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CONSUMPTION CORRELATION AND RISK MEASUREMENT IN ECONOMIES WITH NON-TRADED ASSETS AND HETEROGENEOUS INFORMATION

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ABSTRACT

The consumption beta theorem of Breeden makes the expected return on any asset a function only of its covariance with changes in aggregate consumption. It is shown that the theorem is more robust than was indicated by Breeden. The theorem obtains even if one deletes Breeden's assumptions that (a) all risky assets are tradable, (b) investors have homogeneous beliefs, (c) other assets can be traded without transactions costs and (d) that all assets have returns which are Ito processes.

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Consumption Correlation and Risk Measurement in Economies with Non-Traded Assets and Heterogeneous Information

by

Sanford J. Grossman and Robert Shiller

1. Introduction

Douglas Breeden [1979] has recently considered an intertemporal capital asset pricing model which implies that the expected return on an asset depends on its "consumption beta." His result is that asset i's mean excess return can be explained by the covariance of asset i's return with aggregate consumption. His model has attracted interest because it solves the problem of extending the simple one-period Sharpe-Lintner portfolio model to the intertemporal case without assuming an unchanging investment opportunities set. The kind of solution shown earlier by Robert Merton, Breeden argued, implies that the return on an asset depends on its covariance with $s+2$ portfolios where s is the number of state variables in the economy. Since s is likely to be large, Merton's approach does not suggest a simple empirical regularity as does Breeden's.

We show here that Breeden's result is even more powerful than is suggested by his paper. We assume that there is a single consumption good and that an individual's consumption is a diffusion, i.e. an Ito process. Under this assumption, if asset i is freely tradable and if its return follows an Ito process, then its mean excess return will be explained by the covariance of its return with aggregate consumption. This theorem is true even when we delete Breeden's assumptions that (a) all risky assets are tradable, (b) that investors have homogeneous beliefs about future returns, (c) that all other assets can be traded with no transactions costs, and (d) that all other assets have returns which are an Ito process.[1/] This extension of Breeden's result is important because (i) human capital cannot be traded, but has an uncertain payoff, (ii) other assets, such as housing, are very lumpy, and can be traded only subject to a significant transaction cost, and (iii) when information is costly, traders will, in equilibrium, have different beliefs about returns or consumption.

Our method of proof is more direct and simpler than Breeden's, in that it is not necessary to completely solve the consumer's intertemporal continuous stochastic optimization problem. Our argument can be summarized as follows. For the consumer to be at an optimum, the distribution of excess returns between two traded assets r_1-r_2, given his information I_i, must satisfy $E[U_i'(C_i)(r_1-r_2)| I_i] = 0$. That is, each trader must expect his marginal utility of consumption to be uncorrelated with excess returns on traded assets. This is true even if there are untraded risky assets. In continuous time, the solution to individual i's portfolio problem is as if he has quadratic utility. It is as if consumer i's marginal utility is linear in his consumption: $u_i'(C_i) = a_i-b_iC_i$. When marginal utility is linear, this relationship can be written as $(a_i/b_i)E[r_1-r_2| I_i] = E[C_i(r_1-r_2)| I_i]$. If we let I_c denote the information which all consumers hold in common, such as past prices, past aggregate consumption, and past asset payoffs, then we can take the expectation of both sides of the last stated equation conditional on I_c. Since I_i contains strictly more information than I_c, the above procedure yields the same equation except that I_i is replaced by I_c. This equation can now be summed over the consumers to yield $E[r_1-r_2| I_c] = (\sum_i (a_i/b_i))^{-1}E[(\sum_i C_i)(r_1-r_2)| I_c]$. This equation holds for any subset of the information set which is common to all traders. Hence it is also true using the unconditional distributions.

Thus this model states that covariance with aggregate consumption is the appropriate measure of risk even when traders have different beliefs. The consumption correlatedness model works even when there are nontradable assets and heterogeneous beliefs because it does not rely on there being one portfolio which is optimal for everyone. It holds even when individuals do not have perfectly correlated consumption. The linearity of each consumer's "first order" condition for optimality implies that idiosyncratic components of consumption are irrelevant for asset pricing. This is true even though there does not exist spanning or a complete set of markets which traders can use to insure against fluctuations in the idiosyncratic components of their non-tradable assets.

