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ABSTRACT

In this paper we derive the standard error of a price index when both prices and tastes or technology are treated as stochastic. Changing tastes or technology are a reason for the weights in the price index to be treated as stochastic, which can interact with the stochastic prices themselves. We derive results for the constant elasticity of substitution expenditure function (with Sato-Vartia price index), and also the translog function (with Töörnqvist price index), which proves to be more general and easier to implement. In our application to Asian growth, we construct standard errors on the total factor productivity (TFP) estimates of Hsieh (2002) for Singapore. We find that TFP growth is insignificantly different from zero in any year, but cumulative TFP over fifteen years is indeed positive.

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1. Introduction

In the stochastic approach to index numbers, prices are viewed as draws from some distribution, and the price index is viewed as a measure of the trend change in prices, with an estimable standard error. The most comprehensive treatment of this problem is by Selvanathan and Rao (1994), but the idea dates back to Keynes (1909) and earlier writers, such as Jevons and Edgeworth. Keynes points out that the price changes reflect both a common trend (generalized inflation) and commodity-specific trends,[1] which make the common trend difficult to identify. Selvanathan and Rao (1994, pp. 61-67) attempt to solve this problem using purely statistical techniques, as we describe in section 2, and the standard error of their price index reflects the precision of the estimate of the trend. Keynes does not offer a solution, but elsewhere he observes that changes in the purchasing power of money can occur for three distinct reasons: "The first of these reasons we may classify as a change in tastes, the second as a change in environment, and the third as a change in relative prices."[2]

The first factor identified by Keynes — changing tastes — can be expected to affect the weights in a price index and not just the prices. Accordingly, we will derive the standard error of a price index when both prices and tastes (or technology) are treated as stochastic. The rationale for our treatment of stochastic tastes (or technology) comes from the economic approach to index numbers, (e.g. Diewert, 1976), which shows that certain price indexes, known as *exact* indexes, equal the ratio of expenditures needed to obtain a fixed level of utility at two different prices.

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[1] In Keynes (1909) essay on "Index Numbers," section VIII deals with the "Measurement of General Exchange Value by Probabilities," which is the stochastic approach. He writes: "We may regard price changes, therefore, as partly due to causes arising from the commodities themselves raising some, lowering others, and all different in degree, and, superimposed upon the changes due to these heterogeneous causes, a further change *affecting all in the same ratio* arising out of change on the side of money. This uniform ratio is the object of our investigations." (from *The Collected Writing of John Maynard Keynes,* volume XI, p. 106).

[2] Keynes (1930), cited from *The Collected Writing of John Maynard Keynes,* volume V, p. 85.

This ratio of expenditures depends on the tastes of the consumer, so if the taste parameters are stochastic, then the exact index number is also. Section 3 describes how we allow for *both* random prices and random tastes, thereby integrating the stochastic and economic approaches to index numbers.

We use our integrated approach to stochastic index numbers to derive estimators for index number standard errors for two well-known models of tastes or technology. The first of these is the constant elasticity of substitution (CES) expenditure function (for a consumer) or cost function (for a firm). In section 4 we suppose that the CES taste parameters are random, and obtain a simple specification for demand that depends on the random parameters and on prices. The estimated error from this demand equation can be used to infer the standard error of the exact price index. Inverting the demand equation, we also obtain a simple specification in which price changes depend on a trend (the price index) and a component (log changes in expenditure shares) that has an average of zero, just as is supposed for the error term in an identical set of equations derived under the stochastic approach. The CES case therefore provides a good comparison to the specification used by Selvanathan and Rao (1994). In section 5 we extend our treatment of the CES case to deal with both random prices and random tastes, allowing an additional comparison to the Selvanathan and Rao results.

We next apply our integrated approach to stochastic index numbers to the translog case, considering the effects of random tastes in section 6 and the effects of both random tastes and random prices in section 7. The demand equations estimated are the familiar translog expenditure share equations, and again, the error in this system of regressions is used to infer the standard error of the exact price index. Although the CES system provides a particularly clear comparison with the conventional stochastic approach, a linear relationship between the shares and the taste parameters makes the translog system easier to implement than the non-linear CES system, and we recommend the translog for future use.

In section 8 we provide an application of our results to Asian productivity growth, and in particular, productivity growth in Singapore. The extent to which the East Asian countries are "exceptional" or not in terms of their productivity growth has been a topic of debate between the World Bank (1993) and Young (1992, 1995). Citing the estimates of zero or negative productivity growth in Singapore found by Young and also Kim and Lau (1994), Krugman (1994) popularized the idea the growth in some East Asian countries is mainly due to capital accumulation, and in that respect, is not much different than the former Soviet Union: certainly not a miracle. Recently, however, Hsieh (2002) has re-examined the productivity performance of several East Asia countries using dual measures of total factor productivity (TFP), and for Singapore finds positive productivity growth, contrary to Young. The difference lies in Hsieh's use of "external" rates of return for capital computed from three different sources, which are then used in a dual calculation of productivity growth; this contrasts with Young's calculation of primal productivity growth, which implicitly uses an "internal" return on capital.[3]

We use Hsieh's three different rates of return on capital to compute the standard error of that series, and of estimates of TFP, where we also incorporate the error in fitting the translog function. In the results, we find that TFP growth in Singapore is insignificantly different from zero for any single year in the sample. The same holds true when estimating cumulative TFP growth over any five-year or ten-year period of the sample. For the 15-year period, however, we find that cumulative TFP growth in Singapore is significantly positive. Thus, the estimates of

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[3] That is, the difference between these authors is not in the primal versus dual methodology for computing TFP, but in their use of differing rates of return for capital. We thank Charles Hulten for this observation.

Hsieh (2002) are indeed statistically different from those of Young (1992, 1995), provided that cumulative TFP over a long enough time period is considered.

2. The Stochastic Approach to Index Numbers

An example of the stochastic approach to price indexes is a model where price changes satisfy the equation:

$$\ln(p_{it} / p_{it-1}) = \pi_t + e_{it}, \quad i = 1,..., N, \tag{1}$$

where the errors are independent and heteroskedastic satisfying $E(e_{it}) = 0$ and $\text{var}(e_{it}) = \sigma^2/w_i$, where w_i are some exogenous values that sum to unity, $\sum_{i=1}^{N} w_i = 1$. Under these conditions, an unbiased and efficient estimate of trend change in prices π_t is,

$$\hat{\pi}_t = \sum_{i=1}^{N} w_i \ln(p_{it}/p_{it-1}), \tag{2}$$

which can be obtained by running weighted least squares (WLS) on (1) with the weights $\sqrt{w_i}$. An unbiased estimate for the variance of $\hat{\pi}_t$ is

$$s_{\pi}^2 = \frac{s_p^2}{N-1}, \tag{3}$$

where $s_p^2 = \sum_{i=1}^{N} w_i (\Delta \ln p_{it} - \hat{\pi}_t)^2$.

Diewert (1995) criticizes the stochastic approach and argues that: (a) the common trend π_t in (1) is limiting; (b) the variance assumption $\text{var}(e_{it}) = \sigma^2/w_i$ is unrealistic; (c) some choices of w_i (such as budget shares) will not be exogenous. In assessing these criticisms, we believe that a distinction should be made between *lower-level* and *higher-level* aggregation. At higher levels, these criticisms seem apt, and simple extensions to the model in (1) are unable resolve them in completely satisfactory ways. In particular, to avoid the assumption of a single common trend for all prices, Selvanathan and Rao (1994, pp. 61-67) add commodity-specific trends δ_i to equation (1):

$$\ln(p_{it} / p_{it-1}) = \pi_t + \delta_i + e_{it}, \quad i = 1, ..., N, \tag{1´}$$

where again the errors are independent and satisfy $E(e_{it}) = 0$ and $var(e_{it}) = \sigma^2/w_i$, with $\sum_{i=1}^{N} w_i = 1$. For the estimator of the common trend π_t to be identified, some assumption is needed on δ_i. Selvanathan and Rao show that the estimator for π_t is still given by (2) under the assumption that the commodity-specific trends have a weighted average of zero:[4]

$$\sum_{i=1}^{N} w_i \delta_i = 0. \tag{4}$$

The justification for (4) is purely statistical, i.e. it allows π_t to be identified, though we will suggest an economic interpretation in section 4.

In contrast, at the lowest level of aggregation, prices from different sellers of the same commodity are typically combined into indexes for individual commodities, or for narrow classes of closely related commodities. At this level, the assumption of a common trend, such as π_t in equation (1), will often be realistic. The stochastic approach in (1)-(3) can then be used to form the elementary indexes that are combined at higher levels of aggregation into indexes for all commodities, or for broad classes of commodities. If the expenditure shares needed to compute

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[4] The standard error of $\hat{\pi}_t$ when the commodity effects are used is less than that in (3), since the residual error of the pricing equation is reduced.

the weights w_i are unavailable for lower-level aggregates, so that $w_{it} = 1/N$ is used, the stochastic approach in (1)-(3) amounts to using a simple average of log-changes for prices, and its variance.

3. Integrating the Economic and Stochastic Approaches to Index Numbers

Despite Keynes' early contribution to the literature on the stochastic approach to index numbers, he ultimately rejected it. Keynes wrote,

> I conclude, therefore, that the unweighted (or rather randomly weighted) Index-Number of Prices — Edgeworth's indefinite index number — which shall in some way measure the value of money as such or the amount of influence on general prices exerted by 'changes on the side of money' or the 'objective mean variation of general prices' as distinguished from the 'change in the power of money to purchase advantages', has no place whatever in a rightly conceived discussion of the problems of Price-Levels.[5]

We also believe that index numbers with weights that reflect expenditure patterns are more interesting and informative than index numbers that have a purely statistical motivation. To motivate the incorporation of expenditure information in our index, we assume that the objective is to estimate an *economic index*, which, for the consumer problem, is defined as the ratio of the expenditure function evaluated at current period prices to the expenditure function evaluated at reference period prices.

Adopting an economic index as the goal of estimation makes the link between the

[5] Keynes (1930, pp. 87-8.) Keynes' argument against stochastic price models with independent commodity-specific shocks was that linkages between prices in an economy preclude shocks that affect a single price in isolation: "But in the case of prices a movement in the price of one commodity necessarily influences the movement in the prices of other commodities." (p. 86.)

stochastic properties of the data and the stochastic properties of the estimator less straightforward than when the goal is simply to estimate a mean price change. Nevertheless, any kind of index number calculated from stochastic data is itself stochastic. Moreover, a model of the stochastic processes reflected in the data used to calculate the economic index should allow the derivation of an estimator for its standard error.

