NBER WORKING PAPER SERIES

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Working Paper No. 432

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge MA 02138

January 1980

This research is supported by the National Science and by the Center for Research in Government Policy & Business at the Graduate School of Mangement of the University of Rochester. An earlier version of this paper was presented at the Charles Haywood Murphy Symposium on Government Debt, Fiscal Policy, and Expectational Theory at Tulane University, October 1979. I have benefited from comments by Stan Engerman, Donald Hester, Bob King, Levis Kochin and Charles Plosser and from the research assistance of Gary Gorton and Chirta Ramaswami. The research reported here is part of the NBER's research program in Economic Fluctuations. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

NBER Working Paper #432
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ABSTRACT

Because of a small direct negative effect on private spending, temporary variations in government purchases, as in wartime, would have a strong positive effect on aggregate demand. Intertemporal substitution effects would direct work and production toward these periods where output was valued unusually highly. Defense purchases are divided empirically into "permanent" and "temporary" components by considering the role of (temporary) wars. Shifts in non-defense purchases are mostly permanent. Empirical results verify a strong expansionary effect on output of temporary purchases, but contradict some more specific expectational propositions.

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Macroeconomic analysis typically assigns government purchases an important role in influencing aggregate demand and thereby in affecting output and employment. Bailey (1971) points out that these expansionary effects are offset to the extent that governmentally-provided goods and services are close substitutes for private consumption or investment expenditures. Hall (1979) argues that temporary changes in government purchases can have a substantial business cycle role because they stimulate intertemporal substitution of work and production. These effects are most important in the case of transitory expenditures that are not close substitutes for private spending--notably for wartime spending--but would not apply to long-run changes in government purchases.

The present analysis focuses on the theoretical and empirical distinction between temporary and permanent variations in government purchases. A simple theoretical framework is used to illustrate the aggregate demand role of the temporary part of these purchases. Some consideration is given also to direct effects on aggregate supply, which arise to the extent that government services constitute productive inputs for private firms.

The empirical section estimates the division of defense purchases into permanent and temporary components by considering the effect of war and of war expectations. Defense spending associated with wars is largely transitory, while other changes in defense spending turn out to be predominantly permanent. Shifts in non-defense federal plus state and local purchases are also mostly permanent in character. Analysis of output reveals a significant expansionary effect of temporary defense purchases, a weaker but still highly significant expansionary effect of permanent defense purchases, and no significant effect of non-defense purchases. These findings reject the polar hypotheses that either permanent defense purchases have no output effects or that permanent defense purchases are as important for output as temporary purchases. Some discrepancies between theory and evidence arise for detailed hypotheses that concern the formation of expectations on future values of defense purchases. Overall, the empirical results are mixed in the sense of verifying some aspects of the underlying theory--notably, in supporting the usefulness of separating government purchases into temporary and permanent components--but in contradicting some more specific expectational propositions.

I. Theoretical Considerations

A. Aggregate Demand

The impact effect of government purchases on aggregate demand involves the positive one-to-one effect of public expenditures net of any directly induced contraction of private demand. For given values of prices, including anticipated real rates of return, the reaction of private demand to changes in government purchases involves several channels: 1) alterations in permanent private disposable income corresponding to shifts in the perceived amount of resources absorbed by the government in a long-run average sense; 2) direct substitution of public spending for private consumption or investment, as stressed by Bailey (1971, pp. 152-55); and 3) substitution effects associated with changes in current or anticipated future tax rates.

The first effect involves the perceived long-run average level of government purchases. Since an increase in this average implies a corresponding increase in the normal level of taxes--whether ocurring as explicit taxation, inflationary finance, or in a deferred form involving public debt issue--there would be a roughly one-to-one downward adjustment of private consumer demand (see below). On this ground aggregate demand would be associated primarily with deviations of government purchases from their perceived long-run average value, rather than with the level of government purchases. The channel of direct expenditure substitution--as often illustrated by a publicly supported school lunch program that replaces private spending on lunches--tends to lessen the aggregate demand implications of government purchases even when those purchases differ from the long-run norm. The third channel involves substitutions away from activities taxed by the government. In the case of labor earnings taxes, the principal aggregate demand influence would be a negative effect of the normal level of government purchases on private demands, which would correspond to a reduced average incentive to work rather than consume leisure. (Note that an offsetting wealth effect on leisure would arise in consideration above of the first channel of effects.) The aggregate demand influence of consumption taxes, investment tax credits, or the like would depend more closely on the timing of these levies.

The aggregate demand effects outlined above can be examined within the following formal model. Suppose that the representative household obtains utility from a stream of non-durable consumption expenditures $C(t)$ and government purchases $G(t)$. I do not distinguish here or in the empirical work between current government purchases, which involve partly gross investment, and the flow of government-provided services, which involve partly rental income on the publicly-owned capital stock. In order to highlight the main arguments of the first two channels of effect mentioned above, the analysis abstracts initially from the labor-leisure choice and assumes lump-sum taxation. I assume that a portion of government purchases substitutes directly for contemporaneous private consumption expenditure so that the effective consumption flow at date t is given by $C^*(t) = C(t) + \theta G(t)$, where $\theta \geq 0$.[1] With C and G assumed to be measured in equivalent commodity units per capita,[2] $\theta > 1$ would require a form of efficiency advantage from public sector activity (as well as close utility substitution between public and private spending). In the main analysis I assume that $0 \leq \theta < 1$ applies. At this stage government purchases are assumed to have no impact on private production functions; this type of supply effect is introduced below in section I.B. The utility function is assumed to take the separable form

$$(1)\quad U = V_1[C^*(t\ldots)] + V_2[G(t\ldots)],$$

where $t\ldots$ represents a continuum of time starting at date 0 and extending over an infinite horizon, and V_2 measures the effect on utility of the part of the stream of government purchases that does not substitute directly for the contemporaneous value of C. The inclusion of this term allows one to distinguish substitutability between C(t) and G(t) from the issue of whether G(t) is valued by the private sector. The function V_1 is increasing with C* at any date and likewise for V_2 with respect to G.

Under lump-sum taxation the intertemporal budget equation for the representative household over an infinite horizon beginning at date 0 can be written as

$$(2)\quad K(0) + \int_0^\infty L(t)\nu(t)dt = \int_0^\infty C(t)\nu(t)dt + \int_0^\infty G(t)\nu(t)dt,$$

where K(0) is initial assets (accumulated real capital) per capita; L(t) is the exogenous amount of real labor income per capita; and $\nu(t)$ is a real discount factor: $\nu(t) \equiv \exp[-\int_0^t r(\tau)d\tau]$, where $r(\tau)$ is the instantaneous real rate of return on assets (capital) at date τ. The last term in equation (2) is the real present value of government purchases per capita, which corresponds to the real present value of tax collections per capita possibly including inflationary finance.[3] For convenience I assume that real labor income per capita grows at a constant rate λ (perhaps reflecting the rate of labor-augmenting technical progress): $L(t) = L(0)e^{\lambda t}$, and that the "average" real rate of return is constant at a value $\bar{r} > \lambda$. It is assumed that a departure of the current return $r(\tau)$ from $\bar{r}$ represents a transistory opportunity for a high or low return that may have strong intertemporal substitution effects, but nevertheless represents a weak income effect. In particular, $\nu(t) \simeq e^{-\bar{r}t}$ is assumed for convenience to be a satisfactory approximation in formulating the budget condition of equation (2).

