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MONETARY ACCOMMODATIONS OF SUPPLY SHOCKS UNDER RATIONAL EXPECTATIONS

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ABSTRACT

The following policy question is addressed: When the economy is subject to a "supply shock," such as a rise in the price of energy, is there an exploitable tradeoff between inflation and unemployment? That is, would a more inflationary monetary policy lead to a shallower recession?

Two theoretical macro models are used to address this question; both assume rational expectations. It is found that such a tradeoff always exists where responses to unanticipated supply shocks are concerned. Conclusions with respect to anticipated supply shocks tend to be more model specific; but some exploitable tradeoff emerges if either money wages or the nominal price of oil is "sticky."

Attention is paid to the question of whether parameters of the model might depend on the policy rule being followed, in accord with Lucas' celebrated critique of econometric policy evaluation.

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1. Introduction

To my mind, the leading question of macroeconomic policy since 1973 has been this: When the economy is buffeted by inflationary shocks from the supply side, should monetary policy turn "accommodative," that is, should the money supply be expanded in order to "finance" the higher prices? Clearly the question is normative. Rather than try to answer it directly (a fruitless task), this paper addresses a related question in positive economics: If a policy of monetary accommodation is pursued, what will be the effects on output and prices? More specifically, is there an inflation-unemployment tradeoff that policy can try to exploit?

Theoretical analysis of the Keynesian variety generally leads to the conclusion that there is such a tradeoff in the short run.[1] And empirical estimates of the dimensions of this tradeoff typically find large output effects and small price effects from monetary accommodation in the short run.[2]

But a number of economists have questioned these theoretical and empirical results on the ground that they fail to deal properly with inflationary expectations. Often appealing to some sort of "rational expectations" arguments, these economists deny that there is an exploitable tradeoff and oppose monetary accommodation. For example, Barro (1976, p. 26) writes:

> Adverse shifts like the oil and agricultural crises will reduce output and cause painful relative adjustments no matter what the reaction of the monetary authority. Added monetary noise would only complicate and lengthen the process of adjustment.

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[1]See, for example, Gordon (1975), Phelps (1978), Gramlich (1979), Blinder (1979, Chapter 2).

[2]See, for example, Perry (1975), Ando and Palash (1976), Eckstein (1978), Blinder (1979, 1980).

2. Basic Ideas

The essence of the rational-expectations case against accommodation is best understood by juxtaposition with the Keynesian case in favor. According to Keynesian analysis (Figure 1), an inward shift of the supply curve results in both higher prices and lower output in the short run. Demand management, by shifting the aggregate demand curve upward, can cushion the output effects while exacerbating the price-level effects. Therein lies the tradeoff.

At least where anticipated monetary policy is concerned, standard rational expectations models imply that the aggregate supply curve is vertical, even in the short run (Figure 2). As in the Keynesian case, it is clear that a leftward shift of the aggregate supply curve lowers output and raises prices. But now the case for accommodation has apparently vanished because the only thing an upward shift in the aggregate demand curve could accomplish would be to raise the price level--not raise real output. In fact, the optimal policy seems to be to reduce aggregate demand enough to prevent the price level from rising.

I will argue here, however, that this simple story leaves out at least three pertinent aspects of the OPEC shock. The first simply applies a point made already by Fischer (1977), Phelps and Taylor (1977), and others: if wages (or prices) are sticky for some reason, then the short-run aggregate supply curve retains some upward slope despite rational expectations. In such a case, Figure 1 again becomes relevant, and a potential case for accommodation arises. The second point is that the OPEC shocks also shifted the aggregate demand curve to the left by redistributing income toward people with very low marginal propensities to consume. This suggests a potential role for stabilization policy in offsetting the demand shift. Finally, I suggest that so long as OPEC posts a nominal price of oil, the short-run aggregate supply curve retains some positive slope despite rational expectations even without wage or price stickiness. Since this third aspect seems to be the most controversial, I devote the remainder of this section to spelling out its rationale.

