, but less than ten percent in the $20,000 - $40,000 groups would find their average tax rates reduced by more than two percentage points. The number of significant gainers increases to fifteen percent in the $40,000 - $50,000 class, forty-five percent in the $50,000 - $100,000 class, eighty-six percent of the $100,000 - $200,000 class, and peaks at ninety percent of the over $200,000 class. Conversely, the percentage of households which would experience significant increases in tax burden has an inverted U shape, peaking a fifty-eight percent in the $25,000 - $30,000 group, and being low at either end of the income spectrum. Notice, though, that there is a significant amount of dispersion in the impact of a flat-rate tax within income classes. For example in the $50,000 - $100,000 group, twenty-one percent of the households would experience a reduction of five percent or more in their

TABLE 1

CHARACTERISTICS OF OPTIMAL FLAT-RATE INCOME TAX SCHEDULES

Case	Form of Minimand	Flat-Rate System	Tax Base	a ($)	b ($)	t	Value of Minimand*	Efficiency Cost** ($ billions)
1	1	3	A.G.I.	944	489	.254	61.6	19.6
2	1	3	E.I.	886	424	.236	67.2	16.9
3	1	4	A.G.I.	1705		.233	78.7	16.6
4	1	5	A.G.I.		712	.239	73.1	17.6
5	2	3	A.G.I.	802	429	.242	2.21	18.0
6	2	4	A.G.I.	991		.204	2.93	13.1
7	2	5	A.G.I.		771	.244	3.06	18.5
8	1'	3	A.G.I.	3099	1077	.382	948329.	39.1

Form of Minimand:

1. $\sum_i |T_i^C - T_i^F|$

1'. $\sum_i (T_i^C - T_i^F)^2$

2. $\sum_i \left|\frac{T_i^C}{Y_i} - \frac{T_i^F}{Y_i}\right|$

Flat-Rate System:

3. Tax = Max $(-a - bX_i + tY_i, 0)$

4. Tax = Max $(-a + tY_i, 0)$

5. Tax = Max $(-bX_i + tY_i, 0)$

Tax Base: A.G.I.; adjusted gross income

E.I.; extended income (defined in text)

* The unit of measurement for Cases 1-4 is billions of dollars, for Cases 5-7 it is millions, and for Case 8 is billions of dollars squared. The results are only comparable among cases which use the same minimand.

** Defined in Section II.

TABLE 2

DISTRIBUTION OF CHANGES IN AVERAGE TAX LIABILITY DUE TO CHANGING TO A FLAT-RATE INCOME TAX SYSTEM -- BY INCOME CLASS

Adjusted Gross Income ($1000)	Number of Returns (Millions)	Change in Average Tax Rate (Tax Liability Divided by AGI): Decreases in Tax							Increases in Tax			
		Less than -.20	-.15 to -.20	-.10 to -.15	-.05 to -.10	-.02 to -.05	+.02 to -.02	+.05 to +.02	+.10 to +.05	+.15 to +.10	+.20 to +.15	More than +.20
0 - 5	16.7	.000	.001	.002	.023	.148	.729	.006	.086	.004	.000	.001
5 - 10	14.5	.000	.003	.009	.130	.332	.456	.032	.039	.000	.000	.000
10 - 15	12.2	.001	.000	.012	.046	.201	.636	.062	.027	.015	.000	.000
15 - 20	9.4	.000	.000	.006	.041	.079	.556	.221	.073	.022	.002	.000
20 - 25	8.4	.000	.000	.001	.013	.051	.442	.364	.109	.012	.005	.002
25 - 30	7.9	.000	.000	.001	.012	.020	.393	.403	.159	.013	.001	.000
30 - 40	10.4	.000	.000	.005	.013	.065	.412	.327	.166	.009	.002	.001
40 - 50	4.9	.000	.000	.003	.028	.120	.468	.255	.115	.007	.002	.002
50 - 100	4.4	.003	.005	.029	.173	.235	.355	.131	.047	.013	.003	.005
100 - 200	0.65	.034	.104	.298	.323	.104	.051	.026	.020	.015	.008	.016
More than 200	0.16	.175	.346	.251	.099	.032	.026	.027	.016	.011	.006	.012
TOTAL	89.5	.001	.002	.009	.052	.150	.523	.167	.086	.010	.001	.001

average tax rates, while at the same time more than six percent would have an increase in their tax rate of more than five percent. The dispersion is due to the fact that the high income households differ substantially in the amount of deductions they take. Thus while for a given income class the average tax rate on taxable income does not vary much under the current system, the average tax rate as a fraction of adjusted gross income does vary quite a bit. In this case switching to a system where the fraction of AGI that tax liability comprises is virtually fixed hurts some households and helps others.