Defining the "average" ratio of government purchases to labor income as

$$(3)\ (\overline{G/L})(0) \equiv (\bar{r}-\lambda)\int_0^\infty [G(t)/L(t)]e^{-(\bar{r}-\lambda)t}dt$$

and using the conditions $C^*(t) = C(t) + \theta G(t)$, $L(t) = L(0)e^{\lambda t}$, $\nu(t) = e^{-\bar{r}t}$, equation (2) can be rewritten as

$$(4)\ (\bar{r} - \lambda)\int_0^\infty [C^*(t)/L(t)]e^{-(\bar{r} - \lambda)t}dt = (\bar{r} - \lambda)K/L + 1 - (1 - \theta)\overline{G/L},$$

where zeroes in parentheses are implicit in the symbols K, L and $\overline{G/L}$. The consumer's problem amounts to choosing the path of "effective consumption" $C^*(t)$, subject to given values for the time paths of $G(t)$, $r(t)$ and $L(t)$, and subject to the form of the budget constraint shown in equation (4).

Neglecting private investment,[4] current aggregate demand corresponds to

$$(5)\ Y^d \equiv C^d + G = C^* + (1-\theta)G,$$

where zeroes in parentheses have again been omitted. With $\overline{G/L}$ fixed, an increase in G has no wealth effect on the right side of equation (4). Therefore, for given values of real rates of return including the current value $r(0)$, C^* would be unchanged initially. Aggregate demand rises with G in equation (5) in accordance with the coefficient $(1-\theta)$, where $0 < (1-\theta) \leq 1$. The greater the utility substitution between C and G, as measured by θ, the smaller the aggregate demand impact of this type of temporary movement in government purchases. Note that the implicit value associated with G, which would include the separable utility influence measured by the V_2 term in equation (1), is not directly pertinent.

If $\overline{G/L}$ rises along with the increase in the current ratio G/L, there is a negative wealth effect as shown on the right side of equation (4). C^* declines in the normal case, which implies a reduced overall impact on aggregate demand in equation (5). The quantitative relationship involves the marginal propensity to consume out of "wealth." (Note that leisure is not allowed to vary in the present setup.) Of particular interest is the net effect on demand for the case where G/L and $\overline{G/L}$ rise by equal amounts; that is, when the change in the government purchases ratio is permanent.

In some simple cases there is a steady state value of K/L that is determined by parameters defining the production function, the utility function in equation (1) including the "rate of time preference," and the growth rate λ.[5] In this situation a change in $\overline{G/L}$ leaves unchanged the steady state value of K/L.[6] At least when the initial value of K/L equals its steady state value, the response in current C^*/L would be one-to-one inversely with the change in $(1-\theta)(\overline{G/L})$. Aggregate demand as shown in equation (5) would then be invariant with the shift in government purchases. In this situation aggregate demand depends positively on $(1-\theta)(G/L - \overline{G/L})$--that is, on the value of temporary government purchases--but is insensitive to shifts in the long-run government purchases ratio $\overline{G/L}$. This behavior implies in particular a strong aggregate demand response to temporary movements in government purchases, such as expenditures associated with wars, but no response to a secular change in the government purchases' share of gross national product.

Variable Labor Supply, Non-Lump Sum Taxation

An immediate amendment for the case of variable labor supply concerns the negative wealth effect associated with $\overline{G/L}$ in equation (4) when $0 \leq \theta < 1$. On this count an increase in $\overline{G/L}$ would tend to reduce leisure, which implies that C* declines by less than that calculated above. Accordingly, aggregate demand would tend to rise with equal increases in G/L and $\overline{G/L}$, unlike in the previous example where only the temporary component of government purchases influenced aggregate demand. However, this response is likely to be offset by substitution effects of taxes, as discussed next.

Suppose that taxes apply to labor earnings, rather than being lump sum. If there is no strong utility interaction between consumption and leisure at particular dates (for example, contemporaneously) or if tax rates are based on the normal government purchases ratio $\overline{G/L}$,[7] then substitution effects of taxes on the current value of C* would involve primarily an average of tax rates over time, which would influence the choice of the present value of labor earnings. In particular, an increase in $\overline{G/L}$ would raise this average of tax rates and thereby induce substitution toward leisure and away from consumption at various dates. This inverse effect of $\overline{G/L}$ on

C^* offsets the wealth effect discussed above. Overall, the analysis of consumption from the previous section is modified in accordance with the net wealth and substitution effects on labor supply of a long-run change in earnings taxes. If a long-run expansion of the government's spending ratio, $\overline{G/L}$, motivates a decrease in work effort, then aggregate demand would tend to decline with $\overline{G/L}$, and vice versa for the case where work effort rises.[8]

The analysis would be altered if tax rates do not always correspond to the average value determined by $\overline{G/L}$. Variations in current earnings taxes could have an important substitution effect on current consumption if there were a strong utility interaction between contemporaneous values of consumption and leisure. However, the sign of this interaction is not apparent. The analysis requires more serious modification for cases of temporary taxes on consumption, temporary credits for housing or other investment, and the like. The intertemporal substitution effects of these fiscal instruments would become a central element of the analysis. However, if the levying of these types of taxes is not closely related to the contemporaneous value of government purchases, then these effects would be basically separable from the present analysis. From an empirical standpoint, if substantial variations in temporary taxes occur (which has not been demonstrated empirically in terms of an overall fiscal package), it would be desirable to hold these effects fixed separately.

The main thoeretical conclusion remains as the major positive effect of temporary government purchases on aggregate demand, assuming that the direct substitution parameter θ is well below unity. There are a number of channels through which permanent changes in government purchases could influence aggregate demand, but the overall presumption is for a substantially weaker effect (of indeterminate sign), as compared with the effect of temporary movements.

B. Aggregate Supply

Aside from substituting for private consumer expenditure, government purchases may represent intermediate products that serve as inputs for private firms. For example, police and fire services, highways, and even a system of laws and national defense can be viewed in part as these types of intermediate products. As discussed by Kuznets (1948, pp. 156-57) and Musgrave (1959, pp. 186-88), a consistent difinition of national product would treat these items as inputs that would not be double-counted as elements of final product. Problems with dividing government purchases into final and intermediate categories make this adjustment difficult to make in practice. Even if these adjustments were accomplished it would remain necessary to consider the possible effects of various public services, such as national defense, on private aggregate supply.

To model the simplest possible case (see Barro and Grossman, 1976, Ch. 1, for some further discussion), suppose that a constant fraction ϕ, where $0 \leq \phi < 1$, of current government purchases contributes one-to-one to aggregate commodity supply; that is,

$$(6)\quad Y^S = \ldots + \phi G.$$

In some cases, such as national defense, it may be more appropriate to model Y^S as responding to the long-run average value of purchases $\overline{G}$, rather that G, but the analysis is not greatly affected by this change. I do not consider here dimininshing marginal product of public services, the interplay between these services and private productive inputs, or the possibility that government purchases would exert substantial intertemporal substitution effects on private commodity supply. (Compare the treatment of aggregate demand, as discussed above in n. 1.) Therefore, the portions of aggregate supply denoted by ... in equation (6) are assumed not to be affected directly by G.

