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AN INTEGRATED VIEW OF TESTS OF RATIONALITY, MARKET EFFICIENCY AND THE SHORT-RUN NEUTRALITY OF MONETARY POLICY

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ABSTRACT

This paper analyzes an important class of models in which expectations play an important role. Topics included in the analysis are tests of: (1) rationality of forecasts in either market or survey data, (2) capital market efficiency, (3) the short-run neutrality of monetary policy and, (4) Granger causality in macroeconometric models. The common elements of these tests are highlighted. In particular, cross-equation tests for rationality or the short-run neutrality of money are shown to be equivalent to more common regression tests in the literature. Also discussed are the conditions for identification and the implications for whether hypotheses are testable.

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I

INTRODUCTION

In this paper we develop a framework for analyzing and integrating a broad class of models in which expectations play an important role. One reason for studying these models is that they have strong implications for policy.[1] Among the topics included in this analysis are tests of: (1) rationality of forecasts in either market or survey data, (2) capital market efficiency, (3) the short-run neutrality of monetary policy, i.e., that anticipated monetary policy has no effect on output or employment, and (4) Granger (1969) causality in macroeconometric models. In this paper, we highlight the common elements of these different tests and make clear the relations among them. We find that these tests can be used for inference under quite general conditions. We also demonstrate the equivalence of Granger causality tests and a test of cross-equation restrictions in a particular model which embodies the short-run neutrality of money. Finally, we examine the conditions for identification and the implications for whether various hypotheses are testable.

The paper is organized to begin with the simplest case and to treat increasingly complex cases. The simplest case, discussed in Section II, involves cross-equation tests of rationality when some measure of expectations is available. In the absence of directly observable expectations, some model of market behavior is needed to make inferences about expectations. This case is discussed in Section III. Section IV develops cross-equation tests of the short-run neutrality of money, and Section V discusses the conditions under which coeffients are identified and restrictions are testable. A final section contains a summary of the results.

II

TESTS OF RATIONALITY[2]

Rationality of expectations implies that the market's subjective probability distribution of any variable is identical to the objective probability distribution of that variable conditional on available information. Following the literature, we restrict our attention to linear models and focus only on the first moments of distributions.

Let ϕ_{t-1} denote the set of information available at the end of period t-1, and let $E(\cdots|\phi_{t-1})$ denote the objective expectation conditional on ϕ_{t-1}. Suppose that X_t is generated by the following linear model.[3]

(1) $$X_t = Z_{1,t-1}\alpha_1 + Z_{2,t-1}\alpha_2 + u_t$$

where $Z_{1,t-1}$ and $Z_{2,t-1}$ are vectors of variables known at time t-1 and are thus contained in ϕ_{t-1},

u_t is an error term which is assumed to have the property that $E(u_t|\phi_{t-1}) = 0$.

The distinction between $Z_{1,t-1}$ and $Z_{2,t-1}$ is that $Z_{2,t-1}$ includes variables relevant for forecasting X_t but which are ignored by the econometrician in conducting tests of rationality. Of course $Z_{2,t-1}$ could be empty. It is clear from (1) that the objective expectation of X_t, conditional on ϕ_{t-1}, is

(2) $$E(X_t|\phi_{t-1}) = Z_{1,t-1}\alpha_1 + Z_{2,t-1}\alpha_2 \, .$$

Now consider a one-period-ahead forecast X_t^e which is some observable measure of an expectation of X_t made at time t-1. Rationality of expectations requires that the forecast X_t^e must equal the objective expectation of X_t conditional on ϕ_{t-1}. Thus in the following equation

$$(3) \qquad X_t^e = Z_{1,t-1}\alpha_1^* + Z_{2,t-1}\alpha_2^* + v_t$$

rationality implies that $\alpha_1 = \alpha_1^*$, $\alpha_2 = \alpha_2^*$ and v_t is identically zero. However, in dealing with actual data on expectations, we allow for a nonzero observation error v_t and use the following weaker definition of rationality:

$$(4) \qquad E(X_t - X_t^e | \phi_{t-1}) = 0 \quad .$$

This definition still requires that $\alpha_1 = \alpha_1^*$ and $\alpha_2 = \alpha_2^*$, yet it allows v_t to be non-zero with the restriction that $E(v_t | \phi_{t-1}) = 0$.[4]

Observe that (4) implies that the forecast error is uncorrelated with information in ϕ_{t-1}. This implication of rational expectations is the basis for one test procedure in which $X_t - X_t^e$ is regressed on past information. The null hypothesis of rationality is rejected whenever the estimated coefficient $\hat{\omega}$ differs significantly from zero in the regression below:

$$(5) \qquad \widehat{X - X^e} = Z_1\hat{\omega}$$

where $\widehat{X - X^e}$ is the least squares projection of $X - X^e$ on Z_1, and $\hat{\omega}$ is the coefficient estimated with ordinary least squares (OLS). (Note that X and X^e are $n \times 1$ vectors with X_t and X_t^e, respectively, in row t. Similarly, Z_1 is a matrix of n rows which contains the vector $Z_{1,t-1}$ in row t.) This is the most common test of rationality used to study forward rates in the foreign exchange market.[5]

The effect of ignoring relevant information in this test is made clear by subtracting equation (3) from (1) to obtain the following equation for the forecast error.

$$(6) \qquad X_t - X_t^e = Z_{1,t-1}(\alpha_1 - \alpha_1^*) + Z_{2,t-1}(\alpha_2 - \alpha_2^*) + u_t - v_t.$$

Recall that rationality implies that $\alpha_1 - \alpha_1^* = 0$, $\alpha_2 - \alpha_2^* = 0$ and $E(u_t - v_t | \phi_{t-1}) = 0$. Therefore, under the hypothesis of rationality, the coefficient $\hat{\omega}$ estimated from the OLS regression of $X_t - X_t^e$ on $Z_{1,t-1}$ in (5) will be a consistent estimate of $\alpha_1 - \alpha_1^*$ and should not be significantly different from zero. Note that under rationality, $\hat{\omega}$ is a consistent estimate of $\alpha_1 - \alpha_1^*$ even if Z_2, which is the set of relevant variables excluded from the regression, is not empty. Thus leaving out relevant variables from the OLS regression (5) will not affect the rationality implication that $\hat{\omega}$ should not differ significantly from zero.

