

NBER WORKING PAPER SERIES

THE USE OF VOLATILITY MEASURES  
IN ASSESSING MARKET EFFICIENCY

Robert J. Shiller

Working Paper No. 565

NATIONAL BUREAU OF ECONOMIC RESEARCH  
1050 Massachusetts Avenue  
Cambridge MA 02138

October 1980

This paper was presented at the American Finance Association Meetings, Denver, September 1980. The former title was "The Relationship Between Financial Asset Prices and Subsequent Earnings." This research was supported by the National Science Foundation under Grant #SOC 79-07561. I am indebted to Fischer Black, Olivier Blanchard, Sanford Grossman, Franco Modigliani and Jeremy Siegel for helpful discussions. The research reported here is part of the NBER's research program in Financial Markets and Monetary Economics. The views expressed here should not be attributed to the National Science Foundation or the National Bureau of Economic Research.

The Use of Volatility Measures in Assessing Market Efficiency

ABSTRACT

A number of recent papers have claimed that security prices are too volatile to accord with simple efficient market models. This paper discusses the theoretical foundations of the volatility tests used in these papers. The basic structure common to these models and its implications for volatility measures are discussed. Alternative hypotheses are explicitly considered which might justify the volatility. The volatility tests are compared with the more standard regression tests.

Robert J. Shiller  
Department of Economics  
University of Pennsylvania  
Philadelphia, Pennsylvania 19104

(215) 243-8483

CONTENTS

I.	Introduction .....	1
II.	The Simple Efficient Markets Hypothesis .....	2
III.	Derivation of Variance Inequalities .....	6
IV.	Some Alternative Hypotheses .....	11
V.	Regression Tests of the Model .....	16
VI.	Conclusion and Summary .....	21
	Figure 1 .....	23
	References .....	24

## THE USE OF VOLATILITY MEASURES IN ASSESSING MARKET EFFICIENCY

### I. Introduction

Recently a number of studies have used measures of the variance or "volatility" of speculative asset prices to provide evidence against simple models of market efficiency. These measures were interpreted as implying that prices show too much variation to be explained in terms of the random arrival of new information about the fundamental determinants of price. The first such use of the volatility measures was made independently by LeRoy and Porter [1981] in connection with stock price and earnings data, and myself [1979], in connection with long-term and short-term bond yields. Subsequently, further use of these measures was made to study efficient markets models involving stock prices and dividends (Shiller [1981]), yields on intermediate and short-term bonds (Shiller [1980], Singleton [1980]), preferred stock dividend price ratios and short-term interest rates (Amsler [1980]) and foreign exchange rates and money stock differentials (Huang [1981], Meese and Singleton [1980]). My intent here is to interpret the use of volatility measures in these papers, to describe some alternative models which might allow more variation in prices, and to contrast the volatility tests with more conventional methods of evaluating market efficiency.

My initial motivation for considering volatility measures in the efficient markets models was to clarify the basic smoothing properties of the models to allow an understanding of the assumptions which are implicit in the notion of market efficiency. The efficient markets models, which are described in section II below, relate a price today to the expected present value of a path of future variables. Since present values are long weighted moving averages, it would seem that price data should be very stable and smooth. These impressions can be formalized in terms of inequalities describing certain variances (section III). The results ought to be of interest whether or not the data satisfy these

(from Shiller [1981])  $p_t$  for 1871-1979 is the Standard and Poor Composite Stock Price Index for January (deflated by the wholesale price index scaled to base year  $T = 1979$ ) times a scale factor  $\lambda^{T-t}$  (see figure 1). The parameter  $\lambda$  equals one plus the long-term growth rate of the real value of the portfolio (1.45% per annum) and the scale factor has the effect of eliminating heteroscedasticity due to the gradually increasing size of the market. The variable  $d_t$  for 1871-1978 is the total real dividends paid over the year times  $\lambda^{T-t-1}$  (figure 1). The discount rate  $r$  was estimated as average  $d$  divided by average  $p$ , or 4.52% (while the average dividend price ratio was 4.92%). In LeRoy and Porter [1981] the approach is similar except that real earnings rather than real dividends are used. As is well known, one cannot claim price as the present value of expected earnings without committing a sort of double counting unless earnings are corrected for investment. Usually, such a correction means subtracting retained earnings from earnings, which would convert the earnings series into the dividend series (Miller and Modigliani [1961]). Instead, LeRoy and Porter correct both the price and dividend series by dividing by the capital stock, which is estimated as the accumulated undepreciated real retained earnings. Since the resulting series appear stationary over their shorter (postwar) sample period, they make no further scale correction.

