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REGENERATION, LABOUR SUPPLY AND THE WELFARE COSTS OF TAXES

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ABSTRACT

This paper sets out alternatives to the traditional model of labour supply used to analyse the welfare costs of income and/or sales taxes when preferences are defined over goods and leisure and the market wage yields the slope of the budget constraint. The innovation in our work is to assume that some or all of non market time is used to regenerate the productivity of labour through rest and relaxation. This model has no closed form solution, but we can work with the first order conditions numerically for specific functional forms using non linear solution software. We generate a number of alternative parameterizations of this model through a series of calibrations to the same synthetic base case data set. Across the resulting parameterizations the welfare costs of taxes vary substantially (by a factor of twenty fold in some counterfactual analyses), even though they all involve calibration to the same base case data and labour supply elasticity. These results thus suggest that a small and seemingly plausible departure from a standard model (even if not in closed form) that has dominated the economic literature for many years can yield substantial change for perspectives on policy interventions.

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1. Introduction

This paper sets out an alternative to the standard model of labour supply in which preferences are defined over goods and leisure and the slope of the budget constraint is given by the wage rate. This allows us to explore how the composition of non market time can influence both labour supply responses to tax and wage rate changes, and the welfare cost of labour income or sales taxes. These are ideas that have seemingly been neglected in analytical work since the days of Marshall and Fisher when physical regeneration and degeneration of the human body entered discussion of labour markets (see quotes above).

In its most simple form regeneration is the use of non market time to improve the efficiency or quality of labour (i.e. regeneration of labour productivity) rather than the use of time as leisure to directly yield utility. In this case preferences are defined only over goods consumption. The allocation of time to non market activities is still endogenously determined but this reflects income rather than utility maximization (which has only one argument; consumption). Choices between market and non market time allocation are undistorted by taxes in this case, since either form of time allocation generates a taxable income return (which, in turn, is used to buy consumption goods). This yields two observationally equivalent models (traditional and regeneration models) each with endogenously determined market time allocation but with different implications for the distortionary costs of taxes. One dominates the literature, the other seems to be little discussed.[1]

We then develop a generalization which blends both traditional and regeneration models into a combined model which includes both leisure in the utility function and regeneration possibilities from the use of non-leisure time. This model does not yield closed form solutions to either individual or household optimisation problems, but we can work with it numerically solving the first order conditions for agent constrained utility maximizing behaviour. Changing share parameters on leisure (from smaller to larger)

[1] Widely cited pieces on the welfare costs of labour income taxes using the traditional framework are Hausman (1984) and Blundell and MaCurdy (1999). See also Blundell, Duncan and Meghir (1998), and Ham and Reilly (2002). See, however, the paper by Biddle and Hammermesh (199)) which discusses sleep and the allocation of time which has a formulation of labour supply decisions related to our approach. A recent real business cycle piece by Fisher (2001) discusses cases where household capital affects market productivity and is also related to our discussion. Neither of these pieces discusses welfare effects of taxes.

affects how of additional regeneration time influences labour efficiency and this allows us to move between the two extreme forms (traditional/pure regeneration) of the model.

Treating only market time as observable and regeneration time as unobservable, we show how many parameterizations can be constructed for this generalized model each of which is consistent with the same base case data on time allocated to market labour supply and consumption of goods as used in traditional models of labour supply and the same point estimate of the (uncompensated) supply elasticity of market time (labour) with respect to the wage. Since regeneration time is not directly observable there is no test available to discriminate between alternative parameterizations, generated in this way on any objective basis. However, the notion that there is non market activity regeneration seems intuitively plausible. It is a formulation of human behaviour with respect to labour supply seemingly closer to that used by to psychologists in their studies of behavioural response than is the conventional economics approach. It is also an approach used in business literature on the management of human resources.

For observationally equivalent parameterisations of this model there are different welfare costs of the same tax when parameterizations which are calibrated to different values of unobservable regeneration time are used in counterfactual mode. For model parameterizations that are observationally equivalent in the sense above, we are able to compute ranges of estimates of the welfare costs of the same tax (the ranges are 20 to 1 in some cases).

We also use a number of elaborations on this structure and approach to take the analysis further. We discuss two of these in detail in the text. One is where some market provided goods enter the regeneration process while others directly provide utility. In this elaboration, the usual presumption for a broadly based VAT breaks down and an added tax distortion between market provided goods and non market time enters the analysis. Here, the optimal tax on the market provided good entering regeneration is zero, but we can still evaluate the welfare costs of alternative configurations of tax rates across market goods. A second elaboration is a household model where time spent by one household member with others in the household regenerates both household members; effectively cross regeneration between the members of a household. For both of these model variants, we again compute a range of welfare cost estimates for the same tax across alternate combinations of data/elasticity admissible parameterizations.

We conclude by briefly discussing empirical implementation of the regeneration approach and its wider implications both for empirical work on tax distortions of labour supply and for policy. We note that direct observation of regeneration time (i.e. separately from time devoted to pure leisure) appears to be difficult if not impossible at present. Time use survey data allows for no simple division of time between leisure and regeneration activities since all time allocation seemingly has both leisure (consumption) and regeneration (production) elements (sleeping, for instance).

Empirical implementation of regeneration based models for now may thus be hard to undertake. But the findings we report are disturbing since they seem to provide a plausible recasting of analysis of the effects of taxes on labour supply yielding wide ranges of estimates of welfare costs and yield contrasting implications for policy from standard analysis. Even if a model is hard to operationalize it is not clear that it should play no role in economic policy debate. Here the model is simple, plausible, neglected and with differing predictions to the literature dominant model. Also, simple labour supply elasticities as conventionally estimated have a different interpretation under this approach, seemingly also with implications for econometric studies of labour supply behaviour we do not discuss here in detail.

The broader theme of the paper, therefore, is that a range of observationally equivalent (even if non operational) models are available for one of the classical issues in the public finance literature (the effects of taxes on labour supply) for which the welfare implications of policy interventions are sharply different. Some of these models have no simple closed form solution and may seem unappealing to theorists if there are no general results or properties, but they are plausible, relatively simple to use, and can be worked with numerically. Importantly they seem to yield quite different results compared to those generated by models conventionally used in the literature and also appear to be unexplored. This raises the issue of why economists should adhere so strongly to simple analytic structures with closed form solutions giving strong results when closely related and seemingly equally (or even more) plausible models incorporating relatively small departures from the simple structure (even if not yielding results in closed form) seem to change policy perceptions significantly.

