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EVALUATING THE TAXATION OF RISKY ASSETS

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ABSTRACT

This paper explores the taxation of risky assets, both from the theoretical perspective of optimal taxation and from the practical one of measuring "the" tax rate on an asset when, as under existing practice, its stochastic returns are subject to differential tax treatment across states of nature. The results suggest that it may be "appropriate" for tax rates to vary systematically with the riskiness of an asset, but that use of the expected tax rate to evaluate the characteristics of any particular tax system may be very misleading.

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I. Introduction

Suppose that two investment projects have the same expected rate of return after tax but different expected rates before tax. Which project is taxed more heavily? Is it inefficient for tax burdens to differ, if indeed they do?

While we are accustomed to answering these questions for the certainty case, the task of doing so becomes more complicated when there is uncertainty and investors are risk averse. For a number of reasons, there need be no well-defined measure of "the" tax rate on an asset, at least not in any sense familar from the certainty context. First of all, if there does not exist an efficient pooling of private risks in society, then competitive equilibrium is not Pareto efficient. In such an environment, taxation can act to mitigate this externality. As a result, the excess burden of imposing a tax on asset returns may very well be negative; it may be possible for government to raise a substantial amount of revenue and yet make no asset holder worse off. Thus, in the sense of resource cost imposed, a tax may not really be a tax.

Even if efficient markets for risk exist, each asset is actually a bundle of state-contingent commodity claims. Unless returns in different states are taxed uniformly, the tax rate on the entire bundle must be calculated using some weighting scheme for combining these state-contingent tax rates. One obvious candidate, corresponding to that used implicitly in some empirical studies, is the expected tax rate, based on probability weights for the different states of nature. However, this particular measure will be seen to be of limited value unless the risk characteristics of the taxes collected and the net return that remains are also known. This aggregation problem is not merely of theoretical interest. Current methods of capital income taxation impose markedly different relative tax burdens on any given asset in different states of nature, because of such characteristics as imperfect loss offsets and mismeasurement (often intentional) of economic depreciation.

Finally, the question of whether asset returns are taxed efficiently reduces to one concerning the optimal taxation of the underlying state-contingent returns themselves. Just as in the certainty case, Ramsey rules may be derived for the taxation of different commodities under uncertainty. While these tax rates should be different across the states, in general, the conditions required for the optimality of uniform taxation and the direction of divergence when such conditions are not met have a special interpretation when applied to the taxation of risky claims. Not surprisingly, these results are closely related to those of the earlier literature on taxation and risk-taking. However, the present analysis explores not how taxes affect risk-taking, but how they should affect risk-taking, from an efficiency perspective.

In the next section of the paper, we discuss the optimal taxation of risky assets and the state-contingent claims of which they are composed. Section III explores the problem of calculating a meaningful '"effective" tax rate for a risky asset when its underlying returns are taxed at rates differing across states, and points out the difficulties involved in using expected or "average" tax rates. In Section IV, we apply this analysis to a tax system resembling that of the U.S. to determine the biases involved in the use of expected tax rates.

II. Optimal Taxation of Risky Assets

Many contributions to the literature on taxation and risk-taking have utilized the fact that risky assets are, themselves, bundles of contingent commodities, in applying standard results from the theory of demand (see, for example, Fischer 1972, Diamond and Yaari 1972, and Sandmo 1977). From this perspective, the portfolio choice problem may be seen simply as a choice among the underlying contingent claims subject to a budget constraint. When the number of assets is not sufficient to allow purchase of all state-contingent claims independently, the consumer faces the additional constraint of having to choose among bundles in the commodity subspace spanned by existing assets. Such analysis assumes, either explicitly or implicitly, that competitive trading exists for all risky assets. In such a case, the resulting allocation of societal risk, subject to the constraints imposed by the incomplete spanning of the entire comodity space, is Pareto optimal (Diamond 1967).

We may approach the optimal tax problem for risky assets from this same perspective. For simplicity, we assume that the entire space of contingent claims is spanned by existing assets. A more general treatment would relax this restriction, but would lead to a difficulty with the usual fixed revenue constraint assumption. For example, if there were three uncertain states and two assets offering combinations of returns in these states, arbitrary combinations of receipts in the different states could not normally be achieved without raising an excess amount of revenue in at least one state; this additional welfare cost should be accounted for. It then would be preferable to allow explicitly for the societal preferences for public expenditures (as in

Atkinson and Stern 1974) and choose the level of public output in the different states along with the optimal taxes, subject to the spanning constraints.

Consider a simple two-period two-state model in which the representative individual is endowed with I units of labor in period 1, out of which he can consume leisure, or work and purchase assets that provide different combinations of returns, which are consumed in the two states in period 2. The individual's optimization problem is

(1) $\quad$ maximize $W(C_0, C_1, C_2)$ such that $p_1C_1 \; p_2C_2 = (I-C_0)$

where C_0, C_1 and C_2 are first period leisure and consumption in each of the second period states, and p_1 and p_2 are the implicit prices, in terms of labor, faced by the consumer for the state-contingent claims C_1 and C_2, derived from the prices of assets used in combination to obtain returns in the two states. It is possible to restrict the form of the general utility function $W(\cdot)$ by adopting the Van Neumann-Morgenstern axioms that imply expected utility maximization. We return to this point below.

Let the indirect utility function that results from (1) be $v(p_1, p_2, I)$, and suppose that the government wishes to raise revenue sufficient to acquire the vector $\underset{\sim}{S} = (S_0, S_1, S_2)$ of the three commodities using taxes θ^1 on C_1 and θ^2 on C_2.[1/] If we let q_1 and q_2 be the producer prices for C_1 and C_2, and $R = S_0+q_1S_1+q_2S_2$ be the revenue required to purchase the bundle $\underset{\sim}{S}$, then the government's optimal tax problem may be written:

(2) maximize $v(p_1, p_2, I)$ such that $\theta^1 p_1 C_1 + \theta^2 p_2 C_2 = R$

Since the same optimal tax formula results whether we assume a constant returns to scale production function (as in Diamond and Mirrlees 1971) or simply that producer prices are fixed, we make the latter assumption for the sake of simplicity.