First some notation. Let R_t be the (log of the) nominal price of oil. I assume throughout that OPEC sets the price of oil before period t begins. Thus they cannot observe the (log of the) general price level, P_t, but can form an expectation of it, ${}_{t-1}\hat{P}_t$. If OPEC sets a target real price of oil in view of these expectations, then:

$$(2.1) \quad R_t = {}_{t-1}\hat{P}_t + r_t ,$$

where r_t is the (log of the) exogenous target real price. Throughout Section 4, I will assume that OPEC adheres to (2.1) and therefore cannot be victimized by anticipated inflation. But Section 3 will consider rules of the form:

$$(2.2) \quad R_t = r_t + \omega_{t-1}\hat{P}_t + (1-\omega)Z_t ,$$

where Z_t is some exogenous nominal variable. If $\omega = 1$, this is the same as (2.1). At the other extreme, if $\omega = 0$, then OPEC's nominal price becomes exogenous, and monetary policy can--in principle--push the real price as low as it wants. Intermediate cases with $0 < \omega < 1$ imply partial adjustment of the nominal OPEC price to anticipated inflation. Let me now provide a rationale for such partial adjustment.

think we may safely assume that purchasing power parity does not hold in the very short run. In this case, the fraction ω in (2.2) can be expected to be less than unity.

To summarize this discussion, the parameter ω in (2.2) reflects two factors: the extent to which other countries follow suit in pursuing an accommodating monetary policy, and the extent to which purchasing power parity holds. For short-run analysis, it seems likely that both of these will be quite incomplete, so ω will be substantially below unity. In the longer run, however, $\omega=1$ and (2.2) reduces to (2.1).

3. A Model with a Nominal OPEC Pricing Rule

The first model to be considered is an extension of the famous Lucas (1973) model in which all markets are spot markets and prices are perfectly flexible. The potential case for accommodation here is based on distribution effects and on the notion, embodied in (2.2), that OPEC may not react fully to anticipated domestic inflation. Section 4 will show that similar results can be obtained when OPEC cannot be "fooled" even in the short run, if money wages are sticky.

3.1 Dimensions of the "Present Disaster Argument"

It is worth recalling the basic rationale for the Lucas supply function: agents do not know the current price level, and instead must infer it from the information available to them. In Lucas' (1973) analysis, this information set consisted of the agent's own price and his knowledge of the distribution of P_t as of time t-1. As he showed, optimal use of this information leads to a supply function:

$$(3.5)\quad p_t - {}_{t-1}p_t = \frac{m_t - {}_{t-1}m_t}{1+\alpha+\delta-c} + \frac{\delta-c}{1+\alpha}(r_t - {}_{t-1}r_t)$$
$$+ \frac{\delta-c}{1+\alpha}\ \frac{{}_{t-1}\hat{m}_t - {}_{t-1}m_t}{1+\alpha+\delta-c} + \eta_t$$

where $\eta_t = \frac{v_t - e_t}{1+\alpha+\delta-c}$,

and where the last term in (3.4) and (3.5) allows OPEC's prediction of m_t to differ from the domestic prediction because OPEC knows what r_t will be.

One important aspect of the OPEC shock emerges already from these equations. An increase in the relative price of oil decreases both aggregate supply and aggregate demand. It will be inflationary only if δ exceeds c , an assumption I make hereafter.

To take up the issue of "accommodation," I introduce the following class of monetary rules:

$$(3.6)\quad m_t = k_t + \beta(R_t - {}_{t-1}R_t) + b\,{}_{t-1}R_t\ :$$

If $b = \beta = 0$, this is the Friedman rule of letting money grow at the growth rate of potential GNP. The parameters b and β indicate the elasticity of the money stock with respect to anticipated and unanticipated OPEC price increases respectively. Any rule with b and/or β positive will be considered "accommodating."