2. The Neutrality of Taxes in a Simple Regeneration Model

The conventional labour supply model considers a single individual or household who maximizes utility defined over consumption, (*C*), and leisure, (*L*), subject to a budget constraint in which the wage rate, *w*, and the goods price, *p*, are given, i.e.

$$\max\ u(C,L) \tag{1}$$
$$\text{s.t. } p \cdot C = w(\overline{L} - L)$$

where $\overline{L}$ is the labour or time endowment.

A tax on labour supply at rate t distorts individual behaviour by changing the slope of the budget constraint, and problem (1) becomes,

$$\max\ u(C,L) \tag{2}$$
$$\text{s.t. } p \cdot C = w(1-t)(\overline{L} - L) + R$$

where R is tax revenue recycled in lump sum form to the individual paying the tax. The tax distortion of labour supply decisions in this model has a welfare cost associated with it.

Now suppose that non market time is instead treated as devoted to regenerating the efficiency of labour, such that with more rest labour becomes more efficient[2]. While a decision by the individual or household on how much time to allocate to market labour supply must still be made, leisure will no longer enter the utility function if this is the only permissible use of non market time (our simplest regeneration case).

In this case, we can use the alternative simple household optimisation problem

$$\max\ u(C) \tag{3}$$
$$\text{s.t. } p \cdot C = w(L_R)(\overline{L} - L_R)$$

to determine labour supply decisions, where $w(L_R)$ reflects the feature that the wage rate of the individual is now an increasing function of time devoted to regeneration, L_R. This

[2] An alternative formulation not explored here would be where labour productivity progressively degenerates the more time is spent each day in the work force. Fisher's formulation of degeneration cited above involved physical degeneration from particular non market activities (consumption of alcohol).

can be interpreted as implying that time spent on regeneration today increase the number of efficiency units of labour an individual can supply each hour the next day[3].

This formulation implies income maximizing behaviour since only goods appear in the utility function. From (3) the first order condition is

$$w(L_R) = w'(L_R)(\overline{L} - L_R) \tag{4}$$

or the wage obtained by supplying labour to the market equals the marginal return to regeneration time in the form of the higher wage times labour supply. In this event, a tax on labour supply applied to the budget constraint in (3) is neutral and has no impact since the individual simply income maximizes and both forms of time allocation (market and non-market labour supply) imply a taxable return. Thus, rest and relaxation improve labour efficiency and after some point diminishing returns to regeneration occur. In such a model, the amount of time devoted to market labour supply will still be endogenously determined, but in the simple case above seemingly distortionary labour income taxes are non distorting.

While identifying time devoted to regeneration is difficult in practice since time use survey data and other sources only report time devoted to particular activities (sleeping, eating, personal care, etc.) which may be interpreted as part leisure-part regeneration, the point remains that how we interpret time spent on non market activity affects model implications for policy. The alternative approach suggested here appears to have been little discussed in contemporary literature whether in the simple form of out early discussion or in the more complex forms that follows.

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[3] This idea is discussed at some length in Marshall's Principles (1895) (see p.694, Chapter XIII, 8th Edition), and is also implicit in Irving Fisher's well known calculation of the large benefits to the US economy from prohibition (Fisher (1926)). Since these writings by these two eminent economists, the idea that non market time regenerates labour through rest and relaxation seems to have disappeared from the literature. One could argue that it is implicit in human capital literature, where time invested raises productivity, and these formulations are clearly closely related, although it should be noted that Marshall also discusses human capital (see P. 561, where the heading is "investment of capital by parents in children") and treats this discussion as entirely distinct from regeneration as do we. Here, there is no intertemporal structure, and time input every day is needed to maintain or enhance labour productivity. The more recent literature on efficiency wages also seems relevant to this discussion (see Akerloff and Yellen (1986)), but typically in this literature there is only an exogenous distribution of efficiencies across workers, different to the formulation employed here of one individual with endogenously determined labour productivity.

3. A Generalized Model of Labour Supply and Taxes

The two models set out in section 2 can be combined to yield a generalized (or combined) model of labour supply and regeneration in which taxes still have distorting effects but these effects vary with leisure parameters in preferences and the strength of regeneration effects. For this generalized model we can construct parameterizations such that each is consistent with the same base case data and target (literature based point estimates of) labour supply elasticities. These parameterizations all meet a requirement of observational equivalence in the sense that they are each the result of calibration to the same base case data set and each have the same model labour supply elasticity as a point estimate around the base case model solution. But each parameterization yields different welfare cost estimates for the same labour income or sales tax rate (and in some cases sharply) even though in the absence of direct estimates of labour efficiency regeneration parameters there seems to be no clear procedure to distinguish between these specifications on empirical grounds.

This generalized model cannot be solved in closed form and so we use particular functional forms for preferences and the regeneration function and work with it numerically using GAMS[4] solution software. We assume preferences over goods and leisure are CES and the regeneration function $w(L_R)$ can be written as $\left(\overline{w} + \beta L_R^{\alpha}\right)$, where β and α are parameters. $\left(\overline{w} + \beta L_R^{\alpha}\right)$ is the hourly wage rate received by the individual and consists of two components; a base case wage rate $\overline{w}$ if labour quantity is not regenerated, and a regeneration component βL_R^{α} which is increasing in L_R but at a decreasing rate α $(\alpha < 1)$. β is a units term denominated in dollars per unit time. We use this form for the regeneration function as convenient and easy to work with numerically in the absence of any prior literature that we are aware of.

In this case, the optimizing problem for the household can be written, , as

$$\max \left[\gamma C^{\rho} + (1-\gamma) L^{\rho}\right]^{\frac{1}{\rho}} \tag{5}$$

$$\text{s.t.} \quad p \cdot C = \left(\overline{w} + \beta L_R^{\alpha}\right)\left(\overline{L} - L - L_R\right)$$

where γ and ρ are share and substitution elasticity parameters in CES preferences.