2. The Model

We assume that a consumer has a time-additive utility function over a single consumption good

$$(1) \qquad U = \sum_{j=0}^{T/h} \delta^{hj} u(C_j^h) h$$

where T is his time horizon, C_t^h is consumption at time t and δ^h is the discount factor between utility at t and t+h. (Note that consumers may differ in their utility functions and discount factors and time horizons.) $u(C_t^h)$ is the flow of utility during the period of length h. Our approach to the continuous time optimization problem will be by taking a limit of discrete time problems as the time h between periods goes to zero.[2/] To define the budget constraint, suppose the consumer arrives at time t with a portfolio of N traded assets $\underline{X}_{t-h}$, where the date the consumer last trades is denoted by t-h. Let $\underline{P}_t$ be the vector of prices for these assets in terms of the single consumption good. Let $\underline{D}_t^h$ be the vector of per share payouts on each of these assets which is accumulated by time t when the asset is purchased at t-h. Let H_t be the consumption that the consumer gets at t from nontraded or imperfectly traded assets, and let I_t be the consumer's information at time t. Let $\underline{X}_t(I_t)$ denote the portfolio he decides to hold at t; then his budget constraint is

$$(2) \qquad hC_t^h = H_t - \underline{P}_t \underline{X}_t + \underline{P}_t \underline{X}_{t-h} + \underline{D}_t^h \underline{X}_{t-h} \quad .$$

Thus the consumer's problem in a competitive market is to act as a price taker and maximize the expected value of (1) subject to (2) using the controls $\{\underline{X}_t(I_t)\}$. Note that when the consumer chooses $\underline{X}_t$, he does not know H_{t+h}. H_t can be interpreted as the consumption derived from human capital and housing at time t, i.e. stochastic wage income and stochastic imputed rental income.

It is notationally convenient to eliminate the dividend payment as a component of holding period return. We thus assume that all companies use dividend income to buy back shares of firms rather than paying out dividends. Let V_{it} be the market price of asset i at time t. Clearly the real returns are unchanged in this model when dividends are paid out to shareholders instead of being used to make share repurchases (since we assume no taxes).

Let $(C_t^h, \underline{X}_t^h)$ be a solution to the discrete time problem for the consumer when the trading interval is h. Assume that $C_t^h > 0$. For this to be optimal, it must be the case that the consumer cannot raise his expected utility by selling some of asset i at time t, planning to buy it back at time t+h. At any time t, the consumer can sell s units of asset i; this would increase his consumption at t by sV_{it}. Let him make no other change in his portfolio. Suppose he buys the s units back at time t+h, by reducing his consumption at t+h, but keeping his other assets as before, and from then on makes the same trades as previously. Thus the consumer's consumption only changes at t and t+h. Hence his total change in expected utility from selling s units at t and buying them at t+h is

$$(3) \qquad u(C_t^h) + E_t\delta^h u(C_{t+h}^h) - [u(C_t^h + sV_{it}) + E_t\delta^h u(C_{t+h}^h - sV_{it+h})] \quad .$$

For $\underline{X}_t^h, C_t^h$ to be optimal, the proposed trading plan must not raise expected utility. Hence (3) must be minimized at $s = 0$ (since (3) must always be positive for $s \neq 0$ and equals zero at $s = 0$). Assuming u is strictly concave and differentiable, this means that the derivative of (3) with respect to s must be zero at $s = 0$. Hence

$$(4) \qquad u'(C_t^h)V_{it} = \delta^h E_t[u'(C_{t+h}^h)V_{it+h}] \quad .$$

This is just the statement that a trader equalizes his marginal rate of substitution between current consumption and shares to the relative current price of shares. Let τ be a time after t such that $\tau - t$ is an integer multiple of h. Then τ is a feasible trading date. The reader can verify that, for any feasible trading date τ, the consumer must also be indifferent between selling a little of the asset at t and buying it back at τ. This implies that

$$(5) \qquad u'(C_t^h)V_{it} = \delta^{\tau - t} E_t u'(C_\tau^h) V_{i\tau} \quad \text{for all } \tau = t+h,\ t+2h,\ t+3h,\ \ldots$$

