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ENERGY AND GROWTH UNDER FLEXIBLE  
EXCHANGE RATES: A SIMULATION STUDY

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ABSTRACT

This paper offers a theoretical framework for studying the interactions of energy prices and economic growth. The incorporation of energy prices and quantities in a macroeconomic setting focuses on (1) the aggregate technology; (2) the interdependence of energy producers and consumers in the world economy; and (3) the asset markets as the channel through which energy price changes affect output and capital accumulation. While several existing studies consider aspects of these issues, none provides a synthesis. In this analysis, a theoretically sound model of an oil price increase in the world economy is presented, carefully treating topics (1) - (3). The model is solved with computer simulation, as it is far too complex to yield analytical solutions.

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# ENERGY AND GROWTH UNDER FLEXIBLE EXCHANGE RATES:

## A SIMULATION STUDY

### 1. INTRODUCTION

The macroeconomic events of recent years have placed a very heavy strain on existing models of the macroeconomy. The dramatic rise in real energy prices, in particular, requires re-thinking of current models along a number of lines. First, higher energy prices raise new issues for aggregate supply in the short- and long-run. Energy price increases induce a short-run production decline for given capital stock, as well as a decline in net capital accumulation and long-run capital intensity. Second, the energy price increases raise a host of international economic issues. Worldwide interest rates and global production shift when increases in energy prices transfer income to the high-saving OPEC region. The repercussions of the global shifts must be studied in a global context. Finally, energy price increases interact with asset market prices in important ways. Increased attention must be paid to the effect of flexible exchange rates in reacting to, and facilitating adjustment to higher energy prices.

Existing theoretical and econometric models are deficient in addressing these important effects. Progress has been made in each area, but an adequate synthesis of results is wanting. An important reason for this limitation is that theoretical models that adequately treat the supply side, capital accumulation, international repercussion effects, and asset markets, are too complex to solve analytically. The strategy in this paper is to present a theoretically sound model of an oil price increase under flexible exchange rates,

and to solve the model by taking refuge in computer simulation. The goal of this paper is methodological -- to demonstrate the feasibility of an integrated approach to growth and energy -- rather than empirical. No attempt is made here to calibrate precisely the simulation model, though such an attempt is now underway.

Let us consider each of the three problem areas. Economists have increasingly recognized the need to integrate the production technology into models of aggregate output (cf. [5], [10], [16]), and to explore the effects of alternative technological assumptions on adjustment. There is still, however, an inadequate integration of short-run adjustment and long-run growth in most applied macroeconomic analyses, and with respect to energy the deficiency is particularly acute. While demand or monetary disturbances may influence short-run output with little effect on the growth path, energy prices by nature affect both short-run and long-run output, the latter through the effects on capital accumulation and savings. As higher energy prices reduce the profitability of investment, the long-term effect of higher energy prices is a reduction in capital intensity of production. The long-run decline is reflected immediately in the short-run investment rate.

Equity markets provide the link between the long-run capital intensity of production and the short-run investment decisions of firms. In the simulation model, the investment function is built upon Tobin's [24] insight that value-maximizing firms will increase their investment rate when the valuation of equity claims to their capital rises relative to the replacement cost of capital. The ratio of these two prices is widely known as Tobin's  $q$ ; the link between investment and  $q$  will be formally justified. Because  $q$  is itself

a function of the expected stream of future capital earnings, a decline in anticipated earnings causes a fall in current  $q$  and current investment. We shall see that an increase in energy prices depresses  $q$  and causes a sharp fall in capital accumulation. The extent of the fall depends on the nature of technology.

The second concern for sound modelling of OPEC disturbances is that worldwide variables will change with a worldwide shock. A "small economy" analysis that takes world interest rates and income as given following a price shock is partial at best, and probably misleading. Similarly, we cannot very well study the post-shock exchange rate between two oil-importing economies by studying a model of only one economy. Yet various authors have argued that the exchange rate of a non-OPEC economy vis-a-vis other such economies will move in a particular direction after an OPEC price shock, even though the conclusion cannot be true for all such economies. In our simulation study, we include two growing, industrialized economies in addition to OPEC, so that we may explicitly analyze movements of their bilateral exchange rate.

The multilateral aspect of the model also permits us to address the important issues of policy co-ordination and repercussion effects between large, open economies. We reach some surprising conclusions about the transmission of policies across national boundaries. In particular, a fiscal expansion may well be beggar-thy-neighbour if real wages are sufficiently rigid in the industrial economies.

Our third concern is the proper treatment of asset market prices in a study of the OPEC shocks. The recent advances in "efficient market theory" of exchange rates, equity prices, and the commodity price level demonstrate that asset prices aggregate market

expectations of future economic developments. I have mentioned  $q$  in this context. Similarly, the current bilateral exchange rate can be written as a discounted stream of the relative money stocks in the two economies (cf. [17]), and the price level is tied to the discounted stream of expected nominal money services in a single economy (see [2]). Because asset prices translate expected future economic developments into current decisions, it is now well appreciated that a sound model of expectations is crucial. Therefore, I adopt the perfect foresight assumption throughout the simulation model. This approach has several advantages: (1) it avoids the "Lucas critique" by making explicit how agents' behavior will change with shifts in policy regime; (2) it rules out the possibility of systematically profitable arbitrage given known market conditions; and (3) it is the appropriate base case for optimizing agents in a non-stochastic environment.

As I will indicate, the perfect foresight assumption raises interesting methodological issues for simulation. Because equity prices, the exchange rate, and the consumption price level are valued according to the entire future path of various endogenous and exogenous variables, it is difficult to find the initial conditions for these prices. Technically, the models pose two-point-boundary-value problems, for which final but not initial asset prices are known. Elsewhere, I have helped to implement the method of "multiple-shooting," common in certain physical sciences, for the solution of economic simulations (see [12]). This method, I will show, is very powerful in solving the large two-country growth model in this paper.

A complete description of a related two-country growth model, but, without energy inputs or money balances, may be found in [13]. At various points, I will refer to that study for detailed discussion. A similar two-country model of trade, with perfect foresight but without energy and growth is Obstfeld [19]. In this paper, the general equilibrium model is set forth in Section 2, some analytical comparative static results are described in Section 3. In Section 4, the full dynamic effects of an OPEC shock on output and growth are studied through simulation under a variety of labor market assumptions. We will reconfirm in a general equilibrium setting the results of an earlier study by this author, on the importance of the labor market setting for adjustments to the oil shock. Furthermore, we will illustrate how alternative technological assumptions alter the growth paths of the economies. Importantly, we will analyze the transmission of macroeconomic policies across national borders. Conclusions and extensions to the model are discussed in Section 5.

## 2. A MODEL OF ENERGY, FLEXIBLE EXCHANGE RATES, AND TWO-COUNTRY GROWTH

We consider two growing industrial economies, linked through international commodity and financial markets. Each economy produces a single output, which it consumes, sells abroad for consumption, and uses for domestic capital formation. The domestic and foreign goods are imperfect consumption substitutes, with each household's consumption bundle depending on the relative price of the two goods. Each country uses only its own output for capital formation.

Households save in the form of home money balances and domestic and foreign equity claims to capital. We assume that each country's money balances yield transaction services to the country's agents.

These services are represented by the inclusion of real money balances in the household utility function. Foreign money balances yield no utility. Implicitly international transactions are mediated in equity rather than money (see [19] for a similar approach). Equity claims are valued for the stream of real income attached to their ownership. With no transactions costs in international financial markets and perfect foresight instantaneous holding yields of home and foreign equity are always equalized. We abstract from outside interest-bearing nominal securities, such as government bonds.

Current production uses labor inputs, energy, and the existing stock of reproducible capital. Households supply labor inelastically to the labor market, and competitive firms hire labor to the point where the marginal product of labor equals the wage. Short-run rigidities in nominal or real wages may induce labor market disequilibrium and temporary, involuntary unemployment (in [13] we reinterpret the employment fluctuations as movements along the household's (intertemporal) labor-leisure trade off). Firms similarly purchase energy inputs according to profit-maximizing criteria. Investment over time in reproducible capital proceeds according to the value-maximizing program of the firm.

Energy requirements are satisfied through imports from OPEC. OPEC in turn uses its oil revenues to consume the output of the two developed economies, and to accumulate equity claims in the two economies for later consumption. We abstract from OPEC's own development strategy, and indeed from the nature of OPEC's pricing decision itself. The real price of oil, in terms of OPEC's consumption bundle, is exogenous.



Equilibrium requires that output supply and demand balance, that the ex ante yields of home and foreign equities be the same, and that the home and foreign demands for outside money balances equal their respective supplies. Loosely speaking, the world real rate of return adjusts to balance global demand and supply, while the terms of trade (or real exchange rate) shifts demand between the two economies to balance each country's output market. Home and foreign equity prices adjust to equate yields on equity claims today and in the future. Finally, the foreign and home price levels adjust to equate money supplies and demands. The model as written solves for home ( $P$ ) and foreign price levels ( $P^*$ ), and the terms of trade ( $\Pi = P/EP^*$ ), (where  $E$  is the exchange rate, in units of home currency per unit of foreign currency, and  $*$  indicates "foreign"). The nominal exchange rate is given simply at  $E = (P^*/P) \cdot (1/\Pi)$ . Obviously, the model can be re-written to solve for  $E$ , one price level, and the terms of trade, with the other price level determined residually.

It is important to remember that equilibrium in this model is a full, intertemporal Nash equilibrium as characterized in Brock [2]. Agents make current decisions based upon their anticipations of the entire future paths of prices, which they take as given. By the perfect foresight assumption, all agents' anticipations must be the same, and must be equal to the prices that actually unfold over time (barring future, unanticipated shifts in exogenous variables). Thus, to solve for  $q_0$ ,  $q_0^*$ ,  $E_0$ ,  $P_0$ , and  $P_0^*$  at the initial time, we must solve for sequences  $\{q_i\}$ ,  $\{q_i^*\}$ ,  $\{E_i\}$ ,  $\{P_i\}$  and  $\{P_i^*\}$ ,  $i=0, 1, \dots, \infty$ . By examining the intertemporal maximization problems of households and firms, the dependence of today's action on all future prices can be made clear.

In setting forth the model, I will present the home country equations only, with the understanding that comparable equations exist for the foreign economy. The entire model, for both countries and OPEC is presented in Table 1, p. 20. Aside from the wage equation and the OPEC consumption equation, the model is written in continuous time. All equations are discretized for the simulations.