(5) can then be rewritten as the Legrangian

$$\mathcal{L} = \left[\gamma C^{\rho} + (1-\gamma)L^{\rho}\right]^{\frac{1}{\rho}} + \lambda\left[\left(\overline{w} + \beta L_R^{\alpha}\right)\left(\overline{L} - L - L_R\right) - p \cdot C\right] \tag{6}$$

for which the first order conditions are

$$\left.\begin{array}{l}
\gamma C^{\rho-1}\left[\gamma C^{\rho} + (1-\gamma)L^{\rho}\right]^{\frac{1}{\rho}-1} - \lambda p = 0 \\
(1-\gamma)L^{\rho-1}\left[\gamma C^{\rho} + (1-\gamma)L^{\rho}\right]^{\frac{1}{\rho}-1} - \lambda\left(\overline{w} + \beta L_R^{\alpha}\right) = 0 \\
-\left(\overline{w} + \beta L_R^{\alpha}\right) + \left(\overline{L} - L - L_R\right)\alpha\beta L_R^{\alpha-1} = 0 \\
\left(\overline{w} + \beta L_R^{\alpha}\right)\left(\overline{L} - L - L_R\right) - p \cdot C = 0
\end{array}\right\} \tag{7}$$

Given exogenous parameters γ, ρ, α, and β, this system of four equations can be solved numerically for the four unknowns C, L, L_R, and λ. The hourly wage rate the individual faces, $\left(\overline{w} + \beta L_R^{\alpha}\right)$, is endogenously determined. Even though the individual is a price taker in market transactions, the endogeneity of the individual specific hourly wage reflects the endogeneity of L_R. Unlike in the simple regeneration model set out above, if we introduce a tax on labour income at rate t into this model, this tax will be distorting since leisure enters the preferences of the individual (or household).

The consumption possibilities (for budget) set for this model is no longer linear as implied by the budget constraint for the simple labour supply model. As drawn in Figure 1 this set has a shape for which (as leisure falls) there is a segment of the budget set convex to the origin and then concave. This occurs since as more time is progressively devoted to regeneration, the resulting increase in the wage rate for the individual applies to a smaller amount of market time. As drawn, optimal behaviour involves the tangency point A. As ρ changes the curvature of the budget set changes, and beyond some critical value of ρ the budget set can become more steeply sloped than preferences in its portion which is convex to the origin resulting in problems in computation of optimal solutions. These problems are discussed below.

The equation system (7) does not yield a closed form solution, and hence wage elasticities of labour supply cannot be derived analytically. We can, however, parametrically vary $\overline{w}$ and/or β (and hence the market wage), and numerically determine

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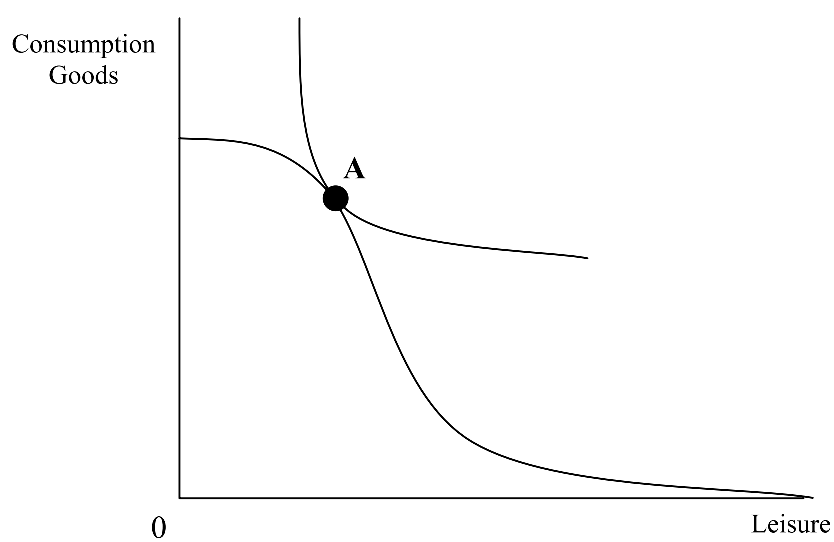
[4] GAMS denotes the Generalized Algebraic Modelling System (see Brooke, Kendrick, and Meeraus (1997)).

the response in both non market time $\left(\overline{L} - L - L_R\right)$ and the resulting hourly wage $\left(\overline{w} + \beta L_R^{\alpha}\right)$, and use these two responses to calculate a wage elasticity of labour supply. We use this method to determine a point estimate this elasticity for this model, i.e. the response in market labour supply with respect to the hourly wage rate. We compute this in the neighbourhood of the model solution implied by this equation system, even though the wage rate is endogenously determined and not exogenous in the elasticity calculation and the elasticity is not constant. We use this to relate the model's behaviour to existing literature on labour supply.

Figure 1

The Budget Set and Optimal Behaviour in a Generalized Model of Labour Supply and Regeneration



Relative to the simple optimizing structures used in existing labour supply literature, this equation system will tend to generate lower values of labour supply elasticities for given values of ρ, because wage increases generate a positive response in time devoted to regeneration as well as increased market labour supply as leisure consumption falls. Current labour supply elasticity estimates only measure the market time response not the regeneration response, which appear in available data as a higher hourly wage rate. The interpretation of existing estimates of labour supply elasticities thus needs to be qualified if one accepts that regeneration activities do in fact occur.

The equation system (7) can also be used to calculate the welfare costs of a labour income or sales tax using numerical simulation methods. We calibrate to a base case model solution and analyse a counterfactual solution under a tax change as in the applied equilibrium modelling literature (see Shoven and Whalley (1992)), except that for now we do not use an empirically based model admissible base case data set for an actual economy. We do this both because time spent on regeneration is unobservable, and also because our paper is expository rather than seeking to provide actual estimates of welfare costs of taxes for real economies. We construct a synthetic base case data set (in model admissible form) which we argue has loose empirical plausibility given the non observability of regeneration time.

Specifically, we take $\overline{w}$, C, p, the market wage rate $\left(\overline{w}+\beta L_R^{\alpha}\right)$ and labour supply $\left(\overline{L}-L-L_R\right)$ as observables and treat them as reflecting a base case optimisation solution in the presence of a given labour income tax rate we then set values for ρ and L_R to identify the equation system used in calibration. We assume L_R, time devoted to regeneration, to be unobservable and set L_R at some arbitrary specified value. This allows us to calibrate the equation system (7) to the resulting synthetic base case data set and determine values for the model parameters γ, α, β, and λ. Since ρ is also unobservable it can be iteratively adjusted to be consistent with values adopted for estimates of labour supply elasticities (from a literature survey). The resulting calibrated model parameter values determine a point estimate for the wage elasticity of labour supply in the neighbourhood of the base case model solution and we can adjust our choice of ρ so as to yield our target value for the labour supply elasticity. We can then compute a counterfactual model solution in the absence of the tax using a model parameterization generated in this way.