The first order conditions with respect to t_1 and t_2 may be combined to yield the familar ratio:

$$(3) \qquad \frac{\theta^1}{\theta^2} = \frac{\varepsilon_{12} + \varepsilon_{21} + \varepsilon_{20}}{\varepsilon_{12} + \varepsilon_{21} + \varepsilon_{10}}$$

where ε_{ij} is the compensated elasticity of demand for good i with respect to price j.

The formula in (3) is no different from that for any three-good optimal tax problem in the absence of uncertainty. However, more may be learned if we accept the hypothesis of expected utility maximization; then, the objective function $W(\cdot)$ may be written

$$(4) \qquad W(C_0,C_1,C_2) = \pi_1 U(C_0,C_1) + \pi_2 U(C_0,C_2)$$

where π_1 and π_2 are the subjective probabilities the consumer assigns to states 1 and 2.

Solving for ε_{10} and ε_{20} , we obtain:

$$(4) \qquad \varepsilon_{ij} = - \frac{(\pi_1 U^1_2)(\pi_2 U^2_2)}{\Delta C_1 C_2}\left[-\left(\frac{U^j_{22} C_j}{Y^j_2}\right) + p_j C_j \frac{d\log(U^j_2/U^i_2)}{dC_0}\right] \qquad (i,j) = (1,2)$$

where Δ is the determinant of the bordered Hessian, which must be negative, $U^i = U(C_0, C_i)$, U^i_j is the partial derivative of U^i with respect to its jth argument, and U^i_{jk} is the second partial derivative of U^i with respect to its j^{th} and k^{th} arguments. From a comparison of (3) and (5), it is apparent that there are two factors that might lead to differential taxes on C_1 and C_2, corresponding to the two terms in brackets in (5). The tax on C_1, θ^1, will be greater than that on C_2, θ^2, to the extent that

$$\text{(6.a)} \qquad -\left(\frac{U^1_{22}\, C_1}{U^1_2}\right) > -\left(\frac{U^2_{22}\, C_2}{U^2_2}\right)$$

$$\text{(6.b)} \qquad \frac{d\log\, (U^1_2/U^2_2)}{dC_0} > 0$$

Each of these effects has an intuitive interpretation. The second is the derivative of the logarithm of the marginal rate of substitution between C_1 and C_2 with respect to C_0.2/ If it exceeds zero, then an increase in the consumption of C_0 increases the marginal valuation of C_1 relative to C_2.

If the marginal rate of substitution between C_1 and C_2 is unaffected by the level of C_0, then the function $W(\cdot)$ is, by definition, weakly separable into goods and leisure; that is,

$$\text{(7)} \qquad W(C_0, C_1, C_2) \;=\; F(C_0, \phi(C_1, C_2))$$

for some well-behaved functions F and ϕ. However, by (4), this weak separability implies strong separability, so that expected utility takes the form:

$$\text{(7)} \qquad \begin{aligned} W(C_0, C_1, C_2) &= \pi_1[U_I(C_0) + U_{II}(C_1)] + \pi_2[U_I(C_0) + U_{II}(C_2)] \\ &= U_I(C_0) + \pi_1 U_{II}(C_1) + \pi_2 U_{II}(C_2) \end{aligned}$$

where $U_I(\cdot)$ and $U_{II}(\cdot)$ are the utility subfunctions that apply to consumption in periods 1 and 2, respectively. This intertemporal separability is a common assumption in the literature.

In this case, equation (6.a) can be rewritten

$$(8) \qquad -\left(\frac{U_{II}''(C_1)C_1}{U_{II}'(C_1)}\right) > -\left(\frac{U_{II}''(C_2)C_2}{U_{II}'(C_2)}\right)$$

which says that the tax on returns in state 1 should be higher than those in state 2 if and only if the degree of relative risk aversion (Arrow 1965) is higher in state 1 than in state 2, given the commodity bundle (C_1, C_2) that is being purchased. The intuitive explanation for this result is that as individuals become more risk averse, their behavior becomes more inelastic; hence, a tax is less distortionary.

The implications of this result for the taxation of risky assets depends on how such assets difer in their combination of returns in the two states as they get "riskier", and whether individuals display constant, decreasing or increasing relative risk aversion. Arrow has argued that relative risk aversion ought to be constant or increasing with respect to wealth. If relative risk aversion is constant, then from (8) it follows that uniform taxation is optimal. This is unsurprising for two reasons, Most directly, since $U_{II}(C)$ must take the form

$$\alpha + \beta \left(\frac{C^{1-\gamma}}{1-\gamma}\right)$$

where γ is the degree of relative risk aversion, the function $\pi_1 U_{II}(C_1) + \pi_2 U_{II}(C_2)$ is homogenous in C_1 and C_2. Thus, the function $W(\cdot)$ satisfies the condition of weak, homothetic separability that is sufficient for the optimality of uniform taxation (Sandmo 1974, Auerbach 1979). In addition, because of the separation of portfolio and savings decisions present under constant relative risk aversion (Merton 1969, Samuelson 1969), it makes sense that a "second-best" distortion of portfolio choice would not help offset the overall disincentive to save introduced by the uniform taxation of savings.

If individuals display increasing relative risk aversion, then the degree of relative risk aversion, and hence the optimal tax rate, will be higher in the state with a higher level of consumption. In a two-state model, this corresponds to the "good" state when returns are high in the aggregate. However, taxes typically would be applied to asset returns rather than state-contingent commodities, so it is important to know what this result implies for the taxation of the risky assets themselves.

