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ABSTRACT

The symmetry restriction in a system of financial asset demands has frequently been employed to reduce the number of independent parameters to be estimated. The theoretical implications of the symmetry restriction are examined in this paper, and it is found that symmetry implies a particular type of risk averse portfolio behavior. The symmetry restriction is also examined empirically, and the evidence supports symmetry only in cases where coefficients on cross-asset yields are insignificantly different from zero.

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SYMMETRY RESTRICTIONS IN A SYSTEM OF FINANCIAL ASSET DEMANDS: A THEORETICAL AND EMPIRICAL ANALYSIS

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Financial model builders have for some time sought restrictions in systems of financial asset demand equations to reduce the number of independent parameters to be estimated. One possible restriction is to specify a symmetric coefficient matrix on the N yields in the simultaneous system of N asset demand equations.1/ In the absence of the symmetry restriction, the coefficient matrix on the vector of yields has N(N-1) independent parameters after taking the wealth constraint into account. By imposing symmetry the number of independent parameters is reduced from N(N-1) to (N-1)(N+2)/2, or a reduction of (N-1)(N-2)/2 independent parameters in the system of equations.

The motivation for imposing the symmetry restriction appears to be based on consumer demand theory. In a linearized system of equations representing the demands for goods and services, however, the coefficient matrix on the vector of prices is not necessarily symmetric. In particular, this coefficient matrix may be represented as a Jacobian matrix with each element comprising both symmetric Slutsky substitution and generally asymmetric Slutsky income effects. The analogue in portfolio theory is a Jacobian matrix representing both substitution and wealth effects, where wealth now takes the place of income in the budget constraint. Thus, as in consumer demand theory, the imposition of the symmetry restriction represents a testable constraint that does not necessarily hold for all reasonable cases of utility maximizing behavior.

The object of this paper is to determine the implications of the symmetry restriction not only in terms of the wealth effect associated with Slutsky equations for financial asset demands, but more importantly in terms of investors' risk aversion. While others have recognized the similarity between systems of demand equations derived from consumer and portfolio theories,[2/] the implications of the symmetry property with respect to investors' risk aversion have not been examined. In this paper, the implications of the symmetry restriction are investigated using the mean-variance model of expected utility maximization, which is briefly reviewed in Section I. In Section II, propositions relating symmetric Jacobian (or coefficient) matrices with risk aversion are presented. The symmetry property is empirically tested in Section III using the disaggregated structural model of the U.S. Treasury securities market developed by Roley [24,25]. The main conclusions of this paper are summarized in the final section.

I. A Model of Financial Asset Demands

To investigate the implications of the symmetry restriction it is necessary to base the analysis on a formal model of portfolio choice. The model most often used to form empirical representations of financial asset demands is the discrete-time version of the mean-variance model of expected utility maximization.[3/] The principal advantage of this framework is that it reduces the number of parameters in the portfolio choice problem while retaining the essential risk-return trade-off fundamental to portfolio choice. Specific mean-variance models follow from simplifying assumptions such as the use of quadratic utility or joint-normally distributed asset holding-period yields.[4/] Another set of assumptions yielding mean-variance analysis is that portfolio rate of return is distributed lognormally and the utility function is logarithmic.[5/] However, the model will be considered here in its most general form which is in principle consistent with exact representations using any one of the above assumptions.

The one-period mean-variance expected utility maximization problem used to derive financial asset demands is

$$\max_{\underline{a}_t,\ \lambda} L = U[E_t(X_t),\ V_t(X_t)] + \lambda(W_t - \underline{a}_t' \underline{1}) \tag{1}$$

where

$U[\ ,\ ]$ = mean-variance expected utility function

$E_t(X_t)$ = expected value at time t of the argument of the utility function X_t

$V_t(X_t)$ = variance at time t of the argument of the utility function X_t

$\underline{a}_t$ = N×1 vector of the investor's holdings of assets in dollar amounts at time t ($\underline{a}_t' \underline{1} = W_t$), with typical element a_i (i = 1, ...,N)

