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THE TAXATION OF RISKY ASSETS

Jeremy I. Bulow

Lawrence H. Summers

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Cambridge MA 02138

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ABSTRACT

This paper reconsiders the effects of taxation on risky assets, recognizing the importance of variations in asset prices. We show that earlier analyses which assumed that depreciation rates are constant and that the future price of capital goods is known with certainty are very misleading, as guides to the effects of corporate taxes. We then examine the concept of economic depreciation in a risky environment, and show that depreciation allowances, if set ex-ante, should be adjusted to take account of future asset price risk. Some empirical calculations suggest that these adjustments are large, and have important implications for the burdens of, and non-neutralities in, the corporate income tax.

Jeremy I. Bulow
Graduate School of Business
Stanford University
Stanford, CA 94305

(415) 497-2160

Lawrence H. Summers
National Bureau of Economic Research
1050 Massachusetts Avenue
Cambridge, MA 02138

(617) 253-4426

Introduction

The enormous volatility in the prices of capital assets in the American economy has been widely noted. The standard deviation of real stock market returns appears to be about 20 percent per year. Over the last 15 years the price of used capital goods as measured in the stock market relative to consumption goods has varied by a factor of more than two. Comparable volatility is observed in the pricing of used capital goods such as airplanes and office building where an active second-hand market exists. It seems clear that realistic positive or normative analysis of the effects of capital income taxation requires models in which there is substantial scope for variation in the price of capital assets. Yet, the substantial literature on capital income taxes and risk taking has focused almost entirely on models where the return from capital goods is variable but their relative price is certain.

This paper reconsiders the effects of taxes on the return from risky assets, recognizing the importance of variations in the price of capital goods. The results suggest that the burden of the corporate income tax is much greater than that implied by analyses such as Gordon (1981), Stiglitz (1969) and Feldstein (1971), suggesting that it falls primarily on the risk premium on corporate equity. The essential error in earlier analyses is in the treatment of depreciation. The observations that capital asset prices are far more volatile than earnings streams suggests that most of the risk associated with capital assets is in their rate of economic (though perhaps not physical) depreciation rather than in their contemporaneous marginal product. The tax laws' use of ex ante depreciation schedules rather

than ex post depreciation means that depreciation or "capital" risk is not shared by the tax collector. Therefore, a tax like the U.S. corporate income tax absorbs a much greater part of the return than of the risk on capital investments.

The implications of capital risk for depreciation policy are then considered. We argue that traditional concepts of what constitutes economic depreciation are likely to lead to serious errors in realistic settings. In particular, the notion which pervades theoretical and empirical work on depreciation policy, that economic depreciation can be measured by the expected decline in the price of capital assets, is shown to be wrong. We derive a new measure of economic durability which can be calculated from observable market data on asset rentals and prices, and use it as a benchmark for evaluating current tax policies.

The implications of using this new measure of economic durability for a number of tax policy questions is analyzed. Using two alternative empirical methods based on financial market data, we show that prior calculations of appropriate economic depreciation represent serious underestimates because of their failure to consider risk. We also show that proper risk accounting suggests that previous calculations of effective tax rates are very misleading, and that their implications for the problem of neutrality between assets of different durability are not valid. A final implication of the results is that true rates of economic depreciation have increased very substantially over the last decade due to increasing uncertainty.

The first section of this paper examines the effects of capital income taxes on investment within standard models where the price of capital goods is known with certainty. A number of serious logical and empirical problems which arise in applying this analysis to actual corporate income taxes

are then pointed out. The crucial distinction between accounting income on which the corporate tax is levied, and real economic income which includes capital gains and losses is emphasized. The second section shows that these difficulties can be avoided by recognizing the distinction between income and capital risk. The observation that most risk is of the latter variety leads to a reconsideration of tax depreciation policy. In the third section our new economic durability measure is introduced, and the concept of economic depreciation in a risky environment is analyzed. The implications of the analysis for the evaluation of tax policy are taken up in the fourth section. Empirical estimates based on financial market data, suggesting the importance of taking account of asset price risk in estimating rates of economic depreciation are also presented. A fifth section of the paper summarizes the results and suggests direction for subsequent theoretical and empirical research.

I. Taxes and Risk Taking When Depreciation
is Known with Certainty

This section reviews previous results regarding the effects of taxation on investment in risky assets. We show that the seemingly paradoxical conclusion of much of this literature, that taxes on risky assets actually encourage investment in them, is a consequence of the fact that the claim taken by the government has a negligible or negative market value. We then assess the relevance of standard models for the problem of evaluating the effects of the U.S. corporate income tax. The models are found wanting because of their failure to take account of changes in the relative price of capital goods.

We begin by considering a simple model of corporate investment in a mean variance setting similar to the one employed in earlier work on taxation and risk taking. While the model has only one period, Hamilton's (1981) analysis suggests that results very similar to those reported here could be obtained in the context of a continuous time intertemporal capital asset pricing model. While the expected return from risky assets is uncertain, we assume that their terminal price is known with certainty. This assumption (which we relax below) although implicit, plays a crucial role in the analyses of the effects of taxes on risky assets presented in Stiglitz (1969), Feldstein (1971) and Gordon (1981) among others. For simplicity, we initially ignore inflation and personal taxes.

We assume that the perfect capital market assumptions necessary for the validity of the Capital Asset Pricing Model¹ are valid. In this case, individuals are compensated for only the systematic risk which they

bear. Diversifiable risk is not compensated so that private and social risk are equal. This implies that there is no gain from risk spreading through government taxation of risky assets. The private capital market is able to accomplish all feasible risk sharing.² In the context of the Capital Asset Pricing Model in the absence of taxes, the corporate sector will invest up to the point where:

$$(1.1) \quad f'(K)^e - \delta = r + \alpha$$

where $f'(K)^e$ is the expected marginal product of capital, assumed to be a declining function of K , δ is the rate of geometric decay of capital goods,³ r is the real interest rate, and α is the risk premium on corporate sector investments. In the context of the CAPM, $\alpha = \frac{\text{cov}(f'(K), r_m)}{\text{var}(r_m)} \cdot \bar{r}_m$ where r_m is the excess rate of return on the market portfolio, and \bar{r}_m is its expected value. Note that because of the assumption of a constant relative price of capital, no term reflecting capital gains or losses appears in (1.1). This assumption is relaxed below.

Now introduce a corporate income tax at rate τ with full loss offsets which allows depreciation deductions of δK each period.⁴ Such a tax reduces both the expected return and standard deviation of returns on corporate investment by 100τ percent.