Suppose there are two assets, A and B, that span the two states of the world. If r_A^i and r_B^i are the returns per dollar of investment in state i, then by solving the system of equations

$$(9) \qquad \begin{bmatrix} r_A^1 & r_B^1 \\ r_A^2 & r_B^2 \end{bmatrix} \begin{bmatrix} x_A^i \\ x_B^i \end{bmatrix} = \begin{bmatrix} \delta_{i1} \\ \delta_{i2} \end{bmatrix} \qquad (i=1,2)$$

where $\delta_{ij} = 0$ for $i \neq j$ and 1 for $i = j$, we obtain the amount of each asset, X_A^i and X_B^i, that must be purchased to get a unit return in state i, as well as the implicit prices of such returns,

$$(10) \qquad q^i = x_A^i + x_B^i \qquad (i=1,2)$$

It is then easy to solve for the tax on returns to asset A and that on returns to asset B that together yield the desired taxes on the state-contingent returns. (In fact, there are an infinite number of other solutions to this transformation problem if we allow tax rates to differ not only across assets but also across states for a given asset, for then there are four, rather than two instruments available.) If we let the state-contingent taxes be θ^1 and θ^2, as above, and the taxes on asset returns be t_A and t_B, then the asset combination needed to achieve a unit return in state i in the presence of taxes, (y_A^i, y_B^i), must satisfy:

$$(11) \qquad \begin{bmatrix} (1-t_A)r_A^i & (1-t_B)r_B^i \\ (1-t_A)r_A^i & (1-t_B)r_B^i \end{bmatrix} \begin{bmatrix} y_A^i \\ y_B^i \end{bmatrix} = \begin{bmatrix} \delta_{i1} \\ \delta_{i2} \end{bmatrix} \qquad (i=1,2)$$

and

$$(12) \qquad (y_A^i + y_B^i)\ (1-\theta^1) = x_A^i + x_B^i \qquad (i=1,2)$$

Together, (11) and (12) represent six equations in the unknowns t_A^i, t_B^i, y_A^i, y_B^i, y_A^i and y_B^i, given the posited values of θ^1 and θ^2 and the values of X_A^1, X_B^1, X_A^2 and X_B^2 obtained from (9). Solving for t_A and t_B yields:

(13a) $$\frac{1}{1-t_A} = \frac{1}{\Gamma}\left[\frac{r_A^1(r_B^2-r_A^1)}{(1-\theta^1)} - \frac{r_A^2(r_B^1-r_A^1)}{(1-\theta^2)}\right]$$

(13b) $$\frac{1}{1-t_B} = \frac{1}{\Gamma}\left[\frac{r_B^1(r_B^2-r_A^1)}{(1-\theta^1)} - \frac{r_B^2(r_B^1-r_A^1)}{(1-\theta^2)}\right]$$

where $\Gamma = r_A^1 r_B^2 - r_A^2 r_B^1$. Combination of (13a) and (13b) yields:

(14) $$\frac{t_A-t_B}{(1-t_A)(1-t_B)} = \frac{1}{\Gamma}\left[\frac{\theta_1 - \theta_2}{(1-\theta^1)(1-\theta^2)}\right]\left[(r_A^1-r_B^1)(r_B^2-r_A^2)\right]$$

Since in equilibrium there can be no stochastic dominance of one asset over another, the last term on the right-hand side of (14) is positive. Thus,

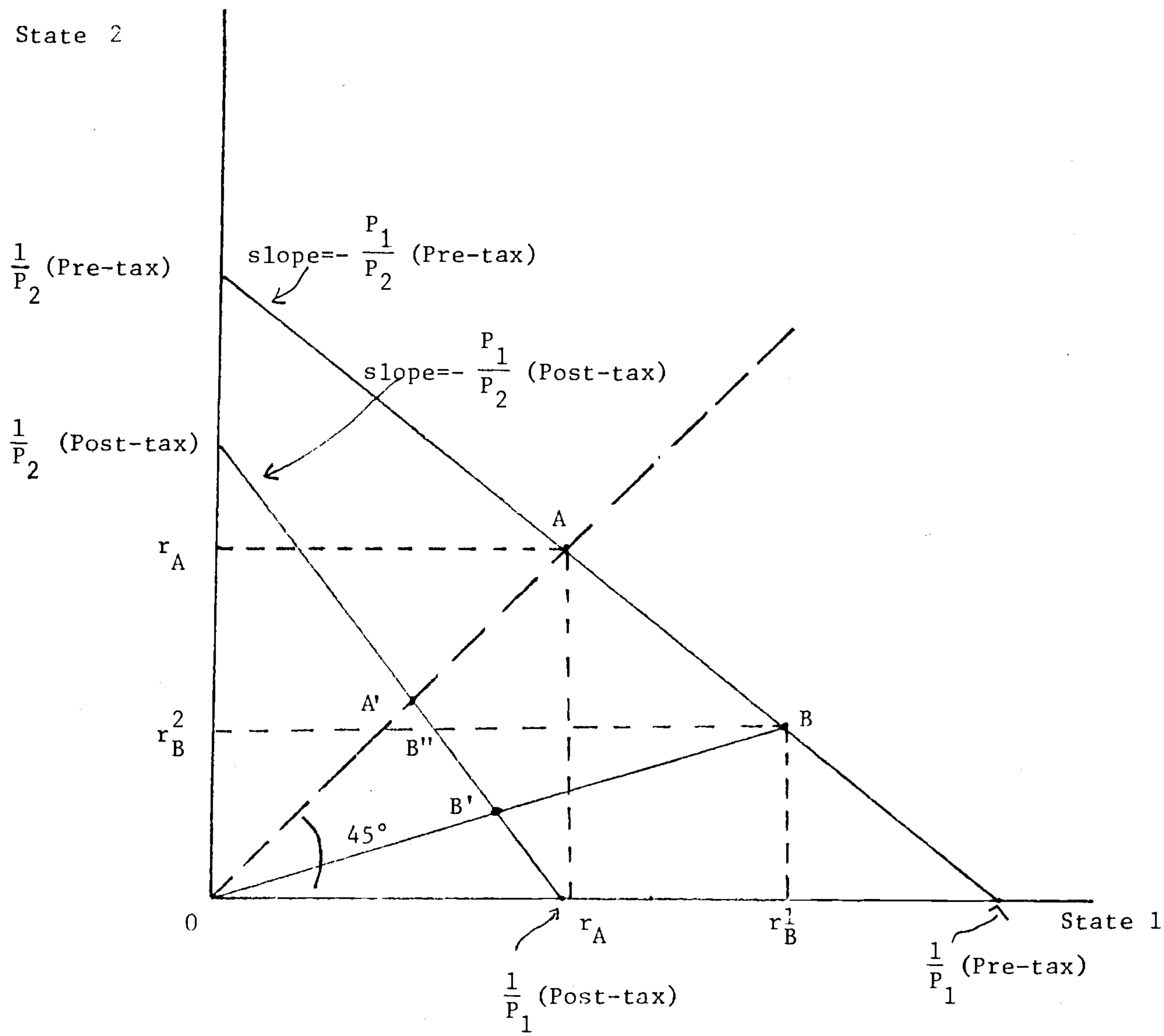
(15) $$\text{sgn}\ (t_A-t_B) = \text{sgn}(\Gamma) \times \text{sgn}(\theta_1-\theta_2)$$

