NBER WORKING PAPER SERIES

TAX POLICY, THE RATE OF RETURN, AND SAVINGS

Lawrence H. Summers

Working Paper No. 995

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge MA 02138

September 1982

I am grateful to Andrei Shleifer for research assistance, and to the National Bureau of Economic Research and the National Science Foundation for financial support. The research reported here is part of the NBER's research programs in Economic Fluctuations, Financial Markets and Monetary Economics, and in Taxation. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

NBER Working Paper #995
September 1982

Tax Policy, the Rate of Return, and Savings

Abstract

The theoretical and empirical results in this paper make a strong prima facie case for the proposition that increases in the after tax rate of return caused by tax policy are likely to bring forth significant increases in saving. Theoretical analysis using a variety of standard models tends to suggest that the aggregate response to savings incentives is likely to be substantial. It is argued that the existing empirical evidence sheds little light on the question. Empirical analyses are then conducted using three alternative approaches. All three confirm the hypothesis of a significant positive response of savings to changes in the rate of return.

Lawrence H. Summers
National Bureau of Economic Research
1050 Massachusetts Avenue
Cambridge, MA 02138

(617) 868-3909

The effects of the rate of return on the level of savings and the rate of capital formation are of central concern to both economists and policymakers. Although the welfare effects of tax reforms do not directly depend on their impact on savings, the effects of taxes on savings is crucial to considerations of tax incidence and equity and to the issue of long run growth. The impact of the rate of return on consumption and savings decisions also bears on questions regarding the appropriate government discount rate, the short run crowding out effects of fiscal policy, and the effects of public indebtedness on capital intensity.

The traditional view among economists is that changes in the rate of return are likely to have only a small effect on the savings rate. This consensus is supported by theoretical arguments pointing to the opposing income and substitution effects associated with changes in the rate of return. The ambiguous implications of theory are matched by empirical studies which yield conflicting estimates as to the size of the impact of changes in the rate of return. The polar empirical estimate is Michael Boskin's (1978) suggestion that the interest elasticity of savings is .4. This estimate is widely regarded as too high.

This paper re-examines the theoretical arguments and presents new empirical evidence regarding the interest elasticity of savings. Both the theoretical analysis and the empirical work demonstrate the strong likelihood that increases in the real after-tax rate of return received by savers would lead to substantial increases in long run capital accumulation. While it is not possible to quantify the impact with any precision, the econometric results here suggest that a shift towards expenditure taxation would lead to substantial increases in the private savings rate.

The theoretical analysis emphasizes the importance of recognizing heterogeneity among savers in examining the effects of tax changes which raise the rate of return available to savers. It begins by demonstrating that even if all savings decisions are determined by rule of thumb, savings are likely to be elastic with respect to the rate of return, as long as the rules of thumb differ persistently across households. The effects of changes in the rate of return on savings is then considered in a realistic multi-period life-cycle framework. Within such a framework, the importance of recognizing future labor income in analyzing savings is stressed. It is shown that for a wide range of utility function parameters, the interest elasticity of savings is likely to be positive. Since recent research suggest the importance of bequests in determining aggregate capital formation, models of intergenerational transfers are also considered. It is shown that as long as any part of the economy is comprised of households with operative intergenerational transfer motives, the long run elasticity of savings is infinite. Some illustrative calculations suggest that the short run impact is also likely to be substantial. While the sign of the response of savings to a change in the interest rate cannot be determined unambiguously from theoretical considerations, consideration of several models leads to a presumption in favor of a positive response.

No single empirical model can possibly capture all the many factors which influence aggregate consumption. This paper utilizes three different empirical approaches to examine the effects of changes in the rate of return on the level of capital accumulation. The fact that all three methods, each based on quite different simplifying assumptions, yield quite similar results suggests the robustness of the conclusion that rate of return effects on savings are both substantively and statistically significant. The first method is based on a modification of the standard consumption function specification to take account of the effects of changes in the interest rate on the level of human wealth.

This type of estimation is necessarily dependent on a procedure for modelling expectations. The second method follows recent work by Hall (1981), Hansen and Singleton (1981), and Mankiw (1981) in attempting direct estimation of the utility function of the representative consumer. Earlier work is extended by allowing for demographic changes and the existence of liquidity constrained consumers. A third approach involves estimating reduced forms linking the wealth-labor income ratio to the after tax real rate of return. This procedure allows for a more direct test of the impact of changes in the rate of return on capital intensity. The plan of the paper is as follows.

The implications of economic theory for the relation between the rate of return and savings are reviewed in the first section. The link between theory and previous empirical work on rate of return effects on savings is also considered. The second, third and fourth sections utilize three alternative empirical approaches in order to test the theoretical predictions. A final section summarizes the paper and considers the implications of the results for future research.

I. Taxation and Savings

In a closed economy, it is not possible to imagine how the rate of return to savers could change without other relevant economic variables also changing. Thus, it is necessary to be clear about the nature of the shock causing the rate of return to change. Discussions of the "interest elasticity of savings" are apt to be misleading since the change in savings associated with any given change in the rate of return to savers will depend on what caused the rate of return to change. The analysis here focuses on the effects of tax policies which alter the rate of return available to savers. Any tax change will affect revenue collections and so must be associated with changes in either government spending, public borrowing or other tax collections. The analysis here is all based on a differential incidence approach, where it is assumed that spending and total revenue collections remain constant so that changes in capital income taxes are offset by adjustments to payroll or consumption taxes. All the discussion is, therefore, about compensated effects. An effort is made to maintain this distinction in drawing implications from the empirical work reported below.[1]

The discussion here focuses on the "partial equilibrium effects" of a change in the rate of return. It is assumed that factor prices are unaffected by changes in the savings rate. Thus the analysis addresses the supply of savings schedule rather than the reduced form relationship between tax changes and capital intensity. In the special cases of a small open economy or a

[1] As noted below, "compensated savings effects" are not well defined because savings are not a commodity. As illustrated in Summers (1981), the effect of a tax change will depend on the timing of compensation.

production function with an infinite elasticity of substitution, the assumption of constant factor prices will be valid. Otherwise, it would be necessary to consider the aggregate production function in assessing the effect of a change in tax policy on savings.

It is natural to ask why the effect of tax policy on the level of capital accumulation is an interesting question, since the government can always neutralize the effect of any tax change by enlarging or contracting the deficit. For this reason, Stiglitz (1978) advocates the use of a "Balanced Growth Incidence" approach. As already noted, the excess burden associated with a capital income tax is not directly dependent on its impact on savings. At a minimum, in order to carry out an appropriate offsetting debt policy, it is necessary to know the savings effect of a given tax reform. Equally important, the government has multiple targets and may be unable or unwilling to assign debt management policy to the goal of attaining optimal capital intensity. Beyond its utility in determining optimal tax policy, an analysis of the effect of changes in the rate of return on private savings decisions is of scientific interest since it may help to explain differences in savings rates across time and space. The sensitivity of savings to the rate of return is also of central importance to macro-economic questions such as the relative short run efficacy of fiscal and monetary policies.

Rule of Thumb Savings

We begin by examining the simplest possible model of savings behavior, one which assumes that individuals have fixed savings propensities. Even in such a setting, compensated reductions in capital income taxes will increase capital accumulation as long as there is heterogeneity among income earners.

At the outset it is useful to review the effects of changes in tax policy in a simple economy where all individuals are identical and save a fixed s of disposable income. It is assumed that the labor force grows at rate n, and that the rate of labor augmenting technical change is g. In steady state, the ratio of capital to effective labor supply is constant. This condition implies that steady state capital intensity is determined by the condition:

$$s(w(1-t_w) + r(1-t_r)k) = (n+g)k, \tag{1}$$

where w is the wage, r is the rate of return on capital, and t_w and t_r are respectively the tax rates on labor and capital income. It is useful to record for future reference that government revenue R is given by:

$$R = t_w w + t_r rk. \tag{2}$$

Equation (1) implies that steady state capital intensity can be written explicitly as a function of the exogenous variables as:

$$k = \frac{sw(1-t_w)}{n+g-s(1-t_r)r} . \tag{3}$$

At this point we are ready to compare the effects of wage and capital income taxation on the level of capital accumulation. Two types of comparison are of interest. The short-run effect can be gauged by considering a revenue preserving tax cut on the level of savings holding the size of the capital stock constant. The long-run impact can be gauged by examining the change in the steady state capital stock following a change in tax structure. In the simple model described here it is easy to verify that any change from wage to capital income taxation which holds the level of revenue collections constant will have no impact on savings in the short run on capital accumulation in the long run. Note, however, that if the specification is taken literally it implies that a switch to expenditure taxation would raise savings. This is an artifact of the definition of disposable income and is of little economic significance.