Our starting point is the stochastic process for the expenditure shares used to calculate the weights in the index. To estimate an economic index requires an assumption about the form of the expenditure function that describes tastes. (For simplicity, our discussion will be in terms of the consumer problem, although the approach is equally applicable to the producer problem.) This assumption implies a functional form for the equations relating expenditure shares to prices. Since these equations generally do not fit the data on expenditure shares precisely, they imply the existence of an error term. We interpret changes in expenditure shares not explainable by changes in prices as arising from stochastic tastes. If we were able to take repeated draws from the distribution of the taste parameters in the expenditure function while holding prices constant, we would observe a range of outcomes for expenditure shares: this is one source of variance in an economic index.

A second source of variance in the price index is sampling error in the price data.[6] When prices are treated as stochastic, we need to decide whether the expenditure shares are determined by *observed* prices or by *expected* prices. Griliches and Grunfeld (1960, p. 7), for example, recognize that models of consumer behavior may be specified using either expected prices (and income) or observed prices (and income). We shall take the latter approach, and assume that

[6] Sampling error in the expenditure shares changes the interpretation and derivation of the estimator of the index standard error, but not the estimator itself.

observed prices determine expenditure shares.[7] In that case, the error term for prices influences the expenditure shares, so a component of the variance of expenditure shares comes from the variance of prices. In our translog results below, we include components representing the variance of expenditure shares that comes from the variance of prices (see proposition 7). However, for the non-linear CES model with prices treated as stochastic, we are unable to include these components in our variance estimator.

In addition to the *indirect* effects arising from the equations relating prices to expenditure shares, which are small, price variances have an important *direct* effect on the index's variance. We therefore extend our results for both the CES model and for the translog model to take account of the direct effect on the index of sampling error in the measures of prices used to construct the index. We assume that the lower-level aggregates in the index are price indexes for individual commodities, or for narrow categories of items that are homogeneous enough to be treated as a single commodity. Each commodity has its own non-stochastic price trend, but rather than summing to zero, as in equation (4), with properly chosen weights these commodity price trends sum up to the true index expressed as a logarithm. As sample estimators of these trends, the lower-level aggregates used to construct the index are subject to sampling error. Equation (1) describes the process generating the changes in individual price quotes that are combined in a lower-level aggregate. If all the quotes have identical weights and variances, the variance of the lower-level aggregate can therefore be estimated by equation (3).

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[7] Under the alternative hypothesis that expected prices are the correct explanatory variable, regression equations for expenditure shares using observed prices must be regarded as having mismeasured explanatory variables. Measurement error in the prices used to explain expenditure patterns may imply a bias in the estimates of the contribution of the taste variance to the index variance, but it does not necessarily do so. If sampling error in the price variables reduces their ability to explain changes in expenditure shares, too much of the variation in expenditure shares will be attributed to changes in tastes, and the estimate of the taste variance will be biased upward. Nevertheless, even if such a bias exists, and it will usually be negligible.

Finally, in addition to sampling error in tastes and in estimates of commodity prices, another source of inaccuracy in an estimate of a cost of living index is that the model used to describe tastes might be misspecified. This was thought to be an important problem until Diewert (1976) showed that by using a flexible functional form, an arbitrary expenditure function could be approximated to the second order of precision. We do not explicitly estimate the effect of possible misspecification of the model of tastes on the estimate of the cost of living index, but our standard error estimator does give an indication of this effect. An incorrectly specified model of tastes will likely fit the expenditure data poorly, resulting in a high estimate of the component of the index's variance that comes from the variance of tastes. On the other hand, our estimator will tend to imply a small standard error for the index if the model fits the expenditure data well, suggesting that the specification is correct, or if growth rates of prices are all within a narrow range, in which case misspecification does not matter.

One source of misspecification that can be important is an incorrect assumption of homotheticity. (Diewert's approximation result for flexible function forms does not imply that omitting an important variable, such as income in nonhomethetic cases, is harmless.) In the economic models considered in this paper, homotheticity is assumed for the sake of simplicity. In applications to producers, or in applications to consumers whose income changes by about the same amount as the price index, this assumption is likely to be harmless, but when consumers experience large changes in real income, the effects on their expenditure patterns are likely to be significant. If a model that assumes homotheticity is used and changes in real income cause substantial variation in expenditure shares, the estimate of the variance of the index's weights is likely to be elevated because of the lack of fit of the model. Knowing that the estimate of the economic index has a wide range of uncertainty will help to prevent us from having too much confidence in results based on an incorrect assumption, but relaxing the assumption would, of course, be preferable.[8]

4. The Exact Index for the CES Model with Random Technology or Tastes

A. Exactness of the Sato-Vartia Index in the Non-Random Case

Missing from the stochastic approach is an economic justification for the pricing equation in (1) or (1´), as well as the constraint in (4). It turns out that this can be obtained by using a CES utility or production function, which is given by,

$$f(\mathbf{x}_t,\mathbf{a}_t) = \left(\sum_{i=1}^{N} a_{it} x_{it}^{(\eta-1)/\eta}\right)^{\frac{\eta}{(\eta-1)}},$$

where $\mathbf{x}_t = (x_{1t},\ldots x_{Nt})$ is the vector of quantities, and $\mathbf{a}_t = (a_{1t},\ldots a_{Nt})$ are technology or taste parameters that we will allow to vary over time, as described below. The elasticity of substitution $\eta > 0$ is assumed to be constant.

We will assume that the quantities $\mathbf{x}_t$ are optimally chosen to minimize $\sum_{i=1}^{N} p_{it} x_{it}$,

subject to achieving $f(\mathbf{x}_t, \mathbf{a}_t)=1$. The solution to this optimization problem gives us the corresponding unit-cost function ,

[8] Caves, Christensen and Diewert (1982) find that the translog model that we discuss below can be extended to allow the log of income to affect expenditure patterns and that the Törnqvist index still measures the cost of living index at an intermediate utility level. However, to model non-homothetic tastes for index number purposes, we recommend the use of Deaton and Muellbauer's (1980) "Almost Ideal Demand System (AIDS)." Feenstra and Reinsdorf (2000) show that for the AIDS model, the predicted value of expenditure shares at the average level of logged income and logged prices must be averaged with the Törnqvist index weights (which are averages of observed shares from the two periods being compared) to obtain an exact price index for an intermediate standard of living. However, the variance formulas that we derive below for the translog model can still be used to approximate the variance of the AIDS index as long as logged income is included among the explanatory variables in the regression model for expenditure shares.

$$c(\mathbf{p}_t, \mathbf{b}_t) = \left(\sum_{i=1}^{N} b_{it} p_{it}^{1-\eta} \right)^{\frac{1}{(1-\eta)}}, \tag{5}$$

where $\mathbf{p}_t = (p_{1t},\ldots p_{Nt})$ is the vector of prices, and $b_{it} \equiv a_{it}^{\eta} > 0$, with $\mathbf{b}_t = (b_{1t},\ldots b_{Nt})$.

Differentiating (5) provides the expenditure shares s_{it} implied by the taste parameters $\mathbf{b}_t$:

$$s_{it} = \partial \ln c(\mathbf{p}_t, \mathbf{b}_t) / \partial \ln p_{it} = c(\mathbf{p}_t, \mathbf{b}_t)^{\eta-1} b_{it} p_{it}^{1-\eta}. \tag{6}$$

Diewert (1976) defines a price index formula whose weights are functions of the expenditure shares s_{it-1} and s_{it} as *exact* if it equals the ratio of unit-costs. For the CES unit-cost function with *constant* $\mathbf{b}_t$, the price index due to Sato (1976) and Vartia (1976) has this property. The Sato-Vartia price index equals the geometric mean of the price ratios with weights w_i:

$$\prod_{i=1}^{N} \left(\frac{p_{it}}{p_{it-1}} \right)^{w_i} = \frac{c(\mathbf{p}_t, \mathbf{b}_t)}{c(\mathbf{p}_{t-1}, \mathbf{b}_t)}, \tag{7}$$

where the weights w_i are defined as:

$$w_i = \frac{(s_{it} - s_{it-1})/(\ln s_{it} - \ln s_{it-1})}{\sum_{j=1}^{N} (s_{it} - s_{it-1})/(\ln s_{it} - \ln s_{it-1})}. \tag{8}$$

The weight for the i^{th} commodity is proportional to $(s_{it} - s_{it-1})/(\ln s_{it} - \ln s_{it-1})$, the logarithmic mean of s_{it} and s_{it-1}, and the weights are normalized to sum to unity.[9] Provided that the

9 The logarithmic mean of the expenditure shares in the two periods approximately equals an average of their arithmetic and geometric means with a 2/3 weight on the geometric mean. If s_{it-1} equals s_{it}, then the logarithmic means is defined as s_{it}.

expenditure shares are computed with *constant* taste parameters $\mathbf{b}_t$, the Sato-Vartia index on the left side of (7) equals the ratio of unit-costs on the right, also computed with constant $\mathbf{b}_t$. In addition to its ability to measure the change in unit-costs for the CES model, the Sato-Vartia index is noteworthy for its outstanding axiomatic properties, which rival those of the Fisher index (Balk, 1995, p. 87).