In Table 3, the distribution of the change in average tax liability is again shown, but this time the rows group households according to their average tax rate under the current system. The large majority of households are situated along the diagonal running from top right to lower left. What this implies is that people who were paying low average tax rates tend to face higher taxes under the flat system, and those who were paying high average tax rates receive a tax reduction. Comparison of Tables 2 and 3 suggests that the proper interpretation of the redistributional impact of switching to a flat-rate tax depends on how one defines rich and poor. As Table 3 shows, if by rich we mean those that currently pay high tax rates as a fraction of AGI, then a flat-rate tax substantially reduces the tax paid by the rich in an unambiguous manner. Table 2 makes clear, though, that among those with high AGI, there is significant dispersion in the average rate of tax paid, due to greatly varying use of deductions. If AGI is the index of income, then the rich still tend to gain under a flat-rate tax, but not to the universal extent that Table 3 would indicate.[10]

Table 4 presents the disaggregated breakdown of how marginal tax rates are changed by switching to the flat-rate tax of Case 1. There are only two possible marginal tax rates under the flat-rate tax: zero, if one's credits are sufficient to offset any tax liability on income, or the one positive marginal rate, which in this case is 0.254. Because the credits under the flat-rate system

TABLE 3

DISTRIBUTION OF CHANGES IN AVERAGE TAX LIABILITY DUE TO CHANGING TO A FLAT-RATE INCOME TAX SYSTEM -- BY CURRENT AVERAGE TAX RATE

Current Average Tax Rate	Number of Returns (Millions)	Change in Average Tax Rate (Tax Liability Divided by AGI): Decreases in Tax							Increases in Tax			
		Less than -.20	-.15 to -.20	-.10 to -.15	-.05 to -.10	-.02 to -.05	+.02 to -.02	+.05 to +.02	+.10 to +.05	+.15 to +.10	+.20 to +.15	More than +.20
Less than .05	25.1	.000	.000	.000	.000	.239	.605	.031	.097	.020	.005	.004
.05 - .10	18.2	.000	.000	.000	.139	.235	.399	.103	.108	017	.001	.000
.10 - .15	24.4	.000	.000	.012	.016	.017	.490	.334	.129	.002	.000	.000
.15 - .20	14.1	.000	.003	.003	.020	.024	.648	.289	.011	.000	.000	.000
.20 - .25	5.2	.000	.000	.003	.046	.328	.619	.003	.000	.000	.000	.000
.25 - .30	1.6	.001	.000	.004	.544	.447	.003	.000	.000	.000	.000	.000
.30 - .35	0.59	.000	.005	.385	.611	.000	.000	.000	.000	.000	.000	.000
.35 - .40	0.26	.056	.180	.763	.000	.000	.000	.000	.000	.000	.000	.000
More than .40	0.17	.628	.372	.000	.000	.000	.000	.000	.000	.000	.000	.000
TOTAL	89.5	.001	.002	.009	.052	.150	.523	.167	.086	.010	.001	.001