#### A. The Household

On the household side, we postulate a Sidrauski-Brock infinitely lived and growing household, which maximizes an additively separable intertemporal utility function in goods and real money balances:

$$\int_0^{\infty} e^{-\delta t} L_F U(C, C^M, M/P) dt.$$
  $\delta$  is the rate of pure time discount, and  $L_F$  is the number of household members at any time (with  $\dot{L}_F/L_F = n$ ).  $C^M$  is per capita domestic consumption of the foreign final good.

(All quantity variables will be written in intensive form, per unit of  $L_F$ , unless otherwise noted). There is no utility to leisure, so that notional labor supply is  $L_F$ . Since labor markets do not necessarily clear instantaneously, however, we may have total manhours  $MH < L_F$ . We define  $L = MH/L_F$  as the employment rate, and  $U = 1-L$  as the unemployment rate. Households have full knowledge of the rationing in the labor market, and they optimize according to their labor market constraints. For simplicity, we assume that rationing is uniform across agents, and appears in the form of reduced hours. Since  $L$  does not directly enter  $U(\cdot)$ , there are no goods-market spillover effects of labor rationing, aside from the direct effects caused by a reduction in human wealth.

Let  $A^M$  represent the households' per capita stock of equity wealth in units of the home good, and  $r$  be the instantaneous yield

on equity claims. The familiar intertemporal budget constraint is:

$$(1) \quad \dot{A}^M = rA^M + (1-\tau)\frac{W}{P}L - C - C^M/\Pi - nA^M.$$

where  $\tau$  is the rate of labor taxation (there are no other taxes). We assume that per capita government expenditure on final consumption always adjusts to keep  $G = \tau\frac{W}{P}L$ . In addition to direct expenditure, the government makes lump-sum transfer payments to households, financed by money creation:  $T = (\dot{M} + nM)/P$ . The discounted value of all future transfers,  $W^T$ , may be written as

$$\int_0^{\infty} e^{-\int_0^t (r-n)ds} [(\dot{M} + nM)/P] dt.$$

Integration by parts reveals in the Cobb-Douglas case that  $W^T$  is given simply by  $(i/(\delta-n) - 1)(M/P) \cdot \frac{1}{\delta-n}$

Now, let  $A$  be total per capita wealth, with

$$(2) \quad A = A^M + \frac{M}{P} + H + W^T$$

with

$$H = \int_0^{\infty} e^{-\int_0^t (r-n)ds} (1-\tau)\left(\frac{W}{P}\right)L dt \quad (\text{Human Wealth})$$

$$W^T = \int_0^{\infty} e^{-\int_0^t (r-n)ds} T dt \quad (\text{Present Value of Transfers})$$

Using (1) and (2), and the definitions of  $H$  and  $W^T$ , we can re-write the budget constraint as:

$$(3) \quad \dot{A} = (r-n)A - [C + C^M/\Pi + (r+\dot{P}/P)M/P]$$

We now see that "full" consumption  $C_F$  in any period is equal to goods consumption, plus the opportunity cost of real balances. The

shadow price of real balances is the nominal interest rate  $i = r + \dot{P}/P$ , which represents the income foregone in holding wealth in the form of money balances.

We may specialize further, by writing  $U(\cdot)$  as a constant-relative-risk aversion function of a linear homogeneous function  $\psi$  of  $C$ ,  $C^M$  and  $M/P$

$$(4) \quad U(\cdot) = \frac{[\psi(C, C^M, M/P)]^{1-\sigma}}{1-\sigma} \quad \sigma \neq 1$$

$$= \log[\psi(C, C^M, M/P)] \quad \sigma = 1$$

(for details see [13]; for a similar approach, see [19]). Now, it is easy to show that intertemporal optimization makes

$$(5) \quad \sigma(\dot{\Psi}/\Psi) = (r - \delta - \dot{P}_\Psi/P_\Psi)$$

where  $P_\Psi$  is a true price index for  $\Psi$ . <sup>2/</sup> Note that  $C_F = P_\Psi \psi$ .

In this study, we proceed with the Cobb-Douglas case for  $\Psi$  and  $U(\cdot)$ :  $\psi = C^{\alpha(1-s)} (C^M)^{(1-\alpha)(1-s)} (M/P)^s$ , and  $\sigma = 1$ . In this case, consumption expenditures on the three goods in each period is a fixed proportion of  $C_F$ , and full consumption  $C_F$  is linear in wealth:

$$(6) \quad (a) \quad C_F = (\delta-n)A$$

$$(b) \quad C = \alpha(1-s)C_F$$

$$(c) \quad C^M = (1-\alpha)(1-s)C_F/\Pi$$

$$(d) \quad i(M/P) = sC_F$$

Very importantly, (6)(d) is the utility-maximizing money demand schedule. Note finally that by the value of  $W^T$  shown above, total wealth is

$$(7) \quad A = [i/(\delta-n)] \cdot (M/P) + A^M + H.$$

## B. The Firm

Under certainty and perfect competition, the strategy of value maximization for the firm is unanimously agreed upon by shareholders.

We follow Hayashi's [8] argument that such a strategy, under technical conditions now described, leads to the investment rule

$$(8) \quad J = J(q) \cdot K$$

where  $J$  is gross (per capita) capital formation, and  $K$  is the pre-existing capital stock (per capita).  $q$  is, again, the price of equity relative to the price of physical capital, and the latter in this model is just equal to the price of output. The investment schedule arises from an assumed cost of adjustment to new investment which makes the marginal cost of investment rise with the investment rate  $J/K$ .

To be specific, suppose that total investment expenditure  $I$  equals  $J \cdot [1 + \phi(J/K)]$  where  $\phi(\cdot)$  is the per unit adjustment cost,  $\phi' > 0$ . Since the market values new capital according to the real equity price  $q$ , the firm's instantaneous change in value due to capital formation  $J$  is simply:

$$(9) \quad dV = qJ - J \cdot [1 + \phi(J/K)] .$$

If (9) is maximized with respect to  $J$ , we see that the firm equates  $q$  with  $1 + d/dJ[(J \cdot \phi)]$ , the marginal adjustment cost. Equation (8) is then simply derived. In the specific and useful case of  $\phi(J/K) = (\phi_0/2) \cdot (J/K)$ , i.e., linear adjustment costs with constant  $\phi_0$ ,  $J$  and  $I$  are given by:

(10)

$$(a) \quad J = (q-1) \cdot K / \phi_0$$

$$(b) \quad I = (q-1)^2 \cdot K / (2 \phi_0) .$$

For a given  $K$ , the firms should behave as a simple short-run profit maximizer, equating marginal products and factor costs of all variable factors (this assumes that these factors are strictly variable, i.e., instantaneously and costlessly adjustable). In this study we assume that there are two variable factors, labor and energy, and that total output is produced according to the CRS, neoclassical production function  $Q = F[L, N, K]$  (note that the function is written in intensive form, per unit  $L_F$ ). A number of recent studies have investigated empirically the form of this three-factor production function to determine the degree of substitutability of energy with the other factors, (see [1], [6], [7] and [14]). Some authors have suggested that  $K$  and  $L$  are weakly separable from  $E$ , writing  $Q$  as a function of value-added  $V(K, L)$  and  $N$ :  $Q = F[V(K, L)N]$ . Others argue that the appropriate assumption is  $Q = F[K^+(K, N), L]$ . In this form, effective capital  $K^+$  is composed of physical capital with a particular energy rating. The  $K^+$  function may be putty-putty, putty-clay, or strictly fixed proportion. In the simulations, we will investigate a variety of putty-putty models; I take up the far more complicated perfect foresight putty-clay model in [22].

For the general case, we require:

$$(11) \quad Q_L = \frac{\partial F}{\partial L} = \frac{W}{P}$$

$$Q_N = \frac{\partial F}{\partial N} = \frac{P_N}{P}$$

In the important special case of  $Q = \min[V(K, L), N]$ ,  $Q_N$  does not exist; (11) becomes  $\frac{\partial F}{\partial L} = \frac{W}{P - P_N} = \frac{W}{P_V}$  where  $P_V$  is the value-added deflator.

C. Asset Market Equilibrium

Domestic and foreign equity claims to capital are perfect substitutes in portfolios. Their ex ante instantaneous yields must be equated throughout time. Unexpected shocks may induce differing extraordinary capital gains and losses on the assets only at the instant of the disturbance, as equity prices re-adjust to equalize all future rates of return. The real instantaneous yield on domestic equity is the sum of the dividend yield and capital gains:

$$(12) \quad r = \dot{q}/q + \text{Div}/qK .$$

Similarly,

$$(13) \quad r^* = \dot{q}^*/q^* + \text{Div}^*/q^*K^* .$$

Now  $r$  and  $r^*$  are the pure yields in home and foreign good units respectively. The foreign yield in home good units is  $r^* - \dot{\Pi}/\Pi$  ( $\Pi = P/EP^*$ ). World asset market equilibrium requires:

$$(14) \quad r = r^* - \dot{\Pi}/\Pi .$$

Note that with  $i \equiv r + \dot{P}/P$  and  $i^* \equiv r^* + \dot{P}^*/P^*$ , (14) implies the standard uncovered interest arbitrage condition of perfect foresight and perfect capital mobility:

$$(15) \quad i = i^* + \dot{E}/E .$$

It remains to specify  $\text{Div}$  and  $\text{Div}^*$ . For convenience (and without loss of generality in the absence of interest and corporate income taxes) I assume that all investment is equity financed.

For gross capital formation  $J$  and geometric depreciation  $d \cdot K$ , new equity issues raise  $q(J-dK)$ . Total dividends in domestic goods units are:

$$(16) \quad \text{Div} = q(J-dK) + Q - \left(\frac{W}{P}\right) L - \left(\frac{P_N}{P}\right) N - I .$$

One of the central relationships of the model emerges from (12) and (16). If we assume that the real price of equity does not explode at  $t \rightarrow \infty$  (i.e., if we rule out speculative bubbles), Tobin's  $q$  may be written as the discounted value of future cash flow per unit of today's capital.<sup>3/</sup>

$$(17) \quad q \cdot K = \int_0^{\infty} e^{-\int_0^t (r(s)-n)dt} [Q - (W/P) L - (P_N/P) N - I] dt$$

#### D. Output Market Equilibrium

Households in all countries grow at rate  $n$  (with different growth rates, one economy would asymptotically dominate the world economy). Thus, the ratio of potential labor in all economies is a constant, and without confusion we may normalize all per capita variables in all countries in terms of home potential labor  $L_F$ . Consider for example foreign imports of the home good.  $C^M^*$  will represent foreign imports per  $L^F$ . With  $\theta \equiv (L_F^*/L_F)$ ,  $C^M^*/\theta$  is foreign per capita imports per unit of foreign potential labor.

