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ANTICIPATED INFLATION, THE FREQUENCY OF TRANSACTIONS AND THE SLOPE OF THE PHILLIPS CURVE

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ABSTRACT

This paper examines the effects of expected inflation on the responsiveness of output to nominal disturbances in the framework of a localized markets model. The mechanism described in the theoretical part of the paper is that expected inflation has a positive effect on the transaction frequency, which in turn increases the flow of price information across markets. More information implies less misperception of monetary shocks as relative shifts in excess demand, resulting in lower sensitivity of real output to these socks.

The empirical implication of this proposition - namely, that expected inflation reduces the coefficient of nominal shocks in an output equation - is tested first using data across countries, and then with time series data from the United States. The first test uses Lucas's and Alberro's estimates of Phillips Curve coefficients from different countries and the corresponding average inflation rates. The second test involves data from the post - World War II period. It uses nominal rates of return on Treasury Bills and corporate bonds as measures of anticipated inflation and Barro's estimates of unanticipated money. In general, results in both tests provide support (stronger than expected) for the implication of the theory.

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This paper was motivated by two independent hypotheses in the macroeconomic literature. The first is Lucas's hypothesis on the slope of the Phillips Curve. Lucas's (1973) study indicated a negative correlation between the variance of the nominal disturbances and the responsiveness of real output to these disturbances. The idea advanced by Lucas is that the higher this variance, the more likely are agents to attribute locally observed price movements to general inflation rather than relative shifts in excess demand. Therefore, as the variance of nominal shocks increases, supply becomes less responsive to the general price movements generated by these disturbances.

The second hypothesis is from the literature on the inventory approach to the demand for money (e.g. Baumol (1952), Tobin (1956), Barro (1970), Feige and Parkin (1971) and Grossman and Policano (1975)). It can be summarized as the existence of a positive effect of expected inflation, or the nominal interest rate, on the frequency at which cash receipts are exchanged for commodities or earning assets.

In Lucas's model, the nonnueutral effect of nominal shocks occurs because of the lack of complete current information. Agents estimate the unobserved price level using an information set that contains lagged values of the nominal aggregates. Now, if expected inflation affects the frequency of transactions, it must also affect the frequency at which other prices are observed. Thus, the structure of the information set--and hence the responsiveness of output to nominal shocks--will depend also on the expected or systematic rate of inflation.

This paper was also motivated by the empirical observation that highly volatile aggregate demand countries are also those with high average inflation rates. Therefore, Lucas's finding that the Phillips Curve is more vertical in those countries is also consistent with the informational effects of the velocity of transactions.

The theoretical part of this paper discusses the proposition that expected inflation affects the slope of the Phillips Curve in two separate sections. The first deals with the relationship between the rate of inflation and the frequency of transactions in a deterministic framework. The model presented in section I summarizes previous results related to the frequency of transactions, especially from Barro (1970) and Grossman and Policano (1975). The purpose of this section is to discuss these results in a general equilibrium framework in which individuals maximize utility from consumption and leisure, given a production function, transaction costs and government spending financed by money issue. Agents decide how much to produce and sell, and for how long to accumulate the resulting inflow of cash until a shopping trip is made. In this framework, the inflation rate determines the relative price relevant for production, which is the current price relative to the price level at the time of the next shopping trip. The inflation rate also affects the decision about the frequency of the visits to the other markets. This section also derives aggregate demand and supply and characterizes the general equilibrium.

The second section is a version of Lucas's and Barro's (1976) localized market models with partial information. The agents are assumed here to behave similarily as in the deterministic model; they face a current price which is deflated by a future (expected) price of the consumption bundle, and accumulate cash until these goods are purchased. In this section however, the frequency of transactions is treated as exogenous. It is viewed as determined in a similar way as discussed in the previous section. The focus here is on the informational implications of the frequency of transactions and its relationship with the responsiveness of real output to nominal shocks.

Section III reports empirical tests of the hypothesis that expected inflation diminishes the effect of nominal shocks on output. A first test uses Lucas's and Alberro's (1980) estimates of Phillips Curve coefficients for different countries and the corresponding average inflation rates. A second test involves times series data from the United States after World War II, using nominal rates of return on Treasury Bills and corporate bonds as measures of anticipated inflation and Barro's (1980) estimates of unanticipated money.

In general, the results in both tests indicate a fairly substantial negative correlation between anticipated inflation and the coefficients relating output to nominal shocks.

Section I. Inflation and the Frequency of Transactions

Consider an agent producing and selling a good, or a service, whose rate of production is a function of the input of labor. With the continuous stream of supply, the agent accumulates cash that depreciates until it is exchanged for consumption goods. A key assumption is that this exchange, or "shopping," involves a cost. These transaction costs--assumed to be independent of the amount purchased--can be viewed as the time or direct expenses involved in traveling to other markets. Given these costs and the rate of depreciation of the currency, there is a tradeoff between economizing on shopping trips and minimizing the effects of the depreciation of the currency received.

The individuals, who have infinite lives and zero rate of time preference, maximize utility from consumption and leisure:

$$U = U(C,L) \qquad U_c, U_1 > 0, \qquad U_{cc}, U_{11} < 0$$

C and L are the uniform, instantaneous flows of consumption and leisure respectively. C can be viewed as a composite good whose components have constant relative prices resulting from a steady state equilibrium.

The production function that expresses the relationship between the input of labor and the rate of production and sale--which occur simultaneously--of good S is

$$S = (1/a)\ (H - L)$$

where a is a positive constant and H is the ceiling to the flow of labor. At time t the individual sells S at the price P(t), which grows at the instantaneous rate π. The purchasing power of the cash accumulated between two shopping trips is

$$\frac{1}{P(T+\tau)} \int_{T}^{T+\tau} SP(t)dt$$

where $P(T+\tau)$ is the general price level at the end of the period of length τ that begins at date T. In the steady state equilibrium all prices grow at the same rate, namely

$$(1)\quad P(T+\tau) = P(T)e^{\pi\tau} \quad \text{and } P(t) = P(T)e^{\pi(t-T)}$$

Given the shopping cost b, the consumption flow per unit of time is

$$(2)\quad C = \frac{1}{\tau P(T+\tau)} \int_T^{T+\tau} SP(t)dt - \frac{b}{\tau}$$

Substituting (1) into (2) yields

$$C = \frac{S}{\tau} \int_T^{T+\tau} e^{\pi(t-T-\tau)}dt - \frac{b}{\tau}$$

Solving the integral, the expression for the consumption flow becomes

$$(3)\quad C = \frac{S}{\pi\tau}(1-e^{-\pi\tau}) - \frac{b}{\tau}$$

One can substitute now (3) and the expression for L from the production function into the utility function to obtain

$$(4)\quad U = U[\frac{S}{\pi\tau}(1-e^{-\pi\tau}) - \frac{b}{\tau},\ H - aS]$$

Utility is maximized with respect to the shopping period τ and the supply level S. The constraint that supply has a uniform level over time is a somewhat restrictive assumption made for simplicity. In a more general setup, the higher real return on supplying closer to the purchasing time would induce an increasing pattern of supply between two shopping points. A corner solution of selling only just before buying can be ruled out if it is assumed that the marginal utility of leisure becomes extremely large as leisure approaches zero, and that it becomes extremely small as leisure approaches H. These assumptions would insure a positive supply at all times, producing the same type of tradeoff between the costs of transacting and that of holding cash. Therefore, given the focus of the analysis here, nothing essential is lost by adopting a uniformity constraint on S.

