NBER WORKING PAPER SERIES

GOLD MONETIZATION AND GOLD DISCIPLINE

Robert P. Flood

Peter M. Garber

Working Paper No. 544

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge MA 02138

September 1980

This research was partly supported by NSF grant SES-7926807. This paper was presented at the research meeting of the NBER Program on Economic Fluctuations, University of Rochester, July 24-25, 1980. The research reported here is part of the NBER's research program in Economic Fluctuations. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

NBER Working Paper #544
September, 1980

Gold Monetization and Gold Discipline

ABSTRACT

The paper is a study of the price level and relative price effects of a policy to monetize gold and fix its price at a given future time and at the then prevailing nominal price. Price movements are analyzed both during the transition to the gold standard and during the post-monetization period. The paper also explores the adjustments to fiat money which are necessary to ensure that this type of gold monetization is non-inflationary. Finally, some conditions which produce a run on the government's gold stock leading to the collapse of the gold standard and the timing of such a run are examined.

Robert P. Flood, Jr.
Department of Economics
University of Virginia
Charlottesville, Virginia 22901

(804) 924-7894

Peter M. Garber
Department of Economics
University of Rochester
Rochester, New York 14627

(716) 275-4320

"Money" is a theoretical concept whose imperfect realization has assumed the form of a sequence of particular physical objects. Since the supply process for each particular physical object is unique, an object which is "money" in one epoch may be supplanted in another epoch by an object whose supply process is (perhaps temporarily) superior. While there exists a flow of objects through the category called "money", the sequence of "money" objects in modern times has been so limited that it can readily be characterized as a "gold-fiat-gold" cycle.

Given the recurrent use of gold as a monetary standard, economic agents will believe that gold may be restored by a government as a money, even in an epoch in which gold is demonetized. Fluctuations in the intensity of such beliefs must then be reflected in gold price fluctuations.

Recently, large movements in the price of gold have initiated two lines of thought concerning government gold market policy. First, Salant and Henderson 1978 (S-H hereafter), have developed a partial equilibrium gold market model to analyze the interaction between rapid gold price rises and government gold auctions. Second, Laffer (1979) has advocated a return to the monetary gold standard.[1] Simultaneously, Barro (1979) has constructed a model to study money and price dynamics under a gold standard. The present paper integrates these approaches, together with a method recently devised by Krugman (1979) to study exchange crises, to construct a framework for simultaneously analyzing government gold and monetary policy. The model which we develop is suitable both for the study of historical gold standards and for the analysis of a future gold standard.

Specifically, we wish to study three questions which arise in the context of transition to and from a monetary gold standard:

(i) What is the inflationary effect of a policy to monetize gold at some future date?

(ii) Given that a gold standard will be adopted, which monetary policy is consistent with price stability and minimum disruption of the gold market during the transition to monetization?

(iii) If a monetary authority implements a gold standard but does not adhere to the "discipline" of the standard, what is the anatomy of the gold standard's collapse?

Questions (i) and (ii) address recent plans for monetization while question (iii) is posed to provide both some perspective for realistic policy discussion and some tentative steps toward understanding the breakdown of the gold standard in the 1930's. In addressing question (iii) we also propose a technical definition of the concept called "the discipline of the gold standard"; the definition allows us to determine if a particular monetary policy is consistent with "gold discipline".

We present our analysis in four sections. In section I we introduce the components of our model and develop equilibrium price paths for a world where agents believe gold will never be money. In section II we study gold's price path following an announcement that gold will be monetized at a fixed future date, with gold's price pegged at the prevailing market level on that date. Section III contains our analysis of the price level effects of monetization along with our model's prescription for monetization without inflation. In section IV we study the forced demonetization of gold, the collapse of a gold standard. Section IV is followed by some concluding remarks and our technical appendix. To maintain clarity of exposition most results will be introduced in the text without proof; the detailed algebra involved in the proofs will be left either to the appendix or to footnotes.

I) The World Without Monetary Gold

We will first analyze a world in which gold has no monetary use. Such a world is very similar to that studied by S-H, except there will be no government auctions. Throughout the paper we employ a continuous time model in which agents have perfect foresight. To minimize technical complexity, we present our ideas in the context of a specific, linearized example.

We divide our economy into two sectors, a gold sector and a monetary sector. Both the output of goods other than gold and the real rate of interest r are fixed at constant levels exogenously to the gold and monetary sectors.[2]

a) The Gold Sector

The operation of the gold market can be described by equations (1)-(3):

(1) $I = D(t) + G(t)$

(2) $\dot{D}(t) = v[D^*(q(t)) - D(t)] \quad , \quad D^{*\prime} < 0$

(3) $\dot{q}(t) = rq(t) \quad \text{for} \quad G(t) \neq 0.$

I is a fixed, total world stock of gold.[3] D(t) is the quantity of gold which has been transformed for industrial or consumption purposes (or which remains in the ground). G(t) is the quantity of gold privately held as ingots or coins in speculative hoards. $D^*(q)$ is a target for the sum of consumption and industrial demand, which depends negatively on q(t) the relative price of gold in terms of other goods.[4,5] v is a positive, constant speed of adjustment which may, if desired, depend on r; and we assume $v \neq r$.[6]

Equation (1) states that the total supply of gold privately available is held either in speculative hoards as ingots or in the form of jewelry or other consumption goods. Equation (2) is an adjustment equation which indicates the rate at which actual non-speculative holdings adjust to desired non-speculative holdings. Equation (3) is a requirement that the relative price of gold must increase at the real rate of interest for any gold to be held in speculative hoards.[7]

Our gold sector model is similar to the gold model developed by S-H. We depart from S-H in our equation (2) which allows consumers and producers to disgorge gold from final uses. The S-H model precludes the possibility of gold's return to speculative hoards after its conversion to final use. We have adopted this modification both because it captures the recently observed disgorging of gold from consumption stocks and because it is technically convenient to allow $\dot{D}$ to bear no prior constraints.[8]

Equations (1)-(3) form a system of differential equations in D(t) and q(t). We assume that

$$(4) \qquad D^*(q(t)) = \frac{\delta}{q(t)}$$

where δ is a constant.

In the remainder of this section, we solve (1)-(4) for q(t) and D(t). Equation (3) yields

$$(5) \qquad q(t) = q(0)e^{rt} \qquad \text{when } G(t) > 0.$$

Substituting from (5) and (4) in (2) we derive

$$(6) \qquad \dot{D}(t) = -vD(t) + \frac{v\delta}{q(0)} e^{-rt} \ ,$$

which has the solution

$$(7) \qquad D(t) = D(0)e^{-vt} + ve^{-vt} \int_0^t \frac{\delta}{q(0)} e^{(v-r)\tau} \, d\tau$$

To complete the solution in (7) we require an initial condition for D(0) and a condition on speculative gold holding to determine q(0). First, we assume that D(0) = 0.[9] Second, if T is the date when $\dot{D}(t) = 0$, G(T) must equal zero, i.e. the speculative gold stock must be exhausted when there is no further accumulation of non-speculative gold. This requirement is the terminal condition imposed by S-H, where speculative gold is held only in anticipation of future non-speculative gold accumulation.[10] Since $G(T) = 0$,

$$(8) \qquad I = D(T) = D^*(q(T))$$

From equations (4) and (8), we can determine the relative price of gold at the time that speculative hoards are exhausted,

$$q(T) = \frac{\delta}{I} ,$$

which, together with equation (5), yields

$$(9) \qquad q(0) = \frac{\delta}{I} e^{-rT} .$$

In addition, the condition that $I = D(T)$, the non-speculative gold accumulation equation, (7), and our assumed value $D(0) = 0$ imply

$$(10) \qquad I = ve^{-vT} \int_0^T \frac{\delta}{q(0)} e^{(v-r)\tau} d\tau = \frac{ve^{-vT}\delta}{q(0)} \frac{(e^{(v-r)T}-1)}{(v-r)} .$$

Substituting for $q(0)$ in (10) we can solve for T,[11]

$$(11) \qquad T = \frac{\log r - \log v}{r-v} ,$$

which with equation (9) implies

$$(12) \qquad q(0) = \frac{\delta}{I} e^{-r[\frac{\log r - \log v}{r-v}]} .$$

b) The Monetary Sector

These results indicate that the relative price of gold and the quantity of gold held in various categories are determined by the operation of the gold sector alone. In order to determine nominal prices, we must append a monetary sector to our model. The operation of this sector will have no real effect in a model in which gold has no monetary function; we introduce the monetary sector at this point to serve as a preliminary to the analysis in Section II.