Under this rule, ${}_{t-1}m_t$ is always a perfect forecast of m_t and does differ from ${}_{t-1}m_t$, viz:

$$(3.7)\quad {}_{t-1}\hat{m}_t - {}_{t-1}m_t = \frac{\beta(1+\alpha+\delta-c)}{1+\alpha-\beta}(r_t - {}_{t-1}r_t)\ .$$

A fair amount of tedious algebra establishes that:

$$(3.8)\quad {}_{t-1}p_t = \frac{b+\delta-c}{1-b}\ {}_{t-1}r_t$$

$$(3.9)\quad {}_{t-1}\hat{p}_t = {}_{t-1}p_t + \frac{\beta+\delta-c}{1+\alpha-\beta}(r_t - {}_{t-1}r_t)$$

Why does a case for activist monetary policy appear in this model, whereas none exists in the standard Lucas model (the case $\delta = c = 0$)? The reason is pretty clear. By hypothesis, OPEC introduces a disturbance which is recognized contemporaneously by the monetary authority.[1] Since R_t pushes P_t up, this additional information gives the authority some ability to react to the contemporaneous price-level disturbance--the righthand side of (3.5). This is an ability the central bank would use even in the Lucas model without OPEC--if only it had it.

I now turn to the opposite extreme case: where $\omega = 0$ so that R_t is an exogenous variable. I emphasize that this does not mean that R_t is fixed for long periods of time in nominal terms, but only that R_t does not react contemporaneously to either P_t or its expectation. Under the same monetary rule as before, the model with $\omega = 0$ produces the following solutions:

$$(3.13)\quad {}_{t-1}P_t = \frac{b+\delta-c}{1+\delta-c}\,{}_{t-1}R_t$$

$$(3.14)\quad P_t - {}_{t-1}P_t = \left(\frac{\beta+\delta-c}{1+\alpha+\delta-c}\right)(R_t - {}_{t-1}R_t) + \eta_t$$

$$(3.15)\quad y_t = k_t - \frac{\delta(1-b)}{1+\delta-c}\,{}_{t-1}R_t + \left[\frac{(\alpha+\delta)(\beta+\delta-c)}{1+\alpha+\delta-c} - \delta\right](R_t - {}_{t-1}R_t) + u_t .$$

The dimensions of the policy problem with respect to unanticipated OPEC shocks are much the same as they were in the previous case (with $\omega = 1$), except that OPEC is now characterized by its nominal price, rather than its real price. Nothing further need be said about this.

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[1]And also by everyone else. There is no issue of informational asymmetry here.

Secondly, it was just noted that an activist monetary response to unanticipated OPEC price changes is advisable because of the informational content of R_t as a predictor of the price level. This informational content, however, depends critically on the correlation between R_t and P_t; and this correlation, in turn, depends in a fundamental way on the policy rule being followed. Lucas' (1976) econometric policy critique makes us suspect that the parameters of the aggregate supply function (3.1) might not be invariant to the policy rule.

This possibility is explored in the appendix, where the following is established. Under the class of monetary rules considered here (equation (3.6)), neither α nor δ in equation (3.1) is structural in Lucas' sense; that is, they both depend on the policy rule. More specifically, it is shown that as policy becomes more accommodating toward unanticipated shocks (i.e., as β rises), α falls and δ rises (the sum $\alpha + \delta$ remains the same). Since the inherent difficulties caused by OPEC shocks depend directly on the size of the parameter δ, the future disaster argument clearly undercuts the case for accommodation.

How important is this problem quantitatively? The appendix establishes the following result:

$$(3.16)\quad \frac{\partial\delta}{\partial\beta} = \frac{a\left(\frac{\sigma_z^2}{\sigma_z^2+\sigma_\eta^2}\right)}{1+d-c}$$

where a and d are respectively the elasticities of output with respect to the perceived relative price of output and the relative price of oil in the micro supply function (see the appendix for details); σ_z^2 is the variance of relative prices; and σ_η^2 is the portion of the

$$y_{1t} = k_{1t} - \alpha({}_{t-1}W_t - P_t) - \delta(R_t - P_t) + e_{1t}$$

where ${}_{t-i}W_t$ is the (log of the) contract wage for period t set in period $t-i$. To interpret these parameters, suppose the underlying production function is Cobb-Douglas:

