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TAX NEUTRALITY AND THE SOCIAL DISCOUNT RATE:  
A SUGGESTED FRAMEWORK

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ABSTRACT

There is probably no specific problem in tax analysis which has generated as much study and discussion among economists as the question of how to formulate "neutral" tax incentives for investment. Yet no consensus has been reached concerning the proper approach to take when adjusting taxes.

Comparing the two fundamental notions of neutrality found in the literature, referred to here as "present value" rules and "internal rate of return" rules, we argue that there is both a single appropriate neutrality criterion (the latter) and a framework which can be used to evaluate the performance of a tax system with respect to this criterion.

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## I. Introduction

There is probably no specific problem in tax analysis which has generated as much study and discussion among economists as the question of how to formulate "neutral" tax incentives for investment. This concentration of research effort may be traced to the importance and direct relevance to policy design of the issues under investigation. In this light, it is especially distressing to the economist and government planner alike that no consensus has been reached concerning the proper approach to take when adjusting taxes. On the contrary, authors continue to analyze the problem of investment incentives using distinct criteria, each calling markedly different tax schemes "neutral."

Our own view, stated previously,<sup>1</sup> is that this difficulty is due in part to the fact that the very concept of tax neutrality is a fairly limited one when viewed in the broader context of optimal tax theory. However, given this concept's demonstrated institutional relevance, it is important that we have a clear idea of what we are talking about in its regard. This paper represents an attempt to serve this purpose. We first present and compare the two fundamental notions of neutrality found in the literature, and then argue that, given the inherent limitations of this type of analysis, there is both a single sensible neutrality criterion and a framework which can be used to intelligently evaluate the performance of a tax system with respect to this criterion.

## II. Tax Neutrality

The problem normally posed is that the government planner confronts an existing and, perhaps, woefully ill-designed tax system and must, within some

bounds, decide how to change the tax treatment of a specified group of assets to accomplish the objective of greater capital investment. The bounds imposed on the designer usually involve short-run and possibly long-run revenue-loss limits, and the reason for the proposed stimulus is a perceived societal need for more capital (because the existing tax law has heretofore unduly discouraged accumulation).<sup>2</sup> The analysis compares the effect of proposed tax changes on assets of different durability to ascertain whether the resulting tax system will "favor" long-lived or short-lived assets, the reference point being either the initial tax system or else the hypothetical no-tax case. The final tax scheme is deemed "neutral" if it favors neither more durable nor less durable assets, and it is in the way in which the tax system's effects are measured that the two approaches differ. In each case, satisfaction of the neutrality criterion is argued to lead to an efficient allocation of capital, although this can't, in fact, be simultaneously true.

To facilitate the development and comparison of these approaches, we introduce the familiar "user cost of capital" (Jorgenson 1963) which describes the implicit rental price of a unit of capital which depreciates exponentially. While we later relax this restrictive assumption of geometric decay, it is perfectly acceptable for the present illustrative purpose. Without loss of generality in the current context, we ignore personal taxes and inflation, and assume all equity finance at the margin (or, equivalently, no deductibility of interest). Letting  $\delta$  be the decay rate of capital,  $p(\delta)$  the price of such capital in units of the output good,  $\tau$  the corporate tax rate,  $k(\delta)$  the tax credit on gross investment (with no corresponding basis adjustment, following current U.S. practice),  $r$  the required return on equity (taken to be the

interest rate) and  $z(\delta)$  the present value of depreciation allowance arising from a dollar of new investment, the user cost of capital is described by the familiar expression:

$$c(\delta) = p(\delta)(r+\delta)(1-k(\delta)-\tau z(\delta)) / (1-\tau) \quad (1)$$

As stated above, the approaches found in the literature may be placed in one of two categories, although this is less than apparent from a first reading because of differences in assumptions and exposition. Our purpose in the present section will be to describe and compare these two approaches, deferring any evaluation of their merits until this has been accomplished.

#### A. Present Value Rules

This approach to neutrality has been formulated in two equivalent ways in the literature. Some authors (Sunley 1973, Sandmo 1974) have formulated the criterion in terms of the tax system's impact on user cost, others (Black 1959, Boadway 1978) have made reference to the present value of taxes (net of incentives) associated with each asset.