Tobin (1958) effectively considers the case where government spending is invariant to corporate tax collections, individuals consider government debt part of net wealth, and the rate of return on the riskless asset is zero. In this case risky investment rises to $\frac{1}{1-\tau}$ times the level it was before the tax, so that each investor has the same expected return and standard deviation as before. The 100τ percent of the systematic risk on corporate investment is effectively eliminated from the economy by the government's

ability (by issuing government bonds when tax collections are low and repurchasing the bonds when collections are high) to create new wealth to offset any risk in tax collections.

Stiglitz (1969) and Feldstein (1971) continue along the same vein with models where the government is able to eliminate from the economy any risk in tax collections. They consider the case where the riskless rate exceeds zero. In their models, if a tax were levied strictly on the risk premium (α , or the actual return less riskless rate) the Tobin result of risky investment rising to $\frac{1}{1-\tau}$ of previous levels is attained. However, if the tax is on the entire return to the risky asset the effect on risky investment is ambiguous: If the risky asset earns the riskless rate pre-tax it yields its owner a lower after-tax return than the riskless asset. This extra tax deters risky investment while the tax on the risk premium encourages it.

Gordon (1981) in his provocative analysis of the effects of the corporate income tax assumes that all revenue risk is ultimately borne by the private sector. In the Gordon model a tax on the risk premium is non-distortionary: unlike the earlier models, risky investment is unchanged because after the tax is imposed in aggregate investors are still bearing as much risk as they wanted to before the tax.

If there is a tax on the entire return the corporate income tax reduces corporate capital accumulation. For example, in the case where investor preferences were such that they demanded the same after-tax capital market line (i.e. risk/return possibilities) as before the tax then corporate investment would take place up to a point where:

$$(1.2) \quad (1-\tau)(f'(K)^e - \delta) = r + \alpha(1-\tau)$$

Gordon points out that the change in the marginal product of corporate capital caused by the imposition of the corporate tax is equal to:

$$(1.3) \quad \Delta f'(K) = \frac{\tau}{1-\tau} r.$$

Given the very low level of real interest rates available in the American economy, this expression seems to imply that the corporate income tax has only a negligible impact on corporate investment. Assuming a two percent real interest rate and a Cobb-Douglas production function, this expression implies that the corporate income tax reduces the size of the capital stock by only about 13 percent.⁵ This low estimate is obtained without considering the effects of accelerated depreciation, the investment tax credit, the tax advantages of debt finance, or the government's ability to spread risk. Introduction of these factors as in Gordon and Fullerton (1981) could even lead to the conclusion that the corporate income tax encourages corporate capital investment. One paradoxical result that arises in this formulation is that the corporate tax encourages investment despite the fact that it raises a significant amount of revenue. The expected revenue yield from the corporate tax is $\tau K \frac{(r + \alpha)}{1-\tau}$ which may be quite large. Plausible American magnitudes suggest that expected annual tax revenues would represent about five percent of the market value of capital stock.

Gordon's provocative paper has several other striking implications. It seems to imply that the corporate tax is very desirable, having little effect on behavior but raising significant revenue. Equation (1.3) would also seem to imply that changes in the tax treatment of the non-corporate sector would have only small effects on the level of corporate investment, unless the real after-tax interest rate was substantially altered. Likewise, the results suggest that differences in the corporate tax rate between sectors are unlikely to have important allocative effects.

These results seem violently counterintuitive. How can a tax which raises significant revenue encourage the activity which is being

taxed? One way to examine this question is to look at the value of the government's claim on the private sector. It is useful to begin with the case where $r = 0$, and the corporate tax is totally non-distortionary. In this case, the government's claim has a zero value. Any individual investor can replicate it costlessly by borrowing and holding the risky asset. The government's tax revenues are fair compensation for the risk that it takes on, but no more. When an individual receives an endowment of part of government revenues, it makes him no wealthier or poorer, since he can costlessly restore his original risk-return position. There is no free lunch here. The non-distortionary character of the tax is a concomitant of its zero market value.

What about the case where the government is able to reduce risk? In this case, as Tobin (1958) and Stiglitz (1969) noted, the corporate tax tends to encourage corporate investment. Unless bonds are net wealth, this situation can only arise if private capital markets are imperfect and so unable to fully diversify risk. By providing a valuable diversification service the government is able to increase the slope of the capital market line faced by investors, even though expected revenues are positive. The net effect can be an increase in risky investments. In this case, the government is providing to taxpayers a form of insurance which is not provided by the private sector.

These results need to be modified slightly in the case where r is positive. The results just stated would then be valid for a tax which had its base the excess return $(f'(K) - \delta - r)$ on corporate capital. As Mintz (1981) and others have observed, a tax on an asset's risk premium has zero market value and no incentive effects.⁶ A standard corporate income tax would then have an effect insofar as it fell on the certain component of the return to corporate investment. The market value of the

government's claim (for a given year) is $\frac{r_0K}{1-\tau}$. In this case where the government's claim is positive, it does have some negative effect on corporate investment.

Introducing inflation and personal taxes would not have any substantial effect in the preceding conclusions. The corporate tax in the foregoing analysis can be treated as an integrated tax including the effects of dividend and capital gains taxes. Introducing inflation would alter the conclusions slightly because of the taxation of nominal as well as real capital gains and interest. However, Gordon's analysis which includes both inflation and personal taxes reaches conclusions similar to those obtained here. The introduction of risk into the models which are normally used in public finance appears to have the dramatic implication that a corporate income tax has only very small allocative effects. In the next section we show that this is a consequence of standard models' failure to account for fluctuations in the relative price of capital goods.

II. Fluctuations in the Price of Capital Goods

The foregoing discussion makes it clear that there are no free lunches in corporate taxation. Only taxes which extract a claim of zero value fail to discourage the taxed activity. In this section we examine the applicability of analyses of the type reviewed in the previous section to the U.S. corporate income tax. We conclude that they are not applicable because the vast majority of the risk borne by corporate investors involves capital gains and losses as the relative price of corporate capital goods changes. The corporate income tax is levied on accounting income which excludes these capital gains and losses. Hence it does not share in this type of risk.