Now we may write the equilibrium conditions. Let OPEC consumption per  $L_F$  of the domestic good equal  $\text{OPEC}^D$ , and of the foreign good equal  $\text{OPEC}^*$ . Total demand for the home good is then  $C^D + I + G + C^M^* + \text{OPEC}^D$  which must equal  $Q$ . Similarly,  $Q^* = C^D^* + I^* + G^* + C^M + \text{OPEC}^*$ . The nature of OPEC demand is described in Part E.



E. Balance of Payments

At any moment, world financial claims on the domestic economy change because of current account imbalances and capital gains and losses on existing assets. Let  $W^F$  represent foreign claims on the domestic income stream (the precise composition of  $W^F$  is described below). Then along the transition path, the change in  $W^F$  equals domestic income  $[Q - rW^F - (P_N/P)N]$  less domestic expenditure  $(C + C^M/\Pi + G + I)$ . Suppose now that  $W^F$  is held in the form of domestic equity:  $W^F = q \cdot Z$ . Thus  $(Z/K)$  of the home capital stock is foreign owned. Then  $\dot{W}^F = q\dot{Z} + \dot{q}Z$ , and

$$(18) \quad \dot{W}^F = [C^M/\Pi - C^{M*} - OPEC^D - (P_N/P)N] + \text{Div}(Z/K) + \dot{q}Z$$

or

$$\dot{W}^F = \text{Trade Deficit} + \text{Service Account Deficit} \\ + \text{Capital Gains.}$$

Because of the assumption of perfect capital mobility there is no guide as to how the various economies will hold their financial wealth. With all rates of return equalized, there is no motive for one asset preference over another; along any perfect foresight adjustment path all portfolios earn rate  $r$ . The portfolio composition only matters at the time of unexpected shocks, when assets experience differing, extraordinary capital gains and losses. Both to simplify bookkeeping, and to mimic in a stylized way the underdiversification of international portfolios, I assume that OPEC holds  $Z$  and  $Z^*$  claims on domestic and foreign capital, while domestic and foreign portfolios contain only equity claims on the own economy.

Since OPEC owns  $Z$ , the equity wealth of the home economy is  $q(K-Z)$ , and of the foreign economy,  $q^*(K^*-Z^*)$ .

#### F. OPEC Demand

The only matter of concern here is OPEC's savings and consumption decision. Models of OPEC pricing, cartel behavior, and oil depletion are crucial adjuncts to this study, if we are to accurately forecast long-run trends. Unfortunately, these issues are beyond the scope of this paper, and beyond the expertise of the author (for a sophisticated recent discussion, see Nordhaus [18]). The principal fact to be captured here is that OPEC consumption appears to lag significantly behind increases in OPEC revenue, following a rise in oil prices. There are a number of reasons for this, including costs of adjustment to rapid changes in consumption levels, and perhaps more important, OPEC members' awareness that current high oil revenues are transitory. Both factors induce short-run wealth accumulation, in order to smooth future consumption.

OPEC's per capita real oil wealth  $W^N$  in home good units, may be written as the discounted value of future revenues:

$$(19) \quad W^N = \int_0^{\infty} e^{-\int_0^t (r-n)ds} (P_N/P) (N+N^*) dt .$$

OPEC equity wealth is  $qZ + q^*Z^*/\Pi$ , and total OPEC wealth  $W^{OPEC}$  is  $W^N + qZ + q^*Z^*/\Pi$ . To capture lags in OPEC short-run consumption, while allowing long-run consumption to match that of the developed economies, we write a stock adjustment equation of the form:

$$(20) \quad OPEC = \lambda (OPEC)_{-1} + (1-\lambda)(\delta-n)W^{OPEC} .$$

As  $\lambda$  approaches 1, OPEC has more prolonged trade surpluses following a rise in oil prices. When  $\lambda = 0$ , the equation matches (6)(a) exactly.

The division of OPEC spending between home and foreign goods is given according to:

$$(21) \quad \begin{aligned} (a) \quad & \text{OPEC}^D = \varepsilon(\Pi) \cdot \text{OPEC} \\ (b) \quad & \text{OPEC}^* = (\text{OPEC} - \text{OPEC}^D)\Pi \end{aligned}$$

#### G. Labor Market Equilibrium

The short-run macroeconomic adjustment to an oil price increase depends crucially on wage behavior. To the extent that workers attempt to preserve real consumption wages, firms' profit margins are squeezed and unemployment results. If nominal wages are highly indexed, any attempts by the monetary authority to reduce real wages through inflation will be vitiated. On the other hand, if nominal wage growth is sticky, an inflationary policy will successfully moderate unemployment, while a contractionary response to the price hike will exacerbate the employment shortfall. These points have been extensively discussed in [4], [20], and [21].

These results may be interpreted in terms of exchange rate policy. To the extent that wages are indexed to the consumer price level, an exchange rate depreciation through monetary expansion will simply raise home wages and prices in equiproportion; there will be no expansionary gain in real output. If, contrariwise, the rate of real wage growth can be slowed through exchange rate depreciation, output will expand, with increases in investments and exports.

At most, nominal and real wage rigidity are temporary phenomena, as shifts in the employment rate will drive wages towards full-employment levels. The empirical evidence on OECD wages during 1973-79 strongly suggests that real wage growth moderated after the first OPEC price hike, but only after years of high inflation and sustained unemployment in most countries (see [20] for evidence). To model short-run rigidities and long-run labor market clearing, I propose the following expression:

$$(22) \quad W_t/W_{t-1} = (P_t^C/P_{t-1}^C)^\rho (P_{t-1}^C/P_{t-2}^C)^{(1-\rho)} L^\gamma$$

Nominal wages are indexed to consumer prices,  $P^C$ , but with a partial lag;  $100\rho$  percent indexation is on current inflation, and  $100(1-\rho)$  percent on lagged inflation. The overall rate of wage change is also responsive to the employment rate, with elasticity  $\gamma$ . Consider a few special cases. With  $\gamma = 0$  and  $\rho = 1$  the real wage is fixed, for  $W_t/P_t^C = W_{t-1}/P_{t-1}^C$ . With  $\gamma > 0$  and  $\rho = 1$ , the real wage responds to employment, but not to a change in inflation:

$W_t/P_t^C = (W_{t-1}/P_{t-1}^C) L^\gamma$ . In general, real wage change is a negative function of accelerating inflation as well as unemployment. (22) can be re-written as:

$$(23) \quad (W_t/P_t^C) = (W_{t-1}/P_{t-1}^C) \cdot [(P_{t-1}^C/P_{t-2}^C)/(P_t^C/P_{t-1}^C)]^{(1-\rho)} L^\gamma$$

Each one percent acceleration in inflation, for given  $L$ , reduces real wages by  $100(1-\rho)$  percent. Finally, note that as  $\gamma \rightarrow \infty$ , we will have  $L \rightarrow 1$ , with full employment guaranteed.

The true consumer price level for each economy can be written according to the underlying household  $U(\cdot)$  function. In the specific

Cobb-Douglas case treated here, we write  $P_C = P^\alpha (P^*E)^{(1-\alpha)}$ , with  $\alpha$  the share of domestic consumption on the domestic good.<sup>4/</sup>

#### H. The Entire Model

The full model is presented in Table 1. A list of variable definitions appears at the end of the table.

The model is written in a special form in Table 1, to anticipate the simulations. Consider a variable such as human wealth  $H$ , described in (2) in the text, and equations (5) and (7) in the table.  $H$  may be defined directly as the present value of the future stream of labor income, as in the text. Alternatively, it may be defined by two equations, in time derivative form, as in the table. Differentiating (2), we find  $\dot{H} = (r-n)H - \left(\frac{W}{P}\right)(1-\tau)L$ . This equation and the transversality condition  $\lim_{t \rightarrow \infty} H e^{-rt} = 0$ , are equivalent to the original equation for  $H$ . In effect, the transversality condition imposes the initial condition on the differential equation to insure that the solution for  $H$  is equal to its value in integral form.

In this model, the "asset variables"  $q$ ,  $q^*$ ,  $H$ ,  $H^*$ ,  $P$ ,  $P^*$  and  $W^N$  all may be written in integral form, or in differential form plus a transversality condition. In almost all cases, it is easier to simulate the model using the latter representation. As I describe in Section 4, the model in time-derivative form poses a standard two-point boundary value problem, for which solution techniques are known.

Table 1. The Complete Model

A. The Household

$$(1) \quad C = (\delta - n)A$$

$$(2) \quad C^* = (\delta - n)A^*$$

$$(3) \quad A = q(K - Z) + (M/P)(i/(\delta - n)) + H$$

$$(4) \quad A^* = q^*(K^* - Z^*) + (M^*/P^*)(i^*/(\delta - n)) + H^*$$

$$(5) \quad \dot{H} = (r - n)H - (W/P)(1 - \tau)L$$

$$(6) \quad \dot{H}^* = (r - n)H^* - (W^*/P^*)(1 - \tau^*)L^*$$

$$(7) \quad \lim_{t \rightarrow \infty} e^{-(r-n)t} H = 0$$

$$(8) \quad \lim_{t \rightarrow \infty} e^{-(r^*-n)t} H^* = 0$$

$$(9) \quad C^D = C^D(\Pi, i) \cdot C$$

$$(10) \quad (M/P) = m(\Pi, i) \cdot C$$

$$(11) \quad C^m = (C - C^D - iM/P)\Pi$$

$$(12) \quad C^{D*} = C^D(\Pi, i^*) \cdot C^*$$

$$(13) \quad (M^*/P^*) = m^*(\Pi, i^*) \cdot C^*$$

$$(14) \quad C^{m*} = (C^* - C^{D*} - i^*M^*/P^*)/\Pi$$

B. The Firm

$$(15) \quad Q = F[K, L, N]$$

$$(16) \quad Q^* = F^*[K^*, L^*, N^*]$$

$$(17) \quad Q_L = W/P$$

$$(18) \quad Q_N = P_N/P$$

$$(19) \quad Q_{L^*}^* = W^*/P^*$$

$$(20) \quad Q_{N^*}^* = P_N/P^*E$$

$$(21) \quad I = J(1 + \phi(J/K))$$

$$(22) \quad I^* = J^*(1 + \phi^*(J^*/K^*))$$

$$(23) \quad J = J(q) \cdot K$$

$$(24) \quad J^* = J^*(q^*) \cdot K^*$$

$$(25) \quad \dot{K} = J - \delta K - nK$$

$$(26) \quad \dot{K}^* = J^* - \delta K^* - nK^*$$

C. Asset Market Equilibrium

$$(27) \quad r = \dot{q}/q + \text{Div}/qK$$

$$(28) \quad r^* = \dot{q}^*/q^* + \text{Div}^*/q^*K^*$$

$$(29) \quad E = (P/P^*) \cdot (1/\Pi)$$

$$(30) \quad r = r^* - \dot{\Pi}/\Pi$$

$$(31) \quad i = r + (\dot{P}/P)$$

$$(32) \quad i^* = r^* + P^*/P^*$$

$$(33) \quad \lim_{t \rightarrow \infty} e^{-rt} (M/P) = 0$$

$$(34) \quad \lim_{t \rightarrow \infty} e^{-r^*t} (M^*/P^*) = 0$$

$$(35) \quad \text{Div} = q(J-dK) + Q - (W/P)L - (P_N/P)N - I$$

$$(36) \quad \text{Div}^* = q^*(J^*-dK^*) + Q^* - (W^*/P^*)L^* - (P_{N^*}/P^*)N^* - I^*$$