We assume that money market equilibrium is given by[12]

$$\frac{M(t)}{P(t)} = \beta - \alpha[r + \frac{\dot{P}(t)}{P(t)}] \quad (13)$$

or

$$M(t) = (\beta - \alpha r)P(t) - \alpha\dot{P}(t) \quad (14)$$

where P(t) is the price level for all goods, M(t) is the exogenous money supply and $(\beta-\alpha r) > 0$.

In this section we assume that $M(t) = \bar{C}$, where $\bar{C}$ is the (constant) quantity of currency in circulation. The solution to (14) dictated by market fundamentals is[13]

$$P(t) = \frac{\bar{C}}{\beta-\alpha r} . \quad (15)$$

The nominal price path for gold is obtained by multiplying q(t), from equation (5), by P(t), from (15) with q(0) given by equation (12).

II) The World with an Announced Gold Monetization

The monetization policy studied in this section will consist of a government's unanticipated announcement that at a future time t=w it will fix the price of gold at the nominal level prevailing at w and that from w onward speculative gold holdings can be used as money.[14,15] The announcement is assumed to occur at time ε, where $\varepsilon<w<T$.[16]

This policy differs from the relative gold price pegging policy studied by S-H in that gold, which can be used for monetary purposes, will generate the same service return that any money yields and in that we assume that the nominal price is pegged. However, the method which we employ to analyze the problem will be quite similar to that of S-H: we first determine a set of conditions which must prevail at the time of monetization and then solve backward to find the initial relative price at time ε implied by profit maximizing behavior.

a) The World after Gold is Money

Our goal in this part is to derive an expression for the relative price of gold at the time of monetization w; such an expression will be essential in linking the gold standard world to the basic system studied in section I. Because of the tedious algebra required to determine the path of the relative price of gold, we will only present a sketch of the required steps in the main text and then skip to the relative price solution. We leave the details to the appendix.

We assume that only gold held in speculative hoards is a perfect substitute for currency. If $Q(w) = \bar{Q}$ is the nominal price of gold at t=w, equilibrium in the money and gold markets for $t \geq w$ requires

$$\frac{\bar{C} + \bar{Q}G(t)}{P(t)} = (\beta-\alpha r) - \alpha\frac{\dot{P}(t)}{P(t)} \tag{16}$$

(17) $\dot{D}(t) = v[\frac{\delta}{q(t)} - D(t)]$

Equation (16) is equation (13) with the additional gold component added to the money supply; the nominal price of gold is pegged at $\bar{Q} = Q(w)$ for $t \geq w$. Substitution from equation (4) into equation (2) produces equation (17). It will be convenient to transform (16)-(17) into

(18) $\dot{P}(t) = (\frac{\beta-\alpha r}{\alpha})\ P(t) - \frac{\bar{Q}}{\alpha} G(t) - \frac{\bar{C}}{\alpha}$

(19) $\dot{G}(t) = \frac{-v\delta}{\bar{Q}} P(t) - vG(t) + vI$,

which are two linear differential equations in G(t) and P(t).[17] Equation (19) is derived by substituting for D(t) from equation (1).

Since the determinant of the system (18)-(19) is negative, the characteristic polynomial has a positive root λ_1, and a negative root λ_2; therefore, the system exhibits saddlepoint stability. Steady-state values of the system are $P^* = [\bar{C} + \bar{Q}I]/(\beta-\alpha r+\delta)$ and $G^* = [I(\beta-\alpha r) - \delta\bar{C}/\bar{Q}]/(\beta-\alpha r+\delta)$. The stable solution for $t \geq w$ obeys

(20) $P(t) = \frac{1}{\Lambda_2} G(t) + [P^* - \frac{1}{\Lambda_2} G^*]$

where $\Lambda_2 \equiv -v\delta/(\bar{Q}(v+\lambda_2))$. Equation (20), which we depict in Figure 1 as the line ss is the stable branch of (18) and (19). (See appendix for these derivations.)

We are interested in finding the relative price of gold at time w. Equation (20), with t=w, provides one equation in the unknowns P(w), $Q(w) = \bar{Q}$, and G(w). The system is completed by using, in addition to (20), equation (17) which describes the evolution of non-speculative gold holding and equation (16), money market equilibrium. In solving the above we use the requirement $\dot{P}(w) = \frac{1}{\Lambda_2} \dot{G}(w)$. Some steps in the solution of this system are carried out in the appendix. Presently, we require only the expression for the relative

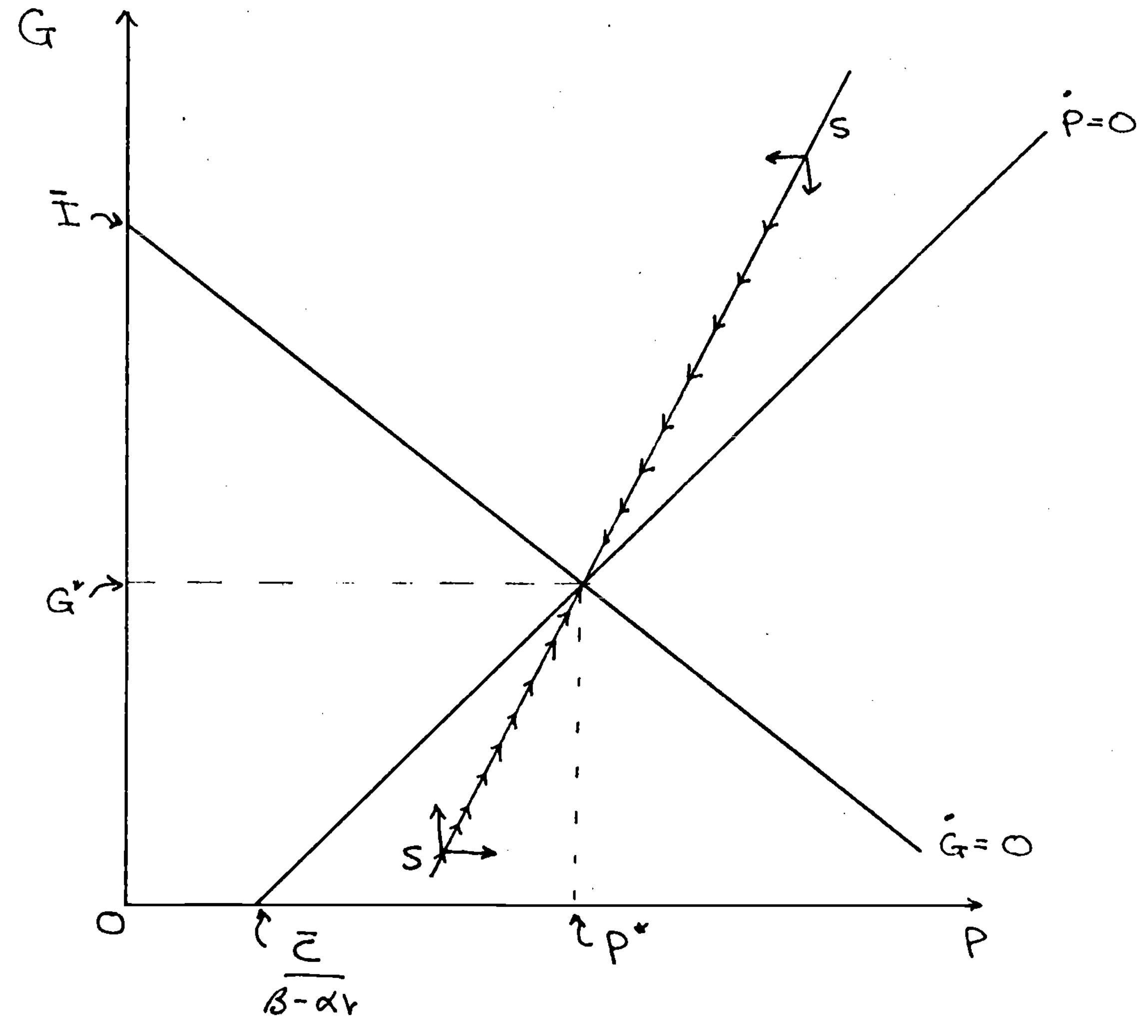

Figure I

Showing the Stable Branch (Equation 20) After Gold is Monetized price of gold at time w, which is

$$\frac{Q(w)}{P(w)} = \frac{v\delta e^{(r-v)(w-\varepsilon)} - r\delta}{(v-r)D(\varepsilon)e^{-v(w-\varepsilon)}} \tag{21}$$

The relative price of gold depends on the quantity of gold used for non-speculative purposes at the time of the monetization announcement, on the speed of adjustment of non-speculative gold holdings, on the real rate of interest, and on the long run non-speculative gold holding parameter, δ.