W_t = the investor's total portfolio size (wealth) at the end of time t

X_t = the argument of the utility function defined as either W_t or r_{pt}, where r_{pt} equals portfolio rate of return at time t

$\underline{1}$ = N×1 vector with each element equal to unity

λ = Lagrangian multiplier

The argument of the utility function is most commonly defined as portfolio wealth (W_t), and this representation will be used in the derivations that follow. However, specifying the argument of the utility function in terms of portfolio rate of return leads directly to asset demands that are linear homogeneous in wealth which is often considered a particularly appropriate form for equations representing the behavior of aggregate categories of investors in a time series context.[6/] Furthermore, those relevant specifications of utility of end-of-period wealth which generate linear homogeneous asset demand equations may be equivalently described in terms of portfolio rate of return independent of initial wealth (that is, utility of end-of-period wealth under constant relative risk aversion). For either of the possible arguments of the utility function, the propositions relating risk aversion to the symmetry restriction are valid as long as the measures of risk aversion are appropriately defined in terms of the argument of the utility function and not strictly in terms of end-of-period wealth.

Using wealth as the argument of the utility function implies that the first-order conditions of the constrained maximization problem (1) may be written as [7/]

$$\begin{bmatrix} 2U_V\Sigma_t & -\underline{1} \\ -\underline{1}' & 0 \end{bmatrix} \begin{bmatrix} \underline{a}_t \\ \lambda \end{bmatrix} + \begin{bmatrix} U_E\underline{\mu}_t \\ W_t \end{bmatrix} = \underline{0} \qquad (2)$$

where

$\underline{\mu}_t$	=	N×1 vector of expected holding-period yields at time t independent of the vector of current market asset prices,[8/] with typical element μ_i (i=i,...,N)
Σ_t	=	N×N variance-covariance matrix of holding-period yields at time t, with typical element σ_{ij} (i,j=1,...,N)
U_E	=	marginal utility of the mean of end-of-period wealth
U_V	=	marginal utility of the variance of end-of-period wealth
$\underline{1}$	=	N×1 vector with each element equal to unity
$\underline{0}$	=	(N+1)×1 vector with each element equal to zero

The second-order condition—which is used later when considering Slutsky equations—is that the principal minors of the (N+1) × (N+1) bordered Hessian

$$H = \begin{bmatrix} \frac{\partial^2 L}{\partial \underline{a} \partial \underline{a}'} & \frac{\partial^2 L}{\partial \underline{a} \partial \lambda} \\ \frac{\partial^2 L}{\partial \lambda \partial \underline{a}'} & \frac{\partial^2 L}{\partial \lambda^2} \end{bmatrix} = \{hij\} \qquad (3)$$

alternate in sign starting positive.

The system of equations (2) is comprised of implicit functions of financial asset demands. At this stage, assumptions concerning the functional form of the utility function U may be made to allow the explicit representation of the vector of financial asset demands. Not all assumptions, however, lead to asset demands that are linear in expected holding-period yields.[9/] Thus, to empirically estimate the system of demand equations (2) it is useful to estimate a linearized version such as

$$\underline{a}_t = \underline{b} + B\underline{\mu}_t \qquad (4)$$

where (4) may be derived from a Taylor's expansion with all terms second order and higher deleted. Thus, in this sense the "coefficient" matrix B is simply the Jacobian matrix $\partial\underline{a}_t/\partial\underline{\mu}_t$. The relationship between the symmetry of this Jacobian matrix and measures or risk aversion is the topic of the next section.