$$(37) \quad \lim_{t \rightarrow \infty} e^{-rt} q = 0$$

$$(38) \quad \lim_{t^* \rightarrow \infty} e^{-r^*t} q^* = 0$$

#### D. Output Market Equilibrium

$$(39) \quad Q = C^D + I + G + C^M + \text{OPEC}^D$$

$$(40) \quad Q^* = C^{D^*} + I^* + G^* + C^M + \text{OPEC}$$

#### E. Balance of Payments Equations

$$(41) \quad q\dot{Z} = [C^M/\Pi - C^{M^*} - \text{OPEC}^D - (P_N/P)N] + \text{Div} (Z/K)$$

$$(42) \quad q^*\dot{Z}^* = [C^{M^*}/\Pi - C^M - \text{OPEC}^* - (P_{N^*}/P^*)N^*] + \text{Div}^*(Z^*/K^*)$$

#### F. OPEC Savings and Consumption

$$(43) \quad \text{OPEC}_t = (1-\lambda)(\delta-n)W^{\text{OPEC}} + \lambda \text{OPEC}_{t-1}$$

$$(44) \quad \text{OPEC} = \text{OPEC}^D + \text{OPEC}^*/\Pi$$

$$(45) \quad \text{OPEC}^D = \text{OPEC}^D(\Pi) \cdot \text{OPEC}$$

$$(46) \quad W^{\text{OPEC}} = qZ + q^*Z^*/\Pi + W^N$$



$$(47) \quad \dot{W}^N = (r-n)W^N - (P_N/P)(N+N^*)$$

$$(48) \quad \lim_{t \rightarrow \infty} e^{-(r-n)t} W^N = 0$$

$$(49) \quad P_N = S \cdot P^\beta (P^*E)^{(1-\beta)}$$

#### G. Labor Market Equilibrium

$$(50) \quad W_t/W_{t-1} = (P_t^C/P_{t-1}^C)^\rho (P_{t-1}^C/P_{t-2}^C)^{(1-\rho)} L^\gamma$$

$$(51) \quad (W_t^*/W_{t-1}^*) = (P_t^{C*}/P_{t-1}^{C*})^{\rho^*} (P_{t-1}^{C*}/P_{t-2}^{C*})^{(1-\rho^*)} L^*\gamma^*$$

$$(52) \quad P_c = P^\alpha (P^*E)^{(1-\alpha)}$$

$$(53) \quad P_c^* = P^{*\alpha} (P/E)^{(1-\alpha)}$$

#### H. Government Spending

$$(54) \quad G = \tau(W/P)L$$

$$(55) \quad G^* = \tau^*(W^*/P^*)L^*$$

Variable Definitions

The following variables are measured per potential labor:

A	Wealth
C	Consumption
C <sup>D</sup>	Home Consumption of Home Goods
C <sup>M</sup>	Home Consumption of Foreign Goods
Div	Total Dividend Payments
E	Energy
G	Government Spending
H	Human Wealth
I	Gross Investment Expenditures
J	Gross Capital Formation
K	Capital
L	Employment
N	Energy Input
OPEC	OPEC Consumption
OPEC <sup>D</sup>	OPEC Consumption of Home Goods
Q	Output
W <sup>N</sup>	OPEC Oil Wealth
W <sup>OPEC</sup>	OPEC Wealth
Z	OPEC Holdings of Home Equity

Other Variables:

E	Exchange Rate
i	Nominal Interest Rate
P	Home Good Price
$P_c$	Consumer Price
$P_N$	Home Price of Energy
q	Equity Price
r	Real Interest Rate
W	Wage
$\Pi$	Final Good Terms of Trade (P/EP*)
$\tau$	Labor Tax Rate

Parameters:

n	Labor Force Growth Rate
$\delta$	Rate of Time Discount
d	Depreciation
$\phi_0$	Investment Adjustment Cost
$\alpha$	Share of Home Goods in Consumption Basket
$\sigma$	Coefficient of Relative Risk Aversion

The model has 55 equations, but in fact only 54 are independent. This can be demonstrated by reducing the system to its minimal state-space representation. Let  $X$  be the vector of exogenous variables, and  $S$  be the vector  $\langle Z, Z^*, K, K^*, P, P^*, q, W^N, H, H^* \rangle$ . After discretizing the dynamic equations, it may be shown that the model reduces to a nonlinear system of the form:<sup>5/</sup>

$$(24) \quad S_{t+1} = F(S_t, X_t, X_{t+1}) .$$

All remaining variables of the model are implicit functions of  $S_t, X_t, X_{t+1}$ , and in particular,  $q_t^*$  can be written as such a function. Thus, the transversality condition on  $q_t^*$  automatically holds when the transversality conditions on the other asset prices in  $S_t$  are satisfied.

The transversality conditions are not needed to reduce the system to the form in (24). In Table 1, equations (7), (8), (33), (34), (37), and (48) impose additional constraints on the dynamics. The six constraints impose implicit initial conditions on  $P, P^*, q, W^N, H, H^*$ . These asset variables always adjust in any period to guarantee that (24) will satisfy the transversality conditions as the system integrates forward. It is readily seen that the initial conditions are functions of the current values of  $Z, Z^*, K,$  and  $K^*$ , and the entire future path of  $X_t$ . Note that the initial conditions of  $Z, Z^*, K,$  and  $K^*$  are given from past history. Since they are stock variables rather than asset prices they cannot jump discretely at any instant.

### 3. COMPARATIVE STATICS

The steady-state growth path of the model depends on the equilibrium distribution of wealth. Higher domestic per capita holdings of equity claims raise home wealth and the home terms of trade ( $\Pi$ ) in the long-run. Because the steady-state distribution of wealth depends upon the entire adjustment path, it can be found only in simulation. Fortunately, important aspects of the steady state such as the long-run capital stock, rate of return, and product wage, do not depend on wealth or  $\Pi$ . To discuss comparative statics, I proceed in two steps, first asking how a parameter change alters the key variables that are not functions of the wealth distribution, and then how a shift in wealth affects the steady state. The discussion here is brief; a more detailed analysis of many of these points may be found in [13].

The most important anchor in equilibrium lies in the savings behavior of the Sidrauski-Brock infinitely-lived household. From (4) and (5) we note that per capita consumption is in steady-state equilibrium ( $\dot{\psi}=0$ ) only when  $r = r^* = \delta$ , which is the modified golden rule in this economy. Since the rate of return on equity equals the rate of time discount, we may readily determine the long-run capital-labor ratio in each economy. To do so, note that  $\dot{K}=0$  requires  $J/K = n+d$ . From (10)(a), we have  $\bar{q} = 1 + \phi(n+d)$ . (A bar over any variable signifies its steady-state value.) Since  $r=\delta$ , and  $\text{Div}/\bar{q}K = r$  in steady state,  $K$  adjusts until the dividend yield equals the required rate of return.

What is that level of  $K$ ? From the dividend expression (16), we may show that steady-state  $\text{Div}/qK$  equals  $[Q - (W/P)L - P_N/P]/qK + \xi(\bar{q})$ , where the latter term is a function in  $\bar{q}$ . Using Euler's equation,

the first part of this expression is simply  $F_K/\bar{q}$ . Since  $F_K/\bar{q} + \xi(\bar{q}) = \delta$ ,  $F_K$  is constant across steady states. We may proceed further. Full-employment is a condition of steady state; the product wage adjusts to force  $L=1$ , and  $Q = F[L,K,N]$ . Since  $\partial Q/\partial N = P_N/P$ , it is straightforward to use the output equation and this first-order condition to derive the dual expression  $F_K = G(P_N/P, K)$ , with  $G_1 \leq 0$ ,  $G_2 < 0$ . Since  $d\bar{F}_K = 0$ ,  $dK/d(P_N/P) = -G_1/G_2 \leq 0$ .

Consider the long-run decline in capital intensity, following an energy price increase, for a variety of aggregate technologies. In Chart 2, some common functional forms are examined. The analytical expressions are log-linear approximations for the percentage decline in  $K$ . The numerical illustrations are for a doubling of  $P_N/P$ , assuming: initial labor share, .57, capital share, .38, and energy share, .05. Observe that the long-run decline in  $K$  is greatest in the case of capital-energy complementarity (cf. [1], [9]).

We assume in the simulations that OPEC sets the relative oil price  $S$  in terms of a bundle of home and foreign goods. Since  $P_N = S \cdot P^\beta (P_E^*)^{1-\beta}$ , we see that  $P_N/P = S \cdot \Pi^{(1-\beta)}$ . For given  $S$ , an improvement in the terms of trade reduces the real cost of energy inputs. Thus, a rise in long-run  $\Pi$  can moderate the decline  $K$  following an oil-price hike.