An additional restrictive assumption is that production and sale take place simultaneously. With respect to services, this is no constraint. Regarding commodities one could argue that under inflationary conditions, agents can do better by producing during the period and selling only at its end. However, this plan would not be optimal if the act of selling also requires the input of labor. Then, if the marginal utility from leisure behaves as mentioned above, sales will have some dispersion over the period. Sales close to production could be rationalized also on the basis of unincluded factors like storage costs or risk of damage, spoiling etc.[1]

The maximization of (4) with respect to τ and S implies the following first order conditions

(5) $U_\tau = U_c \dfrac{\partial C}{\partial \tau} = 0$ where $\dfrac{\partial C}{\partial \tau} = \dfrac{S}{(\pi\tau)^2} [\pi^2 \tau e^{-\pi\tau} - \pi(1-e^{-\pi\tau})] + \dfrac{b}{\tau^2}$

(6) $U_s = U_c \dfrac{\partial C}{\partial s} - U_1 a = 0$ where $\dfrac{\partial C}{\partial S} = \dfrac{(1-e^{-\pi\tau})}{\pi\tau}$

Equation (5) means that, given S, the stream of consumption is maximized by equalizing the marginal benefit from enlarging the shopping period,--which reduces the transaction costs--with the marginal depreciation costs. Equation (6) expresses the standard optimal labor supply condition that the utility from the additional consumption obtainable by a marginal increase in labor supply equals the marginal disutility from reducing leisure.

The second order conditions are

(7) $U_{\tau\tau}$, or $U_{ss} < 0$ and

(8) $U_{\tau\tau}U_{ss} - U_{s\tau}^{2} > 0$

The detailed expressions for (7) and (8) are included in appendix A. (7) is shown to hold and (8) is assumed.

Comparative Statics

The next step is to see how changes in the main exogenous parameter, π, affects the decision variables. Consider first the partial effects of π on τ and S. Differentiating (5) with respect to τ and π and rearranging terms yields

$$\left.\frac{d\tau}{d\pi}\right|_{S} = -\frac{U_{\tau\pi}}{U_{\tau\tau}}$$

This expression is negative from (A.6) and (A.10) in appendix A. This is a standard result which says that the higher the rate at which cash receipts depreciate, the sooner they will be exchanged for commodities (or real valued assests).

Differentiating now equation (6) with respect to S and π yields

$$\left.\frac{ds}{d\pi}\right|_{\tau} = -\frac{U_{s\pi}}{U_{ss}}$$

Since $U_{ss} < 0$ from (A.7), $dS/d\pi$ has the same sign as $U_{s\pi}$. $U_{s\pi}$ is negative or positive according to whether the substitution or income effect is stronger.

S and π moving simultaneously involves interaction between the two changes. Differentiating now both (5) and (6) with respect to S, τ and π and dividing by $d\pi$ yields

$$U_{\tau\tau}\frac{d\tau}{d\pi} + U_{\tau s}\frac{ds}{d\pi} = -U_{\tau\pi}$$

$$U_{s\tau}\frac{d\tau}{d\pi} + U_{ss}\frac{ds}{d\pi} = U_{s\pi}$$

The solutions for $d\tau/d\pi$ and $dS/d\pi$ are

$$\frac{d\tau}{d\pi} = \frac{-U_{\tau\pi}U_{ss} + U_{s\pi}U_{\tau s}}{U_{\tau\tau}U_{ss} - U_{\tau s}^2}$$

$$\frac{dS}{d\pi} = \frac{-U_{s\pi}U_{\tau\tau} - U_{s\tau}U_{\tau\pi}}{U_{\tau\tau}U_{ss} - U_{\tau s}^2}$$

Since the denominators are positive from (A.9), the signs of $d\tau/d\pi$ and $dS/d\pi$ depend on the expressions in the numerators. Except for $U_{S\pi}$ all the cross derivatives are unambiguously negative. Consider first $d\tau/d\pi$. If the income effect on labor supply is stronger than the substitution effect--$U_{S\pi} < 0$--$d\tau/d\pi$ is unambiguously negative. In this case, an increase in inflation induces more supply, which means more cash accumulation. The depreciation costs increase and therefore agents transact sooner. This effect reinforces the standard one expressed by $U_{\tau\pi}$. Also when $U_{S\pi} = 0$, $d\tau/d\pi$ is unambiguously negative. However, if the substitution effect is stronger than the income effect--$U_{S\pi} < 0$--, the sign of $d\tau/d\pi$ becomes ambiguous. It will be assumed that in the "normal" case $d\tau/d\pi$ is negative.

Consider now $dS/d\pi$. If $U_{S\pi} < 0$, the partial effect of π on S is negative, as seen before. However, $dS/s\pi$ includes the offsetting term $U_{S\tau}U_{\tau\pi}$ which comes from the negative partial effect of inflation on τ; when τ declines, the relative price increases inducing more supply.[2]

If U_{SS} and $U_{S\pi}$ are sufficiently negative, both $dS/d\pi$ and $d\tau/d\pi$ can also be negative. In this case, the depreciation costs have a stronger effect on the frequency of transactions than the reduced flow of supply, and the substitution effect on supply is stronger than both the income effect and the offsetting shortening of the shopping period.

Aggregate Demand, Supply and Equilibrium.

This part of the section deals with the equilibrium of an economy composed by individuals who behave as seen above. Given the setup used here, general and market equilibrium conditions are the same. This discussion provides some foundations for the specification of commodity demand adopted in the next section.

The demand side of the commodity market is composed by those agents who are currently in their shopping trips, and the government--which finances its purchases by issuing new money. Government services are assumed not to affect the utility of the public. These can be seen as police or defense services that are designed to neutralize negative effects on utility from crime, foreign threats, etc.

The private nominal demand at time t, denoted by $M(t, \tau)$, is determined by the cash accumulated for a period of length τ times the number of individuals currently purchasing

$$M(t,\tau) = \frac{N}{\tau} \int_{t-\tau}^{t} SP(x)\ dx = \frac{NSP(t)}{\pi\tau}\ (1-e^{-\pi\tau})$$

where N is the total number of agents in the economy. Government spending is assumed to require a constant growth rate, μ, of the money stock, $M(t)$. Thus, aggregate nominal demand is given by

$$(9) \quad M(t,\tau) + \mu\, M(t) \equiv M(t)\ V^{d}(\pi,\mu)$$

where $V^{d}(\pi,\mu) \equiv \dfrac{M(t,\tau)}{M(t)} + \mu$ is defined as the desired velocity of money circulation.

Aggregate supply in nominal terms is $NSP(t)$, and in equilibrium it holds

$$M(t)V(\mu) = NSP(t)$$

which is a quantity theory type of equation.

Since commodities and money are the only two goods in the model, commodity market clearing implies equality of demand and supply of money, and vice versa. The demand for money, $M^d(t)$, equals the summation of the cash receipts willingly held by all agents

$$M^d(t) = \int_0^\tau M(t,\tau')\,d\tau' = \frac{NSP(t)}{\pi^2\tau}(\pi\tau + e^{-\pi\tau} - 1)$$

The equality $M^d(t) = M(t)$ implies that M(t) and P(t) must grow at the same rate: $\pi = \mu$.

Also, from $M^d(t) = M(t)$ follows

$$M(t)\,\frac{\mu^2\tau}{\mu\tau+e^{-\mu\tau}-1} = NSP(t)$$

This equality is identical to the commodity market clearing condition and

$$V(\mu) = \frac{\mu^2\tau}{\mu\tau+e^{-\mu\tau}-1}.$$

Finally one can compute the effect of a change in μ on velocity. Differentiating $V(\mu)$ with respect to μ yields

$$\frac{dV(\mu)}{d\mu} = \underset{(+)}{\frac{\partial V(\mu)}{\partial\mu}} + \underset{(-)}{\frac{\partial V(\mu)}{\partial\tau}}\;\underset{(-)}{\frac{d\tau}{d\mu}} > 0$$

The exact form of $\partial V/\partial\mu$ and $\partial V/\partial\tau$ are reported in (A.12) and (A.13) in appendix A. $\partial V/\partial\mu$ represents the direct effect of inflation on real balances; the higher the inflation rate, the lower is the current real value of the cash accumulated for some period. The second term is the standard relationship between the frequency of transactions and the velocity of money circulation.