Table 1 presents an example of both a calibrated and counterfactual solution for this model for both tax and no tax cases. Here we have chosen (somewhat arbitrarily) the values of base case observables to reflect a 40 hour work week, 70 hours as the weekly time endowment, goods prices of 1, $\overline{w}$ of 1.3, and a market hourly wage of 2 for the individual in the presence of a 10% labour income tax with revenues recycled to the consumer in lump sum form. We calibrate the generalized model set out above to this data to determine the model parameter values γ, α, and β and then compute the welfare costs of a 10% labour income tax by comparing the base case data to a counterfactual model solution for a zero tax rate. In this case, ρ has been chosen such that the point estimate of the labour supply elasticity implied by the calibrated model parameters is 0.3. This is in the range of second generation labour supply elasticity parameter values suggested by Killingworth (1984) and is also consistent with more recent literature estimates discussed in Blundell and MaCurdy (1999) and Ham and Reilly (2002). The money metric measure of the welfare cost of a 10% income tax for this model specification is 0.116% of market labour income.

Table 1
The Welfare Cost of a Labour Income Tax in a
Generalized Regeneration Labour Supply Model

A. Assumed Observable Base Case Values used in Calibration				
$\overline{w} = 1.3$	$C = 80$	$P = 1$	$\overline{L} = 70$	$\left(\overline{L} - L - L_R\right) = 40$
A. Additional Assumptions Made on Parameters and Base Case Values				
$\rho = 0.479$		$L_R = 5$		
C. Model Parameter Values Determined By Calibration				
$\gamma = 0.502$	$\alpha = 0.357$	$\beta = 0.394$	$\lambda = 0.373$	
D. Point Estimate of (Uncompensated) Labour Supply Elasticity at Calibrated Equilibrium			$\eta = 0.300$	
E. Estimated Welfare Cost of a 10% Income Tax Calculated as Hicksian Equivalent Variation as % of Market Labour Income.			EV = 0.116	

4. Computing a Range of Welfare Cost Estimates for the Same Tax from Observationally Equivalent Model Parameterizations

Because both ρ and L_R are prespecified in the calibration procedures set out in Section 3 and L_R is unobservable, the generalized model set out in the previous section can be calibrated so as to yield a range of observationally equivalent model parameterisations. We limit ourselves to those parameterizations that are observationally equivalent in the sense that they each reproduce the same base case data as a model solution in the presence of the same tax, and also each imply the same point estimate of the labour supply elasticity around the same base case model solution. This allows us to generate a range of estimates of the welfare cost of a given labour income or sales tax by iteratively generating pairs of values for ρ and L_R under observational equivalence. This means that we calibrate the model to the same base case data for any (ρ , L_R) pair such that both the base case model solution and the implied point estimate of the labour supply elasticity are unchanged.

For each of these (ρ , L_R) pairs and the associated model parameterizations, we can remove the 10% labour income tax and compute the welfare cost of the tax in terms of the Hicksian equivalent variation as a % of base case labour income. These welfare cost estimates will differ as we change parameterisations, even though they all remain consistent with the same base case solution and the same point estimate of the labour supply elasticity at this model solution. For these observationally equivalent parameterizations estimates of the welfare cost of the same tax on labour supply can differ and potentially sharply.

To explore how large these differences in cost estimates can be we take different values for L_R and recalibrate the model, iteratively adjusting ρ in each case so as to preserve the point estimate of the labour supply elasticity at its initial value at the base case solution. This yields us a range of alternative model parameterizations, each generated by different ρ and L_R combinations and each observationally equivalent in the sense of being consistent with the same base case data and elasticity estimates. We then compute the welfare costs of a 10% labour income tax using each of these parameterisations, yielding ranges of welfare cost estimates for a series of observationally equivalent model parameterisations which we report below.

We begin by assuming that we have a target labour supply elasticity of 0.3 (again set somewhat arbitrarily). We again use a base case value of the market wage of 2, the same base case values for C, $\left(\overline{L} - L - L_R\right)$ and $\overline{L}$ as in Table 1, and assume an initial tax on labour income of 10%. We then vary both ρ and L_R, and for each pair maintain observational equivalence in the sense set out above, and compute the welfare gain from removing the 10% tax for each (ρ , L_R) pair. We report the Hicksian Equivalent Variation as a % of base case labour income in Table 2, both for this case and alternatives where $\overline{w}$ changes. This yields a range of L_R and ρ values for a given $\overline{w}$ value for which we can compute welfare cost estimates for the same tax. In computing these ranges, when we increase ρ beyond some critical value the problems discussed above with regard to the convexity of the budget set arise and we stop computation at this point.

Table 2

Ranges of Estimates of the Welfare Cost of a 10% Labour Income Tax from Observationally Equivalent Models with Labour Supply Elasticity = 0.3

Case 1			**Case 2**			**Case 3**		
$\overline{w}$ = 0			$\overline{w}$ = 0.5			$\overline{w}$ = 1		
L_R	ρ	EV(%)	L_R	ρ	EV(%)	L_R	ρ	EV(%)
0.1	0.425	0.110	0.1	0.425	0.110	0.1	0.425	0.110
1	0.433	0.111	1	0.433	0.111	1	0.433	0.111
2	0.442	0.112	2	0.442	0.111	2	0.443	0.112
6	0.483	0.114	6	0.484	0.115	6	0.487	0.116
10	0.531	0.113	10	0.535	0.114	10	0.545	0.118
14	0.591	0.106	14	0.598	0.108	14	0.620	0.116
18	0.666	0.093	18	0.680	0.096	18	0.718	0.108
22	0.762	0.071	22	0.779	0.074	22	0.847	0.090
27	0.947	0.021	25	0.877	0.048	23	0.889	0.081

Case 4			**Case 5**			**Case 6**		
$\overline{w}$ = 1.3			$\overline{w}$ = 1.5			$\overline{w}$ = 1.7		
L_R	ρ	EV(%)	L_R	ρ	EV(%)	L_R	ρ	EV(%)
0.1	0.425	0.110	0.1	0.425	0.110	0.1	0.425	0.110
1	0.434	0.111	1	0.434	0.111	1	0.434	0.111
2	0.443	0.112	2	0.444	0.112	2	0.446	0.113
6	0.491	0.118	6	0.503	0.123	3	0.461	0.117
10	0.565	0.126	8	0.551	0.134	4	0.484	0.123
14	0.681	0.143	10	0.632	0.163	5	0.518	0.136
16	0.772	0.171	10.5	0.655	0.181	6	0.596	0.180