Let us turn to the long-run effects of money growth in this model. Since  $\bar{K}$  is determined by the long-run required rate of return, higher  $\mu (= \dot{M}/M)$  and higher inflation have no effect on the long-run capital intensity of production. Steady-state changes in  $\mu$  cause equal increases in  $i$  ( $d\mu = di$ ), with real money balances  $M/P$

Table 2. Long-Run Decline in Capital Intensity Following Doubling of Input Price: Alternative Technologies

Technology	$F_K$ at Full Employment <sup>1</sup> (L = 1)	Percent Decline in K after Oil Price Shock <sup>2</sup>
$Q = F[K, L, N]$ $Q = \min[V(K, L), N]$ $V(K, L) = K^{1-\alpha}L^\alpha$	$(1-\alpha)(1-P_N/P)K^{-\alpha}$	8.9
$Q = (K^+)^{1-\alpha}L^\alpha$ $K^+ = \min[K, E]$	$(1-\alpha)K^{-\alpha} - (P_N/P)$	17.5
$Q = K^{1-\alpha}(L^+)^\alpha$ $L^+ = \min[L, E]$	$(1-\alpha)K^{-\alpha}$	0.0
$Q = [\mu_1 L^\rho + \mu_2 K^\rho + (1-\mu_1-\mu_2)N^\rho]^{1/\rho}$ (General CES with $\sigma = 1/(1+\rho)$ )	$\Delta \mu_2 [\mu_1 K^{-\rho} + \mu_2]^{(1-\rho)/\rho}$ where $\Delta = [1 - (P_N/P)^{-\sigma\rho} (1-\mu_1-\mu_2)^{\sigma\rho}]^{(1/\rho)}$	

<sup>1</sup>  $F_K = [Q - (W/P)L - (P_N/P)N]/K$ , where  $W/P$  adjusts to guarantee  $\partial Q/\partial L = W/P$  at  $L = 1$ , and where  $\partial Q/\partial N = (P_N/P)$ .

<sup>2</sup> Assuming initial shares for labor (.57), capital (.38), energy (.05),  $L = 1$ , and  $F_K = .11$ .

adjusting to keep  $iM/P$  constant. The result can be made much stronger. As long as nominal wages are fully indexed, and utility is Cobb-Douglas in money and goods, the path of money growth has no effect on any real variables besides  $M/P$  and the inflation rate itself! This means that changes in inflation cannot affect the path of the real exchange rate. A one-shot unanticipated increase in the rate of money growth causes an initial jump in  $P$  and  $E$  of equal proportion, and subsequent higher inflation and exchange rate depreciation. The decline in real balances does not lead to increased hoarding, as in some models of the open economy.<sup>6/</sup>

Finally, we come to the determination of the long-run terms of trade. Here the crucial determinants are the distribution of world equity claims and the division between public and private spending. Since we assume that government consumption falls exclusively on home goods while private consumption falls on home goods and imports, an increase in  $G$  that crowds out equal  $C$  causes an excess demand for home goods at the initial  $\Pi$ ; a labor-tax-financed fiscal expansion raises  $\Pi$ . Starting in a steady state, with fully flexible real wages, a fiscal expansion raises  $\Pi$  with no consequences over time for the capital stock or international wealth flows.<sup>7/</sup>

The second determinant of  $\Pi$  is the world distribution of wealth, which determines the world distribution of spending (as different countries have differing marginal propensities to spend on home and foreign goods). Home country wealth is given in (7) as  $q(K-Z) + H + [i/(\delta-n)](M/P)$ . In steady state,  $\bar{H}$  is the capitalized value of after-tax wage payments:  $\bar{H} = (\bar{W}/P)(1-\tau)/(\delta-n)$ , and the product wage  $W/P$  is a function of  $P_N/P$  and  $\bar{K}$ . Since  $\bar{K}$  is itself a



function of  $P_N/P$ , the main determinants of home wealth are  $Z$  and  $P_N/P$ . In general, a drop in  $Z$  relative to  $Z^*$  induces a fall in  $\Pi$ . A rise in  $P_N/P$  shifts wealth to OPEC, and the effect on  $\Pi$  depends on OPEC consumption preferences.

#### 4. SIMULATION RESULTS

A variety of simulations are now presented, aiming at:

(1) an understanding of the general equilibrium effects of an OPEC price increase; and (2) an analysis of policy responses to an input price increase. Five simulations are treated in detail, though a number of further studies have been made. In the first, the price hike is studied assuming identical developed economies and fully flexible wages and prices. In the following simulation, I point out the implications of significant domestic oil production in one of the two developed economies. Next, the mirror-image assumption for the two economies is re-introduced but the assumption of sluggish real wage adjustment is added. The final two simulations continue with the assumption of short-run wage rigidity, and consider the possibility of expansionary monetary and fiscal policies to combat the unemployment following an input price rise. Attention is paid to the repercussion effects on the other economy of domestic macroeconomic expansion.

In all cases, the simulation model is solved by the method of multiple shooting. As I described above, six transversality conditions must be employed to find the initial conditions for  $q$ ,  $M/P$ ,  $M^*/P^*$ ,  $H$ ,  $H^*$ , and  $W^N$ . For any initial vector of these variables the model may be integrated forward, but only for a unique starting vector will the system converge to a steady-state. For all other starting conditions, the models diverge explosively from the steady-state. Technically, the difference equation system possesses strict saddlepoint stability. When we linearize the transition matrix  $F$  in  $S_{t+1} = F(S_t, X_t, X_{t+1})$  of equation (24), six eigenvalues are found to be outside of the unit circle. These correspond to the six implicit initial conditions on the asset price vector required for convergence.

The general method of "shooting" provides a straight-forward technique for finding the initial conditions. We divide the state vector  $S_t$  into components  $\Gamma$  and  $\Sigma$ , with  $\Gamma = \langle q, M/P, M^*/P^*, H, H^*, W^N \rangle$ , and  $\Sigma = \langle Z, Z^*, K, K^*, \dots \rangle$ , with the precise specification of  $\Sigma$  depending on parameter values in the model. Using the transition function in (24), there is an implicit relationship between  $\Gamma_N$  and  $\Gamma_0$ ,  $\Sigma_0$ , and  $\{X_i\}_{i=0, \dots, N}$ :

$$(25) \quad \Gamma_N = G(\Gamma_0, \Sigma_0, \{X_i\}_{i=0, \dots, N}).$$

Now a condition for convergence to the steady state is

$\lim_{N \rightarrow \infty} \Gamma_N = \bar{\Gamma}$ . Our goal is to find  $\Gamma_0$  such that

$\bar{\Gamma} \approx \Gamma_N = G(\Gamma_0, \Sigma_0, \{X_i\}_{i=0, \dots, N})$  for  $N$  very large. This is a set of non-linear equations, which can be solved by numerical procedures. In "shooting," a guess is made for  $\Gamma_0$ , the system is integrated

forward until  $N$ , and  $\bar{\Gamma} - \Gamma_N$  is evaluated. A new guess for  $\Gamma_0$  is made based on Newton's method:  $\Gamma_0^1 = \Gamma_0 + \left[\frac{\partial G}{\partial \Gamma_0}\right]^{-1}(\bar{\Gamma} - \Gamma_N)$ .

This iterative procedure is continued until  $\bar{\Gamma} \approx \Gamma_N$ , within the specified tolerance. Note that  $\frac{\partial G}{\partial \Gamma_0}$  is numerically calculated.

Because of the explosive nature of the dynamic system, it is often impossible to integrate from 0 to  $N$ , if the guess for  $\Gamma_0$  is poor. To bring the explosiveness under control, the interval  $[0, N]$  may be subdivided, with trial vectors chosen for  $\Gamma$  and  $\Sigma$  at intermediate points. The system is then integrated over the sub-intervals rather than over  $[0, N]$ , so that the instability does not cumulate over the entire interval. The iterations proceed until  $\Gamma_N$  equals  $\bar{\Gamma}$  with the intermediate guesses mutually consistent. This variant of shooting is known in the physical sciences as "multiple shooting," and is used here. Details and references for this solution technique may be found in [12].

The baseline steady-state conditions for the simulations are given in Table 3. In all cases, we assume that the technology for gross output  $Q$  is given by  $\min[V(K, L), N]$ , with  $\sigma_{KL} = 1$  in value added. I have devoted a separate note to analyzing the dynamic implications of alternative technologies (see [22]). The long-run effects of a price rise with this technology were shown in Section 2.  $\bar{K}/L$  falls approximately      percent for a one percent rise in  $P_N/P$ , starting from  $P_N/P = .05$ .

In the flexible wage and price case,  $W/P$  adjusts at all points to keep  $W/P = F_L(K, L)$  at  $L = 1$ . Under this condition, the marginal

Table 3. Key Parameter Values at Initial Steady-State Equilibrium

Labor Share in Gross Output ( $WL/PQ$ )	.57
Energy Share in Gross Output ( $P_N N/PQ$ )	.05
Capital Share in Gross Output	.38
$GDP = Q - P_N N = C^D + I + G + C^{M*} + OPEC^D - P_N N$	
$C^D/GDP$	.36
$I/GDP$	.17
$G/GDP$	.20
$C^{M*}/GDP$	.24
$OPEC^D/GDP$	.08
$P_N N/GDP$	.05
$n$	.02
$\delta$	.11
$d$	.05
$\tau_W$	.33
$\sigma_{KL}$ in value added	1.0

product of capital may be written as a function of  $K$  (or  $K/L$ , since  $L = 1$ ) and the real price of energy  $P_N/P$ :  $F_K = F_K(K, P_N/P)$ . In the case at hand,  $W/P = \alpha K^{1-\alpha} \cdot (1 - P_N/P)$ , where  $\alpha$  is the share of labor in value added. Since  $F_K = [Q - (W/P)L - (P_N/P)N]/K$ , we have  $F_K = (1 - \alpha - (1-\alpha)P_N/P)K^{-\alpha}$ . Thus, even with fully flexible real wages,  $F_K$  is a negative function of  $P_N/P$  at any level of  $K$ . It is clear that both  $W/P$  and  $F_K$  absorb some of the oil shock. This is to be contrasted with cases of capital-energy or labor-energy perfect complementarity in which only one factor price is affected by  $P_N/P$ . Indeed with the assumptions here, the percentage changes in  $F_K$  and  $W/P$  following a rise in  $P_N/P$  are equal, (assuming  $L = 1$ ).