We wish to compare the relative price of gold in a world with monetary gold to the relative price derived in section I. To do this we consider the relative prices at time ε. In particular, we will show that $Q(\varepsilon)/P(\varepsilon)$ jumps at the time of the announcement as long as $w<T$.

From equation (5) we need only discount $Q(w)/P(w)$ by $e^{-r(w-\varepsilon)}$ to find

$$\frac{Q(\varepsilon)}{P(\varepsilon)} = \frac{v\delta e^{(r-v)(w-\delta)} - r\delta}{(v-r)D(\varepsilon)e^{(r-v)(w-\varepsilon)}} , \tag{22}$$

the relative price of gold in the instant after the impending gold standard is announced. From equation (9), the relative price at the instant before the announcement is $q(\varepsilon) = \frac{\delta}{I} e^{-r(T-\varepsilon)}$; and from equation (7) we have $D(\varepsilon) = \frac{vI}{v-r} e^{rT}[e^{(v-r)\varepsilon}-1]e^{-v\varepsilon}$. We may use these expressions to write the ratio of the post- and pre-announcement relative prices as

$$\frac{Q(\varepsilon)/P(\varepsilon)}{q(\varepsilon)} = \frac{ve^{(r-v)(w-\varepsilon)} - r}{v[1-e^{(r-v)\varepsilon}]e^{(r-v)(w-\varepsilon)}} \tag{23}$$

This ratio equals unity when $w = \frac{\log r - \log v}{r-v} = T$. Thus, if gold is to be monetized after all gold enters non-speculative uses, the solution is identical to that in section I. It can easily be shown that $\frac{d[Q(\varepsilon)/P(\varepsilon)}{dw} < 0$; so if $w<T$, $\frac{Q(\varepsilon)/P(\varepsilon)}{q(\varepsilon)} > 1$, i.e. the announcement of monetization causes the relative price of gold to jump upward.[18]

III) Inflation and the Gold Standard

In this section we analyze the inflationary effects of the gold monetization policy proposed in section II. In addition we consider the monetary policy which must accompany gold monetization in order to prevent price level movements.

To begin, we note in Figure I that the pre-announcement price, $P(\varepsilon)$, is $\bar{C}/(\beta-\alpha r)$, the horizontal intercept of the $\dot{P}=0$ schedule. The line ss is the system's saddlepoint path after gold monetization; therefore, it indicates the path for $G(t)$, $P(t)$ for $t \geq w$.

With these facts we can readily establish that $P(w)>P(\varepsilon)$. Since $P(w)$ is on ss and ss lies entirely to the right of $P(\varepsilon)$ it follows that $P(w)>P(\varepsilon)$.

Next, we consider the behavior of P at time ε and during the period $(\varepsilon,w]$. While P may jump in an unforeseen manner at time ε, agents cannot expect it to move discontinuously in the future; otherwise, the expected rate of price change would be infinite at the time of a jump, producing money market disequilibrium. Therefore, P moves smoothly (differentiably) to $P(w)$ during $(\varepsilon,w]$.

We can easily show that during the period $(\varepsilon,w]$, $\dot{P}>0$. If, on the contrary $\dot{P}\leq 0$ at some time in the interval, the money market can clear only if $P\leq P(\varepsilon)$ since $\bar{C}$ is unchanged before w. This precludes the smooth rise of price to $P(w)$. Since $\dot{P}(t)>0$, $t\varepsilon(\varepsilon,w]$, $P(t)$ must lie between $P(\varepsilon)$ and $P(w)$; hence, the initial jump after the monetization announcement is also to a price between $P(\varepsilon)$ and $P(w)$.

To summarize, we find that the price level jumps simultaneously with the monetization announcement and rises smoothly until the date of monetization. However, after time w, the price level and monetary gold holdings will follow the path ss in Figure I. If $G(w)<G^*$, then price will rise in a gold inflation,

ultimately converging to P^*. If $G(w)>G^*$, the price level will decline in a gold deflation as gold is extracted from circulation and used for consumption. The price level will be stable after time w only if $G(w)=G^*$.

Monetization without Inflation

Evidently, a government announcement of eventual gold monetization cannot succeed as a policy to avoid inflation in the absence of other measures. Therefore, we wish to employ our framework to explore how a government simultaneously can announce the monetization of gold, avoid inflation and minimize the disruption of the gold market.

The policy variable available to the government is the amount of non-gold money, C, which we have previously set at the constant level $\bar{C}$. We require a time path for C which will cause the nominal prices of gold and goods to follow

(24a) $\dot{P}(t) = 0 \qquad t \geq \varepsilon$

(24b) $\dot{Q}(t) = rQ(\varepsilon)e^{r(t-\varepsilon)} \qquad \varepsilon \leq t \leq w$

(24c) $\dot{Q}(t) = 0 \qquad t \geq w$

given monetization at w. Condition (24a) requires P to remain constant indefinitely. (24b) and (24c) require Q both to move continuously at the rate r between ε and w and to remain constant after w; in particular, (24b) prevents Q from jumping discontinuously at time ε.

We can use the money market to determine the required path for C:

(25a) $C(t) = C(\varepsilon) \qquad , \varepsilon \leq t < w$

(25b) $C(t) = P(\varepsilon)[\beta - \alpha r] - Q(\varepsilon)e^{r(w-\varepsilon)}G(w) \qquad t=w$

(25c) $\dot{C}(t) = -Q(\varepsilon)e^{r(w-\varepsilon)}\dot{G}(t) \qquad t \geq w$

In the above equations, $G(w)$ equals $I-D(w)$; and $D(w)$ can be derived from (7) as

$$(26) \qquad D(w) = D(\varepsilon)e^{-v(w-\varepsilon)} + \frac{P(\varepsilon)v\delta e^{-vw}\{e^{(v-r)w}-e^{(v-r)\varepsilon}\}}{Q(\varepsilon)(v-r)}.$$

In addition, for $t \geq w$, $G(t)$ can be determined from equation (19) by setting $P(t)=P(\varepsilon)$.

From time ε until w, the policy requires that C remains constant. At time w, C must decline by the amount $Q(w)G(w)=Q(\varepsilon)e^{r(w-\varepsilon)}G(w)$ to accommodate monetized gold. Following w, the government stabilizes price by exactly balancing with currency injections or withdrawals the value of gold which leaks into or out of consumption and industrial use.

The only component of the policy lacking in simplicity is (25b); this element's complexity arises from the goal of minimizing the disruption to the gold market. Indeed, any discrete reduction in C at time w would, when combined with (25a) and (25c), stabilize P. However, any reduction in C other than (25b) will disturb the gold market.[19]

IV) Anatomy of a Crisis (Demonetization) in the Gold Market

In previous sections we did not account for government gold reserves since, except for fixing the price of gold, the government was passive; merely its willingness to intervene was sufficient to produce the derived results. However, if the government adopts additional policies which potentially conflict with the maintenance of a gold standard, we may no longer treat gold market intervention as implicit; rather, we must directly consider the government's role in the gold market.

In this section we introduce government gold reserves, R, and initially assume that the government sets the overall money growth rate at the positive constant g. With finite R such a money growth policy must lead to a breakdown of the gold standard.[20] In addition, it is possible for the gold standard to break down even if money is decreasing, providing that prior to the inception of the gold standard there was "too much" money in existence.

A Breakdown with Constant Money Growth

We can readily show that the gold standard must collapse when money grows at a constant, positive rate. If there were no breakdown and money grew at the rate g, then P must rise at the rate g. Since P rises, $q=\bar{Q}/P$ must decline at the rate g; then $D^*=\delta/q$ rises at the rate g. Therefore, D^* and D increase toward infinity, which is incompatible with a finite R. It follows that the gold standard must collapse at some time.

Our analysis of the dynamics of a gold standard collapse draws directly from Krugman's (1979) study of the breakdown of a fixed exchange rate system. In Krugman's paper, a small country's monetary authority fixes the price of foreign exchange and expands the domestic component of the money supply at a positive, constant rate. Eventually, a crisis must occur; there is a run on the monetary authority's foreign exchange holdings and an end to the fixed exchange rate.