Dividing both sides of this equation by V_{it} (which is known at time t and can therefore be taken inside the expectations operator), we find that, for any two assets i and j:

$$(6) \qquad 0 = E_t \frac{u'(C_\tau^h)}{u'(C_t^h)} \left[\frac{V_{i\tau}}{V_{it}} - \frac{V_{j\tau}}{V_{jt}} \right] \qquad \tau = t+h,\ t+2h,\ t+3h,\ \ldots$$

In order to take limits as h goes to zero, we assume that we are looking at a given consumer and only varying the length of time h between his trades. That is, we will take prices as being unaffected by the change in trading opportunities (i.e., the change in h) to which this consumer is subjected. This paper will say nothing further about the equilibrium determination of prices. Our goal is simply to derive a simple relationship between rates of return and consumption. (See Cox, Ingersoll and Ross [1978] for a model with endogenous prices.)

If we take the limit of (6) as h goes to zero, we conclude that:

$$(7) \qquad \text{For all } h > 0,\ 0 = E_t \frac{u'(C_\tau^h)}{u'(C_t^h)} \left[\frac{V_{i\tau}}{V_{it}} - \frac{V_{j\tau}}{V_{jt}} \right] \text{ for } \tau = t+h, t+2h, t+3h\ldots$$

Note that each h defines a stochastic process $\{C_t^h\}$ over t which are integer multiples of h, assuming that the initial trading date is $t = 0$.

We assume that $C_t^* \equiv \lim_{h \to 0} \{C_t^h\}$ exists and is a diffusion, i.e. an Ito process. (Note that $\lim_{h\to 0} \{C_t^h\}$ is some stochastic process which is defined all over real t.) This is surely a strong assumption. Further we do not know exactly which stochastic processes $\{H_t, \underline{V}_t\}$ will lead the consumer's optimal consumption policy to be a diffusion. However, the reader should be aware that all of the work which uses the stochastic calculus (such as Breeden) to solve for the consumer's optimal consumption policy also assumes that the optimal policy is a diffusion, rather than proving that it is best in a larger class of policies.[3/] We also assume that the log of the price of the assets follows a diffusion. These two assumptions imply:

$$\frac{dV_i}{V_i} = \mu_i dt + \sigma_i d\eta_i \qquad i = 1,\ldots,N \tag{8}$$

$$\frac{dC_t^*}{C_t^*} = \rho dt + \tau d\varepsilon \quad , \tag{9}$$

where μ_i, σ_i, ρ and τ can depend on all current state variables including V, $\underline{C}^*$, where ρ, τ and ε can be different among consumers, and where if $d\eta \equiv (d\eta_1, d\eta_2, \ldots, d\eta_N)$ then (η,ε) is an N+1 dimensional browian motion with mean zero.

Since (7) holds for all h, it is true of the limit process C_t^*:

$$0 = E_t \frac{u'(C_\tau^*)}{u'(C_t^*)} \left[\frac{V_{i\tau}}{V_{it}} - \frac{V_{j\tau}}{V_{jt}} \right] \qquad \text{for all} \quad \tau > t \quad . \tag{10}$$

We now show that (10) implies that, if $A_t \equiv -(u''(C_t^*)/u'(C_t^*))C_t^*$ is the coefficient or relative risk aversion, then

$$E_t \left[\frac{dV_i}{V_i} - \frac{dV_j}{V_j} \right] = A_t \, \mathrm{Cov}_t \left(\frac{dC_t^*}{C_t^*} , \frac{dV_i}{V_i} - \frac{dV_j}{V_j} \right) \tag{11a}$$

equivalently

$$\mu_i - \mu_j = A_t \, \mathrm{Cov}_t \left(\tau \frac{d\varepsilon}{dt} , \sigma_i \frac{d\eta_i}{dt} - \sigma_j \frac{d\eta_j}{dt} \right) \quad , \tag{11b}$$

which simply states that the mean excess return between any two assets is proportional to the covariance between the growth rate of consumption and excess returns.