TABLE 4

DISTRIBUTION OF CHANGES IN MARGINAL TAX LIABILITY DUE TO CHANGING TO A FLAT-RATE INCOME TAX SYSTEM -- BY INCOME CLASS

Adjusted Gross Income ($1000)	Number of Returns (Millions)	Change in Marginal Tax Rate: Decreases in Tax							Increases in Tax			
		Less than -.20	-.15 to -.20	-.10 to -.15	-.05 to -.10	-.02 to -.05	+.02 to -.02	+.05 to +.02	+.10 to +.05	+.15 to +.10	+.20 to +.15	More than +.20
0 - 5	16.7	.000	.000	.258	.007	.016	.622	.004	.089	.004	.000	.000
5 - 10	14.5	.104	.070	.191	.006	.022	.068	.000	.481	.053	.000	.005
10 - 15	12.2	.000	.008	.102	.000	.002	.017	.281	.548	.029	.000	.011
15 - 20	9.4	.000	.011	.003	.042	.016	.201	.252	.436	.025	.003	.011
20 - 25	8.4	.000	.003	.019	.224	.056	.169	.387	.119	.011	.002	.009
25 - 30	7.9	.000	.003	.022	.159	.226	.416	.132	.034	.003	.002	.000
30 - 40	10.4	.006	.022	.100	.261	.437	.129	.030	.014	.000	.001	.000
40 - 50	4.9	.032	.041	.560	.269	.082	.008	.003	.001	.000	.003	.000
50 - 100	4.4	.492	.187	.280	.027	.006	.001	.001	.001	.000	.004	.001
100 - 200	0.65	.953	.011	.004	.001	.002	.018	.002	.004	.000	.002	.002
More than 200	0.16	.941	.004	.003	.011	.002	.023	.000	.001	.000	.011	.004
TOTAL	89.5	.052	.026	.153	.088	.089	.219	.117	.231	.017	.001	.005

are more generous than under the current tax system, substantial numbers of taxpayers would find their marginal tax rate reduced (to zero) under a flat-rate system. However, since the marginal rate of 0.254 is higher than the rate currently applicable to many low-income taxpayers, many others would find their marginal rate to be increased. The bottom row of Table 4 indicates that about twenty-two percent of all taxpayers would experience a small (between +0.02 and -0.02) change in marginal tax rates. Of those that would experience a significant change in their marginal tax rate, about half (forty-one percent overall) would have a decline, and half (thirty-seven percent overall) would see an increase. The U-shaped distributional pattern is evident here as it was in Table 2. It is the middle income groups that for the most part would face higher marginal rates. Many with very low income and the majority of high income taxpayers would face a lower marginal tax rate.

It is worth emphasizing that these changes in the vertical distribution of tax liabilities occur even though the flat-rate system under study is the best one that can be designed in terms of minimizing the particular definition of tax burden changes. Thus they are not the result of an imprecise choice of the flat-rate system's parameters. Of course, a new minimand could be designed that would include a measure of the redistribution of the tax burdens among broadly defined income groups. Given the constraints of the flat-rate system, though, it is not clear in which direction this would change the optimal flat-rate parameters.

All of the optimization exercises discussed so far have been done with the implicit assumption that there is no behavioral response caused by the change to a flat-rate tax. If, however, the change was accompanied by a large increase in labor supply, for example, then the increased tax base would make possible either a lower marginal tax rate or higher credits (or both) than these exercises indicated. The results presented in Table 4, though, indicate that the magnitude of any induced behavioral response may be very small because there are as many households who would face a higher marginal tax rate as would face a lower one. To check that impression, we performed the following simple calculation. Assume that labor supply responds only to its net-of-tax price, and that the wage rate does not adjust when the tax system changes. Assume also that the (uncompensated) elasticity of response of labor supply is 0.2. Given these assumptions we can approximate the percentage increase in labor income due to switching to a flat-rate tax system as one hundred times

$$(6) \qquad \frac{\sum_i W_i \{(1 - t^F_{mi})/(1 - t^C_{mi})\}^{.2} - \sum_i W_i}{\sum_i W_i},$$

where W_i refers to the ith person's current labor income, and t^F_{mi} and t^C_{mi} are the marginal tax rates on labor income under a flat-rate system and the current tax system, respectively. For the flat-rate tax system of Case 1, expression (6) equals 0.0186. That is, only a 1.9 percent increase in labor income could be expected. The expected increases for the other flat-rate systems are also not large. For this reason, we have chosen not to incorporate behavioral responses directly into the optimizations. However, even in the absence of large behavioral response the efficiency implications of changing marginal tax rates may be significant. In Section II we explicitly introduce efficiency into our optimization exercises.