The implausibility of the standard framework's interpretation of the corporate income tax is easily exhibited. If corporate capital requires a significant risk premium, it follows that there must be a sizable risk that the return from holding corporate capital is less than the risk free rate. Otherwise, this asset would dominate the safe asset. The earnings-price ratio measures the rate of return investors would receive if the relative price of capital goods remained constant as assumed in previous analysis of the effects of taxation on risk taking. The lowest value of the earnings price ratio observed since 1948 was 4.62 percent, far in excess of any estimate of the riskless rate.⁷ An alternative way to view the problem is in terms of the after-tax net marginal product of capital. Holland and Myers (1979) report that the lowest value of this statistic was 3.6 percent in 1974. Many studies have estimated the pre-tax marginal product of capital, and have found that it consistently lies above eight percent. These figures imply that if the relative price of capital goods were fixed, corporate investments would dominate acquisitions of the safe asset. In order to make the same point in yet another way, note that the real interest rate has averaged close to zero. Yet, corporate tax collections have always been significantly positive, implying that the tax base has always been significant and positive. It seems clear that any effort to model taxes taking effect of risks borne by investors cannot tell a consistent story if the fiction that the relative price of capital remains constant is maintained.⁸

The importance of changes in the relative price of capital goods may be seen directly in a number of ways. At the aggregate level, the q ratio of the market value of the corporate capital stock to an estimate of

its "replacement cost" has varied between .56 and 1.24 over the past 20 years. This measure involves the ambiguous concept of the replacement cost of capital. An alternative approach is through an examination of the variance in stock market returns. Ibbotson and Sinquefeld (1979) report that the standard deviation of annual stock market returns is about 20 percent. The standard deviation of the earnings price ratio is less than three percent, and the standard deviation of the marginal product of capital was less than two percent. This implies that variations in the earnings price ratio accounts for between one and 17 percent of the variation in stock market returns.⁹ It seems fair to conclude that most of the risk borne by corporate investors is capital risk, involving changes in the price of their asset rather than income risk, involving changes in the current return on assets.

The enormous volatility in the relative price of capital shows up clearly in the markets for used capital goods. Table 1 presents some information on the variability in depreciation rates, as inferred from used car and truck prices. The data are drawn from published guides to used asset prices, and so undoubtedly understate the volatility of actual transactions prices. Since they reflect nationwide averages, they also understate the extent of uncertainty about the rate of decline in the price of any individual capital good. Nonetheless, the data exhibit very significant year-to-year variations in the rate of real price change. For example, the annual rate of real price decline for two year-old Ford F600 trucks varied between 25.8 percent in 1971 and 7.8 percent in 1976. Overall, for most models and vintages of cars and trucks, the standard deviation of annual rates of real price decline was between five and ten percent. It is reasonable to expect that because

TABLE 1
STANDARD DEVIATIONS OF DEPRECIATION RATES

Asset	Age							
	1	2	3	4	5	6	7	8
<u>Cars</u>								
Pinto		8.8	10.7	6.3	5.2			
Malibu		3.9	5.3	10.2	12.7			
Impala		3.1	3.7	6.9	14.1			
<u>Trucks</u>								
Ford F600	6.9	6.5	6.5	5.9	9.6	6.0	6.2	5.1
Ford C8000	1.2	3.2	5.6	5.6	5.9	7.2	11.3	0.9
International Harvester 1600	6.7	5.8	6.4	6.5	7.6	7.8	14.5	13.3
Chevrolet CE61003	7.8	5.2	6.0	5.8	6.8	6.6	10.5	11.5
Dodge D600	6.2	5.6	6.4	4.8	5.7	6.6	7.3	13.1

Note: Data on cars were kindly provided by James Kahn. Data on trucks were provided by Dean Amel. All numbers in the table are percentages at annual rates. Data on cars were for 1972-77 models. Data on trucks were for 1971-78 models, with the prices of the various models tracked through 1980.

of their short lifetimes and easy reproducibility, used car and truck prices should be much less volatile than those of other capital assets.

While data are not available on the rents earned by owners of cars and trucks, it seems fair to conclude that most of the risk borne by holders of these assets is capital risk involving changes in the price rates than income risk involving changes in the current return. These data corroborate the inference drawn from aggregate data that most of the risk borne by corporate investors involves changes in the relative price of corporate capital, rather than movements in the marginal productivity of capital.

Income Risk vs. Capital Risk

It will be useful in what follows to distinguish carefully between income risk and capital risk. An investor demands a premium for holding a risky asset both because the value of the rental services produced this period are risky and because the asset's value at the end of the current period is uncertain. More formally, the holding return on an asset is given by $\frac{f_K}{P_K} + \frac{P_K}{P_K}$. Income risk refers to the uncertainty in the first term while capital risk refers to uncertainty in the second term.

Assets with pure "income risk" would yield uncertain profits in the current period but have a predetermined end-of-period market value. For example, consider an asset that was always supplied perfectly elastically at a price of p^* and exhibited no physical depreciation. In equilibrium the asset would be supplied to the point where the expected return would equal the riskless rate plus any premium necessary to allow for the fact that the current period's income was risky. The asset's terminal value will be p^* for certain, and therefore in a futures market the owner could contract to sell the asset at the end of the period for its expected terminal value of p^* with no premium necessary for any risk in the capital value.

A second type of asset may yield a current income that is predictable with virtual certainty, but the asset's end-of-period value (the present value of subsequent income) will fluctuate substantially. For example, an investor who buys a long-term bond knows that at the end of the current period he will receive a certain amount of income, but the capital value of his bond may change dramatically because of changes in the interest rate and changes in the probability that the bond issuer will be able to make subsequent payments.

For a capital asset, uncertainty about the current period's demand curve and uncertainty about the cost of inputs in the current period cause income risk. Uncertainty about future demand and input prices, plus a less than perfectly elastic capital supply curve, enable asset prices to fluctuate and cause capital risk. Summers (1981a, 1981b) shows how the assumption of rational expectations can be used to model the evolution of asset prices in a situation where the adjustment of the capital stock is costly. The size of fluctuations in asset prices is negatively related to the elasticity of supply of capital goods. Alternatively, and more simply, there may be uncertainty about the rate at which an asset depreciates physically or becomes obsolete.

The distinction between income and capital risk is not of fundamental economic significance, since it refers to forms rather than the size of real economic returns. However, it is crucial to an analysis of a corporate income tax like that in most countries which is levied on accounting measures of income, rather than real economic income. Because accounting income excludes capital gains and losses, the corporate income tax provides a much better hedge against income risk than it does against capital risk. An extra dollar in current income will yield something like an extra dollar in taxable profits. Therefore, the government taxes a share equal to the tax rate in any

unanticipated income gains or losses. This means that the government takes an approximately equal share of risk and return so that the preceding discussion of tax effects is applicable.