We first give an intuitive proof and then give a very short formal argument using Ito's lemma. Intuitively, (11) follows from (10) because over short time intervals the marginal utility of consumption can be approximated as a linear function. That is, take a Taylor expansion of $u'(C^*_\tau)$ about C^*_t to get, from (10),

$$(12) \qquad 0 = E_t\left[\frac{u'(C^*_t) + (C^*_\tau - C^*_t)u''(C^*_t) + \varepsilon}{u'(C^*_t)}\right]\left(\frac{V_{i\tau}}{V_{it}} - \frac{V_{j\tau}}{V_{jt}}\right) \quad ,$$

where ε is the error in the Taylor expansion.

Multiply both sides of (12) by $\frac{1}{\tau - t}$, and assume that

$$(13) \qquad \lim_{\tau \downarrow t} E_t\left[\frac{\varepsilon}{\tau - t}\left(\frac{V_{i\tau}}{V_{it}} - \frac{V_{j\tau}}{V_{jt}}\right)\right] = 0 \quad ,$$

then (12) becomes

$$(14) \qquad -\frac{u''(C^*_t)}{u'(C^*_t)} C^*_t E_t\left[\frac{1}{\tau - t}\left(\frac{C^*_\tau - C^*_t}{C^*_t}\right)\left(\frac{V_{i\tau}}{V_{it}} - \frac{V_{j\tau}}{V_{jt}}\right)\right] = E_t\left[\frac{1}{\tau - t}\frac{V_{i\tau}}{V_{it}} - \frac{1}{\tau - t}\frac{V_{j\tau}}{V_{jt}}\right]$$

Therefore we have the same first-order condition as with quadratic utility. As we noted in the introduction, quadratic utility immediately gives a consumption beta model.[4/] This can be seen as follows. If we take the limit of (14) as $\tau \downarrow t$ (i.e. as τ goes to t from above), then (14) is identical to (11). This is because, for diffusions,

$$(15) \qquad \frac{1}{\Delta t} E_t \frac{\Delta V_i}{V_i} \rightarrow E \frac{1}{dt}\frac{dV_i}{V_i} = \mu_i$$

and

$$(16) \qquad \frac{1}{\Delta t} E \frac{\Delta C}{C} \frac{\Delta V_i}{V_i} = E \left(\frac{1}{\sqrt{\Delta t}} \frac{\Delta C}{C}\right)\left(\frac{1}{\sqrt{\Delta t}} \frac{\Delta V_i}{V_i}\right) \rightarrow \mathrm{Cov} \left(\frac{1}{\sqrt{dt}} \frac{dC}{C}, \frac{1}{\sqrt{dt}} \frac{dV_i}{V_i}\right) .$$

That is, (14) essentially states that the expected value of excess rates of return per unit time is proportional to the covariance between the rate of growth in consumption and excess returns per unit time. Note that, for diffusions, the expected cross product on the left-hand side of (14) is the same as the covariance of the two terms involved, since $(E\frac{dC}{C})(E\frac{dV}{V})$ is of order $(dt)^2$.

Intuitively, the Taylor expansion term in (13) is zero because it equals

$$(17) \qquad C_t E_t \left[\frac{C_\tau - C_t}{C_t\sqrt{\tau-t}}\right] \left[\left(\frac{V_{i\tau}}{V_{it}} - \frac{V_{j\tau}}{V_{jt}}\right) \frac{1}{\sqrt{\tau-t}}\right] \left[\frac{\varepsilon}{C_\tau - C_t}\right] ,$$

The random variables in the first two brackets converge to Normal random variables as $\tau \downarrow t$ since they are each diffusions, i.e. since $(d\eta, d\varepsilon)$ is jointly Normal. The term in the last bracket converges to zero along every sample path because it is the Taylor expansion error and $C_\tau - C_t \rightarrow 0$ along every sample path as $\tau \downarrow t$.

A rigorous proof is much shorter, with the asterisk on C_t^* eliminated for convenience.

<u>THEOREM</u>: If <u>V</u>,<u>C</u> is an Ito process, then (10) implies (11).