If OPEC savings behavior is identical to that of the developed economies, the oil price increase causes a fall in capital accumulation at the time of the shock. This seems intuitive, since the fall in  $F_K$  and  $F_{K^*}$  should push down  $q$  and  $q^*$ , and reduce the rate of capital formation. But to nail down the argument, we must understand why savings falls and consumption rises on a global scale, as is implied by the fall in global capital accumulation. The higher oil price per se does not reduce global savings, since the increase in OPEC consumption is matched, at a constant  $r$ , by a decline in household consumption in the developed economies. But  $r$  does not remain constant. With an investment decline, constant total world consumption, and output fixed in the short run, there is an excess supply of goods.  $r$  falls pushing up consumption and moderating the decline in  $q$ , until output markets clear. Remember that a fall in  $r$  raises both  $q$  and

human wealth, since each is a discounted flow of future income streams. In the initial period,  $q$  and  $q^*$  fall, household consumption declines, and OPEC consumption rises by more than the household decline.

If OPEC has a higher short-run propensity to save out of wealth than do the developed economies, the rise in OPEC consumption does not match the fall in household consumption at constant  $r$ . The real interest rate must fall even more in the initial period to clear output markets. If OPEC spends very little of its new wealth initially,  $r$  may fall enough to actually raise  $q$  and  $q^*$  in the short-run. Even though  $K$  and  $K^*$  must be lower in the new steady-state, they rise temporarily in response to OPEC's high saving propensity. Once OPEC consumption catches up with the revaluation of its oil wealth, the short-term real interest rate is pushed up,  $q$  and  $q^*$  fall, and the rise in capital accumulation is reversed.

Thus, the short-run investment response to OPEC under flexible wages and prices depends crucially on OPEC savings behavior. Remember from Section 2 that the parameter  $\lambda$  describes the lag in OPEC spending.  $\lambda = 0$  implies identical OPEC and non-OPEC saving behavior, and  $\lambda = 1$  implies fixed real OPEC expenditure independent of oil prices, (i.e., a zero marginal propensity to consume out of new wealth). In general, a high  $\lambda$  implies a long lag in adjustment of OPEC consumption to higher wealth. Table presents simulation results for a doubling of energy prices under two cases of OPEC savings behavior. The developed economies are taken to be identical, so the terms of trade between them are fixed at 1.0, with the two countries' variables adjusting identically to the OPEC shock. The table therefore lists only the home variable values.

Table 4. Dynamic Adjustment with Alternative OPEC Savings Behavior

Period	$\lambda = .1$	$\lambda = .75$
Period 1		
Q	0.0	0.0
I	-10.7	0.8
W/P <sub>C</sub>	-5.4	-5.4
$\dot{P}$	5.3	2.6
Period 5		
Q	-1.1	-0.3
I	-11.6	-7.0
W/P <sub>C</sub>	-6.5	-5.7
$\dot{P}$	0.5	0.8
Period 10		
Q	-2.0	-1.2
I	-10.5	-9.9
W/P <sub>C</sub>	-7.5	-6.6
$\dot{P}$	0.3	0.5
Steady-State		
Q	-3.6	-3.6
I <sup>1</sup>	-8.9	-8.9
W/P <sub>C</sub>	-9.0	-9.0
$\dot{P}$	0.0	0.0

All variables except  $\dot{P}$  are measured as percentage deviations from the pre-shock steady-state growth path; specifically  $\log(x/\bar{x}) \cdot 100$ .  $\dot{P}$  is  $[(P_t - P_{t-1})/P_{t-1}] \cdot 100$ .

<sup>1</sup>Note that the long-run proportional decline in I equals the proportional decline in  $\bar{K}$ , since  $J = (n+d)\bar{K}$ , and  $I = J \cdot (1+\phi/2 \cdot (n+d))$ .



Since real wages are fully flexible, full employment is continuously maintained. Real wages in each country fall 5.4 percent at the time of the shock, and decline even more over time as  $K$  and  $K^*$  fall. Given our assumption of no technical substitution between  $V$  and  $N$  in production, gross output is wholly unaffected by the higher energy prices at the initial point. There is, however, a major switch in composition of uses of the final output. In the  $\lambda = .1$  case,  $q$  and  $q^*$  fall sharply, as does household consumption, while OPEC consumption rises steeply. The fall in home consumption reduces real money demand, and with given nominal stock, the price level jumps sharply, by 5.3 percent. Over time,  $K$  and  $K^*$  fall, household consumption falls more, and the price level creeps upward. As we have seen, the long-run decline in capital intensity is governed by the return of  $(1-\alpha)(1-P_N/P)K^{-\alpha}$  to its pre-shock level.  $\bar{K}$  falls 8.9 percent, and  $\bar{Q}$  therefore falls  $(1-\alpha) \times 8.9$  percent, or 3.6 percent. Of course, the higher energy price has no persistent effect on  $\dot{P}$ , but only on the price level itself. With  $\lambda = .1$ , long-run  $C$  falls 12.0 percent, and  $P$  rises in equiproportion.

A higher  $\lambda$  has no long-run effect on  $K$  or  $Q$ , but does affect short-run investment, and long-run  $C$  and  $P$ . For a higher  $\lambda$ , the initial interest rate falls more, and the short-run investment decline is moderated or reversed. Also, as  $r$  falls more upon the shock, households dissave to a greater extent, driving down their stock of assets.  $\bar{C}$  is therefore lower for higher  $\lambda$ , making long-run  $\bar{P}$  higher for given  $M$ . With  $\lambda = .75$ , long-run  $C$  falls 14.1 percent.

Suppose now that the home economy satisfies a portion of its oil inputs through domestic production of energy.

To simplify, we assume an exogenous, permanent, constant flow of domestic oil equal to one-half of initial energy inputs. Furthermore, we abstract from all domestic costs of production, assuming a costlessly producible stream of resources. The oil price hike now induces a windfall in domestic energy wealth (i.e., the discounted value of future energy production) that in part compensates for the fall in human wealth and physical wealth occasioned by the rise in  $P_N/P$ . The consumption demand in the oil-producing home economy falls less than in the foreign economy, and given that output is fixed in the short-run, its real exchange rate appreciates 1.1 percent. Also, the nominal exchange rate rises by 6.8 percent since demand for real money balances falls less in the oil-rich than in the oil-poor economy. The results are given in Table 5.

The smaller energy holdings of the foreign economy do not lead to larger current account deficits in that economy, even though it is more "dependent" on OPEC. Simply, its larger oil imports more fully crowd out other forms of consumption, so that the income/absorption balance is no different for the two economies.<sup>8/</sup> In principle, the higher terms of trade for the oil-rich economy slightly reduces its real cost of imported oil (see p. 24), so its long-run capital stock should be marginally higher. This is the opposite of the "Dutch disease" conclusion that higher domestic energy holdings reduce an economy's capital stock. The effect on the capital stock, however, turns out to be wholly unimportant in the simulations, given the mere 0.9 percent long-run terms-of-trade improvement.

So far I have assumed that the labor market is continuously in balance. However, I have argued at great length elsewhere

Table 5. Dynamic Adjustment with Home Production of Oil

	Period 1	Period 5
Q	0.0	-0.5
I	-2.7	-8.0
W/P <sub>C</sub>	-4.8	-5.4
$\dot{P}$	-1.3	0.8
CA/GDP	-3.5	-1.7
Q*	0.0	-0.5
I*	-2.9	-8.4
(W/P <sub>C</sub> )*	-5.9	-6.4
$\dot{P}^*$	3.6	0.7
CA*/GDP*	-3.3	-1.5
$\dot{E}$	-6.8	0.0
$\Pi$	1.1	0.9
	Period 10	Steady-State
Q	-1.4	-3.6
I	-10.0	-8.8
W/P <sub>C</sub>	-6.3	-8.5
$\dot{P}$	0.5	0.0
CA/GDP	-0.9	-0.8
Q*	-1.5	-3.6
I*	-10.3	-9.0
(W/P <sub>C</sub> )*	-7.3	-9.4
$\dot{P}^*$	0.4	0.0
CA*/GDP*	-0.8	-0.8
$\dot{E}$	0.0	0.0
$\Pi$	0.9	0.9

---

See note at the bottom of Table 4.  $\dot{E}$  is  $[(E_t - E_{t-1})/E_{t-1}] \cdot 100$ .

(see [20], for example) that after the OPEC price increase of 1973-74; real wages in most developed economies did not fall sufficiently in the short-run to keep labor fully employed. If real wages respond sluggishly to unemployment, a major initial effect of oil price hike is a reduction in employment and output. In the simulation reported in Table 6, I assume that each 1.0 percent decline in employment reduces real wages from the previous period by 1.2 percent. Now, a doubling of oil prices causes an initial drop in output of 2.0 percent, and investment falls by 7.3 percent. This compares with a zero initial output drop and a 0.1 percent rise in investment in the case of fully flexible real wages. The steeper investment decline derives from two factors. Higher W/P reduces  $F_K$ , and thus pushes down  $q$ . Also, the decline in aggregate supply reduces or reverses the fall in short term interest rates, also driving  $q$  down. Finally, note that prices jump by more than in the flexible real wage case since the demand for real money balances drops.

What is the scope for policy in moderating the short- and long-run output declines following an oil price shock? To maintain steady-state per capita output levels, the profitability of capital at the initial K/L ratio must be restored. Demand management policies will in general be useless for this purpose, though tax- and subsidy policies might play a role (of course the desirability of such policies is another matter). In the short-run there is far more scope for maintaining output close to potential through monetary and fiscal policies. Bruno and I, [(4), (21)] have emphasized the following aspects of short-run policy. If nominal wages are fully indexed and real wages adjust sluggishly to unemployment, a monetary expansion or exchange rate depreciation will

Table 6. Dynamic Adjustment with Real Wage Rigidity

Period 1	Rigid Wages <sup>1</sup>	Flexible Wages
Q	-2.0	0.0
I	-7.3	0.1
W/P <sub>C</sub>	-4.1	-5.4
$\dot{P}$	3.9	2.7
Period 5		
Q	-0.6	-0.3
I	-6.9	-7.0
W/P <sub>C</sub>	-5.9	-5.7
$\dot{P}$	0.8	0.8
Period 10		
Q	-1.4	-1.2
I	-10.1	-9.9
W/P <sub>C</sub>	-6.7	-6.6
$\dot{P}$	0.5	0.5
Steady-State		
Q	-3.6	-3.6
I	-8.9	-8.9
W/P <sub>C</sub>	-9.0	-9.0
$\dot{P}$	0.0	0.0

---

See note at the bottom of Table 4.