II. The Symmetry Restriction

A sufficient condition for a symmetric Jacobian matrix $\partial\underline{a}/\partial\underline{\mu}$ may be obtained using the properties of the bordered Hessian (3). Totally differentiating the first-order conditions (2) with respect to $\underline{a}_t$, λ, and $\underline{\mu}_t$, and then inverting the bordered Hessian implies

$$\begin{bmatrix} da_1 \\ \cdot \\ \cdot \\ \cdot \\ da_N \\ d\lambda \end{bmatrix} = H^{-1} \begin{bmatrix} \sum_{j=1}^{N} -[U_{EE}\mu_1 a_j + U_{VE} \sum_{k=1}^{N} \sigma_{1k} a_k a_j] d\mu_j - U_E d\mu_1 \\ \cdot \\ \cdot \\ \sum_{j=1}^{N} -[U_{EE}\mu_N a_j + U_{VE} \sum_{k=1}^{N} \sigma_{Nk} a_k a_j] d\mu_j - U_E d\mu_N \\ 0 \end{bmatrix}$$

Thus, the partial derivative $\partial a_n / \partial \mu_m$ may be written as

$$\frac{\partial a_n}{\partial \mu_m} = -a_m \left[\sum_{j=1}^{N} h^{nj} (U_{EE}\mu_j + 2U_{VE} \sum_{k=1}^{N} \sigma_{nk} a_k) \right] - h^{nm} U_E$$

where

h^{nj} = the (n,j) element of H^{-1}

$U_{EE} = \partial^2 U / \partial E^2$

$U_{VE} = \partial^2 U / \partial V \partial E.$

Since the bordered Hessian is symmetric, its inverse is also symmetric. This further implies that if $U_{EE} = U_{VE} = 0$, the Jacobian matrix $\partial \underline{a} / \partial \underline{\mu}$ is symmetric. Following Royama and Hamada [26], the expression $-h^{nm}U_E$ is the substitution term S_{nm} in the Slutsky equation

$$\frac{\partial a_n}{\partial \mu_m} = S_{nm} + a_m \left[\frac{\partial a_n}{\partial E_t(W_t)} \right]_{d\underline{\mu}=0}$$

In this case, the Slutsky substitution term reflects the compensation to the investor enabling him to attain the same expected wealth with the same risk. Thus, the condition for a symmetric Jacobian matrix $\partial \underline{a} / \partial \underline{\mu}$ is that the wealth effects $\partial a_i / \partial E_t(W_t)$, i = 1,...N, equal zero. In the Appendix it is shown that $U_{EE} = U_{EV} = 0$ is also a necessary condition for symmetry. Thus, the relationship between restrictions on mean-variance expected utility functions and symmetry may be summarized by the following proposition:[10/]

Proposition 1: A necessary and sufficient condition for a symmetric Jacobian matrix $\partial \underline{a}/\partial \underline{\mu}$ is $U_{EE} = U_{EV} = 0$

This necessary and sufficient condition for symmetry may be related directly to a measure of risk aversion based on the mean-variance concept first presented by Tobin [29]. Tobin's measure of risk aversion concerns the characteristics of indifference curves relating risk and return. Following Tobin [29], an investor is risk averse if his indifference curves have positive slopes implying

$$R = dE/dV = -U_V/U_E > 0.$$

A risk averse investor is a diversifier if his indifference curves are concave to the E axis implying

$$-U_{VV}U_E + U_VU_{EV} > 0$$

and a plunger if the opposite holds. This mean-variance measure of risk aversion was related to the Pratt [23]-Arrow [2] concept by Adler [1] and Miller [21], and it is this form that is used here. In particular, following Miller [21], an investor exhibits increasing/constant/decreasing absolute mean-variance risk aversion with respect to $E_t(W_t)$ according to[11/]

$$\frac{\partial R}{\partial E} = \frac{-U_{VE}U_E + U_VU_{EE}}{(U_E)^2} \gtreqless 0. \qquad (5)$$

Similarly, an investor exhibits increasing/constant/decreasing absolute mean-variance risk aversion with respect to $V_t(W_t)$ according to

$$\frac{\partial R}{\partial V} = \frac{-U_{VV}U_E + U_V U_{VE}}{(U_E)^2} \gtreqless 0. \qquad (6)$$

It was further demonstrated by Miller [21] that a sufficient condition for constant mean-variance risk aversion with respect to $E_t(W_t)$ is that the Pratt [23]-Arrow [2] measure of absolute risk aversion is constant.