These strong conclusions disappear once heterogeneity among savers is recognized. It is immediately apparent that in a Kaldorian world where the savings propensity out of capital income exceeds that out of labor income, a shift from capital income taxes towards labor income taxes will raise the savings rate. Here we consider the case where the savings rates among different individuals differ, but do not depend on the source of income. A variety of considerations suggest that there are likely to be large variations in individuals' propensity to save. Many individuals are permanently liquidity constrained. One estimate by Hall and Mishkin (1982) suggests this description fits 20 percent of the population. Individuals also differ in their desire to leave bequests, and in their subjective rates of time preference. These are further reasons to believe that savings propensities differ. This inference is strongly supported by the available empirical evidence. Cross section studies typically find substantial variations in individual consumption and wealth holding even after taking account of a variety of economic variables and individual characteristics. For example, Feldstein (1980) finds that after taking account of a large number of individual characteristics, the explanatory power of an equation explaining the wealth/lifetime income ratio of aged married couples was quite low.

There is little evidence available on the persistence of individual differences in savings propensities. In order to illustrate the effects of heterogeneity, I consider the polar case where all differences are permanent. To the extent that differences do not persist, the analysis here will overstate the effects of heterogeneity. Suppose that a fraction α_i of effective labor input is supplied by individuals in class i, who identically have savings rates s_i. Then it is clear that in steady state:

$$k_i = \frac{\alpha_i w(1 - t_w)s_i}{n + g - s_i r(1 - t_r)} \tag{4a}$$

$$k = \sum_{i=1}^{N} k_i , \tag{4b}$$

where k_i represents the capital owned by class i. The share of the capital stock γ_i owned by individuals of class i is given by:

$$\gamma_i = \frac{\dfrac{\alpha_i s_i}{n + g - s_i r(1 - t_r)}}{\sum\limits_{i=1}^{N} \dfrac{\alpha_i s_i}{n + g - s_i r(1 - t_r)}} . \tag{5}$$

Equation (5) implies that high savings classes own a disproportionate share of national wealth. We consider the effects of changing the tax rates on capital and labor income. Equations (2) and (5) imply that in the short run, with k constant:

$$\frac{\frac{ds}{dt_w}}{\frac{dR}{dt_w}} = -\sum \alpha_i s_i \tag{6a}$$

$$\frac{\frac{ds}{dt_r}}{\frac{dR}{dt_r}} = -\sum \gamma_i s_i . \tag{6b}$$

Inspection of equation (5) shows that the expression in (6b) is greater than that in (6a). Since capital is disproportionately owned by individuals with a high marginal propensity to save, taxes on capital income reduce savings more than taxes on labor income.

The potential significance of heterogeneity can be illustrated with a simple numerical example. Suppose there are two classes with equal labor incomes, so $\alpha_1 = \alpha_2 = .5$, and that $s_1 = .15$ and $s_2 = .05$. Suppose for approximate realism that $n + g = .03$, and that $r = .1$, $t_w = .2$ and $t_r = .5$. Then $\sum_{i=1}^{N} \alpha_i s_i = .10$ and $\sum_{i=1}^{N} \gamma_i s_i = .13$. This implies that replacing one dollar of capital income tax with that of labor income tax would raise national savings by about three cents. In this example, a switch to complete wage taxation which held revenue constant would raise the savings rate by about five percent in the short run.

This calculation describes the short-run impact of a change in tax policy. The long-run effect of substituting capital income taxes for labor taxes is to shift the distribution of after-tax income towards the high savings classes, further increasing the impact on capital accumulation. This is easily seen using the preceding numerical example. A switch to wage taxation raises the steady state capital stock and savings rate by 12 percent. It raises the share of capital owned by the high savings class from .78 and .83. This calculation underestimates the likely actual effect of heterogeneity because the dispersion of savings rates is quite narrow. A more realistic calculation which allowed for liquidity constrained consumers with zero marginal propensities to save would suggest even larger effects.

The basic result here would continue to be valid in any model of heterogeneous consumers. Tax changes which redistribute income towards "high savers" will raise capital accumulation. It is likely that individuals' wealth holdings are positively related to their savings propensities. Hence reduction in capital income taxes redistributes income towards persons with high savings propensities and increase total savings.

The same argument implies that any tax measure which redistributes income toward high income individuals is likely to encourage savings, since high income earners are likely to have high savings propensities, if only because the return on wealth is one component of income. A final application of the argument is to the effect of a tax levied at the corporate level, if individuals cannot completely pierce the corporate veil.

Life Cycle Savings

This section reviews and extends the argument in Summers (1981) suggesting that realistic formulations of the life-cycle hypothesis imply a very substantial long run response of capital accumulation to tax measures which change the after tax rate of return. The simulation results reported there are extended by considering a wider class of utility functions and transitory as well as permanent changes in the rate of return.

I work here with a simple continuous time formulation of the life-cycle hypothesis. It is assumed that all workers at a point in time receive the same real wage which grows at rate g, and that capital markets are perfect so that individuals can both borrow and lend at the riskless interest rate r. Also, I assume that individuals maximize a constant elasticity of substitution utility function with a fixed discount rate, subject to the constraint that the present value of future lifetime consumption equals the sum of assets and the present value of future labor income. That is, they solve the problem:

$$\text{Max} \int_t^T \frac{C_s^\gamma}{\gamma} e^{-\delta(s-t)} \, ds \tag{7}$$

$$\text{s.t.} \int_t^T \frac{C_s}{1-t^c} e^{-r(1-t^r)(s-t)} \, ds =$$

$$= A_t + \int_t^{T'} w_t \, (1-t^w) \, e^{-[r(1-t^r)-g](s-t)} \, ds.$$

where T is the individual's certain age of death, T' is the retirement age, w_t is the wage and t^r, t^w and t^c are the tax rates of interest income, labor income and consumption. Solving the maximization problem (1) yields the consumption and savings functions:

$$C_t = \frac{(A_t + HW_t)\left[\frac{r(1-t^r)-\delta}{1-\gamma} - r(1-t^r)\right] (1-t^c)}{e^{\left(\frac{r(1-t^r)-\delta}{1-\gamma} - r(1-t^r)\right)T}_{-1}} \tag{8a}$$

$$S_t = w_t(1-t^w) + rA_t\ (1-t^r) - \frac{Ct}{1-t^c} \tag{8b}$$

where HW_t equals the second term on the right hand side of (7). Note that the after tax interest rate affects consumption in two ways. The marginal propensity to consume out of total wealth $(A_t + HW_t)$ is a positive function of the rate of return if $\gamma<0$ and a negative function if $\gamma>0$. In the $\gamma=0$ case, which corresponds to the case of Cobb-Douglas utility, the propensity to consume is independent of the interest rate. Inspection of (7) shows that human wealth HW is unambiguously negatively related to the interest rate since increases in the rate of return reduce the discounted present value of future labor income. It follows immediately that if $\gamma\geq 0$, the uncompensated value of $\frac{dC}{dt^r}$ is positive, for consumers of all ages so that aggregate private savings is negatively related to the rate of capital taxation. If changes in capital tax rates are financed from permanent changes in consumption rates, the same result unambiguously holds for compensated changes.

In the more plausible case where $\gamma<0$, the effect of a reduction in capital income taxation is ambiguous depending on whether the "human wealth" effect outweighs the effect on the propensity to consume. Note that the human wealth effect is likely to be greatest for young workers since their labor income lies furthest in the future. Below, this is illustrated numerically for some plausible utility function parameters.