Section II. The frequency of transactions and the Phillips Curve

On the basis of the analysis in Section I, which dealt with expected inflation and the frequency of transactions, this section focuses on the informational implications of the transaction frequency and its relationship with the effects of money movements on real output in a stochastic environment. The framework of this discussion is a multimarket model with partial current information of the type used by Lucas (1973) and Barro (1976) and incorporates the basic features of the economy discussed in section I. The agents, located in different markets, sell goods or services and accumulate cash for a period of time of length τ. Then they make a trip to the other markets to purchase consumption goods.

The frequency of transactions is exogenous in this discussion, but it is viewed as determined optimally in a similar fashion as discussed in Section I.[3] The comparative statics exercise carried out below by changing τ is rationalized by an underlying change in expected inflation in the opposite direction.

In this framework, the future price level is not known with certainty and agents form their optimal forecasts using all available information, which includes neither current prices--other than the local one--nor the current money stock. The (log of the) relative price on which supply decisions are made is $P_t(z) - EP_{t+i}$, where $P_t(z)$ is the local price at time t and EP_{t+i} is the mathematical expectation of the consumption bundle's price at the next shopping time ($1 \leq i < \tau$). The information set that conditions the expectation contains $P_{t-\tau+i}$, which is the price level that was observed by agents the last time they visited the other markets.

The importance of the trade period in this context relies primarily on its informational implications. The more often agents visit other markets, the more recent and relevant is the information on which they base their expectations.

Turn now to the specification of the model. The supply behavior in market for commodity z at time t is given in log-linear terms by

$$(10)\quad y_t^s(z) = h + \alpha[P_t(z) - EP_{t+i}] + \varepsilon_t^s(z) \qquad \alpha > 0$$

h is a constant term which can be seen as related to H and b of section I. α is the relative price elasticity of supply. The assumption that α is positive can be interpreted as a net positive partial effect of the relative price on supply, that is, holding constant the trade cycle, the relative price and supply move in the same direction. $\varepsilon_t^s(z)$ is a random productivity shock specific to z. I will return below to the process associated with this term and to the details of the expectation EP_{t+i}.

The log-linear demand for commodity z at time t follows the general form of equation (9) in section I.

$$(11)\quad y_t^d(z) = M_t - P_t(z) + \log(V) + \varepsilon_t^d(z)$$

M_t is the log of the money stock (divided by the number of markets) and V is the fraction of the current money stock being exchanged for commodities, both by shopping agents and by the government. This functional form also corresponds, approximately, to a Cobb-Douglas utility function with equal shares for all commodities. $\varepsilon_t^d(t)$ is a random term associated with shifts in preferences or in the pattern of government spending.

Turn now to the stochastic properties of the model. Money growth follows the process

$$M_t - M_{t-1} = \mu + m_t \qquad \mu > 0$$

that is, the money stock grows at a combination of a constant rate μ and a random term m_t of zero mean and variance σ_m^2. For convenience in the calculations, it is assumed that m_t is serially uncorrelated.[4] The fraction V which is treated as a function of μ would in fact depend positively also on m_t--if the new money is spent by the government. However, since demand depends on m_t through M_t, the model approximately captures that effect.

The other stochastic term involves $\varepsilon_t^d(z)$ and $\varepsilon_t^s(z)$. The excess demand relative shock, $\varepsilon_t^*(z) \equiv \varepsilon_t^d(z) - \varepsilon_t^s(z)$ is assumed to follow a moving-average process of order n

$$\varepsilon_t^*(z) = \varepsilon_{t-n}(z) + \ldots + \varepsilon_t(z)$$

where the series $\varepsilon_t(z)$ is serially uncorrelated and has zero mean and common variance σ_ε^2. The serial correlation in $\varepsilon_t^*(z)$ expresses the assumption that it takes n periods to arbitrage away shifts in relative excess demand. It will be assumed that $n > \tau$.

Turn now to the structure of the information set available to agents. Suppliers and demanders have access to different information. Agents who are currently in their shopping trips observe all prices. On the other hand, suppliers observe only the local price; their information set is the following

$$\left\{M_{t-k},\ P_{t-\tau+i},\ P_{t-\tau+i-1},\ \ldots,\ P_t(z),\ P_{t-1}(z),\ \ldots\right\}$$

EP_{t+i} denotes the expectation of P_{t+i} conditional on this information set. Monetary statistics are published continuosly with a lag of length k, where $k >> \tau$. The most recent observation on the price level is $P_{t-\tau+i}$, which corresponds to the last shopping trip. Additionally, information on all previous prices are also obtained at that time.[5] Finally, the current and past local prices are known.

It is assumed that the trade periods of agents located in one market are synchronized. This assumption simplifies a great deal the computation of the conditional expectations and the solution of the model, but it presents some problems that are discussed later. The trade periods of producers of different commodities will in general not coincide. At each point in time during a period of length τ a different population of agents will be visiting other markets. Thus, according to the timing of their shopping trips agents can be classified into τ groups. These groups may be seen as having the same size because arbitrage of different real prices of the consumption bundle will generate a uniform flow of demand.

The solution of the model follows the same procedure used by Lucas (1973) and Barro (1976). First, from equations (10) and (11) clearing of market z implies

$$(12)\quad P_t(z) = \frac{1}{1+\alpha}[\log(V) - h] + \frac{\alpha}{1+\alpha}\ EP_{t+i} + \frac{1}{1+\alpha}\ [M_t + \varepsilon_t^*(z)]$$

Given the log-linearity of the model, the solution for the average price level P_t will have the form

$$(13)\quad P_t = \xi_0 + \xi_1 M_{t-k} + \pi_{k-1} m_{t-k+1} + \dots + \pi_{\tau-1} m_{t-\tau+1} + \pi_{\tau-2} m_{t-\tau+2} + \dots$$

$$+ \pi_1 m_{t-1} + \pi_0 m_t$$

where the ξ-s and π-s are constant coefficients. The variables m_{t-j}-s are assigned a priori different coefficients according to their date of occurrence. The date of the shock is relevant for its effect on P_t because past price levels are known--and money shocks inferred--by a larger fraction of agents than more recent ones.

Updating now P_t from (13) i periods and taking the expectation operator yields

$$EP_{t+i} = \xi_0 + E[\xi_1 M_{t-k+i} + \pi_{k-1} m_{t-k+i+1} + \dots + \pi_0 m_{t+i}]$$

A more detailed expression for EP_{t+i} is obtained making use of the information set and the stochastic properties of m_t. First, from M_{t-k}, $M_{t-k-1} \dots$ and $P_{t-\tau+i}$, $P_{t-\tau+i-1}$, ... producers of z can infer the values of M_{t-k+1} ..., $M_{t-\tau+i}$. Since money is the only aggregate shock, the prices observed in their last shopping trip provide the information about money growth up to that time. Using also $Em_{t+j} = 0$ for j=i, ..., i, EP_{t+i} can be expressed as

$$(14)\quad EP_{t+1} = \xi_0 + \xi_1 (M_{t-k} + i\mu + m_{t-k+1} + \dots + m_{t-k+i}) + \pi_{k-1} m_{t-k+i+1} +$$

$$\dots + \pi_\tau m_{t-\tau+i} + \pi_{\tau-1} Em_{t-\tau+i+1} + \dots + \pi_i Em_t$$

The next step is to compute $Em_{t-\tau+i+1}, \ldots, Em_t$. From equation (12) it follows that

$$(15) \quad P_t(z)(1+\alpha) + h-\log(V) - \alpha EP_{t+i} - M_{t-\tau+i} - (\tau-i)\mu = m_{t-\tau+i+1} + \ldots + m_t + \varepsilon_t^*(z)$$