<u>PROOF</u>: Consider the random variable $Z_{it} \equiv u'(C_t)V_{it}$. If (V_{it}, C_t) is an Ito process, then by Ito's Lemma Z_{it} is an Ito process and

$$(18) \qquad dZ_{it} = u''(C_t)V_{it}dC_t + u'(C_t)dV_{it} + \tfrac{1}{2}u'''(C_t)V_{it}(dC_t)^2 + u''(C_t)dC_t dV_{it} .$$

Divide both sides of (10) by $\tau-t$ and take the limit as $\tau-t \downarrow 0$. This yields

$$(19) \qquad \frac{1}{Z_{it}} E_t\, dZ_{it} = \frac{1}{Z_{jt}} E_t\, dZ_{it} .$$

Substitute (18) into (19), use the fact that $(dC_t)^2 \equiv (\text{Var } dC_t)dt$, $dC_t dV_{it} \equiv \text{Cov }(dC_t, dV_{it})dt$, and get

$$E_t\left[\frac{u''(C_t)}{u'(C_t)}dC_t + \frac{dV_{it}}{V_{it}} + \frac{u'''(C_t)}{u'(C_t)}(\text{Var } dC_t)dt + \frac{u''(C_t)}{u'(C_t)} C_t \text{ Cov }\left(\frac{dC_t}{C_t}, \frac{dV_{it}}{V_{it}}\right)\right]$$

$$= E_t\left[\frac{u''(C_t)}{u'(C_t)}dC_t + \frac{dV_{jt}}{V_{jt}} + \frac{u'''(C_t)}{u'(C_t)}(\text{Var } dC_t)dt + \frac{u''(C_t)}{u'(C_t)} C_t \text{ Cov }\left(\frac{dC_t}{C_t}, \frac{dV_j}{V_j}\right)\right]$$

Equation (11) follows immediately. Q.E.D.

3. Aggregation Over Consumers

In the last section, we showed that an individual consumer will have as a condition of optimality that the covariance between his own consumption changes and excess returns be proportional to average excess returns. Since we do not have much data on individual consumption, it is important to express (11) in terms of per capita consumption. An aggregation result derived by Breeden for his more restrictive model also applies here.

We first analyze the effects of heterogeneous information. Let $I_{t\ell}$ denote consumer ℓ's information at time t. Then, using the fact that $(EdC)(EdV)$ is of order $(dt)^2$ so that we treat it as zero, we can write (11a) as

$$(20) \qquad E\left[\frac{dV_i}{V_i} - \frac{dV_j}{V_j}\,\middle|\, I_{t\ell}\right] = A_{t\ell}\, E\left[\frac{dC^*_{t\ell}}{C^*_{t\ell}}\left(\frac{dV_i}{V_i} - \frac{dV_j}{V_j}\right)\,\middle|\, I_{t\ell}\right] ,$$

where we use the subscript ℓ to denote a variable specific to consumer ℓ. Since each trader knows his own consumption, $C^*_{t\ell}$ can be moved through the conditional expectation on the right-hand side of (20). Next, multiply both sides by $C^*_{t\ell}/A_{t\ell}$. This yields

$$(21) \qquad E\left[\frac{C^*_{t\ell}}{A_{t\ell}}\left(\frac{dV_i}{V_i} - \frac{dV_j}{V_j}\right)\,\middle|\, I_{t\ell}\right] = E\left[dC^*_{t\ell}\left(\frac{dV_i}{V_i} - \frac{dV_j}{V_j}\right)\,\middle|\, I_{t\ell}\right] .$$

Let I_t denote the information which all traders hold in common (i.e., every trader observes at least I_t). A well-known result in probability theory states that, if I_t is a subset of $I_{t\ell}$, then $E\{E[f(\omega)|I_{t\ell}]|I_t\} = E[f(\omega)|I_t]$, where $f(\omega)$ is any measurable function on the sample space.[5/] Thus, if we take the expectation of (21) conditional on I_t, we obtain

$$(22) \qquad E\left[\frac{C^*_{t\ell}}{A_{t\ell}}\left(\frac{dV_i}{V_i} - \frac{dV_j}{V_j}\right) \Big| \; I_t\right] = E\left[dC^*_{t\ell}\left(\frac{dV_i}{V_i} - \frac{dV_j}{V_j}\right) \Big| \; I_t\right] \quad .$$

We assume that all consumers observe past prices, payoffs, and the aggregate level of consumption in the economy. That is, at least those variables are in the common information set of all traders at t, I_t. In particular, if $C_t \equiv \sum_\ell C^*_{t\ell}$, then we assume that

$$(A1) \qquad C_t \subset I_t \subset I_{t\ell} \quad ,$$

where "$X \subset Y$" means "knowledge of Y implies knowledge of X."