In Shiller [1979],  $p_t$  is the yield to maturity on a long-term bond such as a consol times  $\gamma/(1-\gamma)$ , and corrected for a constant liquidity premium, while  $d_t$  is the one-period yield. This model is derived as an approximation to a number of different versions of the expectations model of the term structure. In Ansler [1980] the same model is applied using instead of the yield on long-term bonds her dividend price ratio series on high quality preferred stock which she argues is a proxy for a perpetuity yield. In Shiller [1980] and Singleton [1980]  $p_t$  is related to the yield to maturity on intermediate term bonds and the model is modified slightly by truncating the summation in (1) at the maturity date.\*

---

\*The former paper (Shiller [1980]) did not find any significant evidence of excess volatility with bonds maturing in less than two years.

are of questionable relevance to modern conditions and of minor interest given possible errors in data from remote times in history.

If we could summarize the results of the tests of the random walk hypothesis with recent data by saying that (2) held exactly, then, using an additional assumption that prices are not explosive, we could conclude that (1) was proven. Rearranging (2) one gets the first order rational expectations model  $p_t = \gamma E_t(p_{t+1} + d_t)$ . One can solve this model by recursive substitution. Substituting the same expression led for one period gives us  $p_t = \gamma E_t(\gamma E_{t+1}(p_{t+2} + d_{t+1}) + d_t) = \gamma^2 E_t(p_{t+2}) + \gamma^2 E_t d_{t+1} + \gamma d_t$ . One may then substitute for  $p_{t+2}$  in this expression and so on. The result is expression (1) as long as the additional requirement that  $\gamma^k E_t(p_{t+k})$  goes to zero as  $k$  goes to infinity is satisfied. That the terminal condition holds in the stock price application seems quite reasonable since the dividend price ratio shows no particular trend, while on an explosive path  $p$  and  $d$  should diverge.

There is a big difference, however, between the statement that (2) holds approximately and (2) holds exactly. Expression (2) can hold approximately if movements  $\Delta p_{t+1}$  are so large as to swamp out movements in  $(rp_t - d_t)$ , even if the movements in  $p$  do not reflect information about future  $d$  at all. For example, suppose stock prices are heavily influenced by fads or waves of optimistic or pessimistic "market psychology." Suppose this means that demeaned  $p_t = \rho p_{t-1} + \varepsilon_t$  where  $0 < \rho < 1$  and  $\varepsilon_t$  is unforecastable noise independent of  $d$ . If  $\rho$  is, say, .95, changes in  $p$  will not be highly autocorrelated, and hence  $\tilde{\Delta}_{t+1} p_{t+1}$  as defined in (2) would not be very correlated with information (the theoretical  $R^2$  of  $\tilde{\Delta}_{t+1} p_{t+1}$  on  $p_t$  and a constant would, for  $r = .045$ , be .08). The fact that in short samples one would have trouble proving the existence of such correlation is a reflection of the fact that with such an alternative it takes a very long time to take effective advantage of the profit opportunity implied by the fad. It may take, say, 10 or 20

variances along an explosive path for  $p$ . On the other hand, the assumption that  $\Delta p/p$ ,  $\Delta d/d$  and  $p/d$  are stationary would rule out an explosive solution to (2). Such an assumption may be more attractive than that made here, and may lead to similar results, but is less convenient analytically because it is a nonlinear transformation of data in a linear model and variance inequalities like those derived in the section below would depend on the distribution assumed for the variables.

Stationarity, and hence the terminal condition, is part of the maintained hypothesis and the motivation for the volatility tests is not that these tests will reject when  $p$  (or  $\Delta p$ ) is explosive in the sample. Quite to the contrary, the stationarity assumption suggests tests which may reject far more decisively than a regression test even when  $p$  (or  $\Delta p$ ) is not divergent in the sample at all, as will be seen in section V below.