From (5) and (6), it is apparent that the necessary and sufficient condition for symmetry in Proposition 1 also satisfies the condition for constant absolute mean-variance risk aversion with respect to $E_t(W_t)$ but not $V_t(W_t)$. Again, this result may be summarized by the following proposition:

> <u>Proposition 2</u>: If $U_{EE} = U_{EV} = 0$, then investors exhibit constant mean-variance risk aversion with respect to $E_t(W_t)$.

Propositions 1 and 2 together imply:

> <u>Proposition 3</u>: If the Jacobian matrix $\partial \underline{a}/\partial \underline{\mu}$ is symmetric, then investors exhibit constant mean-variance risk aversion with respect to $E_t(W_t)$.

The assumption that the coefficient matrix in (4) is symmetric therefore has a distinct behavioral implication in terms of investors' risk aversion. Thus, the symmetry restriction is not a general property and it most notably does not apply to quadratic utility. However, the application of negative exponential utility together with joint normally distributed yields—a model that has been employed frequently because of the empirical tractability of the implied asset demands—does lead to a symmetric coefficient matrix. Other expected utility models that exhibit constant mean-variance risk aversion with respect to $E_t(W_t)$—e.g., $U = (E-V)^2$—do not necessarily imply this result. Thus, in the absence of strong a priori knowledge concerning investors' risk aversion, the symmetry restriction should always be tested when estimating a system of financial asset demands.

III. An Empirical Test of the Symmetry Restriction

The model of the U.S. Treasury securities market developed by Roley [24,25] serves to illustrate in one relevant case the appropriateness of the symmetry restriction. The model itself consists of the demand for two distinct maturity classes of Treasury securities by 10 categories of investors. Because the model represents the behavior of aggregated categories of investors in a time series context, the financial asset demands of each investor category are specified using the linear homogeneous version of (4) which may be obtained by setting $X_t = r_{pt}$ in (1).[12/] It follows that the results in the previous section are still applicable as long as they are interpreted in terms of portfolio rate of return. Rewriting the system of demands (4) to conform with portfolio behavior based on portfolio rate of return implies[13/]

$$\underline{\alpha}^*_t = \underline{b} + B\underline{\mu}_t \qquad (4')$$

where

$\underline{\alpha}^*_t$ = N×1 vector of investors' desired proportional holdings of assets at time t ($\underline{\alpha}^*_t = \underline{a}^*_t/W_t$).

As is common in models of financial asset demands, it is also assumed that portfolio adjustment to desired asset holdings is not instantaneous (or at least not complete within the periodicity of the data) due to transactions costs. To model this feature of portfolio behavior, Friedman's [11] "optimal marginal adjustment" model is applied to (4') to form the final representation of the model:

$$\Delta a_{it} = \sum_{k=1}^{N} \theta_{ik}(\alpha^*_{kt}W_{t-1} - a_{k,t-1}) + \alpha^*_{it}\Delta W_t, \quad i=1,\ldots,N \qquad (7)$$

where the parameters θ_{ik} $(i,k = 1,\ldots,N)$ as well as the $\underline{b}$ vector and the B matrix satisfy the adding-up properties specified by Brainard and Tobin [6].

The explicit substitution of the expression for desired proportional holdings of assets (4´) into (7) implies that the coefficient on each $(\mu_{kt}\Delta W_t)$ term consists of the single parameter b_{ik} which is the (i,k) element of B. Thus, the parameters of the B matrix are identified and the symmetry restriction may be tested directly in terms of the Jacobian matrix $\partial\underline{\alpha}^*_t/\partial\underline{\mu}_t$. Other terms in the final specification of the model typically have coefficients consisting of sums of products of parameters thereby excluding the use of these coefficients in the test of the symmetry restriction.