The sign of the determinant Γ depends on which asset is relatively intensive in state 1. If it is asset A, then $\Gamma > 0$ and sign (t_A-t_B) = sgn $(\theta_1-\theta_2)$. Similarly, if B is relatively intensive in state 1, $\Gamma<0$ and sgn (t_B-t_A) = sgn $(\theta_1-\theta_2)$. In both cases, the tax should be highest on that asset relatively intensive in the state we wish to tax more heavily. Under intertemporal separability of utility and increasing relative risk aversion, this is the state in which consumption is greater, the "good" state. Therefore, the asset to be taxed more heavily is the one yielding a greater fraction of its return in the good state; by any common definition, this is the riskier of the two assets.

This solution is depicted in Figure 1. For simplicity, we assume asset A is

Figure 1

Optimal Taxation of Risky Assets



riskless and that asset B yields a higher return in state 1. Thus, if there are positive quantities of both assets, the market return is higher in state 1 than in state 2. The two budget lines depict the returns possible per dollar of initial investment, before and after tax. A higher tax rate on purchases of state 1 claims is accomplished by placing a higher tax on returns to asset B, (1-OB'/OB), than on the the riskless return, (1-OA'/OA). As stated above, the same outcome would be achieved in a number of ways through the use of state-dependent tax rates on the two assets. For example, a shift in the return per dollar invested in asset B from point B to B", rather than to B', would yield the same post-tax budget line. As depicted, this would involve taxing the return to asset B more heavily in state 1 than in state 2.

Thus, under familar restrictions on the nature of individual preferences, it appears that heavier taxation of the return to risky assets may be appropriate. However, though it would be straightforward to extend the analysis to consider several states of nature, the model is still a very simple one; it would be unwise to draw general conclusions about the optimal tax treament of risk. In particular, the assumption of complete markets is rather extreme, in light of the existence of such important nontraded risks as those associated with human capital. Nevertheless, the analysis does suggest that differential taxation of assets may be optimal.

III. Calculating the Tax Rate on Risky Assets

The previous section was devoted to the derivation of the optimal proportional tax rates on risky assets. However, it was pointed out, in passing, that an infinite number of combinations of state-dependent tax rates on each asset also yield the results desired. For example, a proportional tax that brought the after-tax return on asset B to point B' in Figure 1 would have the same effect on consumer choice as a nonproportional tax, heavier in state 1, that brought the after-tax return to B". As pointed out in the introduction, "real world" tax systems typically do not impose on a given asset the same fractional tax burden in each state of nature, due to mismeasurement of income. Thus, in the derivation of a single "the" tax rate on an asset, the choice of weights used in aggregating tax rates across states is important; there are certain criteria such a choice of weights would satisfy. For example, the tax rate on asset B in Figure 1 should be the same whether the post-tax bundle is B' or B".

One measure that might be suggested is the expected tax rate, defined as the fraction of an asset's gross return the government expects to collect. Aside from having a straightforward interpretation, this measure also lends itself to empirical use. By examining the returns over time of a risky asset, and assuming that they are drawn from the same distribution, one can take average gross and net returns and calculate the sample mean of the annual tax rates to obtain an estimate of the expected tax rate.