### III. Derivation of Variance Inequalities

Since  $p^*$  is a weighted moving average of  $d$ , one can use simple principles of spectral analysis to put restrictions on the spectrum of  $p^*$ : its spectrum must be relatively much more concentrated in the lower frequencies than the spectrum of  $d$ , and must be everywhere below the spectrum of  $d$  except at the zero frequency. Thus, informally,  $p^*$  series ought to be "smoother" than  $d$  series. It was shown in Shiller [1979] that the same formal characterization does not apply to the spectrum of  $p$ , yet there is a sense in which  $p$  ought to be "smooth". The inequality restrictions on the spectra of  $p$  and  $d$  implied by the model in fact concern weighted integrals of the spectra, and these inequality restrictions can be stated more simply in terms of variance inequalities.

The variance inequalities implied by the model (1) can be derived from:

1. the equality restrictions between variances and co-variances imposed by the model,
2. the inequality restrictions regarding variances and covariances implied

inequalities may proceed in much the same way, though the previous papers used a somewhat different derivation. To prove the second, note that the model implies that the innovation  $\Delta p_{t+1} + d_t - rp_t$  is a forecast error which must be uncorrelated with  $p_t$ . Hence,  $\text{cov}(p_{t+1}, p_t) + \text{cov}(d_t, p_t) - (1+r)\text{var}(p_t) = 0$ . Stationarity requires that  $\text{var}(p_{t+1}) = \text{var}(p_t)$ , and hence  $\text{cov}(p_{t+1}, p_t) = \text{var}(p_t) - \frac{1}{2}\text{var}(\Delta p_t)$ . If this is substituted into the restriction we get  $-\text{rvar}(p_t) - \frac{1}{2}\text{var}(\Delta p_t) + \text{cov}(d_t, p_t) = 0$ , or, in terms of standard deviations and the correlation coefficient  $\rho$  between  $d_t$  and  $p_t$ ,  $r\sigma(p)^2 - \rho\sigma(d)\sigma(p) + \frac{1}{2}\sigma^2(\Delta p) = 0$ . Positive semidefiniteness can be interpreted as requiring all standard deviations to be positive and  $|\rho| \leq 1$ . One easily verifies that if  $\sigma(\Delta p)$  exceeds the upper bound of the inequality (I-2) then no values of  $\sigma(p)$  or  $\rho$  can be consistent with the model restriction and positive semidefiniteness. Solving the quadratic expression for  $\sigma(p)$  one finds  $\sigma(p) = (\rho\sigma(d) \pm \sqrt{\rho^2\sigma(d)^2 - 2r\sigma(\Delta p)^2})/2r$ . If a real value of  $\sigma(p)$  is to satisfy the model restriction, then the expression inside the square root operator (discriminant) must be nonnegative,  $\rho^2\sigma(d)^2 \geq 2r\sigma(\Delta p)^2$  which in turn implies, since  $\rho^2 \leq 1$ , the inequality (I-2). To prove the third inequality, note first that if  $\Delta d_t$  is jointly stationary with information variables, then the model (1) implies that both  $\Delta p_t$  and  $rp_t - d_t$  are also jointly stationary. The forecast error  $\Delta p_{t+1} - (rp_t - d_t)$  must be uncorrelated with information at time  $t$  and hence with  $(rp_t - d_t)$ , and so the model implies the restriction  $\text{cov}(\Delta p_{t+1}, (rp_t - d_t)) - \text{var}(rp_t - d_t) = 0$ . Stationarity implies  $\text{cov}(\Delta p_{t+1}, (rp_t - d_t)) = \left\{ 2\text{cov}(\Delta d_t, (rp_t - d_t)) - \text{var}(r\Delta p_t) + \text{var}(\Delta d_t) \right\} / 2r$ . One then finds, converting to standard deviations that the model restriction can be written  $2r\sigma(rp_t - d_t)^2 - 2\rho\sigma(\Delta d_t)\sigma(rp_t - d_t) + r^2\sigma(\Delta p)^2 - \sigma(\Delta d)^2 = 0$ , where  $\rho$  is the correlation coefficient between  $\Delta d_t$  and  $(rp_t - d_t)$ . One can then verify that this restriction cannot be satisfied for  $|\rho| < 1$  and positive standard deviations if (I-3) is violated, by solving the quadratic expression in  $\sigma(rp_t - d_t)$  as with the proof of (I-2).