B. Effect of Random Technology or Tastes

As discussed above, we want to allow for random technology or taste parameters b_{it}, and derive the standard error of the exact price index due to this uncertainty. First, we must generalize the concept of an exact price index to allow for the case where the parameters $\mathbf{b}_t$ change over time, as follows:[10]

<u>Proposition 1</u>

Given $\mathbf{b}_{t-1} \neq \mathbf{b}_t$, *let* $s_{i\tau}$ *denote the optimally chosen shares as in (6) for these taste parameters,* τ = t-1, t, *and define* $\bar{b}_{i\tau} \equiv b_{i\tau} \Big/ \prod_{i=1}^{N} b_{i\tau}^{w_i}$ *, using the weights* w_i *computed as in* (8). *Then there exists* $\tilde{b}_i$ *between* $\bar{b}_{it-1}$ *and* $\bar{b}_{it}$ *such that,*

$$\prod_{i=1}^{N} \left(\frac{p_{it}}{p_{it-1}} \right)^{w_i} = \frac{c(\mathbf{p}_t, \tilde{\mathbf{b}})}{c(\mathbf{p}_{t-1}, \tilde{\mathbf{b}})} \ . \qquad (9)$$

This result shows that the Sato-Vartia index equals the ratio of unit-costs evaluated with

[10] The proofs of all Propositions are in an Appendix, at: <u>http://www.econ.ucdavis.edu/faculty/fzfeens/papers.html</u> .

parameters $\tilde{b}_i$ that lie *between* the normalized values of the b_{it} in each period. Therefore, the Sato-Vartia index is exact for this *particular* value $\tilde{\mathbf{b}}$ of the parameter vector, but the index will change as the random parameters $\mathbf{b}_{t-1}$ and $\mathbf{b}_t$ change. The standard error of the index should reflect this variation. The following proposition shows how the variance of the log Sato-Vartia index is related to the variance of the ln b_{it}, denoted by σ_β^2:

<u>Proposition 2</u>

Suppose that $\ln b_{i\tau}$ *are independently and identically distributed with variance* σ_β^2 *for* $i = 1,\ldots,N$ *and* $\tau = t-1$ *or* t. *Using the weights* w_i *as in (8), denote the log Sato-Vartia index by* $\pi_{sv} \equiv \sum_{i=1}^N w_i \Delta \ln p_{it}$. *Then conditional on prices, its variance can be approximated as:*

$$\text{var}\, \pi_{sv} \approx \tfrac{1}{2}\sigma_\beta^2 \sum_{i=1}^N w_i^2 (\Delta \ln p_{it} - \pi_{sv})^2 = \tfrac{1}{2}\sigma_\beta^2 s_p^2 \overline{\overline{w}}, \tag{10}$$

where $\overline{\overline{w}} \equiv \sum_{i=1}^N w_i^2 (\Delta \ln p_{it} - \pi_{sv})^2 / \sum_{i=1}^N w_i (\Delta \ln p_{it} - \pi_{sv})^2$ *is a weighted average of the* w_i *with the weights proportional to* $w_i(\Delta \ln p_{it} - \pi_{sv})^2$, *and* $s_p^2 \equiv \sum_{i=1}^N w_i (\Delta \ln p_{it} - \pi_{sv})^2$ *is the weighted variance of prices.*

Equation (10), which is *conditional on* the observed prices, shows how the variance of the Sato-Vartia index reflects the underlying randomness of the taste parameters $\mathbf{b}_t$ and $\mathbf{b}_{t-1}$. In effect, we are computing the variance in the price index from the randomness in the weights w_i in (8) rather than from the randomness in prices used in the conventional stochastic approach.

To compare (10) to (3), note that $\overline{\overline{w}}$ equals 1/N when the weights w_i also equal 1/N. In that case, we see that the main difference between our formula (10) for the variance of the price index and formula (3) used in the conventional stochastic approach is the presence of the term $\frac{1}{2}\sigma_\beta^2$. This term reflects the extent to which we are uncertain about the parameters of the underlying CES function that the exact price index is intended to measure. It is entirely absent from the conventional stochastic approach. We next show how to obtain a value for this variance.

C. Estimators for σ_β^2 and for the Index Variance

To use equation (10) to estimate the variance of a Sato-Vartia price index we need an estimate of σ_β^2. Changes in expenditure shares not accounted for by the CES model can be used to estimate this variance. After taking logarithms, write equation (6) in first-difference form as:

$$\Delta \ln s_{it} = (\eta - 1)\Delta \ln c_t - (\eta - 1)\Delta \ln p_{it} + \Delta \ln b_{it}, \tag{11}$$

where $\Delta\ln c_t \equiv \ln[c(\mathbf{p}_t,\mathbf{b}_t)/c(\mathbf{p}_{t-1},\mathbf{b}_{t-1})]$. Next, eliminate the term involving $\Delta\ln c_t$ by subtracting the weighted mean (over all the i) of each side of equation (11) from that side. Then using the fact that $\sum_{i=1}^N w_i \Delta \ln s_{it} = 0$, equation (11) becomes,

$$\Delta \ln s_{it} = \alpha_t - (\eta - 1)\Delta \ln p_{it} + \varepsilon_{it}, \quad i = 1,\ldots,N, \tag{12}$$

where,

$$\alpha_t = (\eta - 1)\sum_{i=1}^N w_i \Delta \ln p_{it}, \tag{13}$$

and,

$$\varepsilon_{it} = \Delta \ln b_{it} - \sum_{i=1}^N w_i \Delta \ln b_{it}. \tag{14}$$

Equation (12) may be regarded as a regression of the change in shares on the change in prices, with the intercept given by (13) and the errors in (14). These errors indicate the extent of taste change, and are related to the underlying variance of the Sato-Vartia index.

Denote by $\hat{\alpha}_t$ and $\hat{\eta}$ the estimated coefficients from running weighted least-squares on (12) over i =1,…,N, using the weights w_i. Unless the supply curve is horizontal, the estimate of η may well be *biased* because of a covariance between the error term (reflecting changes preferences and shifts in the demand curve) and the log prices. We ignore this in the next result, however, because we treat the prices as non-stochastic so there is no correlation between them and the errors ε_{it}. We return to the issue of stochastic prices at the end of this section, and in the next.

The weighted mean squared error of regression (12) is useful in computing the variance of the Sato-Vartia index, as shown by the following result:

Proposition 3

Define $\overline{w} = \sum_{i=1}^{N} w_i^2$ *as the weighted average of the* w_i*, using* w_i *from (8) as weights, with* $\overline{\overline{w}}$ *as in Proposition 2. Also, denote the mean squared error of regression (12) by* $s_\varepsilon^2 = \sum_{i=1}^{N} w_i \hat{\varepsilon}_{it}^2$*, with* $\hat{\varepsilon}_{it} = \Delta \ln s_{it} - \hat{\alpha}_t + (\hat{\eta} - 1)\Delta \ln p_{it}$. *Then an unbiased estimator for* σ_β^2 *is:*

$$s_\beta^2 = \frac{s_\varepsilon^2}{2(1 - \overline{w} - \overline{\overline{w}})}. \qquad (15)$$

To motivate this result, notice that the regression errors ε_{it} in (14) depend on the *changes* in the ln b_{it} minus their weighted mean. We have assumed that the ln b_{it} are independently and identically distributed with variance σ_β^2. This means that the variance of $\Delta \ln b_{it} = \ln b_{it} - \ln b_{it-1}$ equals $2\sigma_\beta^2$. By extension, the mean squared error of regression (12) is approximately twice the variance of the taste parameters, so the variance of the taste parameters is about one-half of the mean squared error, with the degrees of freedom adjustment in the denominator of (15) coming from the weighting scheme.

Substituting (15) into equation (10) yields a convenient expression for the variance of the Sato-Vartia index,

$$\operatorname{var} \pi_{sv} \approx \frac{s_\varepsilon^2 s_p^2 \overline{\overline{w}}}{4(1 - \overline{w} - \overline{\overline{w}})}. \qquad (16)$$

If, for example, each w_i equals 1/N, the expression for var π_{sv} becomes $s_\varepsilon^2 s_p^2 / 4(N-2)$. By comparison, the conventional stochastic approach resulted in the index variance $s_p^2/(N-1)$ in (3), which can be greater or less than that in (16). In particular, when $s_\varepsilon^2 < 4(N-2)/(N-1)$ then conventional stochastic approach gives a standard error of the index that is too high, as will occur if the fit of the share equation is good.

To further compare the conventional stochastic approach with our CES case, let us write regression (12) in reverse form as,

$$\Delta \ln p_{it} = \pi_{sv} - (\eta - 1)^{-1} \Delta \ln s_{it} + (\eta - 1)^{-1} \varepsilon_{it}, \qquad (12')$$

where,

$$\pi_{sv} = \textstyle\sum_{i=1}^N w_i \Delta \ln p_{it}, \qquad (13')$$

and the errors are defined as in (14). Thus, the change in prices equals a trend (the Sato-Vartia index), plus a commodity-specific term reflecting the change in shares, plus a random error reflecting changing tastes. Notice the similarity between (12´) and the specification of the pricing equation in (1´), where the change in share is playing the role of the commodity-specific terms δ_i. Indeed, the constraint in (4) that the weighted commodity-specific effect sum to zero is automatically satisfied when we use (12´) and the Sato-Vartia weight w_i in (8), because then $\sum_{i=1}^{N} w_i \Delta \ln s_{it} = 0$. Thus, the CES specification provides an economic justification for the pricing equation (1´) used in the stochastic approach.

If we run WLS on regression (12´) with the Sato-Vartia weights $\sqrt{w_i}$, the estimate of the trend term is exactly the log Sato-Vartia index, $\hat{\pi}_{sv} = \sum_{i=1}^{N} w_i \Delta \ln p_{it}$. If this regression is run without the share terms in (12´), then the standard error of the trend is given by equation (3) as modified to allow for heteroskedasticity by replacing N with $1/\bar{w}$. The standard error is somewhat lower if the shares are included. Either of these can be used as the standard error of the Sato-Vartia index under the conventional stochastic approach.

By comparison, in formula (16) we are using *both* the mean squared error s_ε^2 of the "direct" regression (12), and the mean squared error s_p^2 of the "reverse" regression (12´) (run without the share terms). The product of these is used to obtain the standard error of the Sato-Vartia index, as in (16). Recall that we are assuming in this section that the taste parameters are stochastic, but not prices. Then why does the variance of prices enter (16)? This occurs because with the weights w_i varying randomly, the Sato-Vartia index $\pi_{sv} = \sum_{i=1}^{N} w_i \ln(p_{it} / p_{it-1})$ will vary if and only if the price ratios (p_{it}/p_{it-1}) differ from each other. Thus, the variance of the Sato-Vartia index must depend on the *product* of the taste variance, estimated from the "direct" regression (12), and the price variance, estimated from the "reverse" regression (12´) without the share terms.

We should emphasize that in our discussion so far, regressions (12) or (12´) are run across commodities i =1,…,N, but for a given t. In the appendix, we show how to generalize Proposition 3 to the case where the share regression (12) is estimated across goods i =1,…,N and time periods t =1,…,T. In that case, the mean squared error that appears in the numerator of (15) is formed by taking the weighted sum across goods and periods. But the degrees of freedom adjustment in the denominator of (15) is modified to take into account the fact that the weights w_i in (8) are correlated over time (since they depend on the expenditure shares in period t-1 and t). With this modification, the variance of the taste parameters is still relatively easy to compute from the mean squared error of regression (12), and this information may be used in (10) to obtain the variance of the Sato-Vartia index computed between any two periods.

Another extension of Proposition 3 would be to allow for stochastic prices as well as stochastic taste parameters. This assumption is introduced in the next section under the condition that the prices and taste parameters are *independent.* But what if they are not, as in a supply-demand framework where shocks to the demand curve influence equilibrium prices: then how is Proposition 3 affected? Using the mean squared error s_ε^2 of regression (12) in equation (15) to compute the variance of the taste disturbances, or in equation (16) to compute the variance of the index, will give a lower-bound estimate. The reason is that running WLS on regression (12) will result in a downward biased estimate of the variance of tastes: $E(s_\varepsilon^2 / 2(1-\overline{w}-\overline{\overline{w}})) \leq s_\beta^2$ if taste shocks affect prices, because the presence of taste information in prices artificially inflates the explanatory power of the regression.