The maximum problem (7) can be solved to determine the effects of transitory as well as permanent changes in the rate of return. The analytic solution is cumbersome and not very revealing. Therefore, Table 1 reports some numerical calculations of the interest elasticity of consumption expenditure and savings based on plausible parameter values. The transitory change in the interest rate is assumed to last for five years. In these calculations, the parameter values assumed are: $T = 50$, $T' = 40$, $\delta = .03$, and $\gamma = .02$ and initial $r = .04$. The initial value of A_t for each age group is calculated from the solution to the maximization problem (7). The results regarding the sensitivity of savings and consumption to the after tax rate of return are not sensitive to these choices of parameter values. In interpreting the table, note that in some cases, which are asterisked, the initial savings rate is negative. These calculations can be interpreted as either uncompensated elasticities, or, since (8) implies that t^c does not affect the level of savings, as compensated elasticities of savings, where the adjustment to restore revenues is made through the consumption tax.

As one would expect, consumption responds more negatively to the rate of return in the cases where the elasticity of substitution between present and future consumption $\left(\frac{1}{1-\gamma}\right)$ is greater. Permanent changes in the rate of return reduce consumption by more, or increase it by less than do transitory changes. This is because of the greater impact of permanent changes in the interest rate on human wealth. The results also show a tendency for the interest elasticity of consumption to rise with age reflecting the diminishing importance of the human wealth effect.

These calculations have serious implications for previous empirical studies of the interest sensitivity of aggregate consumption. First, they show clearly for all age groups the importance of distinguishing between permanent and transitory changes in the rate of return. In all likelihood, the five year long transitory shocks considered here overstate the persistence of the variations in interest rates during the sample periods usually studied in empirical work. Thus, it is clearly possible that even though transitory changes in the rate of return are found to have little or no effect on consumption, permanent changes, such as those caused by tax policy, might have a large effect. The existing empirical evidence sheds no light on this possibility.

There is a second problem with traditional empirical approaches--the use of labor or disposable income rather than human wealth as an explanatory variable. The importance of this error can be seen by considering the $\gamma=0$ rows of Table 1. Since, in this case, the effect of changes in the rate of return on the propensity to consume is zero, the elasticities reflect only the human wealth effect. Clearly, they are quite substantial. It is, perhaps, surprising that, not taking account of these effects, previous empirical work has failed to find a positive relationship between the real rate of return and consumption.

In Summers (1981), I show how the effects of changes in the after tax rate of return on long run capital accumulation can be calculted by aggregating their effects on consumers of different ages. The following expression is derived for the steady state ratio of savings to disposable labor income.

$$\frac{S_t}{w_t(1-t^w)} = \left(\frac{r(1-t^r)-\delta}{1-\gamma} - r(1-t^r)\right)\left(e^{(g-r(1-t^r))T'} - 1\right) \tag{9}$$

$$\times \left(e^{\left(\left(\frac{r(1-t^r)-\delta}{1-\gamma}\right)-g-n\right)T} - 1\right)(n)(n+g)$$

$$\div \left\{\left(\frac{r(1-t^r)-\delta}{1-\gamma} - n-g\right)(g-r)\left(e^{\left(\frac{r(1-t^r)-\delta}{1-\gamma} - r(1-t^r)\right)T} - 1\right)\right.$$

$$\left.\times \left(1 - e^{-NT'}\right)\left(r(1-t^r) - n - g\right)\right\} - \frac{n+g}{r(1-t^r)-n-g}$$

Table 1

Effects of a Change in the Rate of Return on Life Cycle Consumers

Age	Value of γ	Permanent Consumption Elasticity	Permanent Savings Elasticity	Transitory Consumption Elasticity	Transitory Savings Elasticity
0*	0	-.70	-12.1	-.18	-3.18
0*	-1	-.33	- 2.3	-.09	- .65
0*	-5	-.112	- .59	-.03	- .17
0*	-50	- .02	- .09	-.01	- .03
10	0	- .55	15.70	-.18	5.07
10*	-1	- .25	- 5.1	-.10	-1.42
10*	-5	- .08	- .21	-.04	.162
10*	-50	- .001	- .620	-.02	.434
20	0	- .36	2.16	-.165	1.087
20	-1	- .14	.898	-.088	.50
20	-5	.004	- .049	-.054	- .149
20	-50	.061	- 1.341	-.018	- .536
30	-0	- .051	.71	-.114	.623
30	-1	.01	.17	-.041	.297
30	-5	.12	- .15	.005	.101
30	-50	.61	- .0271	.025	.020
40	0	0	.47	.077	.360
40	-1	.094	.270	.162	.195
40	-5	.161	.17	.218	.113
40	-50	.183	.19	.242	.082

Source: Calculations described in the text. In the cases marked *, the initial level of savings is negative.

Table 2 presents calculation of the savings rate given in (9) for a variety of combinations of utility function parameters and rates of return. The earlier assumptions that T=50, T'=40, and g=.02 are maintained and it is assumed that n=.02. The results bear out the conclusion in Summers (1981) that for plausible utility functions which generate savings rates reasonably close to those which are observed, the response of savings to changes in the rate of return is strong and positive. Calculation of elasticities is not very meaningful given that rates of return may be close to zero. In the case where $\gamma=-2$, which is supported by the empirical work reported below, each one percentage point increase in the real after tax rate of return raises savings by about 1.3 percentage points.

As the elasticity of substitution between present and future consumption, $\frac{1}{1-\gamma}$ declines the responsiveness of savings to changes in the rate of return falls off quite sharply. However, it becomes difficult to generate reasonable sized savings rates, without resort to negative rates of pure time preference. Furthermore large negative values of γ ($\gamma \leq -5$) imply an implausibly high degree of risk aversion.[1]

[1]The empirical work reported below provides strong evidence against the hypothesis that ($\gamma \leq -5$).

Table 2

The Interest Sensitivity of Aggregate Savings

(Value of r)

	.01	.02	.03	.04	.05	.06
$\gamma = 0$						
$\delta = .00$	.099	.163	.230	.305	.390	.489
$\delta = .01$	.024	.087	.152	.222	.299	.389
$\delta = .02$	-.051	.012	.075	.141	.212	.292
$\gamma = -1$						
$\delta = 0$	.062	.087	.114	.141	.171	.202
$\delta = .01$	.025	.050	.076	.102	.130	.159
$\delta = .02$	-.013	.013	.038	.064	.091	.119
$\gamma = -2$						
$\delta = 0$	.050	.062	.076	.090	.104	.119
$\delta = .01$	.025	.037	.051	.064	.078	.093
$\delta = .02$	.00	.013	.026	.040	.034	.068
$\gamma = -5$						
$\delta = 0$	.037	.038	.038	.040	.042	.044
$\delta = .01$	.025	.025	.026	.028	.030	.033
$\delta = .02$	.012	.013	.014	.016	.018	.021

Note: The calculations all assume that n = .02, g = .02, T = 50 and T' = 40. The numbers in the table are calculated values of $\frac{S}{(1-t^w)wL}$.

These results suggest that if the life cycle model accurately characterizes aggregate savings behavior, reductions in capital income taxes which increase the after tax rate of return have a significant positive effect on capital formation. The figures in the table give an indication of the consequences of measures to change the rate of return which are financed using consumption taxes. As is clear from (8), measures financed with wage taxes would have a somewhat smaller effect on steady state capital intensity.

The analysis here has considered only the additively separable constant elasticity utility function. While this is the only utility function which is both additively separable and homeothetic, the analysis is not completely general. However, the CES utility function can generate most plausible types of behavior. As $\gamma \to -\infty$, it approaches the "maximin" function. It is also capable of generating a smooth path of consumption with any given slope. The model here is also stylized in its treatment of demographic issues, and its assumption of certainty about the date of death. Tobin (1967) reports some more realistic simulations which suggest that the results are not sensitive to these omissions.

<u>Intergenerational Transfers</u>

Much of the theoretical and empirical analysis of savings has been carried out using models which ignore intergenerational transfers. Recent research suggests that this is likely to be an important omission. Laurence Kotlikoff and I (1981) have estimated that about 80 percent of

U.S. wealth holdings are the result of intergenerational transfers rather than life cycle savings. This finding is corroborated by the work of Mirer (1979) and Bernheim (1982) suggesting that liquid wealth does not decline with age following retirement, in contradiction to the implications of the pure life cycle hypothesis. It is also supported by the fact that the savings rate was quite high in the 19th century before retirement was an important economic phenomenon.