Capital risk is another matter. Capital gains and losses on capital are excluded from the tax base, except in the very rare case where they are realized through the sale of used assets. The only allowance made for changes in the value of capital goods is through certain pre-determined depreciation deductions. Therefore, the corporate income tax does not shield taxpayers from any of the capital risk on their assets.

It is tempting but unwise to think that the fact that taxes will be levied on future corporate income reduces capital risk. The extent of capital appreciation or depreciation depends on the percentage change in the value of future rents. A proportional tax will reduce the variance of absolute but not proportional changes in income.

Realistic analysis of the corporate income tax must distinguish then between the taxes levied on accounting income, and capital gains and losses. This leads immediately to consideration of the difference between allowing ex-post depreciation based on actual market valuations of capital goods, and ex-ante depreciation based on the expected decline in a capital good's market value. The former procedure would hedge taxpayers against capital risk, while the latter does not.

Economic Depreciation

The issues involved in the distinction between income and capital risk can be brought out most clearly by examining the polar case of a depreciating asset that has only capital risk. The asset has an expected terminal value

of $1-\delta$ times its original value. Holders of the asset require a risk premium of α , so in a no-tax world equilibrium would require investment to the point where

$$(2.1) \quad f'(K) = r + \delta + \alpha$$

Note that we can use $f'(K)$ rather than $f'(K)^e$: if an asset has no income risk, its gross rentals for the current period can be predicted exactly, even though its capital value (the present value of future returns as of the end of the period) is uncertain.

In this no-tax setting investors are receiving certain gross rental income equal to $r + \delta + \alpha$ times the initial value of the asset plus a risky claim on the capital at the end of the period with an expected value of $1-\delta$ times the original value. The expected value embodies a risk premium of α , however. If the investor went into the futures market and agreed to sell his capital at a certain price at the end of the period, he would only be able to negotiate a price of $1 - \delta - \alpha$ times the initial value. Such a transaction would lock in the safe rate of return of r .

With a tax, the value of economic depreciation deduction can be calculated using similar analysis. Note that economic depreciation deductions are just the capital losses on holding an asset. In the case above, the firm has uncertain depreciation expected to amount to δ (the asset is expected to decline in value to $1-\delta$ times its initial value).

The firm would be equally happy locking in a certain decline in asset value of $\delta + \alpha$, thereby passing the risk of ownership to the party

it made a futures contract with. That is, the firm is equally happy with certain depreciation deductions of $\alpha + \delta$ as with the uncertain economic depreciation deductions which are expected to be δ . This result may seem paradoxical as it implies that the certainty equivalent of the stream of depreciation tax shields is less than their expected value. The paradox is resolved by noting that the depreciation tax shield is a "negative β " asset. When the market does well, depreciation is low, and conversely when the market does poorly, depreciation is substantial.

Now, the effect of a corporate tax with ex post depreciation deductions can be measured. The investor has a required certainty equivalent rate of return, net of depreciation, of r . This return will require

$$(2.2) \quad f' = \frac{r}{1-\tau} + \delta + \alpha$$

The actual expected after-tax rate of return is

$$(2.3) \quad (f' - \delta)(1-\tau) = r + \alpha(1-\tau)$$

while the certainty equivalent of the rents times $1-\tau$ (i.e., $f'(1-\tau)$) less the certainty equivalent of the economic depreciation to be suffered times $1-\tau$ (i.e., $(\delta+\alpha)(1-\tau)$) equals r .

In equation (2.3) we see that the investor only requires a risk premium of $\alpha(1-\tau)$ because with ex post depreciation the government has taken on 100 percent of the risk. Comparing (2.2) and (2.1) we can see

that because of this risk sharing, the marginal product of capital in the corporate sector need rise by only $\frac{\tau}{1-\tau} r$, which is a small amount given the typically low values cited for r .

Expected tax revenues are quite large, however, equaling $\frac{\tau}{1-\tau} r + \alpha\tau$. The certainty equivalent of those revenues is only $\frac{\tau}{1-\tau} r$ with the remainder being compensation for taking on risk at the market price. Therefore, only $\frac{\tau}{1-\tau} r$ is added in to the required marginal product of corporate capital. This is Gordon's argument reviewed in the first section of the paper. It shows that Gordon implicitly assumes that capital gains and losses are included in the corporate tax base.

However, corporate depreciation deductions do not vary with market valuation of capital goods. Instead, firms receive depreciation deductions according to a fixed depreciation schedule. Contrast the required pre-tax marginal product of capital when investors receive certain depreciation deductions equal to expected depreciation, with the required return when economic depreciation is permitted. With fixed depreciation deductions, the government does not share in the risk associated with capital gains and losses on asset holdings. Therefore, investors will require an after-tax expected return that fully compensates them for all risk in holding the asset.

The firm receives certain deductions equal to expected depreciation of δ . The value of the tax shield produced by those deductions is $\delta\tau$. Given that the government is not sharing in the deviations of capital values from the expected values, investors will require an after-tax return of

$$(2.4) \quad f'(1-\tau) + \tau\delta = r + \delta + \alpha$$

Solving for f' yields

$$(2.5) \quad f' = \frac{r}{1-\tau} + \delta + \frac{\alpha}{1-\tau}$$

Because the government taxes the risk premium by setting taxes on expected economic income, but does not share in the risk, investors require a higher pre-tax return (comparing (2.5) with (2.2)) even though expected tax payments are the same. The difference is very substantial. Notice that (2.5) implies that $(f' - \delta)(1-\tau) = r + \alpha$ which exactly parallels standard results in the certainty model. Data on U.S. stock and bond returns suggests that $\alpha \approx .06$ so that the use of ex-ante rather than ex-post depreciation schedules has very substantial effects. The difference in the required pre-tax rate of return in (2.2) and (2.5) is $\frac{\alpha\tau}{1-\tau}$, which equals the certainty equivalent increase in the tax liability caused by using expected rather than economic depreciation.

III. Ex ante Economic Depreciation

Making Ex ante Depreciation as Favorable as Ex post Depreciation

We have shown that given an asset with expected one period economic depreciation of δ plus a risk premium entirely attributable to capital risk of α an investor would value uncertain ex post depreciation deductions with an expected value of δ as much as certain deductions of $\delta + \alpha$. This result can be extended to calculate the entire ex ante depreciation schedule which is as favorable as ex post depreciation. We derive the result below for an asset with exponential expected depreciation, with all capital risk, and with a constant risk premium of α . Generalization to non-exponential depreciation and fluctuating capital risk premia is transparent.