The first approach defines as neutral a tax system which exerts the same proportional influence on user costs associated with different assets, implying that the ratio

$$\frac{p(\delta)(r+\delta)(1-k(\delta)-\tau z(\delta))/(1-\tau)}{p(\delta)(r+\delta)} = \frac{1-k(\delta)-\tau z(\delta)}{1-\tau} \quad (2)$$

is constant.<sup>3,4</sup> The other approach says a tax system is neutral if, at the margin, all investments of equal initial cost have the same present value when their gross-of-tax flows are discounted at  $r$ . Since, by construction, their

net-of-tax flows all have the same present value when discounted at  $r$ , this is the same as requiring the present value of taxes to be equal. Since the return to the marginal investment equals its cost of capital, this implies that

$$pv(\delta) = \int_0^{\infty} e^{-rt} \left[ \frac{p(\delta)(r+\delta)(1-k(\delta)-\tau z(\delta))}{1-\tau} \right] \cdot \frac{e^{-\delta t}}{p(\delta)} dt \quad (3)$$

is constant, which is equivalent to condition (2).

The clearest justification for this general approach has been offered by Boadway (1978):

Capital at any instant will be allocated efficiently if the value of its marginal product is the same in all uses where the value of its marginal product is the present value of the contribution of an increment of capital investment now to output in the future.<sup>5</sup>

#### B. Internal Rate of Return Rules

Perhaps the more common view of neutrality is based on the internal rates of return on different assets. A tax system is viewed as neutral here if the internal rates of return calculated from the pre-tax flows of each marginal project are equal (Musgrave 1959, Chase 1962, Auerbach 1979a, Harberger 1980). As in the previous case, this criterion may also be phrased in terms of tax liability. Since the internal rate of return of each such project's net flows is  $r$ , the requirement is that the effective tax rate should be equal for all assets, where such a rate is defined as the gross rate of return minus the net rate of return  $r$ , divided by the gross rate. Whereas bias against an asset was represented above by a relatively high present value of taxes per dollar of investment, it is here related to a high effective tax rate.

In terms of the model we have been using, this criterion dictates that

$$\rho(\delta) = \frac{(r+\delta)(1-k(\delta)-\tau z(\delta))}{1-\tau} - \delta \quad (4)$$

should be independent of  $\delta$ .

Though the motivation behind this criterion is undoubtedly related to the idea that uniform taxes prevent distortions in the allocation of capital<sup>6</sup> this is a static concept and its applicability to the current problem is not apparent and has not, to our knowledge, been demonstrated.

### C. A Comparison of Approaches

It should be evident that these two views of what constitutes a neutral tax incentive are not the same, but an example will help demonstrate how different they are. For simplicity, we assume the initial tax system consists of a corporate tax with no investment tax credit, and that depreciation allowances correspond to economic depreciation. It is relatively easy to show, in this case, that the value of a unit of capital of type  $\delta$  initially purchased at time zero for one dollar equals  $e^{-\delta t}$  at time  $t$ , so that economic depreciation at time  $t$  is  $\delta e^{-\delta t}$ , and

$$z(\delta) = \int_0^{\infty} e^{-rt} \delta e^{-\delta t} dt = \frac{\delta}{r+\delta} \quad (5)$$

(The reader should keep in mind that the pattern of economic depreciation is not independent of the actual depreciation allowances permitted.)

The values for the user cost,  $c(\delta)$ , and the two measures  $pV(\delta)$  and  $\rho(\delta)$  are:

$$c(\delta) = p(\delta) \left( \frac{r}{1-\tau} + \delta \right) \quad (6a)$$

$$pv(\delta) = \left( \frac{r}{1-\tau} + \delta \right) / (r+\delta) \quad (6b)$$

$$\rho(\delta) = \frac{r}{1-\tau} \quad (6c)$$

Thus, this system is neutral by the rate of return criterion but favors short-lived assets according to the other view, since  $dpv(\delta)/d\delta < 0$ .

