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ON CONSUMPTION-INDEXED PUBLIC PENSION PLANS

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ABSTRACT

Using the known result that life-cycle investors will optimally hold portfolios whose returns are perfectly correlated with aggregate consumption, this paper uses a simple intertemporal general equilibrium model to explore the merits and feasibility of pension plans where both accumulations and benefits are linked to aggregate per capita consumption. Although the analysis is made within the framework of a public pension plan, it applies equally well to organized private pension plans where participation is virtually mandatory and where individually designed programs are not practical. An additional feature of the plans examined is that they provide for life annuities during both the accumulation and retirement phases of the life cycle.

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## ON CONSUMPTION-INDEXED PUBLIC PENSION PLANS

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### I. Introduction

Most economists using a standard life cycle analysis would probably agree that the primary objective of a pension system is to provide a standard of living in retirement comparable to that enjoyed during the working years. There is, nevertheless, considerable disagreement as to how that objective can best be achieved. Broadly, the disagreements are on the appropriate roles for private pension plans and a public pension plan in the pension system and on whether or not the pension system should also be used for redistribution or transfers. The most elegant approach to the problem would undoubtedly be to solve for the optimal overall pension system with a simultaneous determination of the optimal forms for both the public and private parts. However, the analysis here is more limited in its scope because its focus is principally on the public part of the system and because it examines only one of the many possible functions that such a system might serve in any real-world implementation. That is, the sole intent of the system is assumed to be the retirement objective, and not, for example, to also provide for a redistribution of wealth. The paper should thus be viewed as only a prologue to a more complete functional analysis of the overall pension system including the important issue of the degree of integration between private and public pension plans.

Analysis of the public part of the system is a natural starting place because whatever form the overall pension system takes, it will

surely include a significant public pension plan component. As will be discussed, there are a number of theoretical arguments to support such a component as part of an optimal system. Moreover, as a practical matter independent of any theoretical welfare arguments that economists might provide to the contrary, the public pension system in the United States, after almost half a century of operating experience, is not going to be eliminated, especially when a significant fraction of the population is not covered by any private pension plan. The current problems with Social Security do however, present the possibility for major changes in the structure of the public pension system. It would therefore seem to be somewhat difficult to analyze the optimal design of private pension plans and the associated issue of integration until the structure of the public system is more firmly established.

In theory, the characteristic differences between a public and a private pension system are that participation in a public system is mandatory and that the public system cannot be "custom tailored" to meet the specific preferences of each individual participant. Such a clear distinction is valid if the private system were solely *laisse faire* individual saving. However, as the private system has in fact evolved, the operational significance of this distinction, at least at the level of analysis presented here, is less clear. Participation in most existing private pension plans is virtually mandatory. In a typical defined-contribution plan, individual choice is quite limited as to the amounts contributed and where the funds are invested, and in defined-benefit plans, there is typically no choice at all. The analysis

presented here in the context of a public system is therefore readily adaptable to an "organized" private pension system.

The arguments for a public pension system with mandatory participation fall into two basic categories: externalities and private market "failure." An important example of the former is the utility externality that other people's welfare is one of the arguments of individual utility functions. That is, people care about others and among other things, will not let them starve in retirement. From this, we get a classical example of the "free-rider" problem which cannot be solved by the private markets, but can be solved by an appropriately designed mandatory public pension system. A second example is the possibility of economies of scale in information costs. Virtually everyone faces the decision problem of how much to save for retirement and what to invest those savings in during their working years. The marginal cost of obtaining the education and gathering the necessary data to make informed decisions as well as the time spent implementing these decisions will, of course, vary substantially across individuals as a function of their prior education and their wealth. Presumably, a professor of finance by virtue of his training would have a lower marginal cost than a professor of physics. The cost of buying the service of informed decisions will be lower (as a percentage of wealth) for those who are wealthy than for those of modest means. While this cost could be reduced by "pooling," this solution almost assumes away the problem because such an undertaking requires adequate information and opportunity to form a cohesive group.

If therefore, a pension plan were designed which reasonably approximates the plan which most individuals would choose if they were informed, then by making participation in the plan mandatory, the resources used in individual education and data gathering would be saved and the maximum benefits of pooling to reduce operating costs could be achieved. The benefits of such mandatory participation must, of course, be compared to the cost in terms of loss in individual freedom of choice. As already noted, existing private pension plans permit little choice. Although this data point favors the hypothesis that the benefits outweigh the costs, it is hardly a sufficient basis for a policy decision.

The second basic category of arguments for a public pension plan is that the efficiency of risk-bearing can be improved. That is, the government can provide diversification possibilities which are not available in the private markets, and thereby, issue financial instruments which the private sector cannot. One example would be intergenerational risk sharing which cannot be covered by private markets (cf. Fischer (1982)). Another would be to use either taxes and transfers (cf. Merton (1981)) or taxes and the issue of securities within the pension system to provide diversification of some of the risks of assets which are not tradeable (as is the case for much of human capital).

With these general reasons for a public pension plan as background, the consumption-indexed plan to be studied is briefly summarized before turning to a formal analysis in the context of a simple intertemporal

equilibrium model in Section II. This is then followed in Section III, by a discussion of the merits and feasibility of such plans.