Table 2 reports ranges of welfare cost estimates for the same 10% labour income tax for 6 cases, generated as set out above, and which only differ in the value used for $\overline{w}$. These cases imply ranges of welfare cost estimates which vary by a factor of up to 5 (case 1). We view these results as suggestive of difficulties in both interpreting existing estimates of welfare costs of labour income taxes and labour supply elasticities, since what is usually accepted in most models as leisure might equally well be time devoted to regeneration. Estimates of both welfare costs or taxes and labour supply elasticities obtained from conventional models should thus seemingly be viewed as subject to some unknown variation due to regeneration effects. These ranges behave differently across L_R as $\overline{w}$ changes across cases. In some cases welfare costs behave monotonically in L_R (e.g. rising in case 4, 5 and 6), in behaving non monotonically in others (cases 1, 2 and 3).

For the case where $\overline{w}$ = 1 (column 3 of Table 2) we have used a grid over L_R and again computed welfare costs to yield a welfare cost function over the L_R value used in calibration. This is set out in Figure 2 which shows the behaviour of the welfare cost of the 10% labour income tax as L_R varies in case 3 (where $\overline{w}$ is set equal to one). As L_R approaches zero we asymptotically approach a conventional labour supply model. As both L_R approaches 30 (no leisure consumption) and ρ approaches zero we move to a model in which there is no substitution in preferences between leisure and goods case 1 L_R = 27 is the maximum computable L_R value in this case. Results from these extreme cases are also set out in Table 3, and all welfare cost estimates reported in Table 2 lie between these ranges. These results thus suggest that welfare cost estimates of taxes in a generalized model of labour supply that includes regeneration effects should be thought of as bounded from above by estimates from traditional no regeneration models and from below by zero (as implied by a pure regeneration model).

In Figure 2 the welfare cost of the tax first increases then falls as the L_R value used in calibration increases. This occurs because the generalized regeneration model incorporates two channels through which tax distortions of leisure and consumption operate rather than one as in the traditional simple model. These are directly of labour leisure decisions through changed leisure consumption, and indirectly through changed regeneration activity. Depending on the parameters in the regeneration generated by calibration, tax distortions of the latter can be either more or less costly per dollar of revenue raised, and increasing the L_R value used in calibration can increase or decrease the distortionary costs of taxes in some cases, as Figure 2 shows.

Figure 2

Behaviour of the Welfare Cost Measure for a 10% Labour Income Tax Across (L_R; ρ) Pairs for Case 3 of Table 2 (see text for explanation)

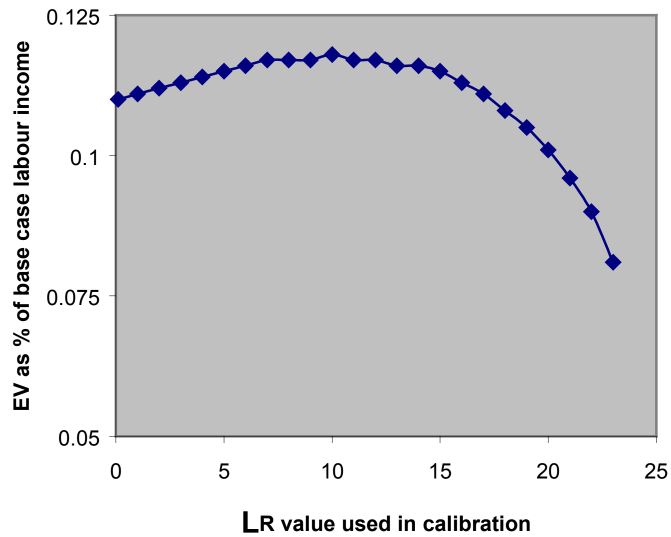


Table 3

The Welfare Costs of a 10% Labour Income Tax in Extreme Observationally Equivalent (Pure Regeneration Traditional) Variants of a Generalized Labour Supply Model

Approximation of Pure Regeneration Model		Traditional Model	
A. Base values used in Calibration	$\overline{w} = 0$ $w = 2$ $L_R = 27$ $\rho = 0.947$	A. Base values used in Calibration	$\overline{w} = 2$ $w = 2$ $L_R = 0$ $\rho = 0.425$
B. Welfare cost of a 10% labour income tax (EV as % of base case market income)	EV(%) = 0.021	B. Welfare cost of a 10% labour income tax (EV as % of base case market income)	EV(%) = 0.191

5. Results from Some Elaborations on the Generalized Leisure-Regeneration Model

We can also work numerically with a number of elaborations on the generalized leisure-regeneration model presented in the last section and for these cases similarly determine ranges of welfare cost estimates for the same tax from observationally equivalent model parameterizations. These cases serve to further highlight the ambiguities in predictions of the welfare effects of taxes using information only on base case allocations of time to market and non-market activities in a conventional labour-leisure model and estimates of labour supply elasticities. More generally, these results are suggestive of the general characterization that for any given issue a hierarchy of models of increasing complexity exists, each plausible and each incorporating unobservable characteristics and each yielding differing implications as to their outcome. While this may perhaps not be surprising, what this implies as to the interpretation of existing empirical estimates of welfare costs of taxes and labour supply elasticities seems both disturbing and little appreciated in existing literature.

Multiple Commodities

The first such elaboration we consider involves the additional specification of a market supplied commodity as an input into regeneration. This commodity is assumed to be distinct from the consumption good which along with leisure once again directly yields utility. Thus, if there are two goods C, R; only one of the goods, C, directly provides utility and the other, R, combines with L_R to yield regeneration services. In this case, the optimal tax structure across the two goods will have a zero tax rate on R, since on efficiency grounds the tax system should not distort between direct labour supply and regeneration activity as a mechanism for generating income. A broadly based sales or value added tax will thus no longer be optimal in this case.