Another way of stating the point made above is that the test described here is a test of rationality no matter what past information is included in Z_1 (or no matter what information is excluded from the regression equation.)[6] That is, plim $\hat{\omega}$ can differ from zero only if there is a violation of rationality. However, it is possible that plim $\hat{\omega}$ could equal zero even in the presence of irrationality. For example, suppose that $\alpha_1 = \alpha_1^*$, $E(u_t - v_t | \phi_{t-1}) = 0$ and Z_2 is orthogonal to Z_1, yet there is irrationality because $\alpha_2 \neq \alpha_2^*$. In this case, plim $\hat{\omega} = 0$. Therefore, a failure to reject the null hypothesis, even asymptotically, does not rule out irrationality.[7,8]

Studies that test for the rationality of survey forecasts [Pesando (1975), Carlson (1977), Mullineaux (1978) and Friedman (1978)] use the following alternative procedure. Consider the following least squares regressions:

(7) $\hat{X} = Z_1 \hat{\gamma}$

(8) $\hat{X}^e = Z_1 \hat{\gamma}^*$

where $\hat{X}$ and $\hat{X}^e$ are the linear least squares projections of X and X^e onto Z_1, and $\hat{\gamma}$ and $\hat{\gamma}^*$ are the OLS coefficient estimates. As pointed out by Modigliani and Shiller (1973), rationality of expectations requires that plim $\hat{\gamma}$ = plim $\hat{\gamma}^*$. This implication of rationality becomes clear if we suppose that Z_2, the set of variables excluded from the regressions in (7) and (8), is empty; that is, the regressions in (7) and (8) contain all information in ϕ_{t-1} relevant for forecasting X_t. In this case, $\hat{\gamma}$ and $\hat{\gamma}^*$ are each consistent estimates of α_1 under the null hypothesis of rationality, and they should not differ significantly from each other.[9] Testing the cross-equation restriction $\hat{\gamma} = \hat{\gamma}^*$ is equivalent to testing $\hat{\omega} = 0$ in (5), since $\hat{\omega}$ is numerically identical to $\hat{\gamma} - \hat{\gamma}^*$.

Now suppose that Z_2 is not empty so that relevant variables are excluded from (7) and (8). In this case, the estimates $\hat{\gamma}$ and $\hat{\gamma}^*$ generally will not be consistent estimates of α_1 and α_1^*, respectively, even if expectations are rational. However, rationality of expectations still implies that plim $\hat{\gamma}$ = plim $\hat{\gamma}^*$ because $\hat{\gamma} - \hat{\gamma}^*$ is numerically equal to $\hat{\omega}$, and plim $\hat{\omega} = 0$. The equality of plim $\hat{\gamma}$ and plim $\hat{\gamma}^*$ reflects the equal asymptotic bias in the two estimates.[10]

This section has analyzed tests of rationality in the presence of some observable measure of expectations. The general conclusion is that a rejection of $\hat{\gamma} = \hat{\gamma}^*$ or, equivalently, $\hat{\omega} = 0$, is a rejection of rational expectations regardless of the completeness of the information set specified by Z_1. The two alternative procedures discussed here are thus tests of rationality under quite general conditions.

In the absence of direct observations on expectations, we must infer information on expectations from observed market behavior. In the next section we discuss the use of security price data to test for the rationality of expectations.

III

TESTS OF RATIONALITY AND MARKET EFFICIENCY

Tests of rationality in capital markets focus on holding-period returns for securities. Let R_t denote the return from holding a particular security from t-1 to t. (This return includes both capital gains and intermediate cash income.) Rationality of expectations, or equivalently, capital market efficiency, implies that the subjective expectation of R_t assessed by the market is equal to the objective expectation conditional on ϕ_{t-1}:

$$(9) \qquad E_m(R_t | \phi_{t-1}) = E(R_t | \phi_{t-1}) \quad ,$$

where $E_m(R_t | \phi_{t-1})$ is the subjective expectation assessed by the market. As in section II, a weaker condition is used in empirical applications:

$$(10) \qquad E(R_t - E_m(R_t | \phi_{t-1}) | \phi_{t-1}) = 0$$

In order to give (10) empirical content, we must specify a model of market equilibrium which relates $E_m(R_t | \phi_{t-1})$ to some subset of past information:

$$(11) \qquad E_m(R_t | \phi_{t-1}) = f(\Omega_{t-1})$$

where Ω_{t-1} is contained in ϕ_{t-1}. The reader is referred to Fama (1976) for a discussion of various models of market equilibrium used to determine $E_m(R_t | \phi_{t-1})$ in empirical work. Combining (10) and (11), we obtain

$$(12) \qquad E(y_t | \phi_{t-1}) = 0$$

where $y_t \equiv R_t - f(\Omega_{t-1})$.

Tests of (12) are tests of the joint hypothesis that 1) expectations are rational (market efficiency) and 2) that the model of market equilibrium is correctly specified in measuring y_t.

Equation (12) above implies that y_t should be uncorrelated with any past information in ϕ_{t-1}. It is the basis for a common test of market efficiency[11] in which y_t is regressed on past information, and the null hypothesis that $\alpha = 0$ is tested in the equation below:

$$(13) \qquad y_t = Z_{t-1}\alpha + \mu_t$$

where Z_{t-1} = an ℓ-element row vector of information contained in ϕ_{t-1},

α = $\ell \times 1$ vector of coefficients,

μ_t = a disturbance where $E(\mu_t | \phi_{t-1})$ is assumed to equal zero.

This procedure is a test of the joint hypothesis of market efficiency and the model of market equilibrium, no matter what past information is included in Z.

A model which satisfies (12) is

$$(14) \qquad y_t = (X_t - X_t^e)\beta + \varepsilon_t$$

where ε_t = a scalar disturbance with the property $E(\varepsilon_t | \phi_{t-1}) = 0$ -- thus ε is a serially uncorrelated and uncorrelated with X_t^e ,

X_t = the k-element row vector containing variables relevant to the pricing of the security at time t ,

X_t^e = the k-element row vector of one-period forecasts of X_t, i.e., $X_t^e = E_m(X_t | \phi_{t-1})$,

β = $k \times 1$ vector of coefficients.

For expositional convenience, we refer to this model (14) as "the efficient markets model." Note, however, that it embodies not only market efficiency (or, equivalently, rational expectations), but also a model of market equilibrium. This model stresses that only when new information hits the market will y_t differ from zero. This is equivalent to the proposition that only unanticipated changes in X_t can be correlated with y_t.

The linear model for the k variables in X can be written as

$$(15) \qquad X_t = Z_{t-1}\gamma + u_t$$

where γ = ℓ x k matrix of coefficients

u_t = k-element row vector of disturbances.

Suppose, for the moment, that $E(u_t|\phi_{t-1}) = 0$, so that an unbiased linear one-period-ahead forecast for the variables in X is

$$(16) \qquad X_t^e = Z_{t-1}\gamma$$

Substituting (16) into (14) we have:

$$(17) \qquad y_t = (X_t - Z_{t-1}\gamma^*)\beta + \varepsilon_t$$

where $\gamma = \gamma^*$.