There are three standard ways of extending the pure life cycle theory to take account of intergenerational transfers.[1] The most straightforward is to assume that the utility function in (7) includes the level of bequests as an additional argument. This modification is exactly equivalent to treating bequests as an additional form of terminal consumption. It does not importantly alter the conclusions from the analysis of the life cycle model. A second possibility is that bequests are involuntary, resulting from life cycle savings in conjunction with the absence of perfect annuities markets. In this case, the preceeding analysis is also applicable. Kotlikoff and Spivak (1981) call the realism of this possibility into question by pointing out the role of family risk pooling in providing annuities.

Almost surely a preferable way of introducing intergenerational transfers is to postulate interdependent utility functions, in which the utility of parents depends on either the utility or the consumption of their offspring. That is:

$$U_t = U(c_t, U_{t+1}, (c_{t+1}, U_{t+2} (c_{t+3} \cdots\cdots\cdots\cdots \tag{10}$$

where the subscript t now indexes generations. This formulation is preferable because a motivation for bequests in terms of the welfare of subsequent generations is introduced. It has the additional virtue of

1) Yet another alternative would be to model "manipulative" bequests as described in Bernheim, Shleifer and Summers (1982). This would lead to results quite similar to those implied by pure life cycle calculations.

resolving the conflict between the very high "income elasticity" of bequests found in cross sectional work, and the much lower elasticity implied by the absence of secular increases in the savings rate.

Adopting this formulation has radical implications for the elasticity of savings with respect to the rate of return. Successive substitution using (10) demonstrates that the problem of generation t, is to choose C_t to maximize some function of the form:

$$U_t = V(C_t, C_{t+1}, \ldots) \tag{11}$$

In what follows, it is assumed that this function is homeothetic. Now suppose that consumption reaches a steady state level $\bar{c}$.[1] It follows immediately from the assumption that (11) is maximized that:

$$\frac{V_{t+1}(\bar{c})}{V_t(\bar{c})} = 1+r(1-t^r) = 1+\delta \tag{12}$$

That is, the marginal rate of substitution and the after tax interest rate are equated. Equation (12) implies that there is a unique after tax rate of return which is compatible with the existence of a steady state. It is not difficult to verify that if $r(1-t^r)$ exceeds δ, wealth grows without bound, and that if $r(1-t^r)$ is less than δ, it shrinks indefinitely. This implies that in the partial equilibrium sense considered here, the long run elasticity of savings with respect to the after tax rate of return is infinite. Of course, infinite responses are not observed in the world, because the accumulation of wealth drives down the rate of return.

---

1) The argument here could easily be modified to take account of the effects of population and productivity growth.

Now suppose that the utility function (10) characterizes the behavior of some but not all economic agents. The remainder can be thought of as life cycle, rule of thumb, or liquidity constrained consumers. The argument just made continues to imply that there is only one after tax rate of return which is compatible with the existence of a steady state. Otherwise, the wealth of the "bequest" class will be growing without bound, and so its consumption will be ever increasing. In a setting with some but not all consumers possessing bequest motives , the general equilibrium rate of return will be determined by the bequest consumers. The distribution of wealth will depend on the savings propensity of the remaining consumers at this interest rate. It follows that as long as any part of the economy is characterized by intergenerational altruism of the type suggested by Barro (1974), the long run partial equilibrium elasticity of savings will be infinite.

Of course, the long run elasticity may not be a very useful guide to the short run response of the economy to a tax change. However, the illustrative calculations of Chamley (1981) suggest that convergence to a new steady state is fairly rapid. He finds that with a Cobb-Douglas production function, about 10 percent of the adjustment to a new steady state occurs within one year, in the Cobb-Douglas utility case, and about 7 percent occurs if $\gamma = -1$. Much more rapid convergence occurs with elastic labor supply. His calculations imply that a permanent increase in the after tax interest rate from 4 to 5 percent would raise savings by 40 percent in the short run in the Cobb-Douglas case and 30 percent in the $\gamma = -1$ case, even without allowing for variable labor supply. These figures would be lower if only a fraction of savings is done by bequest consumers. However, it should be emphasized that even if only a small fraction of consumers have bequest motives, they are likely to account for a large part of the savings. This inference is supported by the great inequality in wealth holdings.

The analysis so far has failed to recognize a variety of complicating factors. Labor supply has been assumed to be inelastic. Relaxing this assumption would strengthen the conclusions reached here, since increases in the rate of return would cause consumers to shift their work effort towards the earlier stages of the life cycle. Liquidity constraints have not been incorporated into the analysis. Including them would also be likely to strengthen the conclusion that compensated increases in the net rate fraction would raise savings. Tax reforms which reduced capital income taxes would redistribute resources away from the consumers who by definition have a marginal propensity to save of zero towards other consumers, raising the aggregate savings rate. A final complicating factor is uncertainty about future labor income and future rates of return. As Sandmo (1970) has shown, uncertainty has an ambiguous effect on the level of savings. There does not seem to be any clear reason for expecting the effects of deterministic changes in the rate of return, such as would be brought about by tax reforms, to be affected by the presence of uncertainty.

The theoretical arguments here suggest that economic theory creates some presumption that savings should respond positively to changes in the after tax rate of return. The standard argument pointing to conflicting substitution and income effects is misleading in several respects. First, for most purposes, it is the compensated effect of changes in interest rates that is relevant. Second, the usual argument implicitly assumes that all income is received in the first period. Allowing for human wealth effects makes it far more plausible that savings respond positively to changes in the after tax rate of return. Third, the standard analysis neglects the effects of heterogeneity emphasized here. As long as some savers have a very elastic response to changes in interest rates, their behavior is likely to determine the aggregate response to changes in the rate of return.

While the considerations stressed here are suggestive, it is certainly possible to construct models in which the rate of savings responds negatively to increases in the rate of return. The question is ultimately an empirical issue. In the next sections, we explore alternative empirical approaches to resolving it.

II. Structural Consumption Function Estimation

A number of studies, including Wright (1969), Weber (1970, 1975), Boskin (1978), Howrey and Hymans (1980), and Blinder (1981), have attempted to estimate the effects of changes in the rate of return on consumption and savings. Despite the profusion of studies, no consensus has been reached. Only Boskin obtains a statistically significant and substantially positive estimate of the interest elasticity of savings. Howrey and Hymans (1980) show that his results are sensitive to the choice of sample period and to issues of data construction. The other studies find insignificant effects, although in some cases nominal pre-tax rather than real after tax returns are used in the estimation. As Feldstein (1970) demonstrates, this is likely to cause serious underestimates of the effects of changes in the properly measured real net yield.

All of these studies attempt to investigate interest rate effects by adding an interest rate variable to a standard life cycle consumption function involving disposable income and wealth as arguments. The coefficient of the interest rate variable is then used as basis for inferring the interest elasticity of savings. There are several serious problems with this type of approach. First, the equation deviates from the life cycle theory discussed in the preceeding section in several respects. The most important of these is the inclusion of disposable income rather than an estimate of human wealth. This obscures an important channel through which the interest rate might be expected to affect savings and consumption decisions. An additional problem is posed by the use of an income measure which includes capital as well as labor income. This leads to a kind of double counting since the wealth variable already represents the present value of current and future capital income. The specification also does not allow the propensity to consume out of wealth to depend on the interest rate as the theoretical model suggests that it should.

The second difficulty is that this specification makes it difficult to interpret movements in the interest rate. It is not possible to conceive of circumstances in which the real net yield would change, but disposable income and wealth would remain constant. This means that the partial derivative of the consumption function equation with respect to the included interest rate variable is not directly informative as to the effect of a change in the interest rate. Since increases in real interest rates are likely to be associated with increases in disposable income and decreases in wealth, there is a strong presumption that the standard procedure will understate their positive impact on savings.

A third problem with standard consumption function approaches is difficulty in measuring accurately all the relevant variables. The variable normally used to proxy the rate of return in most studies is the real after tax bond yield. This involves the construction of a measure of long term inflationary expectations, and the average marginal tax rate on interest income. Neither can be measured with much precision. In any event, bonds represent only a very small fraction of wealth, so it is unclear how well this yield provides the rate of return which will be received on all savings. Insofar as the rate of return is measured with error, the estimated response of savings to changes in the interest rate will be biased towards zero. There are equally serious difficulties involved in measuring expected future income. Lucas (1976) has pointed out the limitations of the standard distributed lag expectational formulations.