The previous analysis of aggregate demand must also be modified because the flow ϕG constitutes income to households (possibly involving firms as intermediaries). The budget expression in equation (4) is therefore altered so that the last term on the right side becomes $-(1-\theta-\phi)\overline{G/L}$; that is, the negative effect of $\overline{G/L}$ on effective private resources is offset first, by the fraction θ that services equivalent to private consumer spending are provided and second, to the extent ϕ that aggregate commodity supplies are enhanced. It is assumed now that $0 < \theta + \phi < 1$ applies. In the case discussed above where the capital-labor ratio was invariant with changes in the government purchases ratio, aggregate demand would now depend on $(1-\theta)(G/L - \overline{G/L}) + \phi(\overline{G/L})$. Therefore, if $\phi > 0$, aggregate demand would now rise with a permanent expansion of the government purchases ratio.

C. Effects on Output and the Real Rate of Return

The translation of aggregate demand and supply effects into output movements involves the determination of real rates of return in order to clear the commodity market. The central assumption is that Y^d is negatively related to the current return $r(0)$, while Y^s is positively related. The latter response involves intertemporal substitution of factor supplies and final products, which has been stressed by Hall (1979, section 2). In particular, periods with relatively high values of r would be unusually rewarding to intensive work effort and production.

Consider a temporary expansion of government purhcases, where G/L rises while the normal purchases ratio $\overline{G/L}$ is held fixed. Aggregate demand rises roughly as $(1-\theta)G$ while supply increases as ϕG. (If supply depended on $\overline{G}$ rather than G, there would be an increase only on the demand side.) Given that $(\theta+\phi) < 1$ applies, there is an excess demand for commodities--that is, a deficiency of currently desired saving--which requires a rise in the current real rate of return $r(0)$ to restore market clearing.[9] Therefore, output and the real rate of return both rise in response to an increase in $(G/L - \overline{G/L})$. The output effect is greater the smaller the value of θ and the larger the value of ϕ. In any event the present setup implies that the output response is less than one-to-one with the movement in G; that is, the model exhibits a dampener rather than a multiplier.

The positive response of output to temporary movements in government purchases would apply especially to wartime periods.[10] The real rate of return can be viewed as the price signal that induces the intertemporal substitution of resources toward periods such as wars in which aggregate output is valued unusually highly. This type of substitution has been stressed by Hall (1979, section 2), who points out also that this behavior differs in some important respects from the response of supply to monetary misperceptions that occurs in some business cycle theories that stress intertemporal substitution on the supply side (for example, Lucas and Rapping, 1969; Lucas 1975; Barro 1980). The effect of government purchases on the time arrangement of work and production does not rely on elements of misperception with respect to the general price level or other variables.

A permanent increase in the government purchases ratio, where G/L and $\overline{G/L}$ expand equally, leads to comparable rises in Y^d and Y^s. The real rate of return would not be affected substantially, but output rises as long as $\phi > 0$. The positive effect on GNP of a permanent change in government purchases is smaller than the effect of an equal, temporary change, because the supply effects are equal and the demand effect is smaller in the permanent situation.[11] Therefore, permanent shifts in government purchases also involve a dampened response of output.

Overall, temporary expansions of government purchases are distinguished from permanent purchases in that 1) the positive effect on real GNP of temporary purchases is larger, and 2) the relative value of the current real rate of return is raised by temporary purchases. The last effect may be difficult to test empirically because of problems in measuring anticipated real rates of return and because tax effects on the return to savings, which would be associated primarily with the long-run average value of the government purchases ratio, may imply an effect of $\overline{G}$ on the overall level of real rates of return. In any event the present empirical investigation deals only with output effects of temporary versus permanent movements in government purchases.

II. Empirical Implementation

The theoretical propositions will be tested by examining the effects of government purchases in a reduced form relationship for output, as measured by real GNP. The analysis is an extension of previous empirical research (Barro, 1977, 1978; Barro and Rush, 1980) that stressed the business cycle influences of monetary disturbances. This earlier work included a government purchases variable (Barro and Rush, 1980, pp. 8,9) or a related measure of military personnel, but did not distinguish temporary from permanent government spending.

Suppose that a relationship has been isolated for the normal government purchases ratio $(\overline{G/L})_t$ in terms of a set of parameters α and currently observed exogenous variables Z_t:

$$(7)\quad (\overline{G/L})_t = F(Z_t;\alpha).$$

A linear reduced form expression for the log of output could be written as

$$(8)\quad \log(Y_t) = \ \ldots + \beta_1(G/L)_t + \beta_2(\overline{G/L})_t,$$

or, using equation (7),

$$(9)\quad \log(Y_t) = \ \ldots + \beta_1(G/L)_t + \beta_2 F(Z_t;\alpha),$$

where omitted variables denoted by ... include current and lagged monetary shocks, as stressed in previous empirical research. Lagged values of G/L and $\overline{G/L}$ could also enter equations (8) and (9), but these effects were not found to be important empirically.

One hypothesis to consider is the polar case where $\beta_1 \doteq -\beta_2 > 0$; that is, the situation where a rise in the temporary part of government purchases increases output through an increase in aggregate demand, but where equal changes in G/L and $\overline{G/L}$ do not affect output. Since this outcome requires a zero supply effect of government purchases (ϕ=0), it is more interesting to test the weaker hypothesis, $\beta_1 > 0$, $\beta_2 < 0$; which implies that temporary changes in government purchases have a larger positive output effect than permanent changes. Since the effect of the latter is given by the sum of the coefficients, $\beta_1 + \beta_2$, the result $\beta_1 > -\beta_2 > 0$ implies that permanent changes in government purchases increase output--as would be expected through the supply channel--and vise versa for $-\beta_2 > \beta_1 > 0$.

Additional hypotheses involve the manner in which the Z-variables appear across equations (7) and (9). If these variables do not appear separately in the list of omitted variables in equation (8), the Z_t variables would appear in equation (9) only as they serve as determinants for $(\overline{G/L})_t$ in the F-function of equation (7). Some cross-equation restrictions would therefore be implied for the parameters of equations (7) and (9). The exclusion of at least some of the Z_t variables from equation (8) is actually needed even to test the hypothesis above that $\beta_1+\beta_2 = 0$; see Barro, 1979b, section V, for a general discussion of these types of cross-equation tests.

The next sections deal with the problem of modeling a form of equation (7) for government purchases in the United States.[12]

A. Government Purchases Equation

The stress on transitory movements in government purchases suggests special attention to war-related expenditures, which are likely to be viewed as largely temporary. I have proceeded empirically by separating total government (federal plus state and local) purchases of goods and services into a "defense" component, G^w, and other purchases, G^p. The present analysis does not attempt to classify components of government purchases in accordance either with their relative substitutabilities with private spending, as reflected above in the θ parameter, or with their role as inputs to private production, as measured above by the ϕ parameter. Differences between defense and non-defense items with respect to these parameters affect the interpretation of some of the empirical findings. Presumably, defense purchases are characterized by a relatively low value of θ and possibly by a relatively high value of ϕ. The former implies a relatively large output effect of temporary defense purchases, while the latter would enhance the output effects of both temporary and permanent defense purchases. The empirical analysis would be sharpened by obtaining a division of non-defense purchases into relatively homogeneous categories with respect to the θ and ϕ parameters, but the feasibility of this classification is unclear.