5. Variance of the Exact Index for the CES Function with Stochastic Prices

Price indexes are often constructed using sample averages of individual price quotes to represent the price of goods or services in the index basket.[11] In these cases, different rates of change of the price quotes for a good or service imply the existence of a sampling variance. That is, for any commodity i, $\Delta\ln p_{it}$ will have a variance of σ_i^2, which can be estimated by s_i^2, the sample variance of the rates of change of the various quotes for commodity i. The variances of the lower-level price aggregates are another source of variance in the index besides the variances of the weights considered in Proposition 3.[12]

In the special case where the commodities in the index are homogeneous enough to justify the assumption of a common trend for their prices and the log changes in commodity prices all have the same variance, an unbiased estimator for this variance is $s_p^2/(1-\overline{w})$, where s_p^2 is defined in Proposition 2.[13] This special case yields results that are easily compared with the results from the conventional stochastic approach.

In the more general case, no restrictions are placed on the commodity-specific price

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[11] Even if every price quoted for a good is included in the sample, we can still adopt an infinite population perspective and view these prices as realizations from a data-generating process that is the object of our investigations.

[12] Estimates of the variance of the Consumer Price Index (CPI) produced by the Bureau of Labor Statistics have long included the effects of sampling error in the price measures used as lower-level aggregates in constructing the CPI. Now they also include the effects of the variances of the weights used to combine these lower-level aggregates, which reflect sampling error in expenditure estimates. See U.S. Bureau of Labor Statistics (1997), p. 196.

[13] The denominator is a degrees of freedom correction derived as follows. Assume for simplicity that $E[\Delta\ln p_{it} - E(\Delta\ln p_{it})]^2 = 1$. Then $E(\Delta\ln p_{it} - \pi_t)^2 = E(\Delta\ln p_{it} - \sum w_j\Delta\ln p_{jt})^2 = E[(1-w_i)\Delta\ln p_{it} - \sum_{j\neq i} w_j\Delta\ln p_{jt}]^2 = (1-w_i)^2 + \sum_{j\neq i} w_j^2 = 1-2w_i + \sum_j w_j^2$. The weighted average of these terms, $\sum_i w_i[1-2w_i + \sum_j w_j^2]$, is $1-\sum_i w_i^2 = 1-\overline{w}$.

trends, but we do assume that the price disturbances are independent of the weight disturbances. Although allowing $\Delta \ln p_{it}$ to have a non-zero covariance with the w_i would be appealing if positive shocks to b_{it} are thought to raise equilibrium market prices, this would make the expression for var π_{sv} quite complicated. With the independence assumption, we obtain Proposition 4:

Proposition 4

Let prices and weights w_i *in (8) have independent distributions, and let* s_i^2 *be an estimate of the variance of* $\Delta \ln p_{it}$, i =1,…,N. *Then the variance of the Sato-Vartia price index can be approximated by:*

$$\operatorname{var} \pi_{sv} \approx \sum_{i=1}^{N} w_i^2 s_i^2 + \tfrac{1}{2} s_\beta^2 s_p^2 \overline{\overline{w}} + \tfrac{1}{2} s_\beta^2 \sum_{i=1}^{N} s_i^2 w_i^2 (1 - w_i)^2 , \qquad (17)$$

where s_β^2 *is estimated from the mean squared error of the regression (12) as in (15). In the special case when every price variance may be estimated by* $s_p^2 /(1-\overline{w})$, *(17) becomes,*

$$\operatorname{var} \pi_{sv} \approx s_p^2 \overline{w} /(1-\overline{w}) + \tfrac{1}{2} s_\beta^2 s_p^2 \overline{\overline{w}} + \tfrac{1}{2} s_\beta^2 \, [s_p^2 /(1-\overline{w})] \sum_{i=1}^{N} w_i^2 (1 - w_i)^2 . \qquad (17')$$

This proposition shows that the approximation for the variance of the price index is the sum of three components: one that reflects the variance of prices, another that reflects the variance of preferences and holds prices constant, and a third that reflects the interaction of the price variance and the taste variance. The first term in (17) or (17′) is similar to the variance estimator in conventional stochastic approach. If each w_i equals 1/N, the first term in (17′)

becomes $s_p^2\overline{w}/(1-\overline{w}) = s_p^2/(N-1)$, just as in (3). The second term in (17) or (17′) is the same as the term that we derived from stochastic tastes, in Proposition 2. The third term is analogous to the interaction term that appears in the expected value of a product of random variables (Mood, Graybill and Boes, 1974, p. 180, Corollary to Theorem 3). The presence of this term means that the interaction of random prices and tastes tends to raise the standard error of the index. If each w_i equals 1/N, then the second term in (17′) becomes $s_\varepsilon^2 s_p^2 / 4(N-2)$ and the third term becomes $\left[s_\varepsilon^2 s_p^2 / 4(N-2)\right]\left[1-1/N\right]$.

6. Translog Function

We next consider a translog unit-cost or expenditure function, which is given by:

$$\ln c(\mathbf{p}_t, \boldsymbol{\alpha}_t) = \alpha_0 + \sum_{i=1}^{N} \alpha_{it} \ln p_{it} + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \gamma_{ij} \ln p_{it} \ln p_{jt}, \tag{18}$$

where we assume without loss of generality that $\gamma_{ij} = \gamma_{ji}$. In order for this function to be linearly homogeneous in prices we must have $\sum_{i=1}^{N}\alpha_{it} = 1$ and $\sum_{i=1}^{N}\gamma_{ij} = 0$. The corresponding share equations are,

$$s_{it} = \alpha_{it} + \sum_{j=1}^{N}\gamma_{ij} \ln p_{jt}, \quad i=1,\ldots,N. \tag{19}$$

We will treat the taste or technology parameters α_{it} as random variables, but assume that the γ_{ij} are fixed. Suppose that $\alpha_{it} = \alpha_i + \varepsilon_{it}$, where the constant coefficients α_i satisfy $\sum_{i=1}^{N}\alpha_i = 1$, while the random errors ε_{it} satisfy $\sum_{i=1}^{N}\varepsilon_{it} = 0$. Using this specification, the share equations are,

$$s_{it} = \alpha_i + \sum_{j=1}^{N} \gamma_{ij} \ln p_{jt} + \varepsilon_{it}\ , \quad i=1,\ldots,N. \tag{20}$$

We assume that ε_{it} is identically distributed with $E(\varepsilon_{it})=0$ for each equation i, though it will be correlated across equations (since the errors sum to zero), and may also be correlated over time. Since the errors sum to zero the autocorrelation must be identical across equations. We will denote the covariance matrix of the errors by $E(\boldsymbol{\varepsilon}_t\, \boldsymbol{\varepsilon}_t') = \boldsymbol{\Omega}$, and their autocorrelation is then $E(\boldsymbol{\varepsilon}_t\, \boldsymbol{\varepsilon}_{t-1}') = \rho\boldsymbol{\Omega}$.

With this stochastic specification of preferences, the question again arises as to what a price index should measure. In the economic approach, with α_{it} constant over time, the ratio of unit-costs is measured by a Törnqvist (1936) price index. The following result shows how this generalizes to the case where α_{it} changes:

Proposition 5

Defining $\overline{\alpha}_i = (\alpha_{it-1} + \alpha_{it})/2$ *, and* $w_{it} = (s_{it-1} + s_{it})/2$*, where the shares* $s_{i\tau}$ *are given by (19) for the parameters* $\alpha_{i\tau}$, τ = t-1, t, *then,*

$$\prod_{i=1}^{N} \left(\frac{p_{it}}{p_{it-1}} \right)^{w_{it}} = \frac{c(\mathbf{p}_t, \overline{\boldsymbol{\alpha}})}{c(\mathbf{p}_{t-1}, \overline{\boldsymbol{\alpha}})}. \tag{21}$$

The expression on the left of (21) is the Törnqvist price index, which measures the ratio of unit-costs evaluated at an *average value* of the taste parameters α_{it}. This result is suggested by Caves, Christensen and Diewert (1982), and shows that the Törnqvist index is still meaningful when the first-order parameters α_i of the translog function are changing over time.

The variance of the Törnqvist index can be computed from the right side of equation (21) expressed in logs. Substituting from equation (18), we find that the coefficients $\bar{\alpha}_i$ multiply the log prices. Hence, conditional on prices, the variance of the log-change in unit-costs will depend on the variance of $\bar{\alpha}_i = (\alpha_{it-1} + \alpha_{it})/2 = \alpha_i + (\varepsilon_{it-1} + \varepsilon_{it})/2$. The covariance matrix of these taste parameters is $E(\bar{\boldsymbol{\alpha}} - \boldsymbol{\alpha})(\bar{\boldsymbol{\alpha}} - \boldsymbol{\alpha})' = (1+\rho)\boldsymbol{\Omega}/2$. This leads to the next result:

<u>Proposition 6</u>

Let the parameters α_{it} *be distributed as* $\alpha_{it} = \alpha_i + \varepsilon_{it}$, *with* $E(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t') = \boldsymbol{\Omega}$ *and* $E(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_{t-1}') = \rho\boldsymbol{\Omega}$, *and denote the log Törnqvist index by* $\pi_t \equiv \sum_{i=1}^{N} w_{it} \Delta \ln p_{it}$. *Then, conditional on prices, the variance of* π_t *is:*

$$\operatorname{var} \pi_t = \tfrac{1}{2}(1+\rho)\Delta \ln \mathbf{p}_t{}'\boldsymbol{\Omega}\, \Delta \ln \mathbf{p}_t\,. \tag{22}$$

Since the errors of the share equations in (20) sum to zero, the covariance matrix $\boldsymbol{\Omega}$ is singular, with $\boldsymbol{\Omega\iota} = 0$ where $\boldsymbol{\iota}$ is a (Nx1) vector of one's. Thus, the variance of π_t is equal to,

$$\operatorname{var} \pi_t = \tfrac{1}{2}(1+\rho)[\Delta \ln \mathbf{p}_t - \pi_t]'\boldsymbol{\Omega}\, [\Delta \ln \mathbf{p}_t - \pi_t]. \tag{23}$$

The variance of the Törnqvist index will approach zero as the prices approach a common growth rate, and this property also holds for the variance of the Sato-Vartia index in Proposition 2 and the conventional stochastic approach in (3). But unlike the stochastic approach in (3), the variance of the Törnqvist index will depend on the fit of the share equations. Our formula for the variance of the Törnqvist index is more general than the one that we obtained for the Sato-Vartia index, because Proposition 6 does not assume that the taste disturbances are all independent and identically distributed.