The system in (15) and (17) can be stacked into one regression system with n(k+1) observations, and estimated by non-linear least squares.[12] The cross-equation constraints implied by market efficiency (rationality), $\gamma = \gamma^*$, can be tested with a likelihood ratio test and are analogous to the rationality constraints for the regressions (7) and (8). Although expectations are not directly observable, we can test their rationality by maintaining, with a model of market equilibrium, the hypothesis that only contemporaneous unanticipated movements in X_t are correlated with y_t. Any rejection of the constraint $\gamma = \gamma^*$ could indicate a failure of either the rationality of expectations about X_t or of the maintained hypothesis. This issue of interpreting tests will be discussed further in Section V.

Two questions arise as to the econometric properties of this procedure. Does the procedure provide a test of market efficiency (rationality) under the maintained hypothesis, even if Z_{t-1} excludes variables relevant to forecasting the variables in X_t? Second, what is the relation of this test to the common test for market efficiency using equation (13)? The following theorem provides answers to these related questions.

<u>Theorem</u>. Consider the system of equations

$$\text{(a)} \qquad \begin{aligned} X_t &= Z_{t-1}\gamma + u_t \\ y_t &= (X_t - Z_{t-1}\gamma^*)\beta + \varepsilon_t \end{aligned}$$

where X_t is a k-element row vector, Z_{t-1} is an ℓ-element row vector, y_t is a scalar, γ and γ^* are ℓ x k parameter matrices, β is a k x 1 parameter vector. Also consider the equation

$$\text{(b)} \qquad y_t = Z_{t-1}\alpha + \mu_t \quad .$$

where α is an ℓ x 1 parameter vector. The quasi-likelihood ratio test of the null hypothesis $\gamma = \gamma^*$ in (a) is asymptotically equivalent to a quasi-F test of the null hypothesis $\alpha = 0$ in (b). (The quasi-likelihood ratio and quasi-F tests are constructed as if the disturbances, u_t, ε_t, and μ_t are i.i.d. normal.)

Outline of Proof:[13] The key insight in the proof of this theorem is to observe that the system (a) can be rewritten as

$$\begin{aligned} X_t &= Z_{t-1}\gamma + u_t \\ y_t &= (X_t - Z_{t-1}\gamma)\beta + Z_{t-1}\theta + \varepsilon_t \end{aligned} \tag{18}$$

where $\theta = (\gamma - \gamma^*)\beta$. The null hypothesis $\gamma = \gamma^*$ will be true only if $\theta = 0$, and this constraint can be tested using the nonlinear least squares estimates of (18). The constraint that γ is the same in both equations in (18) is not binding, so we estimate the parameters in (18) by OLS on each equation. Specifically, the estimate $\hat{\gamma}$ is obtained by OLS on the first equation, and $\hat{\beta}$ and $\hat{\theta}$ are obtained from an OLS regression of y_t on $X_t - Z_{t-1}\hat{\gamma}$ and Z_{t-1}. Since the residuals from the first equation in (18), $X_t - Z_{t-1}\hat{\gamma}$, are orthogonal to Z_{t-1} by construction, the estimate of θ will not be affected if $X_t - Z_{t-1}\hat{\gamma}$ is omitted from the list of regressors when OLS is applied to the second equation in (18).[14] Thus the estimate of θ is numerically identical to, and has the same distribution as, the OLS estimate of α in (b). Although the test statistic associated with the null hypothesis $\alpha = 0$ may differ in small samples from the test statistic associated with the null hypotheses $\theta = 0$, these test statistics will be asymptotically equal.[15]

REMARKS

Observe that $\theta = (\gamma - \gamma^*)\beta$ is an $\ell \times 1$ vector. Thus the test of $\theta = 0$ (or, equivalently, $\alpha = 0$) is a test of only ℓ constraints. However, there are $\ell \cdot k$ constraints in $\gamma = \gamma^*$. Therefore, all of these constraints are testable only if $k = 1$. Even when $k > 1$, imposing the constraint

$\gamma = \gamma^*$ places only ℓ binding restrictions on the system in (a).[16] This issue is discussed in Section V.

If the contemporaneous correlation of u and ε is zero, the OLS regression of y on $\hat{u}$ and Z will provide consistent estimates of both β and θ. However, if the contemporaneous correlation of u and ε is unknown, then β is unidentified. Nevertheless, in this case the OLS estimate of θ is still consistent and the theorem continues to apply. Since β is, in general, unidentified, there is an alternative demonstration of this theorem. The maximized value of the likelihood function is not affected by an arbritrary choice of β. Therefore, set β equal to zero, and observe that we now have a seemingly unrelated system (Zellner (1962)) in which the right-hand side variables are identical in each equation. Therefore, the estimates of γ and θ can be obtained from OLS equation-by-equation.

Even if the time series model generating X_t is incorrectly specified by leaving out relevant available information from Z_1 so that $E(u_t|\phi_{t-1}) \neq 0$, the procedure described above still provides a test of rationality. This is demonstrated by noting that the test of $\gamma = \gamma^*$ is asymptotically equivalent to the test of $\alpha = 0$, which is clearly a test of (12), regardless of what past information is included in Z. However, if the model generating X_t is not correctly specified, then in general, there is an errors-in-variables problem which leads to inconsistent estimates of β and γ. Nonetheless, any asymptotic bias in $\hat{\gamma}$ will be identical to that in $\hat{\gamma}^*$.

IV

TESTS OF THE SHORT-RUN NEUTRALITY OF MONEY

Sargent (1976a) discusses tests of a classical macroeconometric model which displays the neutrality proposition that anticipated countercyclical policy, especially monetary policy, will have no effect on output or unemployment. Thus, in Sargent's model a constant money growth rule is not dominated by any rule with feedback. This controversial policy implication[17] is based on the rationality of expectations and a Lucas supply function of the form

$$(19) \qquad y_t = (X_t - X_t^e)\beta + \varepsilon_t$$

where

y_t is a scalar representing the deviation of output (unemployment) from equilibrium output (unemployment).

X_t is a k-element vector of aggregate demand variables, such as the price level or the money supply.

ε_t is a scalar disturbance term with the property $E(\varepsilon_t | \phi_{t-1}) = 0$ -- hence ε_t is serially uncorrelated.

This equation has the property of "neutrality," i.e., that only unanticipated changes in X_t have an effect on y_t. Note that the supply function (19) has the same form as the "efficient markets model" in (14). As in the previous section, some model of equilibrium behavior is required in order to give the supply function empirical content. The particular model of equilibrium behavior used in the Lucas supply function is that y_t, the deviation of output from its equilibrium level, is

(20) $$y_t = q_t - \sum_{i=1}^{L} \lambda_i q_{t-i},$$ where q_t is output at time t.

Suppose that X_t is generated by the linear model

(21) $$X_t = Z_{t-1}\gamma + \sum_{i=1}^{L'} \psi_i q_{t-i} + u_t$$

where Z_{t-1} is an ℓ-element row vector of predetermined variables other than lagged q.