Defense Purchases

A primary determinant of G^W would be the level of current and anticipated future wartime activity, assuming that at least the timing of wars can be treated as exogenous with respect to expenditure decisions. I have quantified this influence by using a casualty rate measure B_t, which represents battle deaths per 1,000 total population (see table 1) for the wartime years since the Civil War: 1898, 1917-18, 1941-45, 1950-53, 1964-72. Because of improvements in the technology of caring for wounded and offsetting changes associated with the "efficiency" of weapons, it is possible that this variable does not consistently measure the intensity of war at different dates. I considered using a broader casualty measure that included wounded, but the ratio of this concept to battle deaths showed no trend at least since the Spanish American War.[13] Since I was unable to obtain reliable annual data on wounded for World War II, I have restricted my analysis to the narrower, battle deaths concept of casualty rates.

Prospective wars would be likely also to influence current spending, with good information on forthcoming military actions existing prior to at least the U.S. entrances into World Wars I and II. Since I have been unable to construct any instruments for these war expectations, I have introduced some actual future values of B into an equation for current defense spending. This procedure introduces errors-in-variables problems into coefficient estimation, although the present analysis is concerned primarily with obtaining conditional forecasts, rather than with coefficient estimation *per se*. Lagged effects of B on spending are introduced also into the equation.

Since defense expenditures involve a substantial investment component, the amount of current spending would tend to be influenced negatively by the size of existing capital stocks. Accordingly, I have included in a defense spending equation the variable K^W_{t-1}, which measures the beginning-of-period real stock of military equipment, structures and inventories (table 2, col.5). The relation of capital stock to current spending is assumed to be given by

$$(10)\quad K^W_t = bG^W_t + (1-\delta)K^W_{t-1},$$

where δ is a depreciation rate and b measures the fraction of total defense spending that constitutes investment (net of within-year depreciation on this investment). The K^W series was constructed with values of b and δ that varied over time (see the notes to table 2), but I have limited the theoretical discussion below to situations where these parameters are approximated satisfactorily as constants. For generation of forecasts in the empirical analysis I have used the sample average values of $\delta = .16$ per year and $b = .34$.

The estimating equation for G^W takes the form,

$$(11)\ g_t^W = A(L)B_t - \gamma k_{t-1}^W + u_t,$$

where

$g_t^W \equiv G_t^W/Y_t$, G_t^W is real defense expenditure, Y_t is real GNP;

$A(L)$ is a polynominal in the lag operator L, which allows both lags and leads of B_t to affect g_t^W;

$k_{t-1}^W \equiv K_{t-1}^W/Y_{t-1}$;

u_t is a stochastic term.

The form of equation (11) implies that a doubling of Y_t, K_{t-1}^W, and Y_{t-1}, for given values of the B variables, leads to a doubling of G_t^W.

Empirically, two leads, the contemporaneous value, and up to a third lag of B had significant explanatory power for g_t^W. In this case $A(L)B_t = \alpha_0 B_t + \alpha_1 B_{t-1} + \alpha_2 B_{t-2} + \alpha_3 B_{t-3} + a_1 B_{t+1} + a_2 B_{t+2}$ applies in equation (11). The error term was satisfactorily modeled as a random walk, so that equation (11) can be readily estimated in first-difference form,

$$(12)\ Dg_t^W = A(L)DB_t - \gamma Dk_{t-1}^W + \varepsilon_t,$$

where D is the first-difference operator and $\varepsilon_t \equiv u_t - u_{t-1}$ is a white noise error term. A constant is insignificant when added to equation (12) in the empirical analysis; that is, there is no trend in the defense purchases ratio. Moving-average error terms or more complicated autoregressive error structures also did not add to the explanatory value of the equation. The form of equation (12) suggests that a current shock ε_t would have a permanent effect on the mean level of g^W. However, because of the inclusion of the k_{t-1}^W

term with a negative sign, this effect turns out to be less than one-to-one in the "long-run," as derived below.

Determination of Normal Defense Expenditures

Equations (12) and (10), together with a specification for the stochastic structure of the B variable, imply a distribution for future values of g^w, conditional on information available at date t (which is assumed in the present case to include the values of B_{t+1} and B_{t+2}). Equation (12) implies that future values of the spending-GNP ratio are given by

$$(13)\quad g^w_{t+i} = g^w_t + A(L)(B_{t+i} - B_t) - \gamma(k^w_{t+i-1} - k^w_{t-1}) + \text{error term.}$$

Equation (10) can be used repeatedly to eliminate future values of k^w from equation (13), which leads eventually to the condition,

$$(14)\quad g^w_{t+i} = g^w_t \left[\frac{\delta + b\gamma(1-\delta-b\gamma)^i}{\delta + b\gamma}\right] + b\gamma k^w_{t-1}\left[\frac{1-(1-\delta-b\gamma)^i}{\delta + b\gamma}\right] + A(L)(B_{t+i} - B_t)$$

$$-b\gamma[A(L)B_{t+i} + (1-\delta-b\gamma)A(L)B_{t+i-2} + \ldots + (1-\delta-b\gamma)^{i-2}A(L)B_{t+1}] + \text{error term.}$$

The variable of interest for output determination is

$$\overline{g^w_t} \simeq \left(\frac{\rho}{1+\rho}\right)\left[g^w_t + \sum_{i=1}^{\infty} Eg^w_{t+i}/(1+\rho)^i\right],$$

where E is the expectation operator and ρ is a constant discount rate that would correspond to $\bar{r} - \lambda$ in the continuous time formulation of equation (3). The variable $\overline{g^w_t}$ can be determined from summation over i in equation (14) to be

$$(15)\quad \overline{g^w_t} = g^w_t\left(\frac{\rho+\delta}{\rho+\delta+b\gamma}\right) + k^w_{t-1}\left(\frac{\delta\gamma}{\rho+\delta+b\gamma}\right) + \Phi_t\left(\frac{\rho}{1+\rho}\right)\left(\frac{\rho+\delta}{\rho+\delta+b\gamma}\right),$$

where $\Phi_t \equiv \sum_{i=1}^{\infty} E[A(L)(B_{t+i} - B_t)]/(1+\rho)^i$.

The effect of g_t^w is positive but less than unitary. For a given value of g_t^w, the effect of k_{t-1}^w is positive because it indicates that a greater fraction of g_t^w corresponds to a permanent component.

For the case where A(L) includes three lag values and two leads, and where observations on the B variable through B_{t+2} are available at date t, the Φ_t expression in the last term of equation (15) can be written as a function of the variables $(B_{t-3}, \ldots, B_{t+2})$ and of anticipated future casualty rates, which appear in the form,

$$(16)\quad \Psi_t \equiv \sum_{i=1}^{\infty} EB_{t+i+2}/(1+\rho)^i.$$

The remaining work is to relate expectations of future values of B, as entering through the Ψ_t variable, to currently observed variables, including values of B up to B_{t+2}.

Expectations of Future Wars

Calculation of expected future casualty rates is based on the following stationary probability model for wars.[14] First, a 2-by-2 matrix is specified for the probability of war or peace next year (or rather for year t+3 when conditions at t+2 are assumed known at date t), conditional on war or peace prevailing currently. It is assumed that information about the future course of B is contained fully in the most recent observation; earlier values of B and values of other variables not having to be considered. The probability of war during at least part of next year, given peace for the latest observation, is based on data over the 1774-1978 period; namely,

$$(17)\quad p_1 = \text{Prob.}\ (B_{t+1} > 0 \mid B_t = 0) = 9/162 = .06,$$

where 162 is the total number of peacetime years in the sample (where $B_t = 0$) and 9 is the number of these years that were followed by the outbreak of war.[15] Correspondingly, the probability of peace continuing is given by

$$(1-p_1) = \text{Prob.}(B_{t+1} = 0 | B_t = 0) = .94.$$

The value of p_1 is slightly higher if the sample is limited to the more recent period 1889-1978 (the sample for which relatively accurate observations on G^W and B are available), for which the result is $p_1 = 5/68 = .07$.