For this analysis the fiction of the firm considering the sale of its asset in the futures market is again instructive. Consider an asset with an initial value of 1, expected depreciation of δ , and a required risk premium of α . Then the firm could lock in a sales price for one period from now of $1 - \delta - \alpha$. Equivalently, it is equally happy with certain depreciation of $\delta + \alpha$ or uncertain economic depreciation with an expected value of δ .

Now consider what price could be received by agreeing to sell the asset two periods from now. If the risk premium and expected depreciation rate are constant, then the firm would be able to lock in a sales price two periods from now of $(1 - \delta - \alpha)^2$. Equivalently, the firm is equally happy taking economic depreciation in the second period or taking certain depreciation of $(1 - \delta - \alpha) - (1 - \delta - \alpha)^2 = (\alpha + \delta)(1 - \delta - \alpha)$. The analysis could be continued to show that the certainty equivalent of the risky depreciation deductions in period t is $(\alpha + \delta)(1 - \delta - \alpha)^{t-1}$.

To show that such certain depreciation is ex ante as favorable as ex-post economic depreciation we verify that the required rate of return with the proposed stream of certain deductions equals the required rate of return with ex-post economic depreciation. In both cases we verify that for a break-even investment $f' = \frac{r}{1-r} + \delta + \alpha$, as in (2.2).

For the proposed certain deductions the net present value of an extra dollar invested in the asset, including risky cash flows and riskless depreciation allowances can be written as

$$(2.6) \quad \sum_{t=1}^{\infty} \frac{f'(K)(1-r)(1-\delta)^{t-1}}{(1+\alpha+r)^t} + \sum_{t=1}^{\infty} \frac{(\alpha+\delta)(1-\alpha-\delta)r^{t-1}}{(1+r)^t} - 1$$

Equation (2.6) verifies that if the marginal product of capital, $f'(K)$, equals $\alpha + \delta + \frac{r}{1-\tau}$ the expected after-tax gross rents discounted at the risky rate of $\alpha + r$ plus the certain depreciation tax shields discounted at the riskless rate make the net present value of the investment equal zero.

With ex-post economic depreciation, we have

$$(2.7) \quad NPV = \sum_{t=1}^{\infty} \frac{f'(K)(1-\delta)^{t-1}(1-\tau) + \delta t(1-\delta)^{t-1}}{(1+\alpha(1-\tau) + r)^t} - 1$$

In (2.7) the numerator is the firm's expected after-tax income in each period while the denominator is the rate appropriate to discounting these flows, given that the government has taken 100τ percent of the risk. As in (2.6) the required rate of return, $f'(K)$, pre-tax, to make the NPV equal zero, is $\alpha + \delta + \frac{r}{1-\tau}$ -- just as with the previously described certain deductions.

Ex ante Depreciation Rates as a Measure of Economic Durability

Numerous commentators on depreciation policy (e.g., Hulten and Wykoff [1981]) suggest the expected decline in the market value of an asset as a proxy for economic depreciation. Our work indicates that a better measure would be the expected depreciation rate plus the portion of the risk premium attributable to capital risk. (As indicated earlier, this is most of the risk premium.)

Incorporating the capital risk premium in the definition of ex ante depreciation leads to a more satisfactory measure of "economic durability." For example, other things held equal, a greater proportion of the purchase price of a very risky asset will be for near-term cash flows than for a less risky asset. Therefore, one would naturally tend to think of the

risky asset as economically less durable.

Similarly, a change in the riskless interest rate effects the value of a risky asset as though the risk premium were part of the depreciation rate: A one percentage point increase in r decreases the present value of an exponentially depreciating constant risk premium asset by $\frac{100}{r+\delta+\alpha}$ percent - that is, the "duration" of the asset is $\frac{1}{\delta+\alpha}$.

IV. Implications for Tax Policy Analysis

The foregoing analysis suggests that any realistic description of depreciation must recognize its stochastic character. In this section, we show how the concepts developed above can be used to produce empirical estimates of what ex ante tax depreciation schedule is required to correspond to ex-post depreciation. Some crude estimates based on market data of the overall rate of depreciation of the capital stock are then presented. These are compared with standard BEA estimates. Finally, the implications of our results for the measurement of effective tax rates, and for analyses of the effects of taxation of the choice between assets with differing durability are then considered.

We continue to rely on the approximation that all risk is capital risk. In this case the previous discussion demonstrated that it is appropriate to add the asset's risk premium to its expected rate of physical depreciation in order to determine the appropriate rate of ex-ante depreciation. This suggests one empirical method of deriving estimates of appropriate depreciation schedules. If data on a time series of used asset prices can be obtained, and if the assumptions of the capital asset pricing model are accepted, the economic depreciation rate for an asset can be

estimated as:

$$(4.1) \quad \delta_i^* = \delta_i^e + \beta_i (R_m - R_f)$$

where δ_i^* is the rate of ex-ante depreciation which is the certainty equivalent of ex-post depreciation, δ_i^e is the expected rate of depreciation, β_i is the asset's beta, R_m is the return on the market, and R_f is the riskless rate.

Unfortunately, we are unaware of any data set extensive enough to permit estimation of β for any type of used assets. Therefore, it is difficult to use (4.1) as a basis for deriving estimates of economic depreciation for particular types of capital assets. However, it is possible to use (4.1) to make an approximate estimate of the depreciation rate on the total capital stock. Estimates of δ^e prepared by the BEA for the National Income and Product Accounts imply an average depreciation rate of 10.5 percent for the non-financial corporate equipment and structures in 1979. Hulten and Wykoff (1981) obtained lower estimates using data on used asset prices. If it is assumed that the risk characteristics of corporate capital are like those of unlevered equity, the second term in (4.1) is equal to about six percent. Ibbotson and Sinquefeld (1979) report that the risk premium on the stock market, $R_m - R_f$ has averaged about nine percent over the last 50 years. The six percent figure is obtained by assuming a debt-to-market value of equity ratio of one-half as implied by statistics reported in Gordon and Malkiel (1981).

This calculation illustrates that taking account of risk has important implications. These estimates suggest that it raises the appropriate

average ex-ante rate of depreciation from 10 to 16 percent. Stated differently, if double declining balance depreciation is assumed, our risk adjustment reduces the appropriate average tax lives from 20 to 13 years.