To fully appreciate the difficulty faced by the planner uncertain about which approach to take, suppose he is now asked to introduce a neutral investment tax credit to the tax system. The shape of such a credit would be

$$k(\delta) = k(0) - \tau \frac{\delta}{r+\delta} \quad (7a)$$

by the present value criterion, but

$$k(\delta) = k(0) \cdot \left( 1 - \frac{\delta}{r+\delta} \right) \quad (7b)$$

by the rate of return criterion, where  $k(0)$  is the credit which applies if  $\delta=0$ . Table 1 presents sample values for these two functions for  $\tau=.5$ ,  $k(0)=.10$  and  $r=.05$ . Though the results seem extreme, this is no special case--it is based on parameters similar to those characterizing the U.S. economy. It is apparent that these criteria differ radically in their implications for tax policy.<sup>7</sup>

### III. Limitations of the Neutrality Concept

As would be suggested by the two approaches described in the previous section, tax neutrality is by its nature a tool of what has been referred to as piecemeal tax policy, where only a specific sector of the economy is considered



Table 1

A Neutral Investment Tax Credit: Two Views

( $\tau=.5$ ,  $k(0)=.10$ ,  $r=.05$ )

<u><math>\delta</math></u>	criterion	
	<u>pv</u>	<u>IRR</u>
0	.10	.10
.05	-.15	.05
.10	-.23	.03
.20	-.30	.02
$\infty$	-.40	.00

and the appropriate tax policy toward this sector is determined without any consideration of its effects on the broader economy or of the existence of any distortions in other sectors. Specifically, the tax treatment of different capital assets is decided upon without reference to the labor income tax or excise taxes on output. Recent research (Boadway and Harris 1977, Hatta 1977) has discussed the restrictions on the structure of production and demand which must be placed for such a piecemeal approach to be correct. From this research, it is hard to conclude that piecemeal policy design will normally arrive at the socially optimal use of the available instruments.

This limitation is unrelated to the special intertemporal nature of capital which gives rise to the disagreement about which neutrality approach to use. Even if all capital lasted for only one "period," in which case both criteria would agree in calling for the equalization of tax rates on capital income in all uses, this outcome would not generally be the best. Though equal taxes would, in this example, be necessary for production efficiency, such efficient capital allocation will be dictated by the solution to the broader optimal tax problem only if all flows between the production and household sector are taxable and these taxes can be freely adjusted (Diamond and Mirrlees 1971).<sup>8</sup>

One may be prepared to argue for the use of piecemeal policy on more practical grounds, citing both political realities and the informational requirements of a full optimization. However, even if one is willing to accept the general notion of neutrality as valid and ignore all distortions save those directly affected by the instruments to be chosen, it is not a straightforward matter to decide which, if either, of the two basic approaches to take. Here the difficulty does lie in the fact that different capital assets have different service lives and different return streams over these lives.

Consider first the problem with using the internal rate of return criterion in designing the tax system, and suppose there is some social discount rate with which the flows from each asset are to be discounted in arriving at their social value. In general, this value may be the social rate of time preference, which in this example with no personal taxes is the interest rate, or some other rate not generally equal to the gross internal rate of return on any project. This leads to the following criticism:

It is well known that only when the internal rate of return equals the discount rate will the present value of two projects with the same internal rate of return be the same (i.e., zero). Therefore, this criterion of neutrality has no basis in welfare economics. (Boadway 1978)

A simple example verifies this fact. Discounted at  $r$ , the social present value of a one dollar investment in capital of type  $\delta$  with internal rate of return  $\rho$  is  $(\rho+\delta)/(r+\delta)$ , which is different for each  $\delta$  unless  $r=\rho$ .

At first, this argument may seem convincing, but one must examine carefully the assumption upon which it is based--that it is appropriate to discount all flows with a single discount rate, making no further adjustments to the flows according to the source of investment funds or the eventual destination and use of investment proceeds.<sup>9</sup> The justification for this procedure has been explored in the social discount rate literature (Arrow 1966, Kay 1972), with the finding that it is valid only if one assumes that the rate of savings out of project flows is independent of when or in what form these flows occur. We have some question as to whether this is ever a defensible assumption. There may be cases where it applies; for example, the decision between two public projects with the same initial costs, and benefits which are entirely non-pecuniary and have little impact on private decisions (e.g., painting the town hall). However, it seems entirely inappropriate for the analysis of private investment projects,

since it implies that the intertemporal consumption decisions of a firm's owners depend on the durability of its capital stock. If the firm invests in a very short-lived asset which yields a large return almost immediately, the stockholders are presumed to consume the same fraction of this return as they would of a much smaller flow coming at the same time from a more durable asset of equal value. It is more reasonable to assume that the rate of saving would be inversely related to the durability of the asset, in which case the simple present value criterion is insufficient.