The probability of the continuation of war is given for the 1774-1978 sample by

$$(18)\ p_2 = \text{Prob.}\ (B_{t+1} > 0 | B_t > 0) = 33/42 = .79,$$

where 42 is the number of war years (where $B_t \neq 0$) and 33 is the number of these that were followed by another year of war.[16] In other words 9 wars began and ended over the sample 1774-1978. For the 1889-1978 period the result would be $p_2 = 16/21 = .76$. Finally, the probability of no war next year, given its existence this year,[17] is given for the 1774-1978 sample by

$$(1-p_2) = \text{Prob.}\ (B_{t+1} = 0 | B_t > 0) = .21.$$

The expected value of B for the first year of a war is calculated as the mean value for the 5 wars since 1889 (for which accurate data on B are available):

$$(19)\ \tilde{B} \equiv E(B_{t+1} | B_{t+1} > 0,\ B_t = 0) = \frac{1}{5}(.005+.23+.004+.071+.001) = .062.$$

Since war could break out at any time during the year, the annualized value of EB_{t+1}, denoted by $\tilde{B}^A$, would be roughly twice the above figure; that is, $\tilde{B}^A \approx .124$.

Finally, when B_{t+1} and B_t are both positive, the conditional expectation for B_{t+1} is given by

$$E(B_{t+1}|B_{t+1} > 0,\ B_t > 0) = \Theta_0 + \Theta_1 B_t^A,$$

where B_t^A is the current casualty rate expressed at an annual rate if hostilities applied only to a fraction of year t. The parameter Θ_1 is based on the assumption (not refuted by the small sample of U.S. data) that wars tend neither to grow nor contract over time, except that war may end at some time during year t+1 as governed by the parameter p_2. Accordingly, $\Theta_1 \approx 1 - \frac{1}{2}(1-p_2) = .90$. The parameter Θ_0 is set so that the Ψ_t variable in equation (16) corresponding to $B_{t+2} > 0$ converges to the value associated with $B_{t+2} = 0$ as $B_{t+2} \to 0$ (which essentially recognizes that a new war may break out next year even if one is already going on). The value of θ_0 turns out to be $\Theta_0 \approx p_1 \tilde{B}^A = .007$. Accordingly, I use the relation,

$$(20)\quad E(B_{t+1}|B_{t+1} > 0,\ B_t > 0) = .007 + .90\ B_t^A.$$

Equations (17)--(20) allow calculation of the relevant expectation of future casualty rates in equation (16), $\Psi_t \equiv \sum_{i=1}^{\infty} EB_{t+i+2}/(1+\rho)^i$, conditional on observation of B through B_{t+2} and for a given value of the discount rate ρ. Specifically, the result takes the form,

$$(21)\quad \Psi_t = \mu_0 + \mu_1\ B_{t+2}^A,$$

where μ_0 and μ_1 can be determined as functions of the ρ parameter.[18] Specifically, these coefficients are as follows for selected values of ρ:

ρ	μ_0	μ_1
.01	1.95	2.33
.02	0.94	2.26
.05	0.35	2.06
.10	0.16	1.80
.25	0.051	1.30

Since ρ corresponds to the difference between the real rate of return and the real growth rate, the values of the μ coefficients corresponding to the lower values of ρ would seem to be most pertinent.

The combination of equation (21) with equations (16) and (15) allows calculation of the normal government purchases ratio $\overline{g_t^w}$ as a function of the variables $(g_t^w, k_{t-1}^w, B_{t-3}, \ldots, B_{t+2})$ and the parameters $(\rho, \gamma, \alpha_0, \alpha_1, \alpha_2, \alpha_3, a_1, a_2)$, where ρ is the net real discount rate, γ measures the reaction of current defense purchases to existing captial stock, and the α's and a's describe the effect of the array of B variables on defense purchases. The results are therefore expressed in terms of the general form of equation (7), except that government purchases are now expressed relative to GNP, rather than relative to labor income. Other coefficients that appear in the analysis, $(\delta, b, p_1, p_2, \tilde{B})$ (see for example the expression contained in n. 18 above) are treated as fixed at the values specified above: δ = .16 per year, b = .34, p_1 = .06, p_2 = .79, $\tilde{\ddot{B}}$ = .062.

Government Purchases of Non-Defense Items

Statistical analysis of the non-defense portion of government purchases, $g^p \equiv g-g^w$, over samples beginning in 1929 revealed littled predictive value for first differences Dg^p, except for a negative association with the contemporaneous change in the defense component, Dg^w.[19] This association would reflect especially the crowding-out of non-defense government spending during wars. The dependence of Dg^p only on Dg^w means that departures of g^p from the normal value $\overline{g^p}$ are determined entirely by the difference between g^w and $\overline{g^w}$. Therefore, with $g^w-\overline{g^w}$ held fixed, changes in g^p amount entirely to shifts in the permanent component $\overline{g^p}$. Accordingly, with the g^w variables held fixed, the coefficient of the g^p variable in an output equation would reveal the effect of permanent changes in non-defense purchases.

B. Empirical Results

The principal empirical analysis involves joint estimation of the government purchases equation (12) and a relation for output of the form of equation (8) (with government purchases variables now expressed relative to GNP, rather than labor income). Hypothesis tests involving cross-equation coefficient restrictions derive from the calculation of $\overline{g_t^w}$ as determined by the coefficients from equation (12), together with equations (15), (16) and (21). The first set of tests involves the polar restriction that g_t^w and $\overline{g_t^w}$ enter with equal and opposite coefficients in the form of equation (8)--or, equivalently, that the coefficient of $\overline{g_t^w}$ is zero in a form where the variable $g_t^w - \overline{g_t^w}$ is held fixed--and that the coefficient of g_t^p equals zero in this equation. The second set of tests checks whether the explanatory variables for $\overline{g_t^w}$--in this case $B_{t+2}, \ldots, B_{t-3}, k_{t-1}^w$--enter an unrestricted reduced form for output as determined solely by their role in determining $\overline{g_t^w}$ in accordance with the coefficients of equation (12). The analysis is contingent on a value of the discount rate ρ, but results turned out to be relatively insentive to variations in this parameter at least over the range from .01 to .05 per year. The main results refer to a fixed value of $\rho = .02$ per year, which is plausible <u>ex ante</u> and close to a maximum likelihood estimate for this parameter (in forms that calculate $\overline{g_t^w}$ for equation (8) as above).

Jointly estimated equations for defense purchases and real GNP were calculated by means of a non-linear maximum likelihood routine from the TSP regression package, which includes estimation of contemporaneous co-variances for the error terms. The estimation is joint in the sense of incorporating the role of the coefficients from equation (12) in determining the series $\overline{g_t^w}$ and thereby influencing the fit for output in the form of equation (8). Therefore, the coefficients in the equation for Dg_t^w are not determined solely to obtain a best fit of equation (12). I have not carried out joint estimation in the broader context of choosing the order of the A(L) polynominal for the B variable in equation (12), in deciding to omit moving-average error terms in the Dg_t^w equation, in analyzing the Dg_t^p process, etc.