Additional problems with standard consumption functions as vehicles for examining the interest sensitivity of savings include failure to include other relevant variables such as those reflecting the age structure of the population. There is also a problem of simultaneous equations bias. Savings decisions determine the accumulation of wealth which in turn affects future income. In an important recent study, Auerbach and Kotlikoff (1981) illustrate, in a simulation context, the seriousness of the problems discussed here. They simulate the behavior of an economy in which the life cycle hypothesis holds exactly, and then estimate standard consumption function specifications. The results indicate that the parameter estimates are extremely sensitive to the choice of sample period, and that the estimated parameters do not provide a useful guide to the effects of policy interventions.

These considerations suggest that structural consumption function estimates do not provide a very useful basis for estimating the effects of changes in the rate of return on savings behavior. There is some presumption that the effect is underestimated because of errors in the measurement of the rate of return and the failure to take account of interest rate effects on savings and income. In the next section I present estimates of the effects of the rate of return on savings which are based on the direct estimation of first order conditions. These circumvent most of the difficulties raised in the preceeding paragraphs and, I believe, provide the best available method for estimating the effects of tax policy changes. Because structural consumption functions have been used in most recent work in this area, it is perhaps useful to reconsider the role of interest rates in equations of this type. This is done below. It should be clearly recognized that these estimates do suffer from some of the problems just considered, although a number of important difficulties are avoided.

Equation (8a) is approximated by an aggregate consumption function of the form:

$$C_t = \alpha + (\beta_1 + \beta_2 R_t)\left(A_t + \frac{YL^e_t}{R_t+d}\right) + u_t \qquad (13)$$

This equation holds that consumption is a function of total wealth (including human wealth) with a propensity to consume which depends on the real after tax long term interest rate . Human wealth is estimated as permanent disposable labor income discounted at the sum of the real after tax long term interest rate and an econometrically estimated risk premium d.[1] Equation (13) is estimated using non-linear least square with annual data the 1950-1978 period. While it would be possible to estimate this equation using quarterly data, this was not attempted because the primary interest here is in the effects of low frequency changes in the independent variables. The sample period was stopped in 1978 because the disposable labor income series used in the estimation is not available past that point.

Before examining the results, it is necessary to describe the construction of the variables. The dependent variable in the equations reported here is real consumption expenditure per-capita drawn from the National Income Accounts. This measure treats outlays on durable goods as consumption rather than savings. The equations were re-estimated for consumption with the services of durable goods imputed. These modifications had little impact on the results and are not reported here. The value of A_t is the MPS model series on the market value of wealth as reported in Hayashi (1982). The variable is used in per-capita form. Assets are included at market value rather than replacement cost. A variety of measures of permanent labor income, YL^e , were tried.

In the results reported here, YL^e is estimated as a three period

[1] The formulation adopted here thus explicitly models the human wealth effect discussed above. It also avoids pitfalls in simulation by using disposable labor income rather than total disposable income. A final advantage of this formulation is that the interest rate is entered as a factor effecting the marginal propensity to consume.

distributed lag on per-capita disposable labor income, with the weights constrained to sum to one. The data on disposable labor income are also drawn from Hayashi (1982). Alternative lag structures (including relaxing the sum constraint) had little effect on the results and so the estimates using them are not reported here.

Four alternative measures of the real after tax rate of return were tried. The after tax nominal bond yield was proxied alternatively by the municipal bond yield and by AAA yield multiplied by an estimate of the average marginal personal tax rate on interest income. Inflation expectations are proxied using the rolling ARMA procedure described in Summers (1982b), π^e_1 and a weighted average of past rates of inflation, π^e_2. Combining two alternative bond yields, and two alternative measures of expected inflation yields four measures of the real after tax rate of return.

The estimated equations are displayed in Table 3. All the equations yield positive estimates of β_1 implying that $\gamma<0$. This indicates that the marginal propensity to consume out of wealth is a rising function of the rate of return. The estimated risk premium used in discounting future labor income ranges between .09 and .14[1]. While these estimates may seem large, they are only a little greater than the risk premium attached to dividend income. Labor income is likely to be much less easily diversified, so these estimates are not unreasonable. It is encouraging that similar estimates are obtained using each of the rate of return variables. In all cases the explanatory power of the equations is very high, and comparable to that of standard structural consumption functions.

The two effects of interest rate changes in the equations reported in Table 3 conflict. Increases in the interest rate reduce human wealth, but

[1]This terminology may be misleading since d will also capture the effects of trend growth in income.

Table 3

Structural Consumption Function Equations

Equation		α	β_1	β_2	d	$\bar{R}^2$	DW
1	$R_1 = R_{mun} - \pi^e_1$	-.001 (.063)	.405 (.125)	.066 (.014)	.094 (.028)	.994	1.49
2	$R_2 = R_{mun} - \pi^e_2$	-.318 (.090)	.318 (.100)	.090 (.012)	.137 (.029)	.993	1.66
3	$R_3 = R_{AAA} \cdot (1-\theta) - \pi^e_1$	.012 (.074)	.397 (.149)	.068 (.017)	.101 (.036)	.993	1.40
4	$R_4 = R_{AAA} \cdot (1-\theta) - \pi^e_2$	-.337 (.088)	.324 (.097)	.090 (.012)	.137 (.028)	.994	1.63

Note: Estimates obtained using nonlinear least squares for the 1950-1978 period. Distributed lag weights used in forming YL^e are not reported.

Table 4

Effects of a 1 Percentage Point Change in the Rate of Return on Consumption and Saving

	Equation 1		Equation 2		Equation 3		Equation 4	
Initial R	%ΔC	%ΔS	%ΔC	%ΔS	%ΔC	%ΔS	%ΔC	%ΔS
-.02	-2.48	20.73	-1.61	13.44	-1.96	16.42	-1.53	12.82
-.01	-1.61	13.49	-1.18	9.90	-1.25	10.46	-1.12	9.32
0	- .97	8.09	- .84	6.99	- .71	5.94	- .77	6.45
.01	- .48	3.99	- .55	4.59	- .29	2.45	- .49	4.08

Note: These estimates are based on the equations reported in Table 3. All the calculations are done using 1978 values for the independent variables other than R. The calculations assume that no other variable changes when R does.

raise the propensity to consume out of wealth. The total effect of changes in the real rate of return is evaluated in Table 4. The effect of a 1 percent increase in the real rate of return on consumption expediture and savings is calculated using 1978 values of the exogenous variables and various initial real rates of return. These calculations refer to the effect of a change in the real return caused by a reduction in taxes on interest income, which is compensated for by a change in taxes on consumption expenditure. The results almost universally suggest than an increase in the after tax real return would have a significant effect.[1]

For most of the last few years, real after tax bond yields have been significantly below zero. The estimates here thus suggest that a one percent increase in the real rate of return would be likely to raise savings by close to 10 percent in the short run.

These estimates suggest that structural consumption functions estimated in ways consistent with the underlying theory, imply significant effects of interest rates on savings.[2] However, they involve a somewhat unsatisfactory treatment of expectations, and serious problems in measuring expected real returns. An alternative approach is presented in the next section.

---

[1]The numbers in $\%\Delta C$ columns of the table should be interpreted as the change in consumption inclusive of consumption taxes.

[2]It might be objected that this is a consequence of the specification's twinning the labor income and rate of return variables. Efforts to explore this by adding YL_t as a separate variable were not successful because the parameter estimates failed to converge.

III. Direct Estimation of Utility Function Parameters

This section reports on an attempt to directly estimate the parameters of the utility function of the representative consumer. The approach here follows the work of Grossman and Shiller (1981), Hall (1981), Hansen and Singleton (1981), and Mankiw (1981) in attempting to estimate the Euler equation for the representative consumers' stochastic dynamic optimization problem. Their work is extended by allowing for the possibility that some consumers are liquidity constrained.