There is an alternative way of calculating the appropriate ex-ante depreciation rate on assets. Equation (1.1) implies that in the absence of taxation investment will take place up to the point where

$$(4.2) \quad \frac{f'}{p_K} - \delta^e = \alpha + r$$

where p_K is the price of capital goods. This suggests the appropriate rate of ex-ante depreciation on an asset ($\delta^e + \alpha$) can be estimated as:

$$(4.3) \quad \frac{f'}{p_K} - r = \delta + \alpha$$

That is, the appropriate rate of ex-ante depreciation for an asset is given by the difference between its rental price ratio and the risk free rate.¹¹ The analysis is more complex in the presence of taxation since part of the value of a capital asset represents the present value of the depreciation tax shields which it carries. We illustrate this below when a generalization of (4.3) is used to calculate the depreciation rate on the aggregate capital stock. Data on rental price ratios for different capital assets are not readily available. Gordon (1979) estimates rental price ratios of close to 25 percent implying depreciation rates of approximately 22 percent on airplanes. This compares to the BEA rate of 7.5 percent and an estimate of 18.3 percent by Hulten and Wykoff (1981).

A common rule of thumb in real estate is that properties sell for 100 months rent. This implies according to (4.3) a depreciation rate of six percent assuming that cash expenses including property taxes plus the riskless interest rate add to six percent. The National Income Accounts use a much smaller depreciation rate. These examples provide further evidence that current measures of economic depreciation do not provide a good guide to appropriate tax policy, if emulating the effects of ex-post depreciation is the desideratum.

For the economy as a whole, we can make some calculations as to the adequacy of depreciation deductions. We can write the market value of the corporate sector as

$$(4.4) \quad MV_t = R_t + B_t$$

where

MV_t = market value of corporate sector at time t

R_t = present value of after-tax cash flows if no future depreciation were allowed

B_t = present value of depreciation tax shields (equals tax rate times present value of depreciation deductions).

We also have

$$(4.5) \quad R_t(1+r+\alpha) = C_t + E(R_{t+1})$$

$$(4.6) \quad E(R_{t+1}) \equiv R_t(1-\delta^e)$$

where

C_t = after-tax cash flows produced in period t , less the tax savings due to depreciation in the period.

That is, (4.5) says that including the cash payout C_t and the expected terminal value of the capital $E(R_{t+1})$, the investor must have an expected return of $r+\alpha$. Equation (4.6) simply defines the expected depreciation rate as δ^e .

Combining (4.4), (4.5), and (4.6) yields

$$(4.7) \quad \alpha + \delta^e = \frac{C_t}{MV_t - B_t} - r$$

where $\alpha + \delta^e$ is our measure of certainty equivalent economic depreciation. Formula (4.7) allows us to use aggregate data to estimate what we call ex-ante depreciation.

Table 2 shows the ex-post, ex-ante, and National Income Accounts depreciation rates for the years 1950-79. The ex-post depreciation rate is meant to measure the percentage decrease in the real market value of corporate physical assets that were held at the beginning of the year. The ex-post rate was calculated as gross investment in physical assets by non-financial corporations, taken from the National Balance Sheets, less the increase in the real market value of physical corporate assets. For any given year, the market value of physical corporate assets was taken by adding the market value of NFC equity plus short-term and long term debt, and subtracting financial assets. The market value of short-term debt and the value of financial assets come from the National Balance Sheets; the market value of long term debt is from Bulow and Shoven (1981), who took the national balance sheet numbers and multiplied by the ratio of market to book value of New York Stock Exchange bonds. To calculate the increase in the real market value of physical assets in year t , the market value at the end of year t was reduced by the market

value at the end of $t-1$, times one plus the inflation rate for year t as measured by the GNP deflator. The ex-post depreciation rate, reported in the table, was calculated as the depreciation number derived above divided by the market value of physical assets at the end of the prior year, times one plus the inflation rate.

Ex-ante physical depreciation was calculated using (4.7). It is the after-tax cash flow less the value of depreciation tax shields for a given year, C_t , divided by the market value of physical assets at the end of the prior year, MV_t , (calculated above), less the present value of all future depreciation deductions, B_t . C_t was calculated using data from the Economic Report of the President, 1982. The formula used was corporate profits plus capital consumption allowance with capital consumption adjustment, plus net interest, minus corporate profits taxes, minus .48 times the sum of NIA capital consumption allowance and the capital consumption adjustment (the latter usually a negative number).

B was computed by taking B/p_K from Summers (1981) and multiplying by the sum of the current cost value of inventories plus property, plant, and equipment from the National Balance Sheets. The ex-ante depreciation rate was calculated as $\frac{C_t}{MV_t - B_t}$ less the riskless interest rate, calculated by subtracting the inflation rate from the average three-month Treasury bill rate.

Finally, NIA depreciation rates were calculated by dividing NIA depreciation by the current cost value of all NFC tangible assets at the end of the year, as reported in the National Balance Sheets.

In the table we present some data on depreciation at the aggregate level. Since the estimates are derived from market data, they pertain to all the assets of the corporate sector, not only those normally treated as depreciable. These include land and inventories, which are normally

TABLE 2

COMPARATIVE DEPRECIATION RATES

Year	Ex-post Depreciation	Ex-ante Depreciation	NIA Depreciation
1950	-0.5	27.5	5.7
51	11.1	29.2	5.8
52	7.5	22.4	5.6
53	18.7	19.3	5.7
54	-27.2	23.2	5.8
1955	-4.7	16.0	6.0
56	8.1	12.6	6.3
57	27.0	11.7	6.2
58	-23.1	13.6	6.1
59	7.8	10.1	6.2
1960	10.2	9.0	6.1
61	-9.5	9.1	6.1
62	17.5	8.8	6.1
63	-4.9	10.2	6.0
64	-.7	9.3	6.1
1965	3.2	9.6	6.2
66	21.1	9.1	6.3
67	-10.4	10.4	6.2
68	0.9	8.1	6.3
69	25.4	5.8	6.4
1970	11.1	6.7	6.5
71	-1.5	9.4	6.5
72	2.4	8.4	6.7
73	31.9	6.2	6.7
74	43.7	8.5	6.9
1975	-2.6	19.7	6.6
76	2.8	14.6	6.8
77	23.1	14.0	6.9
78	21.6	15.2	6.9
79	14.5	13.7	7.0
1950-59	2.5	18.6	5.9
1960-69	5.3	8.9	6.2
1970-79	14.7	11.6	6.8
1950-79	7.5	13.0	6.3

treated as having a zero depreciation rate, and account for about a third percent of non-financial corporate tangible assets based on information in the National Balance Sheets. This means that the NIA depreciation figures we report are roughly a third percent lower than the composite rate on equipment and structures.