The fit of the share equations will depend on how many time periods we pool over, and this brings us to the heart of the distinction between the stochastic and economic approaches. Suppose we estimated (20) over just two periods, t-1 and t. Then it is readily verified that there are enough free parameters α_i and γ_{ij} to obtain a *perfect fit* to the share equations. In other words, the translog system is flexible enough to give a perfect fit for the share equation (20) at two points. As we noted in section 3, such flexibility is a virtue in implementing the economic approach to index numbers: indeed, Diewert (1976) defines an index to be *superlative* if it is exact for an aggregator function that is flexible.[14] But from an econometric point of view, we have zero degrees of freedom when estimating the share equations over two periods, so that the covariance matrix $\mathbf{\Omega}$ cannot be estimated. How are we to resolve this apparent conflict between the economic and stochastic approaches?

We believe that a faithful application of the economic approach requires that we pool observations *over all available time periods* when estimating (20). In the economic approach, Christensen and Diewert (1982a,b) allow the first-order parameters α_{it} of the translog unit-cost function to vary over time (as we do above), but strictly maintain the assumption that the second-order parameters γ_{ij} are constant (as we also assume). Suppose the researcher has data over three (or more) periods. If the share equations are estimated over periods one and two, and then again

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[14] Diewert (1976) defines an aggregator function to be flexible if it provides a second-order approximation to an arbitrary function at one point, i.e. if the parameters can be chosen such that the value of the aggregator function, and its first and second derivatives, equal those of an arbitrary function at one point. We are using a slightly different definition of flexibility: if the ratio of the aggregator function, and the value of its first and second derivatives, equal those of an arbitrary function at two points.

over periods two and three, this would clearly *violate* the assumption that γ_{ij} are constant. Since this is an essential assumption of the economic approach, there is every reason to use it in our integrated approach. The way to maintain the constancy of γ_{ij} is to pool over multiple periods, which allows the covariance matrix $\boldsymbol{\Omega}$ to be estimated. Pooling over multiple periods is also recommended for the CES share equations in (12), to satisfy the maintained assumption that η is constant, even though in the CES case we do not obtain a perfect fit if (12) is estimated over a single cross-section (provided that N > 2).

Once we pool the share equations over multiple periods, it makes sense to consider more general specifications of the random parameters α_{it}. In particular, we can use $\alpha_{it} = \alpha_i + t\beta_i + \varepsilon_{it}$, where the coefficients β_i on the time trend satisfy $\sum_{i=1}^{N} \beta_i = 0$. Then the share equations become,

$$s_{it} = \alpha_i + \beta_i t + \sum_{j=1}^{N} \gamma_{ij} \ln p_{jt} + \varepsilon_{it} \;, \quad i=1,\ldots,N;\ t=1,\ldots,T\ . \tag{24}$$

We make the same assumptions as before on the errors ε_{it}. Proposition 5 continues to hold as stated, but now the $\bar{\alpha}_i$ are calculated as $\bar{\alpha}_i = (\alpha_{it-1} + \alpha_{it})/2 = \alpha_i + \beta_i/2 + (\varepsilon_{it-1} + \varepsilon_{it})/2$. The variance of these is identical to that calculated above, so Proposition 6 continues to hold as well. Thus, including time trends in the share equations does not affect the variance of the Törnqvist index.

7. Translog Case with Stochastic Prices

As in the CES case, we would like to extend the formula for the standard error of the price index to include randomness in prices as well as taste parameters. For each commodity i we suppose that $\Delta\ln p_{it}$ is random with variance of σ_i^2, which can be estimated by s_i^2, the sample variance of the rates of change of the various quotes for item i. These “lower level” sampling errors are assumed to be independent across commodities, and are also independent of error ε_{it} in the taste or technology parameters, $\alpha_{it} = \alpha_i + \varepsilon_{it}$. Then the standard error in Proposition 6 is extended as:

Proposition 7

Let $\Delta \ln p_{it}$, $i = 1,\ldots,N$, *be independently distributed with mean* 0 *and variances estimated by* s_i^2, *and also independent of the parameters* $\alpha_{it} = \alpha_i + \varepsilon_{it}$. *Using the weights* $w_{it} = (s_{it-1} + s_{it})/2$, *the variance of the log Törnqvist index is approximated by:*

$$\mathrm{var}(\pi) \approx \sum w_{it}^2 s_i^2 + \tfrac{1}{4}\sum_i \sum_j (\Delta \ln p_{it})(\Delta \ln p_{jt}) \left[\sum_k \hat{\gamma}_{ik}\hat{\gamma}_{jk}\, s_k^2\right]$$

$$+ \tfrac{1}{2}(1+\hat{\rho})(\Delta \ln \mathbf{p}_t)' \hat{\mathbf{\Omega}} (\Delta \ln \mathbf{p}_t) + \tfrac{1}{4}\sum_i (s_i^2)\left[\sum_j \hat{\gamma}_{ij}^2 s_j^2\right] + \tfrac{1}{2}(1+\hat{\rho})\left[\sum \hat{\Omega}_{ii}\, s_i^2\right]. \quad (25)$$

In the case where all prices have the same trend and variance, we can estimate σ_i^2 *by* $s_p^2/(1-\overline{w})$ *for all i and (25) becomes:*

$$\mathrm{var}(\pi) \approx s_p^2\, \overline{w}/(1-\overline{w}) + \tfrac{1}{4}[s_p^2/(1-\overline{w})]\left\{\sum_i \sum_j (\Delta \ln p_{it})(\Delta \ln p_{jt}) \left[\sum_k \hat{\gamma}_{ik}\hat{\gamma}_{jk}\right]\right\}$$

$$+ \tfrac{1}{2}(1+\hat{\rho})(\Delta \ln \mathbf{p}_t)' \hat{\mathbf{\Omega}} (\Delta \ln \mathbf{p}_t) + \tfrac{1}{4}[s_p^2/(1-\overline{w})]^2 \left[\sum_i \sum_j \hat{\gamma}_{ij}^2\right]$$

$$+ \tfrac{1}{2}[s_p^2/(1-\overline{w})](1+\hat{\rho})\left[\sum_i \hat{\Omega}_{ii}\right] \quad (25')$$

Thus, with stochastic prices the variance of the Törnqvist index includes five terms. The first term in (25) contains the product of the squared weights times the price variances, the second term reflects the effect of the price variance on the weights, the third term reflects the variance of the weights that comes from the taste shocks, the fourth term reflects the interaction of the price variance component and the component of the weight variance that comes from the price shocks, and the fifth term reflects the interaction between the price variance and the component of the weight variance that comes from the taste shocks. The expression for the variance of the Sato-Vartia index in equation (17) also includes the analogous expressions for the first, third and fifth terms in (25). The terms in (25) reflecting the effect of the price shocks on the weight variance are new, however, and arise because the linearity of the Törnqvist index allow use to include them; they were omitted for the sake of simplicity in the CES case. These terms can be omitted from the estimator of the index variance if the model of consumer behavior is specified with expected prices, rather than realized prices, as explanatory variables.

8. Application to Productivity Growth in Singapore

We now consider an application of our results to productivity growth in Singapore. As discussed in the introduction, Hsieh (2002) has recently computed measures of total factor productivity (TFP) for several East Asian countries, and obtains estimates higher than Young (1992, 1995) for Singapore. The question we shall address is whether Hsieh's estimates for productivity growth in Singapore are *statistically different* from those obtained by Young.

Hsieh uses three different measures of the rental rate on capital for Singapore. They are all motivated by the Hall-Jorgenson (1967) rental price formula, which Hsieh writes as:

$$\frac{r_j}{p} = \frac{p_j^k}{p}(i - \hat{p}_k + \delta_j)\,, \tag{26}$$

where p_j^k is the nominal price of the j^{th} type of capital, p is the GDP deflator, i is a nominal interest rate, $\hat{p}_k$ is the overall inflation rate for capital, and δ_j is the depreciation rate for the jth type of capital. For the real interest rate $(i - \hat{p}_k)$, Hsieh uses three different measures: (i) the average nominal lending rate of the commercial banks, less the overall inflation rate for capital $\hat{p}_k$; (ii) the earnings-price ratio of firms on the stock market of Singapore; (iii) the return on equity from firm-level records in the Singapore Registry of Companies. These are all plotted in Figure 1 (reproduced from Hsieh, 2002, Figure 2), where it can be seen that the three rates are substantially different.

To compute the real rental price, capital depreciation is added to all three series in Figure 1, after which the calculation in (26) is made using the investment price deflators for five kinds of capital for p_j^k. Hsieh weights these five types of capital by their share in payments to obtain an overall rental rate corresponding to each interest rate. We will denote these by r_t^k, k=1,2,3, depending on the three interest rates used. The plot of the real rental prices (not shown) looks qualitatively similar to Figure 1. In Figure 2 we show the *percent change* in the rental prices (computed as the change in the log of (26), times 100), where it is evident that the dip in the commercial bank lending rate in 1974 has a dramatic effect on that rental price. Hsieh (2002, p. 509) regresses the average growth of rentals on a constant and time-trend, and the coefficient of the time trend over each sample – representing the average growth of each rental – is reported in Part A of Table 1.

On the wage side, Hsieh distinguishes eight types of workers, by gender and four educational levels. He uses benchmark estimates for wages and employment in 1966, 1972, 1980 and 1990 and annual data on income and employment from labor market surveys beginning in 1973 to calculate the annual growth rates of wages. The average growth of wages over various times periods is shown in part A of Table 1.[15]

The labor share of 0.511 shown in part A is taken from Young (1995), and is held constant. Then dual TFP growth is computed by the weighted average of the annual growth in the wage and rental price of capital, using the constant labor share as the weight on labor. This results in dual TFP growth ranging from 1.76 percent to 2.46 percent per year, as shown in the second-last column of Table 1. These estimates are comparable to the estimates for other Asian countries, but they contrast with the *negative* estimates of primal TFP for Singapore from Young (1995), shown in the final column.

The question we wish to address is whether the Hsieh's estimates in the second-last column of part A are *significantly different* from Young's estimates in the last column. Hsieh (Table A2, p. 523) computes confidence intervals on the average growth of *each of the real rental prices* in part A using the standard errors from the coefficient on each time trend. For two of the three alternative measures, the 95 percent confidence interval includes a decline of nearly 1 percent per year. Hsieh uses the bounds of these confidence intervals for rental price growth to calculate confidence intervals for TFP growth. The confidence intervals for TFP growth all lie above 1 percent per year, so according to his calculations, TFP growth is significantly greater than zero.