Under (A1), both sides of (22) can be divided by C_t, which can be brought across the expectation operator. Next sum (22) over all agents ℓ and use the fact that, for any two random variables, $EXY = \text{cov}(X,Y) + (EX)(EY)$ to get

$$(23) \qquad \text{Cov}\left(A_t^{-1}, \frac{dV_i}{V_i} - \frac{dV_j}{V_j} \Big| \; I_t\right) + E[A_t^{-1} | \; I_t]E\left[\frac{dV_i}{V_i} - \frac{dV_j}{V_j} \Big| \; I_t\right]$$
$$= E\left[\frac{dC_t}{C_t}\left(\frac{dV_i}{V_i} - \frac{dV_j}{V_j}\right) \Big| \; I_t\right] \quad ,$$

where

$$A_t \equiv \left[\sum_\ell \frac{C^*_{t\ell}}{C_t} \frac{1}{A_{t\ell}}\right]^{-1}$$

We next assume that

$$(A2a) \qquad \text{Cov}\left(A_t^{-1}, \frac{dV_i}{V_i} - \frac{dV_j}{V_j} \Big| \; I_t\right) = 0 \quad .$$

This assumption will hold under many conditions. For example, (A2a) will hold under any one of the following assumptions: (a) each trader knows A_t at time t, i.e. $A_t \subset I_t$; (b) all traders have the same relative risk aversion and know it - since in this case $A_t = A_{t\ell} \subset I_t$; (c) all traders know each others' risk aversions and know each others' current consumption - since again $A_t \subset I_t$. It is important to note that (A2a) is consistent with traders having different information about future returns. To get (A2a) to hold, it is sufficient that all traders know the current A_t which is generated by the current consumptions and risk aversions of traders. For our purposes, this will be sufficient:

(A2b) $$A_t \subset I_t \quad .$$

Thus, under (A1) and (A2b), we obtain from (23)

(24) $$E\left[\frac{dV_i}{V_i} - \frac{dV_j}{V_j} \;\middle|\; I_t\right] = A_t E\left[\frac{dC_t}{C_t}\left(\frac{dV_i}{V_i} - \frac{dV_j}{V_j}\right) \;\middle|\; I_t\right] \quad .$$

Equation (24) is the basic equation for explaining mean returns. If, for example, asset j is commercial paper, then (24) states that any tradable asset's ex ante expected excess return over commercial paper's return can be explained by the ex ante covariance of the excess return with aggregate consumption changes. Testing (24) for tradable assets requires that we know the common information of traders at each date t. However, a great simplification is possible if A_t is constant. If A_t is a constant, or independent of I_t, then (24) must hold for any subset of the common information of agents I_t^s. That is, take the conditional expectation of (24) conditional on I_t^s. Then, by the previously mentioned result on iterated conditional expectations, we obtain

(25) $$E\left[\frac{dV_i}{V_i} - \frac{dV_j}{V_j} \;\middle|\; I_t^s\right] = A_t \; E\left[\frac{dC_t}{C_t}\left(\frac{dV_i}{V_i} - \frac{dV_j}{V_j}\right) \;\middle|\; I_t^s\right] \quad .$$

Note that A_t will be a constant if all traders have the same constant relative risk aversion, or if the traders have different but constant relative risk aversions which are uncorrelated with the consumption share $C^{*}_{t\ell}/C_t$ across traders in the population. 6/

Equation (25) is extremely useful for empirical implementation because it now involves the distribution of observables and a single constant for each date A_t. Thus, for example since all consumers observe current prices and rates of return, we can replace I^s_t by just those variables, since (25) holds for any subset of consumers' common information.