The plan is a mandatory fully-funded savings plan of the "defined-contribution" type where required contributions by each member of the plan are a fixed proportion of his consumption. As with current private defined-contribution plans, each member has an individual account which is credited with his contributions (less any deduction for operating expenses of the plan).

Contributions and earnings in each member's account are invested in "aggregate per capita consumption-indexed life annuities" where these are defined to be an instrument which pays a constant fraction of aggregate per capita consumption to its holder (the member) each period and such payments begin at a prespecified commencement date (the date at which the member begins to receive his benefits) and continue until the member dies. If the member dies before the commencement date, the annuity is worthless. Benefits, therefore, are in the form of a life annuity indexed to aggregate per capita consumption.

The commencement date for benefits is at a specified age (e.g., age 60), independent of whether or not the recipient has retired. This provision is to avoid possibly undesirable distortions of the decision to retire. However, provision could be made for delaying the receipt of benefits to a later age. Contributions are mandatory from some statutory beginning age (e.g., age 21) until the commencement date.

One way to administer such a plan would be to create a public corporation which would be responsible for issuing the indexed

life-annuities to plan members where these annuities would constitute its senior liabilities. The US government would be the residual liability or equityholder of the corporation and would have unlimited liability. The assets of the corporation come from member contributions and are invested in the broadest available portfolio of marketable securities.

The number of units of life annuities issued to an account is on a "mark-to-market" basis at the time each contribution is received. That is, the value of a unit of a life annuity issued is determined by current market prices and mortality tables. To make this possible, it would be necessary for the government to issue aggregate per capita consumption-indexed bonds of various maturities.

To prevent attempts to circumvent mandatory participation in the plan, retirement benefits are assumed to be neither assignable nor attachable. For similar reasons, integration of private pension plans with the public plan are permitted, but only to the extent that the combined benefits received by the individual are no less than he would have received from the public plan alone.

## II. A Simple Intertemporal Equilibrium Model

In this section, a continuous-time consumption choice model of the type presented in Merton (1971) and (1973) is used to analyze the system of mandatory saving and consumption-linked retirement benefits.

Consider an economy where all people have the same lifetime utility of consumption which is given for a person born at time  $t_0$  by

$$(1) \quad E_{t_0} \left\{ \int_{t_0}^{t_0 + \tilde{t}} \frac{[c(s; s - t_0)]^\gamma}{\gamma} e^{-\rho(s-t_0)} ds \right\},$$

$$\gamma < 1$$

where  $c(t; \tau)$  is consumption at time  $t$  of a person of age  $\tau$  and  $E_t$  is the conditional expectation operator conditional on knowing all relevant information available as of time  $t$ . Each person has an uncertain lifetime where  $\tilde{t}$  denotes the random variable age of death, and the probability that the person will die between  $\tau$  and  $\tau + d\tau$ , conditional on being alive at age  $\tau$  is given by  $\lambda(\tau)d\tau$  where  $\lambda(\tau) > 0$ . Each person acts so as to maximize (1) subject to his initial wealth  $w_0$ .

If the event of death is independent of other economic variables, then along the lines of the proof of Theorem VI in Merton (1971, p. 400), we can rewrite (1) as:

$$(2) \quad E_{t_0} \left\{ \int_{t_0}^{\infty} f(s - t_0; 0) e^{-\rho(s-t_0)} \frac{[c(s; s - t_0)]^\gamma}{\gamma} ds \right\}$$

where  $f(\tau; \tau')$  is the probability that the person will be alive at age  $\tau$  conditional on being alive at age  $\tau'$ . By the definition of  $\lambda(\tau)$ ,  $f$  satisfies

$$(3) \quad f(\tau; \tau') = \exp \left[ - \int_{\tau'}^{\tau} \lambda(s) ds \right] .$$

By assumption, each person has no bequest function. Hence, it will be optimal for each person to enter into a life annuity contract where his wealth goes to the issuer if he dies and he receives a payment if he lives. One such arrangement would be a series of "short-term" contracts where at age  $\tau$ , he agrees to bequeath his wealth,  $w(\tau; \tau)$ , to the issuer if he dies between  $\tau$  and  $\tau + d\tau$  and the issuer agrees to pay him a "dividend"  $D dt$  if he lives. If there are a large enough number of people in the economy to diversify away completely the risk of individual deaths and if the contracts (like futures contracts) require no side payments between issuer and purchaser, then the competitive equilibrium "dividend" will be  $\lambda(\tau)w(\tau; \tau)dt$ .

In addition to the annuity contract, the person will choose an optimal portfolio allocation of his wealth. As shown for example in Merton (1971), the fractions of his optimal portfolio allocated to the available investments are independent of his wealth or age because his

utility function is of the isoelastic form. Therefore, all investors in the economy will hold identical portfolios (except for scale). Hence, without loss of generality, I assume that all people invest in a single security, and the rate of return on this security,  $dM/M$ , is assumed to follow an Ito Process given by

$$(4) \quad \frac{dM}{M} = \alpha dt + \sigma dz$$

where the instantaneous expected rate of return  $\alpha$ , and the instantaneous variance of the return  $\sigma^2$ , are constants over time. It follows from (4) that the return on this security is lognormally distributed. Moreover, as a necessary condition for equilibrium, this security must be a "market portfolio," i.e., a portfolio which contains all available investments and holds them in proportion to their market values.