Consider a consumer choosing a lifetime consumption plan. She always has the option of consuming one dollar more or less at time t, and investing or disinvesting in any available asset and then consuming the proceeds at time t+1. It follows that:

$$U'(c_t) = E(U'(c_{t+1})(\frac{1+r}{1+\delta}) \mid \Omega_t) \qquad (14)$$

where δ is the subjective discount rate attached by the consumer to future utility and r is the real return on any freely traded risky asset and Ω_t is the full information set at time t. Note that the first order condition given by (14) will be satisfied for any free traded asset even if some assets such as human capital cannot be traded freely. This condition will hold for consumers who expect with certainty to be alive in the next period regardless of the length of horizon of their maximization problem. Note also that this condition does not depend on any assumptions about expectations regarding future labor income or rates of return. The assumption is maintained here that different types of consumption may be aggregated and that the utility function is separable in consumption and leisure. The importance and validity of these assumptions are discussed in Mankiw, Rotemberg and Summers (1982).

In order to exploit (14) it is necessary to assume that aggregate consumption can usefully be modelled as the outcome of utility maximization by a representative individual. It should be clear that this assumption is not exactly accurate. However, it is to be hoped that the estimated utility function for the representative consumer in some sense typifies individual utility functions. Grossman and Shiller (1981) rigorously justify this hope in a continuous time setting. I assume that the instantaneous utility function has the familiar constant elasticity form $U_t = \frac{C^\gamma_t}{\gamma}$. With this assumption, equation (14) implies that:

$$\left(\frac{C_{t+1}}{C_t} \right)^{\gamma -1} \frac{(1+r_t)}{(1+\delta)} = 1 + \varepsilon_{t+1} \qquad (15)$$

where ε_{t+1} is orthogonal to any element of Ω_t and is serially uncorrelated. Given data on consumption and the returns on an asset, the parameters δ and γ in (15) can be estimated using non-linear two stage least squares. Any element of Ω_t which can help to forecast r_t can be used as an instrument.[1]

Before turning to a discussion of the choices made in actually estimating (15), it may be helpful to comment on the underlying economics. Consider the special case where r_t is nonstochastic. Equation (15) then holds that the growth rate in consumption is dependent on the real return r_t. The greater is the responsiveness, the greater is the implied value of γ. Essentially γ is estimated from information on the strength of the relation between consumption and ex-ante real returns.

---

[1]For more details on the estimation procedure, see Hansen and Singleton (1981) or Mankiw, Rotemberg and Summers (1982). Note that the assumption of conditional homeoscedasticity is maintained here. This does not affect the consistency of the estimates but may bias the standard errors.

In estimating (15) one must choose a measure of consumption, and asset return, as well as the sample period and data frequency. In an effort to verify the robustness of the conclusions, a variety of different implementations of (15) were estimated. Three alternative measures of consumption were used, including total real consumption expenditures, consumption of non-durables and services and consumption of non-durables alone. The choice between these concepts involves trading off comprehensiveness, and the problem of expenditure diverging from the service flow for durable goods. Consumption was measured either per adult (16 and over), or "per adjusted capita", weighting each cohort according to its relative consumption. The latter procedure controls for movements in consumption due to the changing age composition of the population.

The estimates were performed with both quarterly and annual data. Since the results are very similar, the more precise estimates obtained with quarterly data are reported here. The real after tax return on corporate stocks, long term government bonds, treasury bills, and savings deposits were used as proxies for r in these estimates. Inflation was calculated using the price deflator for the consumption concept appropriate to each equation. A thirty percent tax rate on interest and dividends was assumed, while capital gains on stocks and bonds were assumed to be untaxed.[1]

The results of estimating (15) using a variety of specifications are shown in Table 5. In each case, two lagged real returns, two lagged values of inflation, and two lagged consumption growth rates

---

[1] Alternative assumptions about taxation had little effect on the results. The tax series in the previous section was not used because weighting by interest income in finding the average marginal tax rate is not appropriate in the context of the current model.

Table 5

Nonlinear 2SLS Estimates of Consumption Euler Equations

Consumption Measure / Asset Return: per capita using the adult pop.	C72 $\frac{1}{1+\delta}$	C72 γ	C72 DW	CN72 $\frac{1}{1+\delta}$	CN72 γ	CN72 DW	CNS72 $\frac{1}{1+\delta}$	CNS72 γ	CNS72 DW
Treasury bills[a]	1.005 (.001)	.396 (.186)	1.60	1.009 (.003)	.090 (.456)	1.70	1.02 (.007)	-.99 (.860)	1.69
Government bonds[a]	1.02 (.005)	-.882 (.787)	1.84	1.017 (.011)	-.337 (1.55)	1.81	1.03 (.019)	-1.67 (2.24)	1.79
Stocks[a]	1.003 (.01)	-2.83 (1.48)	2.08	1.02 (.018)	-4.55 (2.54)	2.14	1.14 (.084)	-18.0 (9.54)	2.03
Saving deposits[c]	1.01 (.004)	-1.05 (.753)	1.88	1.01 (.005)	-.60 (.664)	1.82	1.03 (.01)	-1.76 (1.11)	1.83

Consumption Measure / Asset Return: per capita using adjusted pop.	C72 $\frac{1}{1+\delta}$	C72 γ	C72 DW	CN72 $\frac{1}{1+\delta}$	CN72 γ	CN72 DW	CNS72 $\frac{1}{1+\delta}$	CNS72 γ	CNS72 DW
Treasury bills[b]	1.008 (.194)	.004 (.291)	1.63	1.006 (.002)	-.116 (.408)	1.66	1.01 (.003)	-.98 (.609)	1.86
Government bonds[b]	1.008 (.009)	.630 (1.31)	1.85	1.006 (.007)	.150 (1.50)	1.87	1.01 (.011)	.21 (1.89)	1.86
Stocks[b]	1.02 (.016)	-4.88 (2.28)	2.07	1.01 (.013)	-5.93 (2.83)	2.02	1.08 (.039)	-16.1 (6.61)	2.00
Saving deposits[c]	1.02 (.004)	-.990 (.670)	1.88	1.01 (.003)	-.764 (.608)	1.73	1.02 (.005)	-1.56 (.756)	1.93

Note: Parameters refer to (1) in the text. a. Sample period: 48:4 - 81:4 b. Sample period: 51:4 - 78:4
c. Sample period: 55:3 - 78:4

are used as instruments. The results in Table 5 are not very sensitive to the choice of a consumption concept. However they are quite dependent on which asset is used in the estimation. In general, the point estimate of γ is greater using treasury bills, time deposits or government bonds, in the estimation than it is using stocks. Using assets other than stocks, the data suggest that the utility function is approximately logarithmic ($\gamma=0$). The estimate appears to be fairly precise. In each case one can conclude at a very high level of confidence that $\gamma \geq -3$. The estimated value of γ varies between -2.83 and -18.0 using corporate stocks to represent r_t. However in only one case are the estimates strongly inconsistent with the hypothesis that $\gamma=0$.

The estimates of the intertemporal utility function derived from the equations using treasury bills and time deposits to proxy asset returns are to be preferred on both economic and econometric grounds. The first order condition (14) on which the estimation is based is only valid for assets which are freely priced. The vast majority of consumption is done by persons who do not directly hold any equity or long term bonds. Nor are they short in these assets. While it is conceivable that some individuals are at an interior solution which calls for exactly zero holdings of these assets, it is unlikely that this consideration can account for the fact that most indivudals never buy or sell these assets despite changing economic condition. If the assets used to proxy r is not freely traded by all consumers, inconsistent estimates will result.

The econometric difficulty with using long term bond yields or stockmarket returns involves the problem of "overfitting" in the first stage of two stage least squares. As Merton (1981) and many others have noted, it is extremely difficult to demonstrate that there is any variation in the expected return on the market. The difficulty arises because of the market's extreme volatility. While instrumental variables will yield consistent estimates of the model's parameters, they are unlikely to be unbiased because the first stage of the estimation procedure is likely to find spurious variations in the ex-ante real rate. Hall (1981) demonstrates this by showing that the fitted values of the first stage of the estimation yield very implausible estimates of the ex-ante rate of return. Because of their substantial volatility, a similar point applies to the use of bond returns.

Taken together, these results strongly suggest that the value of γ is not a large negative number. As the simulations in the preceding section illustrate, this means that savings are very responsive to changes in real after tax rates of return. Indeed it is precisely the fact that the growth rate of consumption depends strongly on the expected real rate of return that drives these results.