II. Which Flat-Rate Tax System Best Balances Efficiency Costs With Minimal Changes in the Distribution of Tax Burdens?

In principle, if we can write down an expression for the value of the incentive effects of the tax system, we could calculate the flat-rate tax system which is best in this regard. More generally, we could construct an objective function which placed value on the incentive effects of the tax and the amount of tax burden redistribution that was caused. Considering both objectives simply requires an evaluation of the relative importance of meeting them.

Valuing the effects of changed incentives is not straightforward. Increased saving implies less current consumption, and increased labor supply entails less leisure. The gain to the economy of greater work effort and savings is properly measured as net of the value of the alternative activity, be it current consumption or leisure. However, in the presence of non-lump-sum taxes, the social value of additional units of taxed commodities will not equal the social opportunity cost of producing additional units of the good. Thus the available resources are being used inefficiently. If we ignore savings distortions for the moment, a well-known approximation of the value of the efficiency loss due to taxing labor income is

$$(7) \qquad \sum_i (\tfrac{1}{2}) \varepsilon_i t_i W_i \quad ,$$

where ε is the compensated elasticity of labor supply with respect to the after-tax wage, t is the marginal tax rate, and W is labor income.

It is possible to derive an expression similar to (7) which accounts for the welfare loss caused by the taxation of capital income (see, for example, Feldstein (1978)). However, there is considerable controversy about what are the appropriate substitution terms that enter the expression, and even the proper model within which to calculate this loss (Summers (1981)). Moreover, because the taxation of capital income is so complex, the appropriate value of the marginal tax rate to use in such a calculation is problematic. The effective tax rate on capital gains differs from that on dividends, which differs from that on capital income accumulated in an IRA, and so on. In order to avoid these measurement problems, and not because we believe the welfare costs are relatively small, we have chosen not to consider any distortions other than those concerning labor supply.

Clearly the welfare loss of expression (7) depends critically on the compensated labor supply elasticity. Econometric estimates of labor supply responsiveness have by no means been unanimous. A review of the literature by Borjas and Heckman (1978) put bounds of -0.19 and -0.07 on the uncompensated elasticity of male labor supply. Killingsworth (1982) determines that estimates for the uncompensated elasticity of female labor supply lie between +0.20 and +0.90. These estimates for uncompensated elasticities underestimate the compensated elasticity due to the fact that the income effect on labor supply is negative. On the basis of this evidence, we have chosen to do all the subsequent calculations with a value of ε of 0.4, which is meant to be a weighted average of the compensated elasticities implied by the econometric estimates of uncompensated elasticities of males and females.[11]

Given a particular value of ε, we can calculate the approximate efficiency loss for any flat-rate tax system. In principle, then, we can proceed by solving one of the following optimization problems:

(8) $$\text{Min} \sum_i |T_i^C - T_i^F|^{\alpha} + P_1\left(\sum_i (\tfrac{1}{2})\varepsilon_i t_i^2 W_i\right) \qquad \text{s.t.} \sum_i T_i^C = \sum_i T_i^F$$

or

(9) $$\text{Min} \sum_i |T_i^C/Y_i - T_i^F/Y_i|^{\alpha} + P_2\left(\sum_i (\tfrac{1}{2})\varepsilon_i t_i^2 W_i\right) \qquad \text{s.t.} \sum_i T_i^C = \sum_i T_i^F.$$

Here P_1 and P_2 refer to the relative weight the policy-maker places on the efficiency/incentive effects of the tax system vis-a-vis the measure of tax burden changes, as defined earlier. Setting P to zero corresponds to the case where the policy-maker has no interest in the efficiency implications of the tax system. Clearly then the best flat-rate tax is found as in Section I. If P is very large, then the only consideration the policy-maker has is the incentive effects of the income tax system; approximating the <u>status quo</u> distribution is not a concern. In this case the optimal flat-rate tax features a marginal tax rate of zero. All revenue is raised from lump-sum taxes.

In the general case when P lies between zero and infinity the best flat-rate income tax schedule is a compromise between the two policy goals of minimizing changes in the current distribution of taxes and minimizing the efficiency costs of the tax system. By finding the optimal tax schedule for each of various values of P we can trace out the frontier which represents how well a particular flat-rate tax system can achieve its two goals. In Figure 1 we have drawn the frontier for the flat-rate system of Case 1. On the vertical axis is the measure of how much change in the distribution of tax burdens occurs. Along the horizontal axis is the measure of efficiency cost, or excess burden, developed earlier; this measure depends on the assumption that the compensated elasticity of labor supply is 0.4.