This criticism of the present value approach to neutrality does not constitute a defense of the rate of return approach, so that we are confronted with the possibility that these are both misleading as guides to policy formulation, and that no useful approach to tax neutrality need exist. The technique of analysis we have used thus far, comparing the tax system's impact on individual assets of different types, is incapable of resolving this uncertainty, but a slightly different method will prove more successful.

#### IV. A Suggested Framework

One thing we shall not attempt to remedy is the piecemeal nature of the analysis, continuing to focus on the direct effect of taxes on investment. This appears to be something which must be accepted if one is to use the neutrality concept at all. We continue to focus on the firm's decision among different types of asset, and maintain the simplifying assumptions of all equity finance and no personal taxes or inflation, although each of these may be relaxed without influencing the nature of our results.<sup>10</sup>

The problem we encountered above in attempting a comparison of the effects of taxes on assets of different productive lives was in evaluating their return

streams, which have different patterns over time. However, as we have emphasized, it is illogical to assume that individual consumption decisions should depend on the durability of the assets they own. We shall therefore introduce a construct which eases this difficulty, comparing what we shall call "value-equivalent" investment programs corresponding to the different assets. Such a program will be defined as one in which a dollar is initially invested in a particular type of capital, and after-tax proceeds retained and reinvested in the same type of asset, or distributed to stockholders, in such a way as to keep the total value of the investment at each instant consistent with some predetermined schedule. Throughout our discussion, we shall concentrate on the particular value-equivalent schedule which keeps the value of invested capital constant at one dollar, though this is done for the sake of simplicity and is not a restrictive assumption. We emphasize that the equivalence across different assets is measured by their market value, taking account of tax rules, not any other measure, such as a physical stock.

Since our comparisons are between the marginal investments of each type, where the flow marginal product of capital equals the user cost of the particular asset, the constant value assumption implies a distribution of  $r$  dollars at each point in time to the stockholders, regardless of which kind of capital is utilized. Thus, we can disregard the question of individual savings decisions over which we stumbled above. Moreover, since all returns from capital not reinvested go either to the stockholders as distributions or to the government as taxes, the differences among these value-equivalent projects are fully characterized by the alternative streams of tax receipts which they generate.

While there will still remain the question of how to compare these streams, this may prove an easier task. Moreover, there is one important and instructive

special case in which this decision is simple. This is when, as in our above example, all capital assets decline in productivity at some geometric rate, and the tax system is characterized by a corporate tax with economic depreciation allowances and no investment tax credit. In this case, the cost of capital for an asset of type  $\delta$  is  $c(\delta)$  as described in (6a), and the rate of economic depreciation is just the decay rate,  $\delta$ . Thus, keeping the value of the total investment constant at a dollar will require a constant reinvestment rate of  $\delta$ . To verify that this is correct we note that the distribution equals gross income, less taxes, plus tax deductions for depreciation, less reinvestment, or:

$$\frac{c(\delta)}{p(\delta)} (1-\tau) + \tau\delta - \delta = \left(\frac{r}{1-\tau} + \delta\right)(1-\tau) + \tau\delta - \delta = r \quad (8)$$

as is required.

Next, consider the corresponding stream of tax receipts coming from this asset. The government gets a constant flow of

$$R(\delta) = \tau \left( \frac{c(\delta)}{p(\delta)} - \delta \right) = \left( \frac{\tau}{1-\tau} \right) r \quad (9)$$

Since the flows are equal for different assets and constant over time, it is clear that which investment the firm undertakes should be a matter of indifference to the government. A fortiori, if the tax rate  $\tau$  differed across assets, the government's receipts would be permanently higher for an asset with a higher tax rate and it would clearly prefer more investment in such an asset. Since investors are indifferent, it would be socially preferable to remove such differences by equalizing tax rates. This would then constitute the optimal and, presumably, "neutral" tax policy for the government to use.