The accumulation equation for the wealth of a person of age  $\tau$  at time  $t$  can, therefore, be written as

$$(5a) \quad dw(t;\tau) = [(\lambda(\tau)+\alpha)w(t;\tau)-c(t;\tau)]dt+\sigma w(t;\tau)dz$$

if he does not die between  $t$  and  $t + dt$ , and as

$$(5b) \quad dw(t;\tau) = -w(t;\tau)$$

if he dies between  $t$  and  $t + dt$ .

Along the lines of the derivation in Merton (1971, p. 390), the optimal consumption demand for a person of age  $\tau$  at time  $t$  can be written as

$$(6a) \quad c(t;\tau) = a(\tau)w(t;\tau)$$

where  $a(\tau)$  is a solution to the differential equation

$$(6b) \quad 0 = \frac{\dot{a}(\tau)}{a(\tau)} - a(\tau) + \lambda(\tau) + \mu$$

with  $\mu \equiv [\rho - \gamma\alpha]/(1 - \gamma) + \gamma\sigma^2/2$ . By inspection, optimal consumption is a function of both wealth and age, and the marginal propensity to consume (out of wealth) will be an increasing function of age if  $\dot{\lambda}(\tau) \geq 0$ . Similarly, the distribution of a person's wealth who is alive at time  $t + s$  given his wealth at time  $t$ , will not only depend upon his wealth at time  $t$  and the return experience on his portfolio between  $t$  and  $t + s$  but also on his age at time  $t$ .

Using Itô's Lemma, we have from (6) that

$$(7) \quad \frac{dc(t;\tau)}{c(t;\tau)} = \frac{dw(t;\tau)}{w(t;\tau)} + \frac{\dot{a}(\tau)}{a(\tau)} dt$$

Conditional upon the person not dying between  $t$  and  $dt$ , we have by substitution from (5) and (6) that (7) can be rewritten as

$$(8) \quad \frac{dc(t;\tau)}{c(t;\tau)} = (\alpha - \mu)dt + \sigma dz$$

and of course, if he dies then  $dc(t;\tau)/c(t;\tau) = -1$ . By inspection of (8), the dynamic path of a person's optimal lifetime consumption follows a Markov process, independent of either his wealth or his age (except for the "stopping point"). That is, given his consumption at time  $t$ ,  $c(t;\tau)$ , his consumption (if alive) at time  $t + s$  has a lognormal distribution which can be represented by

$$(9) \quad c(t + s;\tau + s) = c(t;\tau)\exp[(\alpha - \mu)s + \sigma\sqrt{s} \epsilon]$$

where  $\varepsilon$  is a standard normal random variable. Thus, unlike the percentage change in wealth which is age dependent, the percentage change in consumption is the same for all people alive. It follows, therefore, that

$$(10) \quad \frac{c(t+s; \tau+s)}{c(t; \tau)} = \frac{c(t+s; \tau'+s)}{c(t; \tau')}$$

for all people alive at time  $t+s$  and  $\tau, \tau' \geq 0$ .

Armed with (8) and (10), we can now proceed to derive the dynamic properties of aggregate per capita consumption,  $C(t)$ . If  $L(t; \tau)$  denotes the number of people of age  $\tau$  in the economy at time  $t$ , then the total population size,  $L(t)$ , equals  $\int_0^{\infty} L(t; \tau) d\tau$ . Therefore, aggregate per capita consumption is equal to

$$(11) \quad C(t) = \int_0^{\infty} L(t; \tau) c(t; \tau) d\tau / L(t)$$

If the birthrate at time  $t$  is given by  $b(t)$ , then the change in aggregate per capita consumption is given by

$$(12) \quad dC(t) = \int_0^{\infty} L(t; \tau) dc(t; \tau) d\tau / L(t) - H(t)C(t)dt$$

$$\text{where } H(t) \equiv \left\{ b(t)[C(t) - c(t; 0)] - \int_0^{\infty} \lambda(\tau) L(t; \tau) [C(t) - c(t; \tau)] d\tau / L(t) \right\} / C(t).$$

The properties of  $H(t)$  are, of course, dependent upon demographic assumptions. However, they also depend upon the distribution of consumption per capita. If, for example, the distribution of per capita

consumption were uniform [i.e.,  $c(t;\tau) = C(t)$ , for all  $\tau$ ], then

$H(t) = 0$ , independent of demographics. In a stable population

$$\left[ b(t) = \int_0^{\infty} \lambda(\tau)L(t;\tau)d\tau/L(t) \right],$$

$$H(t) = - \int_0^{\infty} \lambda(\tau)L(t;\tau)[c(t;0) - c(t;\tau)]d\tau/[L(t)C(t)], \text{ and the sign of } H$$

will depend primarily on the distribution of per capita consumption between the very young and the very old (where the marginal death rate,  $\lambda(\tau)$ , is largest). If that distribution is approximately equal [ $c(t;0) \approx c(t;\tau)$  for large  $\tau$ ] and the population is growing, then the sign of  $H(t)$  will equal the sign of  $[C(t) - c(t;0)]$ , the difference between the general population per capita consumption and per capita consumption of the very young.