In Section I we assumed that the sole concern in choosing a flat-rate tax system was in minimizing the change in tax burdens. In the context of Figure 1, we determined the point A. From the last column of Table 1 we can see that this system has an efficiency cost of \$19.6 billion and a sum of absolute tax burden changes of \$61.6 billion.[12] Another point of interest is F, which has zero efficiency cost and thus features a zero marginal income tax rate. If the credit per taxpaying unit and credit per exemption level are chosen to minimize the sum of the absolute changes in tax burden, a = -\$3,238, b = \$19, and the change sums to \$276.7 billion, or about four and a half times as large as for point A. This is obviously not a feasible (or, of course, equitable) tax system, since it implies that a family of four with no special exemptions owes \$3,162 in tax regardless of its income. Poor

FIGURE 1

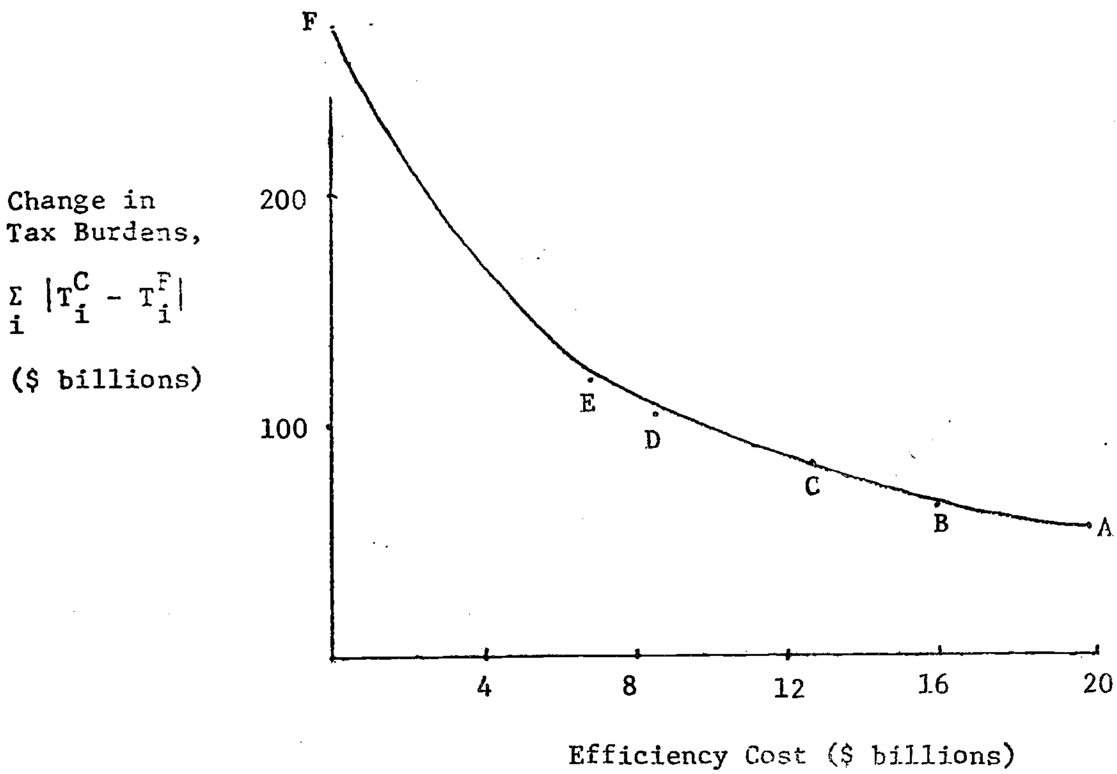
F
Change in
Tax Burdens,
$\sum_i |T_i^C - T_i^F|$
($ billions)
200
100
E
D
C
B
A
4
8
12
16
20
Efficiency Cost ($ billions)