In Krugman's model the exchange rate and the price level are essentially the same variable; no relative price effects arise from the crisis. Our model of a gold market crisis displays the Krugman-type run on government stocks, but it extends his analysis by allowing for real effects of the crisis.

We now define $\bar{I}$ as the total stock of gold inclusive of government reserves,

$$(27) \qquad \bar{I} = R(t) + G(t) + D(t) \ .$$

While gold is money, the money supply is, as before, $M(t) = C(t) + \bar{Q}G(t)$. If the government sets M on a "crisis path", rising at the positive rate g, then at some time z a crisis must occur in the gold market. At the time of the crisis $\bar{Q}G(z)$ will be demonetized; also, $\bar{Q}R(z)$ in currency will be exchanged for government gold during the run on government stocks. In all, the money stock at the instant of the crisis must fall by

$$(28) \qquad \bar{Q}(R(z) + G(z)) = \bar{Q}(\bar{I} - D(z)) \ ,$$

where the equality follows from (27). Thus, the time path of the money stock is

$$(29a) \qquad M(t) = M(0)e^{gt} \ , \qquad t<z$$

$$(29b) \qquad M(t) = M(0)e^{gz} - \bar{Q}[\bar{I}-D(z)], \qquad t=z$$

$$(29c) \qquad M(t) = M(z)e^{g(t-z)} \ , \qquad t\geq z$$

We should emphasize that while gold is money the government may control the growth rate of M but may not control the division of M between C and $\bar{Q}G$. Because the size of G is indeterminate, the size of R is also indeterminate. However, for the problem of determining the time of the crisis , only the allocation of $\bar{I}$ between D and (R+G) is important.

Next we will employ (29a)-(29c) to determine both the pre-crisis price path and the time of the crisis. Crucial to our analysis is the recognition that speculators will force the crisis to take place without any foreseen jumps in P or Q.

The time path for P is obtained by solving the equation $M(t) = P(t)[\beta-\alpha r] - \alpha\dot{P}(t)$ for

(30) $$P(t) = e^{\lambda t}\int_t^{\infty} \frac{M(\tau)}{\alpha} e^{-\lambda\tau} d\tau$$

where $\lambda = \frac{\beta-\alpha r}{\alpha} > 0$. For $t \leq z$ we use equations (29a)-(29c) in (30) to obtain

(31) $$P(t) = \frac{-e^{\lambda t}}{\alpha(g-\lambda)} \{M(0)[e^{(g-\lambda)t}] - \bar{Q}[\bar{I}-D(z)]e^{-\lambda z}\} .$$

For t=z (31) becomes

(31a) $$P(z) = \frac{-M(z)}{\alpha(g-\lambda)} = \frac{-\{M(0)e^{gz} - \bar{Q}[\bar{I}-D(z)]\}}{\alpha(g-\lambda)} .$$

Both (31) and (31a) depend on the, as yet, unknown values of D(z) and z. To find these values we use both our knowledge about consumption gold accumulation and the condition that Q cannot jump at the moment of crisis. After the crisis, the relative price of gold can be determined from the model in section I as

(32) $$q(z) = \frac{\delta}{\bar{I}} e^{-r(T-z)} ,$$

where T, as before, is the choke date. Since $D(T) = \bar{I}$, we require

(33) $$I = D(z)^{-v(T-z)} + ve^{-vT}e^{rz}\int_z^T [\frac{\delta e^{(v-r)\tau}}{q(z)}] d\tau$$

or

(33a) $$1 = \frac{D(z)}{\bar{I}} e^{-v(T-z)} + \frac{v}{v-r} [1 - e^{(r-v)(T-z)}].$$

Equation (33a) follows directly from (33), and (33) is a reinterpreted version of equation (7). Equation (33a) may, in principle, be solved for T as a function of D(z), z, and $\bar{I}$, i.e.

(34) $$T = \tilde{T}(D(z),\ z,\ \bar{I}). \text{ with } \frac{\partial\tilde{T}}{\partial z} = 1,\ \frac{\partial\tilde{T}}{\partial D} > 0,\ \frac{\partial\tilde{T}}{\partial I} < 0.$$

The requirement that gold's price does not jump at time z is

(35) $$q(z) = \bar{Q}/P(z).$$

Since q(z) is identical in (32) and (35), we substitute from (34) to derive

(36) $$\frac{\bar{Q}}{P(z)} = \frac{\delta}{\bar{I}}\, e^{-r[\tilde{T}(D(z),\ z,\ \bar{I})-z]}$$

Equations (31a) and (36) are two equations in the three unknowns P(z), D(z), and z. The third equation, which can be used to complete the solution, is the accumulation equation

(37) $$D(z) = D(0)e^{-vz} + ve^{-vz}\int_0^z \frac{\delta}{q(\tau)}\, e^{v\tau}\, d\tau$$

It is evident that closed form solutions to z are unlikely, even in simple models. In practice, numerical techniques would be required to derive z for particular parameter values.

A Gold Standard Inconsistent Currency Level

That a gold standard must eventually collapse from an inflationary money creation process agrees with intuition. Less obvious is the possibility that a gold standard may collapse in a period of monetary and price decline.

As an example of such a case, we generalize the post-monetization dynamics of equations (18) and (19) by allowing for government reserve holdings $R(t)$. The government changes $\bar{C}$ only by exchanging currency for its reserves at the fixed price $\bar{Q}$; otherwise, it does not intervene in the money or gold markets. The money supply at any time that the gold standard exists is then

$$(38) \qquad M(t) = (\bar{C} - \bar{Q}R_0) + \bar{Q}(R(t) + G(t))$$

where R_0 is the quantity of gold reserves and $\bar{C}$ is the quantity of outstanding currency at the beginning of the analysis $(t=0)$.[21]

The dynamic system analogous to (18)-(19) is

$$(18a) \qquad \dot{P}(t) = \frac{\beta-\alpha r}{\alpha} P(t) - \frac{\bar{Q}(R(t) + G(t))}{\alpha} - \frac{\bar{C}-\bar{Q}R_0}{\alpha}$$

$$(19a) \qquad [\dot{R}(t)+\dot{G}(t)] = \frac{-v\delta}{\bar{Q}} P(t) - v(R(t) + G(t)) + vI.$$

The solution to the system is similar to that for (18)-(19). P and $(R+G)$ converge ultimately to the steady state values P^* and $[R^*+G^*]$; however, nothing guarantees that (R^*+G^*) is positive. Indeed, $[R^*+G^*] = [\bar{I}(\beta-\alpha r) - \delta(\bar{C}-\bar{Q}R_0/\bar{Q}]/(\beta-\alpha r+\delta)$ can be negative if the term $\delta(\bar{C}-\bar{Q}R_0)/\bar{Q}$ is large enough.

We have depicted the case of a negative (R^*+G^*) in Figure II. The $(\dot{R}+\dot{G})=0$ and $\dot{P}=0$ loci are analogous to those in Figure I, except that the steady state value for R^*+G^* is negative. Since the consumption demand for gold under this gold standard eventually exceeds the total available, the system must collapse at some point. At the time of the collapse, currency will be exchanged for all the governments reserves at the fixed rate $\bar{Q}$; and gold will cease to be money. Hence, the post-collapse money stock will be $\bar{C} - \bar{Q}R_0$; and the price level will be $(\bar{C}-\bar{Q}R_0)/(\beta-\alpha r)$, represented by point A in Figure II.