At least in this case, it is clearly optimal to equalize effective tax rates on different types of capital, since the effective and statutory rates are

the same. Thus, the internal rate of return criterion is valid, regardless of the size of the tax. On the other hand, the present value criterion is always incorrect unless two assets have the same value of  $\delta$ , in which case the problem becomes trivial.

Though this result supports the validity of measuring the neutrality of the tax system with effective tax rates, it is nevertheless a special case. However, with only a little more difficulty, we can show that the result holds for a value-equivalent program composed of any type of capital investment, geometric or not.

Let  $A_t$  be the gross return at time  $t$  for an arbitrary such program. Note that  $A_t$  is not the flow from the initial one dollar investment, but that amount plus the flows from all subsequent investments undertaken to keep the total value of capital in the program equal to one dollar. Let  $D_t$  and  $D'_t$  be the corresponding values of economic depreciation and depreciation permitted under law. If  $\tau$  is the applicable tax rate, it must be true that

$$r = (1-\tau)A_t + \tau D'_t - D_t \quad (10a)$$

That is, the after tax distribution equals  $r$ . It follows that tax receipts are

$$R_t = \tau(A_t - D'_t) \quad (10b)$$

If the depreciation deductions correspond to economic depreciation for each investment and, hence, for each program as well, equations (10a) and (10b) become

$$r = (1-\tau)(A_t - D_t) \quad (11a)$$

$$R_t = \tau(A_t - D_t) \quad (11b)$$

so that  $R_t = (\frac{\tau}{1-\tau})r$ , a constant. As is evident from comparing the constants  $A_t$  and  $R_t$ , the statutory tax rate is the effective tax rate whenever economic depreciation is permitted. It is again clear by the same argument as above that the government should seek to set such rates equal on all types of investment.

Lest it appear that this desirability of equal effective tax rates holds only when such rates are also equal to the statutory tax rate, consider the case of immediate expensing, where investment is written off upon purchase and there are no further depreciation deductions. For a typical value-equivalent investment program, the private investor puts up one dollar. Because assets are immediately expensed, this permits a purchase of  $(\frac{1}{1-\tau})$  dollars worth of capital, with the government contributing a fraction  $\tau$  of this amount through the deduction. Thereafter, since in the program economic depreciation and expenditures on new capital are at all times the same, the private distribution and government tax revenue at time  $t$  are still described by (11a) and (11b), respectively.<sup>11</sup> Thus, the government's revenue stream consists of an initial cost,  $(\frac{\tau}{1-\tau})$  followed by a constant revenue flow of  $(\frac{\tau}{1-\tau})r$ , regardless of the type of asset. Once again, it is clear that  $\tau$  should be set equal on all assets. However, here, the effective tax rates, though equal, do not equal  $\tau$ , but zero, since from an initial total investment of  $\frac{1}{1-\tau}$  each program yields a gross annual flow of  $r(\frac{\tau}{1-\tau}) = r(\frac{1}{1-\tau})$ .

A simple extension of the above results is that any effective tax rate between  $\tau$  and zero can be obtained through a combination of economic depreciation and expensing, and it will still be optimal to set statutory and effective tax rates equal across asset types. To see this, note that permitting expensing of a fraction  $\alpha$  of gross investment leads to a constant tax revenue stream of  $(\frac{\tau}{1-\tau})r$  after an initial cost of  $\frac{\alpha\tau}{1-\alpha\tau}$  to the government. This is still independent of asset type, and the effective tax rate is easily calculated to be



$$\tau(1-\alpha)/(1-\alpha\tau).^{12}$$

We have thus far shown that for an important class of tax regimes, and regardless of asset type, it makes sense to equalize the gross-of-tax internal rates of return, or, equivalently, effective tax rates, applicable to different types of capital investment. However, the typical real-world tax code is more formidable in that it is rife with investment tax credits, accelerated depreciation allowances and, in an inflationary environment, the use of nominal rather than real bases to calculate tax liabilities. The upshot of this is that  $D_t$  and  $D'_t$ , as we have referred to them above, need bear little systematic relationship over time or across assets, bringing us to the problem we have thus far avoided--how to aggregate over time and compare streams of government receipts with different patterns.