TABLE 5

OPTIMAL FLAT-RATE SCHEDULES FOR DIFFERENT OBJECTIVE FUNCTIONS

P_1	a	b	t	$\sum_i \lvert T_i^C - T_i^F \rvert$ ($ billions)	Efficiency Cost ($ billions)
0	944	489	.254	61.6	19.6
.5	689	308	.225	67.3	15.8
1	537	115	.198	81.9	12.4
1.5	255	-83	.162	107.7	8.4
2	158	-178	.146	120.9	6.9
∞	-3238	19	.000	276.7	0.0

families will not be able to afford to pay this levy. Table 5 lists the optimal tax schedule for other non-extreme values of P_1. By plotting the values of the last two columns onto Figure 1, we trace out the frontier we can achieve with this kind of flat-rate tax. The slope of the frontier indicates the tradeoff between the two goals that policy-makers face in their choice of a tax schedule. For example, in the region between points C and D, decreasing the efficiency cost by $1 billion must cost $6.4 billion in additional divergence from current tax burdens, as measured by the sum of absolute differences.

Note that the frontier of Figure 1 refers to a particular flat-rate system, tax base, and measure of tax liability changes. If we considered another system such as one with only a per tax-paying unit credit, the frontier would lie entirely above the one in Figure 1, since any one credit tax system is a special case of the system of type (3), and thus can always be at least equalled in meeting the objectives. The frontier for another tax base such as gross income may intersect the frontier of Figure 1. In this case the choice of tax base depends on where on the frontier the policy-maker chooses to be. The frontier for a different measure of tax liability changes, such as the sum of squared differences in tax liability, is not directly comparable to Figure 1 because the unit of measurement along the vertical axis is different. The point analogous to point A is characterized in the last row of Table 1; it features a marginal tax rate of 0.382 and an efficiency cost of $39.1 billion. The optimal marginal tax rate declines from 0.382 to zero as the weight placed on efficiency cost increases.

III. Concluding Remarks

In this section we assess two aspects of this research: the value of the optimization technique to the analysis of taxation, and the choice of the best flat-rate tax to implement, should one be desired. These two assessments must of necessity be related -- our conclusions about the flat-rate tax are worthwhile only to the extent that our methodology is judged to be insightful.

The main advantages of the optimization technique are (i) that it focuses attention on the objectives the tax system is aimed at achieving, and (ii) that it gives specific answers about the optimal tax system designed to meet any formulation of these objectives. This is especially valuable when the objectives can be meaningfully translated into an explicit target function. Thus, in Section I, when several natural alternative definitions of approximating the current income tax distribution exist, the procedure is particularly insightful. In Section II, where the relative weights to be placed on the objectives do not have as natural an interpretation, not as much insight is gained. In this case the procedure still can serve to present the tradeoffs among objectives that are available to the policy-maker. For example, the results of Section II indicate what cost in terms of efficiency loss must result from trying to reduce the changes in the distribution of tax burdens.

What has the application of this technique taught us about the choice of a flat-rate income tax schedule? Section I showed that, if one desired characteristic of a flat-rate tax is to approximate the current distribution of tax burdens, and if the sum of changes in household tax burdens is the measure of how close the flat-rate tax comes to the current system, then the best flat-rate tax features marginal tax rates between 0.204 and 0.254. In particular, a schedule with (approximately) a $1,000 non-refundable credit per taxpaying unit, a $500 credit per exemption, and a marginal tax rate of twenty-five percent applied to all of adjusted gross income with no deductions will raise the same amount of revenue as the current tax system with minimal changes in tax burden. This minimal amount of change in tax burdens amounts to $61.6 billion, or about $688 per taxpaying unit. Moreover, the distribution of tax liability changes is systematically related to current income, with upper-income households paying less tax and middle-income households paying more tax. If the measure of how close a flat-rate tax comes to the current distribution of taxes is the sum of squared changes in tax liability, then the best marginal tax rate should be as high as 0.382. In this case, the flat-rate tax schedule is chosen with more attention to matching the current tax liabilities of the high income households.