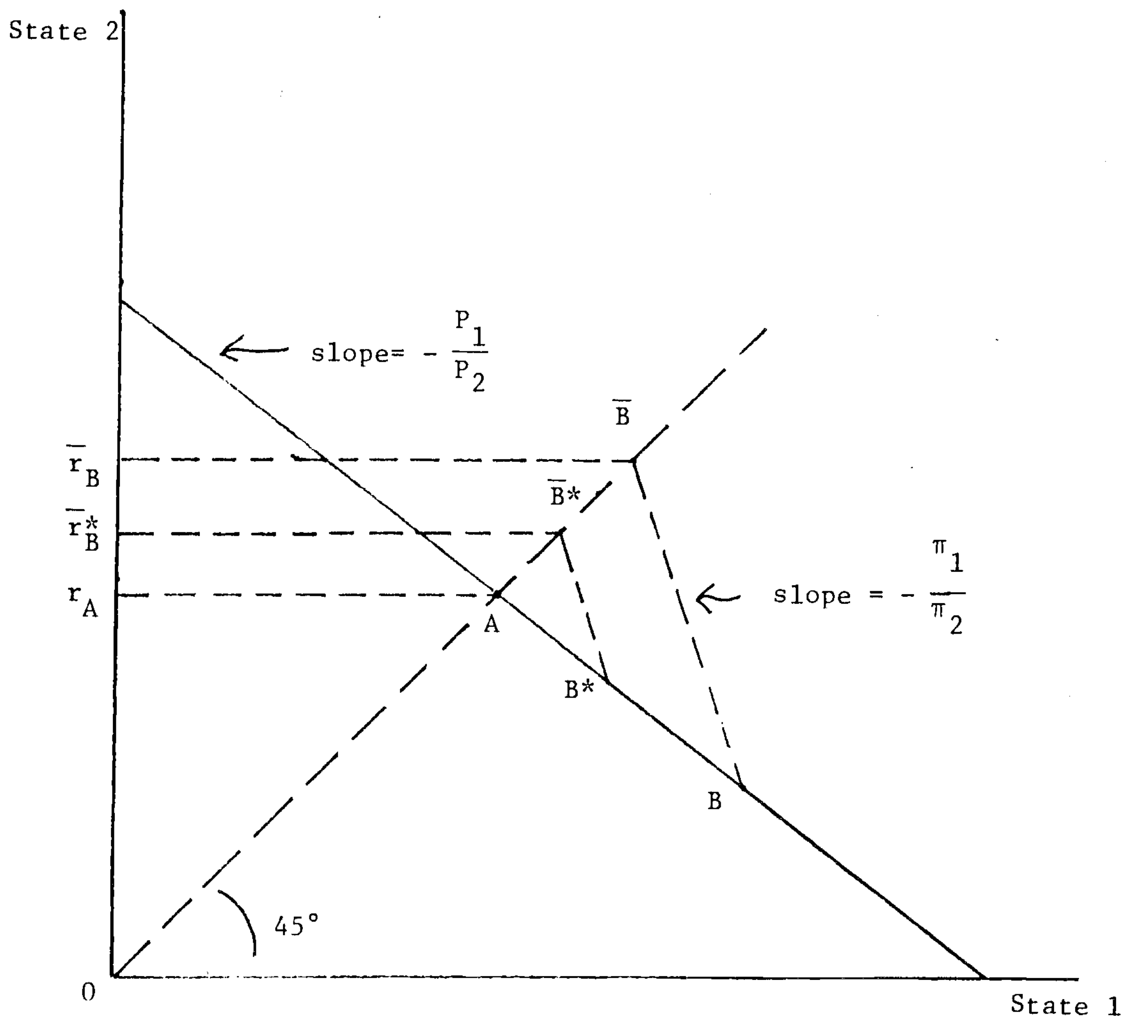
However, the expected tax rate can be a seriously misleading estimate of the real tax burden imposed on an asset. This point is perhaps most easily recognized if one considers a tax that has no real effects at all. As recently pointed out by Gordon (1981),[3/] a tax on risky assets of that portion of the return that exceeds the risk-free rate is completely non-distortionary, and has no effect on the investor's consumption possibilities. This may be seen as an extension of Tobin's (1958) result, which was based on the assumption of a zero safe return. Such a tax is depicted in Figure 2, where the return to asset A is unaffected and that to asset B is moved toward A along the original budget line, to B*, by the fraction of tax on B's excess return.[4/] Such a result would occur if there were no personal taxes and a corporation facing an income tax financed all its investment by the issuance of interest-free debt, with interest payments tax deductible.

To calculate the expected tax rate on asset B, we calculate the change in its expected return resulting from taxation. If π_1 and π_2 are the probabilities attached to states 1 and 2,[5/] then the expected return on asset B before tax equals $\bar{r}_B$, as depicted in Figure 2. The bundle $\bar{B} = (\bar{r}_B, \bar{r}_B)$ lies along the same expected return line as the original bundle. That such a line has a steeper slope than the budget line itself is a condition necessary for equilibrium.[6/] Another way of stating this condition is that the risky asset must carry a positive risk premium.

The post-tax expected yield on asset B is $\bar{r}_{B*}$, and the expected tax rate is therefore $[1-(\bar{r}_{B*})/(\bar{r}_B)]$. Clearly, this is a misleading measure. The error arises because the weights used in calculating the tax rates are probabilities,

Figure 2

A Proportional Tax with Interest Deductibility



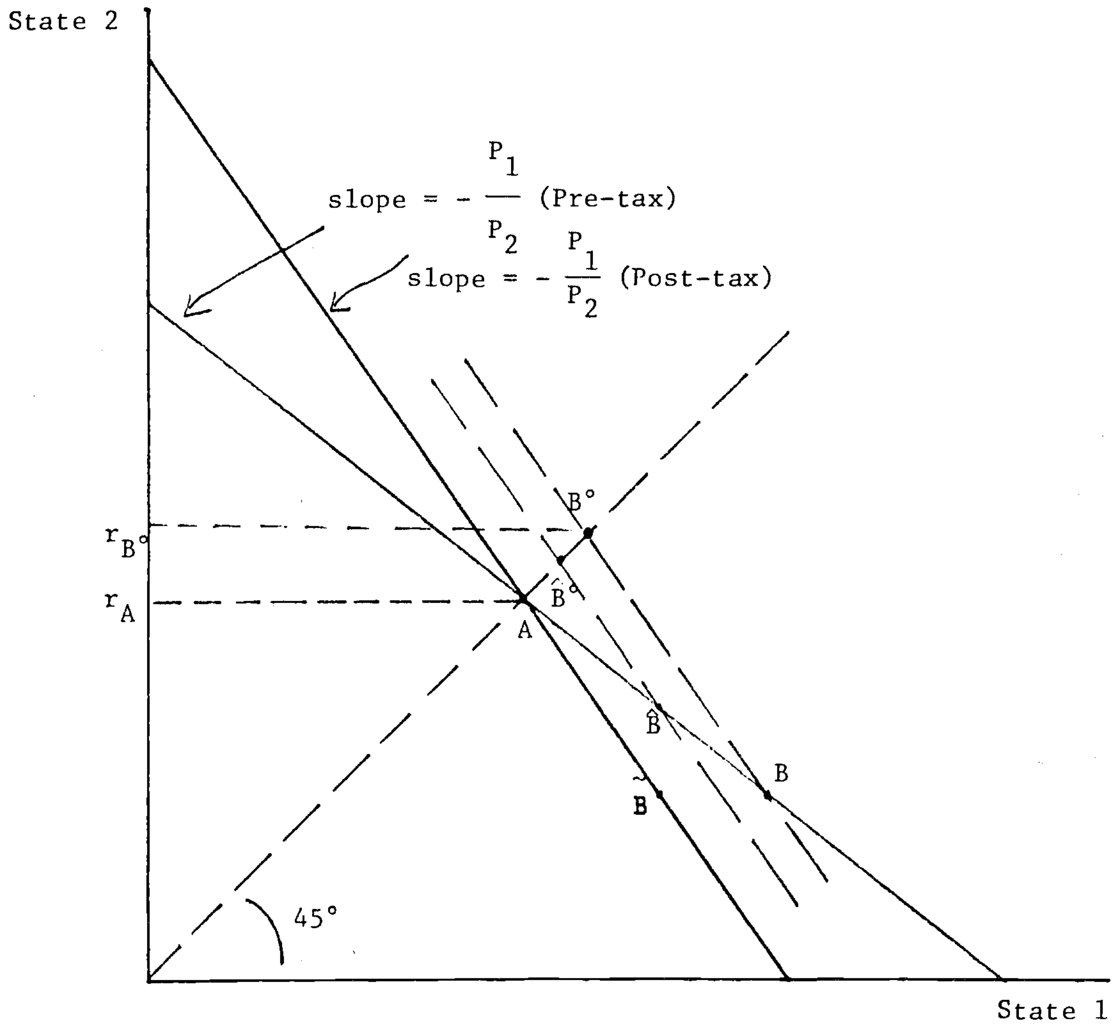
rather than the actual implicit prices for goods in the two states. It would be as if the governement "taxed" a bundle of apples and oranges by purchasing cheaper apples with costlier oranges, at market prices. The value of the bundle would be unaffected, but the number of pieces of fruit would decline. In a similar way, the government exchanges state 1 returns for the scarcer, more expensive (relative to expectations) state 2 returns. This difficulty does not arise when the tax rate on an asset is constant across the states, for then any weighting scheme will do.