We would argue that this procedure fails to convey the true uncertainly associated with the TFP estimates, for two reasons. First, and most important, we should treat each interest rate – and associated rental prices on capital – as an independent observation on the "true" rate, and

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[15] These results are somewhat higher than reported in Hsieh (2002, p. 509), because we have corrected a slight inconsistency in his calculation.

pool across these to compute the standard error of the rentals. Second, we should distinguish this standard error in any one year from that over the entire sample period. Hsieh's procedure is to compute average TFP over the entire sample, along with its standard error, but this does not tell us whether TFP growth in any one year (or shorter period) is significantly positive. We now proceed to address both these points.

8.1 Error in Annual TFP

Before we can construct our estimate of the standard error of dual TFP, we first need to re-measure productivity using annual data on labor shares, wages, and the rental price. These results are shown in part B of Table 1. Annual data for wages and labor shares are available beginning in 1973, and the annual data for all three rentals continues until 1992, so that becomes our sample period. We first aggregate the eight types of labor using a Törnqvist price index, and then compute dual TFP growth using a Törnqvist index over the real wage index and real rental price of capital:

$$\Delta TFP_t^k \equiv \tfrac{1}{2}(s_{Lt} + s_{Lt-1})\Delta \ln(w_t / p_t) + \tfrac{1}{2}(s_{Kt} + s_{Kt-1})\Delta \ln(r_t^k / p_t)\,, \qquad (27)$$

where s_{Lt} is the labor share in period t, s_{Kt} is the capital share with $s_{Lt} + s_{Kt} = 1$, w_t is the wage index, and r_t is the rental price on capital. The labor shares are computed from *Economic and Social Statistics, Singapore, 1960-1982,* and from later issues of the *Yearbook of Statistics, Singapore.* These shares range from 0.36 to 0.47 over 1973-1992, and average 0.418, which is less than the labor share shown in part A of Table 1 and used by Young (1995) and Hsieh.

In addition to the average labor share, we report in part B the average growth rates of the rentals prices and wage index, as well as the computed dual TFP. The average rental price growth differs substantially between parts A and B. This reflects the use of different formulas: as noted above, Hsieh uses a regression-based method to compute the growth rate, whereas we use the average of the difference in logs of (26), times 100. Hsieh states that his method is less sensitive to the initial and end points of the sample period, whereas the average of the difference in log rentals certainly *does* depend on our sample period. It is evident from Figure 2 that the rental price computed with the average bank-lending rate falls by about 200 percent from 1973-74, and then rises by about 300 percent from 1974-75, and these values are the largest in the sample. If instead of using 1973-1992 as the sample period, we use 1975-1992, then the average growth in the rental price computed with the commercial bank lending rate falls from 2.5 percent per year to –1.4 percent per year!

The growth rates of wages reported in Parts A and B also differ slightly because of differences in sample periods and in formulas used.[16] Dual TFP based on the Törnqvist index, reported in part B, shows higher growth for two of the rental price measures, and lower growth for one measure, than dual TFP based on average growth rates, reported in part A. Using the *mean* of the three alternative rental price estimates, the growth of dual TFP based on the Törnqvist index is 2.48 percent per year over 1973-1992. Yet this falls dramatically to 1.37 percent per year if 1973-1974 (when one of the rental prices moved erratically) is omitted.

Our goal is to compute the standard error of the Törnqvist index in (27), where this error arises from two sources: (i) error in measuring the rental prices of capital, based on the three alternative real interest rates used; (ii) error because the annual data will not fit a translog cost function perfectly. Under the hypothesis that the homothetic translog cost function model

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16 As discussed above, we use a Törnqvist price index constructed over the eight types of labor, whereas Hsieh uses an averaging procedure.

describes the process generating the data, the Törnqvist price index exactly summarizes the change in the cost function. Thus, we assume that changes in expenditure shares represent responses to changes in wages and rental prices in accordance with the translog model, plus effects of random shocks to expenditures. *Ceteris paribus*, the greater the variance of the share changes that is unexplainable by the translog model, the greater the variance of the random shocks that affect the weights in the Törnqvist index.

Beginning with error (i), we first construct the mean rental price:

$$\ln \bar{r}_t \equiv \frac{1}{3}\sum_{k=1}^{3} \ln r_t^k ,$$

and its change,

$$\ln(\bar{r}_t / \bar{r}_{t-1}) = \frac{1}{3}\sum_{k=1}^{3} \ln(r_t^k / r_{t-1}^k) , \qquad (28)$$

where k=1,2,3 denotes the three rental prices. Then the sample variance of the change in mean rental price, denoted by s_t^2, is,

$$s_t^2 \equiv \frac{1}{6}\sum_{k=1}^{3} [\ln(r_t^k / r_{t-1}^k) - \ln(\bar{r}_t / \bar{r}_{t-1})]^2 . \qquad (29)$$

In Figure 3 we plot mean TFP growth in each year,

$$\overline{\Delta TFP}_t \equiv \tfrac{1}{2}(s_{Lt} + s_{Lt-1})\Delta \ln(w_t / p_t) + \tfrac{1}{2}(s_{Kt} + s_{Kt-1})\Delta \ln(\bar{r}_t / p_t) , \qquad (30)$$

and the 95 percent confidence interval (with 2 degrees of freedom) constructed as $\overline{\Delta TFP}_t \pm \tfrac{2.9}{2}(s_{Kt} + s_{Kt-1})\sqrt{s_t^2}$. We can see that the confidence interval on mean TFP growth over 1973-1995 is extremely wide, but this is not surprising given the erratic data on rentals shown in Figure 2. In Table 1, we report in parentheses the average standard deviation of the change in rental prices, and the average standard error of mean TFP growth, over the 1973-1992 period. Consistent with Figures 2 and 3, both these are extremely large.

Furthermore, even when we restrict attention to the shorter period of 1975-1992 shown in Figure 4, the confidence interval of mean TFP growth still includes zero in every year. This can also be seen from Table 1, where we report the average standard deviations of the change in rental prices, and mean TFP growth over 1975-1992. The average value of mean TFP growth is 1.4 percent, but it has an average standard error of 5 percent. Accordingly, in every year we cannot reject the hypothesis that mean TFP growth is zero or negative. Thus, on an annual basis, we would be hard pressed to conclude that the positive productivity estimates of Hsieh are significantly different than the negative estimates of Young (1995).

8.2 Error in Cumulative TFP

Nevertheless, our interest is not in the hypothesis that TFP growth in *each year* is positive, but rather, that *cumulative* TFP growth is positive. An erratic movement in a rental price one year might very well be reversed the next year, resulting in a negative autocorrelation that reduces the variance of long-run rental growth. To assess the implications of this, we instead consider longer differences in TFP, such as 15-year growth,

$$\begin{aligned}\overline{\Delta TFP}_{t,15} \equiv {} & \tfrac{1}{2}(s_{Lt} + s_{Lt-15})[\ln(w_t / p_t) - \ln(w_{t-15} / p_{t-15})] \\ & + \tfrac{1}{2}(s_{Kt} + s_{Kt-15})[\ln(\bar{r}_t / p_t) - \ln(\bar{r}_{t-15} / p_{t-15})].\end{aligned} \tag{31}$$

The standard error of this can be measured using the variance of measurement error in the long-difference of rental prices,

$$s^2_{t,15} \equiv \frac{1}{6} \sum_{k=1}^{3} [\ln(r^k_t / r^k_{t-15}) - \ln(\bar{r}_t \; / \bar{r}_{t-15})]^2 . \tag{32}$$

In Figure 5 we plot the mean 15-year TFP growth ending in the years 1988-1992, along with the 95 percent confidence interval $\overline{\Delta\text{TFP}}_{t,15} \pm \frac{2.9}{2}(s_{Kt} + s_{Kt-15})\sqrt{s^2_{t,15}}$. We have five observations for the 15-year cumulative TFP growth, and in four out of five cases the cumulative growth is significantly greater than zero. The only exception is 1989, where the erratic movement in the mean rental from 1974, and its large standard error, makes that observation on TFP growth insignificantly different from zero. In all other end years the confidence intervals on cumulative TFP growth exclude zero. This can also be seen from part C of Table 1, where we report the mean values of the growth in wages, mean rental, and mean TFP growth over the 15-year period, along with their standard deviations. Cumulative TFP growth of 21.8 percent (averaged over the end-years 1990-92) vastly exceeds its standard error of 3.8 percent. Notice that this standard deviation is actually *smaller* than the standard deviation of the *annual* change in TFP growth in part B, indicating some negative correlation in the measurement error of rental price changes.[17] Accordingly, we *cannot* reject that hypothesis that 15-year mean TFP growth is positive, except in 1989.

8.3 Error from Fitting the Translog Function

We still need to check the second source of error in the index, which arises because a translog cost function does not fit the data perfectly. We proceed by estimating the share equations for the translog cost function, using the mean rental price $\bar{r}_t$ and the Törnqvist wage

[17] The standard deviation of the change in rental prices first increases with the lag length, and then falls. That is, let $s^2_{t,T}$ denote the variance of the change in rental prices as in (7), but with a lag length of T. For T=1,2,3,5,10,15, the standard deviation of $\sqrt{s^2_{t,T}}$ (averaged over end-years 1990-1992) equals 5.0, 7.5, 8.7, 18.8, 8.8, 6.6%.

index w_t.[18] Dropping one share equation (since shares sum to unity), we are left with estimating the capital share equation:

$$s_{Kt} = \alpha_L + \gamma_{KK} \ln(\bar{r}_t / p_t) + \gamma_{KL} \ln(w_t / p_t) + \varepsilon_{Kt}, \tag{33}$$

where s_{Kt} is the capital share. We allow for first-order autocorrelation ρ in the error ε_{Kt} when estimating this equation. So losing one observation to allow for estimation of ρ, the sample period becomes t=1974,…,1992, or t=1976,…,1992 when we exclude the erratic change in rentals. Results for both periods are shown in Table 2.

Over the 1974-1992 period, we obtain significant estimates for both γ_{KK} and γ_{KL} in the first regression (row) in Table 2, but with these estimates we strongly reject the homogeneity restriction that $\gamma_{KK} + \gamma_{KL} = 0$. If we go ahead and impose this constraint, then the results, shown in the second regression of Table 2, are quite poor: $\gamma_{KK} = -\gamma_{KL}$ is insignificant, and most of the explanatory power comes from the autocorrelation $\rho = 0.91$. This is likely caused by the erratic movement in the mean rental price over 1973-74, so instead we consider estimation over 1976-1992. In that case, unconstrained estimation in the third regression leads to estimates of γ_{KK} and γ_{KL} that are opposite in sign, and the homogeneity restriction $\gamma_{KK} = -\gamma_{KL}$ is borderline between being accepted and rejected at the 95 percent level. Using this restriction, we obtain the estimates in the final regression, with $\gamma_{KK} = 0.10$, $\rho = 0.10$, and a standard error of the regression equal to 0.012. We shall use these estimates in the calculations that follow.