<sup>1</sup>Wage "rigidity" is specified as  $\gamma = \gamma^* = 1.2$  in wage equation, with  $\rho = \rho^* = 1.0$ .

have no effect on output, but will have a significant inflationary effect on prices. Fiscal policy can raise output with real wage rigidity, by favorably shifting the terms of trade, and reducing the product wage  $W/P$  for given levels of the real wages  $W/P_C$ . The conclusions are reversed for monetary policy if nominal wages are sluggish, with low levels of wage indexation. Now a monetary expansion induces an exchange rate depreciation, drives down the real wage, and causes output to rise. Depending on the nature of indexation, the beneficial effect on output may be very short-lived or highly persistent. Fiscal policy remains effective at low levels of wage indexation.

To illustrate the role of exchange rates and macroeconomic policy following a supply shock, I assume that one developed country has fully flexible wages and prices while the other has sluggish wage adjustment, with lags in indexation. Specifically, for the wage adjustment process in equation (22),  $\rho = .25$  and  $\gamma = .5$ . The first economy will have continuous full employment, while the second will exhibit short-run unemployment following an oil price increase, and will converge to full employment over time. Starting from a steady-state with zero (per capita) nominal money stock growth and constant fiscal expenditure, two policies are tried at the time of an unexpected OPEC shock: (1) an announced permanent increase in nominal money growth to one percent per year, taking effect at the beginning of the second period (i.e., the first period nominal money stock remains unchanged); and (2) a 10% percent increase in per capita fiscal expenditure, financed by a balanced-budget increase in the proportional labor income tax.

The results of these policies are illustrated in Table 7. The simplest and most important point is that the home country can buy a short-run reduction in unemployment, at the expense of a higher price level or steady-state inflation rate. With a shift to one percent permanent money growth, unemployment is eliminated in the first period, but with an 11.4 percent jump in the price level.<sup>9/</sup> The policy works by reducing  $W/P_C$  by 1.6 percent more than in the base case. Note that the exchange rate depreciates 8.6 percent on impact of the announcement of the policy change, even before the nominal money stock changes. The first period terms of trade depreciates 1.0 percent relative to the base case. In the second period, the lagged indexation mechanism induces a substantial catch-up in the real wage, and much of the reduction in unemployment is reversed. The economy converges to full employment with unemployment always slightly below the base case. In the steady-state, of course, domestic inflation equals the rate of per capita nominal money stock growth, while the exchange rate depreciates at the same rate. Long-run output and the capital stock are unchanged by the expansionary monetary policy.

Given the assumption of continuous labor market clearing in the foreign economy, the expansionary policy at home has virtually no effect on output or foreign prices. The most important effect transmitted abroad is a real exchange rate appreciation, which raises foreign real wages relative to the base case, (in the first period,  $W^*/P_{C^*}$  is .5 percent above the base level). If real wage adjustment were also sluggish abroad, the foreign terms of trade improvement would translate into an output expansion, so that the

Table 7. Monetary and Fiscal Policy After an Oil Price Increase

Period 1	Period 2			Period 2	Steady-State				
	B <sup>1</sup>	M	F		B	M	F		
Q	-1.4	0.1	-0.3	-1.8	-1.4	-0.9	-3.5	-3.5	-3.5
L	-2.3	0.2	-0.4	-2.8	-2.4	-1.5	0.0	0.0	0.0
W/P <sub>C</sub>	-3.9	-5.5	-3.7	-3.7	-4.0	-3.1	-8.9	-8.9	-7.5
$\dot{P}$	3.7	11.4	6.3	1.4	2.9	1.5	0.0	1.0	0.0
Q*	0.0	0.0	0.0	0.0	0.0	0.0	-3.5	-3.5	-3.5
L*	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
(W/P <sub>C</sub> )*	-6.0	-5.5	-7.0	-6.1	-5.8	-7.1	-8.9	-8.9	-10.4
$\dot{P}^*$	2.7	2.7	2.7	1.1	1.1	1.2	0.0	0.0	0.0
$\dot{E}$	-0.2	8.6	0.1	0.0	1.0	0.0	0.0	1.0	0.0
$\Pi$	1.2	0.2	3.4	1.5	0.9	3.7	0.0	0.1	3.0
Period 5	Steady-State			Steady-State			Steady-State		
	B	M	F	B	M	F	B	M	F
Q	-1.1	-0.6	-0.5	-3.5	-3.5	-3.5	-3.5	-3.5	-3.5
L	-0.8	-0.2	-0.2	0.0	0.0	0.0	0.0	0.0	0.0
W/P <sub>C</sub>	-5.2	-5.6	-4.2	-8.9	-8.9	-7.5	-8.9	-8.9	-7.5
$\dot{P}$	0.8	1.8	0.8	0.0	1.0	0.0	0.0	1.0	0.0
Q*	-1.2	-0.3	-0.3	-3.5	-3.5	-3.5	-3.5	-3.5	-3.5
L*	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
(W/P <sub>C</sub> )*	-6.1	-5.9	-7.2	-8.9	-8.9	-10.4	-8.9	-8.9	-10.4
$\dot{P}$	0.9	0.9	0.8	0.0	0.0	0.0	0.0	0.0	0.0
$\dot{E}$	0.0	1.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0
$\Pi$	1.1	0.4	3.3	0.0	0.1	3.0	0.0	0.1	3.0

<sup>1</sup>B - Base Case: No change in per capita G or M;

M - Monetary Expansion: 1% growth rate in M, announced in 1970, to begin 1970-71;

F - Fiscal Expansion: 10% balanced budget increase, unanticipated and permanent.



monetary expansion would be positively transmitted. A second transmitted effect, which turns out to be insignificant in simulation, is a slight reduction in the world interest rate following the monetary expansion.

While expansionary monetary policy operates through an inflationary reduction of real wages in the home country, the primary effect of fiscal policy is to raise home output through a terms of trade improvement. Since fiscal expenditure falls solely on home goods, while it crowds out household expenditure on home and foreign goods, a balanced-budget increase in  $G$  induces an excess demand for home goods at the initial prices, and thus raises  $\Pi$ . For a given real wage  $W/P_C$ , the product wage  $W/P$  is reduced, since

$W/P = (W/P_C)\Pi^{1-\alpha}$ . For a ten percent, permanent, unanticipated fiscal expansion in the year of an oil shock,  $\Pi$  is raised 2.2 percent relative to the base case. The nominal exchange rate depreciates .3 percent more than in the base path, while home prices rise by 2.6 percent more. The home fiscal expansion reduces the foreign real wage by 1.0 percent, just as the monetary expansion raised the foreign real wage. If foreign wage adjustments were sluggish, the fiscal expansion at home would increase foreign unemployment. When real wages adjust slowly to unemployment in both countries, then, a fiscal expansion in one is negatively transmitted to the other!

The expansionary effects of a rise in  $G$  are not rapidly reversed in subsequent periods, as with a monetary expansion. Of course in the long-run, output and the capital stock adjust to the steady-state levels in the base case (with a small adjustment due to a

terms-of-trade effect on  $P_N/P$ .) The terms-of-trade shift is permanent, and in the long-run is fully reflected in the difference in real wages in the two economies. Furthermore, there is no persistence in the one-shot jump in prices that accompanies the fiscal expansion. By the third year, domestic inflation is almost identical with the inflation rate in the base case.

## 5. CONCLUSIONS AND EXTENSIONS

The simulations in this paper demonstrate the feasibility of studying applied problems in macroeconomic adjustment in large models with efficient asset markets and intertemporal optimization by economic agents. The assumption of efficient asset markets imposes a great computational burden in theoretical models, because of the difficulty of solving the two-point boundary value problems that result. Thus, studies of flexible exchange rates, or energy prices, or capital accumulation in open economies, typically simplify greatly the structure of the economy in all aspects but the one under study. At the beginning of this essay I suggested that the analysis of the OPEC price hikes demands an integrated approach, with a specification of all major aspects of the macroeconomy. Leaving out capital accumulation, or OPEC savings behavior, for example, would change the patterns of adjustment of all other macroeconomic variables.

The model in this paper illustrates many key facets of adjustment to an energy price rise. First, higher energy prices almost surely require a long-run decline in capital intensity (relative to trend) in the developed economies, in order that pre-shock profitability may be partially or wholly restored. Second, the short-run movement in capital accumulation results from the interplay of many factors, including the short-run profitability decline, differential savings behavior in OPEC and the developed economies, and real wage behavior in the developed economies. The first factor tends to reduce  $I$  at time zero, and to encourage consumption. The second, OPEC's higher short-run propensity to save, reduces world

interest rates, and counterbalances the first effect. Finally, short-run real wage rigidity reduces output at the time of an oil shock, forces up world interest rates, and lowers investment.

The view of macropolicy set forth in Section 4 ties expansion to a reduction in real factor costs, whether through inflation (with monetary and exchange-rate policy) or a terms-of-trade improvement (with fiscal policy). With the parameters assumed for simulation, we were able to quantify the effects of alternative policies.

Obviously the most important next task for this research strategy is a more realistic calibration of the model. Michael Bruno and this author are now undertaking this project with data for the OECD economies. We are using original econometric work as well as published estimates to set the parameters of the model. The plan is to expand the model to include three importing regions (Europe, Japan, the U.S.) as well as OPEC.

The theoretical framework warrants refinement as well. The assumption of continuous output-market clearing should be relaxed for the very short-run, allowing households to make intertemporal decisions with rational expectations of market constraints. In another aspect of the model, more work remains in the specification of aggregate technology. It will be useful to distinguish traded and non-traded goods in the developed economies, as well as to extend the technology to the case of putty-clay capital. Further discussion of alternative technological assumptions is offered in [22]. Finally, the treatment of OPEC can be deepened in a number of ways. OPEC holdings of oil can be treated within a dynamic portfolio model, which stresses the real return to oil and alternative assets.

The nonrenewable nature of oil should be explicitly modeled, as well as the presence of backstop energy technologies in the developed economies.