The joint estimates for real GNP and defense purchases are

<u>1946-78 sample</u>

$$(22)\quad \log(Y_t) = \underset{(.04)}{2.94} + \underset{(.0007)}{.0356 \cdot t} + \underset{(.20)}{.84DMR_t} + \underset{(.20)}{1.04DMR_{t-1}} + \underset{(.16)}{.24DMR_{t-2}}$$

$$\underset{(.20)}{0.98}(g_t^w - \overline{g_t^w}) + \underset{(.11)}{.51g_t^w} - \underset{(.36)}{.19g_t^p}; \quad \hat{\sigma} = .0140,\ DW = 1.2.$$

1932-78 sample

$$(23)\quad Dg^w_t = \underset{(.012)}{.168DB_{t+2}} + \underset{(.012)}{.199DB_{t+1}} + \underset{(.014)}{.270DB_t} + \underset{(.016)}{.233DB_{t-1}} - \underset{(.015)}{.031DB_{t-2}}$$

$$+ \underset{(.015)}{.075DB_{t-3}} - \underset{(.08)}{.20Dk^w_{t-1}}; \quad \hat{\sigma} = .0144, \; DW = 1.8.$$

Asymptotic standard errors are shown in parentheses below the coefficient estimates. The $\hat{\sigma}$ values are asymptotic estimates of the standard errors of the disturbance terms. DW is the Durbin-Watson Statistic. Variables included in equations (22) and (23) are

Y: real GNP (1972 base),

t: time trend,

DMR ≡ DM-$\hat{DM}$ is "unanticipated money growth," as measured in earlier research--Barro (1978, pp. 550-52)--where $\hat{DM}$ is an estimated value of money growth from an equation based on the M1 definition of the money stock,[16]

$g^w \equiv G^w/Y$, where G^w is real defense purchases (1972 base),

$g^p \equiv G^p/Y$, where G^p is real non-defense government purchases (1972 base),

B: casualty rate variable as defined in table 1,

$k^w \equiv K^w/Y$, where K^w is real government defense capital stocks (1972 base).

For present purposes I focus on the role of the government purchases variables in equation (22), neglecting the money shock variables, which have effects similar to those discussed in previous research.[22] The $\overline{g^w_t}$ variable in the output equation is based on the specification for Dg^w_t in equation (23). The main effect isolated in the equation for Dg^w_t is the strong positive spending effect of wars, as measured by the casualty rate variable B. The equation shows a two-year lead effect of the B variable and a lagged effect out to three years. (The negative effect on Dg^w_t of the DB_{t-2} variable is difficult to interpret.)

For present purposes the most important aspect of war spending is its temporary nature, although precise calculations for $\overline{g_t^w}$ involve the distributed lag pattern of DB effects on Dg_t^w and the implications of these responses for the behavior of the capital stock ratio k^w. Equation (23) shows also the expected negative effect of Dk_{t-1}^w on Dg_t^w.

Using equations (15), (16) and (21) and the value $\rho = .02$, the point estimates of coefficients shown in equation (23) can be shown to imply the formula for $\overline{g_t^w}$ as follows:

$$(24)\quad \overline{g_t^w} = .012 + .73g_t^w + .13k_{t-1}^w - .05B_{t-3} + .02B_{t-2} - .17B_{t-1} - .19B_t - .13B_{t+1} - .08B_{t+2}.$$

Equation (24) shows a positive but less than one-to-one effect on $\overline{g_t^w}$ of g_t^w, a positive effect of k_{t-1}^w (for a given value of g_t^w), and a basically negative effect of the casualty rate variables (again given the value of g_t^w). Values of $\overline{g_t^w}$ calculated from equation (24) are shown along with values of g_t^w in table 2, column 4. Because the $g^w - \overline{g^w}$ concept corresponds to a gap between the current and long-run average values of the purchases ratio, rather than to a spread between actual and "anticipated" amounts, it should not be surprising that the variable exhibits a substantial amount of positive serial correlation. In particular, the large number of peacetime years with small negative values of $g^w - \overline{g^w}$ are offset by a small number of wartime years with large excess of g^w over $\overline{g^w}$.

The temporary defense purchases variable, $g_t^w - \overline{g_t^w}$, has a significant expansionary effect on output as shown in equation (22) ("t-value" of 4.8). The normal defense purchases variable $\overline{g_t^w}$ is also significantly expansionary in this equation (t-value of 4.7). The estimated effect for the permanent purchases variable is about half that of the estimated temporary effect. The results permit rejection of two extreme hypotheses: 1) that only the temporary part of purchases affects output (which would require the coefficient of the $\overline{g_t^w}$ variables in equation (22) to differ insignificantly from zero), and 2) that temporary and permanent purchases are of equal importance for output. The latter case would correspond to equal coefficients for the $g_t^w - \overline{g_t^w}$ and $\overline{g_t^w}$ variables; that is, to the proposition that the $\overline{g_t^w}$ variable would be insignificant in an equation that held fixed the value of actual spending g_t^w. For convenience, the results from equation (22) can be expressed in this form as

$$\log(Y_t) = \ldots + \underset{(.20)}{0.98} g_t^w - \underset{(.22)}{.46} \overline{g_t^w}.$$

The coefficient of $\overline{g_t^w}$ differs significantly from zero, as indicated by the t-value of 2.2. This result indicates that temporary defense purchases are significantly more expansionary than permanent purchases.

The estimated coefficient on the $g_t^w - \overline{g_t^w}$ variable in equation (22), 0.98, s.e. = .20, indicates a roughly one-to-one effect of a temporary change in the level of real defense purchases on the level of output. Considering the standard error of the estimated coefficient, this finding is consistent with the theoretical implication of a dampened response of output to a temporary change in real purchases. However, a moderate multiplier relationship would also not be rejected by this evidence. The relatively high estimated output effect would be associated in the theoretical model with a small value of the θ-coefficient, a high value of the ϕ cofficient (see below), and a high real rate of return elasticity of aggregate supply relative to that of demand.

The estimated coefficient on the $\overline{g_t^w}$ variable in equation (22), .51, s.e. = .11, implies that a permanent increase by one unit in real defense purchases leads approximately to a one-half unit rise in real GNP.[23] This result accords with the model's prediction that output would respond less than one-to-one to a permanent change in government purchases; moreover, the estimate is significantly below unity in this case.

The non-defense purchases variable g_t^p is insignificant in equation (22). This finding accords with the view that movements in g_t^p--with the g^w variables held fixed--reflect permanent changes that would not have major consequences for aggregate demand. According to the theoretical analysis, a small output effect of g_t^p would correspond to a small supply coefficient ϕ. Possibly, the larger expansionary effect of $\overline{g_t^w}$ than of g_t^p in equation (22) reflects the more important intermediate "production" role of defense purchases (see, however, n. 23 above). This conclusion is not firm because of the large standard error of the estimated g_t^p coefficient. In any case the estimated coefficient on g_t^p in equation (22) is significantly below unity, which rejects the existence of an output multiplier on non-defense government purchases.

A combination of equation (22) with the formula for $\overline{g_t^w}$ in equation (24) implies a reduced form expression for output in terms of a constant, time trend, DMR variables, g_t^w, g_t^p, the B variables, and k_{t-1}^w. Unrestricted estimation of this reduced form affords a test of the hypothesis that the determinants of $\overline{g_t^w}$--specifically, the B variables and k_{t-1}^w--affect output only in the manner implied by equations (22) and (24). The test is based on the likelihood ratio corresponding to unrestricted and restricted forms of joint estimation. The value of -2·log(likelihood ratio) turns out to be 28.0, which exceeds the 5% critical value for the χ^2 distribution with 6 degrees of freedom (the number of coefficients restrictions in this case) of 12.6.[24] Therefore, the hypothesis that the determinants of $\overline{g}_t^w$ enter only in this indirect manner in influencing output is rejected.