$p$ ,  $\hat{\sigma}(p)$ , was 47.2 and  $\hat{\sigma}(p^*) = 7.51$  so that (I-1) was dramatically violated by sample statistics. The  $\hat{\sigma}(d)$  over the entire sample was 1.28, while  $\hat{\sigma}(\Delta p)$  was 24.3. The upper bound on the standard deviation of  $\Delta p$  allowed by the inequality (I-2) was thus (disregarding sampling error in the estimation of standard deviations) 4.26, so that the upper bound was exceeded by almost six fold. Putting it another way, the standard deviation of  $d$  would have to be at least 7.30, which is greater than the sample mean of  $d$ , in order to justify  $\sigma(\Delta p)$ . \* Figure 1 gives an impression as to how dramatically (I-2) is violated.

On the other hand (I-3) (which was not reported in that paper) was not violated by the data. Since  $\hat{\sigma}(\Delta d)$  was .768 for this data, the upper bound on  $\sigma(\Delta p)$  allowed by (I-3) is 59.0 which is greater than the sample  $\sigma(\Delta p)$ . Of course, we do not expect the data to violate all inequalities even if the model is wrong. Moreover, the choice of annual data was arbitrary, and the inequality is violated when longer differencing intervals are used. The definition of  $p^*$  implies  $p_t^* = \sum_{k=0}^{\infty} \bar{\gamma}^{-k+1} \bar{d}_{t+nk}$  where  $n$  is any integer greater than one,  $\bar{\gamma} = \gamma^n$  and  $\bar{d}_t = d_t(1+r)^{n-1} + d_{t+1}(1+r)^{n-2} + \dots + d_{t+n-1}$ , or the accumulated dividends over  $n$  periods. Defining the  $n$ -period rate  $\bar{r}$  implicitly by  $\bar{\gamma} = 1/(1+\bar{r})$ , we see that for data sampled at  $n$ -period intervals (I-3) becomes  $\sigma(p_t - p_{t+n}) \leq \sigma(\bar{d}_t - \bar{d}_{t+n}) / \sqrt{2\bar{r}^3 / (1+2\bar{r})}$ . \*\* Using  $n$  of ten years, for example, annual data from 1871 to 1969 produces  $\hat{\sigma}(p_t - p_{t+10}) = 69.4$  and  $\hat{\sigma}(\bar{d}_t - \bar{d}_{t+10}) = 16.5$  so that the upper bound on  $\hat{\sigma}(p_t - p_{t+10})$  is 40.8 which is violated by the data.

A sampling theory for these standard deviation and formal tests of the model can be derived if we further specify the model with distributional assumptions for  $d_t$  and  $p_t$ . If  $\Delta p_t$  is assumed normal and serially uncorrelated,

\*Even though dividends cannot be negative,  $\sigma(d)$  could exceed  $\mu(d)$  if the distribution of  $d$  were sufficiently skewed.

\*\*It is easily verified that if  $p_t$  and  $d_t$  are stationary indeterministic processes this inequality approaches (I-1) as  $n$  goes to infinity.

how big its movements will have to be if information about future real rates accounts for the stock price movements not accounted for by dividend movements according to the above analysis. Since with the stock data mentioned above the standard deviation of  $\Delta p$  is 18% of the mean of  $p$ , if we are to attribute most of the variance of  $\Delta p$  to changes in one-period real interest rates, movements in  $r_t$  must be quite large. Expression (3) effectively contains a moving average which ought to average out the movements in the one-period real interest rate. Based on a linearization which makes  $p$  equal to an expected present value of dividends plus a term related to a "present value" of real interest rates, it was concluded in Shiller [1981] that the expected one-year real interest  $r_t$  in percent would have a standard deviation in excess of 5 percentage points to account for the variance of  $\Delta p^*$ . This number is substantially in excess of the standard deviation of nominal short-term interest rates over the sample period and in fact implies a  $\pm$  two standard deviation range of over 20 percentage points for the expected real interest rate.

Because of the very large movements in real interest rates required to explain the observed variance of  $\Delta p$  by this model, some have reacted by claiming that the historical standard deviations of real dividends around their trend are inherently poor measures of subjective uncertainty about dividends. One can propose alternative models for dividends for which, say, the actual  $\sigma(d)$  is infinite, yet of course sample standard deviations will be finite and spurious as a measure of population standard deviations. The most commonly mentioned specific alternative hypothesis is that  $d_t$  is an unforecastable random walk with normal increments. We saw in the preceding section that even such a model, where  $\sigma(\Delta d)$  is finite, can be subjected to volatility tests. In the particular case of an unforecastable random walk,  $p_t = d_t/r$  and  $\sigma(\Delta p) = \sigma(\Delta d)/r$ . With

---

\*An alternative approach to the same issues is used by Pesando [1980].

the possibility of such a disaster has been called the "peso problem", referring to the fluctuation in the peso forward rate in anticipation of a devaluation that did not occur in the sample period, Krasker [1980].