The agents in z perceive the total disturbance--composed by monetary and real factors--that hits the local market. This total disturbance appears on the right hand side of (15). Similarily, the equation (12) corresponding to t-1 implies

$$(16) \quad P_{t-1}(z)(1-\alpha) + h-\log(V) - \alpha E^{-1}P_{t+i} - M_{t-\tau+i} - (\tau-i-1)\mu =$$
$$m_{t-\tau+i+i} + \ldots + m_{t-1} + \varepsilon_{t-1}^*(z)$$

where the superscript -1 indicates that the expectation was taken at t-1. By substracting (16) from (15), the agents can infer the value of $[m_t + \varepsilon_t(z) - \varepsilon_{t-n-1}(z)]$

Given this value, the conditional expectation of m_t is

$$(17) \quad Em_t = \theta[m_t + \varepsilon_t(z) - \varepsilon_{t-n-1}(z)] \text{ , where } \theta \equiv \frac{\sigma_m^2}{\sigma_m^2 + 2\sigma_\varepsilon^2}$$

Likewise, from (16) and equation (12) corresponding to t-2, $[m_{t-1} + \varepsilon_{t-1}(z) - \varepsilon_{t-n-2}(z)]$ can be calculated. Thus, the conditional expectation of m_{t-1} follows as

$$Em_{t-1} = \theta[m_{t-1} + \varepsilon_{t-1}(z) + \varepsilon_{t-n-2}(z)]$$

Repeating the procedure with market clearing conditions back to $t-\tau+i+1$ yields $Em_{t-2} = \theta[m_{t-2} - \varepsilon_{t-2}(z) - \varepsilon_{t-n-2}(z)]$, ..., $Em_{t-\tau+i+1} = \theta[m_{t-\tau+i+1}(z) + \varepsilon_{t-\tau+i+1}(z) - \varepsilon_{t-\tau+i-n}(z)]$

Substituting now (17) and the corresponding expectations of lagged money shocks into (14) yields EP_{t+i} as a function of the exogenous variables. One can then substitute the resulting expression for EP_{t+i} into equation (12) to obtain

$$(18)\quad P_t(z) = \frac{1}{1+\alpha}[\log(V)-h] + \frac{\alpha}{1+\alpha}\Big\{\xi_0 + \xi_1(M_{t-k} + i\mu + m_{t-k+1} + \dots + m_{t-k+i})$$

$$+ \pi_{k-1}m_{t-k+i+1} + \dots + \pi_\tau m_{t-\tau+i} + \pi_{\tau-1}\theta[m_{t-\tau+i+1} + \varepsilon_{t-\tau+i+1}(z) - \varepsilon_{t+\tau-i-n}(z)] +$$

$$\dots + \pi_i\theta[m_t + \varepsilon_t(z) - \varepsilon_{t-n-1}(z)] + \frac{1}{1+\alpha}[M_{t-k} + k\mu + m_{t-k+1} + \dots + m_t$$

$$+ \varepsilon_{t-n}(z) + \dots + \varepsilon_t(z)]$$

The solution of the model proceeds by averaging equation (18) across markets, and then solving for the ξ-s and π-s using the identity of the resulting average price expression and equation (13). Although the full solution for all the coefficients will not be calculated here, the obtained expressions will suffice for analyzing the effects of changing τ. The calculation of the general price level from (18) involves: a) dropping $\varepsilon_t(z)$ terms, since under the assumption of a large number of markets they average out, b) averaging the i that multiplies μ, and c) the calculation of the average coefficient for each money shock, which has different effects across markets. The resulting expression for the unweighted average price level is

$$(19) \quad p_t = \frac{1}{1+\alpha}[\log(V)-h] + \frac{\alpha}{1+\alpha}\Big\{\xi_0 + \xi_1 M_{t-k} + \xi_0[(\tau+1)/2]\mu + \xi_1 m_{t-k+1} +$$

$$[(\pi_{k-1} + (\tau-1)\xi_1)/\tau]m_{t-k+2} + [(\pi_{k-2} + \pi_{k-1} + (\tau-2)\xi_1)/\tau]m_{t-k+3} + \dots +$$

$$[\pi_{k-\tau+1} + \dots + \pi_{k-1} + \xi_1)/\tau]m_{t-k+\tau} + [\pi_{k-\tau+2} + \dots + \pi_{k-1})/\tau]m_{t-k+\tau+1} +$$

$$\dots + [\pi_\tau + \dots + \pi_{2\tau-1})/\tau]m_{t-\tau+1} + [\pi_{\tau-1}\theta + \pi_\tau + \dots + \pi_{2\tau-2})/\tau]m_{t-\tau-2} \dots +$$

$$[(\pi_1\theta + \dots + \pi_{\tau-1}\theta + \pi_\tau)/\tau]m_t\Big\} + \frac{1}{1+\alpha}(M_{t-k} + k\mu + m_{t-k+1} + \dots + m_t)$$

The identity between equations (20) and (13) implies that the solution for the coefficients satisfies

$$\xi_0 = [\alpha(\tau+1)/2 + k]\mu + \log(V)-h$$

$$\xi_1 = 1$$

$$\pi_{k-1} = \dots = \pi_{\tau-1} = 1$$

$$(20) \quad \pi_{\tau-2} = \frac{\alpha}{\tau(1+\alpha)}(\tau-1 + \theta) + \frac{1}{1+\alpha}$$

$$\pi_{\tau-3} = \frac{\alpha}{\tau(1+\alpha)}(\tau+2 + \pi_{\tau-2}\theta + \theta) + \frac{1}{1+\alpha}$$

$$\vdots$$

$$\pi_j = \frac{\alpha}{\tau(1+\alpha)}(j+1 + \pi_{j+1}\theta + \dots + \pi_{\tau-2}\theta + \theta) + \frac{1}{1+\alpha}$$

$$\vdots$$

$$\pi_0 = \frac{\alpha}{\tau(1+\alpha)}(1 + \pi_1\theta + \dots + \pi_{\tau-2}\theta + \theta) - \frac{1}{1+\alpha}$$

The neutrality of fully perceived money appears in the unity value for $\xi_1, \pi_{k-1}, \ldots, \pi_{\tau-1}$ and the $k\mu$ term in ξ_0. This means that the part of the current money stock which is known by all agents in the economy--either from public announcement, from observations on past price levels, or by the knowledge that money grows at an average rate of μ--has a one-to-one effect on the price level. The term $[\alpha(\tau+1)/2]\mu + \log(V)$ in ξ_0 represents the effect of expected money growth and inflation on velocity and the current price level.

Consider now $\pi_{\tau-2}$. Since $\theta < 1$, it holds $\pi_{\tau-2} < 1$. $m_{t-\tau+2}$ has a non-neutral effect because it is unperceived by those agents--the less informed at time t--who visited other markets before τ-1 periods. The next coefficient is $\pi_{\tau-3}$, which corresponds to $m_{t-\tau+3}$. Since $\pi_{\tau-2}\,\theta < 1$, $\pi_{\tau-3} < \pi_{\tau-2}$. The smaller coefficient for $m_{t-\tau+3}$ is due to the larger fraction of the economy that cannot infer $m_{t-\tau+3}$. The fraction consists of the agents who shopped at $t-\tau+1$ and $t-\tau+2$. In general, the more recent the shock the fewer the agents who can infer it and thus the smaller its effects on the price level. The contemporaneous m_t is known only by those agents who are currently in their shopping trips and thus it has the smallest coefficient.[6]

A full solution of π_j in terms of the exogenous parameters is not computed. However, this solution is computable since one can substitute for $\pi_{\tau-2}$ in $\pi_{\tau-3}$ and then both expressions in $\pi_{\tau-4}$ and so on.