FOOTNOTES

1/ To derive the formula for  $W^T$  we begin with

$$(26) \quad W^T = \int_0^\infty e^{-\int_0^t (r-n) ds} (\dot{M} + nM) / P dt .$$

First, let us evaluate  $\int_0^\infty e^{-\int_0^t (r-n) ds} \dot{M} / P dt$ , and then we will add the result to  $\int_0^\infty e^{-\int_0^t (r-n) ds} nM / P dt$ . The first integral is solved by integration by parts. Let  $dv = \dot{M} dt$  and  $u = e^{-\int_0^t (r-n) ds} / P$ . Rewriting  $\int_0^\infty u dv = uv \Big|_0^\infty - \int_0^\infty v du$ , we find

$$(27) \quad \int_0^\infty e^{-\int_0^t (r-n) ds} \dot{M} / P dt = (M/P) e^{-\int_0^t (r-n) ds} \Big|_0^\infty \\ + \int_0^\infty (M/P) e^{-\int_0^t (r-n) ds} (r-n) dt + \int_0^\infty (M/P) (\dot{P}/P) e^{-\int_0^t (r-n) ds} dt .$$

Now add  $\int_0^\infty e^{-\int_0^t (r-n) ds} (nM/P) dt$  to both sides of (31), and evaluate:

$$(28) \quad W^T = -M(0)/P(0) + \int_0^\infty (iM/P) e^{-\int_0^t (r-n) ds} dt$$

But  $iM/P = sC_F$ . Substituting, we have

$$(29) \quad W^T = -M(0)/P(0) + \int_0^\infty sC_F C^{-\int_0^t (r-n) ds} dt .$$

The last integral is nothing but  $s$  multiplying the discounted value of future consumption, or  $sW$ . But  $(\delta-n)sW$  equals current expenditure on  $M/P$ , to-wit:  $iM/P = (\delta-n)sW$ . Thus, from ( ) we may derive:

$$(30) \quad W^T = -M(0)/P(0) + [i(0)M(0)/P(0)] / (\delta-n) .$$

This is given in the text.

2/ The utility function is written to provide an intertemporal homothetic utility function. To derive (5) in the text we solve the problem:

$$(31) \quad \max \int_0^{\infty} e^{-\delta t} L_F U(\Psi(C, C^M, M/P)) dt$$

such that

$$\dot{A} = rA - [C + C^M/\Pi + i(M/P)] .$$

The first order conditions are:

$$(32) \quad (a) \quad L_F U' \Psi_C = \lambda$$

$$(b) \quad L_F U' \Psi_{C^M} = \lambda/\Pi$$

$$(c) \quad L_F U' \Psi_{M/P} = \lambda i$$

$$(d) \quad \dot{\lambda}/\lambda = \delta - r .$$

Now, we use the properties of the true price index for  $P_\Psi$ . In particular,  $P_\Psi$  is linear homogenous in the prices of  $C$ ,  $C^M$ ,  $M/P$ , i.e.,  $P_\Psi = P_\Psi(1, 1/\Pi, i)$ . Also, the derivatives of  $P_\Psi$  with respect to the prices of  $C$ ,  $C^M$ , and  $M/P$  are  $C/\Psi$ ,  $C^M/\Psi$ , and  $(M/P)/\Psi$  respectively. Combining these properties:

$$(33) \quad P_\Psi = C/\Psi + (C^M/\Psi)(1/\Pi) + i(M/P)/\Psi .$$

Now, multiplying ( ) (a), (b), and (c) by  $C$ ,  $C^M$ , and  $M/P$  respectively, adding, and using the condition that

$$\Psi = \Psi_C C + \Psi_{C^M} C^M + \Psi_{M/P} (M/P), \text{ we find:}$$

$$(34) \quad L_F U' \Psi = \lambda \Psi P_\Psi$$

or

$$L_F U' = \lambda P_\Psi .$$

Using ( ) (d), ( ) and the definition of  $U(\cdot)$  from equation (4) in the text, we derive (5) on p. 10.

3/ To derive (17), we re-write (12) using the condition that  $V = qK$ , so  $\dot{V}/V = \dot{q}/q + \dot{K}/K$ .

$$(35) \quad r = \dot{V}/V + Di/V - \dot{K}/K .$$

Also,  $\dot{K}/K = J/K - d - n$ . Using (16) and (35), direct substitution gives:

$$(36) \quad r = n + \dot{V}/V + [Q - (W/P)L - (P_N/P)N - I]/V .$$

This is a simple first order differential equation, with the solution:

$$(37) \quad V = e^{\int_0^t (r-n) ds} V_0 - V_0 e^{\int_0^t (r-n) ds} \cdot$$

$$\int_0^t e^{-\int_0^s (r-n) dz} [Q - (W/P)L - (P_N/P)N - I] ds \cdot$$

Now, if we impose the condition that  $\lim_{t \rightarrow \infty} e^{-\int_0^t (r-n) V} = 0$ , we have

$$(38) \quad V_0 = \int_0^t e^{-\int_0^s (r-n) dz} [Q - (W/P)L - (P_N/P)N - I] ds$$

which was to be proved.

- 4/ The price index is written here in terms of commodities, as is conventional, though the price index for full instantaneous consumption would include the price of money balances, i.
- 5/ Actually, the specific vector of state variables depends on certain key parameters in the model. The vector shown above (24) is correct for  $\gamma = \gamma^* = \infty$ . For  $\gamma$  and  $\gamma^*$  finite, lagged real wages  $(W/P_C)_{-1}$  and  $(W^*/P_{C^*})_{-1}$  and lagged inflation are state variables.
- 6/ See for example [ ]. The difference in result may depend upon the more limited menu of assets available in the economy in [ ]. This cannot be conclusively determined since the savings behavior in that model is not explicitly derived.
- 7/ After the fiscal expansion, the capital/labor ratios remain at the appropriate level for  $F_K/\bar{q} + \xi(\bar{q}) = \delta$ . Also, since the tax change is permanent, "permanent income" of households falls by as much as current income, so there is no motive for saving or dissaving by households over time.
- 8/ This independence of oil holdings and current account balance is sensitive to a number of assumptions. If the domestic oil holdings are depleting, current income exceeds permanent income, and the developed oil-producing country runs a surplus. Also, if the oil price rise is perceived to be temporary, then a similar result obtains.
- 9/ The jump is very large for two reasons. First, the acceleration in M is known to be permanent, so it is fully capitalized in the first period. Second, the interest elasticity of money demand with respect to the interest rate is very high in the model, given the Cobb-Douglas utility assumption.



LIST OF REFERENCES

1. Berndt, Ernst and David O. Wood, (1979). "Engineering and Econometric Interpretations of Energy-Capital Complementarity," American Economic Review, 3, 342-354.
2. Brock, William A., (1974). "Money and Growth: The Case of Long-Run Perfect Foresight," International Economic Review, XV, October, 750-77.
3. Bruno, Michael and Jeffrey Sachs, (1979). "Macro-economic Adjustment with Import Price Shocks: Real and Monetary Aspects," Working Paper No. 340, National Bureau of Economic Research, April.
4. \_\_\_\_\_ . (1979) "Supply versus Demand Approaches to the Problem of Stagflation," Working Paper No. 382, National Bureau of Economic Research, presented at the 1979 Kiel Conference on Macroeconomic Policies for Growth and Stability. Forthcoming in Weltwirtschaftliches Archiv.
5. Findlay, Ronald and C.R. Rodriguez, (1977). "Intermediate Imports and Macroeconomic Policies under Flexible Exchange Rates," Canadian Journal of Economics.
6. Ford, J. and R. Halvorsen, (1978). "Substitution Among Energy, Capital, and Labor Inputs in U.S. Manufacturing," in R.S. Prindyck (ed.) Advances in the Economics of Energy and Resources, Vol. I, JAI Press, Greenwich, Conn.
7. Griffin, J.M. and P.R. Gregory, (1976). "An Intercountry Translog Model of Energy Substitution Responses," American Economic Review, Vol. 66, December 1976, pp. 845-857.
8. Hayashi, Fumio (1980). "The q Theory of Investment: A Neoclassical Interpretation," forthcoming in Econometrica.
9. Hogan, William W. (1979). "Capital-Energy Complementarity in Aggregate Energy--Economic Analysis," Energy and Environmental Policy Center, Kennedy School of Government, Harvard University.
10. Hudson, Edward and Dale W. Jorgenson (1978). "Energy Prices and the U.S. Economy, 1972-76," National Resource Journal, XVIII, October, 877-897.
11. \_\_\_\_\_ . (1979) "The Economic Impact of Policies to Reduce U.S. Energy Growth," in B. Kurnsunoglu and A. Perlumutter (eds.) Directions in Energy Policy, Cambridge: Ballinger, 141-164.
12. Lipton, David, James Poterba, Jeffrey Sachs and Lawrence Summers, (1980). "Multiple Shooting in Rational Expectations Models." NBER Technical Working Paper No. 3, August.

13. Lipton, David and Jeffrey Sachs, (1980). "Accumulation and Growth in a Two-Country Model," NBER Working Paper (in process).
14. Mork, Knut Anton (1978) The Aggregate Demand for Primary Energy in the Short Run and Long Run for the U.S., 1949-1975. Energy Laboratory Working Paper No. 7, Boston, Mass.: M.I.T.
15. \_\_\_\_\_. (1978) The Inflationary Impact of Higher Energy Prices, 1973-75. Energy Laboratory Working Paper No. 14, Boston, Mass.: M.I.T.
16. Mork, Knut Anton and Robert E. Hall, (1979). "Energy Prices, Inflation, and Recession, 1974-75." Energy Laboratory Working Paper No. 79-028WP, Massachusetts Institute of Technology, May.
17. Mussa, Michael (1979). "Empirical Regularities in the Behavior of Exchange Rates and Theories of the Foreign Exchange Market." Vol. 11 of Carnegie-Rochester Conference Series on Public Policy, Supplement to the Journal of Monetary Economics, 1979, pp. 9-57.
18. Nordhaus, William (1980). "The Interaction Between Oil and the Economy in Industrial Countries," draft prepared for Brookings Papers for Economic Activity, meeting of October.
19. Obstfeld, Maurice (1980). "Macroeconomic Policy and World Welfare under Flexible Exchange Rates." Discussion Paper No. 63, Columbia University.
20. Sachs, Jeffrey (1979). "Wages, Profits and Macroeconomic Adjustment: A Comparative Study," Brookings Papers on Economic Activity: 2.
21. \_\_\_\_\_. (1980) "Wages, Flexible Exchange Rates, and Macroeconomic Policy," Quarterly Journal of Economics, Vol. 4, June.
22. \_\_\_\_\_. (1980) "Oil Prices and Economic Growth Under Alternative Technological Assumptions," NBER Discussion Paper (in process).
23. Schmid, Michael (1979). "Oil, Employment, and the Price Level: A Monetary Approach to the Macroeconomics of Imported Intermediate Goods Under Fixed and Flexible Rates," unpublished manuscript, University of Western Ontario, (November).
24. Tobin, James (1969). "A General Equilibrium Approach to Monetary Theory," Journal of Money, Credit, and Banking, 1 (February) 15-29.