It should be acknowledged that the overidentifying restrictions implied by (15) frequently fail. Similar findings have been reported by others including Mankiw (1981) and Hansen and Singleton (1981). This does not seem to me to be a cause for concern. Before embarking on estimation of the type reported here, one knows that the model is not literally true. Therefore with enough data one can be certain of rejecting the model at any desired level of significance. Learning that the model fails tells at least as much about the quantity and quality of data available as it does about the model.

The important question is whether or not there is evidence that consumers are liquidity constrained and so are unable to satisfy the first order condition (14). Flavin (1981), Hall and Mishkin (1982) and Hayashi (1982) have all suggested that some consumers are liquidity constrained. This possibility can be examined by extending the model to allow for the possibility that a fraction of disposable labor income is consumed directly by liquidity constrained or rule of thumb consumers. In this case, equation (15) becomes:

$$\left(\frac{C_{t+1} - \ell YL_{t+1}}{C_{t+1} - \ell YL_t} \right)^{\gamma-1} \frac{(1+r_t)}{(1+\delta)} = 1 + \varepsilon_{t+1} \qquad (16)$$

The parameter ℓ can then be estimated along with γ and $\frac{1}{1+\delta}$.

The results of estimating (16) are displayed in Table 6. They provide little support for the hypothesis that liquidity constraints are important. In every case, the hypothesis that $\ell=0$ cannot be rejected. The estimates of ℓ are as often negative as positive. There is no evidence that supplementing the intertemporal optimization model with a simple Keynesian consumption function improves its performance. Nor is there any indication that the estimated value of γ is affected.

The results in this section provide very strong support for the hypothesis that savings are responsive to real returns. Direct estimation of utility function parameters suggests that the the elasticity of substitution between present and future consumption is quite high. This leads ineluctably to a high long run response of savings to rates of return. Calculations of the type reported in Summers (1981) and Table 2 suggest an interest elasticity of savings greater than unity. Of course these results came from a restrictive model of optimization by a representative consumer. The next section takes a different approach by examining in an atheoretic way the relation between wealth accumulation and real interest rates.

Table 6

Nonlinear 2SLS Estimates of Consumption Euler Equations Allowing for Liquidity Constraints

Consumption Measure: per capita using the adult pop. / Asset Return	C72				CN72				CNS72			
	$\frac{1}{1+\delta}$	γ	ℓ	DW	$\frac{1}{1+\delta}$	γ	ℓ	DW	$\frac{1}{1+\delta}$	γ	ℓ	DW
Treasury bills [a]	1.006 (.002)	.080 (.364)	.045 (.403)	2.01	1.01 (.007)	-1.55 (1.63)	-.185 (.264)	1.76	1.02 (.006)	-2.06 (1.33)	.059 (.208)	1.98
Government bonds [a]	1.007 (.007)	.626 (1.18)	.643 (1.14)	1.83	1.02 (.012)	-2.14 (2.85)	-.327 (.577)	1.71	1.01 (.011)	-.91 (2.24)	.008 (.618)	1.75
Stocks[a]	1.01 (.015)	-4.10 (2.58)	-.088 (.605)	2.13	1.003 (.021)	-6.45 (3.76)	.069 (.216)	2.33	1.04 (.028)	-12.3 (5.68)	.060 (.239)	2.23
Saving deposits[c]	1.01 (.002)	-.032 (.423)	-.117 (.495)	1.75	1.02 (.008)	-1.71 (1.62)	-1.36 (3.63)	1.89	1.02 (.005)	-1.72 (.949)	.019 (.252)	1.93
All assets												

Consumption Measure: per capita using adjusted pop. / Asset Return	C72				CN72				CNS72			
	$\frac{1}{1+\delta}$	γ	ℓ	DW	$\frac{1}{1+\delta}$	γ	ℓ	DW	$\frac{1}{1+\delta}$	γ	ℓ	DW
Treasury bills [b]	1.01 (.002)	.013 (.324)	-.871 (1.70)	1.60	1.02 (.008)	-1.78 (1.71)	-.417 (.583)	1.63	1.01 (.002)	.101 (.240)	-.701 (1.01)	1.43
Government bonds [b]	1.001 (.009)	.371 (1.38)	-10.2 (292)	1.87	1.02 (.016)	-1.97 (3.17)	-.660 (1.53)	1.76	1.01 (.010)	.484 (1.10)	.053 (2.52)	1.85
Stocks [b]	1.03 (.034)	-5.81 (5.00)	.609 (.344)	2.52	1.04 (.029)	-10.1 (4.66)	-.202 (.497)	1.95	1.07 (.030)	-5.04 (3.22)	.298 (.366)	1.63
Saving deposits [c]	1.01 (.003)	-.321 (.391)	-.149 (.513)	1.76	1.02 (.007)	-1.62 (1.28)	-.577 (.971)	1.76	1.02 (.003)	-.239 (.291)	-.117 (.344)	1.09
All assets												

Note: Estimates refer to the parameters of (17) in the text. a. Sample period: 48:4 - 81:4 b. Sample period: 51:4 - 78:4
c. Sample period: 55:3 - 78:4

IV. Wealth Accumulation Equations

The approaches in the preceding two sections focused on effects of the rate of return on consumption decisions. This section examines the effect of changes in the rate of return of desired wealth holding. The hypothesis being tested is that increases in the rate of return raise the desirec level of sealth holding relative to labor income. This relationship is an implication of all the models of savings behavior developed in the first section of this paper; it is tested by estimating the simple partial adjustment model:

$$\left(\frac{A}{YL}\right)^*_t = a + bR_t + u_t \qquad (17)$$

$$\left(\frac{A}{YL}\right)_t = \left(\frac{A}{YL}\right)_{t-1} + \lambda\left[\left(\frac{A}{YL}\right)^*_t - \left(\frac{A}{YL}\right)_{t-1}\right]$$

This leads to the estimating equation:

$$\left(\frac{A}{YL}\right)_t = \alpha + \beta R_t + (1-\lambda)\left(\frac{A}{YL}\right)_{t-1} + u_t \qquad (18)$$

The data and sample period are the same as those used in Section II. Before turning to the results it is important to note that there are strong reasons to expect the estimate of β to be biased downwards. The value of R_t will be negatively correlated with the error term in (18) because increases in the capital stock reduce the real rate of return and because increases in R_t will be associated with downwards recapitalizations of asset values. Also as argued above, R_t is likely to be measured with substantial error.

Estimates of (18) and some variants on it are reported in Table 7.

Table 7

REDUCED FORM WEALTH ADJUSTMENT EQUATIONS

$$\left(\frac{A}{YL}\right)_t = \alpha + \beta R_{it} + (1-\lambda)\left(\frac{A}{YL}\right)_{t-1} + V_t$$

EQUATION	INDEPENDENT VARIABLE	1950 - 1978				1950 - 1964				1965 - 1978			
		α	β	$1-\lambda$	$\bar{R}^2/\rho$	α	β	$1-\lambda$	$\bar{R}^2/\rho$	α	β	$1-\lambda$	$\bar{R}^2/\rho$
1)	R_1	2.41	.119	.504	.795	1.55	.090	.688	.886	3.25	.150	.328	.689
		(.713)	(.035)	(.145)	.054	(.645)	(.035)	(.132)	-.171	(1.28)	(.065)	(.260)	-.081
2)	R_2	4.10	.082	.177	.767	3.47	.093	.328	.870	2.39	.135	.487	.744
		(1.30)	(.027)	(.262)	.654	(1.82)	(.050)	(.356)	-.074	(.843)	(.045)	(.973)	-.203
3)	R_3	2.88	.124	.417	.797	.181	.103	.642	.877	3.86	.156	.212	.743
		(.881)	(.036)	(.177)	.255	(.912)	(.047)	(.184)	.081	(1.27)	(.053)	(.256)	-.067
4)	R_4	4.41	.099	.119	.794	4.36	.118	.163	.876	3.08	.149	.353	.819
		(1.09)	(.027)	(.220)	.713	(2.54)	(.068)	(.493)	.114	(.794)	(.036)	(.162)	-.217

$$\left(\frac{A}{YLTREND}\right)_t = \alpha + \beta R_{it} + (1-\lambda)\left(\frac{A}{YLTREND}\right)_{t-1} + V_t$$