<u>A More Detailed Interpretation of the Money Shocks' Coefficients.</u>

The general form of π_j in (21) represents the average effect of m_{t-j} in the economy. Since agents--and in this specification, also markets--can be classified into groups according to the date of their last shopping trip, π_j

averages the different effects of m_{t-j} across these categories. Consider first the agents whose last trip was before t-j-1 periods. They cannot infer m_{t-j}, but they know that at the time of their next shopping trip--$t+\tau-j-1$--all other agents will know m_{t-j}. This is so because at any date, all money growth from $\tau-1$ periods in the past and backwards can be inferred by all. Therefore, their average estimate of $EP_{t+\tau-j+1}$ in this group includes θm_{t-j}. This is their (average) current best estimate of m_{t-j}, and is expected to be neutral at the time of their next shopping trip. The θ appearing by itself in the π_j expression comes from the effect of m_{t-j} on these markets.

Turn now to the agents in the t-j-2, t-j-3, ..., $t-\tau+1$ groups, which cannot infer m_{t-j} either. Since they are closer to their next visit to the other markets, their corresponding price expectations, $EP_{t+\tau-j-2}$, $EP_{t+\tau-j-3}$, ..., EP_{t+1}, will take into account that at those future times m_{t-j} will still be unperceived by some other agents. In other words, the effect of m_{t-j} on these markets is determined by, first, the shock being locally unperceived, and second, the knowledge of the local agents that at the time they will be purchasing other goods, m_{t-j} will be partially unperceived. Take for example the t-j-2 group. Their next trip is at $t+\tau-j-2$. At that time m_{t-j} will still be unperceived by one group. Thus, the effect of m_{t-j} on $P_{t+\tau-j-2}$ is expected to be the same as that of $m_{\tau-2}$ on P_t, and given by the coefficient $\pi_{\tau-2}$, which appears in the term $\pi_{\tau-2}\theta$ in π_j. The terms $\pi_{\tau-3}\theta$, ..., $\pi_{j+1}\theta$, reflect similar considerations with respect to the traders in the t-j-3, ..., $t-\tau+1$ groups.

Finally, the agents who visited the other markets at t-j, t-j+1, ..., t do infer m_{t-j}. They also know that by the time of their next trips--at $t+\tau-j$, $t+\tau-j+1$, ..., $t+\tau$, correspondingly--all other agents will be able to infer

m_{t-j}. For both reasons the effect of m_{t-j} on these j+1 groups is neutral and this is reflected in the j+1 term in the π_j expression.

Solution for Aggregate Output and the Phillips Curve.

If $P_t(z)$ is the price that clears the market for commodity z, an expression for output of z is obtained from the demand function in equation (11)

$$y_t(z) = M_t - P_t(z) + \log(V) + \varepsilon_t^d(z)$$

Aggregate output is computed now by averaging $y_t(z)$ across markets

$$y_t = M_t - P_t - \log(V)$$

Substituting the solution for P_t yields

$$y_t = \left\{h-[\alpha(\tau+1)/2]\mu\right\} + (1-\pi_0)m_t + (1-\pi_1)m_{t-1} + \dots + (1-\pi_{\tau-2})m_{t-\tau+2}$$

The expression for output at time t has a constant term and a stochastic variation determined by the current and past money shocks. Define the stochastic part as $\tilde{y}_t$, namely

$$(22) \quad \tilde{y}_t = (1-\pi_0)m_t + (1-\pi_1)m_{t-1} + \dots + (1-\pi_{\tau-2})m_{t-\tau+2}$$

The pattern of the coefficients of the money shocks is declining with the length of the lag. This declining effect of money on output mirrors the opposite effect on the price level.

Turn now to the main point in this section, which is to discuss the effects of a change in τ on the stochastic behavior of output. Consider an exogenous increase in the frequency of transactions from one every τ periods to one every τ-1 periods. In the new equilibrium, the resulting coefficients of the money shocks are

$$\pi'_{\tau-2} = 1$$

(23)

$$\pi'_{\tau-3} = \frac{\alpha}{(\tau-1)(1+\alpha)}(\tau-2+\theta) + \frac{1}{1+\alpha}$$

$$\pi'_{\tau-4} = \frac{\alpha}{(\tau-1)(1+\alpha)}(\tau-3 + \pi_{\tau-3}\theta + \theta) + \frac{1}{1+\alpha}$$

$$\vdots$$

$$\pi'_{j} = \frac{\alpha}{(\tau-1)(1+\alpha}(j+1 + \pi'_{j+1}\theta + \ldots + \pi'_{\tau-3}\theta + \theta) + \frac{1}{1+\alpha}$$

Compare the coefficients in (23) tc those in (21). The less informed traders in the shortened trade period case can infer $m_{t-\tau+2}$, and therefore it has now a neutral effect--$\pi'_{\tau-2} = 1$, while previously $\pi_{\tau-2} < 1$. The $m_{t-\tau+3}$ variable is now unperceived only by the less informed group, while previously both the traders who shopped at $t-\tau+1$ and $t-\tau+2$ could not infer it. A larger fraction of the economy has now knowledge of $m_{t-\tau+3}$ and thus its effect is closer to neutrality: $\pi'_{\tau-3} > \pi_{\tau-3}$. This inequality can be easily verified using $\pi'_{\tau-2}\theta < 1$. For the same reason, all the new coefficients in (23) will be larger than those in (21), namely

$$(24) \quad \pi'_{j} > \pi_{j} \qquad j = \tau-2, \tau-1, \ldots, 0$$

A formal argument to show that these inequalities hold is made in appendix B.

A general interpretation of this result is that the shorter the trade period, agents possess--on average--more recent information about the price level and thus, they can estimate the current value of the money stock more accurately. Hence, misperception of nominal shocks for relative shifts is reduced, producing a more neutral effect of monetary disturbances.[7] This implies a weaker correlation between money shocks and real output in equation (22), which can be seen as an inverse Phillips Curve type of relationship.

Aggregation of Ouptut and Money Growth Over Time.

One can define also a single coefficient as the slope of the inverse Phillips Curve by aggregating (22) over a longer period. Since τ is the length of a production and purchase of consumption goods cycle, one can imagine the magnitude of the time unit--indexed by t--as short relative to the basic periods empirically used to investigate output fluctuations (e.g. a quarter or a year). Thus, it seems meaningful to define a period of length T ($T > \tau$) and to consider the deviations of the output flow during this longer period relative to its normal level. In percentage units, this deviation is approximately equal to

$$(25)\quad (\tilde{y}_t + \dots + \tilde{y}_{t-T+1})/T = \Big\{(1-\pi_0)m_t + [(1-\pi_1) + (1-\pi_0)]m_{t-1} + \dots$$
$$+ [(1-\pi_{\tau-3}) + \dots + (1-\pi_0)]m_{t-\tau+3} + [(1-\pi_{\tau-2}) + \dots + (1-\pi_0)]\ (m_{t-\tau+2} + \dots$$
$$+ m_{t-T}) + [(1-\pi_{\tau-2}) + \dots + (1-\pi_1)]m_{t-T+1} + \dots + (1-\pi_{\tau-2})m_{t-T-\tau+2}\Big\}/T$$

Consider now the relationship between the movement of output during the T-period that ends at time t and the deviation of money growth from its trend during the same T-period--$m_t + \dots + m_{t-T+1}$. This monetary variable is also the unexpected money growth for this T-period as to time t-T. Also, one can calculate the effect of the lagged T-period money shock on output. The slope coefficient in the regression of $(\tilde{y}_t + \dots + \tilde{y}_{t-T+1})$ on $(m_t + \dots m_{t-T+1})$ under trade periods of length τ can be interpreted as the slope of the inverted Phillips Curve. This coefficient is calculated as

$$\pi = Cov[(\tilde{y}_t + \dots + \tilde{y}_{t-T+1}), (m_t + \dots + m_{t-T+1})]/Var(m_t + \dots + m_{t-T+1})$$

Using (25) and the stochastic properties of the money growth process yields

$$(26) \quad \pi = \Big\{(1-\pi_0) + [(1-\pi_1) + (1-\pi_0)] + \dots + [(1-\pi_{\tau-3}) + \dots + (1+\pi_0)]$$
$$+ (T-\tau+2)[(1-\pi_{\tau-2}) + \dots + (1-\pi_0)] \Big\} /T$$

Define now π' as the similar regression coefficient under trade cycles of length τ-1. It has the same form of π in (26) but with the π_j-s replaced by the π_j'-s. Since $\pi_j' > \pi_j$ for all the relevant j-s it holds $\pi' < \pi$. In other words, the correlation between output and monetary movements during periods of length T decline with the frequency of transactions.