To evaluate this possibility, suppose that the probability of a disaster ("nationalization") during period  $t$  as evaluated at the beginning of period  $t$  is given by a stochastic process  $\pi_t$ ,  $0 \leq \pi_t \leq 1$ , and suppose  $\pi_t$  is independent of an underlying dividend process  $d_t$  in the absence of disaster. If the disaster occurs, the stock is worthless and  $d_t$  will not be received. This means that the expected value as of time  $t$  of the dividend received at time  $t+k$  is  $E_t \left[ \prod_{j=0}^k (1-\pi_{t+j}) d_{t+k} \right]$  where  $E_t \prod_{j=0}^k (1-\pi_{t+j})$  is the probability that no disaster occurs between time  $t$  and  $t+k$ . Thus, the model (1) should be modified as:

$$p_t = E_t \sum_{k=0}^{\infty} \left( \prod_{j=0}^k (\gamma(1-\pi_{t+j})) \right) d_{t+k} \quad (4)$$

This model is identical to the model (3) if we set  $\gamma_{t+j} = \gamma(1-\pi_{t+j})$ , and collapses to our basic model (1) if the probability of disaster is constant, except that our estimated  $\gamma$  ought to be interpreted as the estimate of  $\gamma(1-\pi)$ . Clearly, then, the disaster model can explain the volatility of stock prices only if the probability of disaster changes substantially from period to period. Indeed, using the earlier conclusion about real interest rate variation the standard deviation of the probability that disaster will occur within the current year must have exceeded .05 if movements in  $p_t$  are to be attributed to new information about current and future  $\pi$ . One wonders whether the probability of disaster in a single year could have changed so much. At the very least the theory relies very heavily on the selection bias for its plausibility since such a standard deviation for  $\pi_t$  implies  $\pi_t$  must often be high. If the probability of disaster is, say, .1 each year then the probability that no disaster occurred over 108 years is roughly 1 in 100,000.

there are  $T$  observations on  $x_t$  (for 1950-60, say) and  $T$  nonoverlapping observations on  $y_t$  (for 1970-80, say). Clearly, it is not possible to test the model by regressing  $y_t$  on  $x_t$ . It does not follow that the model cannot be tested. The likelihood function is then

$$l(\sigma_x, \sigma_y | X, Y) = (2\pi\sigma_x\sigma_y)^{-T} \exp\left(-\frac{\sum x^2}{2\sigma_x^2} - \frac{\sum y^2}{2\sigma_y^2}\right).$$

The model imposes only one restriction on this likelihood function, namely that  $\sigma_x \leq \sigma_y$ . The likelihood ratio test statistic is then:

$$\lambda = \begin{cases} 1 & \text{if } \hat{\sigma}_x \leq \hat{\sigma}_y \\ \frac{\sqrt{\frac{\sum x^2 \sum y^2}{(\hat{\sigma}_x^2 + \hat{\sigma}_y^2)/2}}}{\left(\frac{\sum x^2}{\hat{\sigma}_x^2} + \frac{\sum y^2}{\hat{\sigma}_y^2}\right)^T} & \text{if } \hat{\sigma}_x > \hat{\sigma}_y \end{cases}$$

where  $\hat{\sigma}$  denotes sample standard deviation about zero. The likelihood ratio test which is the unique uniformly most powerful test with this data thus works out to be a volatility test. Since if  $\sigma_x = \sigma_y$ , the distribution of  $-2 \log \lambda$  is asymptotically a mixture of a  $\chi^2$  with one degree of freedom and a spike at the origin, one can easily find an approximate 95% critical value for  $\lambda$  for a given level of significance. The test may enable us to reject the model with confidence. If  $\hat{\sigma}_y = 0$  and  $\hat{\sigma}_x \neq 0$ ,  $-2 \log \lambda$  is infinite, so that the hypothesis would be rejected at any significance level. The power of the test approaches one as  $\sigma_y/\sigma_x$  approaches zero.