[18] The second error can be assessed by either fitting a translog unit-cost function to the data on rental prices and the Törnqvist wage index (2 factors), or to the data on the rental prices and wages for each type of labor (9 factors of production). For convenience, we have uses just 2 factors.

To construct the standard deviation of 15-year TFP growth due to the translog error, we re-write the term on the right of (22) as,

$$\tfrac{1}{2}(1+\rho^{15})[\ln(w_t / w_{t-15}) - \ln(\bar{r}_t / \bar{r}_{t-15})]^2 \omega_{KK}^2 , \tag{34}$$

where ω_{KK} is the standard error of the capital-share regression. We obtain (34) from (22) by using the simple structure of the covariance matrix $\boldsymbol{\Omega} = \begin{bmatrix} \omega_{KK}^2 & -\omega_{KK}^2 \\ -\omega_{KK}^2 & \omega_{KK}^2 \end{bmatrix}$, which follows since the errors in the capital and labor share equations sum to zero.

With autocorrelation of $\rho = 0.10$, the term ρ^{15} is negligible. So taking the square root of (34), the standard error of 15-year TFP growth becomes $\frac{1}{\sqrt{2}}\left|\ln(w_t / w_{t-15}) - \ln(\bar{r}_t / \bar{r}_{t-15})\right|\omega_{KK}$. The 15-year rise in the wage/rental ratio is quite large: 68% from the values in part C of Table 1. part C of Table 1. But then multiplying by the standard error of the capital-share equation, which is 0.012, and dividing by $\sqrt{2}$, we obtain the small standard error of 0.6 percent shown in parentheses in part C. This is about one-sixth the size of the standard error due to measurement error in the rentals, so the imprecision in fitting the translog function does not add very much to the standard error of the productivity index in this case.

Next, we need to check the various interaction terms between the measurement error in the rentals and in error in fitting the translog function; these are the second, fourth and fifth terms on the right of (25). Computing these for the 15-year changes in factor prices, and using the estimated coefficient $\gamma_{KK} = 0.10$, we obtain an additional standard error of 0.1%, also shown in parentheses in part C. Summing the squares of these various sources of error in the TFP index, and taking the square root, we obtain the total standard deviation of the 15-year TFP growth of 3.9%. The 95% confidence interval for 15-year growth (averaged over 1990-1992) is then (10.6%, 33.1%), which easily excludes zero. Therefore, we conclude the even taking into account the errors in computing the dual Törnqvist index, cumulative productivity growth in Singapore has indeed been significantly greater than zero.

9. Conclusions

The problem of finding a standard error for index numbers is an old one, and in this paper we have proposed what we hope is a useful solution. We have extended the stochastic approach to include both stochastic *prices* and stochastic *tastes*. The variance of the taste parameters, which affect the weights in the price index formula, is obtained by estimating a demand system. Our proposed method to obtain the standard error of prices indexes therefore involves two steps: estimating the demand system, and using the standard error of that regression (or system), combined with estimates of the sampling error in the prices measures themselves, to infer the variance of the price index.

While our methods extend the stochastic approach, they also extend the economic approach to index numbers by integrating the two approaches. It is worth asking why standard errors have not been part of the economic approach to indexes. Consider, for example, the problem of estimating a cost of living index. We could estimate the parameters of a model of preferences from data on expenditure patterns and then use these estimates to calculate a cost of living index. Yet, if the data fit the model perfectly, the cost of living index calculated from the parameter estimates would have the same value as an *exact* index formula that uses the data on expenditure patterns directly. Moreover, Diewert's (1976) paper showed that the types of preferences or technology that can be accommodated using the exact index approach are quite general. As a result, econometric modeling was no longer thought to be necessary to estimate economic index numbers.

A consequence of the lack of econometric modeling is that estimates of economic index numbers are no longer accompanied by standard errors, such as those that appear, for example, in Lawrence (1974). Nevertheless, if the model that underlies an exact index number formula has positive degrees of freedom, an error term will usually need to be appended to the model to get it to fit the data perfectly. This will certainly be the case if the consumption or production model is estimated over a panel data set with multiple commodities and years, which is our presumption. Indeed, we would argue that the assumption of the economic approach that taste parameters are constant between two years, when applied consistently over a time series, means that the parameters are constant over *all years* of the panel. This will certainly mean that the demand system must have an error appended, and as a result, the taste parameters and exact price index are also measured with error. We have derived the formula for this error in the CES and translog cases, but our general approach can be applied to any functional form for demand or costs.

In our application to Asian growth, we have contrasted the TFP estimate of Hsieh (2002) to those of Young (1992, 1995). Hsieh argues that the available evidence on returns to capital from financial sources *do not* show the decline that is implicit in the work of Young. Hsieh considers three different measures of the return to capital, and their associated rental prices. While the rentals differ markedly from each other in some years, the error in measuring the true rental is not enough to offset the underlying fact that their decline is *much less* than the cumulative rise in real wages: a 6.8 percent cumulative decline in the average rental over 15 years, as compared to a 61 percent increase in the real wage. Even when including the additional error from fitting a translog function to the data for Singapore, the error on 15-year cumulative TFP growth remains low enough so that its confidence interval is entirely positive. Taken over this sufficiently long time-period, there is compelling evidence that Singapore has enjoyed positive productivity growth, in contrast to the conclusions of Young.

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Table 1: Dual Total Factor Productivity Growth in Singapore

	Labor share	Growth Rate (Percent): Real Rental	Real Wages	Dual TFP	Primal TFP[a]
Part A: Revised from Hsieh (2002)					
Real interest rate used:					
Return on equity (1971 – 1990)	0.511	–0.20	3.64	1.76	–0.69
Bank lending rate (1968 – 1990)	0.511	1.64	2.86	2.26	–0.22
Earnings-price ratio (1973 – 1990)	0.511	–0.50	4.44	2.02	–0.66
Part B: Computed with annual data, Törnqvist index					
Real interest rate used:					
Return on equity (1973 – 1992)	0.418	–0.85	4.33	1.24	
Bank lending rate (1973 – 1992)	0.418	2.50	4.33	3.35	
Earnings-price ratio (1973 – 1992)	0.418	1.62	4.33	2.85	
Average rental price (1973 – 1992)	0.418	1.09	4.33	2.48	
Ave. stand. dev. (1973 – 1992)		(17.4)		(10.5)	
Average rental price (1975 – 1992)	0.424	–0.58	4.02	1.37	
Ave. stand. dev. (1975 – 1992)		(8.7)		(5.0)	
Part C: Computed with 15-year changes, Törnqvist index					
Real interest rate used:					
Average rental price (end-years 1990–92)	0.422	–6.82	61.0	21.8	
Ave. stand. dev. due to error in rentals		(6.6)		(3.8)	
Ave. stand. dev. due to translog error				(0.6)	
Stand. dev. due to interaction between errors				(0.1)	
Total standard deviation [b]				(3.9)	

Notes:
[a] Calculated by Hsieh from primal estimates in Young (1995), which depend on the sample period used.
[b] Computed as the square root of the sum of squared standard deviations listed above.

Table 2: Translog Estimation,
Dependent Variable – Capital Share

Sample	Constant	Rental)	ln(Real Wage)	ln(Real	ρ regression	S.E. of R^2, N
1974 – 1992:						
	0.63	-0.020	-0.14	0.50	0.010	0.92, 19
	(0.02)	(0.011)	(0.022)	(0.23)		
*constrained:**	0.50	-0.015	0.015	0.91	0.015	0.81, 19
	(0.08)	(0.014)	(0.014)	(0.10)		
1976 – 1992:						
	0.65	0.005	-0.14	0.44	0.011	0.89, 17
	(0.02)	(0.048)	(0.027)	(0.23)		
*constrained:**	0.72	0.10	-0.10	0.10	0.012	0.87, 17
	(0.02)	(0.011)	(0.011)	(0.27)		

* The constraint used is that the coefficients of the log real rental and real wage should be equal but opposite in sign. This constraint is rejected over the 1974-1992 period, but not rejected over 1976-1992.

Figure 1: Real Interest Rates in Singapore (Percent)

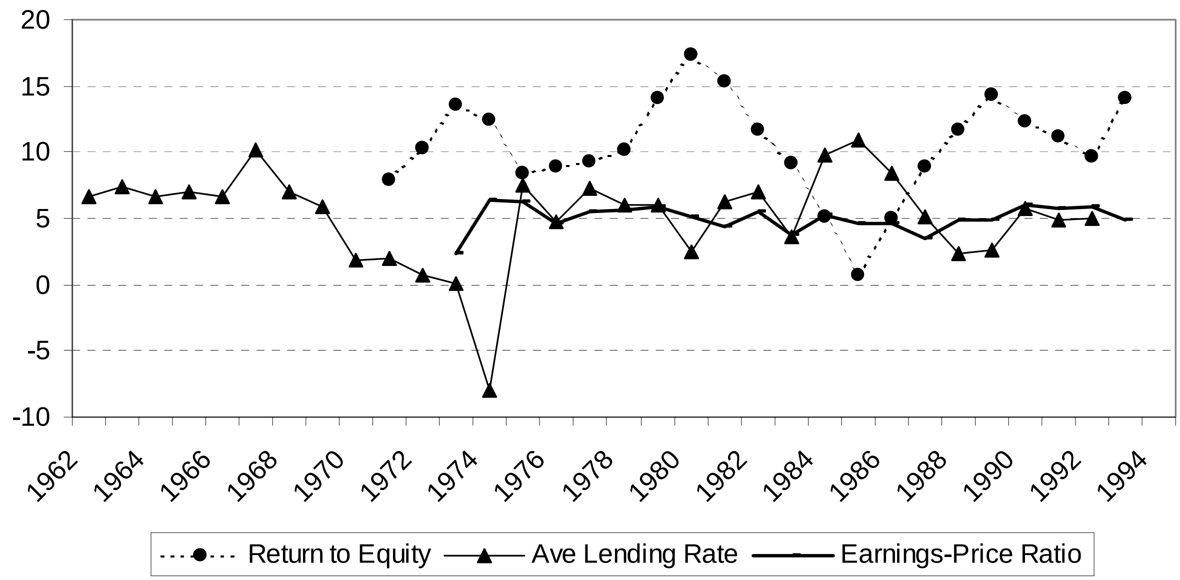
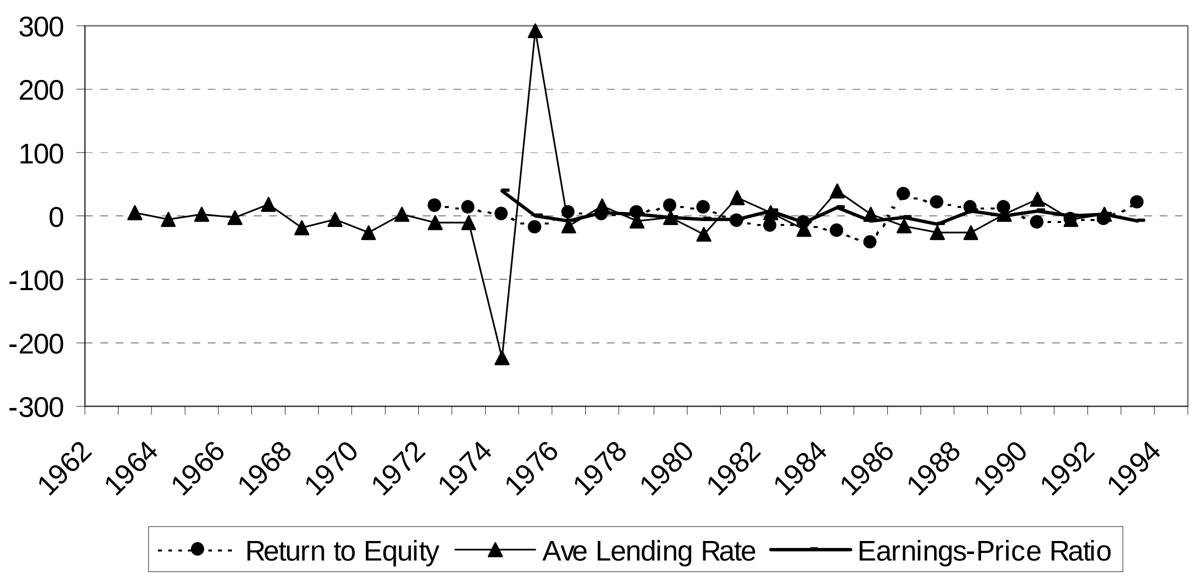