Here, if regeneration involves a similar functional form with R and L_R as Cobb Douglas inputs to regeneration, the individual optimisation in (7) becomes

$$\max \left[\gamma C^{\rho} + (1-\gamma) L^{\rho}\right]^{\frac{1}{\rho}} \tag{8}$$

$$\text{s.t.} \quad P_C \cdot C + P_R \cdot R = \left(\overline{w} + \beta L_R^{\alpha_1} R^{\alpha_2}\right)\left(\overline{L} - L - L_R\right)$$

where P_C and P_R are now the prices of consumption and regeneration input goods.

This yields the Legrangian

$$\mathcal{L} = \left[\gamma C^{\rho} + (1-\gamma)L^{\rho}\right]^{\frac{1}{\rho}} + \lambda\left[\left(\overline{w} + \beta L_R^{\alpha_1} R^{\alpha_2}\right)\left(\overline{L} - L - L_R\right) - P_C C - P_R R\right] \tag{9}$$

for which the system of first order conditions can be written as

$$\left.\begin{aligned}
&\gamma C^{\rho-1}\left[\gamma C^{\rho} + (1-\gamma)L^{\rho}\right]^{\frac{1}{\rho}-1} - \lambda P_C = 0 \\
&(1-\gamma)L^{\rho-1}\left[\gamma C^{\rho} + (1-\gamma)L^{\rho}\right]^{\frac{1}{\rho}-1} - \lambda\left(\overline{w} + \beta L_R^{\alpha_1} R^{\alpha_2}\right) = 0 \\
&\left(\overline{L} - L - L_R\right)\alpha_1 \beta L_R^{\alpha_1 - 1} R^{\alpha_2} - \left(\overline{w} + \beta L_R^{\alpha_1} R^{\alpha_2}\right) = 0 \\
&\left(\overline{L} - L - L_R\right)\alpha_2 \beta R^{\alpha_2 - 1} L_R^{\alpha_1} - P_R = 0 \\
&\left(\overline{w} + \beta L_R^{\alpha_1} R^{\alpha_2}\right)\left(\overline{L} - L - L_R\right) - P_C C - P_R R = 0
\end{aligned}\right\} \tag{10}$$

In contrast to the model set out in section 4 above, this now implies a system of five equations in five unknowns, C, R, L, L_R, and λ. The exogenous parameters are now γ, ρ, α_1, α_2, and β. We can again calibrate this model in the presence of a labour income tax and then eliminate the labour income tax and compute the welfare cost of the tax. This tax will now be distorting both since leisure is in household preferences and two inputs, one market provided and the other non market provided, enter regeneration activities as inputs. An added tax distortion thus enters relative to the model used in the earlier section.

We have executed similar experiments to those which we report on in section 4 above with this model. We calibrate to a similar base case data set and labour supply elasticity estimate for assumed values of ρ, L_R and R. We can once again change the values of L_R or R and iteratively change values of ρ so as to preserve a particular value of the point estimate of the labour supply elasticity in the neighbourhood of the base case.

Table 4 reports estimates of ranges of welfare costs for the same 10% labour income tax using this elaborated version of the leisure-regeneration model. Here we can explore more combinations of variations in observationally equivalent parameterisations than in the case of the earlier model. We report two. In one, we vary $\overline{w}$ and R given values for L_R and C. These results suggest considerably larger ranges of welfare cost estimates because of the added input distortion and its effects in distorting optimizing decisions between L and C. In case 4, where $\overline{w}$ = 0.5, ranges of estimated welfare costs exceed 20:1.

Table 4

Ranges of Welfare Cost Estimates for a 10% Labour Income Tax from Observationally Equivalent Parameterizations for a Multiple Commodity Model with Labour Supply Elasticity of 0.3

A. Varying $\overline{w}$ and L_R given R and C

Case 1			**Case 2**			**Case 3**		
$\overline{w}$ = 0.5			$\overline{w}$ = 1			$\overline{w}$ = 1.7		
w = 2, R=1,C=79			w = 2, R=1,C=79			w = 2, R=1,C=79		
L_R	ρ	EV(%)	L_R	ρ	EV(%)	L_R	ρ	EV(%)
1	0.434	0.118	**1**	0.434	0.118	**1**	0.434	0.119
5	0.473	0.120	**5**	0.476	0.123	**2**	0.449	0.124
10	0.537	0.121	**10**	0.547	0.127	**3**	0.468	0.131
15	0.618	0.112	**15**	0.649	0.125	**4**	0.498	0.144
20	0.727	0.092	**20**	0.790	0.112	**5**	0.556	0.181

A. Varying $\overline{w}$, R (and C) given L_R

Case 4			**Case 5**			**Case 6**		
$\overline{w}$ = 0.5			$\overline{w}$ = 1			$\overline{w}$ = 1.7		
w = 2, L_R =1, C = 80 - R			w = 2, L_R =1, C = 80 - R			w = 2, L_R =1, C = 80 - R		
R	ρ	EV(%)	R	ρ	EV(%)	R	ρ	EV(%)
1	0.434	0.118	**1**	0.434	0.118	**1**	0.434	0.119
10	0.434	0.202	**10**	0.430	0.218	**3**	0.430	0.140
15	0.433	0.280	**15**	0.419	0.342	**5**	0.426	0.205
25	0.427	0.611	**20**	0.399	0.631	**7**	0.396	0.451
35	0.455	2.323	**25**	0.403	1.948	**7.9**	0.430	1.586

The optimal commodity tax structure for this version of the generalized model involves a zero tax rate on R[5], but we can still use this model to compute the welfare costs of non uniform tax structures. We consider 3 cases involving different tax rates for R and C, each of which raises the same revenue in the base case. We report a range of welfare cost estimates for these 3 tax rate configurations, using similar ranges of adjustment in L_R to those set out in Table 4 in each case. Results from these are reported in Table 5. The varying ranges of estimates of welfare costs of taxes adds further ambiguity to any policy prescriptions based on these structures.

Table 5

Ranges of Welfare Cost Estimates of Yield Equivalent Non Uniform Tax Rates on C and R Corresponding to Similar Ranges of (L_R, ρ) Variation in Table 3

Value for $\overline{w}$	Yield Equivalent Tax rate on R	Yield Equivalent Tax rate on C	(L_R,R) values preserving labour supply elasticity of 0.3		EV from tax elimination for each (L_R, ρ) pair (as % of base case income)	
$\overline{w}$ = 1	10%	10%	(1,1)	(20,1)	0.118	0.112
	10%	10%	(1,24)		1.402	
	5%	10.1%	(1,1)	(20,1)	0.116	0.114
	5%	12.1%	(1,24)		0.701	
	0%	10.1%	(1,1)	(19,1)	0.115	0.113
	0%	14.3%	(1,24)		0.412	

Household Behaviour

A second elaboration on the generalized leisure-regeneration model set out above considers regeneration within the household such that there are also cross effects between household members. In this case, the time devoted to regeneration by each household member also regenerates the labour (in efficiency terms) of the other household members. In this case, members of the household benefit positively from each other's company (negative effects could also be considered).