In contrast, if the data were perfectly aligned, the likelihood ratio test would amount to a  $t$  test that the coefficient is 1.00 in a regression of  $y$  on  $x$ .

written in the usual form:  $l(\mu, \Omega | z) = (2\pi)^{-3T/2} |\Omega|^{-T/2} \exp(-\frac{1}{2} \sum_{t=1}^T (z_t - \mu) \Omega^{-1} (z_t - \mu)')$  where  $z$  is the  $T \times 3$  matrix of observations,  $\mu$  is the  $1 \times 3$  vector of means and  $\Omega$  the  $3 \times 3$  covariance matrix. Since  $\Omega$  is symmetric, the model has nine parameters: the three means, the three variances, and the three covariances. However, because of the stationarity assumption  $E(p_t) = E(p_{t+1})$  and  $\text{var}(p_t) = \text{var}(p_{t+1})$  there are only 7 independent parameters, a fact we shall consider below.

Given the vector  $z_t$ , we can by a linear transformation derive the vector  $w_t = [p_{t+1} + d_t, p_t, d_t]$  which has as its first element the dependent variable in the regression described above and the same second and third elements. Since  $z_t$  is normally distributed,  $w_t$  is also. We can therefore write the likelihood function for  $w_t$  by a change of variables from the likelihood function (of  $z_t$ ). This likelihood function can be written in various ways. For our purposes, it is convenient to write the likelihood function for  $w$  in a factored form which makes it easy notationally to consider regression tests of the model. Partitioning  $w_t$  into  $w_t = [w_{1t}, w_{2t}]$  where  $w_{1t}$  is a  $1 \times 1$  and  $w_{2t}$  is  $1 \times 2$ , and letting  $w, w_1$  and  $w_2$  denote the matrices of observations or orders  $T \times 3, T \times 1$  and  $T \times 2$  respectively, and defining a  $T$  element vector  $Y, Y = w_1$  of dependent variables and a  $T \times 3$  matrix  $X, X = [L w_2]$  of independent variables where  $L$  is a column vector of 1's, then the likelihood function is:

$$\begin{aligned}
 l(\sigma^2, \beta, \phi, \mu | w) &= \\
 l(\sigma^2, \beta | w_1, w_2) l(\phi, \mu | w_2) &= \\
 (2\pi\sigma^2)^{-T/2} \exp(-(Y - X\beta)'(Y - X\beta)/2\sigma^2) \cdot \\
 (2\pi)^{-T} |\Phi|^{-T/2} \exp(-\sum_{t=1}^T (w_{2t} - v)\Phi^{-1}(w_{2t} - v)'/2) & \quad (5)
 \end{aligned}$$

The mean restriction becomes  $\beta_1 = (1-\beta_2)\mu_p - \beta_3 \mu_d$  and the variance restriction becomes  $\sigma^2 = (1-\beta_2^2) \text{var}(p) + 2\beta_2(1-\beta_3)\text{cov}(p,d) - (1-\beta_3)^2\text{var}(d)$ . With these restrictions across the two factors of the likelihood function the second factor is no longer irrelevant to the likelihood ratio, and regression tests are no longer appropriate. If we make minus two times the log likelihood ratio our test statistic, then the power of the test approaches one as  $\sigma(d)$  goes to zero so long as  $\sigma(p)$  does not equal zero. If  $\sigma(d)$  equals zero, the above variance restriction reduces to  $\sigma^2 = (1-\beta_2^2)\text{var}(p)$ . Under the null hypotheses,  $\beta_2 = 1 + r > 1$ , so this restriction cannot be satisfied unless  $p$  does not vary, which is rejected with probability one if  $\sigma(p) > 0$ . In this case, as in our simple example with nonoverlapping observations, the sample will give  $-2 \log \lambda = \infty$ , while, whether or not  $p$  appears explosive in the sample  $-2 \log \lambda$  for the regression test will not be infinite. The likelihood ratio test thus has power of one in a region of the parameter space where the regression test does not. In this region, the volatility test, using (I-2) and a test statistic  $\hat{\sigma}(\Delta p)/\hat{\sigma}(d)$  also has power equal to one. Thus, if volatility tests reject decisively and regression tests do not, one may infer that in fact the parameters are likely to lie near this region.