Even without taking into account the interaction between population growth and economic conditions, the analysis of stochastic demographic models is formidable. And, while the death rate (at least in the short run) may be exogeneous, the birth rate is surely affected by economic conditions. Therefore, although explicit consideration of the process for  $H(t)$  is important for many issues in this paper, no such analysis will be undertaken here. Instead, I simply postulate that  $H(t) = 0$ .<sup>1/</sup>

If  $H(t) = 0$ , then we have by substitution from (8) that (12) can be rewritten as

$$\begin{aligned}
 dC(t) &= \left\{ \left[ \int_0^T L(t;\tau)c(t;\tau)d\tau \right] / L(t) \right\} [(\alpha - \mu)dt + \sigma dz] \\
 (13) \qquad &= (\alpha - \mu)C(t)dt + \sigma C(t)dz
 \end{aligned}$$

A comparison of (8) with (13) shows that (except for scale), each person's optimal consumption follows a stochastic process identical to the one for aggregate per capita consumption. That is, conditional on being alive at time  $t + dt$ ,  $dc(t;\tau)/c(t;\tau) = dC(t)/C(t)$ , independent of the person's age  $\tau$ . Therefore, we have for person  $j$  that his consumption (if he is alive) at time  $t$  can be written as

$$(14) \qquad c_j(t) = \beta_j C(t)$$

where  $\beta_j \equiv c(t_j;0)/C(t_j)$  and  $t_j$  is his birth date.

Consider now a mandatory saving and retirement plan where beginning at age  $T_0$ , each person must contribute at rate  $\delta$  times his consumption until at age  $T_1$ , the person begins to receive his life annuity retirement benefits. During the accumulation period of length  $\tau_a \equiv T_1 - T_0$ , each person's contribution is invested in a per capita aggregate consumption-linked life annuity contract matched to his age at the time of the contribution.

Let  $A(t,\tau;T_1)$  denote the equilibrium price at time  $t$  of a life annuity contract which begins its payments at age  $T_1$  and the purchaser is currently age  $\tau$ . The promised stream of payments is equal to  $C(s)$  per unit time from time  $s = t + T_1 - \tau$  until the purchaser dies. Let  $P(t;\tau)$  denote the equilibrium price at time  $t$  of a consumption-linked

pure discount bond of maturity  $\tau$  which pays  $\$C(t + \tau)$  at time  $t + \tau$ . If, as has been assumed, individual death risk can be diversified away, then the competitive equilibrium price for  $A$  can be written as

$$(15) \quad A(t, \tau; T_1) = \int_0^{\infty} f(s + T_1; \tau) P(t; s + T_1 - \tau) ds$$

where, as previously defined,  $f(\tau; \tau')$  is the probability of being alive at age  $\tau$  conditional on being alive at age  $\tau'$ .

For the economy of this section, an explicit formula for the  $P(t; \tau)$  can be derived by competitive arbitrage. From (13),

$$C(t + \tau) = C(t) \exp \left[ \left( \alpha - \mu + \frac{1}{2} \sigma^2 \right) \tau + \sigma \int_t^{t+\tau} dz(s) \right].$$

Therefore, the

realized return on the discount bond between  $t$  and  $t + \tau$  is

$$C(t + \tau)/P(t; \tau) = C(t) e^{-\mu\tau} / P(t; \tau) \exp \left[ \left( \alpha - \frac{1}{2} \sigma^2 \right) \tau + \sigma \int_t^{t+\tau} dz(s) \right].$$

However, from (4), the return per dollar from investing in the market portfolio between  $t$  and  $t + \tau$  is  $\exp \left[ \left( \alpha - \frac{1}{2} \sigma^2 \right) \tau + \sigma \int_t^{t+\tau} dz(s) \right]$ .

Therefore, to avoid arbitrage,  $P(t; \tau)$  must satisfy

$$(16) \quad P(t; \tau) = C(t) e^{-\mu\tau}.$$

It follows from (16), that the instantaneous rate of return on the bond,  $dP/P = \alpha dt + \sigma dz$ , is the same as on the market. Substituting for  $P$  from (16), we can rewrite (15) as

$$(17) \quad A(t, \tau; T_1) = C(t)e^{-\mu(T_1 - \tau)} \int_0^{\infty} e^{-\mu s} f(s + T_1; \tau) ds \quad .$$

Moreover, it is straightforward to show that for  $\tau < T_1$

$$(18) \quad \frac{dA}{A} = [\alpha + \lambda(\tau)]dt + \sigma dz$$

if the owner of the contract is alive at  $t + dt$  and  $dA/A = -1$

if the owner dies between  $t$  and  $t + dt$ .

Let  $V(t; \tau)$  denote the value of the accumulated retirement account for a person of age  $\tau$  at time  $t$ . Under this retirement plan with accumulations in units of a consumption-linked life annuity, the value can be expressed as

$$(19) \quad V(t; \tau) = N(\tau)A(t, \tau; T_1)$$

where  $N(\tau)$  equals the number of units accumulated at age  $\tau$ . By Ito's Lemma,  $dV = N(\tau)dA + \dot{N}(\tau)Adt$  if the person lives to time  $t + dt$  and  $dV = -V$  if he dies between  $t$  and  $t + dt$ . Under the mandatory saving plan,  $\dot{N}(\tau)A(t, \tau; T_1) = \delta c(t; \tau)$  and  $N(T_0) = 0$ . From (14),  $c(t; \tau) = \beta C(t)$ , and if the retirement plan is designed to provide fraction  $\eta$  ( $0 < \eta \leq 1$ ) of the person's optimal retirement period consumption, then  $\delta$  should be chosen so that at retirement, the number of units accumulated,  $N(T_1)$ , equals  $\eta\beta$ .