#### V. Neutrality and the Government Discount Rate

To restate the general problem more formally, suppose two value-equivalent investment programs yield, under a particular tax system, streams of returns  $\{R_t\}$  and  $\{R'_t\}$  to the government in the form of tax revenue. Ruling out the cases where one stream is uniformly bigger than the other, can we still determine which stream the government will prefer? If so, is there any simple criterion concerning the tax system which will lead to an outcome in which neither stream is preferred to the other, so that no reallocation of investment would be desirable? We shall argue in this section that the answer to each of these questions is yes, and that the criterion is the same as that which has proved appropriate in the case of economic depreciation allowances considered above.

Suppose the government can issue debt which is a perfect substitute for real capital, yielding a constant net rate of return equal to  $r$ . The government

uses sales and repurchases of debt to spread tax receipts over time to match public expenditures. Each time the government issues a dollar of debt, this displaces a dollar of private investment in new capital.<sup>13</sup> We assume that the private capital so displaced is always a representative mixture of the investments in society, and take this "composite capital" to be the same over time. By the same argument, repurchase of a dollar of debt by the government will lead to a one dollar purchase of this composite capital by the private sector. It should be clear that the outcome of this process will be identical to one in which, rather than issuing debt, the government purchases this composite capital directly; rather than sell or repurchase debt, it buys less or more capital than it would otherwise. In either regime, the return to the private sector is the same, as is the total return from capital to society, and hence government revenues. Thus, we may safely ignore debt and assume instead that the government invests in this composite capital directly, earning whatever gross returns such an asset generates for private investors before tax. Letting  $u$  be the effective tax rate on this asset, its gross internal rate of return is  $\frac{r}{1-u}$  in equilibrium.

We will compare tax revenue streams coming from different types of asset by comparing the levels of public expenditures they can support. In order to outline our method, we consider first the special case in which the value-equivalent program associated with the composite capital good yields a constant flow over time.<sup>14</sup> This program must have a constant gross rate of return  $\frac{r}{1-u}$ . If  $\{X_t\}$  is a particular stream of expenditures, it follows that it can be financed from a certain stream of tax revenues  $\{R_t\}$  if and only if

$$\int_0^{\infty} e^{-\frac{r}{1-u}t} (R_t - X_t) dt \geq 0 \quad (12)$$

that is, the present value of tax receipts discounted at  $\frac{r}{1-u}$  defines the maximum level (in present value terms) of public expenditures which can be supported by such a stream. Hence,  $\{R_t\}$  is preferable to  $\{R_t^*\}$ , in terms of public goods and services provided, if and only if

$$\int_0^{\infty} e^{-\frac{r}{1-u}t} R_t dt > \int_0^{\infty} e^{-\frac{r}{1-u}t} R_t^* dt \quad (13)$$

Since the total return to each value-equivalent program consists of a constant distribution  $r$  plus the tax revenues generated, we may rewrite (13) as

$$\int_0^{\infty} e^{-\frac{r}{1-u}t} B_t dt > \int_0^{\infty} e^{-\frac{r}{1-u}t} B_t^* dt \quad (14)$$

where  $\{B_t\}$  and  $\{B_t^*\}$  are the total flows generated by the projects net of reinvestment (see (10)).

Now, suppose the gross internal rate of return on every asset, and hence every value-equivalent program, is the same. Then, since the composite capital good is simply a combination of such assets, it must also have this same internal rate of return. It follows that this rate of return is  $\frac{r}{1-u}$ , and that all assets have an effective tax rate of  $u$ . Furthermore, the present value of the streams  $\{B_t\}$ , discounted at  $\frac{r}{1-u}$ , must equal unity, and (14) is satisfied for any two pairs of assets. Thus, no reallocation of investment is warranted, and the tax system may be deemed "neutral." Without making any reference to the way in which the tax system imposes these effective tax rates, we have shown that they should be equal. It is completely irrelevant whether the rate  $u$  results for a particular asset from a statutory tax rate higher than  $u$  coupled with accelerated depreciation or an investment tax credit, or a lower tax rate with allowances which fall short of economic depreciation. The reason is that, though firms

discount after-tax flows at rate  $r$ , they behave as if they were discounting total flows at  $\frac{r}{1-u}$ , which is for this problem the applicable social discount rate.