## VI. Conclusion and Summary

The various papers discussed here, which attempted to provide evidence against simple efficient markets models by showing that prices are too volatile, share a common econometric motivation. By assuming that both  $p_t$  (real stock prices in our main example) and  $d_t$  (real dividends), or some transformations of these, are stationary stochastic processes, it is possible to inquire whether the movements in  $p_t$  are "too big" to be explained by the model. With these assumptions, the conventional regression tests of the model are no longer suggested by the likelihood

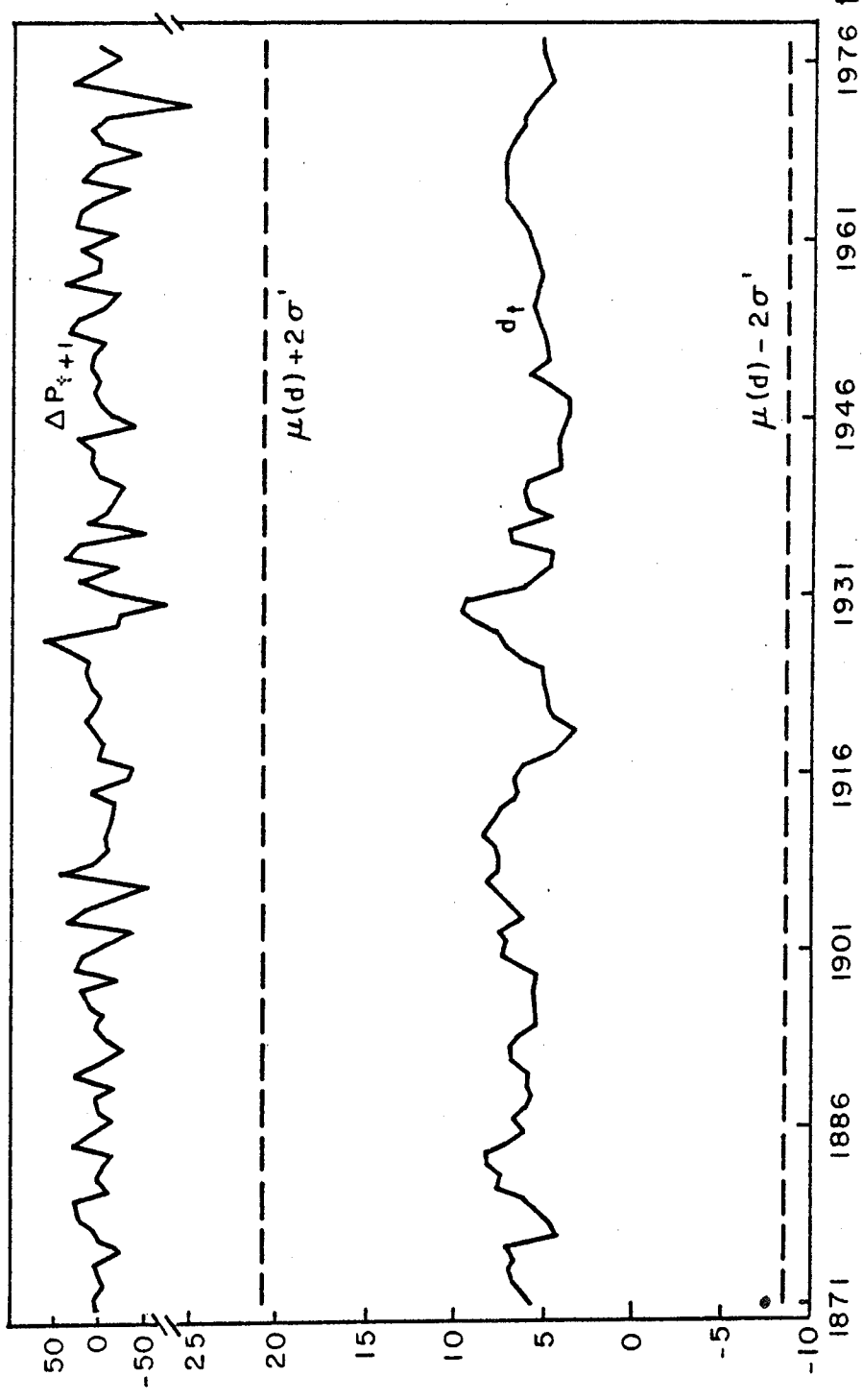


Figure 1: Upper plot:  $\Delta P_{t+1}$  where  $P_t$  is the real Standard and Poor Composite Stock Price Index for January scaled (to eliminate heteroscedasticity) by multiplying  $\exp(.0145(1979-t))$ . Lower plot  $d_t$ : real dividends for the  $P_t$  series times  $\exp(.0145(1978-t))$ . Here,  $\hat{\sigma}(d) = 1.28$ . The dividend series here ranges from 3.31 to 9.62, a range of nearly 5 standard deviations. The dashed lines mark off a  $\pm 2$  standard deviation range from the mean  $\mu(d)$  where the standard deviation is  $\sigma' = 7.30$ , or the lower bound allowed for  $\sigma(d)$  by inequality (I-2), given  $\sigma(\Delta p) = 24.3$ . In order to justify this magnitude of  $\sigma(\Delta p)$ , the true  $\sigma(d)$  would have to be at least this high according to (I-2).

Shiller, Robert J., "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?", forthcoming, American Economic Review, 1981.

Shiller, Robert J., "The Volatility of Long-Term Interest Rates and Expectations Models of the Term Structure," Journal of Political Economy 87: 1190-1219, December 1979.

Singleton, Kenneth, "Expectations Models of the Term Structure and Implied Variance Bounds," forthcoming, Journal of Political Economy, 1980.



<u>Number</u>	<u>Author</u>	<u>Title</u>	<u>Date</u>
529	Claudia Goldin	The Historical Evolution of Female Earnings Functions and Occupations	8/80
530	Daniel Feenberg	Does the Investment Interest Limitation Explain the Existence of Dividends?	8/80
531	Willem H. Buiter and Jonathan Eaton	Policy Decentralization and Exchange Rate Management in Interdependent Economies	8/80