$$y = c + s_1\ell + s_2 e \,,$$

where ℓ is the log of labor input and e is the log of energy input. Assuming the presence of some fixed factor (e.g. capital) and constant returns to scale, $s_1 + s_2$ will be less than unity. Profit maximization will then imply:

$$\alpha = \frac{s_1}{1-s_1-s_2}\,, \qquad \delta = \frac{s_2}{1-s_1-s_2}\,.$$

For plausible empirical values of the factor shares, $\alpha > 1$ and $\delta < 1$.

Assume that nominal input prices are given by (2.1) for energy and the following equation, which is carried over directly from Fischer (1977), for wages:

$$(4.1) \quad {}_{t-i}W_t = {}_{t-i}P_t \qquad i=1,2\,.$$

Then the following supply function is implied:

$$y_{1t} = k_{1t} + \alpha(P_t - {}_{t-1}P_t) + \delta(P_t - {}_{t-1}\hat{P}_t) - \delta r_t + e_{1t}\,.$$

A similar supply function holds for the firms that set their wages in period t-2:

$$y_{2t} = k_{2t} + \alpha(P_t - {}_{t-2}P_t) + \delta(P_t - {}_{t-1}\hat{P}_t) - \delta r_t + e_{2t}\,.$$

Since half the firms fall in each category, the following aggregate supply function follows directly (with obvious notation):

Consider first accommodation of an unanticipated OPEC shock. This is possible, it will be recalled, since both the monetary authority and all private agents are assumed to know R_t at the beginning of period t. From (4.3), the price level can be insulated from unanticipated OPEC shocks by setting $\beta = c - \delta$, which I have assumed to be negative. This is an anti-accommodating stance of the sort considered just above. However, as (4.6) makes clear, such a policy would allow the full negative effect of any unanticipated OPEC shock on production to take place. Instead, a policy of insulating output from the unanticipated portion of any OPEC price increase would set $\beta = \frac{\delta+\alpha c}{\delta+\alpha} > 0$. The tradeoff is pretty clear, and value judgments are necessary to decide on the appropriate β .[1]

The issue of accommodation of "anticipated" shocks is more subtle since, owing to the assumed structure of contracts, it matters whether the shock was anticipated at time t-1 or at time t-2. Equation (4.6) makes it clear that there is nothing to be gained by trying to offset ${}_{t-2}r_t$; and any monetary reaction whereby ${}_{t-2}m_t$ was raised in response to ${}_{t-2}r_t$ would simply cause inflation. Thus there is no case for accommodating such shocks. In the real world, this translates to the statement that there is no case for accommodating (and perhaps a case for anti-accommodating) any OPEC shock that has been anticipated since before the longest existing contract was struck. I leave it to the reader to appraise the practical significance of this result.

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[1]Notice also the importance of the parameter c , which measures the impact of the shock on aggregate demand. Full accommodation (β=1) cannot be optimal so long as $c < 1$.

$$W = \lambda_1 \text{ Var}(y_t - \bar{k}_t) + \lambda_2 \text{ Var}(P_t - {}_{t-1}P_t) +$$

$$+ \lambda_3 \text{ Var}(P_t - {}_{t-2}P_t) + \lambda_4 \text{ Var}({}_{t-1}P_t - P_{t-1})$$

$$= \lambda_1 \text{ Var}(Y_t - \bar{k}_t) + (1-\lambda_1) \text{ Var}(P_t - {}_{t-1}P_t)$$

if $\lambda_1 + \ldots + \lambda_4 = 1$. Minimizing this with respect to β (b is irrelevant since (4.9) eliminates any difference between ${}_{t-1}r_t$ and ${}_{t-2}r_t$) yields the following optimal policy rule:

$$\beta = \frac{\alpha\lambda_1(\alpha c+\delta) - (1-\lambda_1)(\delta-c)}{\alpha\lambda_1(\alpha+\delta) + (1-\lambda_1)}$$

which tends to call for a positive β if:

1. the relative welfare weight attached to output deviations (λ_1) is higher
2. the demand-reducing aspects of the OPEC shock (c) are greater.