Finally, Lucas's hypothesis on the slope of the Phillips Curve also holds in this extension of his model. Each π_j depends positively on θ, which in turn is a positive function of σ_m^2. Therefore, π decreases in σ_m^2.

Section III. Empirical Tests

a. Data across countries.

A first test of the proposition that higher inflation rates reduce the sensitivity of output to aggregate shocks can be carried out using Lucas's (1973, tables 1 and 2) and Alberro's (1980, table 1 and table 2--second column) estimates of an inverted Phillips Curve coefficients and data on average inflation rates and variances of nominal income across countries. First, their results on the Lucas hypothesis can be summarized by regressing $\hat{\pi}_i$--the estimate of the Phillips Curve slope coefficients in country i--on σ^2_{xi}--the variance of the change in nominal income in country i. Using Lucas's coefficients for eighteen countries in the period 1952-1967 the resulting correlation is

$$(26) \quad \hat{\pi}_i = \underset{(.05)}{.58} - \underset{(5.5)}{19.5}\sigma^2_{xi} + \text{unexplained term}$$

where the numbers in parenthesis are the standard errors of the coefficients. Alberro estimated the π_i coefficients for 49 countries--including the 18 in Lucas's sample--using the period 1953-1969.[8] The estimated correlation using these data is almost identical to that reported in (26).

One can test now the correlation between the estimates of the Phillips Curve slopes and DP_i--the average inflation rate in country i. Given the Lucas hypothesis, in order to isolate a partial correlation between inflation and the slope coefficients, the variances must be held constant. An exercise of this type can be carried out by regressing $\hat{\pi}_i$ on both DP_i and σ^2_{xi}. Using Lucas's estimates the resulting equation is

$$\hat{\pi}_i = \underset{(.06)}{.64} - \underset{(1.4)}{2.5}DP_i - \underset{(8.7)}{6.6}\sigma^2_{xi} + \text{unexplained term.}$$

$$R^2 = .55 \qquad F(2,15) = 9.1$$

The coefficients have the expected sign but the standard errors are large. Since the F-statistic is well above the 99% critical value, the standard errors indicate the presence of a strong correlation between DP_i and σ_{xi}^2. When Alberro's data is used, the estimated equation is

$$\hat{\pi}_i = \underset{(.47)}{.63} - \underset{(1.0)}{3.4} DP_i + \underset{(9.3)}{8.8} \sigma_{xi}^2 + \text{unexplained term.}$$

$$R^2 = .39 \qquad F(2,45) = 14.3$$

Here the coefficient of DP_i is significantly different from zero, while the coefficient of σ_{xi}^2 is not.[9] One can also test the existence of this correlation among the relatively moderate inflation countries by excluding Argentina and Paraguay from Lucas's sample, and Argentina, Brazil, Chile, Indonesia, Korea and Uruguay from Alberro's sample. The results from the first sample show lack of statistically significant coefficients both on DP_i and σ_{xi}^2

$$\hat{\pi}_i = \underset{(.19)}{.84} - \underset{(4.5)}{5.3} DP_i - \underset{(140.0)}{142.0} \sigma_{xi}^2 + \text{unexplained term}$$

$$R^2 = .13 \qquad F(2,13) = 1.0$$

However, using the stable price countries in Alberro's sample, the results are more in line with those reported previously

$$\hat{\pi}_i = \underset{(.08)}{.74} - \underset{(1.6)}{5.5} DP_i - \underset{(18.1)}{3.8} \sigma_{xi}^2$$

$$R^2 = .23 \qquad F(2,40) = 5.9$$

In general these results provide support (stronger than expected) for the hypothesis being tested. With respect to the lack of correlation between $\hat{\pi}_i$ and σ_{xi}^2, one can speculate that it may be related to the errors-in-variables bias contained in $\hat{\pi}_i$. Lucas's and Alberro's equations were estimated using data on the change in nominal income rather than the unexpected change. If, for example, velocity in each country is constant the variable used is $m_t + \tilde{\mu}_t$, where $\tilde{\mu}_t$ is the expected deviation of money growth from the constant μ. Then, ignoring the lagged output variable included in their equations, one can represent the relationship between $\hat{\pi}_i$ and the "true" π_i as

$$\hat{\pi}_i = \pi_i \frac{\sigma_{mi}^2}{\sigma_{mi}^2 + \sigma_{\mu i}^2}$$

π_i is theoretically decreasing in σ_{mi}^2 and DP_i, but the ratio of variances increases in σ_m^2. If this ratio is the same across countries, the $\hat{\pi}_i$-s are still biased but they should be negatively correlated with σ_{xi}^2. However, if this ratio increases with σ_{xi}^2 in some segments of the σ_{xi}^2 range, it is possible that the overall correlation between $\hat{\pi}_i$ and σ_{xi}^2 may be negligible.

b. Time series data from the United States

A second test of the proposition presented in this paper uses annual time series data from the United States after World War II. The test consists in the following: the output equation obtained by Barro (1980) is reestimated allowing the coefficients of unanticipated money to depend on a measure of anticipated inflation. The modified equation is

$$(27)\quad y_t = a_0 + a_1(1-bDP_t^e)DMR_t + a_2(1-bDP_{t-1}^e)DMR_{t-1} + a_3\log(G_t) + a_4 t + u_t$$

y_t is the log of real GNP, the DMR-s are Barro's estimates of unanticipated money growth, G_t is real federal purchases, t is time, DP_t^e is a measure of the inflation rate anticipated at time t, and u_t is a random term.

The hypothesis to be tested is whether the b coefficient is positive and whether a_1 and a_2 remain positive. The form of (27) is a linear approximation expressing the hypothesis that the higher the expected inflation rate, the lower the effect of the DMR-s--because they are sooner perceived as money. To preview the results of this test, the statistical significance of the coefficient b turns out to be very sensitive to the starting year of the sample.

The estimation was carried out using the 3-month Treasury Bill and the Aaa corporate bonds nominal rates of return as measures of expected inflation. If the movements in these interest rates are caused mainly by changes in inflationary expectations, the Treasury Bill rate represents the short term expected inflation (Fama (1975)) and the bonds rate reflects expectations over the more distant future. Which of these variables is more adequate in this context may be related to the existence and importance of costs of adjusting the transaction frequency. If these costs are relatively low a short-term rate is more adequate. If these costs are relatively high, the transaction frequency would be affected mainly by expectations of prolonged periods of inflation. Given no a-priori presumption about these adjustment costs, the equation was estimated separately using each one of these interest rates. The variable used in the equations referred to in the paper is the Treasury Bill rate, RT_t, which--from the point of view of the theory being tested--fits somewhat better. Under the assumption that the DMR-s are exogenous, the endogeneity of the interest rate does not seem to be a serious problem in the estimation because it interacts with the money shocks.