The equation presented in (7) representing the demand for financial assets is applied to six of the 10 investor categories, with the remaining four investor categories specified using somewhat more complicated adjustment models that do not allow the identification of the B matrix.[14/] The equations are estimated with the Federal Reserve's seasonally adjusted quarterly flow-of-funds data [4], with the sample period beginning in 1960:Q1 and ending in 1975:Q4.[15/] The available data for disaggregated U.S. Treasury securities holdings consist of four weighted maturity classes defined in terms of four "definite" areas and three "borderline" areas. The definite areas include securities with maturities of (1) within 1 year (short-term), (2) 2 to 4 years (short-intermediate-term), (3) 6 to 8 years (long-intermediate-term), and (4) over 12 years (long-term). Securities with maturities in the borderline areas are allocated to the definite classifications according to a weighting scheme.[16/] The two maturity classes of Treasury securities chosen for empirical estimation consist of the short-intermediate-term and long-term classifications. The corresponding own-yields are the Federal Reserve's published yield series on "3- to 5-year" U.S. Treasury securities for the short-intermediate-term maturity class, and the Federal Reserve's published yield series on "long-term" U.S. Treasury securities for the long-term maturity class. The coefficients on the $\mu_{kt}\Delta W_t$ terms involving these two yields are those used to test the symmetry restriction.

Because of the simultaneous nature of the financial asset demands—due to the endogeneity of the own-yields—the equations are estimated using a simultaneous equations technique. In particular, a limited information instrumental variables technique described by Brundy and Jorgenson [7] is used to obtain consistent estimates for the structural equations.[17/] Using this estimation procedure, a summary of the estimation results pertaining to the six categories of investors considered here are presented in Table 1. As indicated by the multiple correlations ($\bar{R}^2$), these equations explain much of the variation of the volatile net purchases of Treasury securities. The root-mean-square errors of the estimated equations are additionally reported to indicate the accuracy of the estimated equations in dollar amounts. The root-mean-square errors are computed by replacing the variables being instrumented with their historical values, and simulating using historical values in each period. The Durbin-Watson statistic is also reported for each equation as well as the estimates of the identifiable parameters on the terms involving the short-intermediate-term ($r_S\Delta W_t$) and long-term ($r_L\Delta W_t$) Treasury security

Table 1

SUMMARY OF ESTIMATION RESULTS FOR TREASURY SECURITY DEMAND EQUATIONS

Investor Category		$\bar{R}^2$	RMSE	DW	Estimated Coefficients: Own Yield (r_S or r_L)	Estimated Coefficients: Competing Yield (r_S or r_L)
Life Insurance Companies	$\Delta US2^L$	.82	19	2.16	.0299 (5.1)	-.0206 (-4.3)
	$\Delta US4^L$	.92	35	2.42	.0136 (4.0)	--
Mutual Savings Banks	$\Delta US2^M$	.69	47	1.90	.0746 (2.5)	-.0346 (-1.4)
	$\Delta US4^M$	.71	47	1.75	.0039 (1.6)	--
Other Insurance Companies	$\Delta US2^O$	.54	53	1.86	.1107 (4.1)	--
	$\Delta US4^O$	.65	22	2.35	.0158 (2.2)	--
Private Pension Funds	$\Delta US2^P$	.75	42	2.21	.0239 (2.5)	--
	$\Delta US4^P$	.80	35	2.17	.0158 (2.9)	--
Savings and Loan Associations	$\Delta US2^{SL}$	.57	146	1.89	.0732 (2.7)	--
	$\Delta US4^{SL}$	.67	47	2.39	.0023 (1.7)	--
State-Local Retirement Funds	$\Delta US2^S$	.40	24	1.76	.0091 (2.6)	--
	$\Delta US4^S$	.63	104	2.02	.1912 (4.2)	-.1083 (-3.2)

Notes: ΔUS2 and ΔUS4 are net purchases of short-intermediate-term and long-term Treasury securities, respectively. Numbers in parentheses are ratios of estimated coefficients to their standard errors (t-ratios). RMSE in millions of dollars.

yields when they are statistically significant in the preliminary OLS results.