To summarize the results from this model, in a rational expectations model with nominal wage contracts there is a case for providing some degree of monetary accommodation to both anticipated and unanticipated OPEC shocks. This holds even if OPEC knows that a policy of accommodation will be followed, and uses this knowledge in setting its price. The strength of the case for accommodation, of course, depends on value judgments because any such accommodation, while partially insulating real output from OPEC shocks, exacerbates the inflationary consequences of such shocks.

by (4.10) and reworking the model. I report below, with corresponding numeration, those equations that are changed by this substitution:

$$(4.4') \quad P_t - {}_{t-2}P_t = P_t - {}_{t-1}P_t + \frac{b+\delta-\gamma_1-c}{1+\alpha_2-b}({}_{t-1}r_t - {}_{t-2}r_t)$$

$$(4.6') \quad y_t = k_t-(\delta-\alpha\gamma)\,{}_{t-2}r_t + [\frac{\alpha(\beta+\delta-c)}{1+\alpha-\beta} - \delta](r_t-{}_{t-1}r_t)$$

$$+ [\frac{\alpha_2(b+\delta-\gamma_1-c)}{1+\alpha_2-b} - (\delta-\gamma_1)]\,({}_{t-1}r_t - {}_{t-2}r_t) + u_t$$

$$(4.7') \quad {}_{t-2}P_t = \frac{b+\delta-\alpha\gamma-c}{1-b}\,{}_{t-2}r_t$$

$$(4.8') \quad {}_{t-1}P_t = {}_{t-2}P_t + \frac{b+\delta-\gamma_1-c}{1+\alpha_2-b}({}_{t-1}r_t - {}_{t-2}r_t)$$

where $\gamma_1 \equiv \frac{\alpha}{2}\gamma$.

It can be seen from these equations that, wherever δ appears in a term involving variables known before time t, its magnitude is reduced by an amount that depends on γ. Since it is clear from the structure of the model that a reduction in δ lessens the economy's vulnerability to OPEC, it is clear that higher values of γ help insulate the economy from OPEC.

Second, it seems a plausible conjecture that accommodating monetary policy might reduce the sensitivity of expected real wages to OPEC's expected real price, i.e. reduce the value of γ . This possibility is much harder to explore, since it requires a complete micro model justifying (4.10).

The remainder of this section outlines one such model and shows that it implies that γ is in fact

R^* = money price of energy in the future period ($R = \log R^*$)

P^* = price level in the future period ($P = \log P^*$)

$v^* = W^*/P^*$ = real wage ($W - P = \log v^*$)

$r^* = R^*/P^*$ = real OPEC price ($R - P = r = \log r^*$).

The contract sets W^* <u>now</u> with v^* and r^* treated as random variables and L to be chosen by the firm <u>after</u> v^* and r^* are known. The essence of the "future disaster" problem is that, for any chosen W^*, monetary policy influences the joint density of (v^*, r^*) and the issue is whether policy therefore changes the nature of the contract agreement.

I assume that W^* is chosen Pareto optimally, that is, to maximize a weighted sum of expected profits and expected utility of workers. When the future period arrives, assuming a Cobb-Douglas production function, the firm will solve the following <u>certainty</u> problem:

$$\max_{\{E,L\}} L^{s_1} E^{s_2} - v^* L - r^* E \qquad s_1 + s_2 < 1; \quad v^*, r^* \text{ known.}$$

The solution of this problem is:

$$(4.11) \quad \ell = \text{const.} - \frac{s_2}{1-s_1-s_2} r - \frac{1-s_2}{1-s_1-s_2}(W-P) ,$$

which defines ℓ as a random variable from today's perspective.