γ is an ℓ x k matrix of coefficients.

ψ_i is a k-element row vector of coefficients.

Note that (21) has the same form as the linear model (15) except that in (21) we distinguish between lagged values of q_t and other predetermined variables. We assume for the moment that $E(u_t|\phi_{t-1}) = 0$ and combine (19), (20) and (21) to obtain the system

(22) $$X_t = Z_{t-1}\gamma + \sum_{i=1}^{L'} \psi_i q_{t-i} + u_t$$

$$q_t = (X_t - Z_{t-1}\gamma^* - \sum_{i=1}^{L'} \psi_i^* q_{t-i})\beta + \sum_{i=1}^{L} \lambda_i q_{t-i} + \varepsilon_t$$

with the cross-equation rationality constraints $\gamma=\gamma^*$ and $\psi_i=\psi_i^*$, $i=1,\ldots,L'$. Any rejection of these constraints could indicate a violation of the null hypothesis of rationality, or of the maintained hypothesis of the model of equilibrium output and the neutrality of anticipated policy.

Sargent (1967a) has proposed using Granger (1969) causality tests[19] to test the joint hypothesis of rationality of expectations, the model of equilibrium output, and neutrality of anticipated policy as embodied in

(22). This joint hypothesis requires that Z_{t-1} fails to Granger cause q_t. Specifically, if OLS is used to estimate the parameters ϕ_i and α in

$$(23) \qquad q_t = \sum_{i=1}^{L'} \phi_i q_{t-i} + Z_{t-1}\alpha + \mu_t,$$

the estimate of α should not differ significantly from zero.

The relationship between tests of the cross-equation constraints in (22) and the Granger causality test in (23) is made clear by the following corollary.

COROLLARY:

If $L' \geq L$, then a quasi-likelihood ratio test of the null hypothesis $\gamma=\gamma^*$ in (22) is asymptotically equivalent to a quasi-F test of the null hypothesis that $\alpha=0$ in (23).

OUTLINE OF PROOF:

As in the proof of the theorem, the unconstrained system (22) can be rewritten as

$$(24) \qquad X_t = Z_{t-1}\gamma + \sum_{i=1}^{L'} \psi_i q_{t-i} + u_t$$

$$q_t = (X_t - Z_{t-1}\gamma - \sum_{i=1}^{L'} \psi_i q_{t-i})\beta + \sum_{i=1}^{L} \lambda_i q_{t-i} + Z_{t-1}\theta_o + \sum_{i=1}^{L'} \theta_i q_{t-i} + \varepsilon_t$$

where $\theta_o = (\gamma-\gamma^*)\beta$ and $\theta_i = (\psi_i-\psi_i^*)\beta$ for $i = 1,\ldots,L'$.

Note that since θ_i and λ_i are each coefficients of q_{t-i} in (24), the separate parameters θ_i and λ_i are not identified for $i \leq L \leq L'$. Hence, the constraints $\psi_i=\psi_i^*$ for $i \leq L$ are not testable.[20] In order to test the testable cross-equation restrictions, the system (24) can be estimated by OLS on each equation, as explained in the proof of the theorem in Section

III.[21] Since the estimated residuals from the first equation will be orthogonal to Z_{t-1} and q_{t-i} for $i=1,\ldots,L'$, the deletion of this residual vector from the second equation will not affect the OLS estimates of the coefficients on Z_{t-1} and q_{t-i}. Hence, as in the previous proof, the least squares estimates of α and θ_o will be numerically identical, and the test statistics associated with the null hypotheses $\alpha=0$ and $\theta_o=0$ will be asymptotically equal.

REMARKS

It is important to consider the effects of incorrectly specifying the list of variables included in Z_{t-1}. Including irrelevant predetermined variables in Z_t will not lead to inconsistent parameter estimates but in general will reduce the power of tests. On the other hand, excluding relevant variables from Z_{t-1} will lead to a breakdown of the assumption that $E(u_t|\phi_{t-1}) = 0$, and will lead to inconsistent estimates of γ. However, even in this case any rejection of the constraint $\gamma=\gamma^*$ in (24) indicates a failure of rationality, the model of equilibrium output, or of neutrality since a rejection of this constraint indicates that Z_{t-1} Granger causes q_t. As Sargent has shown, rationality of expectations combined with the neutrality embodied in the supply function (19) implies that no vector of predetermined variables Z_{t-1}--even a vector which excludes relevant predetermined variables--can Granger cause q_t.

We have shown that the procedure above provides a test of the joint hypothesis of rationality, the model of equilibrium output, and neutrality even if relevant predetermined variables are omitted from Z_{t-1}. This indicates that, contrary to a statement by Lucas (1972), tests of neutrality can be conducted even when there is a change in the policy regime. A change in a policy regime can be incorporated in a linear model by including an additional set of variables, with dummy variables to indicate the relevant regime for any given time.[22] Neglecting to take account of a change in policy regime is equivalent to omitting the additional set of variables from Z_{t-1}. Thus, even if the variables in Z_{t-1} are chosen without taking account of the change in policy regime, a rejection of the constraint $\gamma=\gamma^*$ indicates a failure of the joint hypothesis of rationality, the model of equilibrium output, and neutrality.

McCallum (1979) and Nelson (1979) have emphasized the point raised by Sargent (1973, 1976b) that the Granger causality tests are valid tests of the neutrality of anticipated policy only if: (a) lagged values of $X_t-X_t^e$ do not enter the supply function (19); or (b) the disturbance ε_t in (19) is serially uncorrelated. That is, if either of these two conditions does not hold, then it is possible for Z_{t-1} to Granger cause y_t even though anticipated policy is neutral.

The analysis of this paper also demonstrates these points. The corollary above breaks down if there are lagged surprises in (19) and hence in (22). Although the contemporaneous residual from the first equation in (24) is, by construction, orthogonal to Z_{t-1} and q_{t-i}, this is not true of lagged residuals. Thus, the test of $\gamma=\gamma^*$ will no longer be equivalent to a Granger-causality test. Therefore, Granger-causality will no longer be a test of the joint hypothesis of rationality, the model of equilibrium output and neutrality.

Now consider the case in which only contemporaneous innovations in X_t appear in (19) and (22), but ε_t is serially correlated, implying that μ_t is serially correlated. Here, the corollary holds and the Granger-causality test is asymptotically equivalent to the test of $\gamma=\gamma^*$. However, since the right-hand side of both (22) and (23) includes lagged dependent variables, the estimates of α and θ_o will no longer be consistent. Thus both sets of tests are invalid in this case.[23]

V

COEFFICIENT IDENTIFICATION AND HYPOTHESIS TESTING

In this section we examine estimation and hypothesis testing in a broader class of models in which expectations, and especially deviations from expectations, are important determinants of behavior. Since this broader class of models contains the models used for tests of market efficiency discussed earlier in this paper, as well as models based on the work of Barro (1977), the analysis of this section will provide a unifying framework for two important branches of the literature.