	INDEPENDENT VARIABLE	1950 - 1978				1950 - 1964				1965 - 1978			
		α	β	$1-\lambda$	$\bar{R}^2/\rho$	α	β	$1-\lambda$	$\bar{R}^2/\rho$	α	β	$1-\lambda$	$\bar{R}^2/\rho$
1)	R_1	3.13	.155	.359	.759	2.59	.105	.478	.757	3.42	.206	.291	.757
		(.731)	(.036)	(.148)	-.027	(.902)	(.042)	(.184)	-.285	(1.12)	(.068)	(.226)	-.159
2)	R_2	4.22	.085	.158	.663	3.96	.076	.227	.650	2.66	.201	.424	.811
		(1.40)	(.032)	(.281)	.584	(3.58)	(.079)	(.705)	-.006	(.703)	(.048)	(.145)	-.406
3)	R_3	3.41	.153	.314	.764	2.51	.101	.502	.726	3.93	.201	.198	.801
		(.801)	(.036)	(.161)	.037	(1.04)	(.048)	(.209)	-.200	(1.07)	(.053)	(.214)	-.162
4)	R_4	4.63	.102	.081	.685	4.07	.078	.212	.638	3.41	.211	.281	.892
		(1.22)	(.033)	(.246)	.647	(4.63)	(.100)	(.905)	.068	(.570)	(.032)	(.117)	-.485

Note: the variables $R_1 - R_4$ are defined in Table 3.

The results uniformly suggest that increases in the long term rate of return are associated with increases in the wealth labor income ratio -- despite the biases noted in the previous paragraph. This conclusion is robust to variations in the sample period and the way in which the real rate of return is measured. They are also insensitive to the choice between actual labor income and its trend value as a scaling variable.

The estimates here imply that wealth accumulation is quite sensitive to the rate of return. Increases of one percentage point in the real after tax rate of return are estimated to increase the desired wealth-labor income ratio by about .15 or 3 percent. Adjustment appears to be quite rapid with mean lags on the order of two years. This suggests that savings are likely to be very responsive for the short run to changes in the real after tax return.

The model here is quite crude in that it does not include many other determinants of wealth-labor income ratio, such as demographic variables and changing expectations of future income growth. It is possible that this leads to biased estimates of the effects of changes in the after tax rate of return, although there is no presumption as to the direction of the bias. Proxies for other factors affecting savings were not included because of the difficulty in estimating the effects of slowly changing variables, using short time series.

V. Conclusions

The theoretical and empirical results in this paper make a strong prima facie case for the proposition that increases in the rate of return are likely to bring forth significant increases in saving. Theoretical analysis indicates that a variety of standard models tend to suggest that the aggregate response to savings incentives is likely to be substantial. It is argued that the existing empirical evidence sheds little light on the question. Empirical analysis is then conducted using three alternative approaches. All three suggest a significant response of savings to changes in the rate of return.

While the work in this paper suggests strong rate of return effects on savings, it is too crude to provide a basis for estimating the effects of savings incentives frequently considered by policymakers. A micro-econometric approach recognizing the non-linearities introduced into consumers' budget by tax incentive schemes such as IRAs, would be necessary for this task. The techniques in Section III of this paper can easily be adapted to the study of individual consumption behavior. Such research would also shed light on the extent of heterogeneity among consumers and the importance of liquidity constraints.

References

Auerbach, A.J. and Kotlikoff, L., "An Examination of Empirical Tests of Social Security and Savings," NBER Working Paper 730, August 1981.

Barro, R., "Are Government Bonds Net Wealth?" Journal of Political Economy, Vol. 82, November-December 1974, pp. 1095 - 1117.

Bernheim, B.D., "Post Retirement Wealth Decumulation," mimeo, 1982.

Bernheim, B.D., A. Shleifer and L.H. Summers, "A Model of Manipulative Bequests," mimeo, 1982.

Blinder, A.S., "Temporary Tax Cuts and Consumer Spending," Journal of Political Economy, Vol. 89, February 1981, pp. 26-53.

Boskin, M.J., "Taxation, Saving, and the Rate of Interest," Journal of Political Economy, Vol. 86, April 1978, pt. 2, pp. S3-S27.

Chamley, C., "The Welfare Cost of Capital Income Taxation in a Growing Economy," Journal of Political Economy, Vol. 89, June 1981, pp. 468-496.

Feldstein, M.S., "Social Security Benefits and the Accumulation of Preretirement Wealth," NBER Working Paper #477, May 1980.

Feldstein, M.S., "Social Security, Induced Retirement and Aggregate Capital Accumulation: A Correction and Update," NBER Working Paper #579, November 1980.

Flavin, M.A., "The Adjustment of Consumption to Changing Expectations about Future Income," Journal of Political Economy, Vol. 89, October 1981, pp. 974-1009.

Hall, R., "Intertemporal Substitution in Consumption," NBER Working Paper #720, July 1981.

Hall, R. and F. Mishkin, "The Sensitivity of Consumption to Transitory Income: Estimates from Panel Data on Households," Econometrica, Vol. 50, March 1982, pp. 461-482.

Hansen, L. and Singleton, K., "Stochastic Consumption, Risk Aversion, and the Intertemporal Behavior of Stock Market Returns," unpublished.

Hayashi, F., "Liquidity Constraints and the Consumption Function," forthcoming in Journal of Political Economy, 1982.

Howrey, E.P. and Hymans, S., "The Measurement and Determination of Loanable Funds Saving," Brookings Papers on Economic Activity, 1978, 3, pp. 655-685.

Kotlikoff, L. and Spivak, A., "The Family as an Incomplete Annuities Market," Journal of Political Economy, Vol. 89, April 1981, pp. 372-391.

Kotlikoff, L. and Summers, L.H., "The Importance of Intergenerational Transfers in Aggregate Capital Accumulation," Journal of Political Economy, Vol. 89, August 1981, pp. 706-732.

Lucas, R.E. Jr., "Econometric Policy Evaluation: A Critique," in Karl Brunner and Allan H. Metzler, eds., The Phillips Curve and Labor Markets, Carnegie-Rochester Conference Series on Public Policy, Vol 1, Amterdam, North Holland, 1976, pp. 19-46.

Mankiw, N.G., "The Permanent Income Hypothesis and the Real Interest Rate," Economic Letters, 7, 1981, pp. 307-311.

Mankiw, G., J. Rotemberg, and L. H. Summers, "Intertemporal Substitution in Macroeconomics," NBER Working Paper #898, June 1982.

Merton, R.C., "On Estimating the Expected Return on the Market: An Exploratory Investigation," Journal of Financial Economics, 1980.

Mirer, T.W., "The Wealth-Age Relation Among the Aged," American Economic Review, Vol. 69, June 1979, pp. 435-443.

Sandmo, A., "The Effect of Uncertainty on Saving Decisions," Review of Economic Studies, Vol. 37, July 1970, pp. 353-360.

Stiglitz, J., "Notes on Estate Taxes, Redistribution, and the Concept of Balanced Growth Path Incidence," Journal of Political Economy, Vol. 86, part 2, April 1978, pp. S137-50.

Summers, L.H., "Capital Taxation and Accumulation in a Life Cycle Growth Model," American Economic Review, Vol. 71, September 1981, pp. 533-544.

Summers, L.H., "Inflation and the Valuation of Corporate Equities," NBER Working Paper #824, 1982.

Tobin, J., "Life Cycle Saving and Balanced Growth," in William Fellner and others, Ten Economic Studies in the Tradition of Irving Fisher, 1967, John Wiley and Co., pp. 231-256.

Weber, W.E., "The Effect of Interest Rates on Aggregate Consumption," American Economic Review, Vol. 60, September 1970, pp. 591-600.

Weber, W.E., "Interest Rates, Inflation and Consumer Expenditures," American Economic Review, Vol. 65, December 1975, pp. 843-858.

Wright, C., "Saving and the Rate of Interest," in Harberger and Bailey, eds., The Taxation of Income from Capital, 1969, Washington: The Brookings Institution, pp. 275-300.

</ref_section>