Assuming normally-distributed errors, the equation is estimated using a non-linear maximum likelihood criterion. First, when the sample used the 1946-1978 period, b turns out to be insignificantly different from zero (see table 1, line 1). Speculating that pegging of the Treasury Bill rates until the Treasury-Federal Reserve Accord of 1951 may be affecting the validity of the interest rates as measures of expected inflation, the equation was reestimated over the sample 1953-1978. The resulting equation is

$$(28)\quad y_t = \underset{(.17)}{4.16} + \underset{(.71)}{1.71}(1-\underset{(3.7)}{10.9}RT_t)DMR_t + \underset{(.72)}{2.11}(1-10.9RT_{t-1})DMR_{t-1}$$

$$+ \underset{(.04)}{.121}\log(G_t) + \underset{(.001)}{.032}t$$

$$\hat{\sigma} = .017 \qquad D.W. = 1.6$$

where the numbers in parenthesis are asymptotic standard errors. The coefficient b appears here with the expected sign and with a relatively small standard error (t-ratio of 2.9).[10]

However, a closer examination indicates that the poor results obtained in the entire 1946-1978 sample may be due mainly to the observations of 1951 and 1952, which correspond to the Korean War. If only these two observations are deleted, the estimated b coefficient is 8.2 with a standard error of 4.1 (see table 1, line (2)). According to this result, it seems that the pegging of the interest rate may not be the main problem that affects the estimate of b in the 1946-1978 sample, but the Korean War years, which correspond to a high level of government spending. Presently, I do not have an explanation of this observation. It could be related to a misspecification of the government spending effects, either on output or in the money growth equation that generated the DMR-s. In any event, in the period beginning in 1953, the negative correlation between RT and the effect of the DMR-s on output is fairly substantial.

I also found interesting to examine whether this correlation has anything to do with the oil price shock, which is accompanied by the highest RT value in the sample--.079 in 1974. To do that, equation (28) was reestimated over the sample interrupted in 1973, and then again deleting only the observations of 1974 and 1975. Both equations--reported in table 1, lines (4) and (5)--show an even stronger negative effect of RT on the coefficients of the money shocks than that appearing in equation (28). This suggests that the effect showing in equation (28) is not related to the oil price shock but, on the contrary, is somewhat obscured by it.

Concluding Remarks

This paper examined the effects of expected inflation on the reponsiveness of output to nominal disturbances. The mechanism described in sections I and II is that expected inflation has a positive effect on the transaction frequency, which in turn increases the flow of price information across markets. More information implies less misperception of monetary shocks as relative shifts in excess demand, resulting in lower sensitivity of real output to these shocks.

The empirical implication of this proposition--namely, that expected inflation reduces the coefficient of nominal shocks in an output equation--was tested first using data across countries, and then with time series data for the United States. Overall, the results in both tests provide support for this implication of the theory. The interpretation of these results as supporting the specific mechanism described in the first two sections is related to two critical assumptions. First, as in the equilibrium business cycles literature, to which this analysis is an extension, the model involves the assumption that agents cannot observe the current value of the money stock. If its value is contemporaneously known, the frequency of price observations--and the money shocks--does not have any effect on output. Secondly, the analysis excluded earning assets. Anticipated inflation does not have to affect the transaction frequency in the commodity markets if there are assets whose return is associated with anticipated inflation. However, this effect may still exist if financial transactions costs preclude agents from using these assets between shopping trips, or if the nominal return on the available assets is, for some reason, sluggish to shifts in the inflation rates.

Summarizing, the theoretical mechanism described in the paper seems consistent with the empirical results, which indicate a substantial negative correlation between anticipated or average inflation rates and the effect of nominal disturbances on output.

Table 1

$$y_t = a_0 + a_1(1-bRT_t)DMR_t + a_2(1-bRT_{t-1})DMR_{t-1} + a_3\log(G_t) + a_4 t$$

SAMPLE	a_0 (const)	b	a_1	a_2	a_3 (log (G))	a_4 (t)	$\hat{\sigma}$	D.W.
(1) 1946-78	4.28 (.05)	3.5 (6.5)	1.13 (.39)	1.12 (.39)	.087 (.013)	.0327 (.0004)	.0170	1.52
(2) 1946-50 1953-78	4.25 (.05)	8.2 (4.1)	1.56 (.46)	1.59 (.48)	.101 (.015)	.0323 (.0005)	.0165	1.47
(3) 1953-78	4.16 (.17)	10.9 (3.7)	1.71 (.71)	2.11 (.72)	.122 (.038)	.0321 (.0005)	.0171	1.61
(4) 1953-73	4.15 (.18)	14.2 (4.0)	1.74 (.82)	2.57 (.90)	.112 (.041)	.0330 (.0009)	.0176	1.28
(5) 1953-73 1976-78	4.14 (.17)	13.7 (2.8)	1.86 (.72)	2.57 (.76)	.123 (.037)	.0323 (.0006)	.0166	1.27

Notes to Table 1

RT is the annual average nominal rate of return on 3 month Treasury Bills (expressed as per cent per year).

$y \equiv \log(GNP)$, GNP is in 1972 dollars.

DMR is the residual from a money growth equation reported in Barro (1980, Table 2).

Appendix A.

This appendix shows the detailed expressions and signs of derivatives used in Section I. The second and cross derivatives correspond to the point where the first order conditions hold

$$C = \frac{S}{\pi\tau}(1-e^{-\pi\tau}) - \frac{b}{\tau}$$

$$\frac{\partial C}{\partial \tau} = \frac{S}{(\pi\tau)^2}[\pi^2 \tau e^{-\pi\tau} - \pi(1-e^{-\pi\tau})] + \frac{b}{\tau^2}$$

(A.1) $$\frac{\partial C}{\partial \pi} = \frac{Se^{-\pi\tau}}{\pi^2 \tau}(1+\pi\tau - e^{\pi\tau}) < 0 \qquad \text{for } \pi\tau > 0$$

(A.2) $$\frac{\partial^2 C}{\partial \tau^2} = \frac{-Se^{-\pi\tau}\pi}{\tau} < 0$$

(A.3) $$\frac{\partial^2 C}{\partial \tau \partial S} = \frac{1}{(\pi\tau)^2}[\pi^2 \tau e^{-\pi\tau} - \pi(1-e^{-\pi\tau})] = \frac{e^{-\pi\tau}}{\pi\tau^2}(1+\pi\tau - e^{\pi\tau}) < 0$$

$$\frac{\partial^2 C}{\partial \tau \partial \pi} = \frac{Se^{-\pi\tau}}{(\pi\tau)^2}[e^{\pi\tau} - 1 - \pi\tau - (\pi\tau)^2]$$

(A.4) $$\frac{\partial^2 C}{\partial \tau \partial \pi} = < 0 \quad \text{if} \quad 1+\pi\tau + (\pi\tau)^2 > e^{\pi\tau}$$

The second inequality holds for $\pi\tau < 1.79$. For realistic values of π and τ, $\pi\tau << 1.79$. Thus (A.4) is assumed given in what follows.

(A.5) $$\frac{\partial^2 C}{\partial S \partial \mu} = \frac{e^{-\pi\tau}}{\pi^2 \tau}(1+\pi\tau - e^{\pi\tau}) < 0$$

(A.6) $$U_{\tau\tau} = U_c \frac{\partial^2 C}{\partial \tau^2} < 0 \qquad \text{from (A.2)}$$

(A.7) $$U_{ss} = U_{cc}\left(\frac{\partial C}{\partial S}\right)^2 - 2U_{c1}\frac{\partial C}{\partial S} a + U_{11}a^2 < 0$$

This inequality holds if the indifference curves between C and L are concave towards the origin.

(A.8) $U_{S\tau} = U_c \frac{\partial^2 C}{\partial\tau\partial S} < 0$ from (A.3)

(A.9) $U_{\tau\tau}U_{SS} - (U_{S\tau})^2 > 0$

This inequality is assumed.