While it is reasonable to assume that investment displaced (encouraged) by public borrowing (lending) is drawn in some fixed proportion from the types of investment found in the private sector, it may appear restrictive to require that the value-equivalent program associated with this composite capital yield a constant return over time. However, we will now show that, regardless of the pattern of the stream,  $\{\bar{B}_t\}$ , which comes from the composite capital stock, equations (13) and (14) are still completely valid.

Consider the value of capital which results from starting with a one dollar value-equivalent program of composite capital and reinvesting all proceeds from this program, and the resulting proceeds, etc., until time  $v$ . We call this value  $S_v$ . (Note that for the preceding example,  $S_v = e^{\frac{r}{1-u}v}$ .) For any horizon  $T$ , a program of expenditures  $\{X_t\}$  can be supported by a stream of revenue flows  $\{R_t\}$  if and only if

$$\int_0^T S_{T-t} (R_t - X_t) dt \geq 0 \quad (15)$$

That is, the value of capital left on hand at time  $T$  from increments to and subtractions from the composite capital stock over time must exceed zero. Expressing this in initial dollars, we divide by  $S_T$ , the value of an initial investment at time zero compounded by continual reinvestment until time  $T$ :

$$\int_0^T \frac{S_{T-t}}{S_T} (R_t - X_t) dt \geq 0 \quad (16)$$

Here  $\frac{S_{T-t}}{S_T}$  is the discount factor applied to time  $t$  expenditures, and equals  $e^{-\frac{r}{1-u}t}$  for the previous example.

Given an infinite horizon, we must take the limit of the expression in (15), obtaining:

$$\lim_{T \rightarrow \infty} \int_0^T \frac{S_{T-t}}{S_T} (R_t - X_t) dt \geq 0 \quad (17)$$

which, assuming each limit is finite, implies that

$$\lim_{T \rightarrow \infty} \int_0^T \frac{S_{T-t}}{S_T} R_t dt \geq \lim_{T \rightarrow \infty} \int_0^T \frac{S_{T-t}}{S_T} X_t dt \quad (18)$$

Thus,  $\{R_t\}$  is preferable to  $\{R'_t\}$  if and only if

$$\lim_{T \rightarrow \infty} \int_0^T \frac{S_{T-t}}{S_T} R_t dt \geq \lim_{T \rightarrow \infty} \int_0^T \frac{S_{T-t}}{S_T} R_t^* dt \quad (19)$$

which, by the same argument as before, is equivalent to the condition that

$$\lim_{T \rightarrow \infty} \int_0^T \frac{S_{T-t}}{S_T} B_t dt \geq \lim_{T \rightarrow \infty} \int_0^T \frac{S_{T-t}}{S_T} B_t^* dt \quad (20)$$

where  $\{B_t\}$  and  $\{B_t^*\}$  are the total streams from each program.

To show that (19) is equivalent to (13), we must demonstrate that  $\frac{S_{T-t}}{S_T} e^{-\frac{r}{1-u}t}$  approaches  $e^{-\frac{r}{1-u}t}$  as  $T$  approaches infinity.

The term  $S_v$  describes the value of the composite capital stock which results from an initial investment of one dollar and the reinvestment of all proceeds until time  $v$  in more composite capital. This process of capital accumulation has the same characteristics as the growth of a population, where the stream  $\{\bar{B}_t\}$  represents the stream of "offspring" each initial investment produces, with the "offspring" themselves having the same "fertility" pattern. What happens to the age structure of the population, or capital stock, over

time? Under the assumptions we have made thus far, the age structure approaches a constant as  $v$  approaches infinity, by the "Strong Ergodic Theorem" of stable population theory.<sup>15</sup> A direct corollary is that the capital stock, and hence  $S_v$ , grows exponentially at a constant rate once  $v$  is sufficiently large. Since this growing capital stock is composed solely of assets with a gross internal rate of return  $\frac{r}{1-u}$ , it too has an internal rate of  $\frac{r}{1-u}$ . Since all proceeds are being reinvested, the exponential rate of growth must therefore be  $\frac{r}{1-u}$ . It follows directly that  $\frac{S_{T-t}}{S_T}$  approaches  $e^{-\frac{r}{1-u}t}$  as  $T$  becomes large, as we set out to prove.