An interesting special case of (25) obtains when we choose to set I^s_t to be the null set. That is, (25) holds for expectations conditioned on no information. Thus, if there is a constant long run joint distribution of aggregate consumption changes and rates of return, this long run distribution can be used to test (25). To be more precise, suppose that returns, consumption and I_t have a stationary joint distribution. (This is consistent with consumers' expected returns changing from period to period as they observe different realizations of their information.) Then we may test (25) by using only the marginal joint distribution of returns and consumption alone (ignoring I_t). Thus, an implication of (25), if $A_t = A$ is constant over time, is

$$(26) \qquad E\left[\frac{dV_i}{V_i} - \frac{dV_j}{V_j}\right] = A\ E\left[\frac{dC_t}{C_t}\left(\frac{dV_i}{V_i} - \frac{dV_j}{V_j}\right)\right] \equiv A\ \mathrm{Cov}\left(\frac{dC_t}{C_t}, \frac{dV_i}{V_i} - \frac{dV_j}{V_j}\right)$$

Equation (26) involves only the unconditional distribution of the variables of interest. Note that (26) will hold for portfolios of traded assets. We are currently empirically testing (26) for assets like the Standard and Poors portfolio of 500 stocks, commercial paper, long term bonds, etc. over long time horizons (1890-1980), using the *ex post* estimated joint distribution of rates of change in consumption and rates of return on those assets.

4. Conclusions

We have shown that cross sectional differences in mean asset returns can be explained by the cross sectional differences in the riskiness of the assets where risk is measured by covariability of the return with rate of change in aggregate consumption. This result is true in the presence of heterogeneous expectations as well as nontradable risky assets. We obtain our result because of the linearity of consumers' first-order condition in a continuous time model, rather than from the condition that all consumers hold the same portfolio of risky assets or have perfectly correlated consumption. The latter two conditions will not obtain in a model with heterogeneous expectations or non-tradable assets. Our result is similar to that of Mayer [1978] who analyzes a discrete time model with nontraded assets under the assumption that returns are Normally distributed. There he is also able to make use of the fact that each consumer's first-order condition is linear and this permits aggregation over consumers.

We have shown that under the assumption of constant relative risk aversion, the relationship between *ex ante* mean returns and covariance may be replaced by an *ex post* (i.e. unconditional) relationship between these quantities. We are currently implementing empirically a test of the consumption correlatedness model based upon that result. We have also shown that the fundamental relationship between *ex ante* returns and covariances with consumption is true conditional on any subset of information which is common to all consumers, such as current interest rates, prices, money supply, etc. This should lead to interesting empirical tests of the model.

FOOTNOTES

1/ Note that equation (3) in Breeden permits labor income to be stochastic. However, in equation (7) only the deterministic component of labor affects the rate of change in wealth. That is, Breeden assumes that when the portfolio decision is made at time t, the investor can act as if he is locally certain about the size of his labor income. See Merton [1971] for an earlier model which explicitly used consumption.

2/ Indeed, we do not solve a continuous time optimization problem, but only take limits of discrete time problems. The continuous time solution which we derive should be interpreted as the approximate solution of a very, very short discrete time problem rather than as the solution to an unstated continuous time problem. See Merton [1978] for a good exposition about this technique.

3/ See Harrison and Kreps [1978] for another approach.

4/ Throughout this paper, we refer to the fact that excess returns are explained by covariance with consumption as "a consumption beta model." Of course this means the following. Let R_p be the rate of return on a portfolio which satisfies $\text{Cov}(R_p, \frac{dC}{C}) = \text{Var}(\frac{dC}{C})$. Let β_i be the slope of the regression of R_i on $\frac{dC}{C}$, so $\beta_i \equiv \text{Cov}\ (R_i, \frac{dC}{C}) \div \text{Var}(\frac{dC}{C})$. Let there be a risk-free asset, i.e. one with return R_o uncorrelated with consumption changes. Then (11) implies that $E(R_p - R_o) = A\ \text{Var}(\frac{dC}{C})$. Hence, for asset i, (11) implies $E(R_i - R_o) = \beta_i E(R_p - R_o)$. The next section deals explicitly with aggregation over consumers.

5/ See Ash [1972, p.260].

6/ That is, suppose

$$\sum_{\ell} \frac{C^*_{t\ell}}{C_t} \frac{1}{A_{t\ell}} = \left(\sum_{\ell} \frac{C^*_{t\ell}}{C_t}\right)\left(\sum_{\ell} \frac{1}{A_{t\ell}}\right) \equiv \sum_{\ell} \frac{1}{A_{t\ell}}$$

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