When another objective is minimizing the efficiency costs of the tax system, the best flat-rate schedule optimally balances the efficiency cost with the change in tax burdens. For any given type of system, choice of tax base, and measure of change in tax burdens, the optimization technique allows us to construct the efficient fontier in the space of the two objectives. The best flat-rate tax depends on the relative weights assigned to the two objectives. Using this frontier we can calculate the tradeoff between conflicting policy goals the choice of a flat-rate tax must confront.

One of the virtues of the optimization technique used here is that the procedure uses the detailed microunit information on income by source, deductions, exemptions, and so on. Thus it is a significant improvement over the analytical treatments which use highly simplified assumptions about the distributions of the variables. However, even the microunit information does not allow investigation of certain issues of interest. First, it is a cross-section of observations in a given year. As such it is not helpful in assessing the implications for the distribution of lifetime tax burdens of a particular change. If fluctuations in annual income are large, then a flat-rate tax may approximate current lifetime tax burdens more closely than a cross-section would indicate. Second, it does not contain information on some types of income, such as interest from state and local securities, which may be taxable under a comprehensive flat-rate tax system. The optimal flat-rate schedule which also included this income in its tax base might look different than the schedules calculated in this paper.

Throughout this paper we have assumed that one desideratum to be considered in the choice of a flat-rate tax is minimal changes in current tax burdens. Of course, the optimization technique is general enough to also consider a problem in optimal tax design, where the current distribution of taxes is irrelevant and the societal standards of equity are considered in the form of the social welfare function. In this case the choice of the optimal flat-rate tax involves trading off equity and efficiency. We hope to consider this problem in future research.

Footnotes

1. For example, see Sheshinski (1972), Atkinson (1973), and Phelps (1973).

2. See Feldstein (1976) for a discussion of tax reform versus tax design.

3. The Treasury Tax File is a stratified random sample of over 100,000 individual tax returns. Because the sampling weights are known, the individual records can be used to make estimates for the population of taxpayers. See Feldstein and Frisch (1977).

4. The updating is accomplished by multiplying all amounts by 1.52 and by increasing the number of taxpaying units by a factor of 1.02.

5. The 1982 tax law used in the calculations takes account of only the major differences from the 1981 law. Specifically, all tax rates are reduced by ten percent, and the top marginal tax rate is reduced to fifty percent.

6. This tax schedule is of interest if, for example, the reason for implementing a flat-rate tax is to achieve administrative/compliance cost savings, which do not depend on the particular flat-rate schedule chosen. The policy-maker wants to achieve these savings while incurring a minimum of tax burden redistribution.

7. With the aging procedure discussed in note 4 and the tax law as presented in note 5, we calculate that revenues from the individual income tax in 1982 would be $295 billion.

8. The optimizations of Cases 5, 6, and 7 were carried out

excluding households with adjusted gross income of less than $1,000. This was done because small changes in tax liability to these households can represent huge changes in tax paid as a fraction of income and thus can unreasonably dominate the measure of tax burden changes. By excluding them from the optimization sample we insure that the optimal flat-rate system approximates the current distribution of tax burdens throughout the whole range of incomes and not just for those with very low incomes.

9. The disaggregated information in Tables 2, 3, and 4 is based on the over 25,000 records of the Treasury Tax File. As mentioned earlier the optimizations were performed on a subsample of 947 records. Tables 2, 3, and 4 computed using the smaller sample yielded qualitatively similar results to those reported, but featured some irregularities due to small sample size of some cells.

10. Feldstein and Taylor (1976) present detailed evidence of the dispersion of marginal tax rates within income classes.

11. The elasticity of 0.2 used in Section I to estimate behavioral response represents an uncompensated elasticity.

12. It is interesting to compare these figures to the current tax system, which has an efficiency cost of $32.7 billion and a sum of absolute tax burden changes equal to, of course, zero. Note that all the flat-rate tax systems lie to the northwest of the current system of Figure 1. If reducing the efficiency cost of the tax system were the only reason for implementing a flat-rate tax and the redistribution of tax burdens its only disadvantage, the policy-maker must decide whether $13.1 billion in efficiency cost reduction is worth $61.6 billion in tax liability changes, (assuming Case 1 is the flat-rate tax option). Of course, these are not the only considerations in the decision of whether to switch to a flat-rate tax.

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