Since a complete set of asset demands is not estimated for the individual investor categories, the test of the symmetry restriction is only applied to the pair of coefficients on cross-maturity class yields. When these terms are not present in the specifications listed in Table 1, they are included for the purposes of the test. The procedure utilized for the test involves stacking the two demand equations for each investor category, and estimating the stacked equations both with and without the symmetry contraint.[18/] Under the null hypothesis of symmetry, the test statistic is asymptotically distributed as χ^2 with one degree of freedom. This is the proper framework for the test due to the two-stage technique employed for the estimation of the structural equations. However, it may be desirable because of the small sample properties of the test statistic to use the F-distribution.[19/] Either test may be performed on the basis of the values given in Table 2.

The test statistics reported in Table 2 indicate that at the 10 per cent level of significance the null hypothesis is rejected for three investor categories (life insurance companies, other insurance companies, and state and local government retirement funds) and it cannot be rejected for the remaining three investor groups. These results hold for either the χ^2- or F-test. At the 5 per cent level of significance, the only change occurs for other insurance companies under the F-distribution.

The estimated coefficients on the cross-maturity class terms with the symmetry constraint imposed are given in the last column of Table 2. The

Table 2

A TEST FOR SYMMETRY IN THE COEFFICIENT MATRIX ON THE VECTOR OF ASSET YIELDS

Investor Category	Test Statistic	Degrees of Freedom (F-test)	Estimated Cross Coefficient with Symmetry Imposed
Life Insurance Companies	14.70*	1,105	0.008 (3.1)
Mutual Savings Banks	1.07	1,107	-0.007 (-0.6)
Other Insurance Companies	3.87*	1.106	0.006 (0.4)
Private Pension Funds	1.06	1,106	0.010 (1.5)
Savings and Loan Associations	0.09	1,103	-0.004 (-0.3)
State-Local Retirement Funds	13.44*	1,110	-0.035 (-2.4)

Notes: Numbers in parentheses are ratios of estimated values to standard errors (t-ratios).
*Significant at the 5 per cent level under the χ^2 test.

t-ratios for the estimated coefficients indicate that for the three investor groups in which symmetry cannot be rejected the relevant coefficients are statistically insignificant. Thus, symmetry in this case takes the form of zero off-diagonal elements corresponding to the yields on short-intermediate-term and long-term U.S. Treasury securities. On the basis of the tests reported above, the evidence appears to favor rejection of the symmetry hypothesis. However, tests concerning this hypothesis would benefit from the inclusion of a larger subset of an individual investor category's asset demands.

IV. Summary and Conclusions

The symmetry restriction in a system of financial asset demands has frequently been employed to reduce the number of independent parameters to be estimated. Despite the usefulness of this constraint in empirical applications, the symmetry restriction imposes a behavioral assumption on the model which may not conform to actual portfolio behavior. In particular, a system of financial asset demands with a symmetric coefficient matrix implies that investors exhibit constant mean-variance risk aversion with respect to the mean of the argument of the utility function. Thus, the symmetry restriction is a testable hypothesis concerning not only the properties of the coefficient matrix itself, but also a particular type of risk averse behavior.

The symmetry restriction was tested in the context of a disaggregated structural model of the U.S. Treasury securities market, where the demands for two different maturity classes of Treasury securities are estimated for 10 different investor categories. The test revealed that the symmetry restriction could not be rejected only in cases where the off-diagonal elements of the constrained coefficient matrix are statistically insignificant. The evidence from this test—which involves only one constraint across two equations—suggests that tests are in order whenever the symmetry restriction is imposed.