Finally, make use of the following approximation:[1]

$$\log \mathcal{E}(Z) \approx \mathcal{E}(\log Z) + \frac{1}{2} C_Z^2$$

where Z is any random variable and C_Z is its coefficient of variation, to write this as:

$$E(\log Z) + \frac{1}{2} C_Z^2 = \text{a constant that depends on } \Theta .$$

Using the definitions of Z, b_1, and b_2, it follows that:

$$\frac{-s_2}{1-s_1-s_2} \mathcal{E}(r) - \frac{s_1}{1-s_1-s_2} [W - \mathcal{E}(P)] = \text{const.},$$

or

$$(4.13) \quad W = \mathcal{E}(P) - \frac{s_2}{s_1} \mathcal{E}(r) + \text{const.},$$

where the constant now depends on Θ and on the coefficient of variation of Z.

Now (4.13) is precisely of the form (4.10) with $\gamma = \frac{s_2}{s_1}$. The model therefore implies that γ is independent of the policy rule despite the clear effect of policy on $Var(P)$ and $Cov(P,r)$.

What was wrong with the intuition that told us that the covariance "mattered?" The answer is that while the covariance between P and r is relevant to the variability of profits and to the variability of utility, it does not effect the expected values of either. Thus, if workers maximize expected utility and firms maximize expected profits, behavior will be independent of monetary policy.

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[1] This follows by expanding $\log Z$ in a second-order Taylor series around $\mathcal{E}(Z)$ and taking expectations.

2. Where anticipated OPEC shocks are concerned, conclusions tend to depend more on the specific model. In the extended Lucas model, the potential case for accommodation rests on OPEC's nominal price not adjusting fully to the anticipated U.S. price level. I argued that such a situation is likely to prevail in the short run, but not in the long. In the extended Fischer model, there is a case for accommodating OPEC shocks that are currently anticipated, but were unanticipated when some existing contracts were struck. Thus both models seem to suggest a similar policy response to an OPEC shock: a temporary "blip" in the growth path of the money supply, but no permanent deflection.

Turning next to the more elusive "future disaster" arguments:

1. Where responses to unanticipated OPEC shocks are concerned, the issues are again quite different in the two models. In the extended Lucas model, accommodation of unanticipated shocks will enhance the usefulness of the nominal price of oil as a signal of the aggregate price level, and hence increase the sensitivity of output to OPEC shocks. In the extended Fischer model, on the other hand, a plausible model of the labor contract leads to the conclusion that the parameters of the model are invariant to the policy rule. While this example hardly exhausts the possibilities of modeling the labor contract, it does suggest that any parameter changes would be second-order effects. Thus the "future disaster" argument seems important in the Lucas model, but unimportant in the Fischer model.

2. Where responses to anticipated OPEC shocks are concerned, it seems likely that knowledge that the U.S. will accommodate shocks

APPENDIX

MICROECONOMICS OF THE AUGMENTED LUCAS MODEL

Assume that all oil is imported, and follow Lucas (1973) in assuming that there are a large number of domestic competitive firms, indexed by a continuous variable z, producing in isolated markets and selling their outputs at prices $P_t(z)$. The absolute price of each product differs from the absolute price level, P_t, according to some relative price shock, Z_t. Letting lower case symbols denote natural logs of upper case symbols, Lucas writes:

$$p_t(z) = p_t + z_t \ ,$$

where z_t is assumed to have mean 0 and variance σ_z^2 . The firm at location z makes an estimate of the (unknown) price level, $E_z(p_t)$, given the information available to it, and produces output as an increasing function of its own estimate of its own relative price, viz.,

$$\text{(A.1)} \quad x_t(z) = a[p_t(z) - E_z(p_t)], \quad a > 0 \ ,$$

where x is the deviation of output from trend. To allow OPEC into the picture, I add the notion that output is a decreasing function of the price of oil relative to the price of the firm's output:

$$\text{(A.2)} \quad x_t(z) = a[p_t(z) - E_z(p_t)] - d[R_t - p_t(z)], \quad d > 0 \ ,$$

where R_t is the log of the nominal price of energy--a random variable with known probability distribution. I assume that firms observe the absolute price of energy before they make production decisions.