Thus, if regeneration activity for each household member involves a similar functional form to that used for L_R in (7) and the household optimisation problem involves a CES utility function defined over each individual's consumption of goods and leisure, optimisation implies

$$\max \left[\gamma_1 C_1^{\rho} + \gamma_2 C_2^{\rho} + \gamma_3 L_1^{\rho} + \gamma_4 L_2^{\rho}\right]^{\frac{1}{\rho}} \tag{11}$$

$$\text{s.t.} \begin{cases} p_c C_1 + p_c C_2 = w_1\left(\overline{L} - L_1 - L_{R1}\right) + w_2\left(\overline{L} - L_2 - L_{R2}\right) \\ w_1 = \left(\overline{w} + \beta_1 L_{R1}^{\alpha_1} L_{R2}^{\alpha_2}\right) \\ w_2 = \left(\overline{w} + \beta_2 L_{R2}^{\alpha_3} L_{R1}^{\alpha_4}\right) \end{cases}$$

where the parameters α_2, and α_4 reflect cross regeneration effects, and L_{R1} and L_{R2} are time devoted to regeneration activities by household members 1 and 2.

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[5] This is because, as discussed above, all returns to regeneration activity are fully taxed as additional labour income.

This yields the Legrangian,

$$\mathcal{L} = \left[\gamma_1 C_1^{\rho} + \gamma_2 C_2^{\rho} + \gamma_3 L_1^{\rho} + \gamma_4 L_2^{\rho}\right]^{\frac{1}{\rho}} + \lambda\left[w_1\left(\overline{L} - L_1 - L_{R1}\right) + w_2\left(\overline{L} - L_2 - L_{R2}\right) - p_c C_1 - p_c C_2\right] \tag{12}$$

$$\text{where} \quad \begin{aligned} w_1 &= \left(\overline{w} + \beta_1 L_{R1}^{\alpha_1} L_{R2}^{\alpha_2}\right) \\ w_2 &= \left(\overline{w} + \beta_2 L_{R2}^{\alpha_3} L_{R1}^{\alpha_4}\right) \end{aligned}$$

for which a system of first order conditions can be written as

$$\left.\begin{aligned}
&\gamma_1 C_1^{\rho-1}\left[\gamma_1 C_1^{\rho} + \gamma_2 C_2^{\rho} + \gamma_3 L_1^{\rho} + \gamma_4 L_2^{\rho}\right]^{\frac{1}{\rho}-1} - \lambda p = 0 \\
&\gamma_2 C_2^{\rho-1}\left[\gamma_1 C_1^{\rho} + \gamma_2 C_2^{\rho} + \gamma_3 L_1^{\rho} + \gamma_4 L_2^{\rho}\right]^{\frac{1}{\rho}-1} - \lambda p = 0 \\
&\gamma_3 L_1^{\rho-1}\left[\gamma_1 C_1^{\rho} + \gamma_2 C_2^{\rho} + \gamma_3 L_1^{\rho} + \gamma_4 L_2^{\rho}\right]^{\frac{1}{\rho}-1} - \lambda w_1 = 0 \\
&\gamma_4 L_2^{\rho-1}\left[\gamma_1 C_1^{\rho} + \gamma_2 C_2^{\rho} + \gamma_3 L_1^{\rho} + \gamma_4 L_2^{\rho}\right]^{\frac{1}{\rho}-1} - \lambda w_2 = 0 \\
&\left(\overline{L} - L_1 - L_{R1}\right)\alpha_1 \beta_1 L_{R1}^{\alpha_1 - 1} L_{R2}^{\alpha_2} + \left(\overline{L} - L_2 - L_{R2}\right)\alpha_4 \beta_2 L_{R2}^{\alpha_3} L_{R1}^{\alpha_4 - 1} - w_1 = 0 \\
&\left(\overline{L} - L_1 - L_{R1}\right)\alpha_2 \beta_1 L_{R1}^{\alpha_1} L_{R2}^{\alpha_2 - 1} + \left(\overline{L} - L_2 - L_{R2}\right)\alpha_3 \beta_2 L_{R2}^{\alpha_3 - 1} L_{R1}^{\alpha_4} - w_2 = 0 \\
&w_1\left(\overline{L} - L_1 - L_{R1}\right) + w_2\left(\overline{L} - L_2 - L_{R2}\right) - p_c \cdot C = 0 \\
&\text{where} \quad \begin{aligned} w_1 &= \left(\overline{w} + \beta_1 L_{R1}^{\alpha_1} L_{R2}^{\alpha_2}\right) \\ w_2 &= \left(\overline{w} + \beta_2 L_{R2}^{\alpha_3} L_{R1}^{\alpha_4}\right). \end{aligned}
\end{aligned}\right\} \tag{13}$$

This yields a system of seven equations in seven unknowns, C_1, C_2, L_1, L_2, L_{R1}, L_{R2}, and λ. This system has 11 parameters γ_1, γ_2, γ_3, γ_4, ρ_1, α_1, α_2, α_3, α_4 and β_1, β_2 (10 if we normalize such that $\sum_{i=1}^{4} \gamma_i = 1$). For this model to be identified and useable in calibration mode identifying restrictions must be used. For simplicity, we have again assumed a value for ρ and then arbitrarily assumed that α*'s* and β*'s* are related in ratio form i.e. we assume a ratio of α_2/α_4, α_1/α_3 *and* β_1/β_2. These identifying restrictions can be set as desired so as to yield stronger or weaker cross regeneration effects, but affect the ranges of welfare cost estimates we can generate.

This model form is more complex to work with numerically than the single agent formulations set out above since two elasticities are involved with cross effects. Thus changes in ρ affect the labour supply elasticity of the second household member and vice versa, and hence iterations on ρ must be made with corresponding variations in L_R for both household members.