This result suggests that a more logical way of calculating effective tax rates is to measure the value of resources extracted by comparing the lines with the slopes of the new budget line running through the before and after tax return bundle for each particular asset.[7/] An example is shown in Figure 3, where it is assumed that A is not taxed and B is taxed only in state 1, the good state. We measure the effective tax rate on asset B by comparing the distance from the origin of the new budget line and that line parallel to it passing through the original bundle B. This measure will be the same along any ray from the origin, but is perhaps clearest intuitively if we use the 45° line and project onto either axis. The effective tax rate would then be $(r_B{}^\circ - r_A)/r_B{}^\circ$, which compares the risk-free components of the before-tax and after-tax bundles. By insisting that the bundles compared lie along the same ray, we avoid confusing changes in risk characteristics with changes in tax burden. Putting everything in risk-adjusted terms seems the most natural method of doing this.

This "risk-adjusted effective tax rate" (henceforth referred to as RET) has

Figure 3

Calculation of Risk-Adjusted Effective Tax Rates



certain desirable properties. First of all, any time the tax burden is uniform across states of nature, the pre-tax and post-tax budget lines are parallel. Thus, the RETs on assets A and B will be equal. In particular, for the special case depicted in Figure 2, the RET on both assets is appropriately measured as zero. A second characteristic of the RET is that any shift in post-tax returns that leaves the post-tax budget line unaffected, such as a movement of the post-tax return on asset B from B' to B" in Figure 1, also leaves unaffected the RET on each asset. Finally, when taxation of state-contingent commodities is not uniform, the RET must be higher on that asset which is relatively intensive in the more highly taxed state.

However, knowledge of the RETs on different assets is not sufficient for determination of the overall tax burden faced by the investor, as measured by the exact tax burden in each state. This is because the post-tax and pre-tax budget lines may be the same in two cases, yet the RETs may differ if the initial bundles differ, as seen in Figure 3 by comparing the RET starting from point $\hat{B}$ rather than B.

Thus, in using the RET, we can say which assets are taxed more heavily, but have no unambiguous measure of the differential tax between assets. Nevertheless, it remains a useful statistic in telling us how the tax system is biased with respect to different assets. It can be useful, therefore, in comparing an existing tax system with one dictated by optimal tax considerations like those discussed above, where a heavier tax burden on riskier assets may be appropriate.

IV. An Application

Rather than fully describing an asset in terms of its returns across states of nature, we may also characterize it in terms of how much its expected return exceeds its risk-adjusted return. This may be done for the return on an asset as a whole or for various components separately. For example, suppose an asset receives a post-tax cash flow, x, plus some certain depreciation allowance, y, as depicted in Figure 4. No adjustment need be made to the latter's expected value. However, the expected cash flow, $\bar{x}$, exceeds its value at post-tax prices, x°. We may summarize this by defining a discount rate ρ such that