In order to prevent the infinite profits associated with price level jumps, the price level must have attained $(\bar{C} - \bar{Q}R_0)/(\beta-\alpha r)$ at the moment of

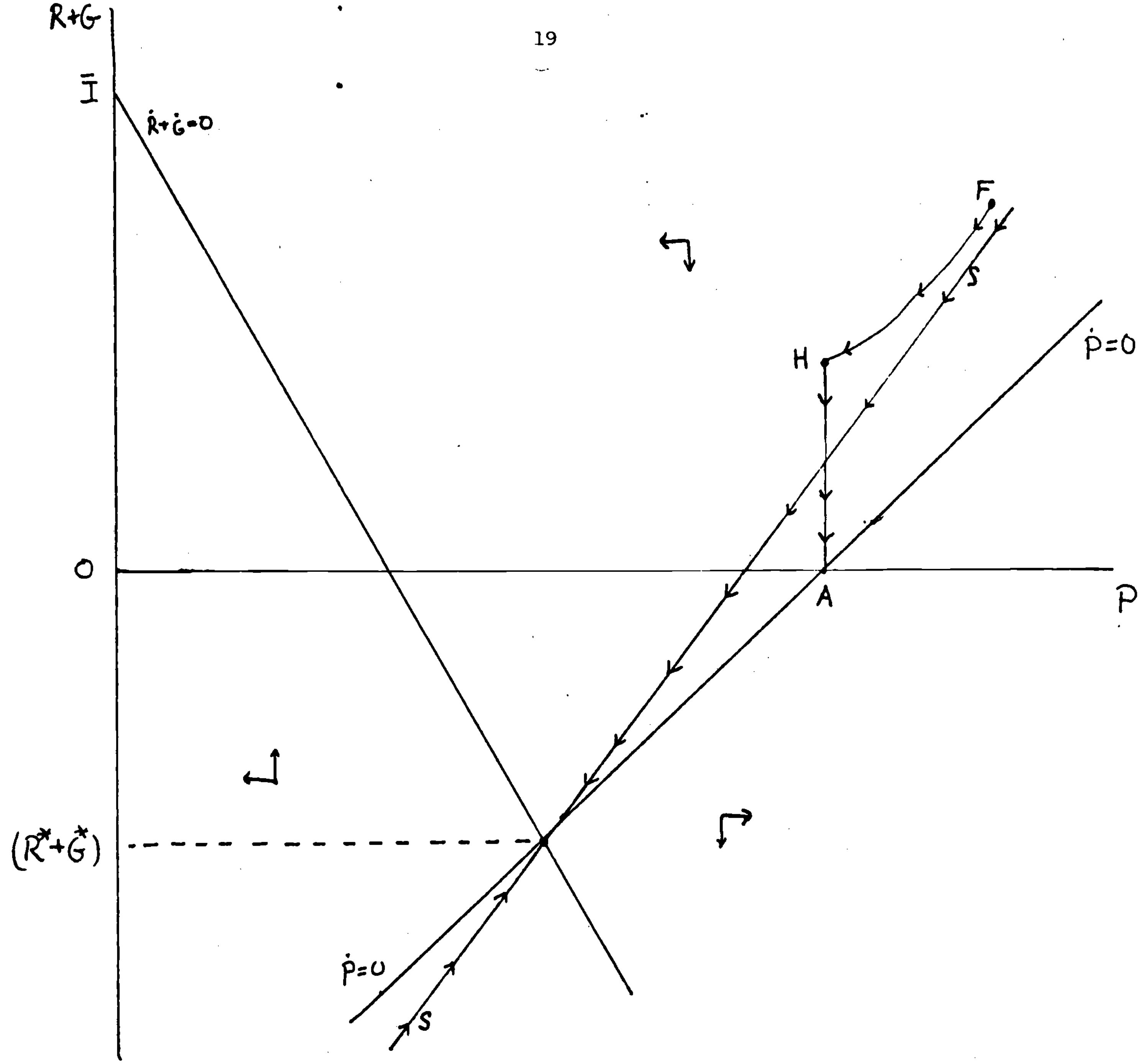

Figure II

the collapse. Similarly, to prevent a jump in the nominal gold price Q, there must still be some gold in the hands of the public at the time of the collapse. The gold price then gradually rises, thereby reducing the slope of the $(\dot{R}+\dot{G})=0$ locus and rotating it counter-clockwise until it intersects the P-axis at A. The lines through the points FHA represent the path followed by the price level and (R+G). The FH segment, located to the left of the stable manifold ss, is a solution path for the system (18a)-(19a). Thus, we have forced a collapse of the gold standard even when the money stock and prices continually fall; formal derivations of these results together with an equation determining the timing of the collapse are presented in the appendix.[22]

Gold Discipline

In the two preceding examples, we have demonstrated that a gold standard can collapse both when an inflationary monetary policy is in effect and when money and prices are declining. In this section we explore the restrictions on the money supply process which guarantee that a gold standard will survive indefinitely. We call such a restricted policy a *strictly gold-disciplined* money policy, and we seek a means of determining whether a particular policy is *strictly gold-disciplined*. One such method, presented in the appendix, consists of solving directly for z, the time of the collapse. If a finite solution for z exists, then the monetary policy can be classified as a *z-year gold-disciplined* policy; if z is infinite, then the monetary policy is strictly gold-disciplined. In the main text, we develop an equivalent method.

A *policy* is a sequence of money growth rates g(t). We model *government caused* money growth, i.e. all changes in the money supply except that discontinuous change associated with the gold standard collapse, as a smooth process. The money supply at any time t is then

(39a) $$M(t) = M(0)e^{\int_0^t g(h)dh} \qquad t<z$$

(39b) $$M(t) = M(0)e^{\int_0^t g(h)dh} - k(z) \qquad t=z$$

(39c) $$M(t) = M(z)e^{\int_z^t g(h)dh} \qquad t\geq z$$

where $k(z)$ is the amount of money destroyed when the gold standard collapses at time z.

Our model requires $\bar{I} \geq D(t)$, $\forall t$, which is a real resource constraint holding independently of the monetary regime. The real resource constraint applicable to the gold standard is $\bar{I} > D(t)$, $\forall t$. A gold standard, once imposed, will collapse if and only if agents foresee that in the absence of a collapse there would be some finite t when $\bar{I} = D(t)$.

We define a strictly gold-disciplined money growth policy as a policy such that

$$\bar{I} > \hat{D}(t) \qquad \forall t$$

where $\hat{D}(t)$ is the value of $D(t)$ constructed from equations (30) and (37) using the money growth policy and setting $k(z)\equiv 0$. We emphasize that $\hat{D}(t)$ need not be less than or equal to $\bar{I}$. The $\hat{D}(t)$ values are, for our model, a sequence of hypothetical values calculated by setting $k(z)\equiv 0$.

From previous results we have

$$P(\tau) = e^{\lambda\tau}\int_\tau^\infty \frac{M(j)}{\alpha} e^{-\lambda j}dj$$

and

$$D(t) = D(0)e^{-vt} + ve^{-vt}\int_0^t \frac{\delta}{\bar{Q}} P(\tau)e^{v\tau}d\tau$$

or

$$D(t) = D(0)e^{-vt} + ve^{-vt}\int_0^t \frac{\delta}{\bar{Q}} e^{\lambda\tau}\{\int_\tau^\infty \frac{M(j)}{\alpha} e^{-\lambda j}dj\} e^{v\tau}d\tau$$

By setting $k(z)\equiv 0$ we find

$$\hat{D}(t) = D(0)e^{-vt} + \frac{v\delta}{\alpha\bar{Q}} e^{-vt}\int_0^t\{ \int_\tau^\infty [M(0)e^{\int_0^j g(h)dh}]e^{-\lambda j}dj\}e^{(v+\lambda)\tau}d\tau$$

Thus, the policy in question is strictly gold-disciplined if and only if

$$\bar{I} > D(0)e^{-vt} + \frac{v\delta e^{-vt}}{\alpha\ \bar{Q}} \int_0^t \{ \int_\tau^\infty [M(0)e^{\int_0^j g(h)dh}]e^{-\lambda j}dj\}e^{(v+\lambda)\tau}d\tau$$

for all $t \geq 0$.

We argued previously that a policy of money growth at the constant positive rate g must lack gold discipline. To gain intuition about the applicability of our definition we will calculate $\hat{D}(t)$ for the policy $g(t)=g>0$ for all t. We find

$$\hat{D}(t) = D(0)e^{-vt} + \frac{v\delta(e^{gt} - e^{-vt})}{\bar{Q}(\lambda-g)(g+v)}.$$

Recall that we require $\lambda>g$. Our expression for $\hat{D}(t)$ grows without bound as t rises. Hence we can not have $\bar{I}>\hat{D}(t)$ for all t.

V) Conclusion and Extensions

A government may announce the fixing of the nominal price of gold in a number of different environments. Gold may currently circulate along with fiat money as part of the money supply with a freely floating nominal price of gold. Alternatively, gold may have no monetary role until the fixing of its nominal price. In this paper we have analyzed the movement of the price of gold in the latter case. In the context of our model we have shown that only a very special money supply process will prevent inflation when gold is monetized. In addition we have devised a method for determining the timing of a gold standard's collapse and produced a formal definition of "the discipline of the gold standard."

The model employed may be developed in a number of ways. The model specifies a fairly simple demand and supply behavior for money and gold rather than invoking explicit maximizing behavior. Also, agents are assumed to have perfect foresight. We think that it is worthwhile to place our model of a gold standard's collapse in a stochastic environment. Given such a framework it is possible to investigate the collapse of the 1920's gold standard to determine if it was viable when it was formed.