Consider the following system of equations:

$$X_t = Z_{t-1}\gamma + u_t \tag{25}$$

$$y_t = \sum_{i=0}^{N} (X_{t-i} - Z_{t-1-i}\gamma)\beta_i + \varepsilon_t$$

where X_t is a k-element row vector of observations at time t on variables whose surprises are correlated with y_t,

Z_{t-1} is a h-element row vector of predetermined variables at time t useful in predicting X_t,

γ is a h x k matrix of coefficients,

y_t is a scalar,

β_i is a k x 1 vector of coefficients.

Observe that the system (25) embodies the exclusion restriction that Z_{t-1-i} does not enter the second equation of (25) except as it enters the term representing X^e_{t-i}. This exclusion restriction is crucial to identification and hypothesis testing as discussed later in this section.

Note that if X_t is interpreted as the growth rate of money and if y_t is the deviation of output from some natural level, then (25) represents the model used by Barro. Alternatively, in the efficient markets model, X_t is a vector of variables relevant for pricing a security at time t, y_t is $R_t - E_m(R_t|\phi_{t-1})$ as defined in Section III, and N = 0.

The system (25) embodies two sets of constraints. Rationality of expectations is imposed since the coefficient γ which appears in the equation for X_t also appears in the equation for y_t. The system (25) exhibits neutrality because the coefficient on X_t^e is constrained to be zero when $X_t - X_t^e$ is included as an explanatory variable. Relaxing the neutrality and rationality constraints, the system (25) becomes

$$(26) \qquad X_t = Z_{t-1}\gamma + u_t$$

$$y_t = \sum_{i=0}^{N} (X_{t-i} - Z_{t-1-i}\gamma^*)\beta_i \quad + \quad \sum_{i=0}^{N} Z_{t-1-i}\gamma^*\delta_i + \varepsilon_t$$

where γ^* is a h x k matrix of coefficients,

δ_i is a k x 1 vector of coefficients.

If all of the coefficients in (26) can be estimated (an issue to be discussed later in this section), then a comparison of the weighted sums of squares from (25) and (26) provides a joint test of both the rationality constraints $\gamma = \gamma^*$, and the neutrality constraints $\delta_i = 0$, conditional on the maintained hypothesis of the model of equilibrium output.

As an alternative to relaxing both the neutrality and rationality constraints, we can relax only one set of the constraints. For example, maintaining the hypothesis of rationality but relaxing the assumption of neutrality, the system (25) becomes

$$(27) \qquad X_t = Z_{t-1}\gamma + u_t$$

$$y_t = \sum_{i=0}^{N}(X_{t-i} - Z_{t-1-i}\gamma)\beta_i + \sum_{i=0}^{N} Z_{t-1-i}\gamma\delta_i + \varepsilon_t$$

Under the maintained hypothesis that expectations are formed rationally, the null hypothesis of neutrality, i.e., $\delta_i = 0$, can be tested by comparing the estimated systems (25) and (27). This test is similar to those conducted by Barro.

Rather than maintain the hypothesis of rationality of expectations and then test for neutrality, one can maintain the hypothesis of neutrality and then test for rationality. To perform this test, the unconstrained system is

$$(28) \qquad X_t = Z_{t-1}\gamma + u_t$$

$$y_t = \sum_{i=0}^{N}(X_{t-i} - Z_{t-1-i}\gamma^*)\beta_i + \varepsilon_t$$

A comparison of the estimated systems (25) and (28) provides a test of the null hypothesis of rationality, i.e., $\gamma = \gamma^*$, under the maintained hypothesis of neutrality. Note that when $N = 0$, so that only the current surprise in X_t appears in the second equation in (28), this test is the efficient markets test discussed in Section III. In the efficient markets case, neutrality is a reasonable maintained hypothesis since the absence of neutrality would indicate the presence of unexploited profit opportunities. Maintaining the hypothesis that unexploited profit opportunities do not exist, the null hypothesis of rationality can be tested. It must be noted, however, that a rejection of the null hypothesis that $\gamma = \gamma^*$ may result from a breakdown of rationality, neutrality, or the model of market equilibrium.

The X^2 statistic for the joint hypothesis of rationality and neutrality can be partitioned into the contribution from each component hypothesis by sequentially relaxing the constraints. The order in which these constraints are relaxed is arbitrary from a statistical viewpoint. However, some a priori economic reasoning may suggest an appropriate sequence for relaxing constraints. For example, in testing whether anticipated monetary policy affects output, it seems appropriate first to relax $\delta_i = 0$ and test neutrality under the maintained hypothesis of rationality. Indeed, this test of neutrality is essentially the Barro test. Then, without maintaining neutrality, the constraint $\gamma = \gamma^*$ can be relaxed, and rationality can be tested, as in Leiderman (1980) and Mishkin (1982).

Under the alternative sequence for relaxing constraints, we first relax the constraint $\gamma = \gamma^*$ and test for rationality under the maintained hypothesis of neutrality. Indeed, in the case in which $N = 0$, this is the test of the efficient markets model discussed in Section III. The next step in relaxing constraints permits a test of neutrality without maintaining the hypothesis of rational expectations. Here the test is conducted on the assumption that the expectations of X_t in the second equation of the system (26) are formed with the same set of information, Z_{t-1}, as the time-series model of X_t in the first equation. Yet if we are not willing to assume that expectations are rational, there seems to be no reason to assume that the same set of variables belongs in Z in both equations in (26). Therefore, it is not clear that this test yields useful information.

IDENTIFICATION

The various tests discussed above depend on estimation of the parameters δ_i and γ^* in the unconstrained system (26). More specifically, neutrality requires that the estimate of δ_i not differ significantly from zero, and rationality requires that the estimate of γ^* not differ significantly from γ. These restrictions are testable only if the relevant parameters are identified. If not all of the parameters are identified, then only some of the restrictions, or linear combinations of restrictions, are testable.

We outline here a procedure for determining identification by analyzing the following interesting special case of systems (25) - (28) where we rewrite Z_{t-1} as shown below for the system (26).[24]

(29)
$$X_t = \sum_{i=1}^{M} Z_{t-i}\gamma_i + u_t$$

$$y_t = \sum_{j=0}^{N} (X_{t-j} - \sum_{i=1}^{M} Z_{t-j-i}\gamma_i{}^*)\beta_j + \sum_{j=0}^{N} (\sum_{i=1}^{M} Z_{t-j-i}\gamma_i{}^*)\delta_j + \varepsilon_t$$

where X_t is a k-element row vector of variables relevant for determining y_t; $k \geq 1$.