If the retirement plan is fully-funded and actuarially fair, then at age  $T_0$ , the present value of the future contributions by the person should be equal to the present value of the annuity payments to be received in retirement. Under the terms of the mandatory saving plan,

the person will contribute at the rate  $\delta c(t; \tau) = \delta \beta C(t)$  (as long as he is alive) until he reaches  $T_1$ . Therefore, at age  $T_0$ , the present value of his future contributions,  $F(t; T_0)$ , is given by

$$\begin{aligned}
 (20) \quad F(t; T_0) &= \int_0^{T_1 - T_0} f(s + T_0; T_0) [\delta \beta P(t; s)] ds \\
 &= \delta \beta \int_0^{\tau_a} f(s + T_0; T_0) P(t; s) ds .
 \end{aligned}$$

If the plan is to provide  $N(T_1) = \eta \beta$  units in retirement, then the present value of these retirement benefits at age  $T_0$  is  $\eta \beta A(t; T_0; T_1)$ . Therefore,  $\delta$  must be chosen such that  $F(t; T_0) = \eta \beta A(t; T_0; T_1)$ , and from (15) and (20), we have that

$$(21) \quad \delta = \frac{\eta \int_0^{\infty} f(s + T_1; T_0) P(t; s + \tau_a) ds}{\int_0^{\tau_a} f(s + T_0; T_0) P(t; s) ds} .$$

Substituting for  $P$  from (16), we can rewrite (21) as

$$(22) \quad \delta = \frac{\eta e^{-\mu \tau_a} \int_0^{\infty} f(s + T_1; T_0) e^{-\mu s} ds}{\int_0^{\tau_a} f(s + T_0; T_0) e^{-\mu s} ds} .$$

By inspection of (22), the required contribution fraction does not

depend upon endowments or the individual contributor's age. It does, of course, depend upon the statutory retirement age  $T_1$ , the accumulation period  $\tau_a$ , and the target fraction of retirement period consumption provided by the plan,  $\eta$ . Therefore,  $\delta$  can be kept constant over time, and still meet the objectives of the plan. The only changes required would be in response to large cumulative changes in the mortality tables  $f$  or  $\mu$ , and these would probably be infrequent. Moreover, because the plan is fully-funded and accumulations earn a fair market return, such changes in  $f$  or  $\mu$  that might occur will cause no significant distortions even if  $\delta$  were not adjusted over time.

To provide a crude estimate of the magnitude of  $\delta$ , I assume the following: (i) the accumulation period  $\tau_a = 45$  years; (ii) during the accumulation period, the mortality rate is a constant,  $\lambda$ , equal to .0138 per year; (iii) during the retirement period, the mortality rate is a constant,  $\lambda$ , equal to .0666 per year and that, in no event, will anyone live longer than thirty years after retirement. The average rate of growth of aggregate per capita real consumption from 1947 to 1981 is approximately two percent per year. If the expected real rate of return on all wealth in the economy,  $\alpha$ , is taken to be four percent, then from (13), we derive an estimate for  $\mu$  of two percent. Substituting these numbers into (22), we have that

$$(23) \quad \delta = .10 \eta \quad .$$

That is, to provide for all of retirement consumption ( $\eta = 1$ ) would require about a ten percent contribution rate. While such a rate may seem large (requiring contributions of the order of \$200 billion in 1981),

ten percent is a common contribution rate (on income) in many existing private defined-contribution plans, and the current (maximum) contribution rate for Keogh Plans is fifteen percent. To provide further perspective, I would also note that the combined employee-employer contributions to Social Security in fourth quarter 1981 was at an annual rate of \$245 billion. It is, of course, unlikely that a public pension plan would be expected to provide for all retirement consumption and therefore, the necessary contribution rate would be considerably less than ten percent.

### III. On the Merits and Feasibility of a Consumption-Indexed Public Plan

While the analysis in the previous section demonstrates a consumption-indexed public retirement plan, it is presented within the context of a model where such plans are redundant. That is, with perfect markets for both assets and annuities, no utility externalities, and rational and informed people, there is, of course, no need for such public intervention. From this base, however, imperfections can be introduced to provide at least a qualitative analysis of the benefits of the plan for comparison with alternative plans if, and when, such intervention were deemed appropriate.

For example, a significant feature of this plan is that contributions be invested in aggregate consumption-linked life annuities. If important assets within the economy such as human capital and real estate are either nontradeable or not available in divisible lots, then even a broad-based portfolio of tradeable assets will not provide a fully-efficient diversified portfolio. However, an individual's consumption is likely to be strongly correlated with his wealth (or permanent income) whether that wealth is tradeable or not, and therefore, a security whose return is perfectly-correlated with aggregate per capita consumption is likely to represent a better-diversified holding than a portfolio containing only marketable securities. Moreover, even when all securities are traded, Breeden (1979) has shown that all efficient portfolios will be perfectly-correlated with aggregate consumption.

If there are systematic differences among large segments of the population as to the types of nontradeable assets they hold, then it

is possible to improve diversification efficiency still further. An example would be that the young in the economy are forced to hold too large a fraction of their wealth in human capital because it is not tradeable while the old hold too small a fraction in human capital because they cannot buy it. As I have shown elsewhere (1981), risk-bearing can be improved by a system that taxes wages and pays wage-linked retirement benefits. However, as that analysis amply demonstrates, such further diversification gains are earned at the expense of having a "pay-as-you-go" retirement system with a risk of significant distortions from the associated taxes and transfers.