References

Barro, Robert J., "Money and the Price Level Under the Gold Standard," Economic Journal, 89, (March, 1979), 13-33.

Flood, Robert and Peter Garber, "Market Fundamentals vs. Price Level Bubbles: The First Tests," Journal of Political Economy, forthcoming, August, 1980.

Friedman, Milton and Anna Schwartz, A Monetary History of the United States, 1867-1960, Princeton: Princeton University Press, 1963.

Krugman, Paul, "A Model of Balance-of-Payments Crises", Journal of Money, Credit, and Banking, XI, no. 3, August, 1979, 311-325.

Laffer, Arthur, "Making the Dollar 'as Good as Gold'", Los Angeles Times, October 30, 1979.

Salant, Steven and Dale Henderson, "Market Anticipations of Government Policies and the Price of Gold," Journal of Political Economy, Vol. 86, no. 4, (August, 1978), 627-648.

Footnotes

[1] Laffer proposes to have the government announce that gold will be monetized at the market price prevailing on a certain future date. To prevent inflation in the interim, the government would follow an "austere" monetary policy and sell a large portion of its gold holdings.

[2] We can easily relax the fixity of other goods, providing that other goods are given exogenously to the model developed here. Allowing for changes in the real rate of interest would greatly complicate our analysis; thus, the development of our models is an exercise in partial equilibrium analysis. In (S-H), r is given exogenously and output of other goods is ignored. In Barro, both r and goods production are exogenous.

[3] Until section IV we ignore government gold stocks.

[4] It is possible to interpret $D^*(q(t))$ and $D(t)$ as desired and actual consumption holdings of gold <u>plus</u> desired and actual gold remaining in the ground, respectively. T en the model allows for gold to be mined at increasing costs.

[5] Our model contains an analogue to the choke price used by (S-H). When $D^*(q(t)) = D(t)$, $\dot{D}(t) = 0$, and there is no further change in speculative gold holding. The $\bar{q}(t)$ for which this condition holds is the analogue of the choke price.

[6] In an optimizing model, the speed of adjustment, v, would depend on r. We avoid the complex problems that this causes by fixing r.

[7] If $\dot{q}/q < r$, there would be no demand for speculative gold holdings, and the price of gold would fall discontinuously. If $\dot{q}/q > r$, speculative demand would cause a discontinuous upward jump in gold price. Since forseeable discontinuous jumps in q are not consistent with speculative equilibrium, q must follow equation (3).

[8] When there is no gold mining, equation (2) is identical to Barro's (1979) adjustment equation for non-monetary gold except that we don't include depreciation. To the extent that there is gold mining in our model, equation (2) differs from Barro's adjustment equation because our $\dot{D}$ is the difference between newly consumed and newly mined gold.

We have also explored an alternative form for the D* function which includes anticipated capital gains to gold holding as an argument; this form does not substantively change our results. See the appendix for this analysis.

[9] This assumption precludes the possibility of costly gold production. Gold exists as a pile of pure bars, all of which are contained initially in speculative holdings. This is the same assumption that S-H employ, and it greatly simplifies the ensuing algebra. An assumption that $D(0) > 0$ allows for the existence of gold mines; but since such mines make no qualitative difference in the effect of a monetization announcement on relative prices, it is helpful to ignore them.

[10] If $G(T) > 0$ equation (5) implies that $q(t)$ continues to rise for $t>T$. If $q(t)$ continues to rise after T, equation (2) indicates that $\dot{D}(t) < 0$, so $G(t)$ will rise for $t>T$. But such speculative hoards will never be used for non-speculative purposes since $\dot{D}(t)$ remains negative. Therefore, $G(T) = 0$.

[11]We report in the text solutions for $r \neq v$. Solutions are easily attainable for the special case $r=v$ but equations (10) and (11) become

$$(10') \qquad I = \frac{ve^{-vT}\delta T}{q(0)}$$

$$(11') \qquad T = 1/v = 1/r \; .$$

[12]An alternative money demand function is $\frac{M(t)}{P(t)} = [\beta-\alpha(r+\frac{\dot{P}(t)}{P(t)})]y(t)^{\delta}$, where where $y(t)$ is the amount of other goods produced in the economy. The solution for price (given a constant $y(t)$ and $M(t)$) would then be $P(t) = [M(t)/y(t)^{\delta}]/(\beta-\alpha r)$. Alternatively, we can assume that $M(t)/y(t)^{\delta}$ moves exogenously; if so then most of the following analysis holds, except that $[M(t)/y(t)^{\delta}]$ is used in place of $M(t)$.

[13]Flood and Garber (1980) test a model very much like equation (14) and cannot reject the hypothesis that price responds only to market fundamentals.

[14]That gold cannot be used as money prior to its monetization is a fairly extreme assumption, but it seems to characterize current gold use.

[15]The fixing of the price of gold at a giving future time has a precedent in the Greenback period of the U.S. In 1875, Congress passed a law requiring a return to the gold standard in January, 1879 at the pre-Civil War parity. In this case gold circulated as money with a fluctuating greenback exchange rate. For details, see Friedman and Schwartz (1963), Ch. 2.

[16]Since Laffer's proposal for monetization includes interim government reserve sales, it is more complex than the policy studied here. It is hard to reconcile Laffer's policy with interim price level stability (see Section III).

[17]We assume here that the quantity of gold in speculative hoards is great enough that merely the willingness of the government to intervene will cause the price of gold to be fixed; however, the government need not actually intervene. Hence, we can ignore movements in government reserves. In section IV we generalize the model to the case in which the government must intervene directly with its reserves to preserve the gold standard.

[18]It is possible to extend the model to an uncertain world in which the timing of the future fixing to gold is unknown. In the appendix, we explore such a case and determine the time ε relative price of gold that must prevail.

[19]Clearly, we must assume that the initial value of C is large enough to accommodate the desired monetization of private gold without an increase in nominal money balances.

[20]A gold standard is said to <u>break down</u> when a private run on government reserves exhausts those reserves. As Krugman has noted a government may divide reserves into primary and secondary reserves with only primary reserves being committed to the price fixing policy. Thus, a gold standard may break down when primary reserves are exhausted but secondary reserves remain intact.

[21] More generally, we could begin the analysis at time ε, the moment that the future monetization of gold is announced. Agents at time ε would realize that the gold standard to be implemented at w will not be permanently viable. They will prepare for the expected demonetization so that at time w the nominal and relative prices of goods and gold and the quantity of consumption gold holdings will be different from the results of section II.

Still more generally, the government may hold secondary and tertiary gold reserves in preparation for new gold standards to be established at some time after the collapse of the current standard; i.e. to defend the current standard in the crisis, it will only expend a known part of its total reserve. This rhythmic withdrawal from and return to the gold standard will reflect itself in yet different paths for the price level and gold consumption from those which we have examined thus far.

[22] A possible (though very conjectural) application of this analysis is to the study of the reestablishment and collapse of the inter-war gold standard. The U.S. continuously used a gold standard until 1933, but other major countries, i.e. Germany (1924), Great Britain (1925), France (1927), fixed their currencies to gold in the 1920's. It is often suggested that the gold parities which were established were "inappropriate". Here we can interpret "inappropriate" to mean that there was too much currency outstanding in the world to maintain the viability of the gold standard.

Appendix

I - Derivation of Equation (21)

Since $G(w) = I - D(w)$ we use equation (7) and the condition $q(\varepsilon) = q(w)e^{-r(w-\varepsilon)}$ with $q(w) = Q(w)/P(w)$ in equation (20) to obtain

$$\text{(A1)} \qquad P(w) = -\frac{(v+\lambda_2)}{v\delta}\{Q(w)[I - D(\varepsilon)e^{-v(w-\varepsilon)}] - \frac{P(w)v\delta}{v-r}[1-e^{(r-v)(w-\varepsilon)}] + \frac{-\lambda_2\delta\bar{C} + v\delta Q(w)\bar{I} + (v+\lambda_2)Q(w)\bar{I}(\beta-\alpha\bar{r})}{v\delta[\beta-\alpha r+\delta]}$$

and we use $G(w) = I - D(w)$, equation (7), and $q(\varepsilon) = q(w)e^{-r(w-\varepsilon)}$ along with $\dot{P}(w) = \frac{1}{\Lambda_2}\dot{G}(w)$ in equation (16) to obtain

$$\text{(A2)} \qquad \bar{C} + \{1+\frac{\alpha(v+\lambda_2)}{\delta}\}[Q(w)(\bar{I}-D(\varepsilon)e^{-v(w-\varepsilon)}) - \frac{P(w)v\delta}{(v-r)}(1-e^{(r-v)(w-\varepsilon)}] = [\beta-\alpha\bar{r}-\alpha(v+\lambda_2)]P(w) + \frac{\alpha(v+\lambda_2)}{\delta}\bar{I}\,Q(w) .$$

Since $w-\varepsilon$ is given, and $D(\varepsilon)$ and I are given, equations (A1) and (A2) are a pair of linear equations in $P(w)$ and $Q(w)$. After a lot of manipulations we found that

$$\text{(A3)} \qquad \frac{Q(w)}{P(w)} = \frac{v\delta e^{(r-v)(w-\varepsilon)} - r\delta}{(v-r)D(\varepsilon)e^{-v(w-\varepsilon)}} ,$$

which is equation (21) in the text. An interesting aspect of (A3), which we do not have much intuition about, is that it is entirely independent of money market parameters.