Z_{t-i} is a $(p+k)$-element row vector of variables dated t-i which are used in predicting X_t. It contains the k elements of X_{t-i} as well as p other variables; $p \geq 0$.

y_t is a scalar.

γ_i and γ_i^* are $(p+k) \times k$ matrices of parameters,

β_i and δ_j are $k \times 1$ column vectors of parameters.

Note that (29) embodies the following simplifying assumptions: (a) the same lag length applies to all variables used to predict X_t in the first equation,and (b) in the second equation, the same lag length, N, is used for both anticipated and unanticipated X_t. These assumptions, which are made for expositional clarity, can be relaxed and the following discussion can be generalized in a straightforward manner. Note also that the row vector Z_{t-i}, which is used in the time-series model for predicting X_t, contains the k-element row vector X_{t-i}, since lagged values of the dependent variable are often useful in prediction. In addition, the row vector Z_{t-i} contains p other variables at time t-i, where $p \geq 0$. We also assume that u_t and ε_t are uncorrelated and that $E(u_t|\phi_{t-1}) = E(\varepsilon_t|\phi_{t-1}) = 0$. Finally, recall that the rationality restriction is $\gamma_i = \gamma_i^*$, $i = 1, \ldots, M$, and the neutrality restriction is $\delta_j = 0$, $j = 0, \ldots, N$.

The first step in determining identification is to analyze the order condition. For example, consider the most unconstrained system (29) in which γ_i, γ_i^*, β_j and δ_j are the free parameters to be estimated. Observe that γ_i can be estimated by OLS on the first equation in (29). The remaining parameters γ_i^*, β_j, and δ_j must be estimated from the second equation in (29). The most unconstrained form of this second equation is

$$(30) \qquad y_t = \sum_{j=0}^{N} \hat{u}_{t-j}\beta_j + \sum_{\ell=1}^{M+N} Z_{t-\ell}\theta_\ell + \varepsilon_t$$

where $\hat{u}_{t-j} = X_{t-j} - \sum_{i=1}^{M} Z_{t-j-i}\hat{\gamma}_i$

θ_ℓ is a $(p+k) \times 1$ column vector of parameters which is zero[25] if $\delta_j = 0$, $j = 0, \ldots, N$ and $\gamma_i^* = \gamma_i$ $i = 1, \ldots, M$.

Note that for $j = 1, \ldots, N$, the residual $\hat{u}_{t-j}$ can be expressed as a linear combination of the other right-hand side variables $Z_{t-1}, \ldots, Z_{t-M-N}$. That is, only the residual at time t, $\hat{u}_t$, is not perfectly correlated with the other right-hand-side variables. Hence, the most unconstrained form of this equation which can be estimated by OLS is

$$(31) \qquad y_t = \hat{u}_t\beta_o + \sum_{\ell=1}^{M+N} Z_{t-\ell}\theta'_\ell + \varepsilon_t \, .$$

Since there are k elements in β_o and (M+N)(p+k) elements in the θ' coefficients, equation (31) can be used to estimate at most k + (M+N)(p+k) parameters. As long as this number of estimable parameters exceeds the number of free parameters contained in the β, δ, and γ^* coefficients, the order condition is satisfied.

Identification depends on the rank condition as well as the order condition. The rank condition is particularly important in the identification of (29) because, in general, it need not be satisfied when the order condition is satisfied. This failure to satisfy the rank condition becomes clear if we rewrite (29) as

$$(32) \qquad X_t^1 = \sum_{i=1}^{M} Z_{t-i}\gamma_i^1 + u_t^1$$

$$\vdots$$

$$X_t^k = \sum_{i=1}^{M} Z_{t-i}\gamma_i^k + u_t^k$$

$$y_t = \sum_{s=1}^{k} \{ \sum_{j=0}^{N} (X_{t-j}^s \beta_j^s) + \sum_{j=0}^{N} (\delta_j^s - \beta_j^s) \quad \sum_{i=1}^{M} Z_{t-i-j}\gamma_i^{*s} \}$$

where X_t^s, γ_i^s, γ_i^{*s}, and u_t^s are the s^{th} columns of X_t, γ_i, γ_i^* and u_t respectively. The scalars β_j^s and δ_j^s are the s^{th} elements of β_j and δ_j respectively.

Note that for any particular s, say s_o, the system will be unchanged by a doubling of all of the elements of $\gamma_i^{*s_o}$ for all i and a halving of $\delta_j^{s_o} - \beta_j^{s_o}$ for all j. Because of this observational equivalence, the parameters $\delta_j^{s_o} - \beta_j^{s_o}$ and $\gamma_i^{*s_o}$ are not identified even when the order condition is satisfied. A restriction on any element of $\delta_j^{s_o}$ or $\gamma_i^{*s_o}$ is sufficient to identify these parameters. Applying this argument to each of the k values of s, it is clear that k restrictions are needed to identify all of the parameters in (29). The restrictions will be provided if either neutrality ($\delta_j = 0$) or rationality ($\gamma_i = \gamma_i^*$) is treated as a maintained hypothesis. Thus, only if neither neutrality nor rationality is maintained will the rank condition fail to be satisfied in situations when the order condition is satisfied.

Tests of hypotheses are conducted by comparing the residual sums of squares from constrained and unconstrained systems. The number of restrictions tested (and hence the number of degrees of freedom in the χ^2 statistic) equals the number of identified parameters estimated in the unconstrained system less the number of identified parameters estimated in the constrained system. To illustrate this calculation using the procedures above, consider the test of rationality, under the maintained hypothesis of neutrality, in the efficient markets case in which $N = 0$. The last equation in the constrained system (where $\delta_o = 0$, $\gamma_i = \gamma_i^*$) contains k parameters (the elements of β), all of which are identified. The last equation in the unconstrained system (where $\delta_o = 0$) contains $k + Mk(p+k)$ parameters. However, as explained above, only $k + M(p+k)$ parameters can be estimated. Only if $k = 1$ will all of the parameters in the unconstrained system be identified. However, even if $k > 1$, there are $M(p+k)$ testable restrictions. These restrictions are linear combinations of the restrictions $\gamma - \gamma^* = 0$. (See footnote 16 for an example.)

An alternative test which may be conducted in the efficient markets framework $(N=0)$, is a test of the null hypothesis of neutrality under the maintained hypothesis of rationality. Recall that the last equation of the constrained system $(\gamma_i = \gamma_i^*,\ \delta_o = 0)$ contains k parameters (the elements of β), and observe that the last equation of the unconstrained system $(\gamma_i = \gamma_i^*)$ contains 2k parameters (the elements of β and δ_o). In both the constrained and unconstrained systems, all of the parameters are identified and all k neutrality restrictions are testable.