(A.10) $U_{\tau\pi} = U_c \frac{\partial^2 C}{\partial\tau\partial\pi} < 0$ from (A.4)

(A.11) $U_{S\pi} = U_{cc} \frac{\partial C}{\partial\pi} \frac{\partial C}{\partial S} + U_c \frac{\partial^2 C}{\partial S\partial\pi} - U_{cl} \frac{\partial C}{\partial\pi} a \gtreqless 0$

The first term on the right hand side is positive, the second is negative and the third uncertain. This expression involves the familiar ambiguous effect of a change in the return to labor on labor supply. This return is here negatively represented by π.

$$V = \frac{\mu^2\tau}{\mu\tau + e^{-\mu\tau} - 1}$$

(A.12) $\frac{\partial V}{\partial\mu} = \frac{(\mu\tau)(\mu\tau + 2e^{-\mu\tau} + \mu\tau e^{-\mu\tau} - 2)}{(\mu\tau + e^{-\mu\tau} - 1)^2} > 0$

I could determine the sign of $(\mu\tau + 2e^{-\mu\tau} + \mu\tau e^{-\mu\tau} - 2)$ only numerically. It is positive for any $\mu\tau > 0$.

(A.13) $\frac{\partial V}{\partial\tau} = \frac{\mu^2 e^{-\mu\tau}(1 + \mu\tau - e^{\mu\tau})}{(\mu\tau + e^{-\mu\tau} - 1)^2} < 0$

Appendix B.

This appendix shows that

$$(B.1) \quad \pi_i' > \pi_i \qquad i = \tau-2,\ \tau-1,\ \dots,\ 0$$

where π_i' is a coefficient corresponding to the τ-1 periods trade cycle case, and π_i to the τ periods case. These inequalities are verified bv showing that

$$(B.2) \quad \pi_i' - \pi_i \geqslant \frac{(i+1)\alpha}{\tau(\tau-1)\ (1+\alpha)} (1-\theta\pi_i') \qquad i = \tau-2,\ \tau-1,\ \dots,\ 0$$

Since the right hand side of (B.2) is positive, this inequality implies (B.1).

(B.2) is now established using induction. Consider first the difference between $\pi'_{\tau-2}$ and $\pi_{\tau-2}$.

$$\pi'_{\tau-2} - \pi_{\tau-2} = 1 - \frac{\alpha}{\tau(1+\alpha)} (\tau-1 + \theta) - \frac{1}{1+\alpha} = \frac{\alpha}{\tau(1+\alpha)} (1-\theta), \quad \text{or}$$

$$\pi'_{\tau-2} - \pi_{\tau-2} = \frac{(\tau-1)\alpha}{\tau(\tau-1)(1+\alpha)} (1-\theta\pi'_{\tau-2})$$

Thus, (B.2) holds for i= τ-2. It has to be shown now that if

$$(B.3) \qquad \pi_j' - \pi_j \geqslant \frac{(j+1)\alpha}{\tau(\tau-1)(1+\alpha)} (1-\theta\pi_j')$$

it also holds

$$(B.4) \qquad \pi'_{j-1} - \pi_{j-1} \geqslant \frac{j\ \alpha}{\tau(\tau-1)(1+\alpha)} (1-\theta\pi'_{j-1})$$

From the expressions for π_j and π_j' in equations (21) and (23), one can write the following equalities

$$\text{(B.5)} \quad \pi'_j - \pi'_{j-1} = \frac{\alpha}{(\tau-1)(1+\alpha)} (1-\theta\pi'_j)$$

$$\text{(B.6)} \quad \pi_j - \pi_{j-1} = \frac{\alpha}{\tau(1+\alpha)} (1-\theta\pi_j)$$

Subtracting (B.6) from (B.5) and rearranging terms yields

$$\text{(B.7)} \quad \pi'_{j-1} - \pi_{j-1} = (\pi'_j - \pi_j) - \frac{\alpha}{\tau(\tau-1)(1+\alpha)} (1-\theta\pi'_j) + \frac{\alpha}{\tau(1+\alpha)} \theta(\pi'_j-\pi_j)$$

Substitute now (B.3) into the first term on the right hand side of (B.7)

$$(\pi'_{j-1} - \pi_{j-1}) \geq \frac{(j+1)\alpha}{\tau(\tau-1)(1+\alpha)} (1-\theta\pi'_j) - \frac{\alpha}{\tau(\tau-1)(1+\alpha)} (1-\theta\pi'_j) + \frac{\alpha}{\tau(1+\alpha)} \theta(\pi'_j-\pi_j)$$

Simplifying yields

$$\text{(B.8)} \quad (\pi'_{j-1} - \pi_{j-1}) \geq \frac{j\ \alpha}{\tau(\tau-1)(1+\alpha)} (1-\theta\pi'_j) + \frac{\alpha}{\tau(1+\alpha)} \theta(\pi'_j - \pi_j)$$

(B.8) can be rewritten as

$$\text{(B.9)} \quad (\pi'_{j-1} - \pi_{j-1}) \geq \frac{j\ \alpha}{\tau(\tau-1)(1+\alpha)} (1-\theta\pi'_{j-1}) + \frac{\alpha\theta}{\tau(\tau-1)(1+\alpha)} [(\tau-1)(\pi'_j-\pi_j) - j(\pi'_j-\pi'_{j-1})]$$

Consider the bracketed expression on the right hand side of (B.9). If it is nonnegative, (B.4) is established. From (B.3) and (B.5) it holds

$$[(\tau-1)(\pi'_j-\pi_j) - j(\pi'_j-\pi'_{j-1})] \geq \frac{(j+1)(\tau-1)\alpha}{\tau(\tau-1)(1+\alpha)} (1-\theta\pi'_j) - \frac{j\ \alpha}{(\tau-1)(1+\alpha)} (1-\theta\pi'_j) =$$

$$\frac{\alpha}{\tau(\tau-1)(1+\alpha)} (1-\theta\pi'_j)\ [\tau-(j+1)] > 0$$

The last inequality holds because $\tau-2 \leq j \leq 0$, and this completes the argument. It can be noted that given (B.3), (B.4) was shown to be a strict inequality, which means that only for $i=\tau-2$, (B.2) holds as an equality.

Footnotes

[1]Storing costs would affect also the behavior of the individual as a consumer. The higher these costs the lower the optimal stock of consumption goods, implying a shorter transactions period.

[2]This effect involves only substitution because at the maximum

$$\partial C/\partial \tau = 0$$

[3]The discussion abstracts from the effects of differential cash flows accruing to different agents on their trade cycles.

[4]This assumption may be seen as unrealistic in the present context, since the length of the time unit is short. However, I do not believe that the results would be affected by introducing serial correlation in m_t.

[5]Imagine each current price posted above the crossed price of last period.

[6]It is assumed that production in these agents' markets continues at that time. Imagine a shopper-spouse visiting other markets while the producer - spouse continues producing in the local market.

[7]At this point it is appropriate to return and discuss the assumption made earlier, that all agents in one market follow synchronized transaction cycles. This specification has the undesired implication that individual prices do not follow a smooth pattern--as the average price level does. At the time of the shopping trip, the local price jumps due to the change in the relevant price deflator from P_t to $P_{t+\tau}$. A more appropriate specification would be to allow producers in one location to shop at scattered dates. I do not believe, however, that the results for the aggregate economy in this case would differ qualitatively from those obtained here. With scattered shopping trips, agents can learn from the local price something about the price observations of others. However, given the unobserved relative shock $\varepsilon_t^*(z)$, they

([7]continued)

still have the problem of estimating the relative and nominal shocks affecting the local price. The confusion between nominal and real shocks in this case will also diminish when the transaction frequency increases because agents will be able to infer more about the money shocks.

[8]For Uruguay the period is 1958-1972.

[9]As in Alberro's calculations, Indonesia was excluded from the sample. If Indonesia is included, the coefficient of DP_i remains positive and significant--with a higher t-ratio--but the coefficient of σ^2_{xi}, still positive, becomes statistically significant.

[10]If the bonds rate is used, the estimated value of b is 7.5, s.e. = 3.3. Using actual inflation, b turns out to be insignificantly different from zero.

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