Diamond (1977) has suggested that one reason for a Social Security system is the absence in the private markets of "real" or "indexed" investments by which people of normal means can accumulate savings for retirement. However, "real" fixed-income bonds would only protect such savers against the uncertainties of inflation. They would not protect the saver against the risk of real increases in the standard of living. As shown in Table 1, real per capita consumption in the United States has increased at an average rate of 1.96 percent per year from 1947 to 1981. Moreover, the annual standard deviation of that growth rate is 1.68 percent. Hence, if a person's sense of "how well off he is" depends not only on the absolute level of his consumption, but also on its level relative to those around him, then the risk in utility-terms of a price-level-linked investment can be considerable, especially over a long accumulation period. A consumption-linked investment protects against both inflation and real changes in the standard of living. It

TABLE 1

Levels and Growth Rates  
 US Aggregate Real Consumption and Over-Age-16 Population<sup>2/</sup>  
 1947 - 1981

Year	Aggregate Consumption (billions/1972 \$)		Population (millions)		Per Capita Consumption (thousands/1972 \$)	
	Level	% Change	Level	% Change	Level	% Change
1947	305.8	---	103.4	---	2.957	---
1948	312.2	2.1	104.5	1.1	2.987	1.0
1949	319.3	2.3	105.6	1.0	3.023	1.2
1950	337.3	5.6	106.6	1.0	3.163	4.6
1951	341.6	1.3	107.7	1.0	3.171	0.3
1952	350.1	2.5	108.8	1.0	3.217	1.5
1953	363.4	3.8	110.6	1.6	3.286	2.1
1954	370.0	1.8	111.7	1.0	3.313	0.8
1955	394.1	6.5	112.7	1.0	3.496	5.5
1956	405.4	2.9	113.8	1.0	3.562	1.9
1957	413.8	2.1	115.1	1.1	3.596	1.0
1958	418.0	1.0	116.4	1.1	3.592	-0.1
1959	440.4	5.4	117.9	1.3	3.736	4.0
1960	452.0	2.6	119.8	1.6	3.774	1.0
1961	461.4	2.1	121.3	1.3	3.802	0.7
1962	482.0	4.5	123.0	1.3	3.919	3.1
1963	500.5	3.8	125.2	1.8	3.999	2.0
1964	528.0	5.5	127.2	1.7	4.150	3.8
1965	557.5	5.6	129.2	1.6	4.314	3.9
1966	585.7	5.1	131.2	1.5	4.465	3.5
1967	602.7	2.9	133.3	1.6	4.521	1.3
1968	634.4	5.3	135.6	1.7	4.680	3.5
1969	657.9	3.7	137.8	1.7	4.773	2.0
1970	672.1	2.2	140.2	1.7	4.794	0.5
1971	696.8	3.7	142.6	1.7	4.887	1.9
1972	737.1	5.8	145.8	2.2	5.056	3.5
1973	768.5	4.3	148.2	1.7	5.183	2.5
1974	763.6	-0.6	150.8	1.7	5.063	-2.3
1975	780.2	2.2	153.4	1.7	5.084	0.4
1976	823.7	5.6	156.0	1.7	5.279	3.8
1977	863.9	4.9	158.6	1.6	5.448	3.2
1978	904.8	4.7	161.1	1.6	5.618	3.1
1979	930.9	2.9	163.6	1.6	5.689	1.3
1980	935.1	0.5	166.2	1.6	5.625	-1.1
1981	958.9	2.5	168.6	1.4	5.688	1.1
Average Growth Rate		3.44%		1.45%		1.96%
Standard Deviation		1.75%		0.32%		1.68%

has the further practical advantage of avoiding the index problem because it is not necessary to distinguish between nominal and real changes.

In another context, Fischer (1982) argues that the government should issue wage-income-linked bonds. While it is likely that such bonds would be superior to price-level-linked bonds for most saving plans, at least in theory, they may not be as efficient as consumption-linked bonds. One reason is that changes in wage income capture the returns to only one segment (albeit an important one) of national wealth while consumption changes depend upon all segments. A second reason is that wage income is more likely to have a significant transient component than is consumption since by the Life Cycle Hypothesis, consumption depends upon permanent income or wealth. How important the difference would be between wage-income and consumption linked bonds is, of course, an empirical matter, and one that warrants further study.

There are relatively limited opportunities in existing private markets to accumulate savings in life annuities and none where those savings are invested in consumption-linked investments. In the absence of such instruments, the individual may be forced to save too much relative to his bequest motive. By investing contributions in life annuities, the proposed plan permits a person to accumulate adequate amounts for retirement with smaller contributions. The additional available funds from this reduced contribution rate can be used either for more current consumption or to purchase life insurance or other

saving instruments to meet bequest motives. This feature is especially important in a mandatory saving plan because, for the same target level of retirement benefits, it reduces the welfare-loss of the plan to those in poor health or those who have no bequest motive.