Figure 2: Change in the Rental Prices (Percent)



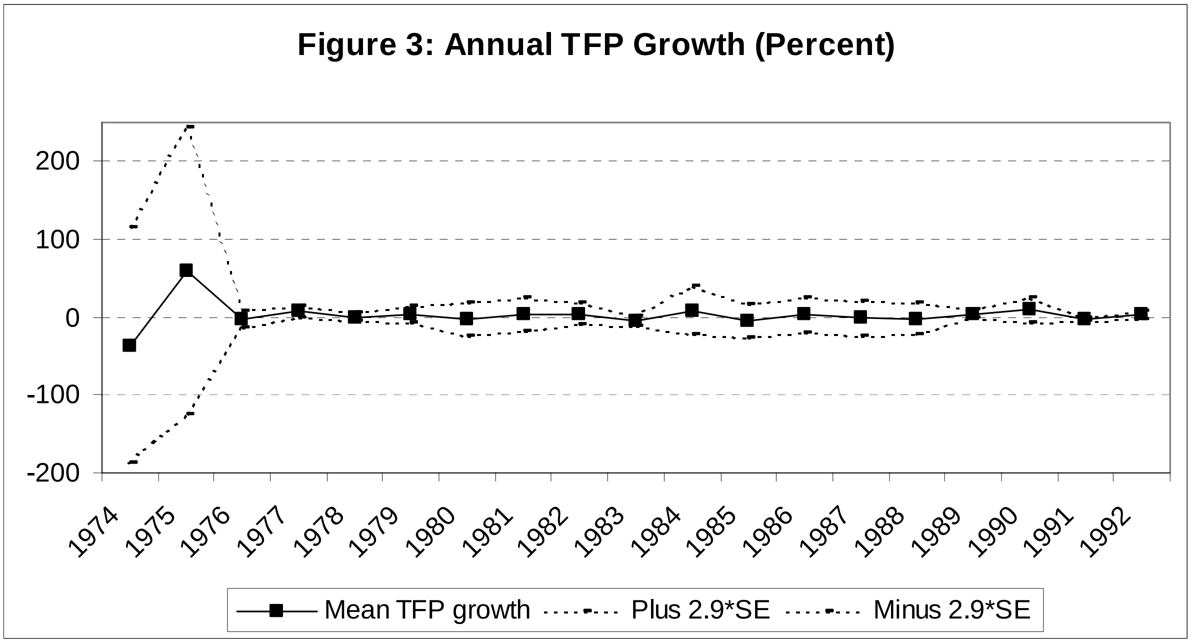


Figure 3: Annual TFP Growth (Percent)
200
100
0
-100
-200
1974
1975
1976
1977
1978
1979
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
Mean TFP growth
Plus 2.9*SE
Minus 2.9*SE

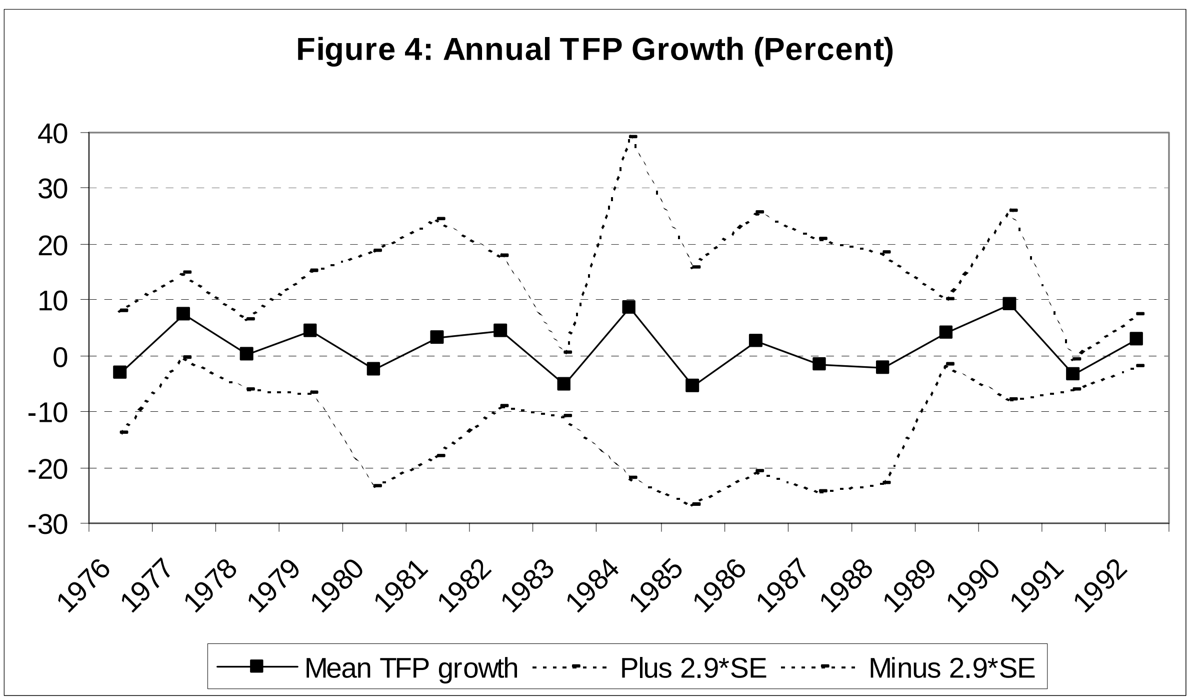


Figure 4: Annual TFP Growth (Percent)
40
30
20
10
0
-10
-20
-30
1976
1977
1978
1979
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
Mean TFP growth
Plus 2.9*SE
Minus 2.9*SE

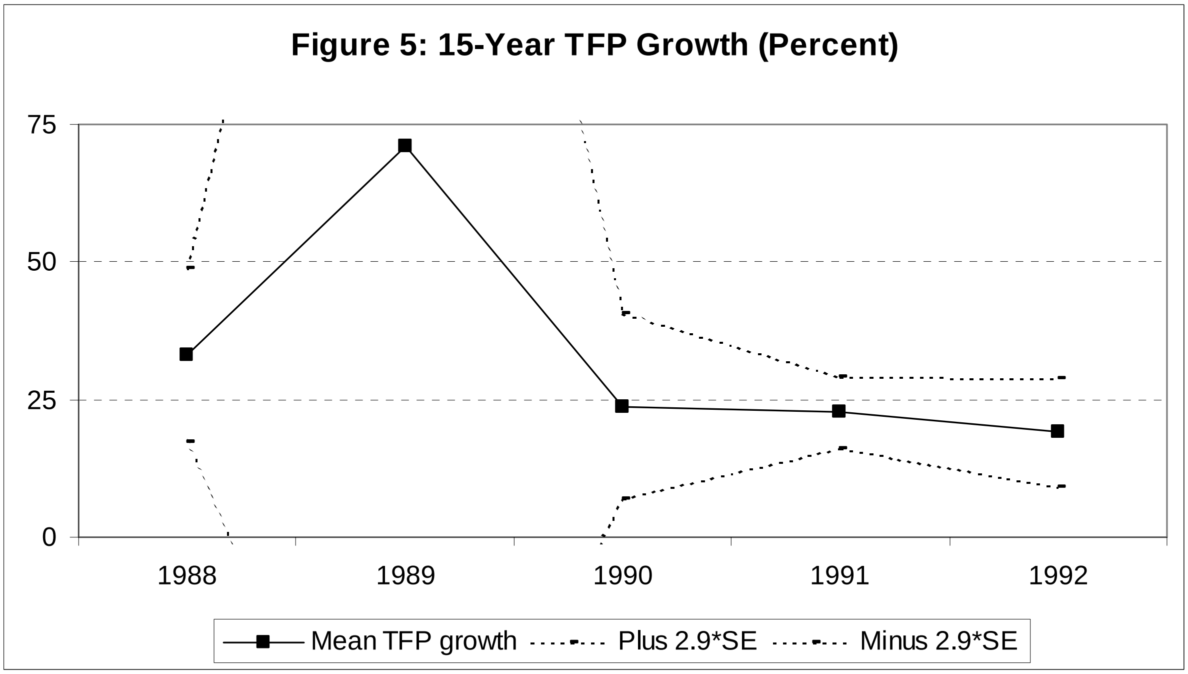


Figure 5: 15-Year TFP Growth (Percent)
75
50
25
0
1988
1989
1990
1991
1992
Mean TFP growth
Plus 2.9*SE
Minus 2.9*SE