Broadly, the results corroborate the calculations presented so far. The mean ex-post depreciation rate is 7.5 which is quite close to the standard estimates of the rate of depreciation. It is extremely volatile ranging from 43.7 to - 27.2. The mean ex-ante depreciation rate is 13.0 reflecting its inclusion of the risk premium.

Including the recent Hulten and Wykoff (1981) used asset price depreciation estimates we now have four alternative measures of depreciation. The highest estimate is ex-ante depreciation of 13.0 percent. This number is much higher than the ex-post rate of 7.5 percent. By contrast, both the NIA and Hulten & Wykoff estimates are significantly lower than the ex-post rate. The NIA rate is 6.3 percent, while Hulten & Wykoff are roughly 20 percent below the NIA estimates for 1949-74.

The 5.5 percentage point differential between ex-ante and ex-post depreciation is remarkably in line with our prediction. Recall that with a risk premium on the market of 9.0 percent and a debt/equity ratio in the corporate sector of $\frac{1}{2}$ we predict a 6.0 percent differential. Of course estimates of the market risk premium include data from 1950-79. Nevertheless, it is of note that two alternative empirical methodologies of estimating the difference between ex-ante and ex-post depreciation give virtually the same answer.

The most striking feature of the data is the increase in the ex-ante rate of depreciation during the last decade. It averaged 7.8 between 1970 and 1974 and 15.4 between 1975 and 1979. This increase reflects the

increased relative price uncertainty in the economy in three ways: first, the increased risk may have led to increased expected returns and higher risk premia. Second, increased relative price uncertainty increases the expected decline in an asset's price because the investor is likely to have made a more costly deviation from the ex-post optimal choice of asset production techniques. Therefore, even if investors were risk neutral they would require higher rental price ratios because of increased expected economic depreciation. Third, the increased uncertainty about relative prices could have led firms to invest in less durable assets-- particularly in the period right after the 1973-74 oil crisis when, with new information coming in rapidly, firms were no doubt leery of committing money to durable irreversible projects. For an excellent analysis and discussion of these issues see Bernanke (1982).

Using the approach developed here to examine the effective tax rate on capital income in the U.S. is beyond the scope of this paper. The data do appear to indicate that current depreciation allowances are much less adequate than is usually assumed.

The importance of the effects considered here, and their implications for the issue of neutrality can be examined by reconsidering standard calculations of effective tax rates. Standard procedures include calculating the expression:

$$(4.8) \quad \tau^e = \frac{R-s}{R}$$

where τ^e is the effective tax rate on a project, R is its pre-tax internal rate of return and s is its post-tax required internal rate of return.

Consider investing in a project which is all equity financed, has a marginal product of capital of $f'(K)$, expected depreciation δ^e and a required

return of $r+\alpha$ where α is the risk premium. Assume depreciation at rate δ^e is permitted by pre-tax law. Equation (1.2) holds that equilibrium requires that the condition $(1-\tau)(f'(K)-\delta^e) = \alpha+r$ hold. This implies that the standard calculation of an effective tax rate would yield

$$(4.9) \quad \tau^e = \frac{\frac{\alpha+r}{1-\tau} - \alpha - r}{\frac{\alpha+r}{1-\tau}} = \tau$$

Now our analysis suggests that it would be more correct to use risk adjusted rates of return in calculating effective tax rates. Reinterpreting R and s as the risk adjusted rate of return yields:

$$(4.10) \quad \tau^e = \frac{\frac{\alpha+r}{1-\tau} - \alpha - r}{\frac{\alpha+r}{1-\tau} - \alpha} = \frac{\tau(\alpha+r)}{\tau\alpha+r}$$

As long as $\alpha > 0$ the risk adjusted effective tax rate exceeds the unadjusted rate. To see the importance of the risk adjustment, suppose that $\alpha = .06$ and $r = .02$, and $\tau = .5$. Standard effective tax rate calculations would yield $\tau^e = .5$ while our procedure yields $\tau^e = .80$. Thus the use of misleading measures of economic depreciation can lead to serious underestimates at the burden of capital income taxation.

Equation (4.10) also suggests that if tax depreciation is economic in the conventional sense, the tax system will be biased against risky (high α) assets. These assets are likely to be ones which are long lived since their greater durability causes their values to be more sensitive to interest rate changes and new information. They are also likely to be assets whose supply is relatively inelastic. Note also that if depreciation allowances were allowed at exponential rate $\delta^e + \alpha$, τ^e would equal τ .

Finally, observe that the understatement of depreciation will be greatest for long lived assets. Land, for example, has a $\delta < 0$. Our work shows, however, that land should be depreciated. In general, $\frac{\delta + \alpha}{\delta}$ will be greatest for long lived assets. We believe that the risk effects are sufficiently important that calculations which ignore them such as those presented in Jorgenson and Sullivan (1981) are likely to be very misleading. In future work, we hope to use data on used asset prices and rents to estimate appropriate ex-ante depreciation rates for different assets.

V. Conclusions

This research echoes, for problems involving risk, the argument of Summers (1981b) that analyses of the effects of capital taxation must recognize the importance of fluctuations in capital asset prices. We argue that most of the risk borne by owners of corporate capital pertains not to the current rentals which are hedged by the corporate income tax, but to changes in the relative price of these assets which are not hedged because the tax is levied on accounting income. This means that the tax takes a much larger fraction of the return than it takes of the risk on corporate investments.

We then analyze the role of tax depreciation recognizing that there is substantial volatility in the rates at which capital assets lose their value. The tax system as currently set up relies on ex-ante depreciation allowances rather than actual ex-post measures of depreciation and so does not share in the associated risk. We show that in a stochastic environment the natural counterpart to economic depreciation involves allowing ex-ante

depreciation at a rate faster than the expected decline in asset values. More precisely if depreciation allowances are to compensate investors for the risks they bear, the portion of the risk premium in the asset's expected return that is attributable to "capital risk" (asset price fluctuation) must be added to expected depreciation.

Our empirical analysis reveals that this adjustment is of substantial importance. Using two alternative methodologies both based on financial market data we conclude that the appropriate rate of ex-ante depreciation to allow in the U.S. non-financial corporate sector is approximately twice that implied by data in the National Income Accounts. This suggests that many previous analyses have significantly understated the burden of taxes on corporate capital. It also suggests the need for further work in order to assess possible non-neutralities between assets of different durability and with different risk characteristics.