Appendix

To demonstrate that $U_{EE} = U_{VE} = 0$ is a necessary condition for a symmetric Jacobian matrix $\partial \underline{a}/\partial \underline{\mu}$, it is convenient to use the expression for the wealth constraint to substitute for the N^{th} asset (a_N). By suitably partitioning the vector of assets $\underline{a}$ and the variance-covariance matrix Σ, this procedure implies that the mean and variance of end-of-period wealth may be represented as

$$E = (\hat{\underline{\mu}} - \mu_N \underline{1})'\hat{\underline{a}} + (1 + \mu_N)\, W$$

$$V = \hat{\underline{a}}' \, [\hat{\Sigma} - \underline{1}\,\hat{\underline{\sigma}}' - \hat{\underline{\sigma}}\underline{1}' + \sigma_{NN}\, \underline{1}\,\underline{1}']\, \hat{\underline{a}} + 2W\, [\hat{\underline{\sigma}}' - \sigma_{NN}\, \underline{1}']\, \hat{\underline{a}} + W^2 \sigma_{NN}$$

where

$$\underline{a} = \begin{bmatrix} \hat{\underline{a}} \\ a_N \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} \hat{\Sigma} & \hat{\underline{\sigma}} \\ \hat{\underline{\sigma}}' & \sigma_{NN} \end{bmatrix}$$

$$\underline{\mu} = \begin{bmatrix} \hat{\underline{\mu}} \\ \mu_N \end{bmatrix}$$

and the time subscript t has been deleted. The maximization problem then becomes

$$\max_{\hat{\underline{a}}} U(E,V).$$

The first-order conditions of this maximization problem are

$$U_E \underline{\mu}^* + 2U_V \Sigma^* \hat{\underline{a}} + 2U_V W \underline{\sigma}^* = \underline{0} \qquad (A1)$$

where

$$\underline{\mu}^* = (\hat{\underline{\mu}} - \mu_N \underline{1})$$

$$\Sigma^* = [\hat{\Sigma} - \underline{1}\,\hat{\underline{\sigma}}' - \hat{\underline{\sigma}}\,\underline{1}' + \sigma_{NN}\,\underline{1}\,\underline{1}']$$

$$\underline{\sigma}^* = \hat{\underline{\sigma}} - \sigma_{NN}\,\underline{1}\,.$$

The Jacobian matrix $\partial \underline{a}/\partial \underline{\mu}$ has zero column sums independent of the utility function. Symmetry therefore implies that this matrix also has zero row sums. In turn, if this matrix has zero row sums, then

$$\frac{\partial \hat{\underline{a}}}{\partial \delta} = \underline{0}$$

where δ is a nonstochastic scalar added to the vector of expected holding-period yields $\underline{\mu}$; i.e., $\underline{\mu} + \delta \underline{1}$. (See Jones [17].) Differentiating the first-order conditions (A1) with respect to δ yields

$$\frac{\partial \hat{\underline{a}}}{\partial \delta} = [U_{EE}\underline{\mu}^*\underline{\mu}^{*\prime} + 2U_{VE}\Sigma^*\hat{\underline{a}}\,\underline{\mu}^{*\prime} + 2U_{VE}W\underline{\sigma}^*\underline{\mu}^{*\prime} + 2U_V\Sigma^*]^{-1} \times (-W)[U_{EE}\underline{\mu}^* + 2U_{VE}\Sigma^*\hat{\underline{a}} + 2U_{VE}W\underline{\sigma}^*]. \qquad (A2)$$

It is apparent that a necessary condition for equation (A2) to equal the zero vector for all joint probability distributions is that $U_{EE} = U_{EV} = 0$. Thus, $U_{EE} = U_{EV} = 0$ is a necessary condition for symmetry.

Footnotes

*Senior Economist, Federal Reserve Bank of Kansas City. This paper is based on my Ph.D. dissertation [24], which benefited from the generous research support of the National Bureau of Economic Research and the National Science Foundation. I am grateful to Richard Abrams, Benjamin Friedman, Orlin Grabbe, Angelo Melino, and especially David Jones and Gordon Sellon for helpful comments on an earlier draft of this paper. The views expressed here are solely my own and do not necessarily represent the views of the Federal Reserve Bank of Kansas City or the Federal Reserve System.