II - Solution to the System (18)-(19)

In this appendix we derive equation (20), the stable manifold for G(t) and P(t) after monetization. Equations (18) and (19) are a system of differential equations in G(t) and P(t). The steady state values of the system are $G^* = \frac{I(\beta-\alpha r)-\delta\bar{C}/\bar{Q}}{\beta-\alpha r+\delta}$ and $P^* = \frac{C+\bar{Q}I}{\beta-\alpha r+\delta}$.

The determinant of the homoegenous system is

$$\text{Det}\begin{pmatrix} \frac{\beta-\alpha r}{\alpha} & \frac{-\bar{Q}}{\alpha} \\ -\frac{v\delta}{\bar{Q}} & -v \end{pmatrix} = -\frac{v}{\alpha}(\beta-\alpha r+\delta) < 0.$$

Since the determinant is negative, the roots of the system are of opposite sign, indicating saddle point type stability. Assume that the roots are $\lambda_1>0$ and $\lambda_2<0$. Complete solutions of (18) and (19) are then of the form

$$G(t) = C_1A_1e^{\lambda_1 t} + C_2A_2e^{\lambda_2 t} + G^*$$

$$P(t) = C_1e^{\lambda_1 t} + C_2e^{\lambda_2 t} + P^*$$

with $A_1 = \frac{-v\delta}{\bar{Q}(v+\lambda_1)}$ and $A_2 = \frac{-v\delta}{\bar{Q}(v+\lambda_2)}$. For stability we require that $C_1 = 0$. Therefore, $P(t) = \frac{1}{A_2}G(t) + (P^* - \frac{1}{A_2}G^*)$.

III - Subjective Probabilities over Monetization

In this section, we generalize the model of section II, to the case in which agents have a probability density function over the timing of monetization. Fortunately, most of the required drudgery has already been done in section II because the solution for the time ε relative price of gold falls easily out of equation (22). This case is similar in form to the various policies studied by Salant and Henderson when agents are uncertain about the timing of policy implementation. The results which we derive here can be employed in an alternative interpretation of the relative gold price movements described in the first pages of Salant and Henderson.

We will assume that our gold speculators are risk neutral and that the subjective p.d.f. over the time w of monetization is the same exponential p.d.f. used by Salant and Henderson in their appendix, i.e.

$$\text{(a)} \qquad f(w) = \gamma e^{-\delta(w-\varepsilon)} \quad \text{for } w > \varepsilon$$

The relative price solution in equation (22) is conditioned on a particular announcement time ε and monetization time w. Here we assume that at time ε the government announces that gold will be monetized at some uncertain future time w with the p.d.f. over w given by (a). Let us denote the relative price solution in (22) as $Q/P(\varepsilon/w)$, to indicate that it is conditional on the time of monetization.

Risk neutral speculators will act to set actual relative price at time ε equal to the weighted average of the $Q/P(\varepsilon/w)$'s, where the weights are given in (a). The expected return to speculative gold holding at ε will then equal the real rate of interest. However, as long as the actual monetization does not occur, the relative price of gold must rise at a higher rate than the real rate of interest.

We proved in section II that if $w>T$, $Q/P(\varepsilon/w) > q(\varepsilon)$. The expected value of $Q/P(\varepsilon/w)$ can be written as

(b) $$E\,\frac{Q}{P}\,(\varepsilon/w) = \int_{\varepsilon}^{T} \frac{v\delta e^{(r-v)(w-\varepsilon)} - r\delta}{(v-r)D(\varepsilon)e^{(r-v)(w-\varepsilon)}}\,\gamma e^{-\gamma(w-\varepsilon)}dw + q(\varepsilon)\int_{T}^{\infty} \gamma e^{-\gamma(w-\varepsilon)}dw$$

$$= \frac{\gamma r\delta}{D(\varepsilon)(v-r)(r-v+\gamma)}\,[e^{-(r-v+\gamma)(T-\varepsilon)}-1] + \frac{v\delta}{(v-r)D(\varepsilon)}\,(1-e^{-\gamma(T-\varepsilon)})$$

$$+ q(\varepsilon)e^{-\gamma(T-\varepsilon)}$$

The determination of the time path for $Q/P(\varepsilon/w)$, conditional on monetization's not yet having occurred, has proven to be intractable (for us).

IV - Determination of z for the Case of too Much Initial Currency

1) The money supply is

(1a) $M(t) = (\bar{C}-\bar{Q}R_0) + \bar{Q}(G(t) + R(t))$ $t<z$

(1b) $M(t) = \bar{C} - \bar{Q}R_0$ $t\geq z$

2) During the gold standard period, price must move along the path

(2a) $P(t) = C_1e^{\lambda_1 t} + C_2e^{\lambda_2 t} + P^*$ $t\leq z$

where $P^* = \dfrac{\bar{C} - \bar{Q}R_0 + \bar{Q}\bar{I}}{\beta-\alpha r + \delta}$. This comes from the original P, G solution notes, and we will determine C_1, C_2 later. After z, the money supply is fixed so price must be

(2b) $P(t) = \dfrac{\bar{C} - \bar{Q}R_0}{\beta-\alpha r}$ $t\geq z$

3) Just after the run, the relative price of gold is $q(z) = \dfrac{\delta}{\bar{I}} e^{-r(T-z)} = \dfrac{\bar{Q}}{P(z)}$ to prevent price jumps. But since $P(z) = (\bar{C}-\bar{Q}R_0)/(\beta-\alpha r)$,

$$e^{-r(T-z)} = \frac{I\bar{Q}(\beta-\alpha r)}{\delta(\bar{C}-\bar{Q}R_0)}$$

Then we can solve for T-z:

$$T - z = -\frac{1}{r}\log\left[\frac{\bar{Q}\bar{I}(\beta-\alpha r)}{\delta(\bar{C}-\bar{Q}R_0)}\right] \equiv F$$

4) We need to solve for C_1 and C_2. From the solution for (G+R) we have:

$$(G+R) = C_1A_1e^{\lambda_1 t} + C_2A_2e^{\lambda_2 t} + (G^*+R^*) \text{ or at } t=0$$

(4a) $I - D(0) = C_1A_1 + C_2A_2 + (G^*+R^*)$

where

$$A_1 = \frac{-v\delta}{\bar{Q}(v+\lambda_1)}, \quad A_2 = \frac{-v\delta}{\bar{Q}(v+\lambda_2)}, \text{ and } (G^*+R^*) = \frac{\bar{I}(\beta-\alpha r) - \delta(\bar{C}-\bar{Q}R_0)/\bar{Q}}{\beta - \alpha r + \delta}$$

From (4a) we solve for C_1 in terms of C_2:

$$C_1 = \frac{-\bar{Q}(v+\lambda_1)}{v\delta}[I-D(0) - (R^*+G^*)] - C_2(\frac{v+\lambda_1}{v+\lambda_2})$$

Using (2a) we can solve for C_2 explicitly:

$$C_2 = \frac{P(z)-P^* + \frac{\bar{Q}(v+\lambda_1)}{v\delta}[I - D(0)-(R^*+G^*)]e^{\lambda_1 z}}{[e^{\lambda_2 z} - \frac{v+\lambda_1}{v+\lambda_2}]}$$

Thus, we can solve for C_1, C_2 in terms of the parameters, the initial, and the terminal conditions. This determines exactly the price path.