The research in this paper could be extended in a number of directions. The current analysis has ignored considerations of personal income taxation and corporate financial policy. We have not yet attempted an analysis of optimal taxation in the presence of capital risk. Such an analysis would need to recognize that the prices of inelastically supplied assets are likely to be more volatile than the prices of more elastically supplied assets. We hope to follow the valuable work of Hulten and Wykoff (1981) in using data on used asset prices, in conjunction with rental price data to derive depreciation rates on different types of assets.

Our analysis also has implication for work on tax reform. The scheme we propose of allowing ex-ante depreciation deductions at a rate which compensates investors for the risks they bear is one of a number of equivalent types of tax reform. Similar goals could be achieved by

including capital gains and losses, as measured on the stock market, in the corporate income tax. Alternatively, a tax on the net worth of the corporate sector could be employed. In the framework considered here, these tax schemes would be very similar. In the context of richer models there would be important differences which seem worthy of study.

The analysis here also has important implications for research on investment. The rate of ex-ante economic depreciation appears in standard expressions for the cost of capital. Our calculations show that rate increased substantially during the 1970's. This suggests a possible explanation for the sharp decline in net investment which has been observed during the 1970's.

Footnotes

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1. These include the absence of transactions costs, limitations on short selling, homogeneous expectations, the existence of a safe asset, and competitive behavior.
2. If these assumptions are not satisfied, the government can increase welfare by serving as a financial intermediary. However, if there are economic reasons for the non-existence of markets, such as moral hazard problems, there is no presumption that tax policy can increase welfare.
3. Note that the gross marginal product of capital is $f'(K)$ while the net product is $f'(K) - \delta$. The depreciation assumption implies that the value of a capital good declines by the factor $(1-\delta)$ each period.
4. This corresponds to standard concepts of economic depreciation in a certainty setting.
5. This calculation assumes that the pre-tax gross marginal product of capital is .20, and the aggregate production function of the corporate sector is Cobb-Douglas with a capital share of .25. It is also assumed that labor is supplied inelastically to the corporate sector.
6. This is because the government's claim can be costlessly replicated by bargaining at the riskless rate to buy equity. The argument is the same as that presented above.
7. Similar conclusions can be obtained using the inflation-adjusted earnings price ratios described in Summers (1981b).

8. There is a remote possibility of a "peso" problem, where the market risk premium reflects a low probability disaster which has not yet taken place. Such a disaster would surely involve a change in the relative price of capital goods, as discussed below.
9. The range reflects ambiguity in the allocation of the covariance term, and the choice of concept. Note that since much of the variation in earnings-price ratios, and marginal products of capital is forecastable, these figures overstate the extent of income risk.
10. This conclusion needs to be modified slightly because of personal taxation of capital gains. However, these taxes are levied at very low effective rates because of the advantages of deferral and the absence of constructive realization.
11. Note that if $\alpha = \beta_i(R_m - R_f)$ this equation is equivalent to (4.1).

References

- Bernanke, Ben S. "Irreversibility, Uncertainty, and Cyclical Investment." Quarterly Journal of Economics, forthcoming, 1982.
- Board of Governors of the Federal Reserve System. "Balance Sheets for the U.S. Economy 1945-80." Washington, D.C., October 1981.
- Bulow, Jeremy I. and John B. Shoven. "Inflation, Corporate Profits, and the Rate of Return to Capital." National Bureau of Economic Research, Conference Paper No. 107, March 1981.
- Feldstein, Martin S. "The Effects of Taxation on Risk Taking." Journal of Political Economy (1969): 755-64.
- Gordon, Robert J. "Energy Efficiency, User Cost Change, and the Measurement of Durable Goods Prices." National Bureau of Economic Research Working Paper No. 408, November 1979.
- Gordon, Roger H. "Taxation of Corporate Capital Income: Tax Revenues vs. Tax Distortions." National Bureau of Economic Research Working Paper No. 687, June 1981.
- Gordon, Roger H. and Burton G. Malkiel. "Corporation Finance." In H. J. Aaron and J.A. Pechman (eds.), How Taxes Effect Economic Behavior. Washington, D.C.: Brookings Institution, 1981. Pp. 131-92.
- Hamilton, Jonathan. "Taxation and Risk-Taking: A Continuous Time Analysis." Mimeo, Tulane University, 1981.
- Holland, Daniel and Stewart C. Myers. "Trends in Corporate Profitability and Capital Costs." In Robert Lindsay (ed.), The Nation's Capital Needs: Three Studies. Committee for Economic Development, 1979.

- Hulten, Charles R. and Frank C. Wykoff. "The Measurement of Economic Depreciation." In C. R. Hulten (ed.), Depreciation, Inflation, and the Taxation of Income from Capital. Washington, D.C.: Urban Institute, 1981. Pp. 81-125.
- Ibbotson, Roger G. and Rex A. Sinquefeld. Stocks, Bonds, Bills, and Inflation: Historical Returns (1926-78) and Supplements. Charlottesville, Va.: Financial Analysts' Research Foundation, 1979.
- Jorgenson, Dale W. "The Economic Theory of Replacement and Depreciation." In W. Sellekaerts (ed.), Econometrics and Economic Theory. New York: Macmillan Publishing Co., 1973. Pp. 189-221.
- Jorgenson, Dale W. and Martin A. Sullivan. "Inflation and Corporate Capital Recovery." In C. R. Hulten (ed.), Depreciation, Inflation, and the Taxation of Income from Capital. Washington, D.C.: Urban Institute, 1981. Pp. 171-237.
- Mintz, J. M. "Some Additional Results on Investment, Risk Taking, and Full Loss Offset Corporate Taxation with Interest Deductibility." Quarterly Journal of Economics (November 1981): 631-42.
- Stiglitz, Joseph. "Effects of Wealth, Income, and Capital Gains Taxation on Risk Taking." Quarterly Journal of Economics 83 (1969): 263-83.
- Summers, Lawrence H. "Taxation and Corporate Investment: A q-Theory Approach." Brookings Papers on Economic Activity 1 (1981): 67-127 (a).
- _____. "Taxation and the Size and Composition of the Capital Stock: An Asset Price Approach." Forthcoming in Carnegie-Rochester series. Journal of Monetary Economics supplement, 1981 (b).
- Tobin, James. "Liquidity Preferences as Behavior Towards Risk." Review of Economic Studies 25, No. 67 (February 1958): 65-86.