1. See, for example, Parkin [22], Gramlich and Kalchbrenner [14], Gramlich and Hulett [13], Courakis [8], Melton [20], and Hendershott [16].

2. See, for example, the analyses of Slutsky equations for financial assets presented by Royama and Hamada [26] and Bierwag and Grove [3].

3. See, for example, Parkin [22], Gramlich and Hulett [13], Courakis [8], Melton [20], and Hendershott [16].

4. See, for example, Borch [5], Feldstein [10], and Tobin [30].

5. If asset holding-period yields are lognormally distributed, however, then it is well known that a linear combination of these holding-period yields is not lognormally distributed. Lintner [19] has investigated this case and has developed an approximation, based on simulation experiments, which is relevant to this problem.

6. For examples of the use of the linear homogeneous form, see DeLeeuw [9], Brainard and Tobin [6], Hendershott [15], and Friedman [11].

7. Using portfolio rate of return instead of end-of-period wealth as the argument of the utility function results in $\underline{\alpha}_t = (\underline{a}_t/W_t)$ replacing $\underline{a}_t$ and 1 replacing W_t in (2). Also, the marginal utilities (U_E, U_V) in this case are in terms of portfolio rate of return.

8. It is assumed throughout that market prices of securities remain constant and expected holding-period yields change only in response to changes in expected returns. This assumption is often adopted implicitly by those considering models of portfolio behavior. For an example of a distinct representation of market prices and expected returns, see Lintner [18].

9. The most prominent example is the quadratic utility case. For the derivation of the system of financial asset demands under the assumption of quadratic utility, see Royama and Hamada [26] and Roley [24].

10. This result also follows when investors are explicitly assumed to have initial endowments of individual securities and they select quantities of assets (e.g., the number of bonds) based on expected and current market

prices. In this case, contrary to Hendershott [16], the Jacobian matrix $\partial \underline{q}/\partial \underline{P}$ is not in general symmetric, where $\underline{q}$ and $\underline{P}$ represent the quantities and market prices of assets, respectively. For an analysis of the Slutsky equations under these assumptions, see Bierwag and Grove [3].

11. Again, these measures of risk aversion may be expressed in general terms using $E_t(X_t)$ and $V_t(X_t)$.

12. As discussed previously, other assumptions may be used to generate asset demands that are linear homogeneous in wealth. See, for example, Friedman and Roley [12].

13. In the empirical implementation of (4′), the common assumption involving the stationarity of the variance-covariance matrix Σ_t is also employed. For examples of others either explicitly or implicitly using this assumption, see the references in footnote 3.

14. However, the adjustment models used in the specifications of the other four investor categories contain the optimal marginal adjustment model as a special case.

15. For other details concerning the data and the complete specification of the model, see Roley [24,25].

16. For a further discussion of the weighted maturity class data, see Taylor and Wood [27].

17. This instrumental variables procedure involves replacing current values of dependent variables appearing in the right-hand side of the structural equations with fitted values obtained from first-stage regressions. The first-stage regression for an individual structural equation has right-hand side variables consisting of a subset of the principal components of the entire set of predetermined variables in the system of equations, augmented by the set of predetermined variables appearing in the individual structural equation. In addition, since the dependent variables being instrumented appear as products with either wealth flows or stocks, the proper procedure is to instrument the entire multiplicative term.

18. In addition, assumptions concerning the error terms—including zero contemporaneous correlation and identical variances for each equation—are employed to simplify the testing procedure. This contemporaneous correlation assumption is consistent with the limited information methodology used in the estimation of the system of structural equations.

19. For comments on a similar problem in statistical hypothesis testing, see Theil [28], pp. 312-16, and pp. 402-3.

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