Results using this model variant are presented in Table 6 using a variant with data similar to above but now for a two member household.[6] In this case calibration to two separate labour supply elasticities, one for each household member, is involved (we assume these are both 0.3). These results show ranges of welfare cost estimates for a 10% labour income tax on both household members. These ranges are smaller than for Table 2 since cross effects between household members now enter.

Table 6

Range of Welfare Costs Estimates for 10% Labour Income Tax on Both Household Members in a Generalized Household Regeneration Model with Labour Supply Elasticity of each household member = 0.3

Identifying Restrictions: $\alpha_2=\alpha_4$, $\alpha_3=\alpha_4$, $\beta_1=\beta_2$								
Case 1			**Case 2**			**Case 3**		
$\overline{w}$ = 0			$\overline{w}$ = 0.5			$\overline{w}$ = 1		
w = 2			w = 2			w = 2		
L_R	ρ	EV(%)	L_R	ρ	EV(%)	L_R	ρ	EV(%)
1	0.433	0.196	1	0.433	0.196	1	0.433	0.196
5	0.472	0.216	5	0.473	0.217	5	0.475	0.219
10	0.532	0.241	10	0.535	0.245	10	0.544	0.258
15	0.608	0.264	15	0.617	0.277	15	0.642	0.320
20	0.711	0.288	20	0.725	0.315	20	0.777	0.456
24	0.823	0.313	22	0.779	0.335	22	0.848	0.595

Case 4			**Case 5**			**Case 6**		
$\overline{w}$ = 1.3			$\overline{w}$ = 1.5			$\overline{w}$ = 1.7		
w = 2			w = 2			w = 2		
L_R	ρ	EV(%)	L_R	ρ	EV(%)	L_R	ρ	EV(%)
1	0.434	0.196	1	0.434	0.196	1	0.434	0.197
5	0.478	0.223	5	0.484	0.231	5	0.518	0.283
9	0.544	0.271	9	0.586	0.350	6	0.596	0.446

[6] Each household member has base case model solution values equal to those used in the model generating results in Table 1.

6. Empirical Implementation of Regeneration Models

While the numerical results we report above are only suggestive rather then definitive, they are in our view disturbing. We have been able to use seemingly plausible modifications and extensions to the traditional model of labour supply widely used to estimate labour supply elasticities and compute a range of welfare cost estimates for the same tax from observationally equivalent model specifications. While not yielding a closed form solution, the model we work with can nonetheless be used numerically and yields differing results from conventional models. In its more general form, we are able to show how there can be a wide range of estimates of the welfare costs of the same tax for observationally equivalent specifications of the same model, in which calibration replicates both base case data and elasticity responses.

The response from an empirically trained economist may be that there must surely be some way of discriminating among these estimates and thus narrow the range. There must surely be an empirical test which can be used for deciding which estimate to accept. To us, the disturbing feature of our analysis seems to be that, while plausible, regeneration based labour supply models seem also to be non-operational on empirical grounds. The notion that a certain amount of non-labour time goes to regeneration rather than pure leisure seems appealing, but no data seems to exist which allows regeneration and leisure attributes of time use to be distinguished. Sleep, for instance, seemingly has both attributes. Eating food similarly likely has both. Time use survey data as currently collected yields no direct information as to how time is employed in regeneration, nor its effects on households and market productivity. Short of direct estimation of the labour productivity effects of time devoted to various regeneration type activities there seems to be no data that can be directly used to parameterise the model.

Snippets of data seemingly exist in the efficiency wage literature for developing countries, where daily caloric intake is related to labour productivity at performing various tasks. But for developed countries such information is, to our knowledge, unavailable and so for now we conclude that the regeneration labour supply model should be viewed as important but effectively non-operational.

Should a non operational model of labour supply yielding different implications for the welfare costs of taxes be rejected as lacking any empirical basis, and work in the area only concentrate on conventional implementable models? Our contention is that there is no basis for this conclusion. The model we present seems plausible and interesting. It generalizes the existing literature dominant model. The reaction we have received to our work from other economists is that this model is both plausible and interesting. In implicit form it is in both Marshall and Fisher. The differences in results we report are striking. Our conclusion, therefore, is that conventional estimates of welfare costs must be evaluated as reflecting a subjective judgement call that other plausible structures which could potentially yield sharply differing results are rejected. The basis for such rejection seems to us unclear and unproven relative to the scientific standards to which much of modern economics aspires.

7. <u>Concluding Remarks</u>

In this paper, we present a model of labour supply behaviour in which the allocation of time to non market activity reflects the impact of time on the regeneration of labour productivity. This can be in place of time which enters preferences as leisure or coexist with leisure time. In a simple model in which all non market time goes to regeneration because households now income maximize a labour income tax has no impact on either welfare or labour supply even though labour supply is endogenously determined. This is in contrast to the conventional model of labour supply in which utility is defined over goods and leisure and no regeneration occurs.

Between these two extremes is a generalization of both conventional leisure based and regeneration models which nests both models time is devoted both to leisure and to regeneration activity. This model has no closed form solution, but we can work with the resulting analytical structure numerically. To do this we assume that time separately allocated to leisure and regeneration are unobservable and by varying weights on leisure and elasticities in CES preferences we generate a series of model parameterizations which are observationally equivalent in the sense that they all yield the same optimizing behaviour and all have the same implied point estimate for the labour supply elasticity around this model solution. Importantly, however, in combination they generate a range of welfare cost estimates for a given tax on labour income.

We also consider several generalizations of the joint leisure/regeneration model the first of which incorporates separate market provided consumption goods and goods which only enter regeneration as inputs. We also incorporate regeneration activities by multiple members of a household into a household based model. Similar exercises can be performed and further ranges of welfare costs computed for those model variants. The same themes of results remain but quantitative dimensions of ranges change.

We conclude by briefly discussing the empirical operationality of regeneration based models. We suggest they are analytically appealing, but little or no data exists in which regeneration and leisure are separated (sleep, for instance, involves both). We also highlight the dilemma that such models, while seemingly non operational, are hard to reject since they generalize conventional models while giving different numerical predictions of outcomes under tax or market wage changes ($\overline{w}$).

We conclude by posing the wider dilemmas this work raises for modern economic analysis. Should non operational but appealing generalizations of simple standard models be used to guide policy debates when their model predictions can differ sharply from those of simple models. What form of data should now be collected to guide model parameterizations of models in this light? How can subjectivity in model use in economics be dealt with? None of these questions have easy answers in our view, and nor does the question what is the welfare cost of a labour income (or sales) tax.

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