5) Now we must find z. From equation (33a),

$$1 = \frac{D(z)}{I}e^{-vF} + \frac{v}{v-r}[1-e^{(r-v)F})]$$

so

(5a) $$D(z) = Ie^{vF}[1 - \frac{v}{v-r}(1-e^{(r-v)F})]$$

From the accumutation equation for D(z),

(5b) $$D(z) = D(0)e^{-vz} + ve^{-vz}\int_0^z \frac{\delta}{\bar{Q}}P(t)e^{v\tau}d\tau$$

We know what D(z) must be from (5a). Then we need only plug in our price path in (5b), integrate and solve for z. The integral is essentially

$$\int_0^z P(\tau)e^{v\tau}d\tau = C_1\int_0^z e^{(\lambda_1+v)\tau}d\tau + C_2\int_0^z e^{(\lambda_2+v)\tau}d\tau + P^*\int_0^z e^{v\tau}d\tau$$

$$= \frac{C_1}{v+\lambda_1}[e^{(\lambda_1+v)z}-1] + \frac{C_2}{v+\lambda_2}[e^{(\lambda_2+v)}-1] + \frac{P^*}{v}[e^{vz}-1]$$

Then (5b) is

$$\text{(5c)} \qquad Ie^{vF}[1 - \frac{v}{v-r}(1-e^{(r-v)F})] = D(0)e^{-vz}$$

$$+ \frac{\delta v e^{-vz}}{\bar{Q}} [\frac{C_1}{v+\lambda_1}[e^{(\lambda_1+v)z}-1] + \frac{C_2}{v+\lambda_2}[e^{(\lambda_2+v)z}-1] + \frac{P^*}{v}[e^{vz}-1]]$$

All we have to do is find the z which solves (5c).

V - Analysis with an Alternative D*

In the text we assumed that the target value of D is $D^* = \delta/q$. This simple form was chosen because of its tractability. In this appendix we demonstrate that none of the conclusions we reach in the text would be altered by changing our assumption about D* to conform with that assumed by Barro. In our notation, one functional form which has the qualitative properties assumed by Barro (1979, pg. 15) is $D^* = \delta_0 + \delta_1 y + \frac{[\delta_2 + \delta_3(\dot{q}/q)]}{q}$ where y is real output, assumed constant in our paper, and the linearization is appropriate only over ranges where $[\delta_2 + \delta_3 \dot{q}/q] > 0$. We differ from Barro in that he assumed unit income elasticity, but this does not seem substantive for present purposes.

With the new specification of D* we may write the differential equation system (18), (19) from the text as

$$(*)\quad \dot{P} = \frac{\beta - \alpha r}{\alpha} P - \frac{M}{\alpha}$$

$$(**)\quad \dot{G} = v[-(\frac{\delta_2}{\bar{Q}} - \frac{\delta_3(\beta - \alpha r)}{\bar{Q}\alpha})P - (\frac{\delta_3}{\alpha} + 1)G] + v[\bar{I} - \delta_0 - \delta_1 y - \frac{\delta_3 \bar{C}}{\bar{Q}\alpha}] .$$

To derive (**) use (*) in place of $\dot{P}$ in the target and note that after monetization $\dot{q}/q = -\dot{P}/P$.

If $(\frac{\delta_2}{\bar{Q}} - \frac{\delta_3(\beta - \alpha r)}{\bar{Q}\alpha})$ is positive then the phase diagram of (*) and (**) is qualitatively identical to Figure I in the text and the analysis of the model is essentially unchanged. However, if $\frac{\delta_2}{\bar{Q}} - \frac{\delta_3(\beta - \alpha r)}{\bar{Q}\alpha} \leq 0$ then the phase diagram appears as in Figure A. The important point about this figure is that, if positive, the slope of the $\dot{G}=0$ schedule is less than that of the $\dot{P}=0$ schedule. The slopes of these schedules are

$$\left.\frac{dG}{dP}\right|_{\dot{P}=0} = \frac{\beta - \alpha r}{\bar{Q}} > 0$$

$$\left.\frac{dG}{dP}\right|_{\dot{G}=0} = \frac{\left.\frac{dG}{dP}\right|_{\dot{P}=0} - \frac{\alpha\delta_2}{\delta_3\bar{\bar{Q}}}}{1 + \frac{\alpha}{\delta_3}}$$

It is evident from Figure A that the stable branch, ss, is qualitatively unchanged from Figure I. Finally note that in the steady state $\dot{q}/q = -\dot{P}/P = 0$ and hence for the steady state including the rate of return variable is irrelevant.

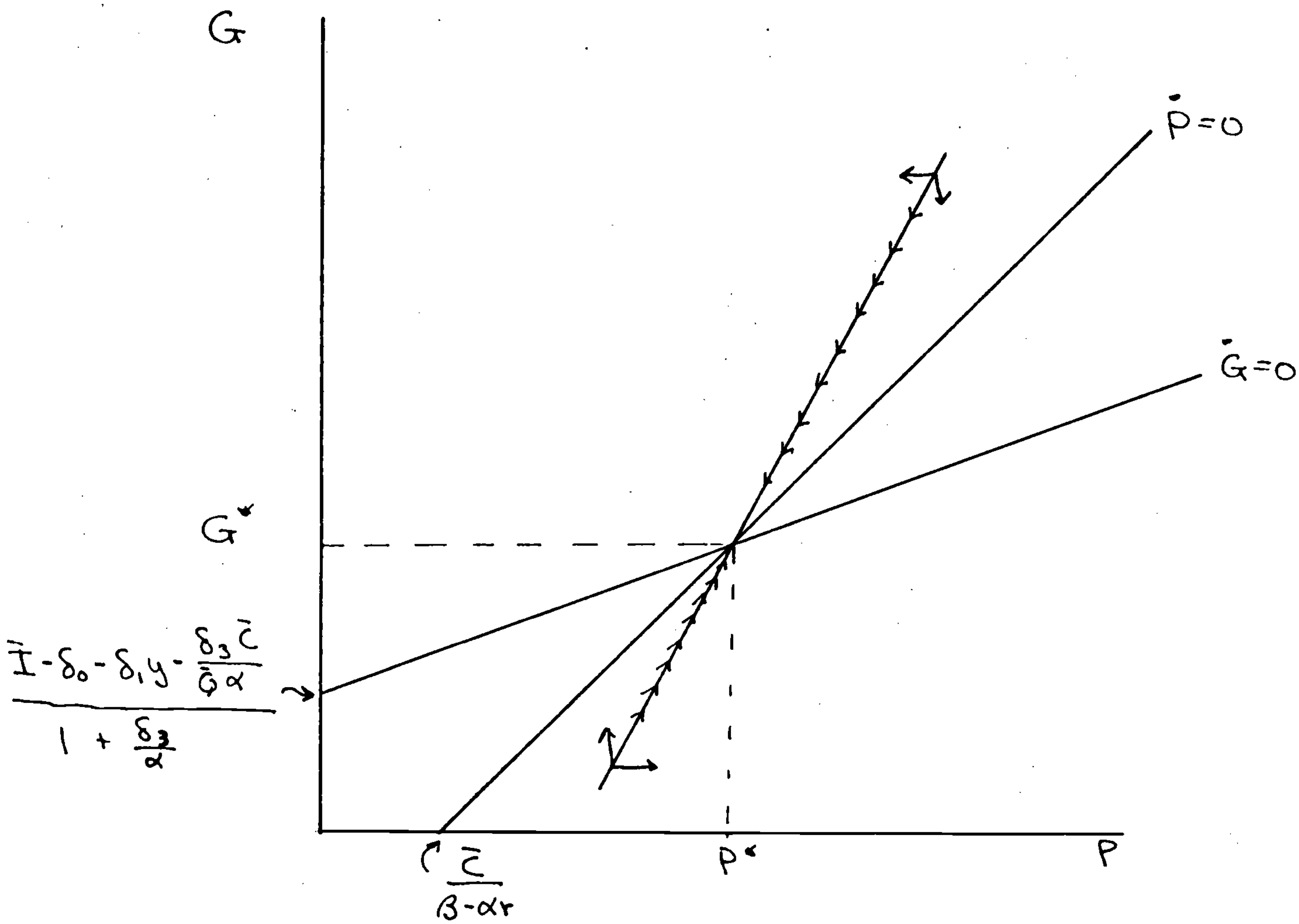
G
$\dot{P}=0$
$\dot{G}=0$
G^*
$\dfrac{\bar{I}-\delta_0-\delta_1 y-\dfrac{\delta_3 \bar{C}}{\bar{Q}\alpha}}{1+\dfrac{\delta_3}{\alpha}}$
$\dfrac{\bar{C}}{\beta-\alpha r}$
P^*
P

Figure A