532	Anne O. Krueger and Baran Tuncer	Microeconomic Aspects of Productivity Growth Under Import Substitution: Turkey	8/80
533	Fischer Black	The Tax Advantages of Pension Fund Investments in Bonds	8/80
534	Herschel I. Grossman	Incomplete Information, Risk Shifting, and Employment Fluctuations	8/80
535	Charles Freedman and David Longworth	Some Aspects of the Canadian Experience with Flexible Exchange Rates in the 1970s	8/80
536	Kenneth L. Stanley, Wilbur G. Lewellen, and Gary C. Schlarbaum	Further Evidence on the Value of Professional Investment Research	8/80
537	Robert E. Cumby and Maurice Obstfeld	Exchange-Rate Expectations and Nominal Interest Differentials: A Test of the Fisher Hypothesis	8/80
538	Ray C. Fair	Estimated Effects of the October 1979 Change in Monetary Policy on the 1980 Economy	8/80
539	Victor R. Fuchs	Time Preference and Health: An Exploratory Study	8/80
540	Maurice Obstfeld	Intermediate Imports, The Terms of Trade, and the Dynamics of the Exchange Rate and Current Account	9/80
541	Eliana A. Cardoso and Rudiger Dornbusch	Three Papers on Brazilian Trade and Payments	9/80
542	David S. Jones	Expected Inflation and Equity Prices: A Structural Econometric Approach	9/80

<u>Number</u>	<u>Author</u>	<u>Title</u>	<u>Date</u>
558	John F. Boschen and Herschel I. Grossman	Tests of Equilibrium Macroeconomics Using Contemporaneous Monetary Data	10/80
559	Bennett T. McCallum	Price Level Determinacy with an Interest Rate Policy Rule and Rational Expectations	10/80
560	Robert E. Hall	The Importance of Lifetime Jobs in the U.S. Economy	10/80
561	Ariel Pakes and Zvi Griliches	Patents and R & D at the Firm Level: A First Look	10/80
562	Alan S. Blinder, Roger H. Gordon and Donald E. Wise	Reconsidering the Work Disincentive Effects of Social Security	10/80
563	Robert J. Shiller	Alternative Tests of Rational Expectations Models: The Case of the Term Structure	10/80
564	Sanford J. Grossman and Robert J. Shiller	The Determinants of the Variability of Stock Market Prices	10/80
565	Robert J. Shiller	The Use of Volatility Measures in Assessing Market Efficiency	10/80

Note: Copies of the above working papers can be obtained by sending \$1.00 per copy to Working Papers, NBER, 1050 Massachusetts Avenue, Cambridge, MA 02138. Advance payment is required on orders totaling less than \$10.00. Please make check payable to National Bureau of Economic Research, Inc.