A second significant feature of the plan is that retirement benefits are linked to aggregate per capita consumption. The arguments in favor of consumption-linked benefits are essentially the same as those given for consumption-linked accumulations. So, for example, while a number of people including Diamond (1977), have argued for real or price-indexed fixed annuities for retirement benefits, per capita consumption-linked benefits are likely to dominate such annuities because they protect the retiree against both the uncertainties in the inflation rate and changes in the standard of living.

The success of a consumption-indexed plan (whether public or private) depends critically on the existence of per capita aggregate consumption-linked bonds. In their absence, administrators of the plan would be required to estimate the "fair market value" of such bonds in order to determine how many units to credit each account with during the accumulation period and to determine how much to pay in benefits during retirement. I need hardly mention the extreme difficulties associated with making these appraisals especially when such instruments have never traded. Moreover, for a public plan, there would likely be times when strong political pressure would be brought to bear on the administrators to "adjust" their appraisals. Even if such pressure were in fact resisted, the mere prospect of a potential "conflict of interest" could taint the entire system.

In theory, the private sector could create a market for per capita aggregate consumption-linked bonds and provide consumption-linked life annuities through financial intermediaries. Some might indeed argue that the fact that such instruments have not been created is strong evidence in favor of the hypothesis that there is no need for them. However, if this hypothesis is correct, then there must already exist close surrogates for these instruments in the market since, as suggested for example by Breeden's (1979) analysis, there is a strong theoretical foundation for the belief that an aggregate consumption-linked security would be widely demanded. I know of no such combination of available securities.

There is, of course, the alternative hypothesis that the nonexistence of such instruments is an example of private market "failure." That is, even though there would be a demand for these instruments, there is insufficient incentive for investment bankers, for example, to undertake the costs of educating both purchasers and issuers especially when the latter have no assets which are naturally matched to this type of liability. Similarly in the absence of a "thick" market for consumption-linked bonds, financial intermediaries would likely be reluctant to issue such annuity liabilities because there is no asset which can be purchased to hedge these liabilities. Of course, some intermediaries might be induced to take some limited amount of risk without being hedged, but this limited amount would surely be inadequate for the scale required for pension plans. On the other hand, it appears that the government is a "natural" intermediary to issue consumption-linked bonds because it has available the power to tax expenditures. That is,

the government could institute a consumption tax proportion to the number of consumption-linked bonds outstanding and the revenues from the tax would exactly match the required liability payments. Moreover, there appears to be no significant social cost to the government issuing consumption-linked bonds and there may be social benefits from the government financing the deficit in this form.<sup>3/</sup> While the principal reason for discussing the creation of such bonds here is their essential role in pension plans, I believe that, independent of pension plans, consumption-linked bonds would be an ideal investment instrument for private saving generally. If this belief is correct and if the government did issue such bonds, then it is likely that private financial intermediaries would introduce consumption-linked annuities and corporations would issue consumption-linked liabilities. The existence of such private-sector financial instruments would serve to make consumption-indexed pension plans more efficient by providing better pricing information for the plans' annuities and by providing a broader-base of securities in which to invest the plans' assets.

Even if the private sector could efficiently provide consumption-linked bonds and life annuities, as noted in the Introduction, private pension plans alone cannot handle either information cost or utility externalities. While it is of course difficult to measure how other people's welfare enter into an individual's utility function, I believe that it is likely to do so in a relative fashion. That is, we are less inclined to "worry about" or make transfers to those who have a relatively high standard of living, and among those with the same current standard of living, we are more sympathetic towards those who

have fallen on "hard times" and experienced a decline from their historical standard of living. If this assessment is correct, then a public plan along the lines discussed here appears to efficiently handle this utility externality for people in retirement. By requiring contributions proportional to individual consumption during his working years and investing these contributions in per capita consumption-linked life annuities, such a plan ensures an accumulated amount sufficient to support a retirement consumption path for the individual at a level (relative to aggregate per capita consumption) similar to that which he enjoyed during the working phase of his life. Linking benefits to per capita aggregate consumption provides for a continuation of this standard of living throughout the retirement years. Thus, a plan with these features meets the objective of ensuring an appropriate relative standard of living in retirement for everyone and it handles the "free-rider" problem.

These features do not, of course, solve the redistribution problem for those people whose relative standard of living is too low during their work years. However, a reasonable argument can be made that it is more efficient to make the necessary transfers by other more-direct means at the time (during the working years) when they are needed instead of attempting to do so indirectly by redistributing future benefits within the retirement plan. There are other good economic arguments for keeping the transfer system and the retirement system separate, but that is not the focus of this paper. I would note however, that the plan analyzed here would automatically handle much of the redistribution

problem for people in their retirement years if a proper transfer system were devised for people during their working years. Transfers received and consumed during the working years will increase future retirement benefits proportionately because the required contributions to the plan are proportional to consumption. Transfers in the form of a total or partial credit for the individual's required contribution to his retirement account would work in a similar fashion, provided that the cost of this transfer is not borne by the retirement plan itself.