A likely reason for rejection of the null hypothesis is the existence of direct positive output effects of war, which do not operate through the channels specified in the present analysis. However, because the war variables are also the prime basis in this work for distinguishing temporary from permanent movements in government purchases, it would not be feasible to allow unrestricted direct wartime output effects and still carry out interesting tests of the underlying hypotheses. Other possible problems include errors in relating $\overline{g}_t^w$ to the present war situation, as determined from equation (23) and the simple model of war probabilities; and errors in the construction of the DMR variables; in particular, in the role of the temporary federal expenditure variable in the equation for constructing anticipated money growth (n. 21 above). The present analysis has important implications for improving on the measurement of this temporary federal spending variable, which may interact substantially with the estimated effects of the g^w variables in an output equation.

Table 1

Casualty Rate Variable

Date	B	Date	B	Date	B
1898	.0052	1945	.603	1966	.025
1917	.23*	1950	.071	1967	.047
1918	.28*	1951	.097	1968	.073
1941	.0044	1952	.030	1969	.046
1942	.162	1953	.021	1970	.021
1943	.205	1964	.0014	1971	.0067
1944	1.090	1965	.0070	1972	.0014

Notes: B is battle deaths per 1,000 total population. Values of zero apply to dates not listed.

Sources for casualty figures: Vietnam (1964-72): Statistical Abstract of the U.S., 1977, p. 369, table 590. World War I (1917-18) and Spanish American War (1898): Historical Statistics of the U.S., 1975, p. 1140, line 880. Korean War (1950-53): yearly data from Dept. of the Army, Battle Casualties of the Army, 1954, were applied to war total from Statistical Abstract of the U.S., 1977, p. 369, table 589. World War II (1941-45): yearly data from Office of the Comptroller of the Army, Army Battle Casualties and Nonbattle Deaths in World War II: Final report, Dec. 1941-Dec. 1946, were applied to war total from Statistical Abstract of the U.S., 1977, p. 369, table 589. Korean War and World War II data were obtained from William Strobridge, Chief, Historical Services Division, Department of the Army.

Orders-of-magnitude values of B (per year) for earlier wars are: Revolution (1775-1783): 0.2, War of 1812 (1812-15): 0.08, Mexican War (1846-48): 0.04, Civil War (1861-65, union only): 1.0. Casualty figures are from: Civil War: Historical Statistics of the U.S., 1975, p. 1140, line 880; other wars: Department of the Army, History of Military Mobilization in the U.S. Army, 1955, Appendix A.

*Yearly data were unavailable. Figures are based on war total assuming equal rate of casualties per month.

Table 2

Government Purchases Variables

Date	g^w	$\overline{g^w}$	$g^w - \overline{g^w}$	k^w	g^p
1889	.0060				.097
1890	.0057				.094
1	.0057				.094
2	.0056				.089
3	.0062				.097
4	.0064				.102
1895	.0053				.092
6	.0059				.097
7	.0080				.091
8	.0192				.094
9	.0165				.087
1900	.0131				.088
1	.0113				.081
2	.0110				.084
3	.0108				.088
4	.0123				.088
1905	.0112				.089
6	.0093				.082
7	.0090				.091
8	.0117				.110
9	.0105				.089
1910	.0100				.091
1	.0101				.105
2	.0094				.100
3	.0096				.096
4	.0139				.106
1915	.0135				.112
6	.0164				.093
7	.076				.085
8	.258				.080
9	.156				.049
1920	.038				.085
1	.033				.125
2	.017				.116
3	.014				.105
4	.014				.116
1925	.012				.115
6	.011				.108
7	.012				.118
8	.013				.121
9	.013			.055	.117
1930	.015	.030	-.015	.056	.141
1	.017	.031	-.014	.057	.158
2	.019	.033	-.014	.063	.175
3	.016	.032	-.016	.063	.176
4	.016	.032	-.016	.056	.187
1935	.017	.032	-.014	.050	.174
6	.018	.032	-.013	.045	.179

Table 2

Government Purchases Variables
(Continued)

Date	g^w	$\overline{g^w}$	$g^w - \overline{g^w}$	k^w	g^p
1937	.016	.030	-.013	.043	.165
8	.019	.031	-.012	.046	.187
9	.017	.030	-.013	.045	.183
1940	.028	.024	.004	.052	.163
1	.120	.066	.054	.074	.122
2	.317	.103	.214	.164	.092
3	.439	.090	.349	.309	.069
4	.463	.071	.392	.431	.064
1945	.410	.069	.341	.470	.063
6	.103	.062	.041	.483	.092
7	.055	.066	-.011	.414	.106
8	.056	.065	-.010	.337	.117
9	.064	.085	-.020	.290	.132
1950	.066	.068	-.002	.239	.117
1	.123	.097	.027	.209	.107
2	.156	.130	.027	.225	.110
3	.156	.143	.012	.260	.118
4	.133	.134	-.001	.291	.119
1955	.115	.132	-.017	.284	.116
6	.112	.129	-.016	.283	.115
7	.116	.133	-.017	.286	.119
8	.115	.133	-.017	.296	.134
9	.108	.128	-.020	.286	.129
1960	.102	.123	-.021	.282	.133
1	.104	.123	-.020	.278	.138
2	.103	.123	-.019	.268	.138
3	.096	.115	-.019	.263	.142
4	.087	.105	-.019	.251	.145
1965	.080	.093	-.014	.235	.147
6	.088	.088	.000	.219	.146
7	.098	.085	.013	.216	.149
8	.096	.080	.016	.211	.151
9	.088	.079	.009	.206	.150
1970	.079	.082	-.003	.201	.154
1	.068	.079	-.011	.173	.157
2	.063	.076	-.013	.169	.153
3	.056	.073	-.017	.153	.148
4	.055	.071	-.016	.147	.157
1975	.055	.070	-.016	.141	.164
6	.051	.067	-.016	.128	.156
7	.050	.064	-.015	.119	.152
8	.047	.061	-.014	.112	.152

Notes to Table 2

$g^w \equiv G^w/Y$, where Y is real GNP (1972 base). G^w is real defense purchases (1972 base). Data since 1929 are from National Income and Product Accounts of the U.S. and recent issues of the U.S. Survey of Current Business. The fraction of nominal defense purchases in total nominal federal purchases was multiplied by figures on real federal purchases (1972 base). Data from 1889-1928 are from Kendrick, 1961, table A-I, col. 5. Figures were multiplied by 4.8, based on the overlap for 1929.

$\overline{g^w}$ is the estimated normal defense purchases ratio, as calculated from eg. (24) in the text.

$k^w \equiv K^w/Y$, where K^w is the end-of-year value of net real stocks of military structures, equipment and inventories (1972 base). Data from 1929-69 are from Kendrick, 1976, table B-24, converted from a 1958 to a 1972 index by a constant multiple (1.72). Fugures were extended to 1978 using data on various expenditure components: military sturctures, AEC structures, military equipment, AEC equipment, inventories--GSA stockpiles, inventories--AEC stockpiles. Depreciation estimates were based on rates used by Kendrick within each category. His calculations assume a higher rate of depreciation during World War II.