$$p_t - E_z(p_t) = p_t - \Theta_1 p_t(z) - \Theta_2 R_t - \Theta_3\ {}_{t-1}p_t$$

$$= (1-\Theta_1)p_t - \Theta_1 z_t - \Theta_2 R_t - \Theta_3\ {}_{t-1}p_t$$

$$= -\Theta_1 z_t - \Theta_2(R_t - p_t) - \Theta_3\ {}_{t-1}p_t + \Theta_3 p_t$$

$$= -\Theta_1 z_t + \Theta_3(p_t - {}_{t-1}p_t) - \Theta_2[(R_t - {}_{t-1}R_t) - (p_t - {}_{t-1}p_t)]$$

since $\Theta_1 + \Theta_2 + \Theta_3 = 1$ and ${}_{t-1}R_t = {}_{t-1}p_t$. Thus,

$$M = \Theta_1^2\ \sigma_z^2 + \Theta_2^2(\sigma_R^2 + \sigma_p^2 - 2\sigma_{RP}) + \Theta_3^2\ \sigma_P^2$$

$$- 2\Theta_3\Theta_2\ \sigma_{RP} + 2\Theta_3\Theta_2\sigma_P^2$$

where $\sigma_P^2 = E(p_t - {}_{t-1}p_t)^2$ and $\sigma_{RP} = E(R_t - {}_{t-1}R_t)(p_t - {}_{t-1}p_t)$.

Minimizing this with respect to Θ_1 and Θ_3 (noting that $\Theta_2 = 1 - \Theta_1 - \Theta_3$) gives:

$$\frac{\partial M}{\partial \Theta_1} = 2\Theta_1\sigma_z^2 - 2\Theta_2(\sigma_R^2 + \sigma_P^2 - 2\sigma_{RP}) + 2\Theta_3\ \sigma_{RP} - 2\Theta_3\ \sigma_P^2 = 0$$

$$\frac{\partial M}{\partial \Theta_3} = 2\Theta_3\ \sigma_P^2 - 2\Theta_2\ (\sigma_R^2 + \sigma_P^2 - 2\sigma_{RP})$$

$$+ 2(\sigma_P^2 - \sigma_{RP})(\Theta_2 - \Theta_3) = 0$$

Eliminating Θ_2 gives:

$$\Theta_1(\sigma_z^2 + \sigma_R^2 + \sigma_P^2 - 2\sigma_{RP}) + \Theta_3(\sigma_R^2 - \sigma_{RP}) = \sigma_R^2 + \sigma_P^2 - 2\sigma_{RP}$$

$$\Theta_1(\sigma_R^2 - \sigma_{RP}) + \Theta_3\ \sigma_R^2 = \sigma_R^2 - \sigma_{RP}$$

Solving for Θ_1 and Θ_3 leads to:

Thus, using (A.4) and (A.6), (A.8) and (A.9) can be written

$$(A.10) \quad \alpha = a\left(\frac{\sigma_z^2}{\sigma_z^2+\sigma_\eta^2}\right)\left(\frac{1+\alpha-\beta}{1+\alpha+\delta-c}\right)$$

$$(A.11) \quad \delta = d + a \frac{\sigma_z^2}{\sigma_z^2 + \sigma_\eta^2} \frac{\beta+\delta-c}{1+\alpha+\delta-c} .$$

Observe first that:

$$\alpha + \delta = d+a\left(\frac{\sigma_z^2}{\sigma_z^2+\sigma_\eta^2}\right)$$

so that the sum, $\alpha + \delta$, does not depend on β, or:

$$\frac{\partial\delta}{\partial\beta} = - \frac{\partial\alpha}{\partial\beta} .$$

Using this fact in taking the derivative of (A.11) with respect to β, yields:

$$\frac{\partial\delta}{\partial\beta} = \frac{a}{1+d-c} \frac{\sigma_z^2}{\sigma_z^2 + \sigma_\eta^2} ,$$

which is equation (3.16) in the text.

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