$$(15) \qquad \frac{\bar{x}}{1+\rho} = \frac{x^{\circ}}{1+i}$$

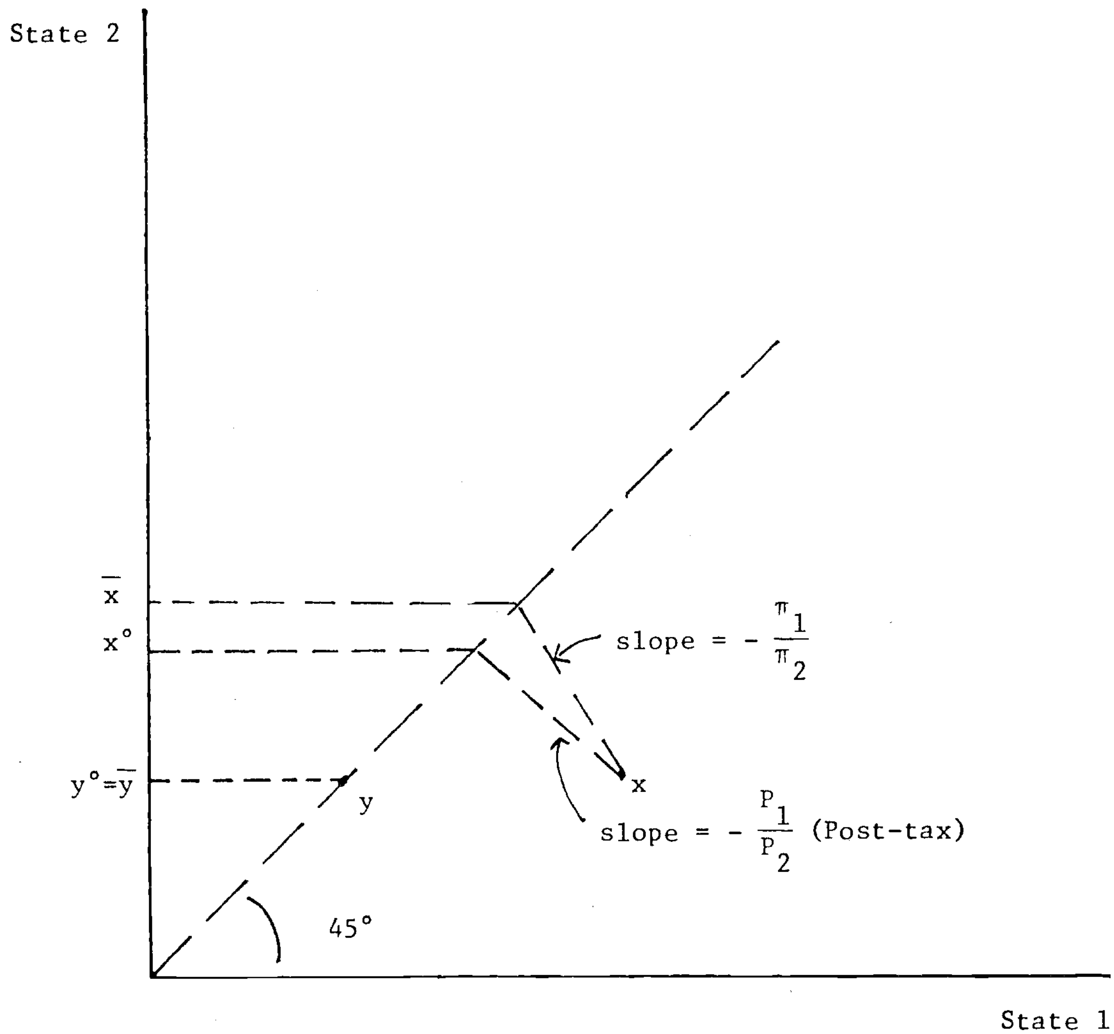
where i is the post-tax risk-free return. ρ is the discount rate that, when applied to the expected return $\bar{x}$, gives the correct value of the uncertain return x. The total value of the asset's return x+y would be

$$\frac{\bar{x}}{1+\rho} + \frac{\bar{y}}{1+i} \ .$$

To use this approach in a multiperiod setting is complicated unless we have some form of stationarity which allows us to examine each period independently. Therefore, let us consider a model where such stationarity obtains. We assume assets depreciate exponentially. In each period, the gross return to dollar of capital of type j is r_j and the rate of physical decay is d_j. Both r_j and d_j are independent and identically distributed over time. Thus, for a dollar invested, the gross return at the end of the period is $r_j - d_j$, plus a certain

Figure 4

Calculation of Discount Rates



return of the original dollar. Once depreciated capital has been replaced,[8] the asset is identical in all respects one period hence and must have the same value.

To maintain this stationarity in the presence of taxes, we must assume unrealistically that all assets of a particular type j, whatever their age, are allowed the same (possibly stochastic) depreciation per dollar of capital, α_j, for tax purposes, as well as the same physical depreciation, d_j. We assume new assets receive an investment tax credit equal to $\tau\lambda_j$,[9] and that taxable profits face a proportional tax at rate τ.

Thus, an investment of one dollar yields an immediate credit of value $\tau\lambda_j$, and, one period hence, a gross return $r_j(1-\tau)$ in after tax flows, less depreciation d_j, plus depreciation deductions $\tau\alpha_j$, plus an investment tax credit on replacement investment, $\tau\lambda_j d_j$. At the end of the period the asset itself, including replacement, is still worth $1-\tau\lambda_j$, the value of the initial net investment. If we let ρ_j, δ_j and γ_j be the discount rates appropriate for the expected values of r_j, d_j, and α_j, then it follows, by construction, that the adjusted return to the one dollar of capital (which costs $(1-\tau\lambda_j)$) is

$$(16) \qquad \left(\frac{\bar{r}_j(1-\tau)}{1+\rho_j} - \frac{\bar{d}_j(1-\tau\lambda_j)}{1+\delta_j} + \frac{\tau\bar{\alpha}_j}{1+\gamma_j} \right)(1+i)$$

Since in equilibrium all assets must yield the same adjusted return after tax, this must, in turn, equal $(1-\tau\lambda_j)i$. That is, the risk-adjusted return after tax to this investment of $(1-\tau\lambda_j)$ is i, the post-tax risk-free return. Rewriting

(16), we have the post-tax, risk-adjusted return on asset j:

$$\frac{\bar{r}_j(1-\tau)(1+i)}{(1-\tau\lambda_j)(1+\rho_j)} - \frac{\bar{d}_j(1+i)}{1+\delta_j} + \frac{\tau\bar{\alpha}_j(1+i)}{(1-\tau\lambda_j)(1+\gamma_j)} = i \qquad (17)$$

To obtain the RET on asset j, we simply construct the risk-adjusted gross of tax return and compare it to (17). The gross adjusted return on asset j is:

$$z_j = \frac{\bar{r}_j(1+i)}{(1+\rho_j)} - \frac{\bar{d}_j(1+i)}{(1+\delta_j)} \qquad (18)$$

Thus, the RET on asset j is $(z_j - i)/z_j$, or

$$\sigma_j = \left(\frac{\tau}{1-\tau\lambda_j}\right)\left[\frac{\dfrac{\bar{r}_j(1-\lambda_j)}{(1+\rho_j)} - \dfrac{\bar{\alpha}_j}{(1+\gamma_j)}}{\dfrac{\bar{r}_j}{(1+\rho_j)} - \dfrac{\bar{d}_j}{(1+\delta_j)}}\right] \qquad (19)$$

For a number of special cases, expression (19) becomes quite simple:

(1) Expensing: Here, λ_j and $\alpha_j \equiv 0$, so $\sigma \equiv 0$.