To summarize the results of this section, we have shown that, regardless of the particular tax structure and the particular types of capital assets which are purchased by investors, the value placed by the government and, a fortiori, the social value of different marginal value-equivalent investment projects between which investors are indifferent will be the same if the gross internal rates of return on all such assets are the same. This in turn requires that the effective tax rates on all such investments be equal.

## VI. Conclusions and Extensions

This paper has attempted to demonstrate that while it is limited by the narrowness of its scope, tax neutrality is not a meaningless concept. Moreover, we have argued that while there are two distinct criteria for neutrality with radically different implications for tax design, only one of these makes any sense at all, and that this one, that effective tax rates should not differ across assets, applies to a very general class of situations.

There are two additional points we wish to make in closing, concerning the generality of our results. We have assumed throughout that each possible

value-equivalent program is composed of homogeneous assets, but this was merely for expositional purposes. The argument does not depend on this homogeneity, and could be applied to heterogeneous programs as well. In such a case, the effective tax rate of a program would depend in a complex way on those of its component assets, but uniformity of such rates across programs would again normally require that each type of asset face the same rate. Similarly, the assumption that the value of a firm's investment program is independent of the type of assets purchased is unnecessary. Our argument really depends on behavior at the household level. Whether a firm actually reinvests a dollar itself, or whether it distributes it to its shareholders, who then reinvest it, is of no importance. That is, just as the value-equivalent programs can include more than one type of asset, they can include investments by more than one firm.

Footnotes

<sup>1</sup>Auerbach (1978)

<sup>2</sup>At least in the past, a second reason for initiating investment incentives has been to stimulate aggregate demand during a recession. See Gordon and Jorgenson (1976). However, as argued by Lucas (1976), if such is the avowed purpose of the incentives, the analysis must take into account the likelihood that investors will anticipate tax changes. Furthermore, the impact of such expectations differs across investments of different durability. See Auerbach (1978).

<sup>3</sup>An alternative way of stating the criterion would be that, starting from a no-tax equilibrium, changes in the tax law exert an equiproportional influence on the different user costs. However, this would require the tax changes to depend on general equilibrium changes in the interest rate. Phrasing the criterion as we have is consistent with the normal approach which does not consider changes in  $r$ .

<sup>4</sup>Feldstein (1979) has phrased this criterion in terms of the proportional effect of the tax system on  $[1-k(\delta)-\tau z(\delta)]$ , which he refers to as the "net cost of investment."

<sup>5</sup>One might also justify this approach by observing that if the production technology is separable into capital and other factors and homogeneous in capital, the ratio of different types of capital in use depends only on the relative values of the different user costs.

<sup>6</sup>As argued in Harberger (1966), for example.

<sup>7</sup>A recent example in the U.S. is the evaluation of the Conable-Jones proposal, which would shorten tax lifetimes to five and ten years for equipment and plant, respectively. Feldstein (1979) finds such a proposal reasonable under the present value criterion (see footnote 4) while Auerbach and Jorgenson find it to be even less neutral than current practice, using the effective tax rate as their metric.

<sup>8</sup>For a treatment of this issue as it relates to the differential taxation of capital, see Auerbach (1979b).

<sup>9</sup>There are other problems with the present value approach, or at least its implementation, discussed in Auerbach (1979c).

<sup>10</sup>All that will really be necessary for these complications to not matter is that there be no systematic relationship between the type of asset and the method with which it is financed or the personal tax rates of its owners.



<sup>11</sup> Note that the specific values of  $A_t$  and  $D_t$  need not be the same as before for this result to apply.

<sup>12</sup> See Auerbach (1979a) or Harberger (1980) for further discussion of this approach to incentives.

<sup>13</sup> This assumption does not imply any "bond illusion" on the part of households, and is perfectly consistent with the results in Barro (1974), where new issues of government debt had no impact on private decisions. The difference lies in the purpose of the debt. In Barro's model, there is no real transaction associated with the debt--it is given to households, and its interest is paid by taxing them. Thus, if households hold the bonds and don't change their behavior, the bonds and taxes cancel and there is no real effect. In our model, the bond is sold to the household to finance real projects.

<sup>14</sup> This would be true if the tax law specified economic depreciation or expensing, or any combination of the two, as just shown.

<sup>15</sup> See Golubitsky et al (1975)

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