A third test in the efficient markets framework is a test of the joint hypothesis of neutrality and rationality. As in the two tests discussed before, all k parameters of the last equation in the constrained system are identified. In the unconstrained system the last equation contains $2k + Mk(p+k)$ parameters(k elements of β, k elements of δ_o and $Mk(p+k)$ elements of γ_i^*, $i = 1, \ldots, M$), but, as explained above, only $k + M(p+k)$ parameters can be estimated. Therefore, under no circumstances will all of the parameters of this equation be identified. However, there are $M(p+k)$ testable restrictions which are linear combinations of the restrictions $\gamma - \gamma^* = 0$ and $\delta_o = 0$.

The interpretation of the tests above depends on what hypothesis is maintained. In particular, the test statistic associated with the joint test of rationality and neutrality is identical to the test statistic for the test of rationality, under the maintained hypothesis of neutrality. This follows from the fact that, although the free parameters in the unconstrained systems are different, the estimated coefficients are identical. Furthermore, the constrained systems are the same. Because of the equivalence of the two tests, one can not determine whether a rejection is due to a violation of rationality alone or a violation of both rationality and neutrality.

Another interesting example arises in tests of policy neutrality under the maintained hypothesis of rationality as in Barro (1977, 1978). In these models it is assumed that the deviation of current output from its natural level is affected only by the current and N lagged surprises in a single policy variable (i.e., $k=1$ and $N > 0$). In order to obtain identification of the coefficients on surprises in the policy variable, these studies implicitly place restrictions on the covariances of ε_t with both u_{it} and with lagged disturbances. There are two alternative conditions which are sufficient for identification of the δ coefficients, i.e., the coefficients on anticipated policy. One condition, discussed and used by Barro (1977,1978, 1979), Leiderman (1980), and Mishkin (1982), is the exclusion restriction $p \geq 1$. That is, the time series model for the policy variable X_t contains at least one variable which is not directly included in the output equation. The output equation in the constrained system (where $\delta_i = 0$ and $\gamma_i = \gamma_i^*$) contains $N+1$ parameters $(\beta_o, \ldots, \beta_N)$, and in the unconstrained system (where $\gamma_i = \gamma_i^*$) it contains $2(N+1)$ parameters $(\beta_o, \ldots, \beta_N$ and $\delta_o, \ldots, \delta_N)$. In each of these systems, all of the parameters are identified because the number of free parameters is less than the number of estimable parameters, $1 + (M+N)(p+1)$. Therefore all of the $N+1$ neutrality restrictions are testable.

The alternative sufficient condition for identification is $M > N$; that is, the number of lags in the time series model for the policy variable X_t exceeds the number of lagged surprises in the output equation. Although this condition does formally lead to identification, it requires strong a priori knowledge of lag lengths. Without this prior knowledge we are faced with the observational equivalence problem raised by Sargent (1976b).

For identification of δ_i it is necessary that at least one of the two conditions above holds. One recent piece of research where this does not occur is in Grossman (1979). His specification of the time-series equation describing his policy variable (nominal GNP growth) does not include any variable other than lagged dependent variables. In addition, the number of lags in the output equation exceeds that in the time-series equation for the policy variable. Therefore, the δ coefficients in his model are not identified, with the result that not all the neutrality constraints can be tested.

VI

SUMMARY AND CONCLUSIONS

The testing framework in this paper facilitates analysis and integration of a wide range of issues in testing rationality, capital market efficiency, and the short-run neutrality of monetary policy. The main points of this analysis are summarized below.

1. Given observations on expectations of a variable, there are two alternative procedures for testing rationality of these èxpectations. One procedure tests for correlation between the forecast error and past information. The other procedure tests the cross-equation restriction that the relation of the forecast to past information is the same as the relation of the realization to past information. These two procedures are equivalent. Furthermore, these procedures provide tests of rationality regardless of what past information is used.

2. In the absence of direct observations on expectations, we can test rationality (market efficiency) by testing cross-equation restrictions similar to those used in rationality tests involving direct observations on expectations. The cross-equation test of the joint hypothesis of market efficiency and the model of market equilibrium is asymptotically equivalent to the common test in which the deviation of the return from the equilibrium return is regressed on past information. Since the common test procedure is a test of market efficiency and the model of market equilibrium even if some relevant past information is ignored, the cross-equation procedure also has this property.

3. Granger causality tests for the short-run neutrality of monetary

policy are asymptotically equivalent to tests of cross-equation restrictions in a model in which only contemporaneous surprises in monetary policy affect unemployment or output. Since these Granger causality tests are tests of the joint hypothesis of rationality, neutrality, and the model of equilibrium output, even if relevant past information is ignored, the cross-equation procedure is also a test of this joint hypothesis. Thus, even if the policy regime changes, and the change is ignored, we still are able to test the joint hypothesis.

4. If lagged surprises in monetary policy affect output and unemployment, then the Granger causality test and the cross-equation test are no longer asymptotically equivalent. If the disturbance term in the output equation is serially correlated, then the two procedures are asymptotically equivalent; however, they are no longer tests of short-run neutrality.

5. There is a straightforward procedure for determining whether coefficients are identified and whether hypotheses are testable. A particular application of this procedure shows why all of the neutrality restrictions are testable in Barro's (1977, 1978) model, but not in J. Grossman's (1979) model.

FOOTNOTES

1. For example, see Sargent and Wallace (1975), Lucas (1976), Poole (1976) and Mishkin (1978).

2. Many of the results in this section are not new. Yet the exposition here sets the stage for the later sections which do contain new results.

3. In this paper we use the convention that the subscript t indicates that a variable is realized at the end of period t. For a variable which describes a security's one-period return, the subscript t indicates that the return occurs between the end of period t-1 and the end of period t.

4. If v_t is identically zero, then X_t^e is a minimum variance unbiased forecast of X_t. Replacing the restriction that v_t be identically zero with the restriction that $E(v_t|\phi_{t-1}) = 0$ will remove the minimum variance property of X_t^e , but not the unbiasedness conditional on ϕ_{t-1}.

5. See the survey in Levich (1979).

6. Recall that the information in $Z_{1,t-1}$ must have been available to the market at time t-1 since we are assuming that $Z_{1,t-1}$ is contained in ϕ_{t-1}.

7. In addition, the power of the test for $\hat{\omega} = 0$ could be low because the reported standard errors of $\hat{\omega}$, which is an estimate of $\alpha_1 - \alpha_1^*$, could be overstated.

8. In this case, the probability of Type II error does not go to zero as the sample size goes to infinity.

9. One way to test for the significance of $\hat{\gamma} - \hat{\gamma}^*$ is to stack (7) and (8) into one regression and perform a Chow test for the equality of

coefficients. (See Pesando (1975)). However, if, as is likely, the variance of residuals in (7) differs from the variance of residuals in (8), a correction must be made for this heteroscedasticity. See Mullineaux (1978).