(2) Economic Depreciation: Here, $\lambda_j = 0$, $\alpha_j = d_j$ and hence $\gamma_j = \delta_j$. Thus, $\sigma_j = \tau$.

(3) First-Year System: As in Auerbach and Jorgenson (1980), the idea is to set $\alpha_j = 0$ and choose λ_j to set $\sigma_j = \tau$. This is accomplished for

$$\lambda_j = \frac{\bar{d}_j}{\bar{d}_j + i\left(\frac{1+\delta_j}{1+i}\right)}$$

which equals the present value of flows of future economic depreciation.[10/]

Typically, the tax system satisfies none of these simple cases. Except for inflation risk, depreciation allowances are certain. With economic depreciation and no investment tax credit, this would yield:

$$(21) \qquad \sigma_j = \tau \left[\frac{\frac{\bar{r}_j}{(1+\rho_j)} - \frac{\bar{d}_j}{(1+i)}}{\frac{\bar{r}_j}{(1+\rho_j)} - \frac{\bar{d}_j}{(1+\delta_j)}} \right]$$

The effective tax rate will be higher or lower than τ depending on whether depreciation carries a discount rate less than or greater than i.

In comparison to the risk-adjusted effective tax rate, the simple expected tax rate on asset j is:

$$(22) \qquad \phi_j = \left(\frac{\tau}{1-\tau\lambda_j}\right) \left[\frac{\bar{r}_j(1-\lambda_j) - \bar{\alpha}_j}{\bar{r}_j - \bar{d}_j} \right]$$

which will be the same in general if and only if the tax is proportional (expensing or economic depreciation) or all components of the net return possess the same characteristics. Even in so simple a case as the first year system, the result is different. Whereas the RET $\sigma_j = \tau$, the expected tax rate is

$$(23) \qquad \phi_j = \tau \cdot [\ 1+ (\frac{1+i}{1+\rho_j})(\frac{\rho_j-\delta_j}{1+\delta_j})\ \frac{\bar{d}_j(1-\tau)}{i}\ ]^{-1}$$

For the simple case where physical depreciation itself is certain (δ_j=i), (23) reduces to

$$(24) \qquad \phi_j = \tau \cdot [\ 1+ (\frac{\rho_j-i}{1+\rho_j})\ \frac{\bar{d}_j(1-\tau)}{i}\ ]^{-1}$$

Thus, riskier assets (ρ_j large) or rapidly depreciating assets ($\bar{d}_j$ large) would appear to be taxed at lower rates, even though $\sigma_j = \tau$ for all assets and the underlying state-contingent commodities are taxed equally.

Thus, if one knows the values of the discount rates ρ_j, δ_j, and γ_j for each asset, as well as the expected values of $\bar{r}_j$, $\bar{d}_j$ and $\bar{\alpha}_j$, calculation of a useful effective tax rate is possible (leaving aside how the above analysis can be extended to a more complicated tax system). Without such information, one might still avoid use of the expected tax rate by calculating the effective tax rate for hypothetical riskless assets with the same values of $\bar{r}_j$, $\bar{d}_j$, and $\bar{\alpha}_j$. But these will generally differ from those of the actual assets, which are risky. Without a proportional tax system, it will not generally be possible to infer anything about the tax rates on risky assets without actually making assumptions about the risk characteristics of such assets.

V. Conclusion

The foregoing analysis has shown that it needn't be optimal to tax the returns to different assets at the same rate. Indeed, it may be optimal to tax risky assets more heavily than safe assets, if individuals possess increasing relative risk aversion. However, a comparison of any particular tax system, current or proposed, is difficult when the taxation of individual assets varies across states. The use of expected tax rates in such cases can be seriously misleading, not only in theory, but in practice as well, as the example in section IV demonstrates. A proposed alternative measure, the "risk-adjusted effective tax rate," performs better, but cannot be computed without more extensive knowledge of an asset's risk characteristics.

Footnotes

1 The tax on first period labor supply, $(I-C_0)$, may be set equal to zero with no loss of generality since equiproportional taxes on C_1, C_2, and $(I-C_0)$ have no real effects.

2 This term appears in the analysis by Atkinson and Stiglitz (1976) of the optimal non-linear tax on labor income with a heterogenous population. In that context, only this term enters into the determination of whether commodity taxes should be different.

3 See also the discussion in Mintz (1981).

4 This involves a subsidy in state 2.

5 We ignore issues arising if individuals have different or incorrect subjective evaluations of π_1 and π_2.

6 With expected utility maximization, $(\pi_1 U_{II}'(C_1)/\pi_2 U_{II}'(C_2)) = (p_1/p_2)$. Since $C_1 > C_2$, $(\pi_1/\pi_2) > (p_1/p_2)$.

7 A similar measure could be constructed using the before-tax budget line and a line of equal slope running through the post-tax bundle.

8 It is merely a convenience to assume that such replacement actually occurs.

9 This may also be thought of as an initial deduction equal to a fraction λ_j of the asset's purchase price.

10 This can be seen as the summation of the infinite series:

$$\frac{d_j}{1+\delta_j} + \left(\frac{1}{1+i} - \frac{d_j}{1+\delta_j}\right)\frac{d_j}{1+\delta_j} + \ldots + \left(\frac{1}{1+i} - \frac{d_j}{1+\delta_j}\right)^n \frac{d_j}{1+\delta_j} + \ldots$$

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