$g^p \equiv G^p/Y$, where G^p is real non-defense purchases of the federal plus state and local government sectors (1972 base). G^p was calculated as total government purchases G less G^w. Sources for G correspond to those above for G^w, except that Kendrick, 1961, table A-IIa was used for data from 1889-1928.

Footnotes

[1]The form for $C^*(t)$ rules out an effect of the time path of $G(t)$ on the relative "price" of C^* at various dates. For example, a multiplicative interaction between $C(t)$ and $G(t)$ in the expression for $C^*(t)$ would generate a form of complementarity where an increase in $G(t)$ would stimulate a substitution toward $C(t)$ from private consumer spending at other dates. Complementarity of this type between $C(t)$ and the long-run average value of G would matter only when the labor/leisure choice was introduced. The interaction of G with C versus that with leisure would then come into the analysis.

[2]This setup abstracts from explicit modeling of a possible public goods-type scale economy for government purchases.

[3]See Barro (1979a) on the implications of deficit finance.

[4]The wealth effects discussed in the present analysis would not apply directly to private investment demand. Direct substitution of public projects for private investment would seem similar to the consumption substitution that is governed by the θ-parameter.

[5]If production is subject to constant returns to scale, labor-augmenting technical progress occurs at rate λ, G grows also at rate λ, and $V_1 = \int_0^\infty u[C^*(t)]e^{-\rho t}dt$, where $\rho > \lambda$, $u[C^*(t)] = a[C^*(t)]^b$, $a > 0$, $0 < b < 1$, then a steady state involves $C^*(t)$ growing at rate λ with the rate of return given by $\bar{r} = \rho + \lambda(1-b)$. Non-zero population growth could also be included, as in Arrow and Kurz (1970, Ch. III). The steady-state value of K/L is determined by an equality between $\bar{r}$ and the marginal product of capital.

[6] In a finite horizon, life cycle-type model, one would expect a decline in the "target value" of K/L. In this case there would be a smaller negative response of the current C*/L to an increase in $\overline{G/L}$. However, this sort of effect need not arise under finite lives if intergenerational transfers are included in the model.

[7] Barro (1979a, pp. 941-45) provides some optimizing rationale for this behavior on the part of the government.

[8] This discussion ignores government transfers, which involve a substitution effect away from work activity, but not the sort of wealth effect discussed above. (Other substitution effects could arise, of course, depending on the nature of the transfers.) Transfer payments also do not involve the direct effect on aggregate demand associated with purchases of goods and services.

[9] The current price level will tend also to rise, possibly generating the expectation of future deflation. A full analysis would require a model of money supply behavior. Since the effects at issue are primarily nonmonetary in nature, it is unnecessary for present purposes to carry out the details of this analysis.

[10] Wartime may also be associated with uncertainties on maintaining property rights, which would tend to reduce private investment demand. The analysis abstracts from this effect and from controls on prices or interest rates. Also excluded are effects of patriotism or coercive behavior, such as conscription.

[11] If Y^s depended on $\overline{G}$ rather than G, this conclusion would require some quantitative restrictions on θ, ϕ, and the real rate of return elasticities for Y^s and Y^d.

[12] Levis Kochin has suggested the attractive alternative of using the current overall tax rate as a proxy for the anticipated long-run average ratio of government purchases to GNP. A rationale for identifying the tax rate with the anticipated government expenditure ratio is given in Barro (1979a). The argument involves a deficit policy that smoothes tax rates over time in order to minimize excess burden costs for a given present value of net tax collections. Some problems with implementing Kochin's suggestion are 1) the distinction between purchases and expenditures implies that a separate model would be required to predict future transfers (including interest payments), which is not obviously easier than modeling purchases directly; and 2) the use of the tax rate to proxy the permanent expenditure ratio may work better for the federal government than for total government. See Benjamin and Kochin (1978), who argue that mobility possibilities would prevent state and local governments from choosing an excess burden minimizing debt policy. However, this issue involves also the federal government's interaction with state and local governments.

[13] The ratio of total casualties (including wounded but excluding deaths that were unrelated to combat) to battle deaths is 5.3 for the Spanish American War, 4.8 for World War I, 3.3 for World War II, 4.0 for the Korean War, and 4.3 for the Vietnamese War. See the notes to table 1 for sources of casualty data.

[14] War probabilities and the distribution of sizes of wars need not be constant over time, although there is no indication of substantial structural change in the small sample of evidence afforded by the 200 years of U.S. history. (The largest value for the B variable would actually apply to the

(continued)

(n. 14 continued)

Civil War; see the notes to table 1.) From the standpoint of constructing the $\overline{g_t^w}$ variable, shifts in the stochastic structure for wars would essentially be an alternative to the present specification that allows for shifts in spending for a given war structure, as represented by the stochastic variable u_t in equation (11). In the context of output analysis, it is unclear that there would be much empirical difference between these alternatives.

[15]The year 1978 is not included in this calculation, although it could have been if peace during 1979 were also included. War years are taken to be 1775-83, 1812-15, 1846-48, 1861-65, 1898, 1917-18, 1941-45, 1950-53, 1964-72. There may be some objection to starting the sample just before a war (which is not independent of the start of U.S. data), but the results are not highly sensitive to this choice.

[16]The probability p_2 refers to the existence of war during at least part of a year following a period of war during at least part of the previous year.

[17]This calculation pertains to the existence of peace over the entire year t+1, conditional on war during at least part of year t.

[18]The general formulae are

$$\mu_0 = [\tilde{B}p_1(1-p_2)(1+\rho+p_2\theta_1) + p_2\theta_0(1+\rho)(\rho+p_1)]/[\rho(1+\rho+p_1-p_2)(1+\rho-p_2\theta_1)],$$

$$\mu_1 = p_2\theta_1/(1+\rho-p_2\theta_1).$$

[19]Past history of the residuals, lagged values of Dg^p or Dg^w, a capital stock measure Dk^p, and a constant were all insignificant.

[20] There are questions about the exogeneity of the g variables especially because of the presence of Y in the denominators. In preliminary analyses the use of instruments for g^w and g^p, which involved the use of estimated rather than actual values of Y in the denominators, had a negligible effect on the results.

[21] $\hat{DM}$ is determined from the equation estimated over the 1941-78 sample:

$$\hat{DM}_t = \underset{(.023)}{.097} + \underset{(.14)}{.48}DM_{t-1} + \underset{(.12)}{.17}DM_{t-2} + \underset{(.015)}{.071}FEDV_t + \underset{(.008)}{.031}\cdot\log(U/1-U)_{t-1},$$

where observations from 1941-45 are weighted by .36. $FEDV_t$ is real federal spending relative to a distributed lag of itself and U is the unemployment rate in the total labor force. See Barro (1978, pp 550-51) for a discussion of this equation.

[22] The deletion of the military personnel variable that was included in pervious analysis does result in an increase in residual serial correlation and also in the elimination of the significance of the DMR_{t-2} variable.

[23] In the simple model from the theoretical section, this coefficient would be an estimate of the ϕ parameter associated with defense purchases. However, this interpretation would be affected by a correlation between the normal level of defense purchases and the perceived long-run external military threat to the United States. Changes in this long-run threat, which were not admitted in the model with stationary war probabilities that was considered above, can be thought of as exerting negative supply effects in equation (6).

[24] If ρ were regarded as a freely estimated parameter, there would be only 5 degrees of freedom, which would imply a critical value of 11.1.

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