10. In this case, plim $\hat{\gamma} = \alpha_1 + (Z_1' Z_1)^{-1} Z_1' Z_2 \alpha_2$ and plim $\hat{\gamma}^* = \alpha_1^* + (Z_1 ' Z_1)^{-1} Z_1' Z_2 \alpha_2^*$. Since $\alpha_1 = \alpha_1^*$ and $\alpha_2 = \alpha_2^*$ under rationality, the asymptotic bias is identical for $\hat{\gamma}$ and $\hat{\gamma}^*$.

11. See Fama (1976).

12. Systems of this type have been estimated in Mishkin (1981a,b).

13. Abel and Mishkin (1980) contain a more detailed proof and discussion of identification. This paper also shows that if appropriate corrections for degrees of freedom are made, tests using (a) versus (b) are not only asymptotically equivalent, but are equivalent in finite samples as well.

14. Observe that the second equation in (18) contains a model of market equilibrium and can be rewritten as

$$(*) \qquad R_t = f(\Omega_{t-1}) + (X_t - X_t^e)\beta + Z_{t-1}\theta + \varepsilon_t \quad .$$

The proof outlined above treats $f(\Omega_{t-1})$ as known. If it were unknown and were assumed to be a linear function of past variables W_{t-1}, then W_{t-1} must also be included as explanatory variables in the time series model for X_t. This will preserve the orthogonality of the residuals in the equations for X_t with the other right hand side variables in equation (*), thereby allowing the proof of the theorem to proceed as in the text. This issue is discussed further in the proof of the corollary in section IV. Of course, if the coefficients of W_{t-1} in the model of market equilibrium are estimated, then we cannot test the rationality restriction that y_t is uncorrelated with W_{t-1}. The question of testability of such restrictions is discussed in section V.

15. Even if the disturbances are not normal, the quasi-likelihood ratio test can be used for valid inference under quite general conditions. See Kohn (1979).

16. Consider the case in which $\ell = k = 2$. The system of equations can be written as:

$$X_{1t} = \gamma_{11} Z_{1,t-1} + \gamma_{21} Z_{2,t-1} + u_{1t}$$

$$X_{2t} = \gamma_{12} Z_{1,t-1} + \gamma_{22} Z_{2,t-1} + u_{2t}$$

$$y_t = \beta_1 X_{1t} + \beta_2 X_{2t} - (\gamma^*_{11}\beta_1 + \gamma^*_{12}\beta_2) Z_{1,t-1} - (\gamma^*_{21}\beta_1 + \gamma^*_{22}\beta_2) Z_{2,t-1} + \varepsilon_t$$

The four parameters γ_{ij} can be estimated from the first two equations. If $\mathrm{Cov}(\varepsilon_t, u_{it})$ is known to be zero, we can estimate β_1, β_2, $(\gamma^*_{11}\beta_1 + \gamma^*_{12}\beta_2)$ and $(\gamma^*_{21}\beta_1 + \gamma^*_{22}\beta_2)$ from the third equation. Since we cannot separately estimate the four elements γ^*_{ij}, we cannot separately test the four restrictions $\gamma_{ij} = \gamma^*_{ij}$. However, we can test $\ell = 2$ linear combinations of the rationality restrictions:

$(\gamma_{i1} - \gamma^*_{i1})\beta_1 + (\gamma_{i2} - \gamma^*_{i2})\beta_2 = 0$ for $i = 1$ and 2. If we do not know the covariances of ε_t and u_{it}, then β_1 and β_2 are not identified. However, we can still test whether the two linear combinations above are equal to zero. To see this, rewrite the third equation as

$$y_t = [(\gamma_{11} - \gamma^*_{11})\beta_1 + (\gamma_{12} - \gamma^*_{12})\beta_2]\, Z_{1,t-1} + [(\gamma_{21} - \gamma^*_{21})\beta_1 + (\gamma_{22} - \gamma^*_{22})\beta_2] Z_{2,t-1}$$
$$+ \beta_1 u_{1t} + \beta_2 u_{2t} + \varepsilon_t \quad .$$

Observe that the coefficients of $Z_{1,t-1}$ and $Z_{2,t-1}$ in the rewritten equation are the testable linear combinations of rationality restrictions.

17. See Modigliani (1977).

18. The model of equilibrium output provides the exclusion restriction that Z_{t-1} does not enter the second equation of (22) except as it is contained in X_t^e. This restriction leads to identification in the system. See section V.

19. The use of the word "causality" in describing the Granger (1969) test is somewhat unfortunate, for it has led to much confusion in the literature. It is really a test of predictive content and not of economic causation. See Zellner (1979) for a discussion of this point.

20. As indicated in footnote 14 and Section V, since we must estimate the coefficients of $q_{t-1}(i = 1,...,L)$ in the model of equilibrium output, we do not obtain testable restrictions on the estimates of ψ_i and ψ_i^* for $i = 1,...,L$. The constraints $\theta_i=0$, and hence $\psi_i = \psi_i^*$, can be obtained only if we impose the identifying restriction that the lag length L in (20) is shorter than the lag length L' in (21). This appears to be a rather strong assumption to impose on the basis of a priori knowledge, and one should be cautious in interpreting results based on estimates of θ_i in this case.

21. Of course, OLS cannot be directly applied to the second equation of (24) as it is written, since the variables q_{t-i} appear twice on the right-hand side. This equation must be rewritten to eliminate the perfect colinearity of right-hand variables; then OLS may be used.

22. If there are two policy regimes in the sample period 1 to T, then we can write

$$X_t = Z_{t-1}\gamma_1 + u_{1t} \qquad \text{for } t = 1 \text{ to } T_1$$

$$X_t = Z_{t-1}\gamma_2 + u_{1t} \qquad \text{for } t = T_1 + 1 \text{ to } T$$

which can be rewritten as:

$$X_t = Z_{t-1}\gamma_1 + Z^*_{t-1}\xi + u_{1t} \quad \text{for } t = 1 \text{ to } T$$

where

$$Z^*_{t-1} \begin{cases} 0 & \text{for } t = 1 \text{ to } T_1 \\ Z_{t-1} & \text{for } t = T_1 + 1 \end{cases}$$

$$\xi = \gamma_2 - \gamma_1$$

23. If (19) is quasi-differenced to obtain a specification with a serially uncorrelated disturbance, the new specification will contain current and lagged residuals from the first equation in (24). Hence, the Granger causality test will not be a true test of the joint hypothesis of rationality, the model of equilibrium output, and neutrality, as explained in the paragraph above.

24. We assume that variables are measured as deviations from sample means so that no constant term appears in any equation.

25. Specifically, $\theta_\ell = \sum_{i+j=\ell} \{(\gamma_i - \gamma^*_i)\beta_j + \gamma^*_i\delta_j\}$, $1 \leq i \leq M$, and $0 \leq j \leq N$.

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