Having reviewed the merits of a consumption-indexed pension plan, I now turn to the issue of its feasibility. Although the idea of investing accumulations in consumption-linked life annuities is new, the basic structure of the plan is simple and is essentially the same as a standard defined-contribution pension plan. It is therefore a relatively easy plan to explain and understand. Its format also has the attraction of "stability" in the sense that neither its basic structure nor the parameters of the structure such as the contribution rate or the period of accumulation would require much change over time even in the face of significant variations in economic conditions. It does however require that an appropriate measure for aggregate per capita consumption be chosen.<sup>4/</sup> To select the proper measure would require further study to determine how consumer durable purchases should be treated and whether or not to include items such as leisure time which are not normally included in measures of consumption. There is also the issue of what population measure to use. While investigation of these issues is surely beyond the scope of this paper, their resolution is as surely not

an insurmountable problem. With this measurement problem solved, there does not appear to be any major difficulty with the government issuing consumption-linked bonds and using their prices to determine the value of consumption-linked life annuities.

The main feasibility problems with a public plan as described here are likely to be associated with the method of collecting the required contributions and the maintenance of the individual accumulation accounts. While I have not investigated in detail the amount of computation and record keeping required in the current Social Security system, it appears that the amount required for individual account maintenance would not be significantly larger for a consumption-linked plan. However, the method of collection in such a plan is likely to be more difficult than for current Social Security because the base is consumption rather than income. As outlined, the plan requires that the amount of each contribution be identifiable in the same way that individual federal income tax payments are identified. Therefore, the method of collection necessary for its implementation would probably be like that of the income tax with consumption determined as the residual from a cash flow analysis. The feasibility of such a collection system is currently a topic of considerable discussion among economists principally in the context of the feasibility of an individual expenditure tax (cf. Aaron and Boskin (eds.) (1980) and Pechman (ed.) (1980)). Although a serious analysis of feasibility will not be undertaken here, I would note that there is an important difference between an expenditure tax and the mandatory contribution part of a fully-funded retirement plan. Because it is a

defined-contribution plan and accumulations earn a competitive rate, cheating is less of a problem to the extent that people treat contributions as saving and not as a tax. Indeed, the rich, high-income, and well-informed people who might be thought to have the greatest incentive and opportunity to cheat on a tax are probably the most likely to view such contributions as saving, since these are the people who now voluntarily enter into deferred compensation and Keogh plans. In general, those who cheat on contributions are primarily cheating themselves. However, one slight modification which might make the collection part of the plan more effective would be to have withholding of the required contribution based upon income as is currently the practice for Social Security, and then to have refunds or additional contributions based upon the computation of consumption made in conjunction with the filing of federal income tax returns.

A more-radical modification of the plan as described here was suggested to be by Lester Thurow of MIT. The collections for the plan would be done at the aggregate level by a value-added tax. The aggregate amount collected would then be distributed as contributions to individual accumulation accounts in proportion to the amount of income reported on the individual's federal tax return. The administrative benefits of this modification depend upon the relative costs of collection for a VAT versus a residual cash flow computation on the income tax return. It does have the attractive feature that those who cheat by underreporting income on their federal tax will lose some of their retirement benefits (which they presumably paid for through the VAT). The principal

disadvantage of this modification is that the aggregate contributions will now be treated as a consumption tax which can distort the labor-leisure decision. However, the credit to individual retirement accounts based upon income will act as a subsidy to wage income which may offset this distortion at least in part.<sup>5/</sup> This modification would become considerably more attractive if the government chooses to use a VAT to finance general government expenditures.

In summary, although the method of collecting contributions poses the principal feasibility problem for such a public plan, a number of different methods would seem to serve as close substitutes provided that it remains essentially a defined-contribution plan which earns a fair rate of return on accumulations and pays benefits indexed to consumption.

If a policy decision were made to adopt a public pension plan with a basic structure like the one analyzed here, there would still be the further critical policy decision of what fraction of retirement period consumption should be the target for the plan. Presumably, those who are most concerned about the plan's success in dealing with information cost and utility externalities would advocate a high fraction and those who are most concerned about preserving individual choice would advocate a low fraction. The correct policy decision will surely depend upon the amount of other retirement saving that people are likely to make, especially in housing and private pension plans. The resolution of this policy issue, therefore, requires an analysis of the overall pension system. Since that was the note on which the paper began, it seems an appropriate place for it to end.

Footnotes

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1. On the matter of the assumed stability of  $H(t)$ , I note that because  $c(t;0)$  depends strongly upon the initial endowments of the very young,  $c(t;0)/C(t)$  is likely to be larger when the value of human capital relative to other factors of wealth is larger. It also seems reasonable that the birth rate will be higher when the relative economic value of children is high. However, if  $c(t;0)/C(t) < 1$ , then comparative statistics reveal that these two effects work in opposite directions on  $H(t)$  in a stabilizing fashion.

2. Consumption data from US Department of Commerce, Bureau of Economic Analysis, National Income and Product Accounts of the United States, Table 1.2. Noninstitutional Population Aged 16 and Over data from US Department of Labor, Bureau of Labor Statistics.

3. Fischer (1982) discusses a number of social benefits from the government issuing wage-income-linked bonds, including possible intergenerational risk sharing that private markets cannot provide. Many of the same benefits would come from consumption-linked bonds, and indeed, if a consumption tax is less distorting than a wage tax, then the consumption-linked bonds may be superior.

4. It is, of course, not true that every model of lifetime consumption choice will lead to an efficient allocation of retirement consumption which depends only upon aggregate per capita consumption. For example, Breedan's (1979) important theorems on this matter will not apply if utility of consumption is state-dependent.

5. As I have shown elsewhere (1981), the distortion of the labor-leisure decision of a consumption tax can be offset by linking future retirement benefits to current wage income.

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